PRESENTATION

Effective Treatment of Schwinger pair production during Axion inflation

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2204.01180 and 2206.12218

with Kume (Padova), Mukaida (KEK), Tada (Nagoya)

7th. Nov. 2024 @KEK Tsukuba Campus

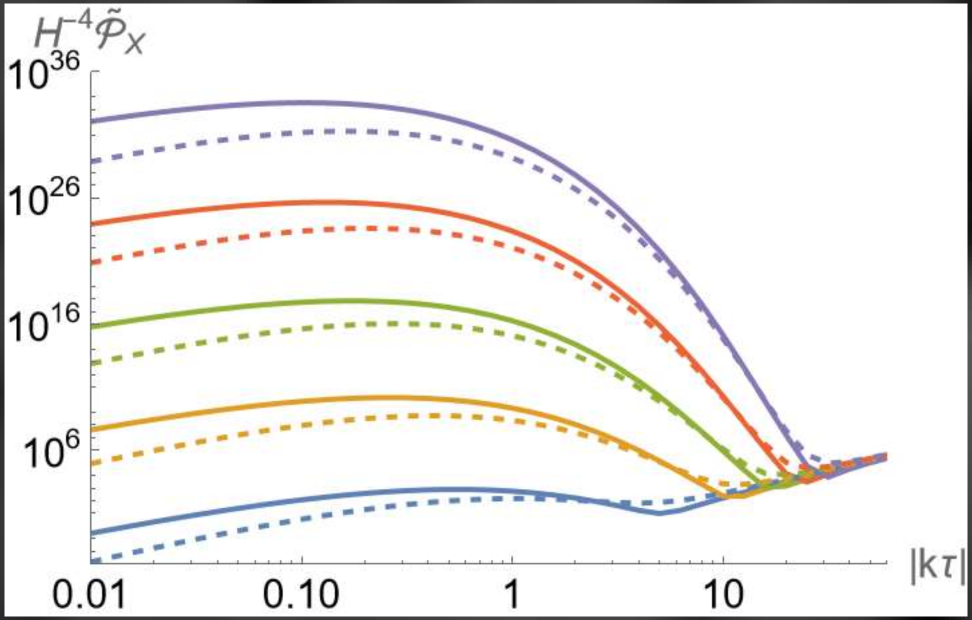

MESSAGE

MESSAGE

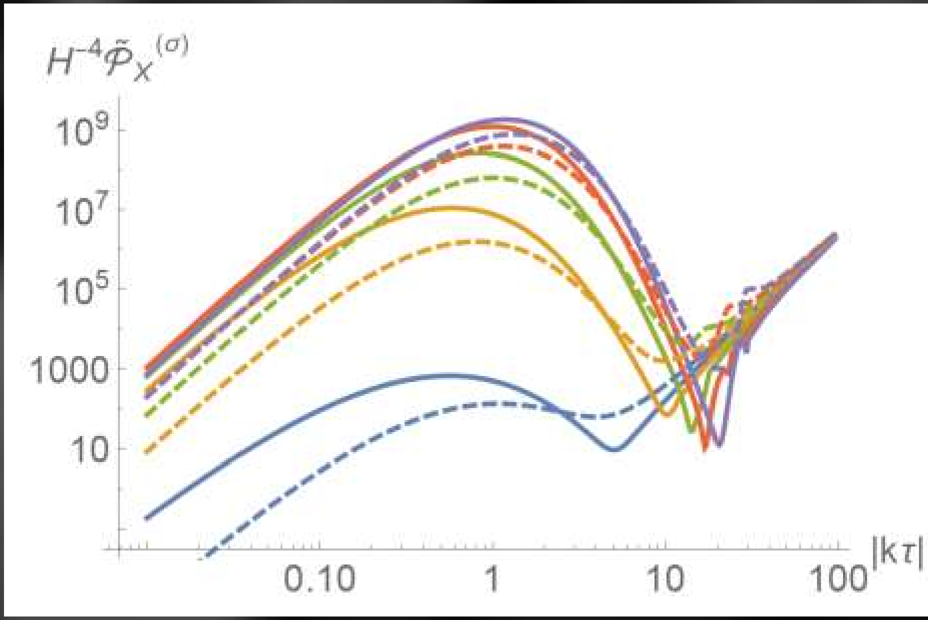
Schwinger effect matters

(How to improve the treatment?)

Without Ψ

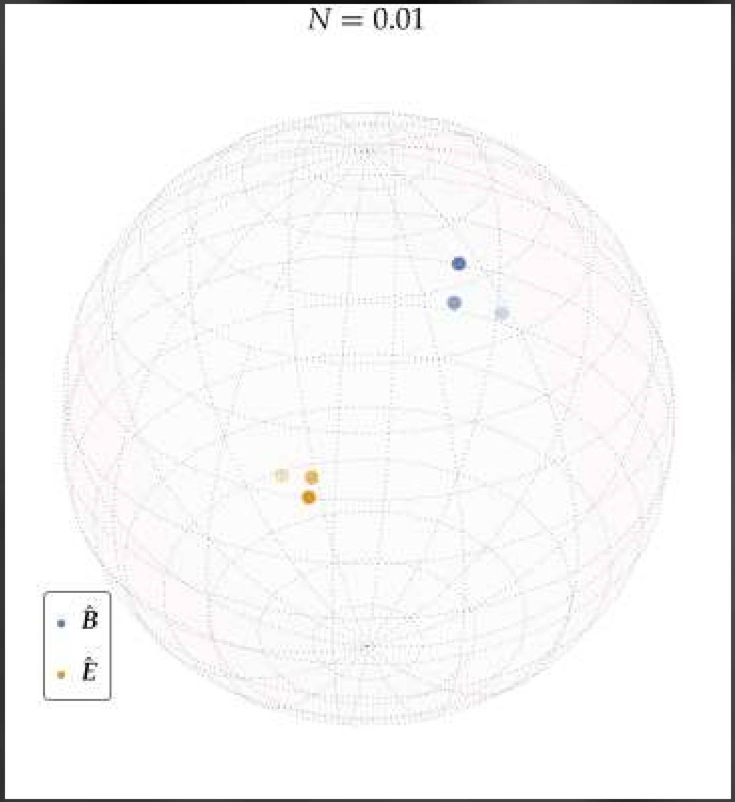


With Ψ

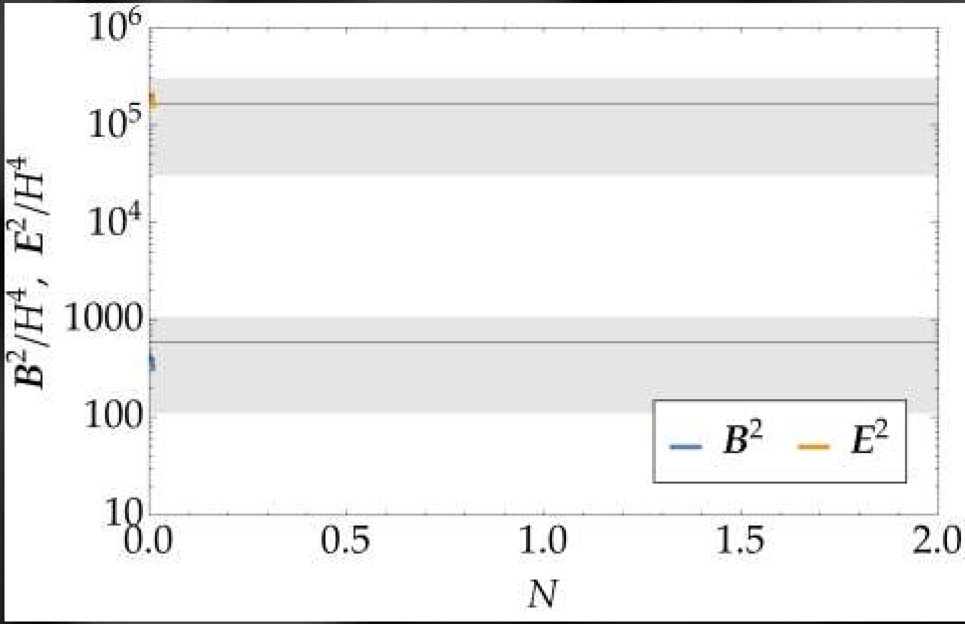


MESSAGE

Future work: EMF dance



EMF Stochastic simulation
without charged particles



Plan of Talk

1. Motivation
2. Review the case without ψ
3. Solve the system of A & ψ
4. Results
5. Summary

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1 Motivation

Setup inflaton ϕ – photon A_μ – fermion ψ coupled system

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - V(\phi)}_{\text{Axionic inflaton}} \underbrace{- \frac{1}{4}FF - \frac{\alpha}{4f}\phi F\tilde{F}}_{\text{U(1) gauge field coupled to } \phi} \underbrace{+ i\bar{\psi}\not{D}\psi}_{\text{Charged fermion}}$$

Axionic inflaton

U(1) gauge field coupled to ϕ

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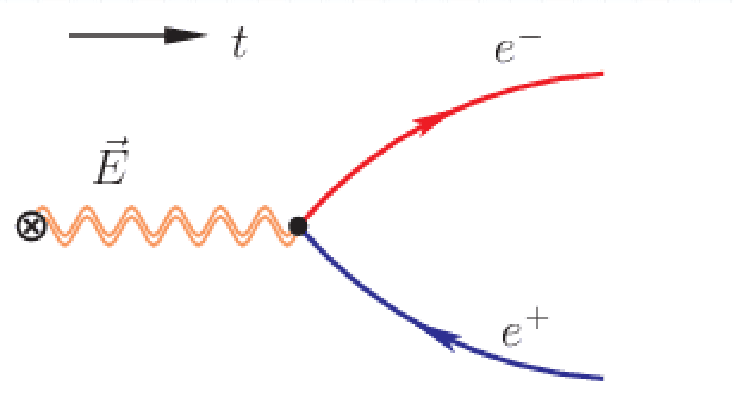
Motivations

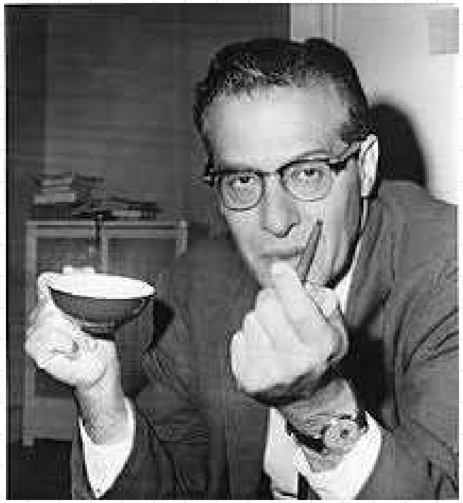
① Particle Physics: Shift symmetry of ϕ ➡ Reheating requires coupling

② Phenomenology: Helical B ➡ Baryogenesis & Magnetogenesis

③ Theoretic interest: Strong E ➡ Schwinger effect

Schwinger effect





Julian Schwinger(1918～1994)

- Sufficiently strong ($eE > m^2$) electric field causes a **pair production** of charged particles. It's a **non-perturbative process** in QED.
- Not yet detected. It may be observed by EBI or X-FEL etc...

G. V. Dunne, Eur. Phys. J. D55, 327-340
A. Ringwald, Phys. Lett. B510, 107-116

- In the early universe, however, It may have played an **important role**.

Is Schwinger effect relevant?

Mass Suppression $\qquad eE \lesseqgtr m_\psi^2 = y^2 h^2$

Yukawa coupling $\qquad y \ll 1 \quad (y_e \sim 10^{-6})$

Higgs vev during inflation $\qquad \langle h \rangle \sim H_{\rm inf},$

Electro-magneto Generation $\qquad eE \gg H_{\rm inf}^2$

⇨ **Schwinger effect** $\qquad \frac{m_\psi^2}{eE} \ll 10^{-12} \left(\frac{y}{10^{-6}}\right)^2$

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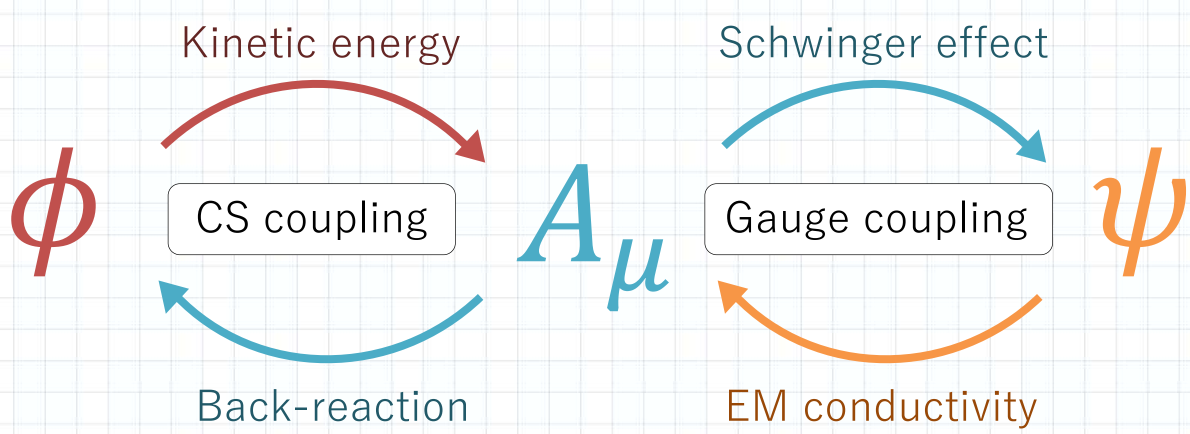
Interactions



1 Motivation

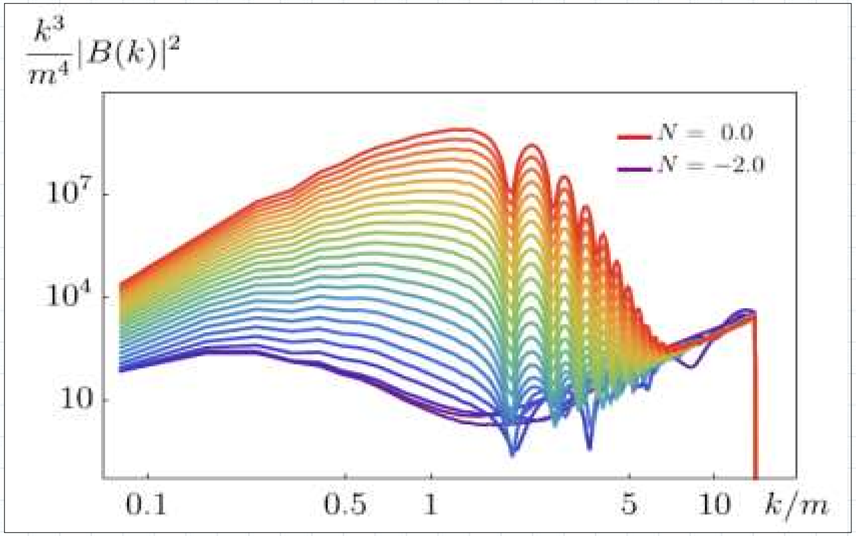
[Garretson+(1992), Field&Carroll(2000), Anber&Sorbo(2006)
Durrer+(2011), Fujita+(2015), Adshead+(2016),⋯]

Previous works

The $\phi - A_\mu$ system well studied

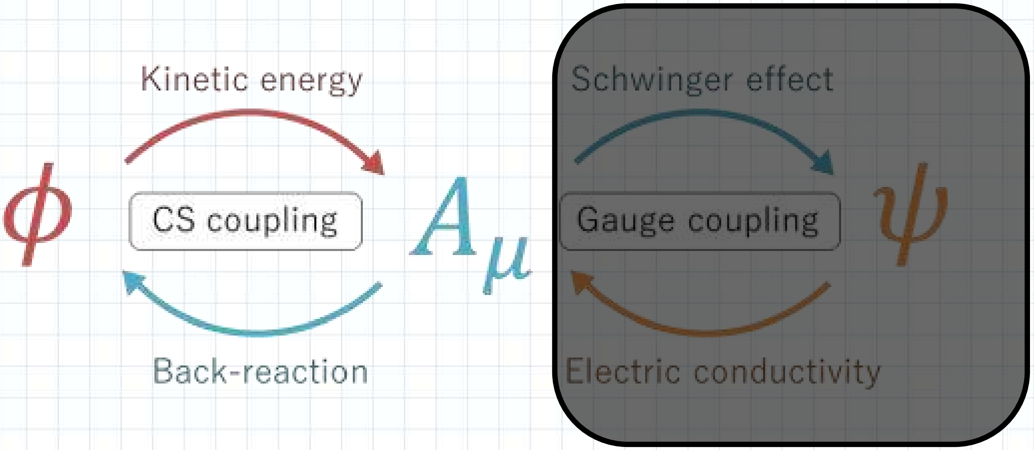
➡ A_μ production at inf. end is dominant

However, ψ is not yet included!



[Lattice simulation by Cuissa & Figueroa (2018)]

Disregarded



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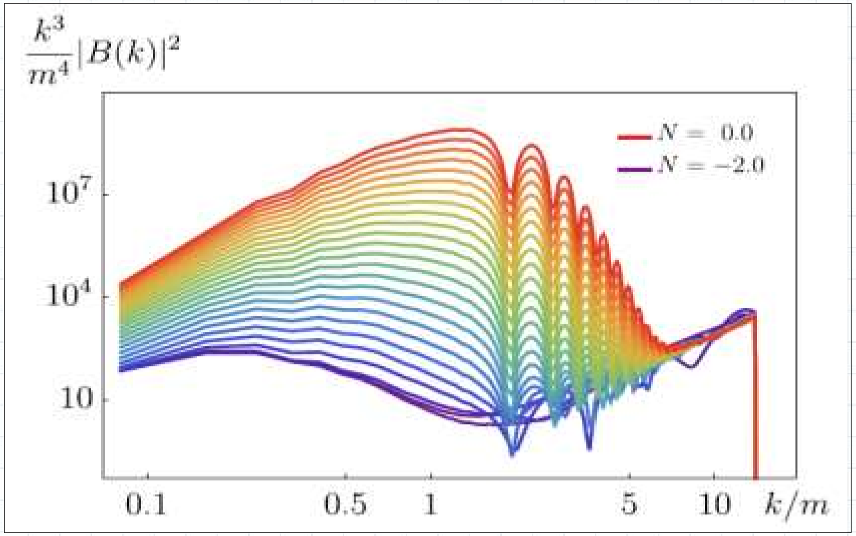

[Lattice simulation by Cuissa & Figueroa (2018)]

Difficulty

Non-linear & non-perturbative Dynamics

➡ $A(k), \psi(k)$: different k-modes are coupled

➡ System is close to neither free mode nor thermal equilibrium

We need a new approach to solve it

[See also Domcke, Ema, Mukaida(2019);
Gorbar, Schmitz, Sobol, Vilchinskii(2021)]

See also
von Eckardstein+(2024)
[2408.16538]

Proposed Methods

	Schwinger Effect	Evolution	Thermalization	Orientation	
Gradient Expansion	σ_E	Yes	No	No	Gorbar+(2021) Gorbar+(2022)
Mean-field Approx.	σ_E, σ_B	No	No	No	TF+(2022)
Kinetic	σ_E	Yes	Yes	No	Sobol+(2018) Gorbar+(2019) Okano&TF(2020)
Stochastic	No	Yes	No	Yes	TF+(2022)

[Garretson+(1992), Field&Carroll(2000), Anber&Sorbo(2006)
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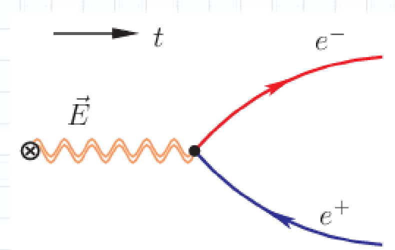
Short summary

1. (Axion inflaton + CS coupling) is interesting.
Strong electromagnetic fields are produced.

2. Charged particles are hardly considered.
But, Schwinger effect must produce them.

3. What happens then? How to analyze?
We develop a new formalism.



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Review no-charged-particle case

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) \; - \frac{1}{4}FF - \frac{\alpha}{4f}\phi F\tilde{F} + i\bar{\psi}D\psi$$

Axionic inflaton

U(1) gauge field coupled to ϕ

Charged fermion

Assumption: the inflaton rolls at a constant velocity $\xi \equiv \frac{\alpha\dot{\phi}}{2fH}$

The EoM for the gauge field mode function $\mathcal{A}_\pm$ is given by

$$\left[\partial_\tau^2 + k^2 \pm 2k\frac{\xi}{\tau}\right]\mathcal{A}_\pm(\tau, k) = 0$$

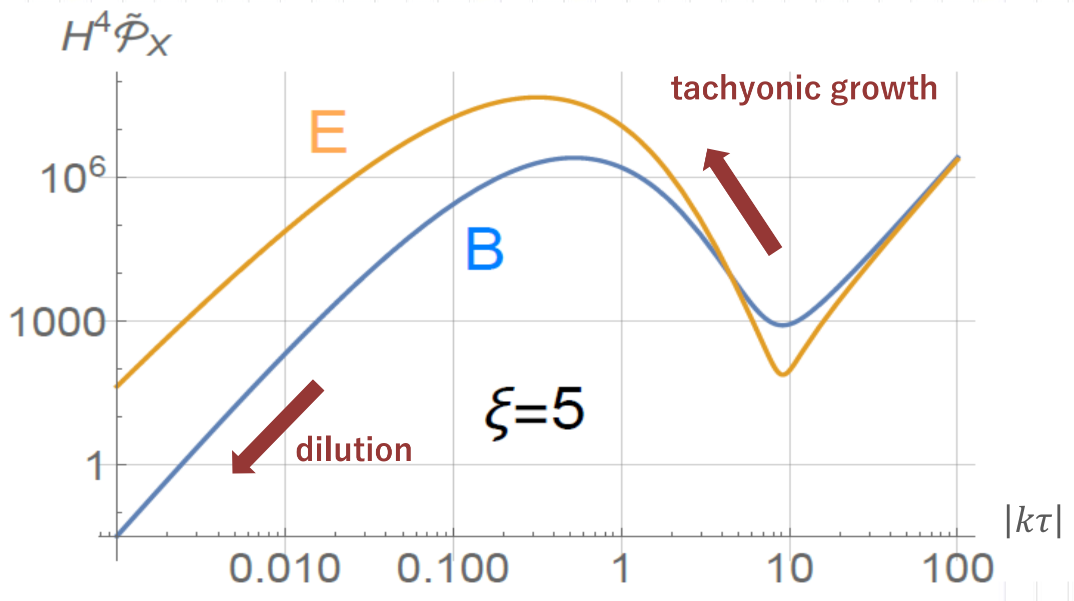
Either $\pm$ mode is amplified by the tachyonic instability.

In the slow-roll phase, an analytic solution is available.

If $\xi \equiv \frac{\alpha\dot{\phi}}{2fH} = const. > 0$ ➡ $\mathcal{A}_+ = \frac{1}{\sqrt{2k}}e^{\pi\xi/2}W_{-i\xi,1/2}\,(2ik\tau)$

EM field production

(without ψ)



1. Due to the exp amplification, very strong EMFs are produced, $E \gg B \gg H^2$.
2. The typical EM length scale is $L_{em} \simeq \xi/H$
3. The typical EM time scale is $t_{em} \simeq 1/H$

$$\tilde{\mathcal{P}}^+_{BB}(\tau,k) = a^{-4}\mathcal{P}^+_{BB}(\tau,k) = \frac{k^5}{2\pi^2 a^4}|\mathcal{A}_+(\tau,k)|^2 = \frac{|k\tau|^4 H^4}{4\pi^2}e^{\pi\xi}|W(-k\tau)|^2, \quad \tilde{\mathcal{P}}^+_{EE}(\tau,k) = a^{-4}\mathcal{P}^+_{EE}(\tau,k) = \frac{k^3}{2\pi^2 a^4}|\partial_\tau\mathcal{A}_+(\tau,k)|^2 = \frac{|k\tau|^4 H^4}{4\pi^2}e^{\pi\xi}|W'(-k\tau)|^2,$$

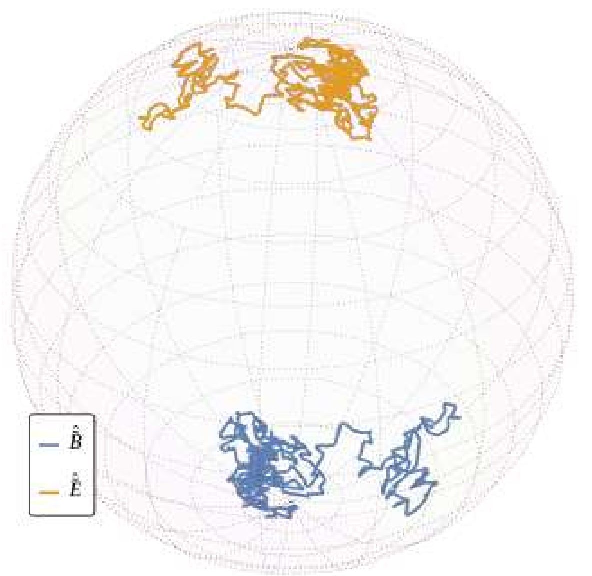
2 Review no-charged-particle case

EM field orientation

(without ψ)

Since the parity is fully violated,
EM fields take an **anti-parallel** configuration.

$$\frac{\tilde{\mathscr{P}}^{+}_{BE}}{\sqrt{\tilde{\mathscr{P}}^{+}_{EE}\tilde{\mathscr{P}}^{+}_{BB}}} \xrightarrow{|k\tau|\ll 2\xi} -1.$$



Evolution of $\hat{\boldsymbol{E}}\cdot\hat{\boldsymbol{B}}$ for 0.5 e-folds

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2. The typical EM length scale is $L_{em} \simeq \xi/H$
3. The typical EM time scale is $t_{em} \simeq 1/H$
4. E and B are anti-parallel, $\hat{\boldsymbol{E}}\cdot\hat{\boldsymbol{B}} = -1$

$$\tilde{\mathscr{P}}^{+}_{BB}(\tau,k) = a^{-4}\mathscr{P}^{+}_{BB}(\tau,k) = \frac{k^5}{2\pi^2 a^4}|\mathscr{A}_{+}(\tau,k)|^2 = \frac{|k\tau|^4 H^4}{4\pi^2}e^{\pi\xi}|W(-k\tau)|^2, \quad \tilde{\mathscr{P}}^{+}_{EE}(\tau,k) = a^{-4}\mathscr{P}^{+}_{EE}(\tau,k) = \frac{k^3}{2\pi^2 a^4}|\partial_\tau\mathscr{A}_{+}(\tau,k)|^2 = \frac{|k\tau|^4 H^4}{4\pi^2}e^{\pi\xi}|W'(-k\tau)|^2,$$

4 properties in the no charged particle case

1. Strong EMFs are produced: $E, B \gg H^2$

2. The EM length scale $L_{\mathrm{em}} \simeq \xi/H$

3. The EM time scale is $\tau_{\mathrm{em}} \simeq 1/H$

4. EMFs are anti-parallel: $\widehat{\boldsymbol{E}} \cdot \widehat{\boldsymbol{B}} = \boldsymbol{-1}$

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3 Solve the system of A and ψ

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}FF - \frac{\alpha}{4f}\phi F\tilde{F} + i\bar{\psi}D\psi$$

Axionic inflaton　U(1) gauge field　Charged fermion

Assumption: the inflaton rolls at a constant velocity $\xi \equiv \frac{\alpha\dot{\phi}}{2fH} = \text{const.}$

The EoMs for the gauge field and fermion are **coupled** and **non-linear**

$$\left[\hat{\gamma}^\mu\left(\partial_\mu + igQ\hat{A}_\mu\right) + \frac{3}{2}aH\hat{\gamma}^0\right]\hat{\psi} = 0$$

$$\partial_\tau^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau}\epsilon_{ijl}\partial_j A_l = a^2 e J_i \qquad J^\mu = \bar{\psi}\gamma^\mu\psi$$

We cannot exactly solve them… Then, we introduce two prescriptions

1. **Integrating out ψ**: Reduce the coupled EoMs into a single non-linear eq.
2. **Mean-field approx**: linear eq. for perturbation and consistency eq.

3 Integrating out ψ

[Domcke&Mukaida(2018)]

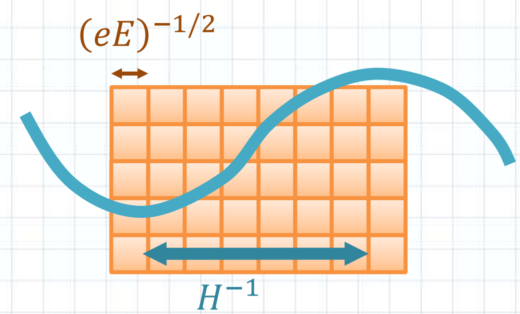
Remember the properties of the produced EMFs

❶ $E, B \gg H^2$ ❷ $L_{\rm em} \simeq \xi/H$ ❸ $\tau_{\rm em} \simeq 1/H$

Typical momentum of the Schwinger produced fermion is $\boldsymbol{p_\psi \simeq \sqrt{eE}}$

Thus, a **hierarchy of scales** exists

$$L_\psi \sim t_\psi \sim (eE)^{-1/2} \ll L_{\rm em} \sim t_{\rm em} \sim H^{-1}$$



For fermions, EMFs look static and homogeneous, $\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{B}} \simeq const.$

Schwinger current induced by static, homogeneous & anti-parallel EMFs is known:

$$\partial_\tau (a^2 e J_i) = \frac{e^3 B E_i}{2\pi^2} \coth\left(\frac{\pi B}{E}\right).$$

NB; this current satisfies the chiral anomaly equation. Assumption: the fermion's mass is negligible, $m_\psi \ll H$

3 Integrating out ψ

We need not $\partial_\tau J_i$ but J_i itself.

Assumption: the physical EMFs are static, $E, B \propto a^2$, for $t \gtrsim H^{-1}$

$$\partial_\tau (a^2 e J_i) = \frac{e^3 B E_i}{2\pi^2} \coth\left(\frac{\pi B}{E}\right). \quad \Rightarrow \quad \boxed{e J_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H} \coth\left(\frac{\pi B}{E}\right)}$$

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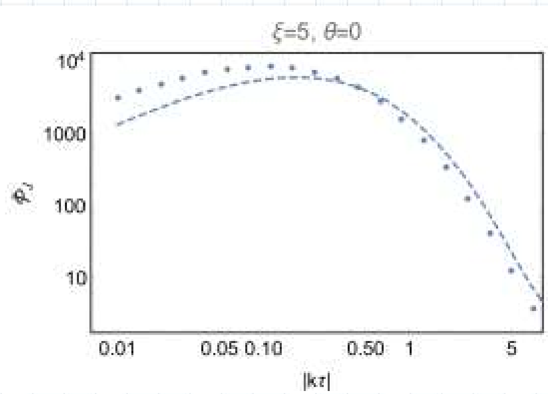
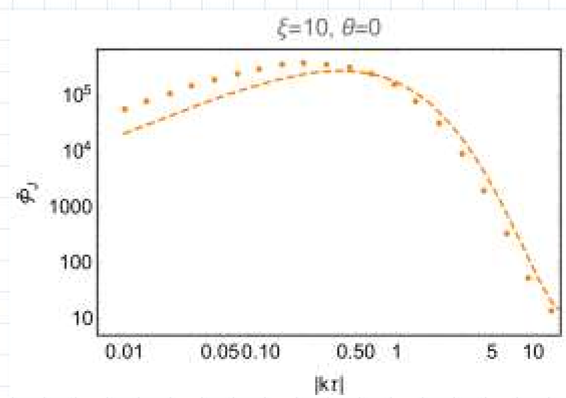
Since $t_{\mathrm{em}} \simeq H^{-1}$, this expression may not be very accurate.
But on average, E and B amplitudes should be almost constant,
because the energy injection from the inflaton is constant, $\xi = const$.

This assumption may lead to O(1) Error?

Consistency Check:
Not bad in the growing phase





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This assumption may lead to O(1) Error?

We obtain a single **non-linear** EoM for A!!

$$\partial_\tau^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau} \epsilon_{ijl} \partial_j A_l = a^2 e J_i$$

3 Schwinger Conductivity?

Ohm's law??

No! J_i is not linear in E_i

$$eJ_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H} \coth\left(\frac{\pi B}{E}\right)$$

None of the followings is correct.

$$\boldsymbol{J}(t,\boldsymbol{x}) = \sigma_E(E_0, B_0)\boldsymbol{E}(t,\boldsymbol{x})$$ ⇒ Effective Friction for **A**

$$\boldsymbol{J}(t,\boldsymbol{x}) = \sigma_B(E_0, B_0)\boldsymbol{B}(t,\boldsymbol{x})$$ ⇒ ξ effectively decreases

$$\boldsymbol{J}(t,\boldsymbol{x}) = \sigma_{EB} B(t,\boldsymbol{x})\boldsymbol{E}(t,\boldsymbol{x})$$ ⇒ Stay non-linear

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$\boldsymbol{J}(t,\boldsymbol{x}) = \sigma_{EB} B(t,\boldsymbol{x}$ [illegible] $,\boldsymbol{x})$ — Stay non-linear

Mean-field approximation

How to solve a full non-linear equation??

$$\partial_\tau^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau}\epsilon_{ijl}\partial_j A_l = a^2 e J_i$$

$$eJ_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H}\coth\left(\frac{\pi B}{E}\right)$$

We introduce **mean-field approximation.**

3 Mean-field approximation

How to solve a full non-linear equation??

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Mean Field Approx

Focusing on one particle in a many-body system, we solve a one-body problem under the mean field created by the other particles.

Then, we also solve the self-consistency equation, whose averaged solution coincides with the mean field.

3 Mean-field approximation

How to solve a full non-linear equation??

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one Fourier mode

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We introduce **mean-field approx**. and split EMFs into a mean and a perturbation

$$\boldsymbol{E}(\tau,\boldsymbol{x}) \simeq \boldsymbol{E}_0 + \delta\boldsymbol{E}(\tau,\boldsymbol{x}), \qquad \boldsymbol{B}(\tau,\boldsymbol{x}) \simeq \boldsymbol{B}_0 + \delta\boldsymbol{B}(\tau,\boldsymbol{x}).$$

The Schwinger current is accordingly decomposed. $(\hat{\boldsymbol{E}}_0 \cdot \hat{\boldsymbol{B}}_0 = -1, \text{but } \delta\boldsymbol{E}\cdot\delta\boldsymbol{B} \neq -1)$

$$a^2 e\boldsymbol{J} = a^2 e(\boldsymbol{J}_0 + \delta\boldsymbol{J}),$$

$$a^2 e\boldsymbol{J}_0 = \frac{e^3 B_0 E_0}{6\pi^2 a H}\coth\left(\frac{\pi B_0}{E_0}\right)\boldsymbol{e}_z,$$

$$a^2 e\delta\boldsymbol{J} = \frac{e^3}{6\pi^2 a H}\left[\left(\frac{B_0^3\delta E_z - E_0^3\delta B_z}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right) + (B_0\delta E_z + E_0\delta B_z)\frac{\pi B_0}{E_0}\operatorname{csch}^2\left(\frac{\pi B_0}{E_0}\right)\right)\boldsymbol{e}_z + \frac{E_0^2 B_0\delta\boldsymbol{E} - B_0^2 E_0\delta\boldsymbol{B}}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right)\right].$$

$\delta\boldsymbol{E}, \delta\boldsymbol{B}$ **may not** $\parallel$ $\boldsymbol{E}_0, \boldsymbol{B}_0$

3 Mean-field approximation

How to solve a full non-linear equation??

$$\partial_\tau^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau}\epsilon_{ijl}\partial_j A_l = a^2 e J_i \qquad eJ_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H}\coth\left(\frac{\pi B}{E}\right)$$

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The Schwinger current is accordingly decomposed. $(\hat{\boldsymbol{E}}_0 \cdot \hat{\boldsymbol{B}}_0 = -1, \text{but } \delta\boldsymbol{E}\cdot\delta\boldsymbol{B} \neq -1)$

$$\delta\boldsymbol{J} \simeq \sigma_E \delta\boldsymbol{E} + \underline{\sigma_B \delta\boldsymbol{B}}$$

3 Linearized eq. for perturbation

The EoM for the perturbation is

$$\left[\partial_z^2 - \frac{\Sigma}{z}\partial_z + 1 - \frac{2\xi_{\rm eff}}{z}\right]\mathscr{A}_+^{(\sigma)} = 0$$

$$z \equiv -k\tau$$

with **the electric and magnetic conductivity**:

$$\Sigma \equiv \Sigma_E + \Sigma_{E'}\sin^2\theta_{\boldsymbol{k}}, \qquad \xi_{\rm eff} \equiv \xi - \frac{1}{2}\left(\Sigma_B + \Sigma_{B'}\sin^2\theta_{\boldsymbol{k}}\right)$$

$$\hat{\boldsymbol{E}}_0 \cdot \boldsymbol{e}^{\pm}(\hat{\boldsymbol{k}}) = -\sin\theta_{\boldsymbol{k}}/\sqrt{2}.$$

$$\Sigma_E \equiv \frac{e^3 B_0}{6\pi^2 a^2 H^2}\left(\frac{E_0^2}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right)\right), \quad \Sigma_{E'} \equiv \frac{e^3 B_0}{12\pi^2 a^2 H^2}\left(\frac{B_0^2}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right) + \frac{\pi B_0}{E_0}\operatorname{csch}^2\left(\frac{\pi B_0}{E_0}\right)\right)$$

$$\Sigma_B \equiv \frac{e^3 E_0}{6\pi^2 a^2 H^2}\left(\frac{B_0^2}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right)\right), \quad \Sigma_{B'} \equiv \frac{e^3 E_0}{12\pi^2 a^2 H^2}\left(\frac{E_0^2}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right) - \frac{\pi B_0}{E_0}\operatorname{csch}^2\left(\frac{\pi B_0}{E_0}\right)\right)$$

Fortunately, an analytic solution is available!

$$\mathscr{A}_+^{(\sigma)}(\tau,\boldsymbol{k}) = \frac{1}{\sqrt{2k}}e^{\pi\xi_{\rm eff}/2}z^{\Sigma/2}\left[c_1 W_{-i\xi_{\rm eff},(\Sigma+1)/2}(-2iz) + c_2 M_{-i\xi_{\rm eff},(\Sigma+1)/2}(-2iz)\right],$$

3 Consistency equation

We impose the **consistency equation** to determine the mean-field value,

Require the integration over the perturbation reproduces the mean field amplitude

$$\left[\partial_z^2 - \frac{\Sigma}{z}\partial_z + 1 - \frac{2\xi_{\rm eff}}{z}\right]\mathcal{A}_+^{(\sigma)} = 0$$

$$\Sigma(E_0, B_0), \xi_{\rm eff}(E_0, B_0)$$

Mean

EoM

$$\langle E_0^2 \rangle, \langle B_0^2 \rangle = \frac{1}{2}\int \mathrm{d}\cos\theta \int \frac{dk}{k}\, \mathcal{P}_{E,B}$$

Solution

$$\boldsymbol{\delta A_k}$$

We numerically find self-consistent values of E_0 and B_0,

under the assumption of $\xi = const.$

3 Consistency equation

We impose the **consistency equation** to determine the mean-field value,

Require the integration over the perturbation reproduces the mean field amplitude

Mean-field

Perturbation

$$\tilde{E}_0 = \sqrt{2\rho_E(\tilde{E}_0, \tilde{B}_0)}, \quad \tilde{B}_0 = \sqrt{2\rho_B(\tilde{E}_0, \tilde{B}_0)},$$

$$\rho_B = \frac{1}{4}\int_{-1}^{1} \mathrm{d}\cos\theta \int_0^{2\xi} \frac{\mathrm{d}z}{z}\, \tilde{\mathscr{P}}_{BB}^{+(\sigma)}(z,\theta), \qquad \rho_E = \frac{1}{4}\int_{-1}^{1} \mathrm{d}\cos\theta \int_0^{2\xi} \frac{\mathrm{d}z}{z}\, \tilde{\mathscr{P}}_{EE}^{+(\sigma)}(z,\theta),$$

$$\tilde{\mathscr{P}}_{BB}^{+(\sigma)}(z,\theta_k) = \frac{H^4}{4\pi^2} e^{\pi\xi_{\rm eff}} z^{4+\Sigma} \left| c_1 W_\Sigma + c_2 M_\Sigma \right|^2, \quad \tilde{\mathscr{P}}_{EE}^{+(\sigma)}(z,\theta_k) = \frac{H^4}{4\pi^2} e^{\pi\xi_{\rm eff}} z^{4+\Sigma} \left| c_1 W'_\Sigma + c_2 M'_\Sigma + \frac{\Sigma}{2z}(c_1 W_\Sigma + c_2 M_\Sigma) \right|^2,$$

We **numerically found** the consistent amplitudes of EMFs for given ξ

(NB: In this iterative numerical process, we allowed **1% error**.)

Plan of Talk

1. Motivation
2. Review the case without ψ
3. Solve the system of A & ψ
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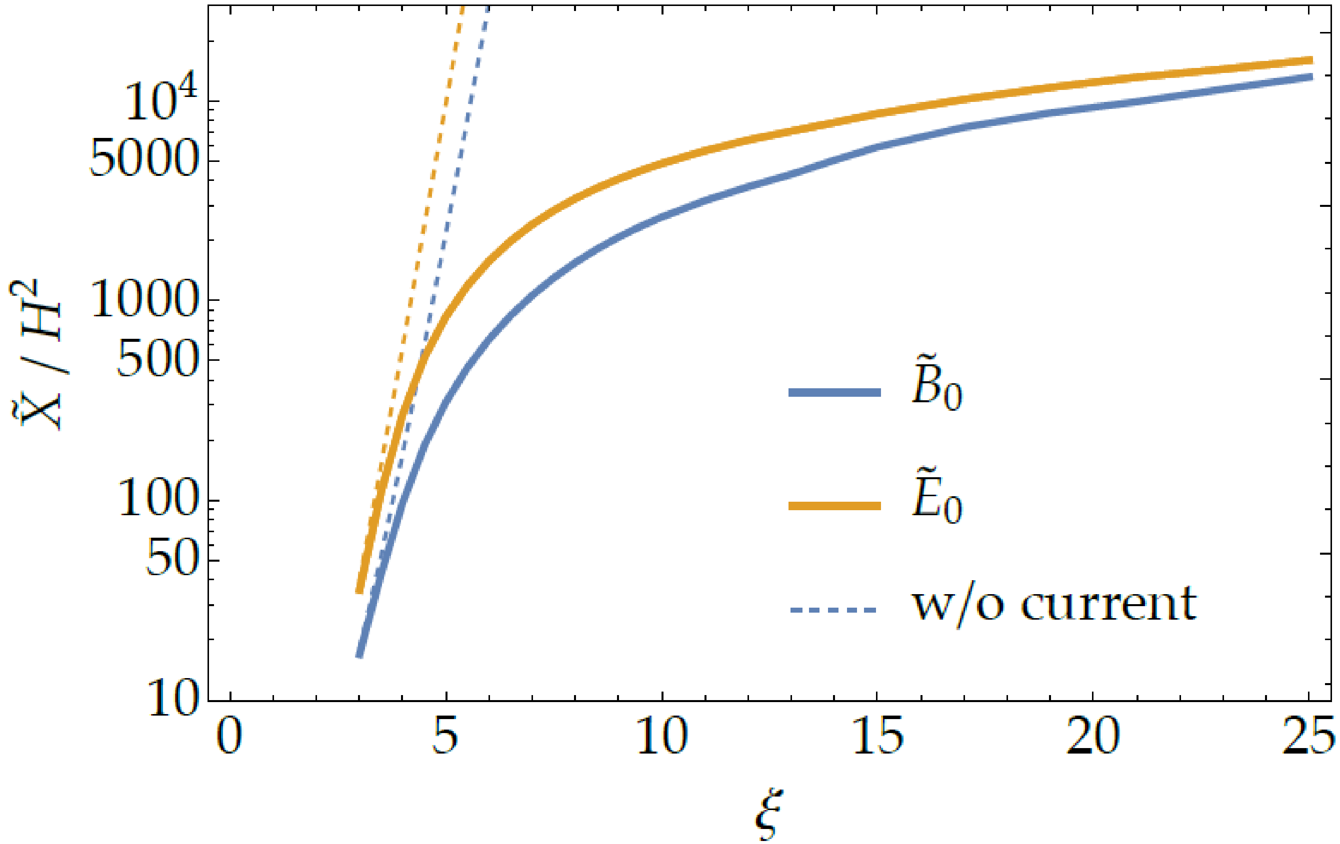
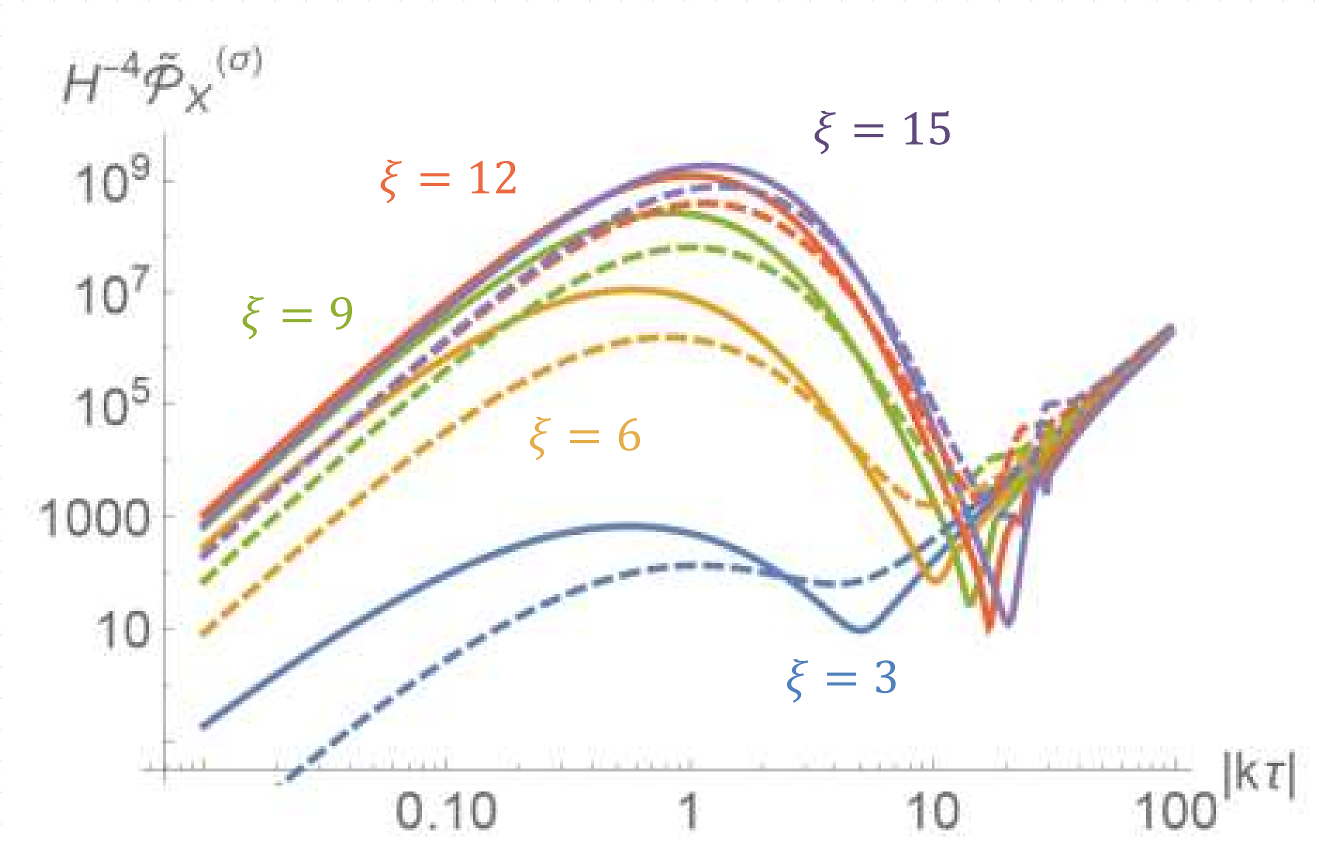
Self-consistent mean-field amplitudes for EMFs



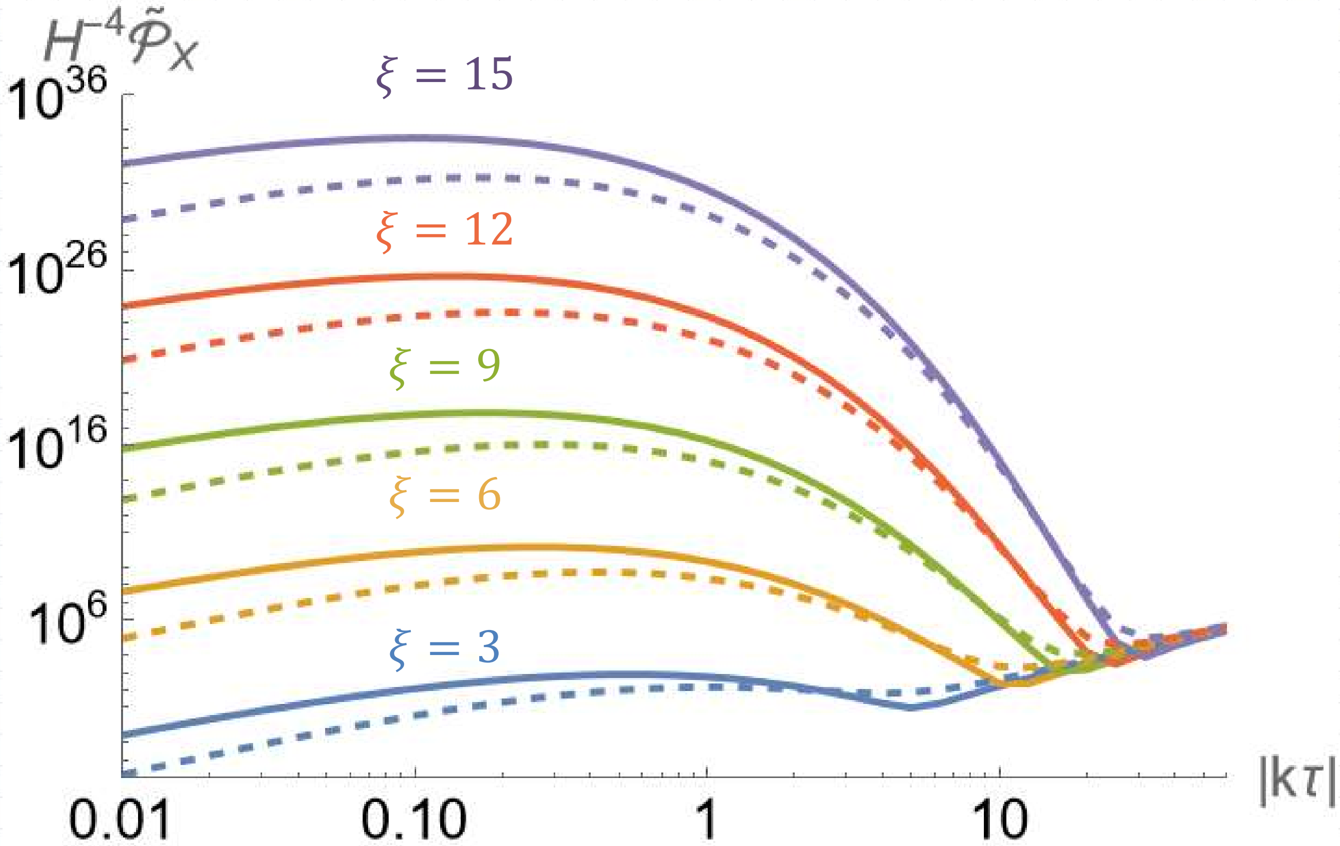
Charged fermions **drastically suppress** the EMF amplitudes.

E,B power spectra

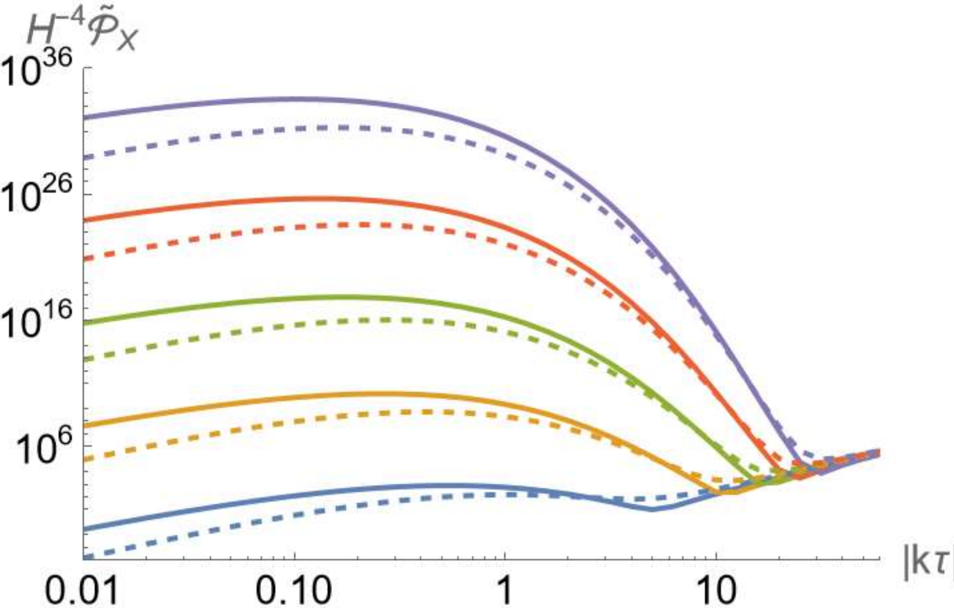


- The spectra reach their peaks **earlier** due to the **effective friction**.
- EMFs keep the **4 properties**, which verifies our argument.

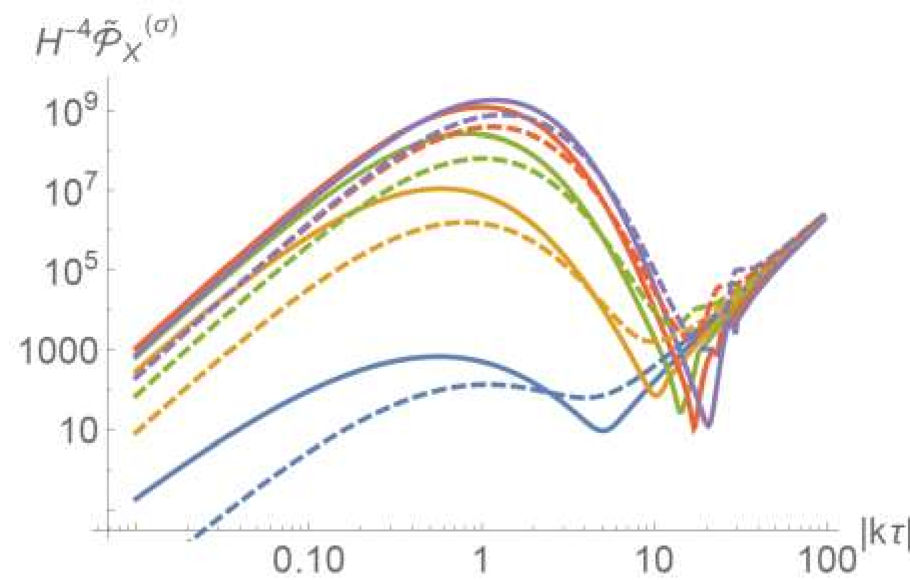
No charged particle case



Without Ψ

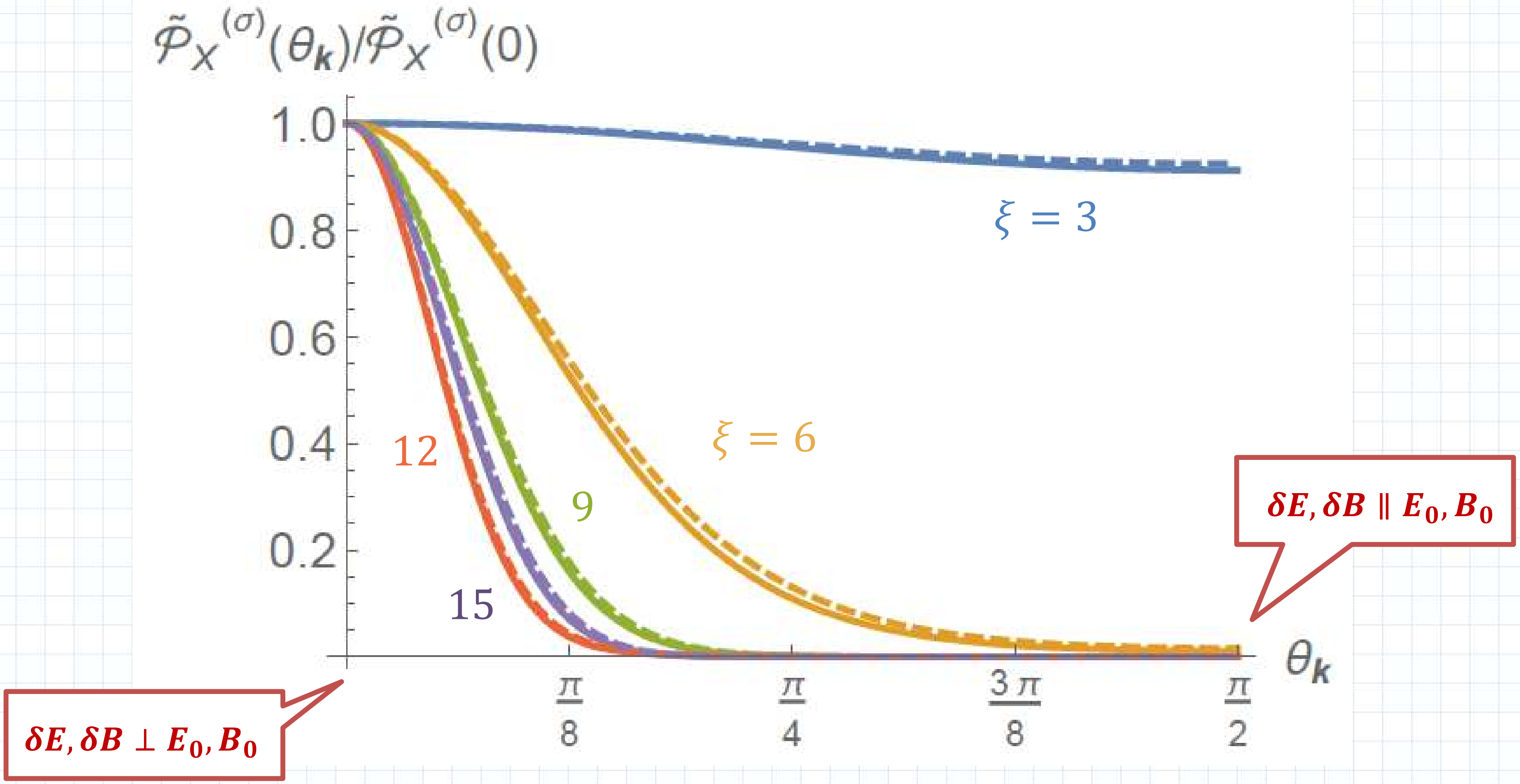


With Ψ



- The spectra reach their peaks **earlier** due to the **effective friction**.
- EMFs keep the **4 properties**, which verifies our argument.

Direction dependence of the power spectra



Schwinger current prevents the EMF production in similar directions

perpendicular production is favored ⇒ **Rotation** of the EMFs??

Energy conservation

The energy density of EMFs evolves as

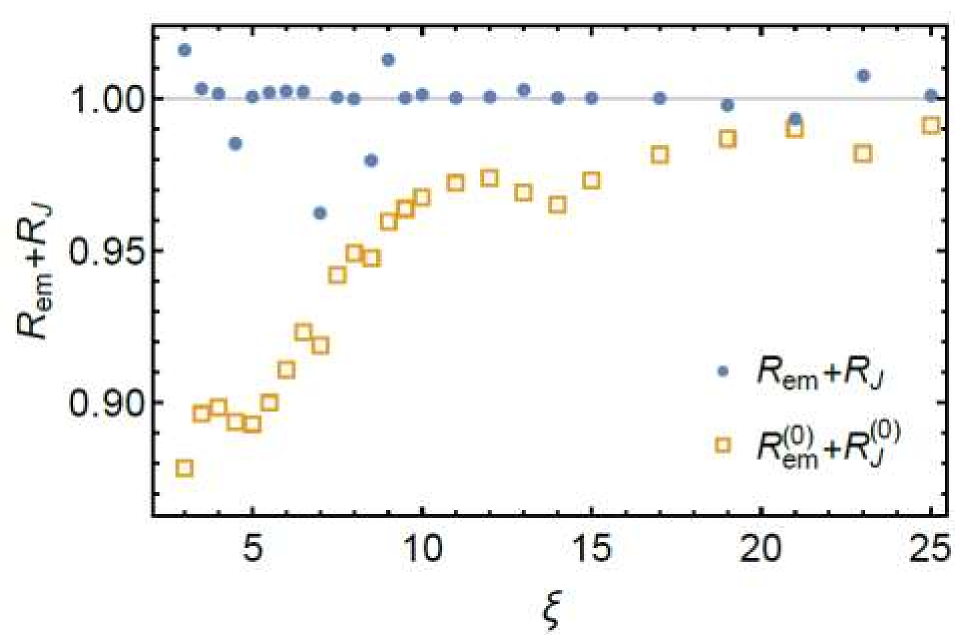
$$\langle \dot{\rho}_A \rangle = \underbrace{-2H\langle \tilde{\boldsymbol{E}}^2 + \tilde{\boldsymbol{B}}^2 \rangle}_{\text{Hubble dilution}} \underbrace{- 2\xi H \langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{B}} \rangle}_{\text{Energy injection from } \phi} \underbrace{- e \langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{J}} \rangle}_{\text{produce\&accelerate charged fermions}},$$

Since we consider a **static** system, $\langle \dot{\rho}_A \rangle$ should vanish and the 3 terms should be **balanced.**

$$R_{\rm em} + R_J = 1,$$

$$R_{\rm em} \equiv \frac{\langle \tilde{\boldsymbol{E}}^2 + \tilde{\boldsymbol{B}}^2 \rangle}{\xi |\langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{B}} \rangle|},$$

$$R_J \equiv \frac{e \langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{J}} \rangle}{2\xi H |\langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{B}} \rangle|}$$

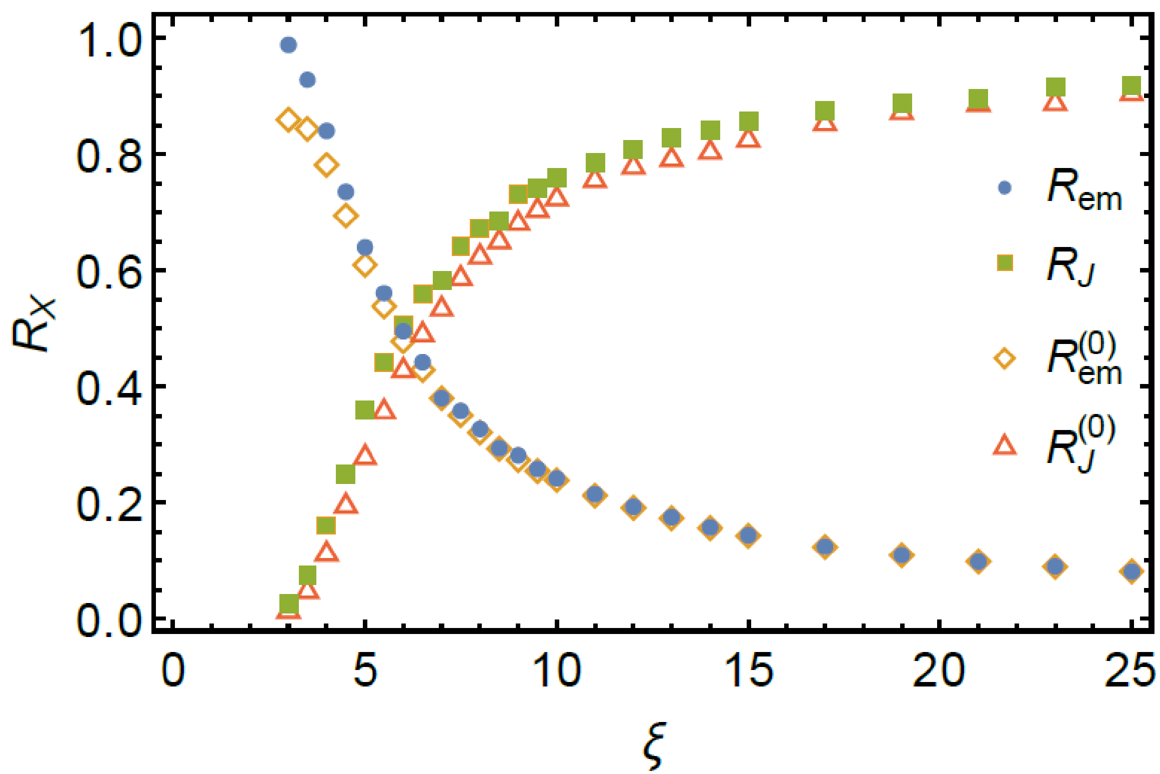


Energy distribution

$$R_{\rm em} + R_J = 1,$$

$$R_{\rm em} \equiv \frac{\langle \tilde{\boldsymbol{E}}^2 + \tilde{\boldsymbol{B}}^2 \rangle}{\xi |\langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{B}} \rangle|},$$

$$R_J \equiv \frac{e \langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{J}} \rangle}{2\xi H |\langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{B}} \rangle|}$$



- For $\xi \gtrsim 10$, the energy transfer to the **fermions is dominant**.
- We don't know **why**… But it may have an interesting consequences.

Plan of Talk

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SUMMARY

- Inflaton ϕ – photon A_μ – fermion ψ coupled system is well motivated but difficult. We need a **new approach** to solve this.

- We **integrated out** ψ by using the scale separation $L_\psi \ll L_{\mathrm{em}}$, and introduced **mean-field approx**. to solve non-linear eq. for A_μ. EM conductivities provide effective friction and reduction of ξ.

- We numerically solve the **consistency equation** to find the mean fields. The EM amplitudes are **drastically suppressed** compared to no-ψ case.

- Interestingly, the **dominant part** of the injected energy from ϕ goes to the charged fermions for $\xi \gtrsim 10$, which changes the conventional picture and may leads **new consequences**.

Future Work

- Relax the $\xi = const.$ assumption.

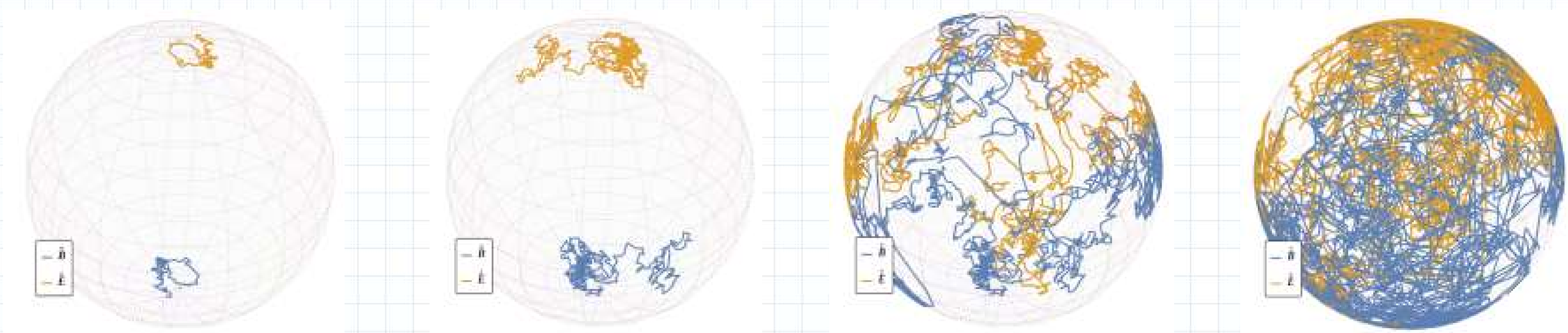
  Then we can explore the **inflation end** where ξ becomes maximum.

- Two unsatisfactory points of this work:

  1. Static EM assumption for $t \gtrsim H^{-1}$
  2. Consistency eq. is imposed only on the EMF amplitude not direction.

  ⇨ We cannot incorporate the **rotation** of EMFs

[TF, Mukaida, Tada, 2206.12218]



$\Delta N = 0.1$ $\Delta N = 0.5$ $\Delta N = 2$ $\Delta N = 10$

THE THEME
OF CHAPTER IS...

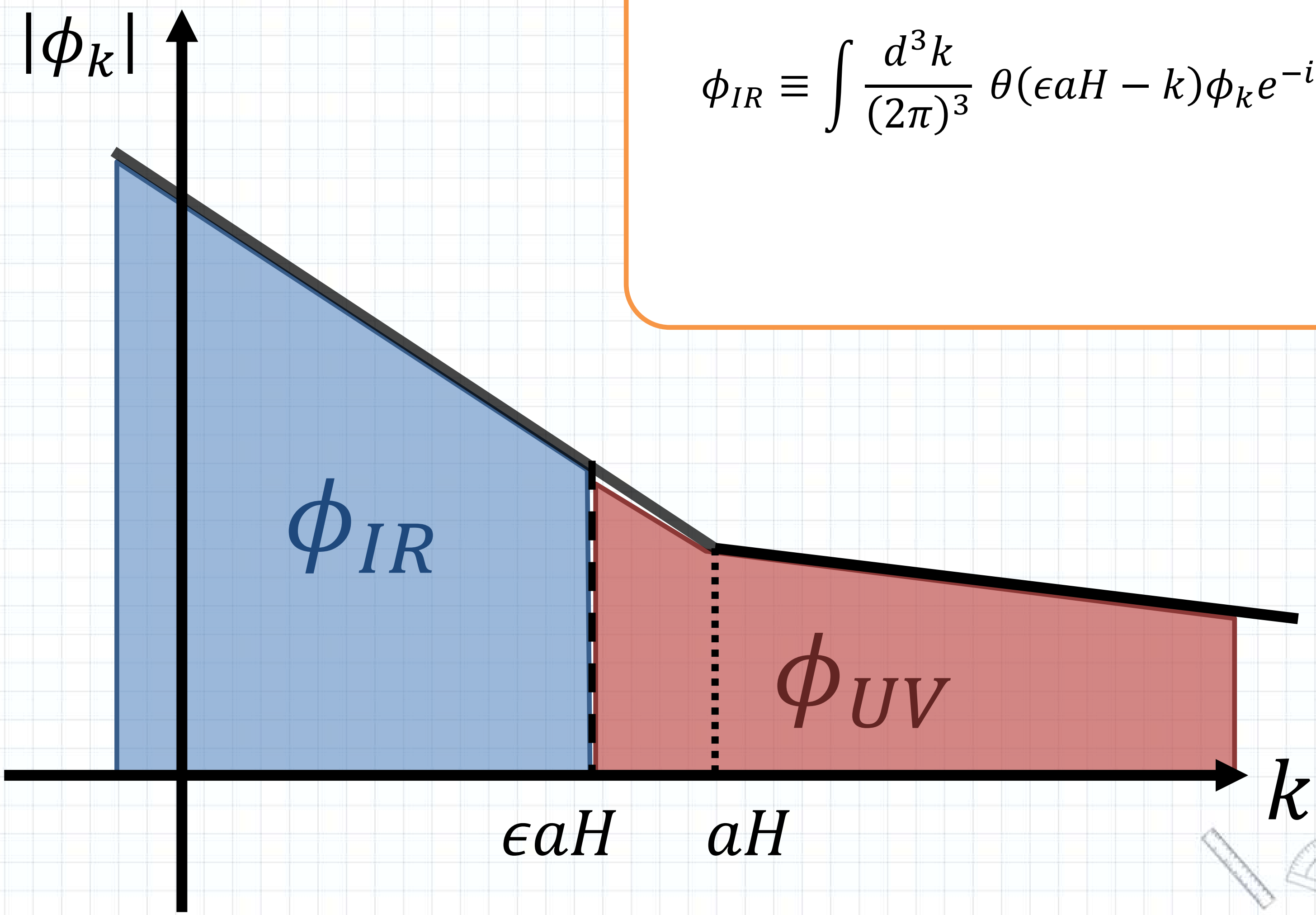
Thank you ！

[Starobinsky(1984), Starobinsky & Yokoyama(1994)]

Original S.F. for ϕ

$$\phi(t,x) = \phi_{IR}(t,x) + \phi_{UV}(t,x)$$

$$\phi_{IR} \equiv \int \frac{d^3k}{(2\pi)^3}\,\theta(\epsilon a H - k)\phi_k e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$



[Starobinsky(1984),
Starobinsky & Yokoyama(1994)]

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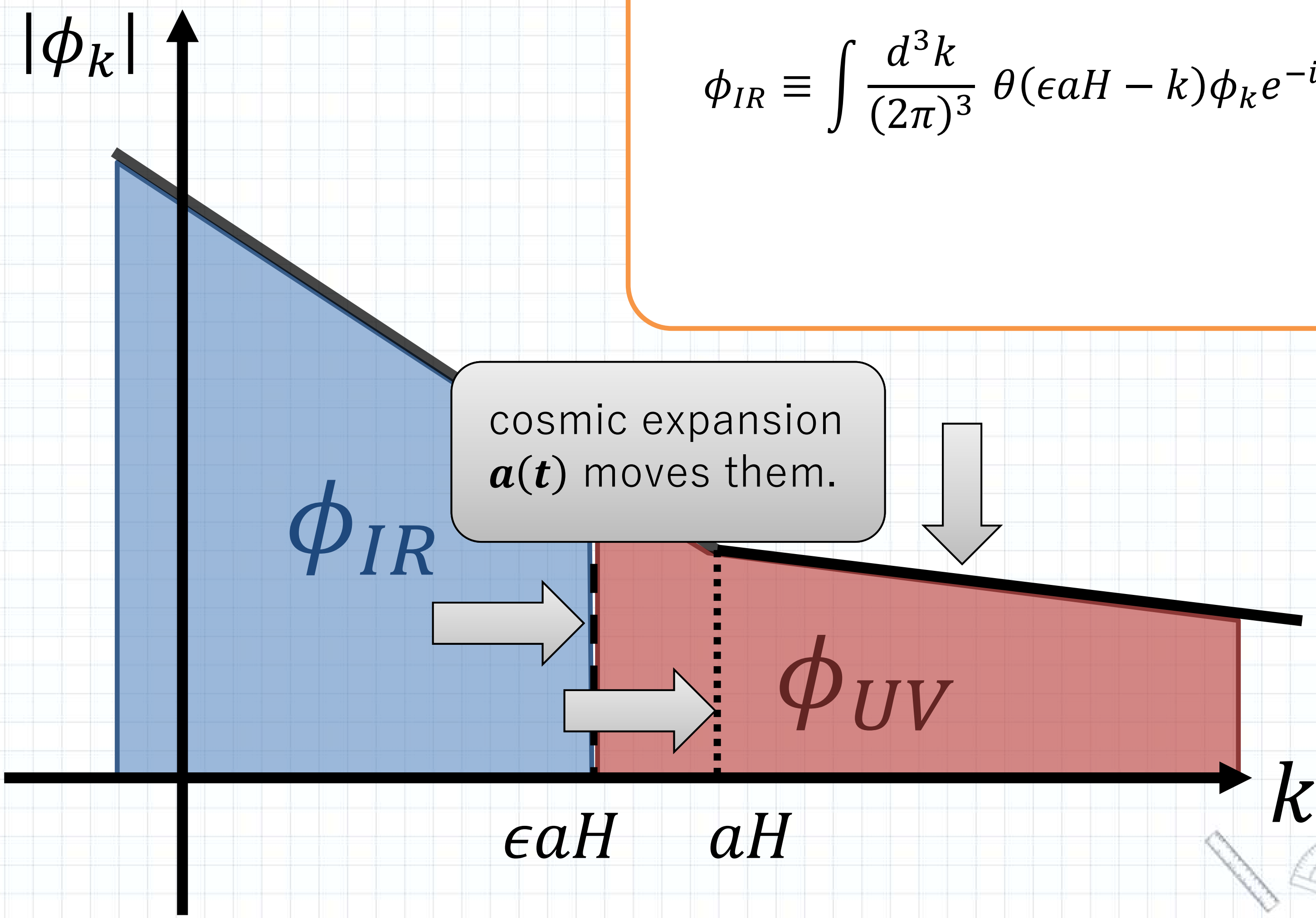


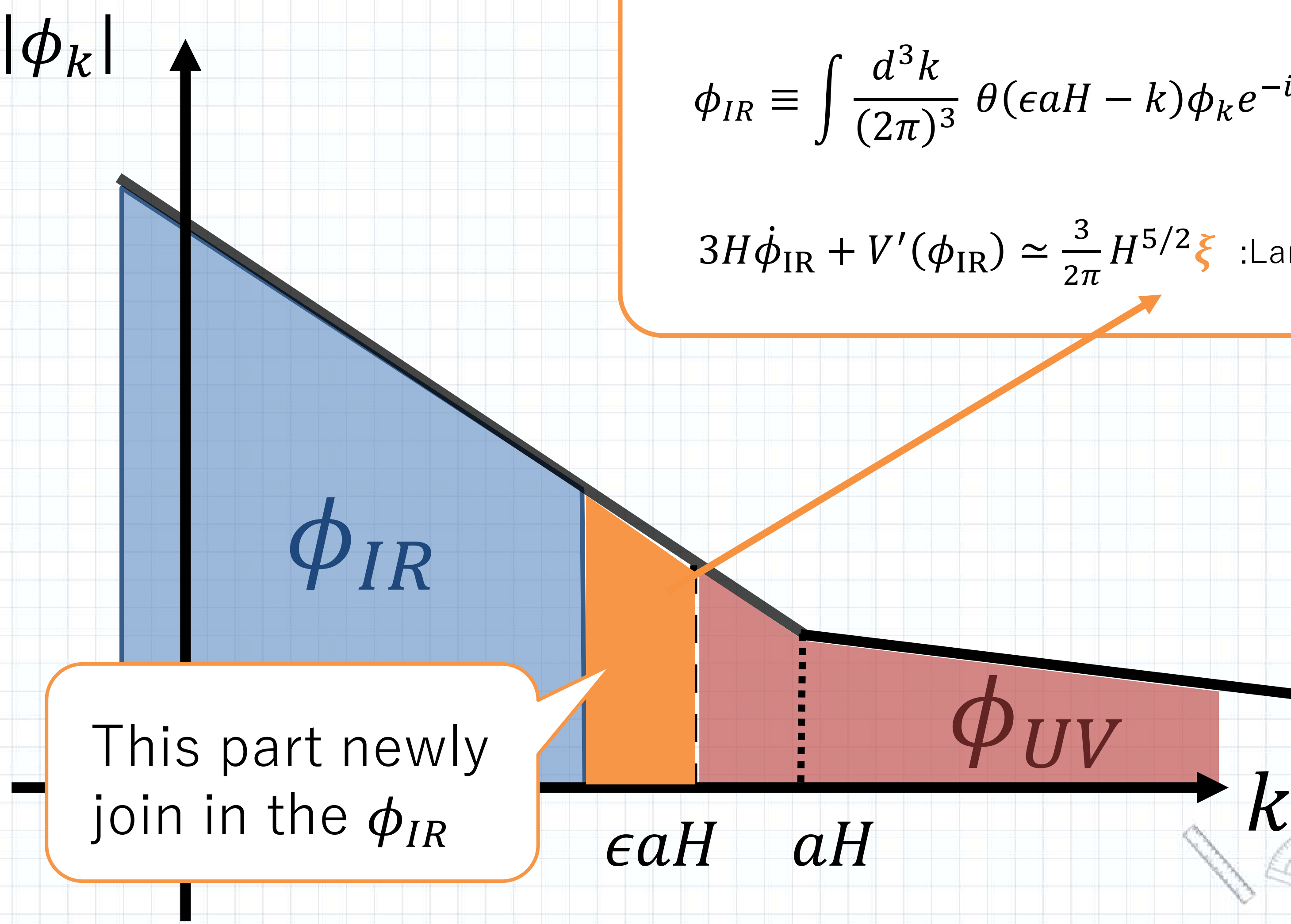
[Starobinsky(1984), Starobinsky & Yokoyama(1994)]

Original S.F. for ϕ

$$\phi(t,x) = \phi_{IR}(t,x) + \phi_{UV}(t,x)$$

$$\phi_{IR} \equiv \int \frac{d^3k}{(2\pi)^3}\,\theta(\epsilon aH - k)\phi_k e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$

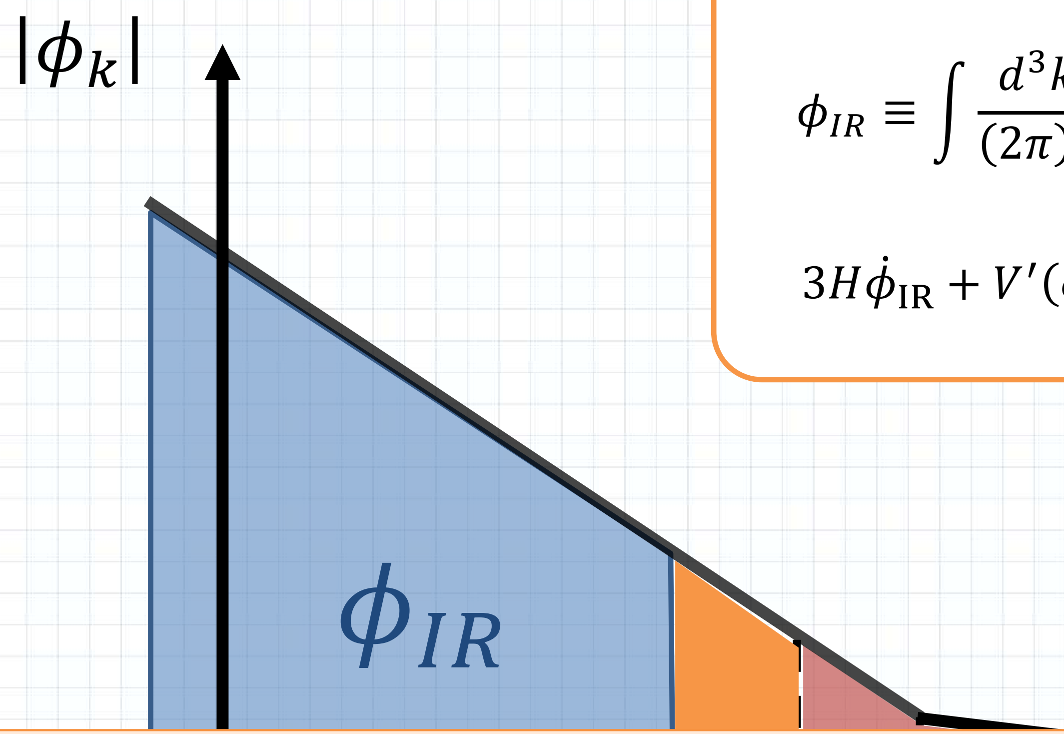
$$3H\dot{\phi}_{\mathrm{IR}} + V'(\phi_{\mathrm{IR}}) \simeq \frac{3}{2\pi}H^{5/2}\xi$$:Langevin eq.



3 Stochastic Formalism

[Starobinsky(1984), Starobinsky & Yokoyama(1994)]

Original S.F. for ϕ



$$\phi(t,x) = \phi_{IR}(t,x) + \phi_{UV}(t,x)$$

$$\phi_{IR} \equiv \int \frac{d^3k}{(2\pi)^3}\,\theta(\epsilon a H - k)\phi_k e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$3H\dot{\phi}_{\mathrm{IR}} + V'(\phi_{\mathrm{IR}}) \simeq \frac{3}{2\pi}H^{5/2}\xi$$:Langevin eq.

BG + fluctuations are solved at once!

S.F. for EM fields (without charged particles)

The Langevin eqs. are given by

$$\begin{pmatrix} \partial_N \tilde{\boldsymbol{B}}_{\rm IR} + 2\tilde{\boldsymbol{B}}_{\rm IR} \\ \partial_N \tilde{\boldsymbol{E}}_{\rm IR} + 2\tilde{\boldsymbol{E}}_{\rm IR} + 2\xi \tilde{\boldsymbol{B}}_{\rm IR} \end{pmatrix} = R^T \begin{pmatrix} 0 \\ \tilde{\xi}(N) \end{pmatrix},$$

Only one noise term appears, because EMFs consist of one DoF.

where

$$\widetilde{\boldsymbol{E}}^{\pm}_{\rm IR} = -a^{-2} \int \frac{{\rm d}^3 k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \theta(\kappa a H - k) \boldsymbol{\epsilon}^{\pm}(\hat{\boldsymbol{k}}) \partial_\tau \mathcal{A}_{\pm}(\tau, k) \hat{a}^{\pm}_{\boldsymbol{k}} + h.c.$$

$$\widetilde{\boldsymbol{B}}^{\pm}_{\rm IR} = a^{-2} \boldsymbol{\nabla} \times \int \frac{{\rm d}^3 k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \theta(\kappa a H - k) \boldsymbol{\epsilon}^{\pm}(\hat{\boldsymbol{k}}) \mathcal{A}_{\pm}(\tau, k) \hat{a}^{\pm}_{\boldsymbol{k}} + h.c.$$

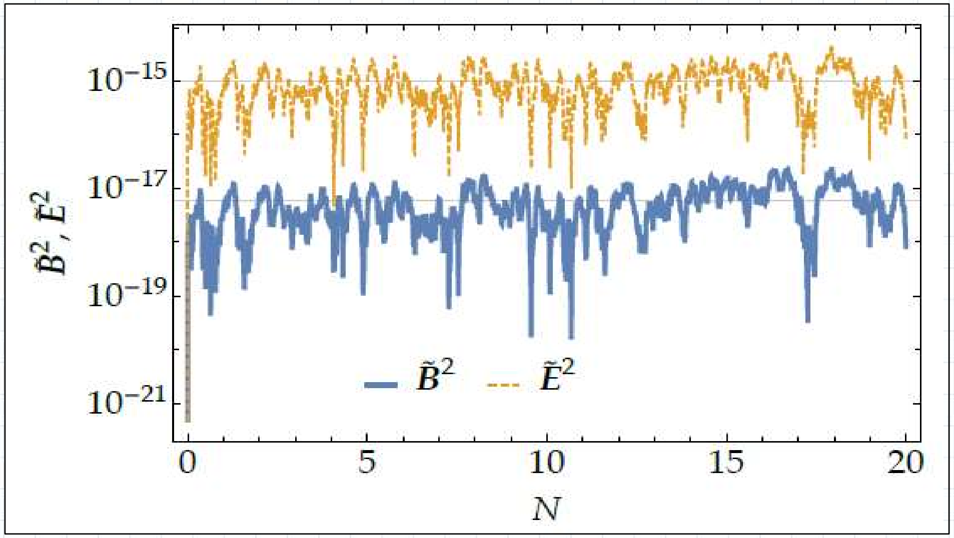
and $R, \tilde{\xi}$ are fixed by the mode function $\mathcal{A}_+(\tau, k)$ at $k = \kappa a H$

We can solve it once $\mathcal{A}_+(\tau, k)$ is obtained.

$$R = \frac{1}{\sqrt{|W|^2 + |W'|^2}} \begin{pmatrix} |W'| & |W| \\ -|W| & |W'| \end{pmatrix},$$

$$\langle \tilde{\xi}_i(N) \tilde{\xi}_j(N') \rangle = \delta_{ij} \delta(N - N') \frac{\kappa^4 H^4}{12\pi^2} e^{\pi\xi} (|W|^2 + |W'|^2),$$

Stochastic Results 1



We can compute

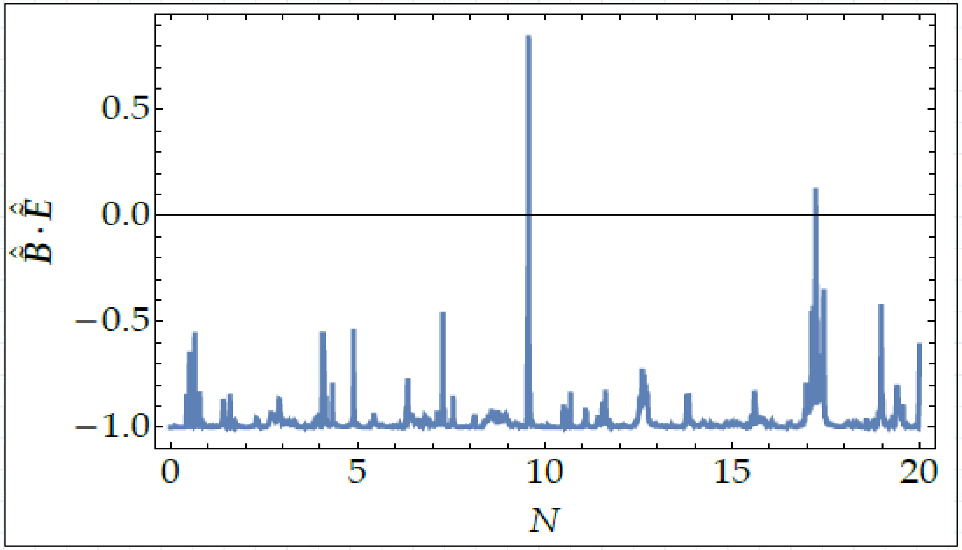
the variance of $\tilde{\boldsymbol{E}}_{\mathrm{IR}}, \tilde{\boldsymbol{B}}_{\mathrm{IR}}$

$$\langle \tilde{\boldsymbol{B}}_{\mathrm{IR}}^2(t)\rangle \simeq \frac{1}{4}\tilde{\mathscr{P}}_{BB}(\kappa),$$

$$\langle \tilde{\boldsymbol{E}}_{\mathrm{IR}}^2(t)\rangle \simeq \frac{1}{4}\tilde{\mathscr{P}}_{EE}(\kappa) - \frac{\xi}{8}\left(\tilde{\mathscr{P}}_{BE}(\kappa) + \tilde{\mathscr{P}}_{EB}(\kappa)\right) + \frac{\xi^2}{8}\tilde{\mathscr{P}}_{BB}(\kappa).$$

Indeed, in a realization, the $\tilde{\boldsymbol{E}}_{\mathrm{IR}}, \tilde{\boldsymbol{B}}_{\mathrm{IR}}$ strengths fluctuate around them.
Clearly, $\tilde{\boldsymbol{E}}_{\mathrm{IR}}$ is much stronger than $\tilde{\boldsymbol{B}}_{\mathrm{IR}}$.

Stochastic Results 2



$\tilde{\boldsymbol{E}}_{\rm IR}$ & $\tilde{\boldsymbol{B}}_{\rm IR}$ are **anti-parallel**, which can be analytically confirmed:

$$\hat{E}_{\rm IR}\cdot\hat{B}_{\rm IR}=\frac{\langle \tilde{E}_{\rm IR}\cdot\tilde{B}_{\rm IR}+\tilde{B}_{\rm IR}\cdot\tilde{E}_{\rm IR}\rangle}{2\,\langle \tilde{B}^2_{\rm IR}\rangle^{1/2}\,\langle \tilde{E}^2_{\rm IR}\rangle^{1/2}}\xrightarrow{\kappa\ll 2\xi}-\frac{|W'|+\xi|W|/2}{\sqrt{|W'|^2+\xi|W'||W|+\xi^2|W|^2/2}}\xrightarrow{2\kappa\xi\ll 1}-1,$$

This is because the coupling $\phi F\tilde{F}=\phi\boldsymbol{E}\cdot\boldsymbol{B}$ sources not $\boldsymbol{E}\perp\boldsymbol{B}$ but $\boldsymbol{E}\parallel\boldsymbol{B}$.

Stochastic Results 3

The coherent time of $\tilde{\boldsymbol{E}}_{\mathrm{IR}}, \tilde{\boldsymbol{B}}_{\mathrm{IR}}$ is

$$\tau_c = (2H)^{-1}$$

$$\langle \tilde{B}_{\mathrm{IR}}(t) \cdot \tilde{B}_{\mathrm{IR}}(t+\Delta t) \rangle \simeq e^{-2H\Delta t} \tilde{B}^2_{\mathrm{IR}}(t),$$

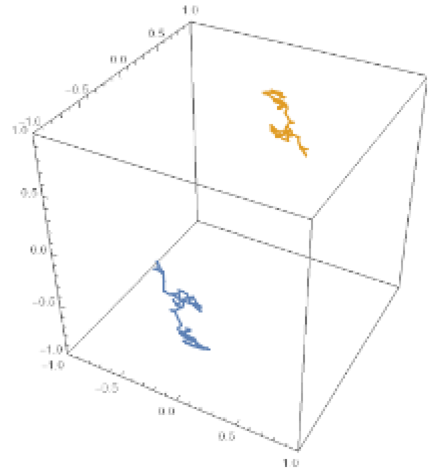
$$\langle \tilde{E}_{\mathrm{IR}}(t) \cdot \tilde{E}_{\mathrm{IR}}(t+\Delta t) \rangle \simeq e^{-2H\Delta t} \left[\tilde{E}^2_{\mathrm{IR}}(t) - 2\xi H \Delta t \, \tilde{E}_{\mathrm{IR}}(t) \cdot \tilde{B}_{\mathrm{IR}}(t) \right].$$

In other words, $\tilde{\boldsymbol{E}}_{\mathrm{IR}}, \tilde{\boldsymbol{B}}_{\mathrm{IR}}$ lose their memories every 0.5 e-folds and they are always dominated by newly produced fluctuations.

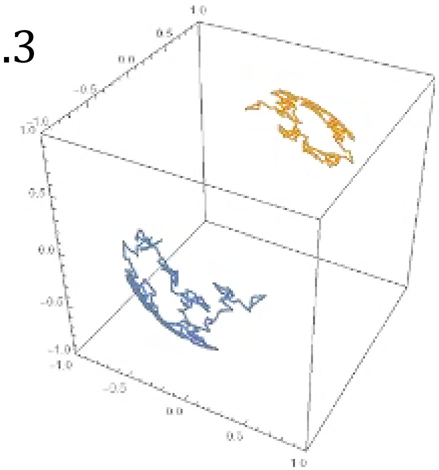
Stochastic Results 3

$\widetilde{\boldsymbol{E}}_{\mathrm{IR}}$ & $\widetilde{\boldsymbol{B}}_{\mathrm{IR}}$ are anti-parallel and randomly rotate for $\Delta N = 0.5$

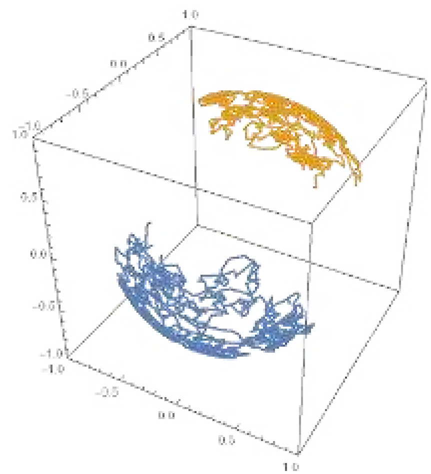
① $N = 0.1$



② $N = 0.3$



③ $N = 1$



④ $N = 3$