

Testing the lepton PDFs of the proton at lepton-proton colliders

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Lepton PDFs

- At zeroth order in perturbation theory the lepton carries all the momentum of the beam
- At high energies, collinear radiation emitted by splitting of the initial state must be taken into account.

- The case of collinear photon emission from an electron gives the

Equivalent Photon Approximation (EPA):
given by the Weizsacker-Williams spectrum

$$f_{\gamma,l}(x) \approx Q_l^2 \frac{\alpha}{2\pi} P_{\gamma,l}(x) \ln \frac{E^2}{m_l^2}$$

Weizsacker, Copenhagen

<https://doi.org/10.1007/BF01333110>

Williams

<https://doi.org/10.1103/PhysRev.45.729>

↓

$$\text{Splitting func: } P_{\gamma,l}(x) = \frac{1 + (1-x)^2}{x}$$

the probability of the lepton to emit a photon
with a fraction x of its longitudinal momentum.

- These radiated photons can furthermore provide other fermion parton content to the colliding lepton
via the $\gamma \rightarrow f\bar{f}$ splitting function $P_{f,\gamma}(z_f) = 1 - 2z_f + 2z_f^2$ $z_f \rightarrow$ the fraction of momentum of the fermion with respect to the photon.

- The EPA has been generalized to describe EW gauge bosons in high-energy collisions, in what is now known as Effective Vector Approximation (EVA)

Lepton PDFs

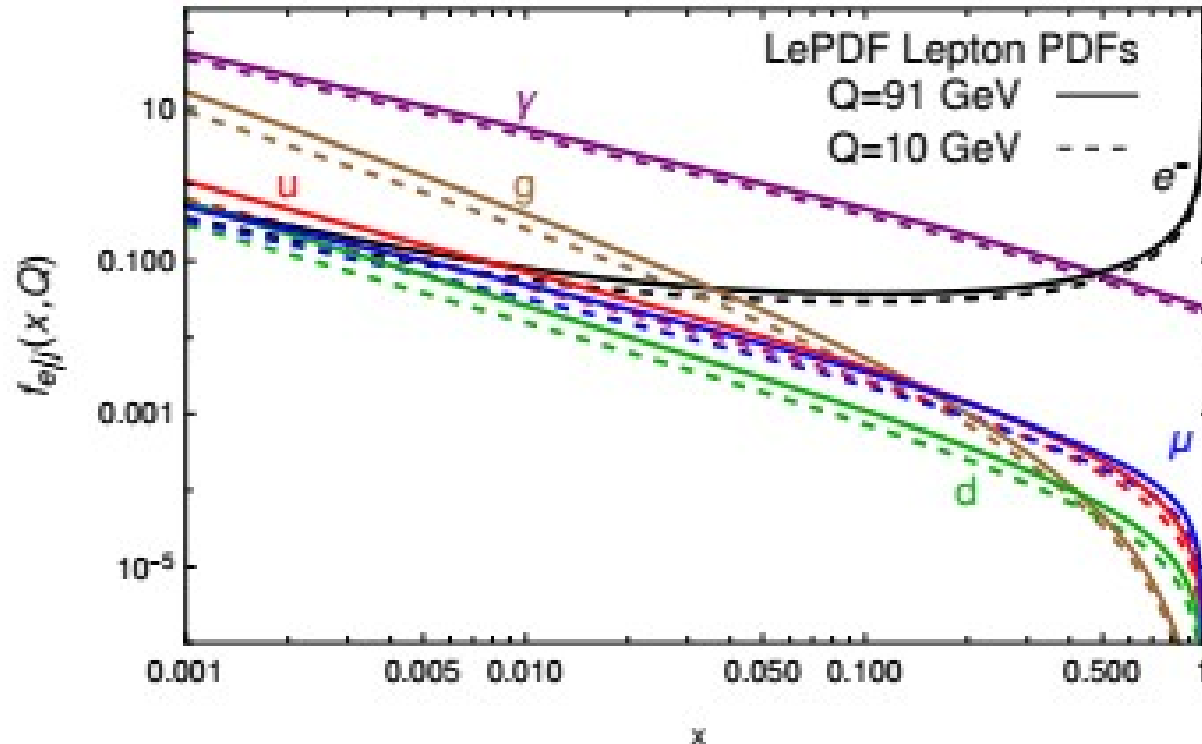
- This previous description becomes inadequate at some high scale

At high energies $E \gg m_\ell$ the collinear logarithmic term can become sizeable and needs to be resummed for reliable predictions.

—▶ Leads to solving QED analog of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations

Generalization of Parton Distribution Functions (PDFs) for charged leptons

- Recently, it has been computed for electron and muons



Garosi, et al.

<https://arxiv.org/abs/2303.16964>

Han, et al.

<https://arxiv.org/abs/2007.14300>

<https://arxiv.org/abs/2103.09844>

<https://arxiv.org/abs/1607.03210>

Lepton PDFs of proton

hep-ph/9412286

hep-ph/0403200

hep-ph/0411040

- A similar description applies to the QED interactions within the proton.

When estimating proton PDFs involving QED-induced processes, hadronization effects lead to a modification

of the logarithmic factor to $\ln(\mu_F^2/\Lambda^2)$ $\mu_F \rightarrow$ factorization scale
 $\Lambda \rightarrow$ typical hadronic scale

arXiv:2005.06477

Buonocore, et al.

- In a pure QCD model, protons consist of valence quarks, with abundant soft and collinear QCD radiation.

Order of quark and gluon PDFs $(\alpha_S L)^n$, $L = \ln(\mu_F/\Lambda)$

If $L \sim 1/\alpha_S$, all the contributions becomes relevant

Notes:

- DGLAP equations account for perturbative evolution (renormalization scale dependence).
- Non-perturbative initial conditions (momentum fraction dependence) are extracted through global fits to DIS and collider data

- Naive expectation from power counting that the lepton density in the proton very small $f_\ell \sim \alpha^2 f_q$

But it could be large due to large logarithmic factors

Buonocore, et al.

Power counting schemes

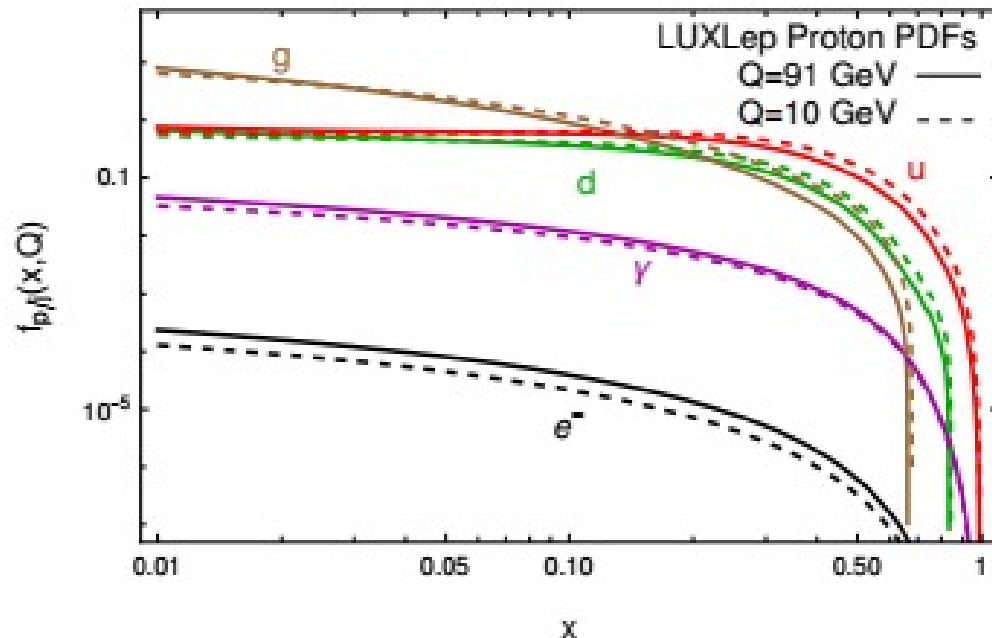
- quark and gluon PDF $(\alpha_s L)^n$, $L = \ln(\mu_F/\Lambda)$

- The photon PDF is reduced by a factor of αL (collinear-enhanced term) at the LO compared to the quark PDF.

$$\alpha \sim \alpha_s^2 \quad L \sim 1/\alpha_s \quad \rightarrow \alpha L \sim \alpha_s$$

- Similarly, leptons will be suppressed by a factor of $(\alpha L)^2 \sim \alpha_s^2$ at the LO

- So, $f_{p/q}$ and $f_{p/g} \sim (\alpha_s L)^n \sim \mathcal{O}(1)$ $f_{p/\gamma} \sim \alpha_s$ $f_{p/\ell} \sim \alpha_s^2 C_\ell$ $C_\ell = 3/20$



- Recently, the lepton PDFs of proton has been computed by Buonocore, et al.

arXiv:2005.06477

- One can think of the LHC as a lepton collider with varying energy. (not in terms of backgrounds)

Claim:

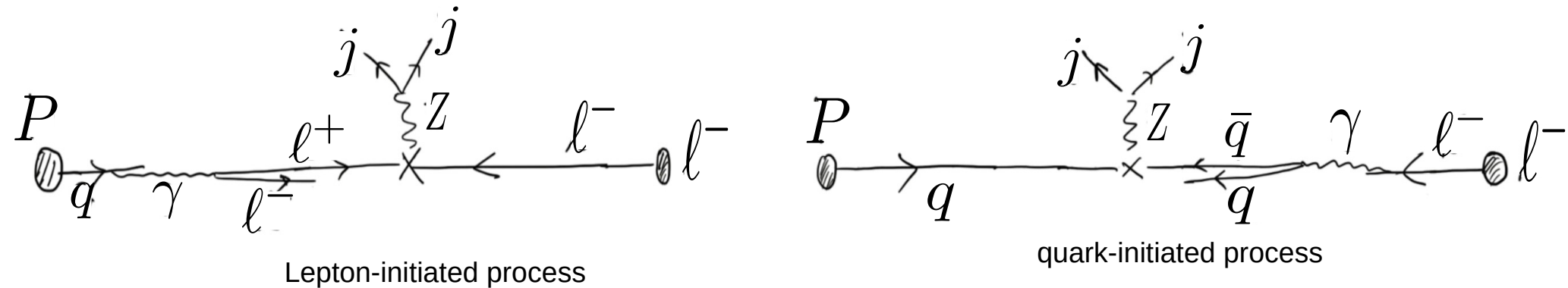
- The LHC can probe lepto-philic Z' up to 600 GeV, leptoquarks and doubly charged scalars up to 2.5--3 TeV with order-one couplings at high luminosity

Resonant Z-boson production at proton-lepton colliders:

- For HERA and EIC, e^+ from P and the e^- from e^- collides and produce Z-boson

For MUIC, the same process would take place involving μ instead of electron

- Another processes: involving quarks coming from the colliding proton and lepton beams



PDF formalism is crucial as leptons and quarks originating from photons are in the collinear regime.

- Cross-sections in pe colliders: $f_{p/i} \otimes f_{e/j} \otimes \hat{\sigma}_{ji}$
PDFs hard scattering between partons i from the p and j from the e^-

- For, $e^+e^- \rightarrow Z$ $\hat{\sigma}_{e^+e^-} \sim \alpha/\hat{s} \sim \alpha_s^2/\hat{s}$, $\hat{s} \sim m_Z^2$, Lepton-initiated process $\rightarrow \alpha_s^4 C_\ell / m_Z^2$

- For, $q\bar{q} \rightarrow Z$ $f_{e/\gamma} \sim \alpha L' \sim \alpha_s$, $f_{e/q} \sim (\alpha L')^2 C_q \sim \alpha_s^2 C_q$ $L' = \log(\mu_f^2/m_e^2)$, $C_q \sim 3Q_q^2/20$
quark-initiated process $\rightarrow \alpha_s^4 (C_u + C_d) / m_Z^2$

● Why Focus on a Lepton-Proton Collider Instead of a Hadron-Hadron Collider Like the LHC?

- Lepton involving process $p(\ell^\pm)p(\ell^\mp) \rightarrow Z$ cannot be tested at hadron colliders, as the quark initiated processes $p(q)p(\bar{q}) \rightarrow Z$ dominate.
- Making the Proton-Lepton collider a unique machine to study lepton initiated Z-resonant production and test the lepton PDFs of proton.
- There is a need to distinguish lepton-initiated processes from quark-initiated processes at the proton-lepton colliders.
- Useful variable: $E - P_z \equiv \sum_i (E^i - P_z^i)$
Sum runs over all detected particles
 $P_z \rightarrow$ momentum along the Z-direction
 $E \rightarrow$ Energy

Kinematics:

Energies of the colliding partons

- Resonance features the relation: $4E_1 E_2 \sim m_Z^2$

$$\rightarrow 4E_{e^-/e^-} E_{p/e^+} \sim m_Z^2$$

$$x_{e^-/e^-} E_e$$

$$x_{p/e^+} E_p$$

- Electron PDFs of electron peaks at $x_{e^-/e^-} \simeq 1$ (at the LO, its a Dirac-Delta func)
- The largest contribution to the resonant Z-production cross section from the lepton-initiated process occurs for

$$x_{p/e^+} = \frac{m_Z^2}{4E_e E_p}$$

- The lepton-initiated process mostly occurs for a fixed value of x_{p/e^+} ,

while for the quark-initiated process, a range of $x_{e^-/q}$ and $x_{p/q}$ exists for which significant contributions

■ HERA:

[arXiv:0911.0884]

Operated from 1992 to 2007

Electrons or positrons colliding with protons

Proton ~ 920 GeV and Electrons ~ 27.5 GeV

Integrated luminosity: 500 Pb^{-1} per collider experiment
(H1 and ZEUS)

$$x_{p/e^+} = \frac{m_Z^2}{4E_e E_p}$$

Lepton-initiated Z production

$$E_{p/e^+} \sim 75 \text{ GeV}$$

$$x_{p/e^+} \sim 0.082$$

■ EIC:

[arXiv:1212.1701, 2103.05419]

Proton ~ 250 GeV and Electrons ~ 20 GeV

Integrated luminosity: $10\text{-}100 \text{ fb}^{-1}$ (expected)

$$E_{p/e^+} \sim 103.5 \text{ GeV}$$

$$x_{p/e^+} \sim 0.412$$

■ MUIC:

(proposed) [arXiv:2107.02073, 2203.06258]

Proton ~ 275 GeV and Muons ~ 1 TeV

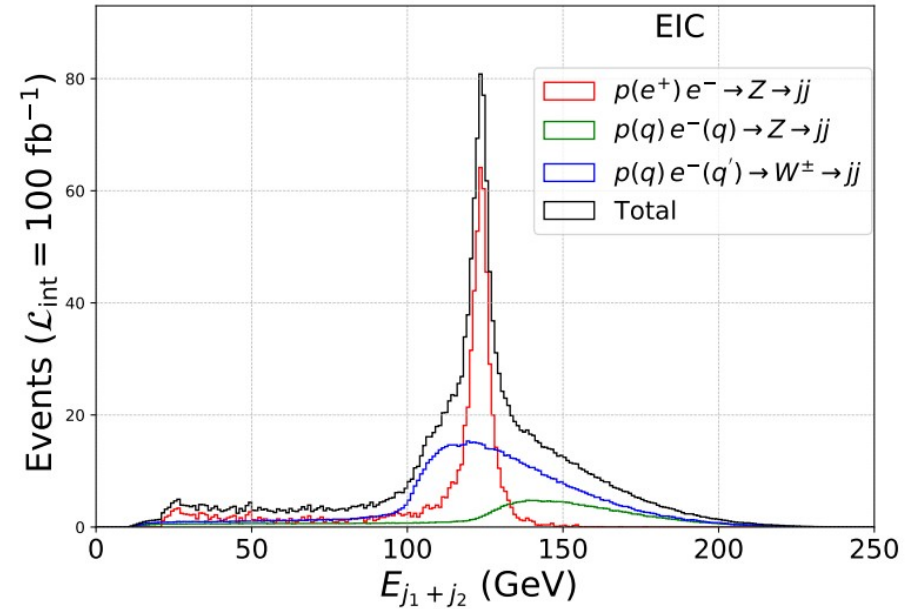
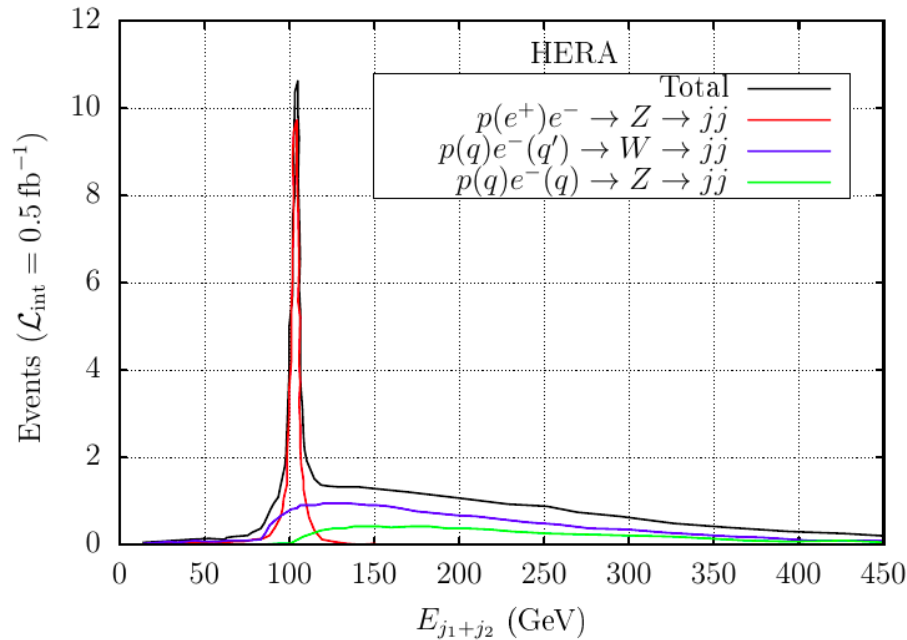
Integrated luminosity: 400 fb^{-1}

$$E_{p/\mu^+} \sim 2 \text{ GeV}$$

$$x_{p/\mu^+} \sim 0.0073$$

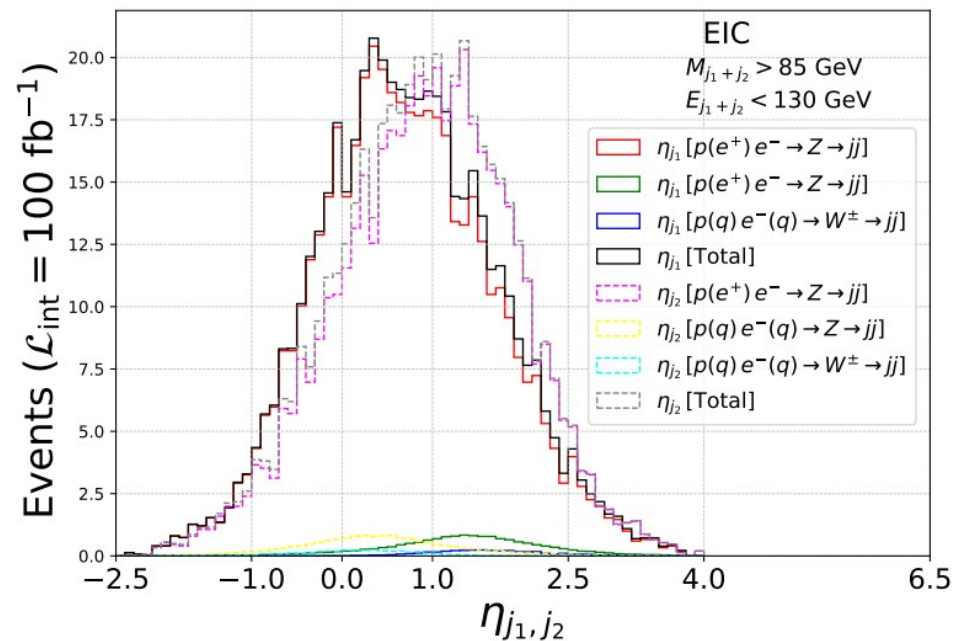
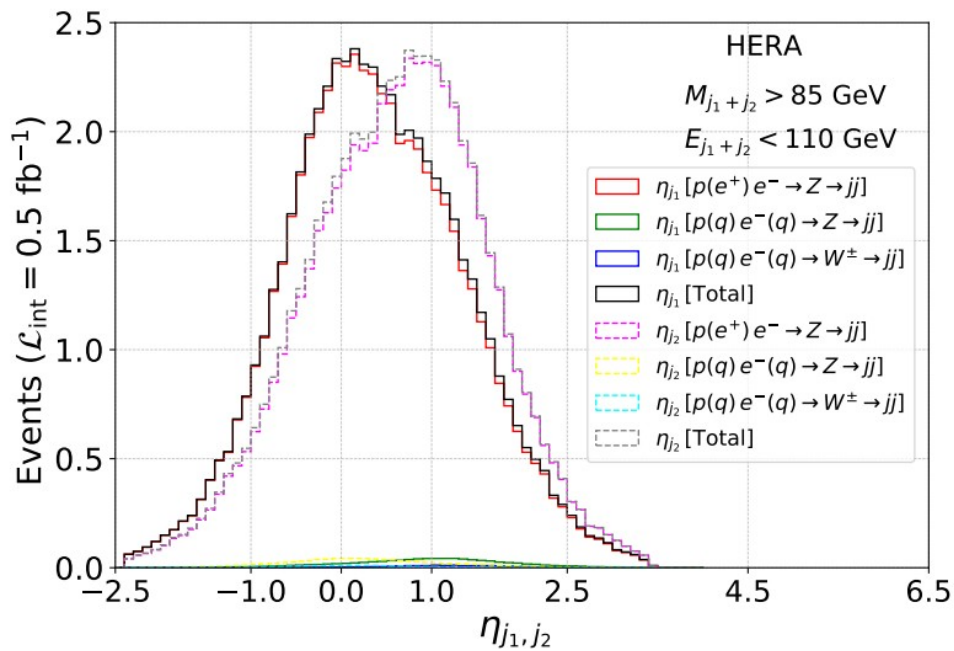
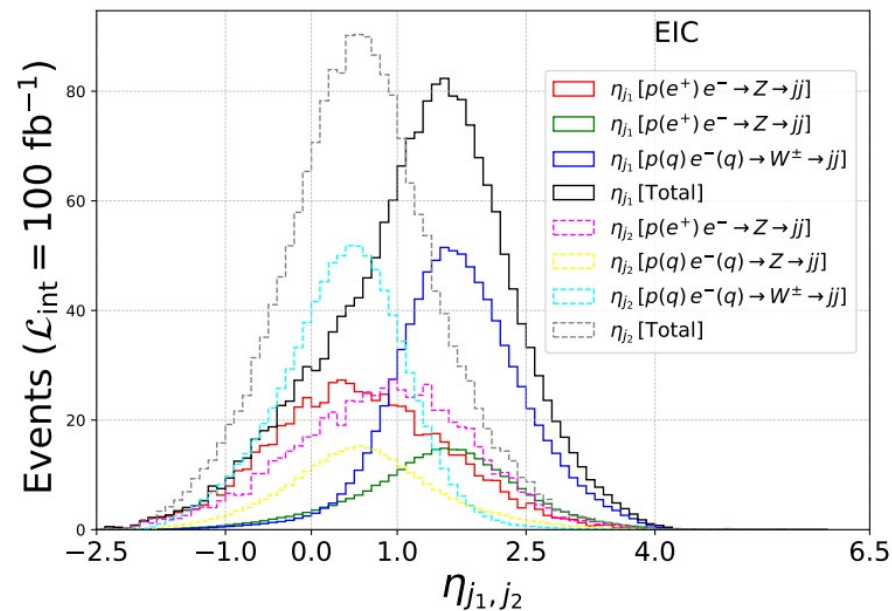
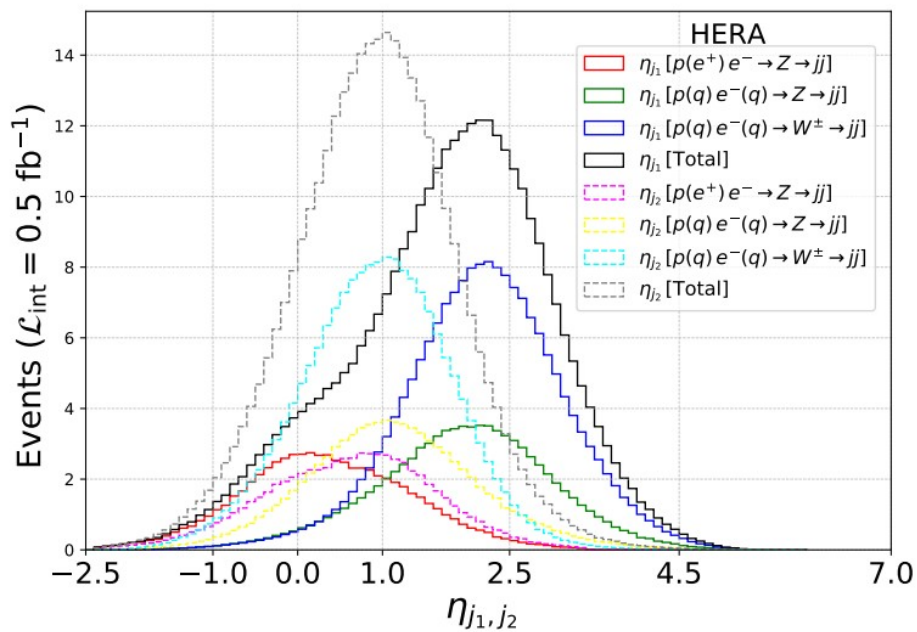
Energy sum of decaying objects coming from the produced Z-boson

Focus to the di-jet decay mode of Z



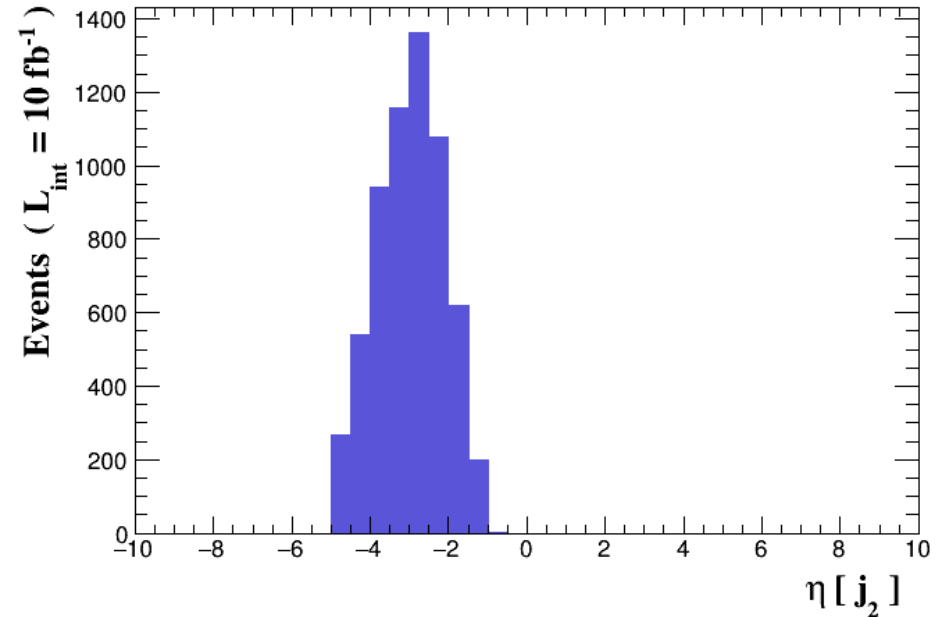
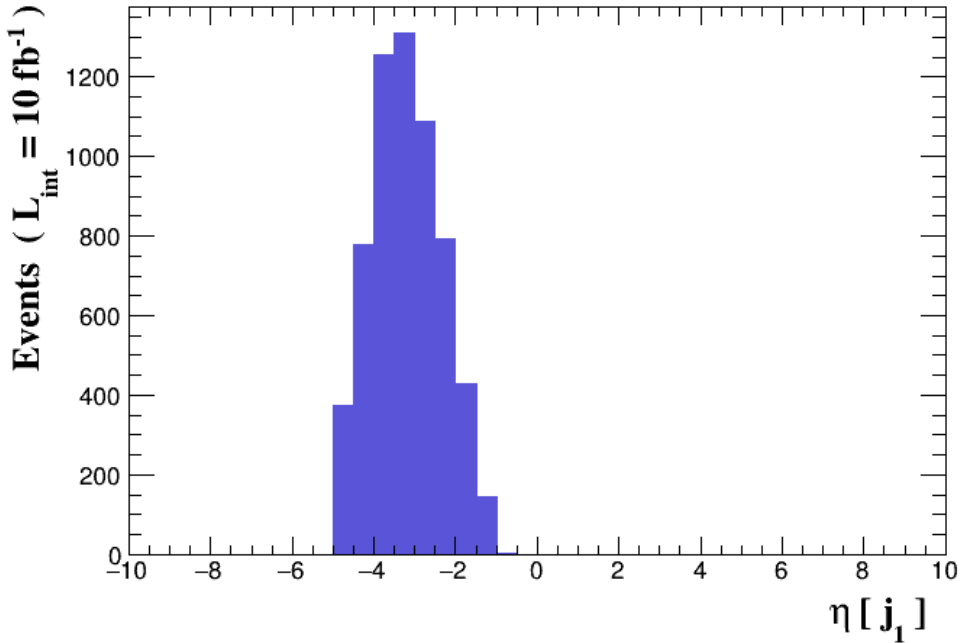
- These distributions suggest that the quark-initiated processes occur for a wide range of longitudinal momentum fraction of the quarks coming from the proton and the lepton.
- Meanwhile, the lepton-initiated process occurs at approximately a fixed value of the longitudinal momentum fraction of the lepton originating from the proton.

η -distributions of the leading two jets



η -distributions of jets from Z, produced through the lepton-initiated process at MuIC

$$P(\mu^+) \mu^- \rightarrow Z \rightarrow jj$$



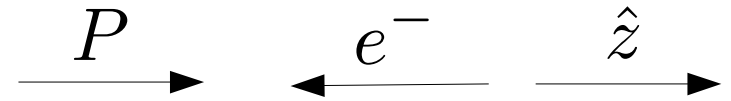
- $E_\mu \sim 1 \text{ TeV}$ $E_p \sim 275 \text{ GeV}$

- $\rightarrow E_{p/\mu^+} \sim 2 \text{ GeV}$

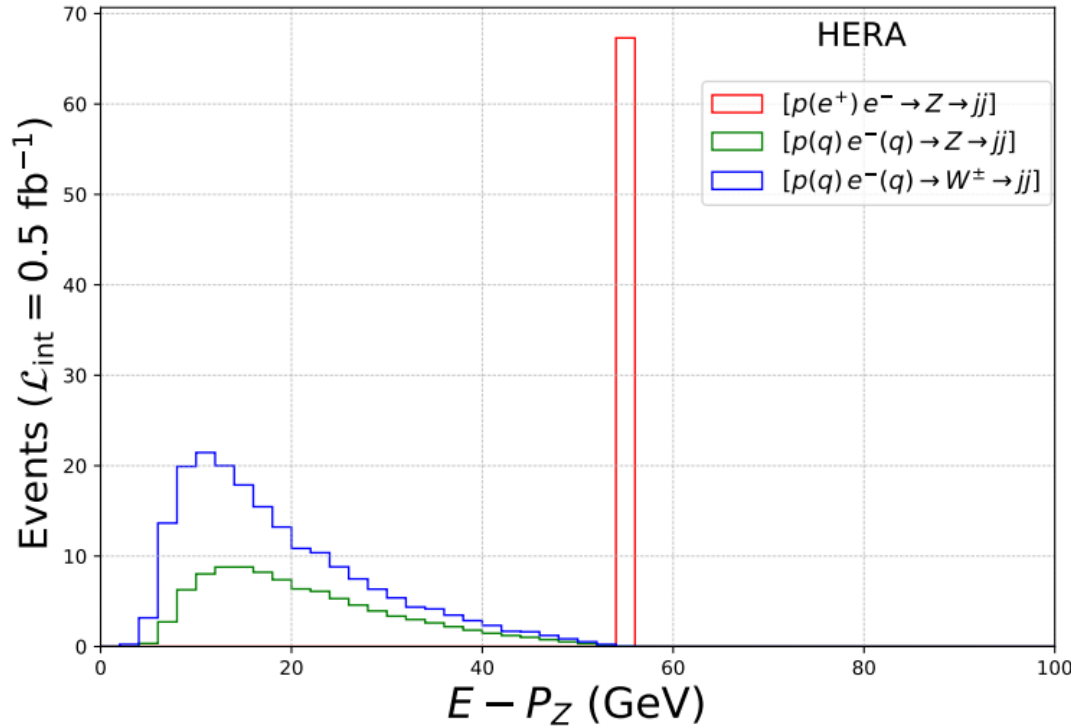
Dominant contribution to Z-production through lepton-initiated process

- Decaying objects carry longitudinal boosts along the muon-beam direction ($-\hat{z}$ -direction)

$$E - P_z \equiv \sum_i (E^i - P_z^i)$$



sum runs over all detected particles



$E \rightarrow$ Energy

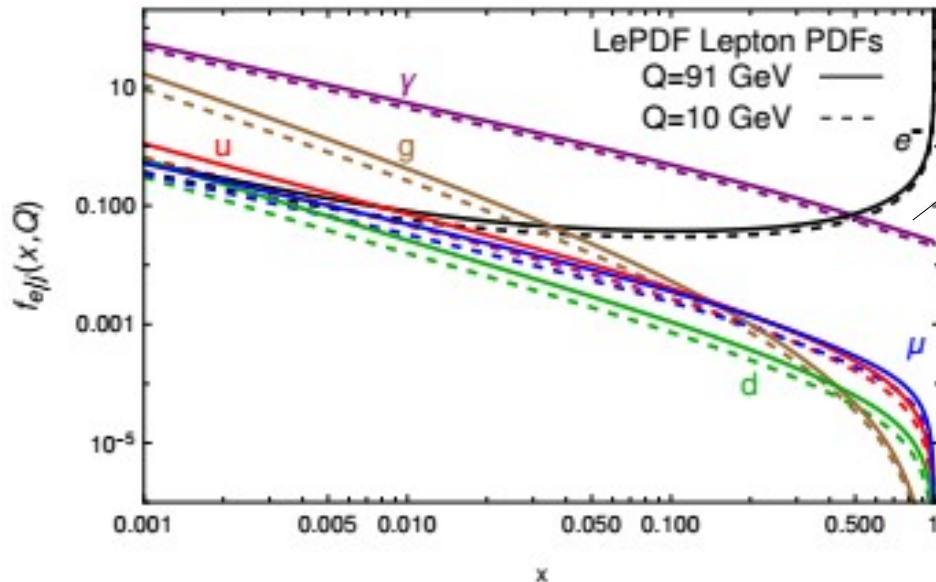
$P_z \rightarrow$ momentum along the z-direction
(proton beam direction)

- According to longitudinal momentum conservation this quantity should be equal to $2x_{e/j}E_e$
 $j \rightarrow$ colliding parton within the electron

- cut of $E - P_z$ close to $2E_e$ leads to enhancing the purely electron initiated process due to the almost Dirac delta-like behaviour of the electron PDFs of electron near $x_{e/e} \sim 1$
- kinematical distributions of the quark-initiated and photon-initiated processes predominantly peak at relatively small $x_{e-/q}$ and $x_{e-/\gamma} \rightarrow$ significantly smaller $E - P_z$
- cut of $E - P_z$ close to $2E_e$ can separate these two contributions

Other relevant SM processes:

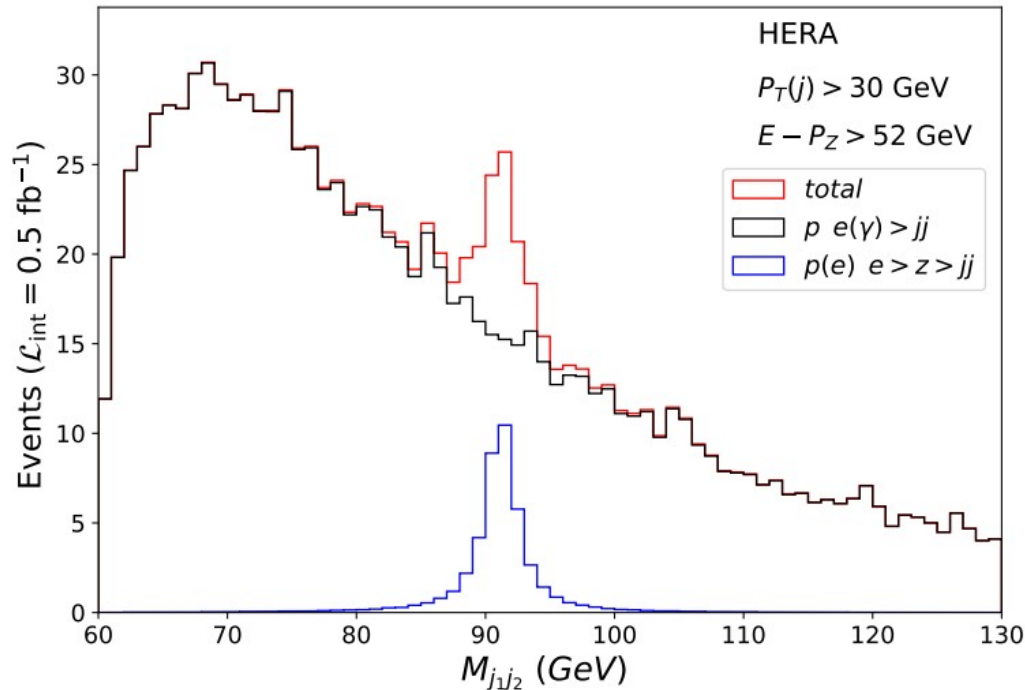
- Di-jet production processes involving photon partons from the lepton beam and gluon/quark partons from the proton
- Selection cuts of $E - P_z > 52 \text{ GeV}$ —————> Only processes with incoming photons or quarks from the electron carrying more than 26 GeV. i.e., $x_{e/\gamma}, x_{e/q} > 0.945$ contribute.



- Photon PDFs of the electron is not too small, even at relatively large x-values.
- Photon from electron involving processes dominate.

- A di-jet background with no lepton can arise from processes such as $pe^- \rightarrow je^-, jj e^-$ and $je^- \gamma$ where the final state electron is misidentified as a jet.
 - > Negligible number of events in the signal region considering 10^{-3} fake rate
- Other production processes are also suppressed.

$M_{j_1 j_2}$ distribution



- Perform a cut-based analysis
selection cuts:
 - $E - P_Z > 52 \text{ GeV}$
 - $P_T(j) > 30 \text{ GeV}$
 - $|\eta(j)| < 2$
- 5 GeV selection window of $M_{j_1 j_2}$ and $E_{j_1 j_2}$ around 91 GeV and 103 GeV, respectively

Applying the selection cuts

- Obtain 73 signal events and 267 background events considering the combined integrated luminosity of 0.98 fb^{-1} of H1 and ZEUS detectors
- Signal significance of around 4.5σ at HERA
- Since our calculations are at LO in QCD and QED, and no showering, hadronization, or detector effects are included, it's better to extract the di-jet background from data by extrapolating from sidebands ---- similar to the approach used for the di-photon background in the Higgs discovery at the LHC.
- Similar analysis can be done in EIC. We obtain 4.3σ signal significance there for the same process.
- Due to the larger luminosity at the EIC, leptonic decay modes (particularly involving muons) of the Z boson can be studied.

NLO corrections

There are virtual corrections at one-loop level and real corrections from radiation to the processes that we discuss.

For resonant Z production,

- NLO QED corrections can be obtained via the photon initiated processes:

$$\gamma e^- \rightarrow Z e^- \quad \hat{\sigma}_{\gamma e^-} \sim \alpha^2/m_Z^2 \sim \alpha_s^4/m_Z^2 \quad \text{After convolution with the PDFs} \quad \sigma \sim \alpha_s^5 C_\ell/m_Z^2$$

- Corrections from processes with virtual particles in a loop, as well as those with a final photon

$$e^+ e^- \rightarrow Z \gamma \quad \sigma \sim \alpha_s^6 C_\ell/m_Z^2$$

- NLO QCD corrections in $Z + j$ production

$$gq \rightarrow Zq \text{ and } q\bar{q} \rightarrow Zg \quad (\text{initial quark or gluon coming from the colliding lepton}) \quad \sigma \sim \alpha_s^5/m_Z^2$$

- The lepton-initiated resonant process has $x_{e^-/e^-} \simeq 1$, while the NLO QCD correction processes peak at low $x_{e^-/q,g}$

- A cut on $E - P_Z$ can reject these corrections.

- Finally, processes with a final photon:

$$q\bar{q} \rightarrow Z\gamma \text{ are suppressed by } \alpha_s^2 \text{ compared with the resonant production.}$$

Another possibility:

- Study of non-resonant di-leptonic final state
- Signal is dominated by photon mediated t/u and s -channel processes involving the lepton PDFs of the proton
- Concentrate on the ee , $e\mu$ and $\mu\mu$ final states. (interesting: same sign different flavor)
- $E - P_Z$ of processes involving electron PDFs of electron beam peaks at $2E_e \sim 55$ GeV at HERA

$$\rightarrow e^- e^+, e^- e^- e^- \mu^\pm$$

exclusively involves muon PDFs of proton

involves positron, electron PDFs of proton

- $e^- e^+$, $\mu^+ \mu^-$ final states can produce via the so-called photoproduction processes involving photon PDFs of proton and electron.

$E - P_Z$ would peak relatively smaller value

- One can compare with this with the existing data of HERA

Comparison with data

<https://arxiv.org/pdf/0907.3627>

Multi-Leptons at HERA (0.94 fb^{-1})

Sample	Data	SM	Pair Production (GRAPE)	NC DIS + QEDC
ee	873	895 ± 57	724 ± 41	171 ± 28
$\mu\mu$	298	320 ± 36	320 ± 36	< 0.5
$e\mu$	173	167 ± 10	152 ± 9	15 ± 3
eee	116	119 ± 7	117 ± 6	< 4
$e\mu\mu$	140	147 ± 15	147 ± 15	< 0.5
$(\gamma\gamma)_e$	284	293 ± 18	289 ± 18	4 ± 1
$(\gamma\gamma)_\mu$	235	247 ± 26	247 ± 26	< 0.5

Multi-Leptons HERA analysis

Selection cuts:

- at least two central leptons (electrons or muons)
($20^\circ < \theta < 150^\circ$)
- Leading lepton $P_T^\ell > 10 \text{ GeV}$, other leptons $P_T^\ell > 5 \text{ GeV}$

- Isolated leptons $\Delta R > 0.5$

- $E - P_Z < 45 \text{ GeV}$ cut to isolate the di-leptonic ($e^+e^-, \mu^+\mu^-$) events from photo-production regime.

- The detector efficiencies for e and μ in the central region:

e – 80% (HI) and 90% (ZEUS), μ – 90% (HI) and 55% (ZEUS).

- $0.005 \lesssim x_{p/e,\mu} \lesssim 0.03$ region contributes under these cuts

- Check cross-section uncertainties by varying the renormalization scale by a factor of 2 up and down.

Obtain large uncertainty which reflects that our results are only accurate at LO.

Our results:

final state	ℓ -init	γ -init	γ -init $_{E-P_z > 45 \text{ GeV}}$
e^+e^-	216^{+34}_{-41}	560^{+54}_{-55}	70^{+7}_{-8}
e^-e^-	346^{+65}_{-64}	—	—
$\mu^+\mu^-$	12^{+2}_{-2}	430^{+41}_{-43}	54^{+5}_{-6}
$e^-\mu^-$	174^{+39}_{-32}	—	—

Comparison with data

- We estimate more di-leptonic events than observed at HERA.
- Agreement in a few percent is found for ee and $\mu\mu$ events for $E - P_z > 45$ GeV region where lepton-initiated processes mostly dominates
- Our results show 632_{-113}^{+105} (ee) and 66_{-8}^{+7} ($\mu\mu$) events vs. 418 (ee) and 63 ($\mu\mu$) reported.
- Agreement is worse in the $e\mu$ channel, with 348_{-64}^{+79} events estimated vs. 173 observed.
- Overestimates are seen in photon-initiated di-lepton pair-production
(The rate of this processes involves the photon PDFs of the electron and the proton.)
with $E - P_z < 45$ GeV: 490_{-47}^{+47} (ee) and 376_{-36}^{+36} ($\mu\mu$) events estimated vs. 284 (ee) and 235 ($\mu\mu$) observed.

These discrepancies could originate from various factors:

- higher order corrections
- showering, hadronization \longrightarrow estimated numbers may drop to lower values
- proper detector simulation
- Alternatively, this may suggest an overestimation of lepton and photon PDFs in the proton.

Further theoretical and experimental analysis is needed to understand these discrepancies

EIC

- Like at HERA, di-leptonic final states from non-resonant processes can be studied at the EIC to test these PDFs.

- Lepton-initiated processes:

$$E - P_Z \sim 2E_e \sim 40 \text{ GeV}$$

- Expected number of di-lepton events at EIC at the LO, considering the selection cuts of $P_T^\ell > 10 \text{ GeV}$ and $|\eta_\ell| < 2.0$ and setting the renormalization scale as P_T of the lepton.

$$e^+e^- \rightarrow 15067$$

$$e^-e^- \rightarrow 23287$$

$$\mu^+\mu^- \rightarrow 730$$

- Additionally, same-flavor, opposite-charge di-lepton final states can arise from photo-production predominantly in the regions of $0.1 \lesssim x_{e^-/\gamma} \lesssim 1.0$ and $0.04 \lesssim x_{p/\gamma} \lesssim 0.2$, under the same phase space cuts

- 23900 e^+e^- total events at the LO under the same cuts, out of which 2780 events occur in the $E - P_Z > 35 \text{ GeV}$ region
similar numbers for $\mu^+\mu^-$

- Once future data is available, a detailed study of these processes can be performed at the EIC to test the lepton PDFs of the proton

MuIC

- Proton ~ 275 GeV and Muons ~ 1 TeV Integrated luminosity: 400 fb^{-1}

- selection criteria: $P_T(\ell) > 5 \text{ GeV}, |\eta_\ell| < 2.5$

- $P\mu^- \rightarrow \mu^+\mu^- \sim 150$ events
 $\rightarrow \mu^-\mu^- \sim 600$ events
 $\rightarrow \mu^-e^\pm \sim 720$ events
 $\rightarrow e^+e^- \sim 30$ events
- } All these events, $E - P_z$ peaks at 2 TeV

- Furthermore, opposite-sign same-flavor di-lepton final states can be produced through photoproduction, involving the photon PDFs of the proton and muon, with $E - P_z$ peaking at relatively low values.

Consider rare SM signatures at hadron colliders like LHC

Non-resonant production of back-to-back same sign and/or different flavour di-leptons

arXiv:2005.06477

Buonocore, et al.

■ At the LO

$$p_{T,\ell} > 20 \text{ GeV}, \quad |\eta| < 2.4, \quad \mu_F = p_{T,\ell}$$

	$e^+ \mu^-$	$e^+ \tau^-$	$\mu^+ \tau^-$	$e^+ e^+$	$\mu^+ \mu^+$	$\tau^+ \tau^+$
$\sigma_{13\text{TeV}}$ [fb]	$0.29^{+0.13}_{-0.10}$	$0.18^{+0.11}_{-0.08}$	$0.16^{+0.10}_{-0.07}$	$0.24^{+0.10}_{-0.08}$	$0.19^{+0.09}_{-0.07}$	$0.08^{+0.06}_{-0.04}$
$\sigma_{27\text{TeV}}$ [fb]	$0.53^{+0.25}_{-0.18}$	$0.34^{+0.21}_{-0.15}$	$0.30^{+0.19}_{-0.14}$	$0.440^{+0.19}_{-0.14}$	$0.34^{+0.16}_{-0.12}$	$0.14^{+0.12}_{-0.07}$

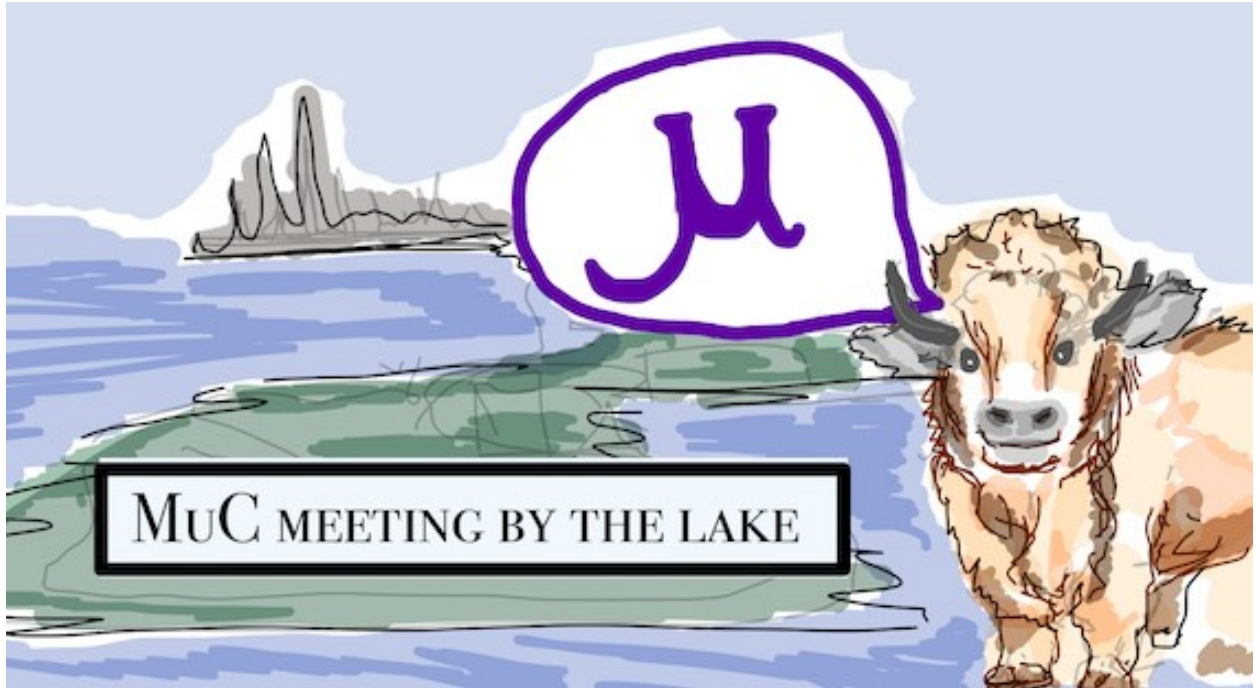
- Number of events of such processes are estimated using the lepton PDFs of the proton

Measurable number of events are expected at HL-LHC

- For example, at 3ab^{-1} of data, $850 e^+ \mu^-$, $550 e^+ \tau^-$ and $500 \mu^+ \tau^-$ events are expected at HL-LHC, considering these phase space cuts
- Analysis on this at HL-LHC can also verify the distributions of these leptonic PDFs of proton
- This could be even crucial for studying BSM scenarios, like flavor-violating scalars, as precise knowledge of the SM background is required.

Conclusions

- PDFs are key to hadron collider phenomenology, describing the momentum distribution of proton constituents.
- The lepton PDFs of the proton have been recently computed (LuxLep).
- This is crucial for LHC searches in BSM scenarios, including hadrophobic and leptophilic Z' , doubly charged scalars, leptoquarks, flavor-violating leptophilic scalars, etc.
- We propose to validate the lepton content of the proton by analyzing both non-resonant di-lepton production and di-jets from resonantly produced Z boson, Using existing HERA data and future measurements from other lepton-proton colliders
 - For Z-resonant production, we propose conducting this study at pe colliders rather than at a hadron collider like the LHC.
 - Focus to di-jet decay modes of Z, due to large branching fractions
 - Feasible to study the leptonic decay modes of the Z boson, particularly in the di-muon final state, at the EIC
 - For non-resonant di-lepton final states, consider all flavor combinations and charge pairings.
 - These can explore a smaller $x_{p/e,\mu}$ region of the proton lepton PDFs compared to resonant Z-boson production.
 - Comparison with HERA data indicates a slight overestimation of di-lepton production rates by a few tens of percent.
 - A dedicated theoretical and experimental study could clarify these discrepancies.



Extra

NLO corrections in non-resonant dilepton processes

- For the non-resonant di-lepton production there are real corrections from photon emission of initial and final particles, with the same initial state as the leading processes, that are suppressed by α_s^2 compared with them.
- Di-lepton plus jet final states can be obtained from $\gamma q \rightarrow \ell\ell q$, being suppressed by α_s only, compared with the leading processes.
- When considering the quark and gluon content of the electron one can also obtain di-lepton plus jet final states which are suppressed by α_s
- The kinematic distributions of processes initiated by partons of the electron, other than the electron itself, are peaked towards low values of $\vec{E} - P_z$, thus the rates of these processes can be controlled by cuts on this variable.
- Virtual QED corrections are expected to be suppressed by α_s^2 compared with the real ones.
- A dedicated analysis containing these corrections, as well as parton showering, is needed for a precise calculation.