

# Higgs Production at $\mu^+ \mu^+$ Colliders

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Based on collaboration with  
Yu Hamada, Ryuichiro Kitano, Ryutaro Matsudo, Ryoto Takai, Lukas Treuer, Hiromasa Takaura  
arXiv:2408.01068

*MuC Meeting by the Lake @ Northwestern University, 18 September 2024*

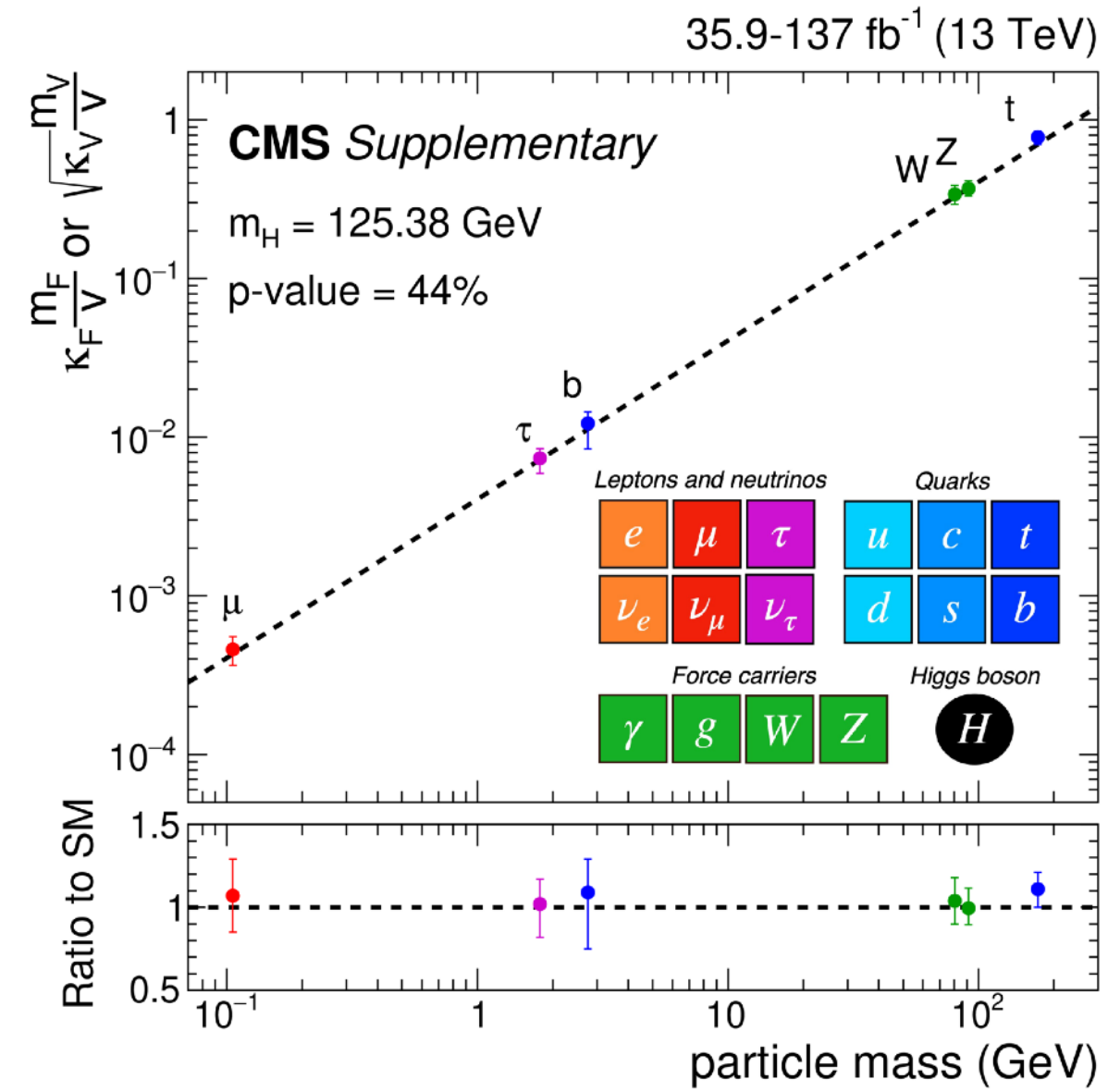
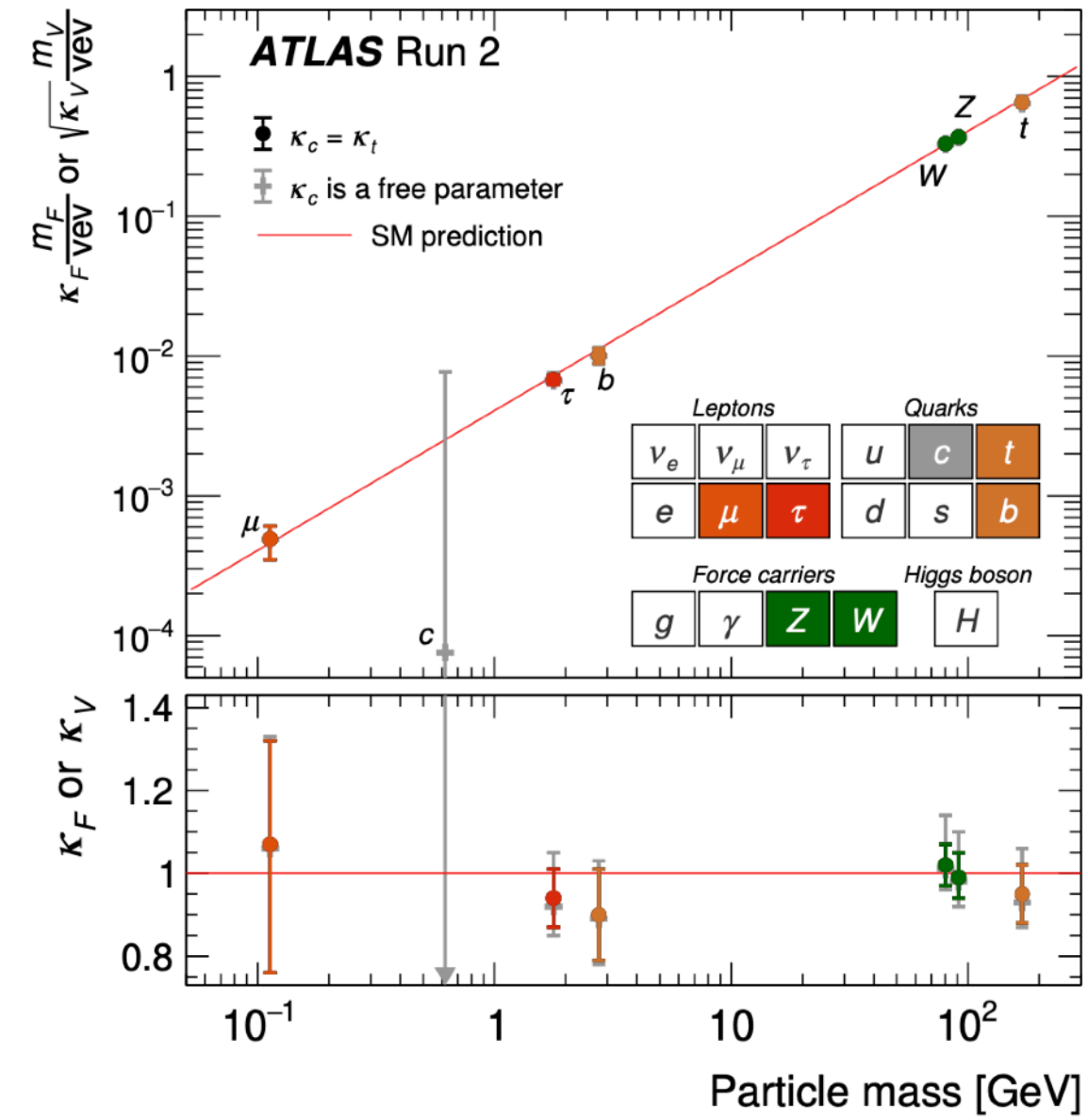
Need next generation colliders

## Need next generation colliders

- To dig deep into the nature of **the Higgs boson**

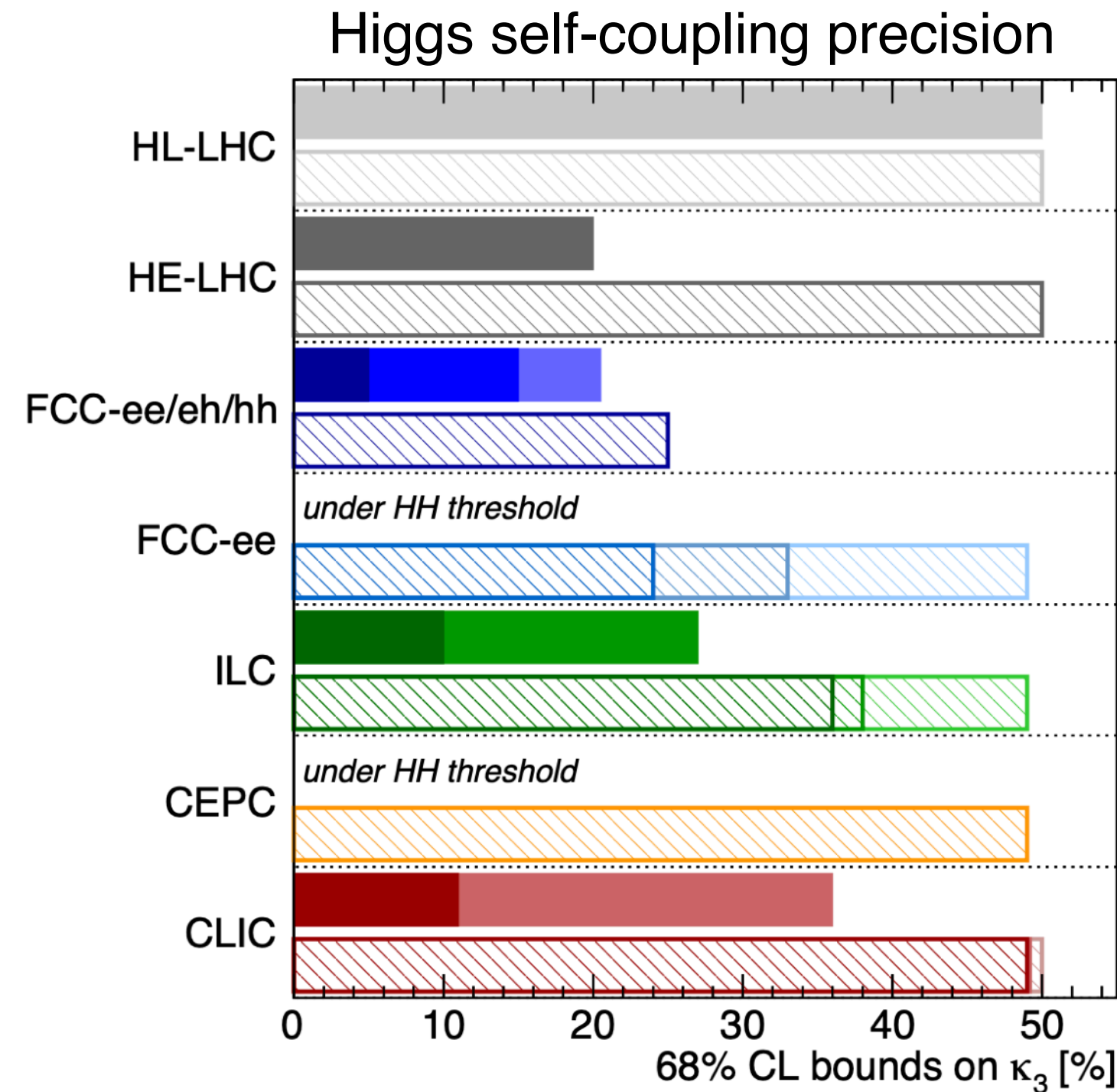
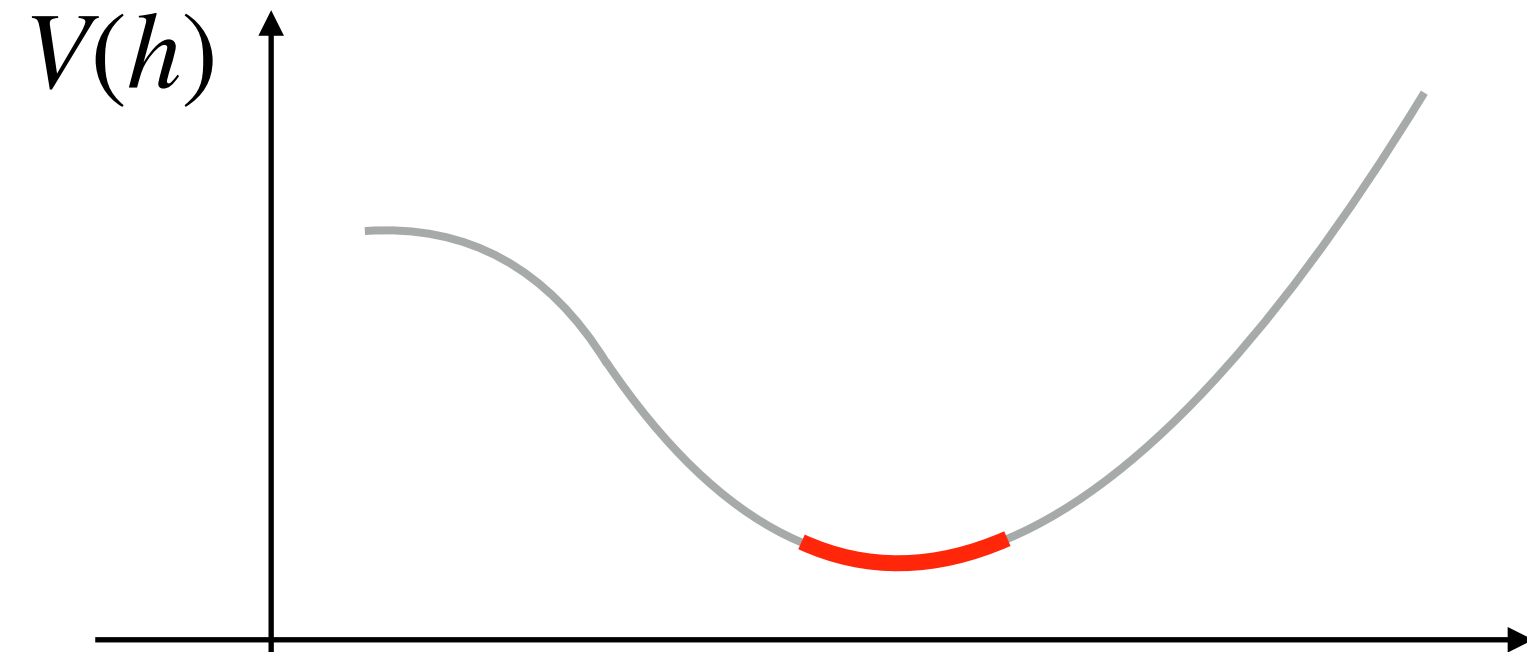
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Higgs@FC WG September 2019

di-Higgs	single-Higgs
HL-LHC 50%	HL-LHC 50% (47%)
HE-LHC [10-20]%	HE-LHC 50% (40%)
FCC-ee/eh/hh 5%	FCC-ee/eh/hh 25% (18%)
LE-FCC 15%	LE-FCC n.a.
FCC-eh <sub>3500</sub> -17+24%	FCC-eh <sub>3500</sub> n.a.
	FCC-ee <sup>4IP</sup> <sub>365</sub> 24% (14%)
	FCC-ee <sub>365</sub> 33% (19%)
	FCC-ee <sub>240</sub> 49% (19%)
ILC <sub>1000</sub> 10%	ILC <sub>1000</sub> 36% (25%)
ILC <sub>500</sub> 27%	ILC <sub>500</sub> 38% (27%)
	ILC <sub>250</sub> 49% (29%)
	CEPC 49% (17%)
CLIC <sub>3000</sub> -7%+11%	CLIC <sub>3000</sub> 49% (35%)
CLIC <sub>1500</sub> 36%	CLIC <sub>1500</sub> 49% (41%)
	CLIC <sub>380</sub> 50% (46%)

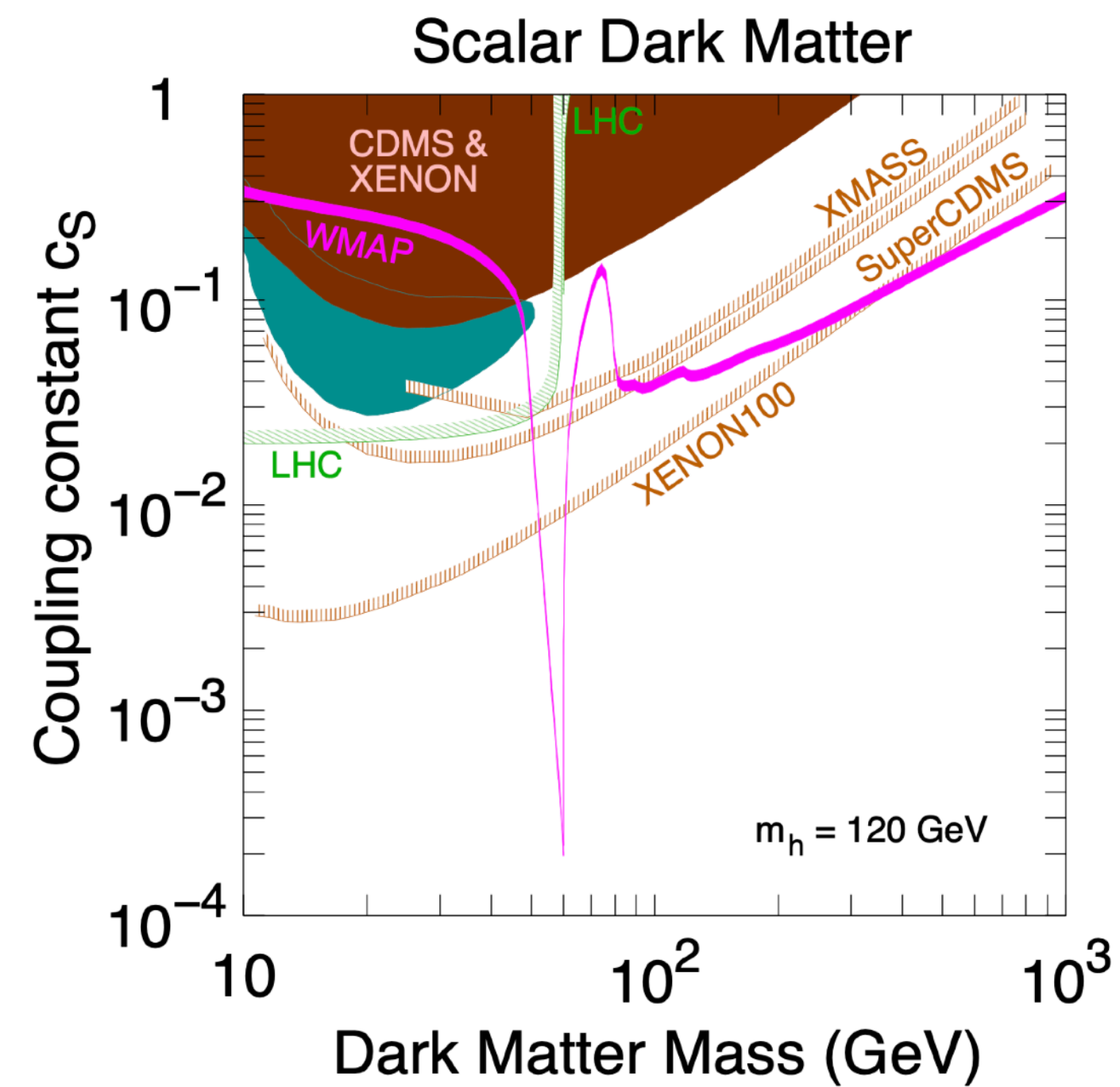
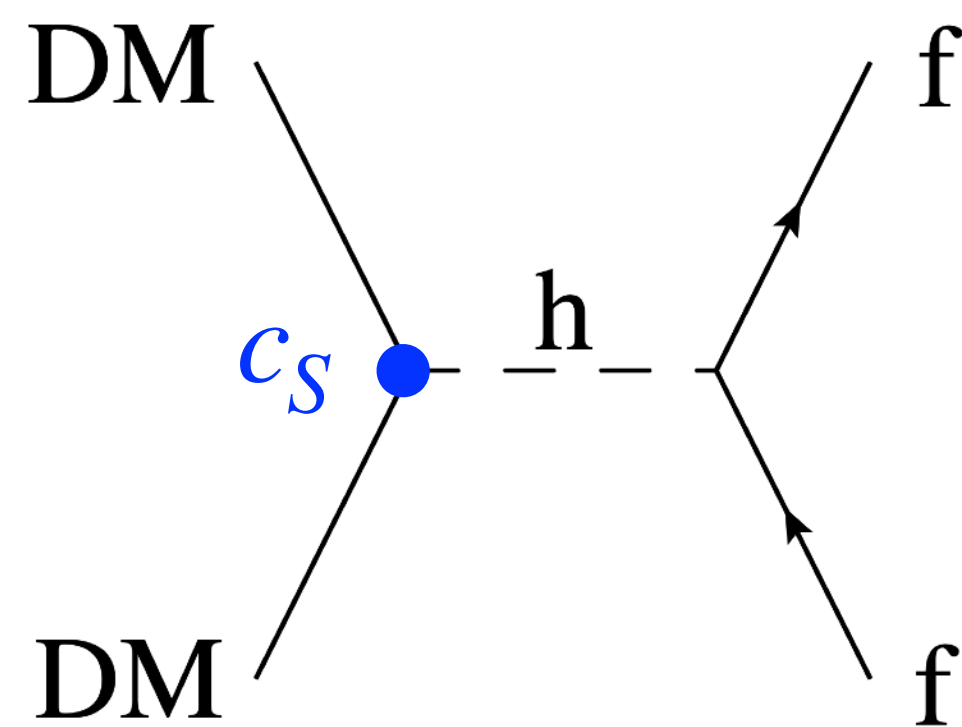
All future colliders combined with HL-LHC

## Need next generation colliders

- To dig deep into the nature of **the Higgs boson**
- To look for **dark matter** that weakly couples to the Standard Model

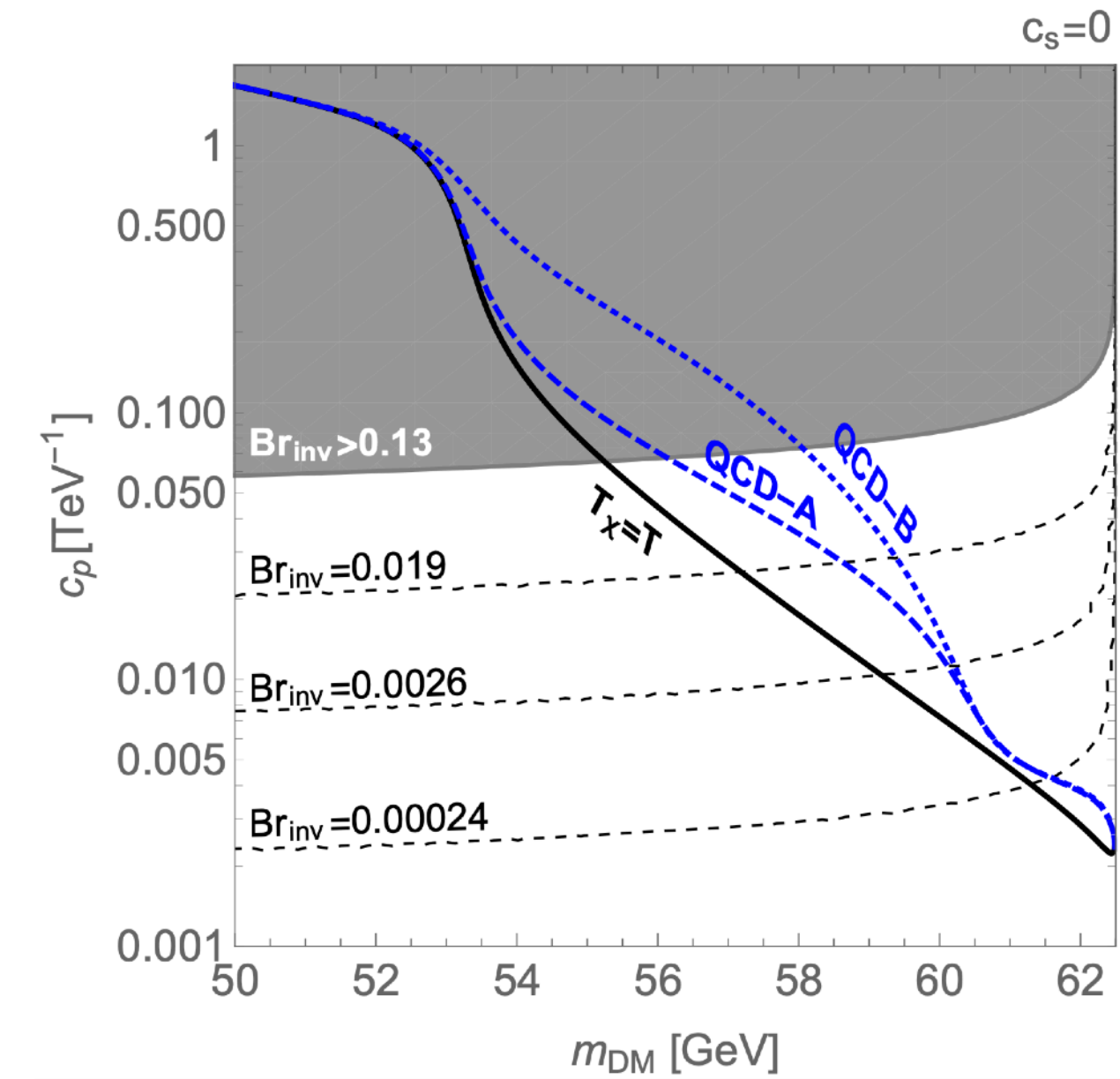
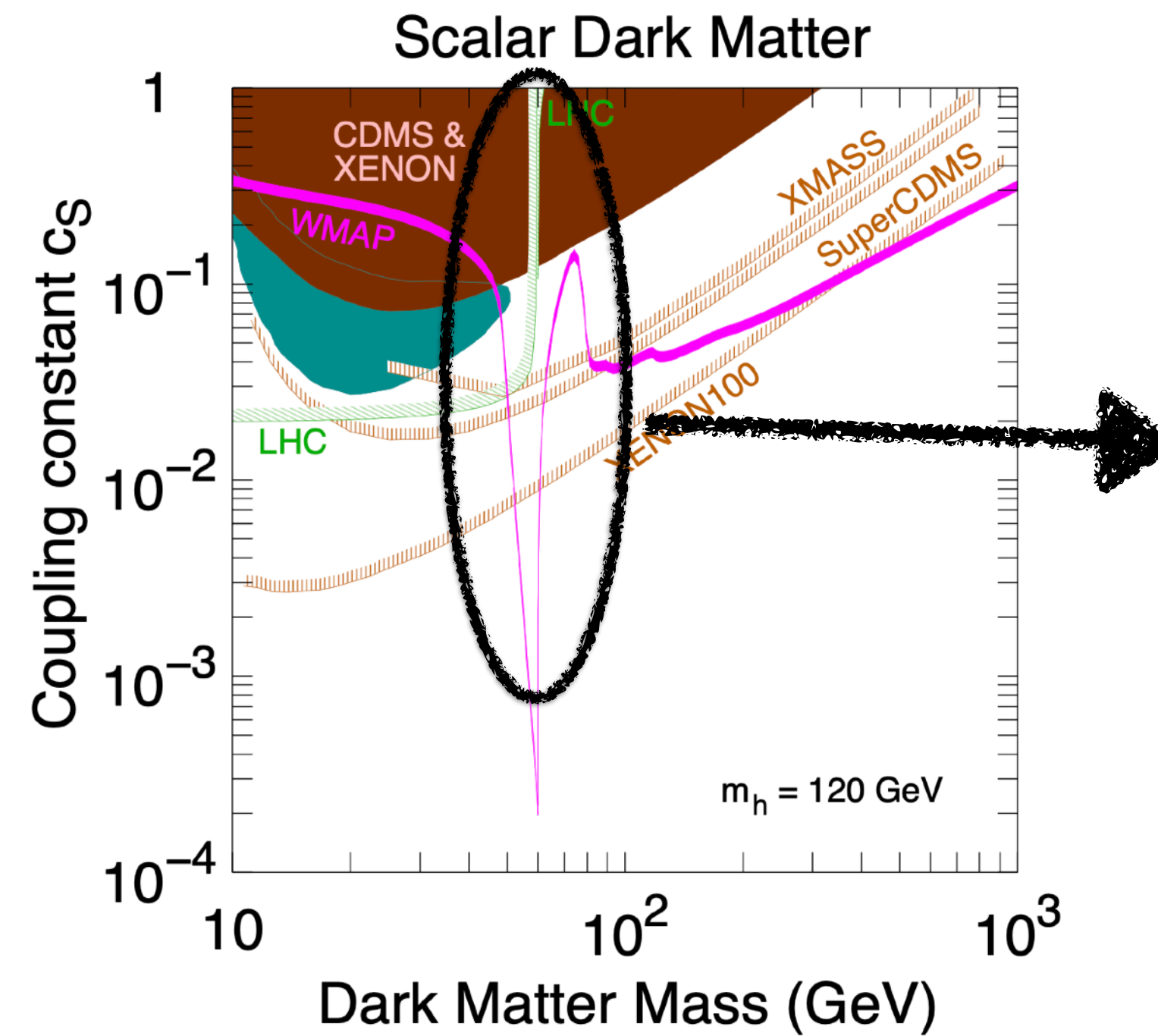
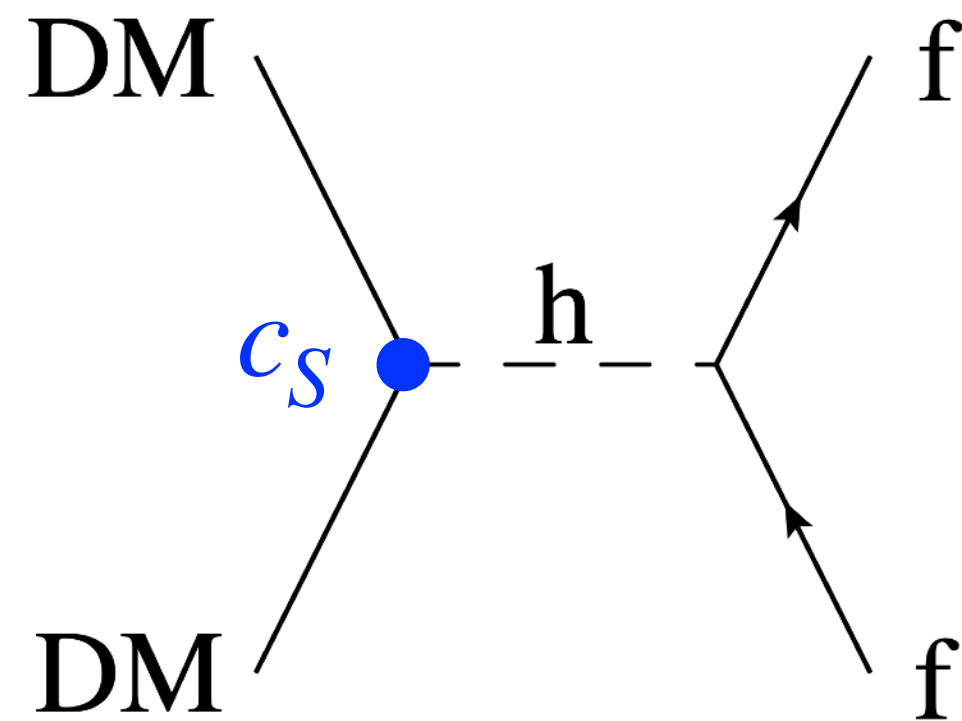
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[Kanemura, Matsumoto, Nabeshima, Okada, '10]

[T. Abe '22]



# Muon Colliders as a future machine

Muon Colliders are a good discovery and precision machine

- as compact as 10km in size to achieve 10TeV parton collisions
- as clean as  $e^+e^-$  colliders

# Muon Colliders as a future machine

MuC is a good discovery and precision machine

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- as clean as e+e- colliders

MuC is a Higgs factory

#(Higgs bosons):  $5 \times 10^5$  at 3TeV ( $\text{ab}^{-1}$ ) and  $10^7$  at 10TeV ( $10\text{ab}^{-1}$ )

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MuC is a Higgs factory

#(Higgs bosons):  $5 \times 10^5$  at 3TeV ( $\text{ab}^{-1}$ ) and  $10^7$  at 10TeV ( $10\text{ab}^{-1}$ )

However, many challenges to make it the real:  $\mu$  cooling (luminosity), BIB, magnet, ...

# μTRISTAN

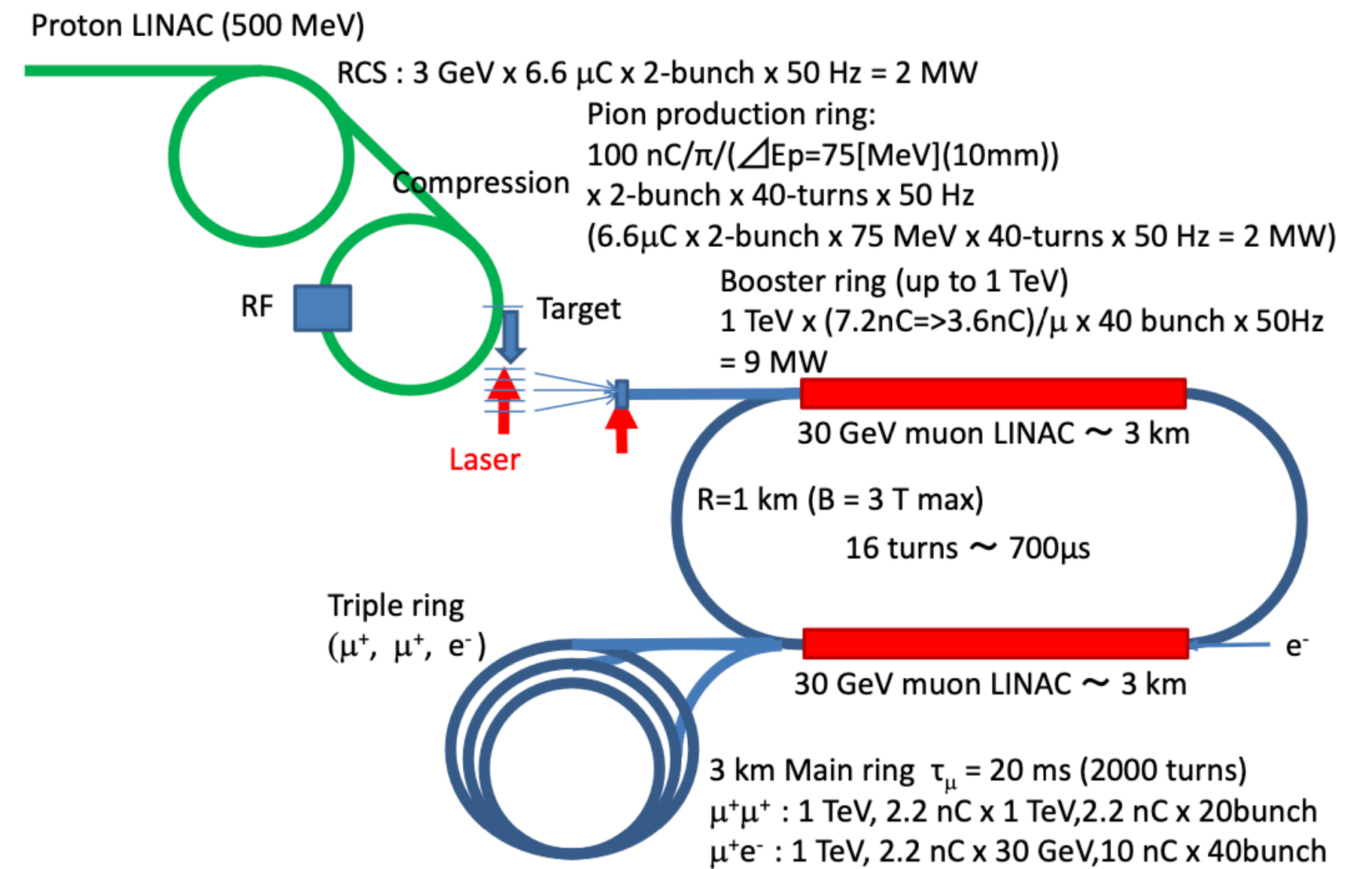
## Prototype Design:

- builds on an existing μ<sup>+</sup> cooling technology at J-PARC
- 3 km main ring
- two options
  - μ<sup>+</sup> e<sup>-</sup> : √s = 346 GeV (1TeV μ<sup>+</sup> / 30GeV e<sup>-</sup>)
  - μ<sup>+</sup> μ<sup>+</sup> : √s = 2 TeV (1TeV μ<sup>+</sup> each)

\*polarized beams also possible (non-trivial though)

- luminosity ~ 10<sup>33</sup> cm<sup>-2</sup> s<sup>-1</sup> (ab<sup>-1</sup> level for 10yrs running)

Hamada, Kitano, Matsudo, Takaura, Yoshida ('22)



Physics Case: SUSY, Precision Tests, (E)WIMPs, LFVs, Neutrinos and more

next talk by Joe Sato

see also Hamada, Kitano, Matsudo, Takaura ('22); Fridell, Kitano, Takai ('23); Fukuda, Moroi, Niki, Wei ('23); Okabe, Shirai ('23); etc.

# Muon Collider Comparison

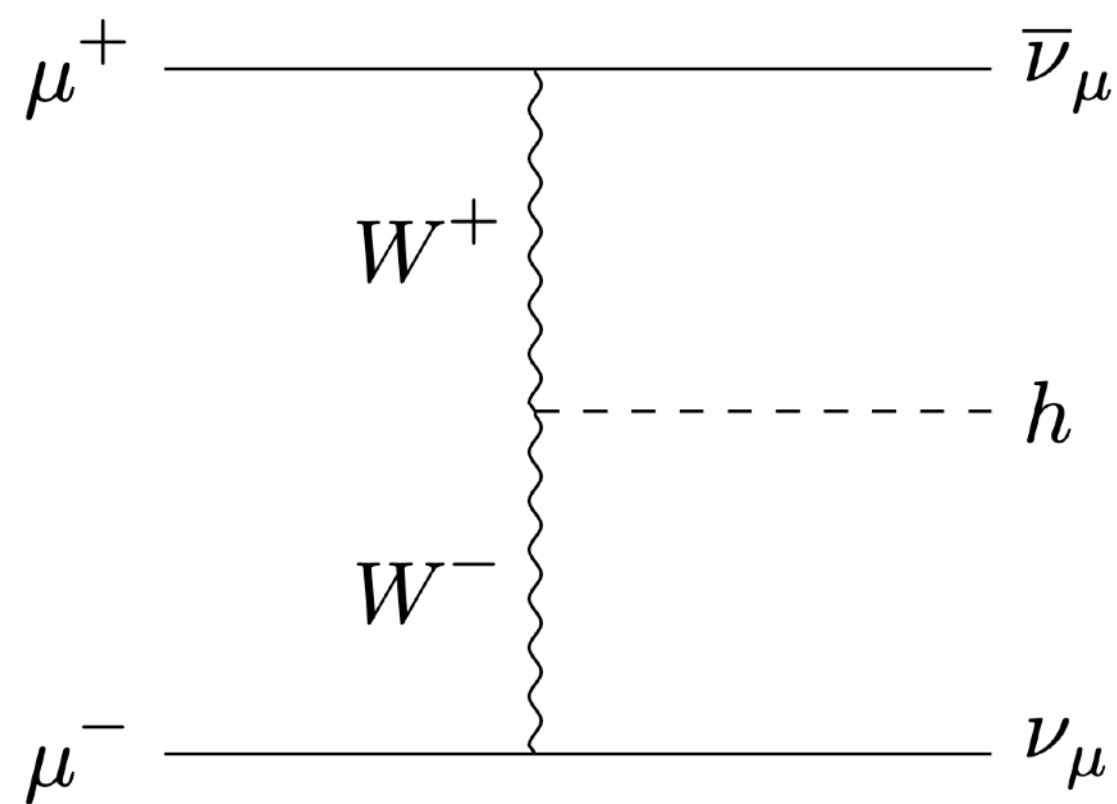
Today's focus

	$\mu^+ \mu^-$	$\mu^+ e^-$	$\mu^+ \mu^+$
Electrically Neutral	✓	✓	✗
Vector Boson Fusion (VBF) w/o s-Channel Background (e.g., $W^+ W^-$ , $q\bar{q}$ )	✗	✓	✓
Special Advantage	Resonant s-Channel Production	Flavor Physics	$\mu^+$ Cooling Technology Already Exists!
Disadvantage	$\mu^-$ Cooling Technology Needed	Lower Energy	Suppression From Extra Vertex...?

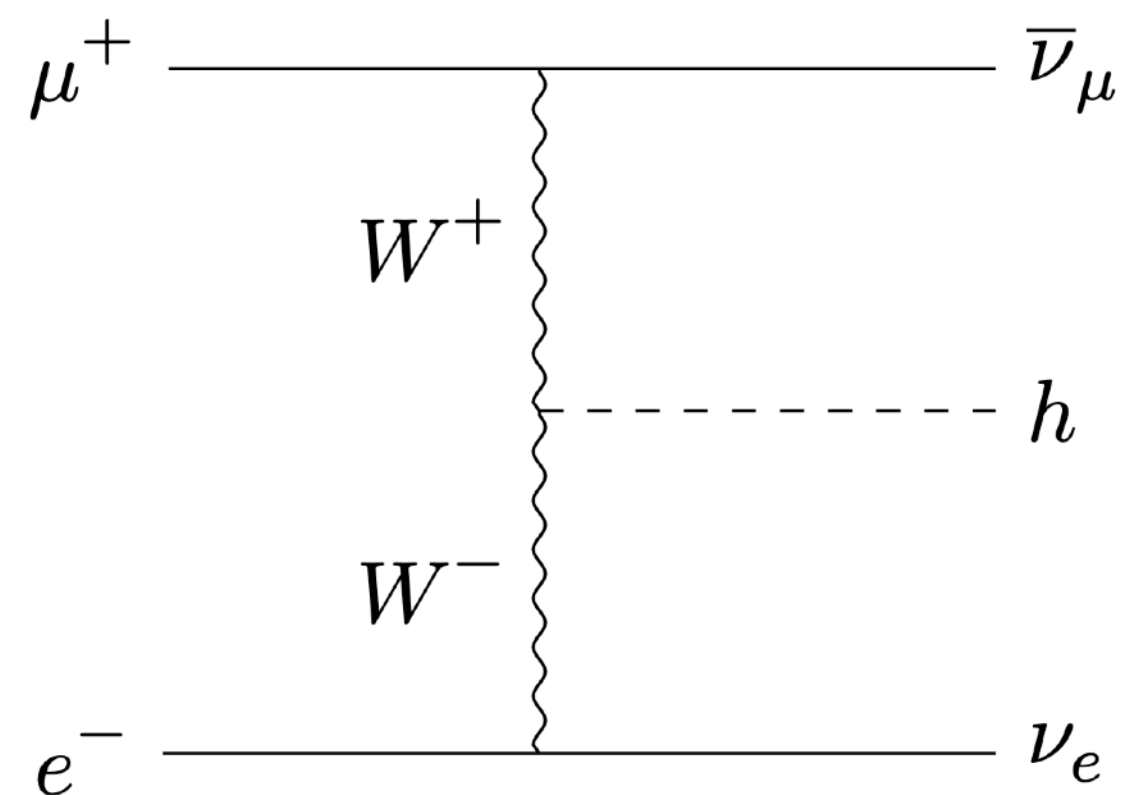
Table from a talk at LCWS2024  
by Lukas Treuer

# Higgs production at muon colliders

W-fusion processes are dominant at high energies at  $\mu^+\mu^-$  and  $\mu^+e^-$  colliders



$\sim 400\text{fb}$  (1TeV x 1TeV)

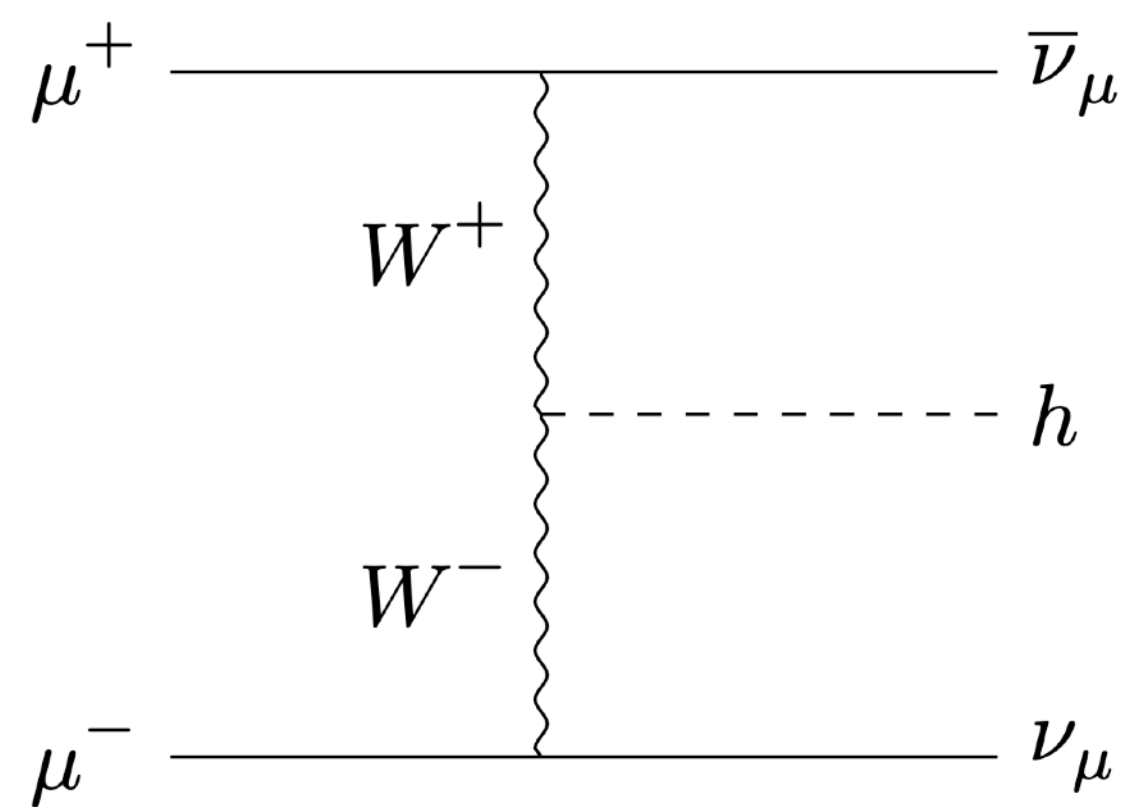


$\sim 90\text{fb}$  (30GeV x 1TeV)

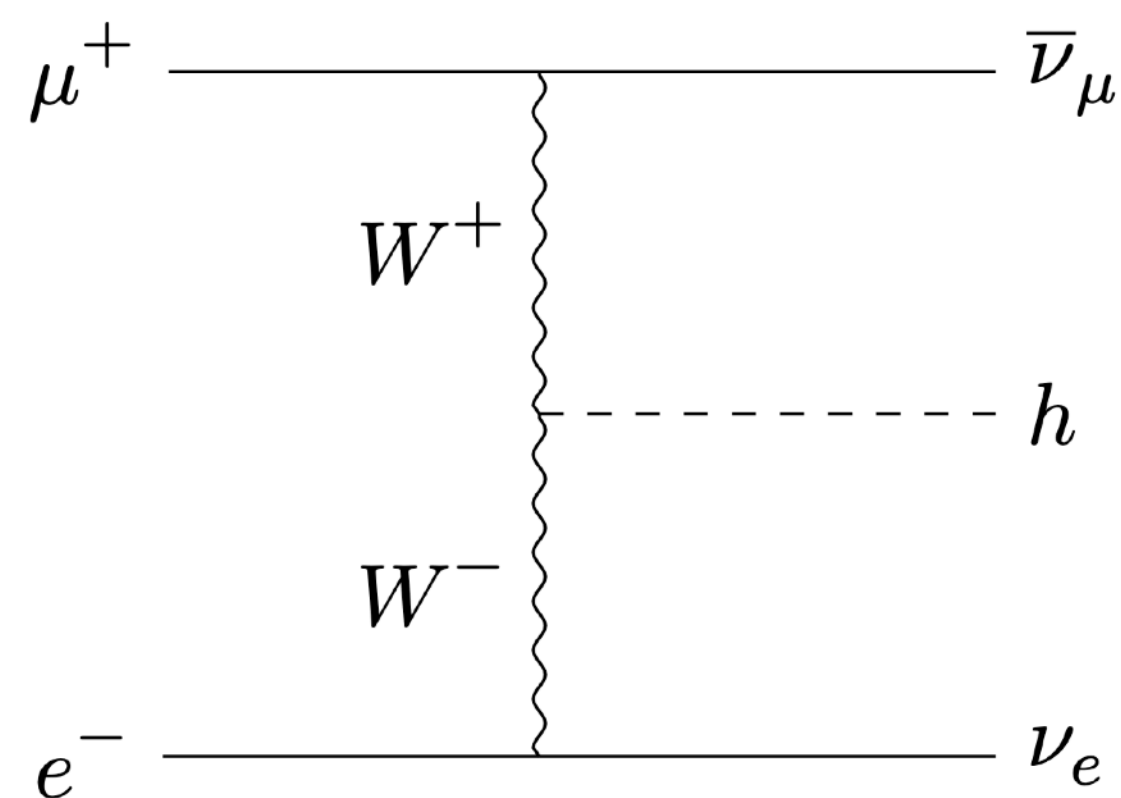
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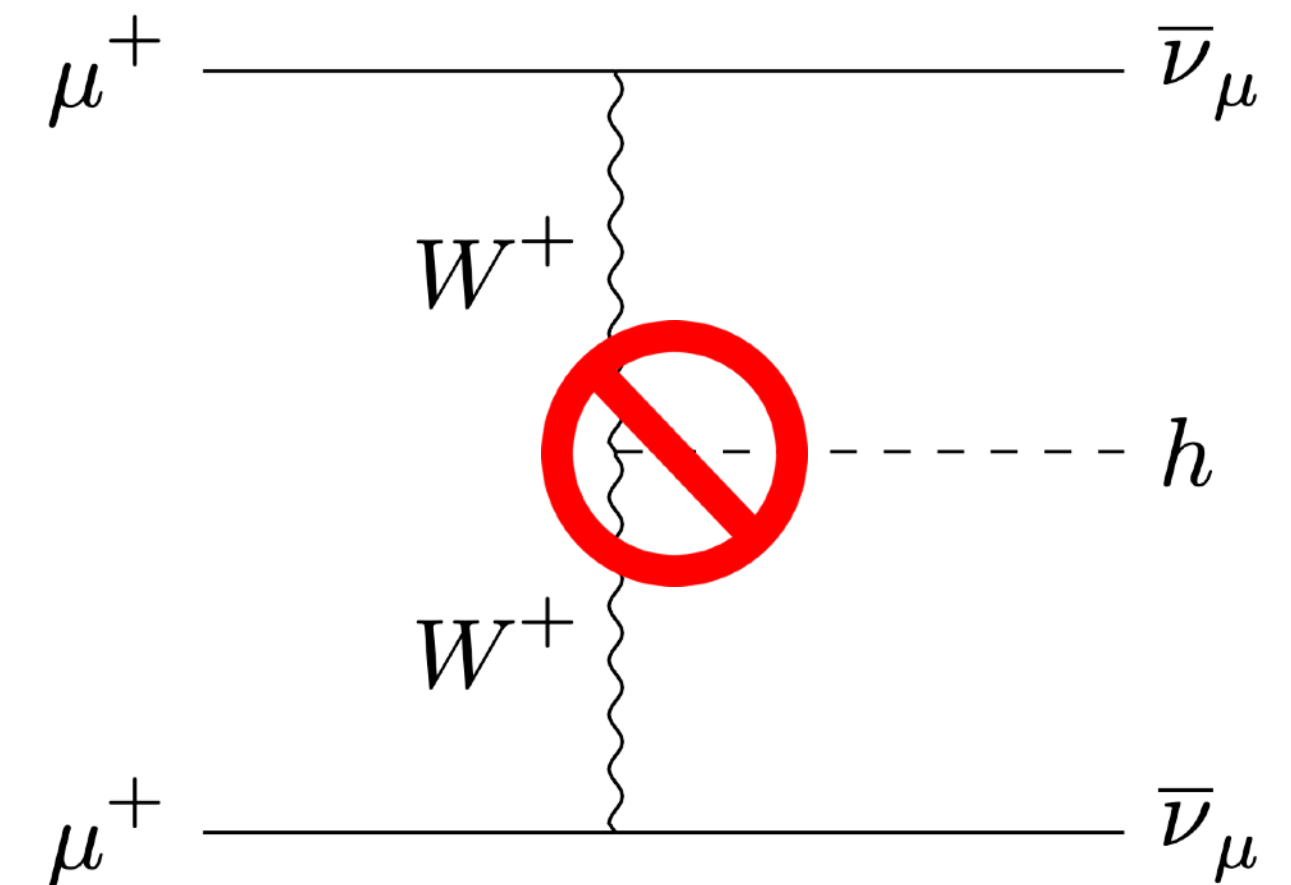
No leading W-fusion at  $\mu^+\mu^+$  colliders



$\sim 400\text{fb}$  (1TeV x 1TeV)

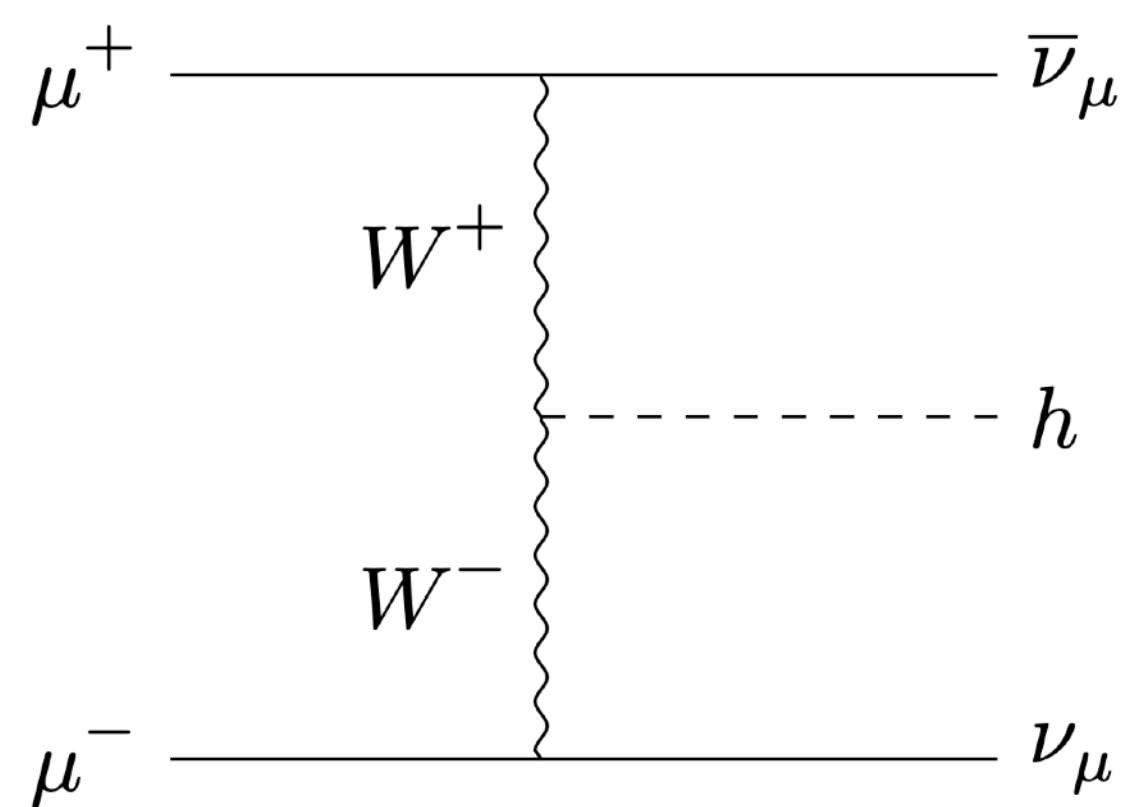


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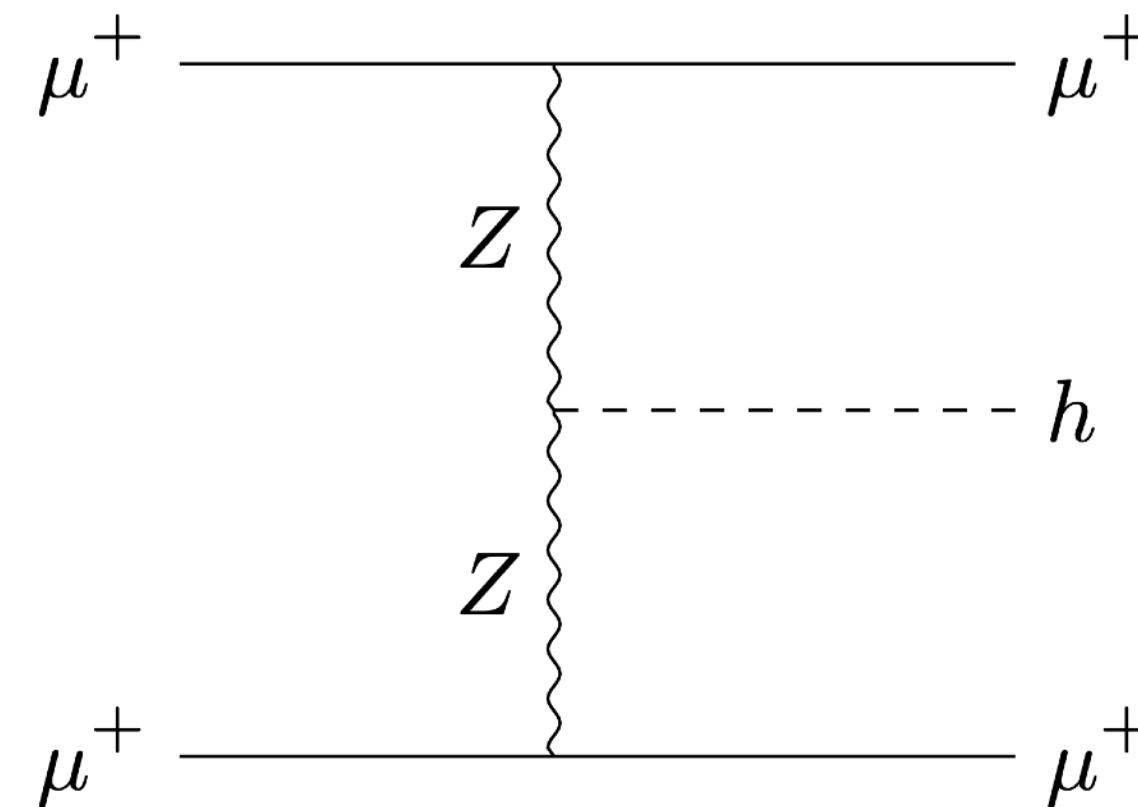
# How to produce Higgs boson at $\mu^+ \mu^+$ colliders?

- Z-fusion process?  $\rightarrow$  suppressed due to small Z coupling to muons



$\sim 400\text{fb}$  (1TeV x 1TeV)

vs



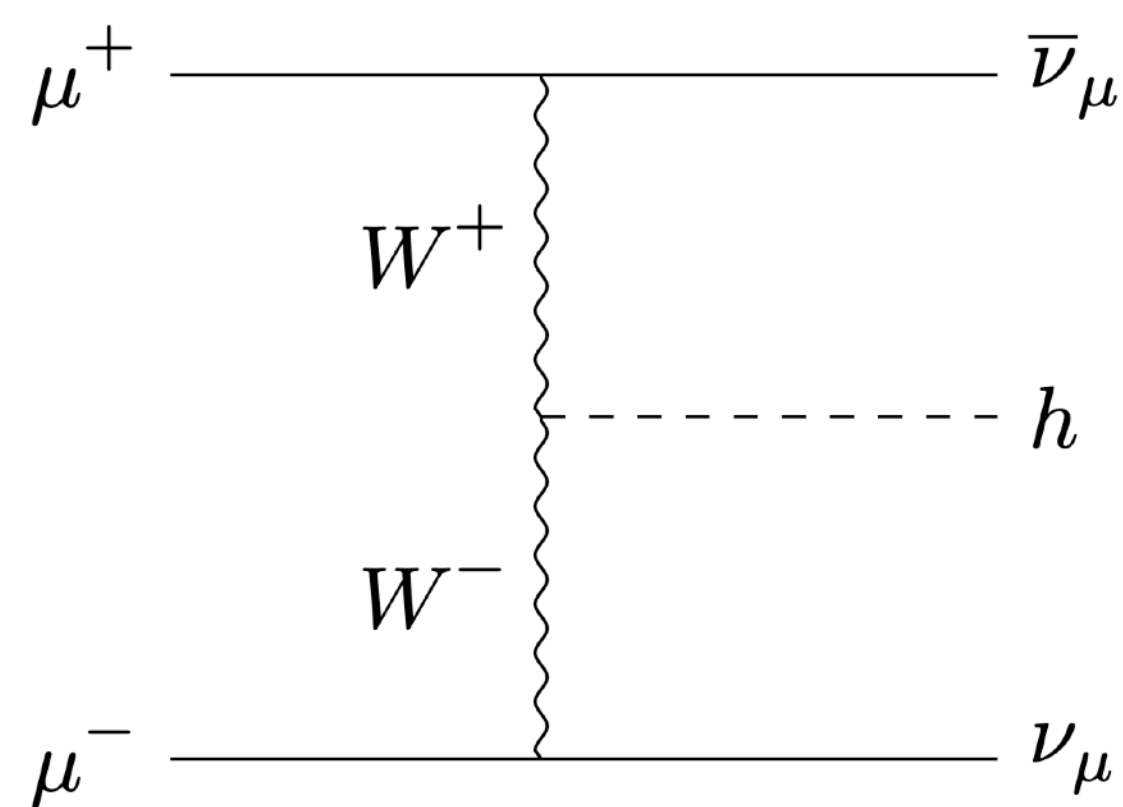
$\sim 40\text{fb}$  (1TeV x 1TeV)

$$\propto (1 - 4s_W^2 + 8s_W^4)^2$$



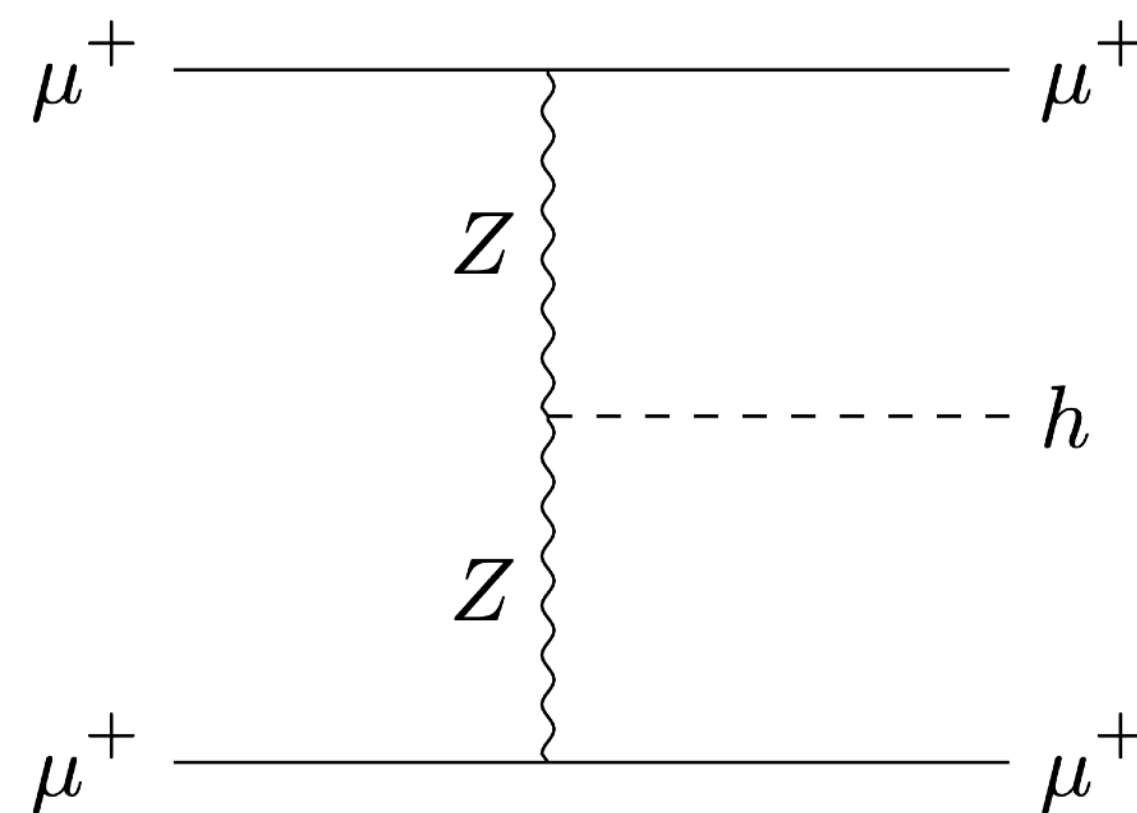
# How to produce Higgs boson at $\mu^+ \mu^+$ colliders?

- ▶ Z-fusion process?  $\rightarrow$  suppressed due to small Z coupling to muons
- ▶ **photon-mediated W-fusion process**



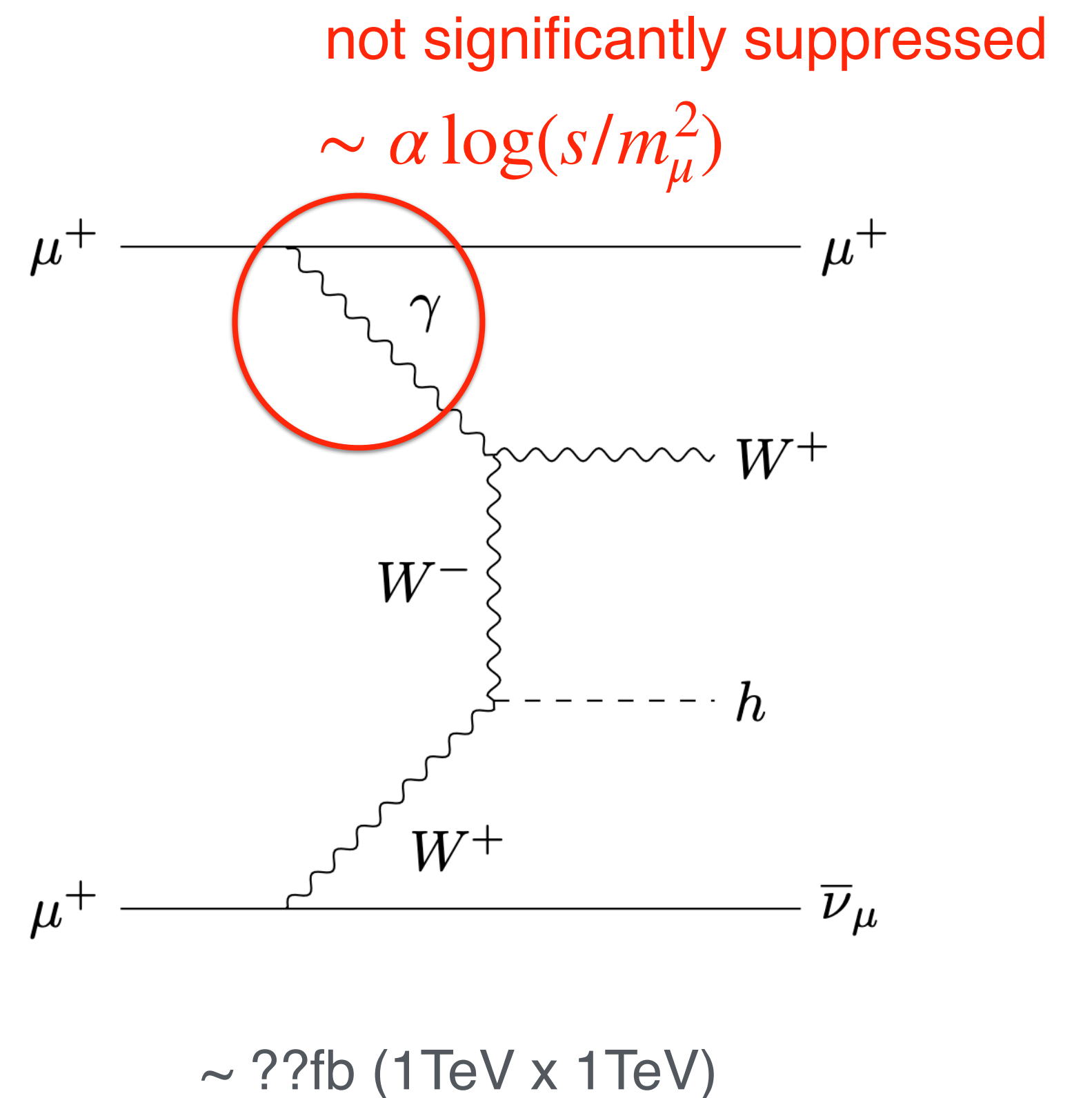
$\sim 400\text{fb}$  (1TeV x 1TeV)

VS

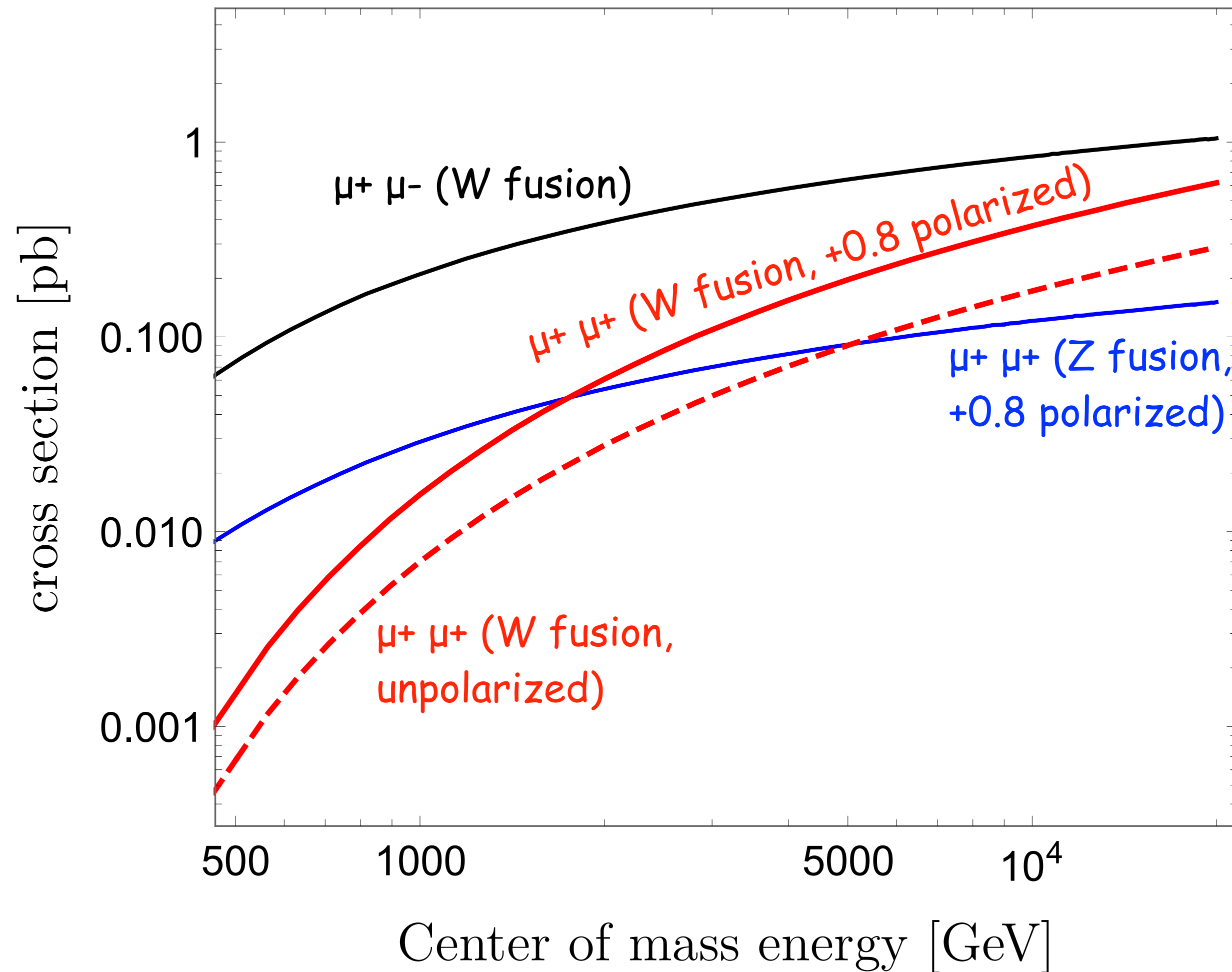


$\sim 40\text{fb}$  (1TeV x 1TeV)

VS



# Comparison of Higgs production



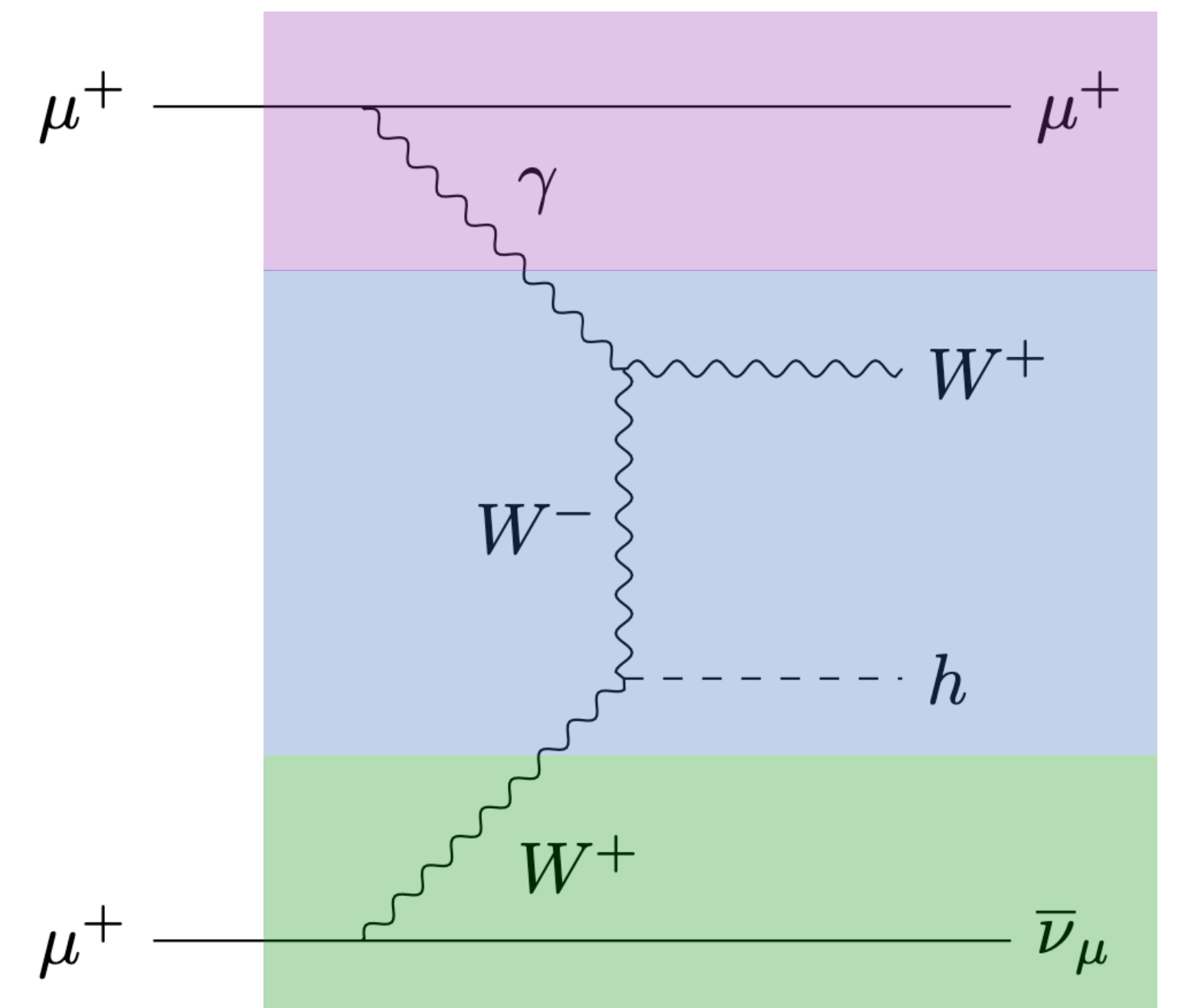
- ▶ Photon-mediated W-fusion can
  - surpass Z-fusion at  $\sim$ TeV
  - be as large as half of  $\mu^+ \mu^-$  W-fusion at  $\sim$ 10TeV
- ▶ Polarization is important

# Why photon-mediated W-fusion is so large?

Cross sections have soft/collinear logarithms at high energies

- Leading log analysis:

$$\sigma(s) \sim \int dx dy \underbrace{\sigma_{\gamma W_L^+ \rightarrow W^+ h}(xys)}_{\text{photon-W xsec}} \underbrace{f_{W_L^+/\mu^+}(y)}_{\substack{\text{W PDF} \\ \sim 1/y}} \underbrace{f_{\gamma/\mu^+}(x)}_{\sim \log(s/m_\mu^2)/x}$$



# Why photon-mediated W-fusion is so large?

Cross sections have soft/collinear logarithms at high energies

■ Leading log analysis:

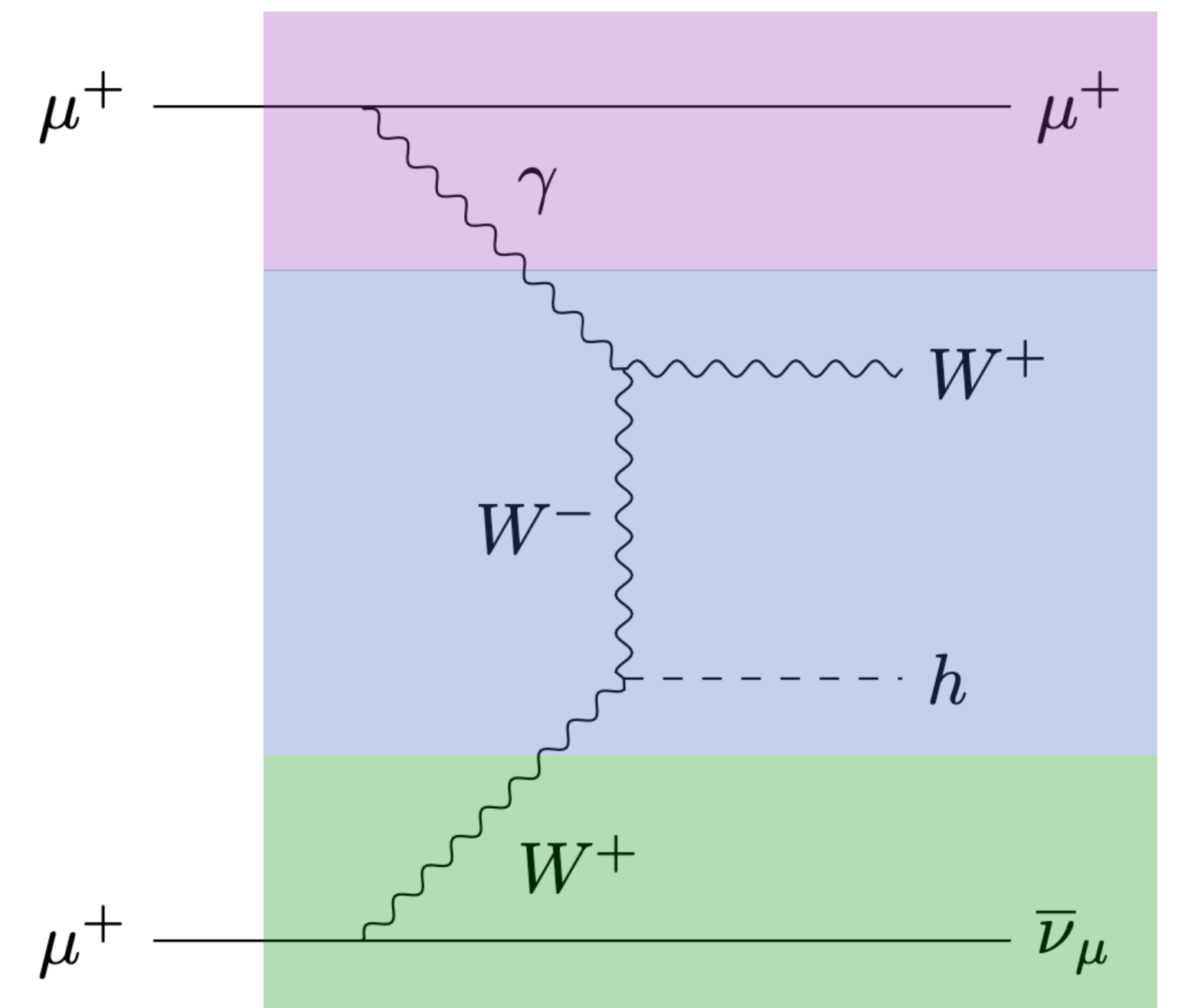
$$\sigma(s) \sim \int dx dy \underbrace{\sigma_{\gamma W_L^+ \rightarrow W+h}(xys)}_{\text{photon-W xsec}} \underbrace{f_{W_L^+/\mu^+}(y)}_{\text{W PDF}} \underbrace{f_{\gamma/\mu^+}(x)}_{\text{photon PDF}}$$

$$\sim \frac{(1 + P_\mu^+) \alpha^4}{4\pi m_W^2 \sin^4 \theta_W} \log \frac{s}{m_\mu^2} \left( \log \frac{s}{(m_h + m_W)^2} \right)^2$$

~1/y                      ~log(s/m<sub>μ</sub><sup>2</sup>)/x

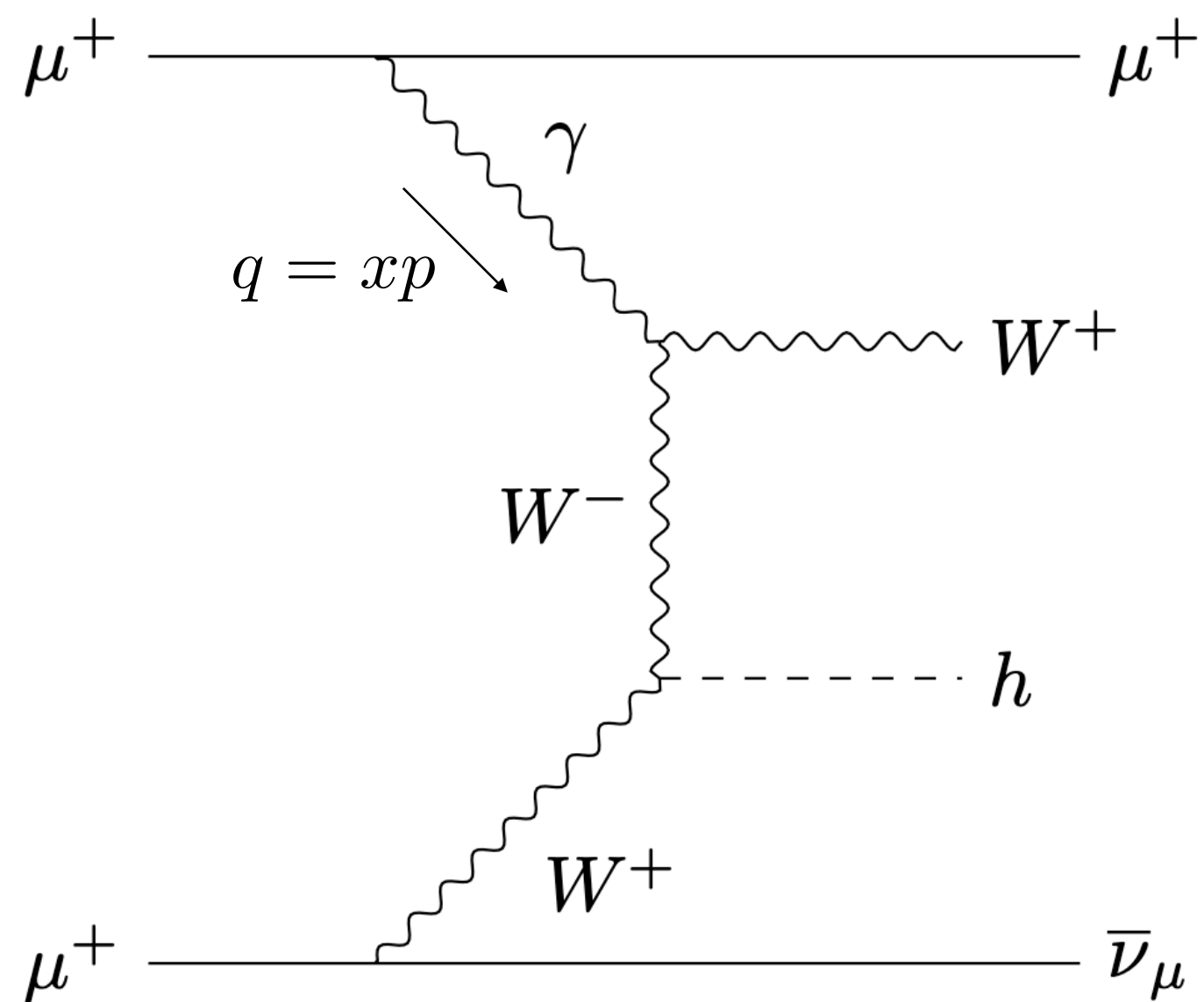
**Triple Logarithms!**

c.f.  $\mu^+\mu^-$  W-fusion  $\sigma(s) \sim \frac{\alpha^3}{16m_W^2 \sin^6 \theta_W} \log \frac{s}{m_h^2} \rightarrow$  only a single log



# A difficulty in numerical calculation

Collinear photon emission causes a problem in MadGraph calculation



$$d\sigma = \frac{1}{8k \cdot p} \frac{\alpha}{4\pi} \frac{W^{\mu\nu} T_{\mu\nu}}{q^4} dq^2 dx$$

$$\simeq dx \frac{dq^2}{q^4} \frac{\alpha}{2\pi} \left[ 2m_\mu^2 + q^2 \frac{1 + (1-x)^2}{x^2} \right] \sigma_{\gamma\mu^+ \rightarrow W^+ \bar{\nu}_\mu h}(xS) \Big|_{q^2 \rightarrow 0}$$

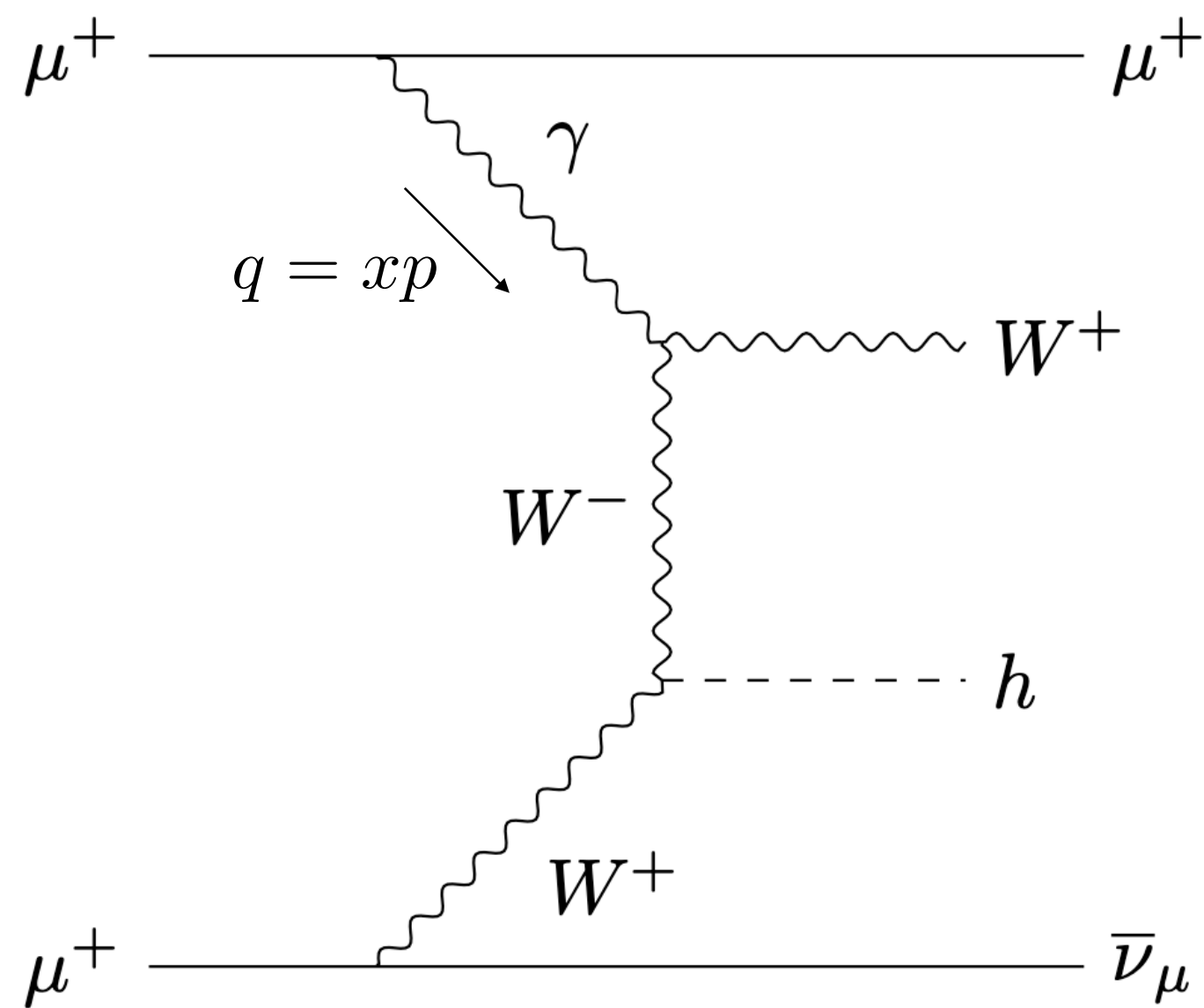
xsec of sub-process

divergent in a forward region ( $q^2 \rightarrow 0$ )

→ instability in numerical phase-space integral

# Equivalent Photon Approximation (EPA)

Analytically perform the  $q^2$ -integral over forward phase-space



$$d\sigma \simeq dx \frac{dq^2}{q^4} \frac{\alpha}{2\pi} \left[ 2m_\mu^2 + q^2 \frac{1 + (1-x)^2}{x^2} \right] \sigma_{\gamma\mu^+ \rightarrow W^+ \bar{\nu}_\mu h}(xs) \Big|_{q^2 \rightarrow 0}$$



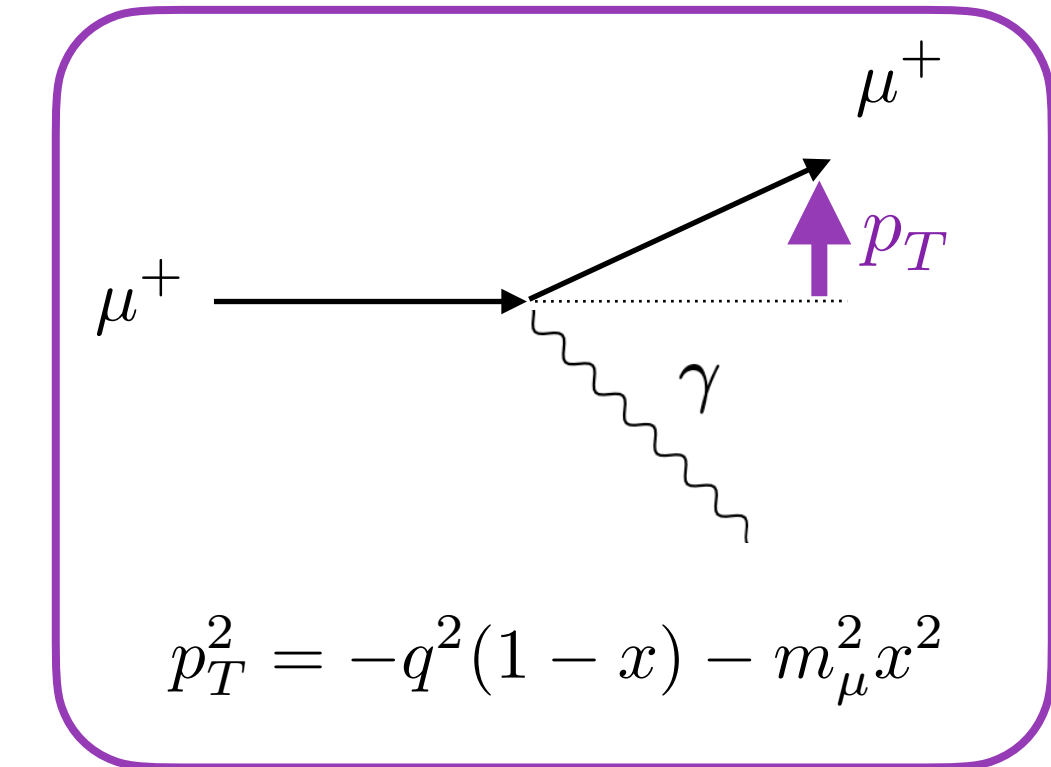
$$\sigma_{\text{EPA}} \simeq \int dx \sigma_{\gamma\mu^+}(xs) \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} \frac{dq^2}{q^4} \frac{\alpha}{2\pi} \left[ 2m_\mu^2 + q^2 \frac{1 + (1-x)^2}{x^2} \right] \underbrace{f_{\gamma/\mu}(x, p_T^{(\text{cut})})}_{p_T^2 = -q^2(1-x) - m_\mu^2 x^2}$$

$$p_T^2 = -q^2(1-x) - m_\mu^2 x^2$$

# pT-cut calculation scheme

phase-space of muon  
emitting photon

$$\sigma = \int d^3 p_\ell \frac{d\sigma}{d^3 p_\ell} = \int dp_T \frac{d\sigma}{dp_T}$$

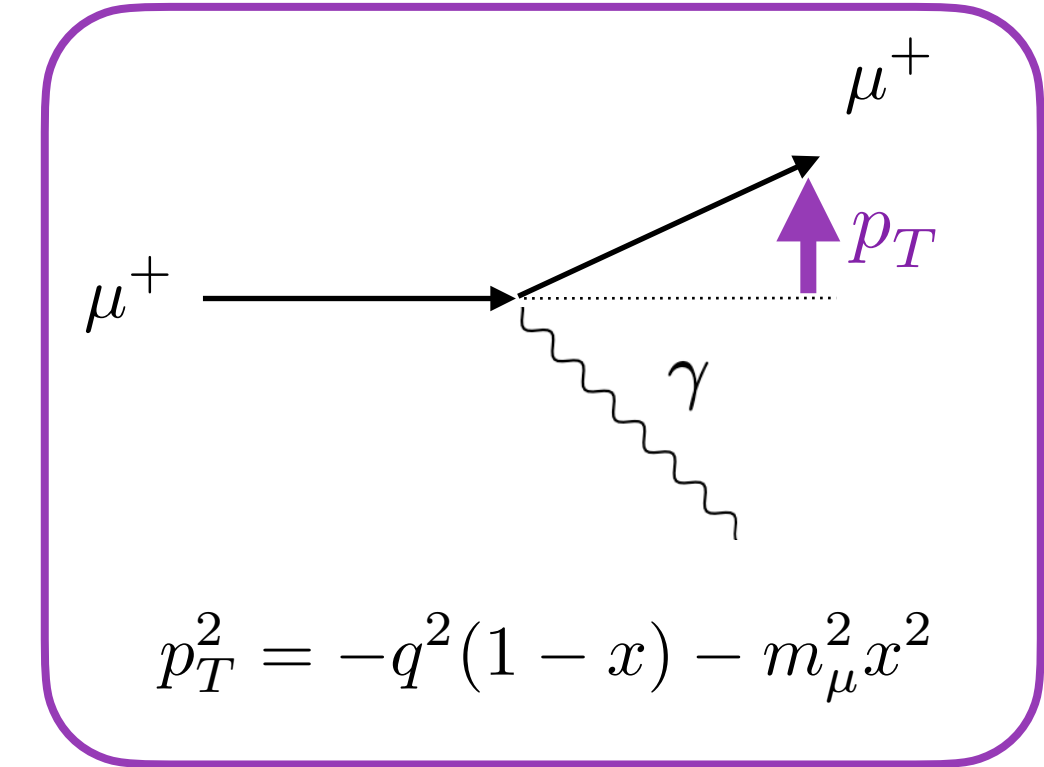


# pT-cut calculation scheme

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$$= \int_{p_T < p_T^{(\text{cut})}} dp_T \frac{d\sigma}{dp_T} + \int_{p_T > p_T^{(\text{cut})}} dp_T \frac{d\sigma}{dp_T}$$





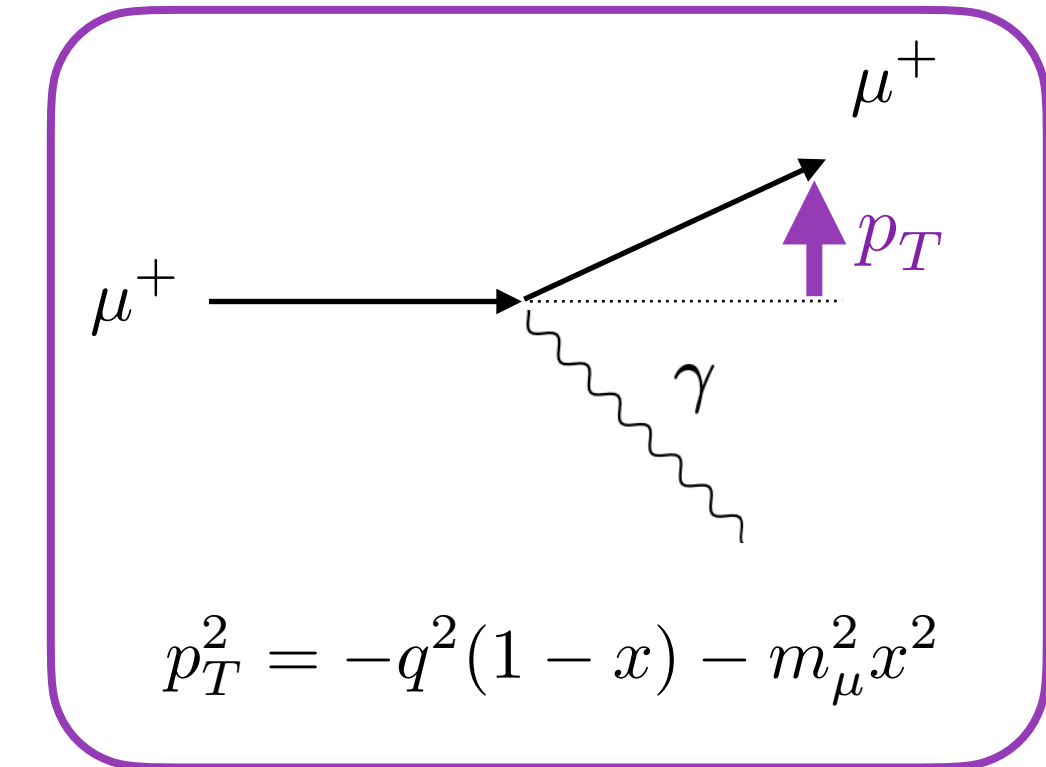
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$$= \underbrace{\int_{p_T < p_T^{(\text{cut})}} dp_T \frac{d\sigma}{dp_T}}_{\text{origin of instability in MG calculation}} + \int_{p_T > p_T^{(\text{cut})}} dp_T \frac{d\sigma}{dp_T}$$

origin of instability in MG calculation  
-> evaluated with EPA



evaluated with MadGraph

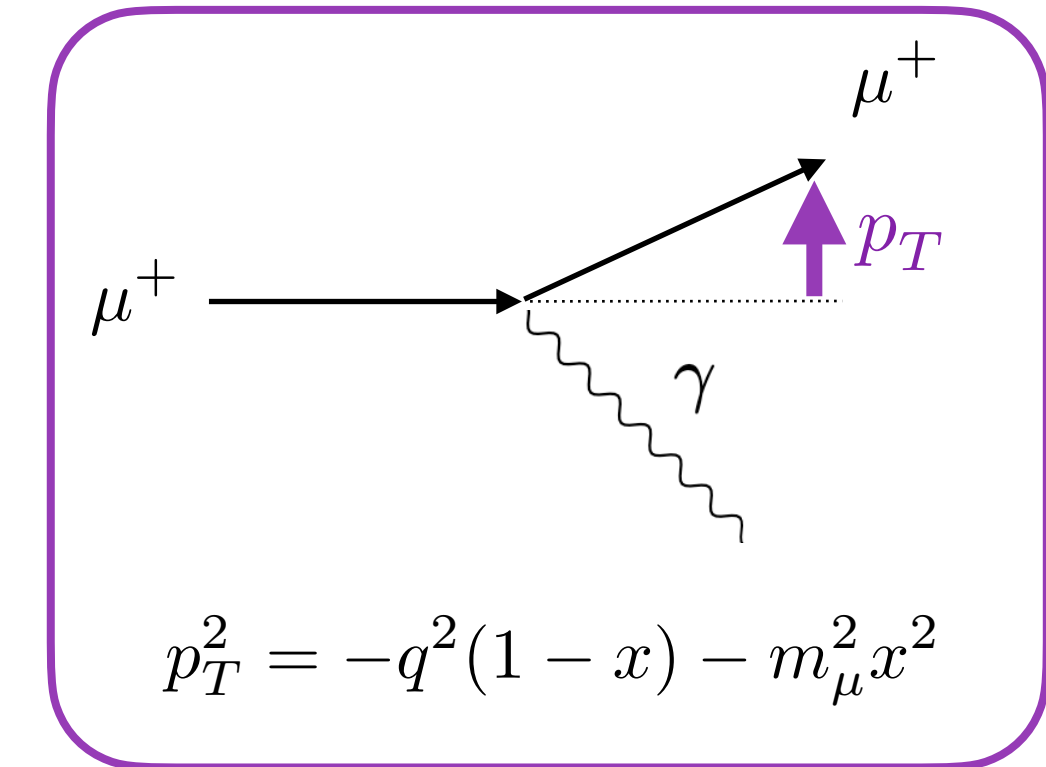
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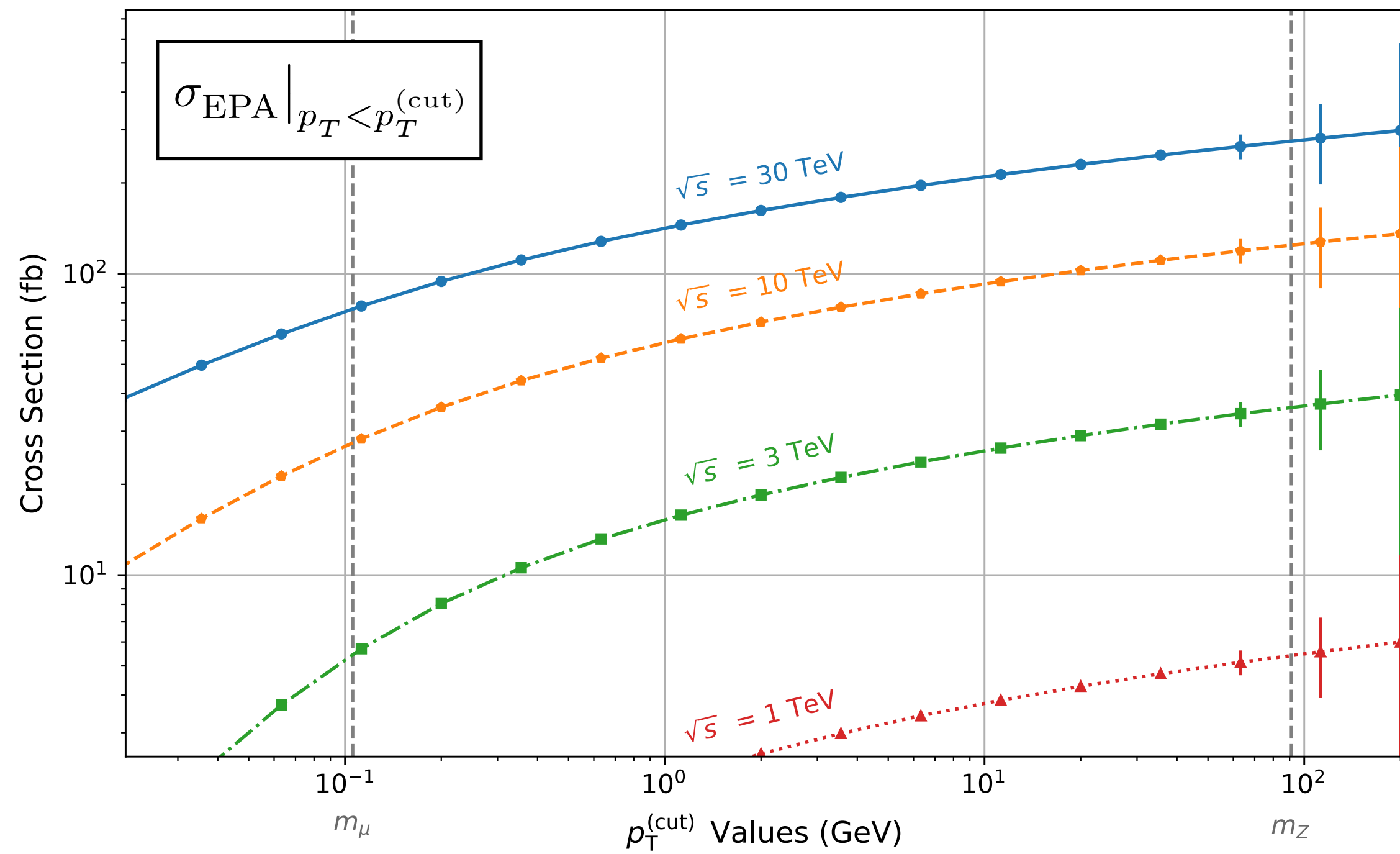
$$\simeq \sigma_{\text{EPA}} \Big|_{p_T < p_T^{(\text{cut})}} + \sigma_{\text{MG5}} \Big|_{p_T > p_T^{(\text{cut})}}$$



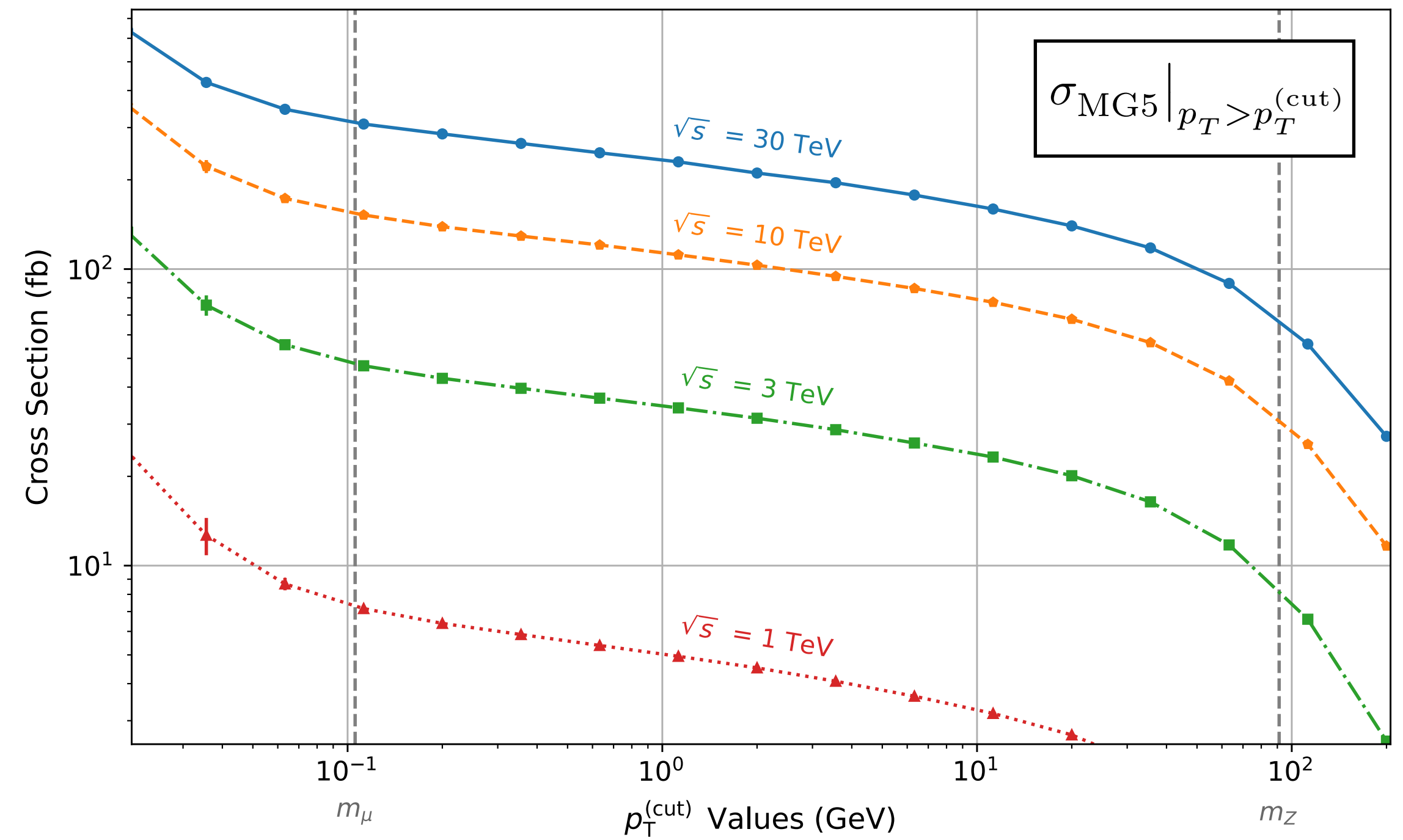
Sum of these two should be independent of pT-cut value if correctly calculated

# Calculation in the pT-cut scheme

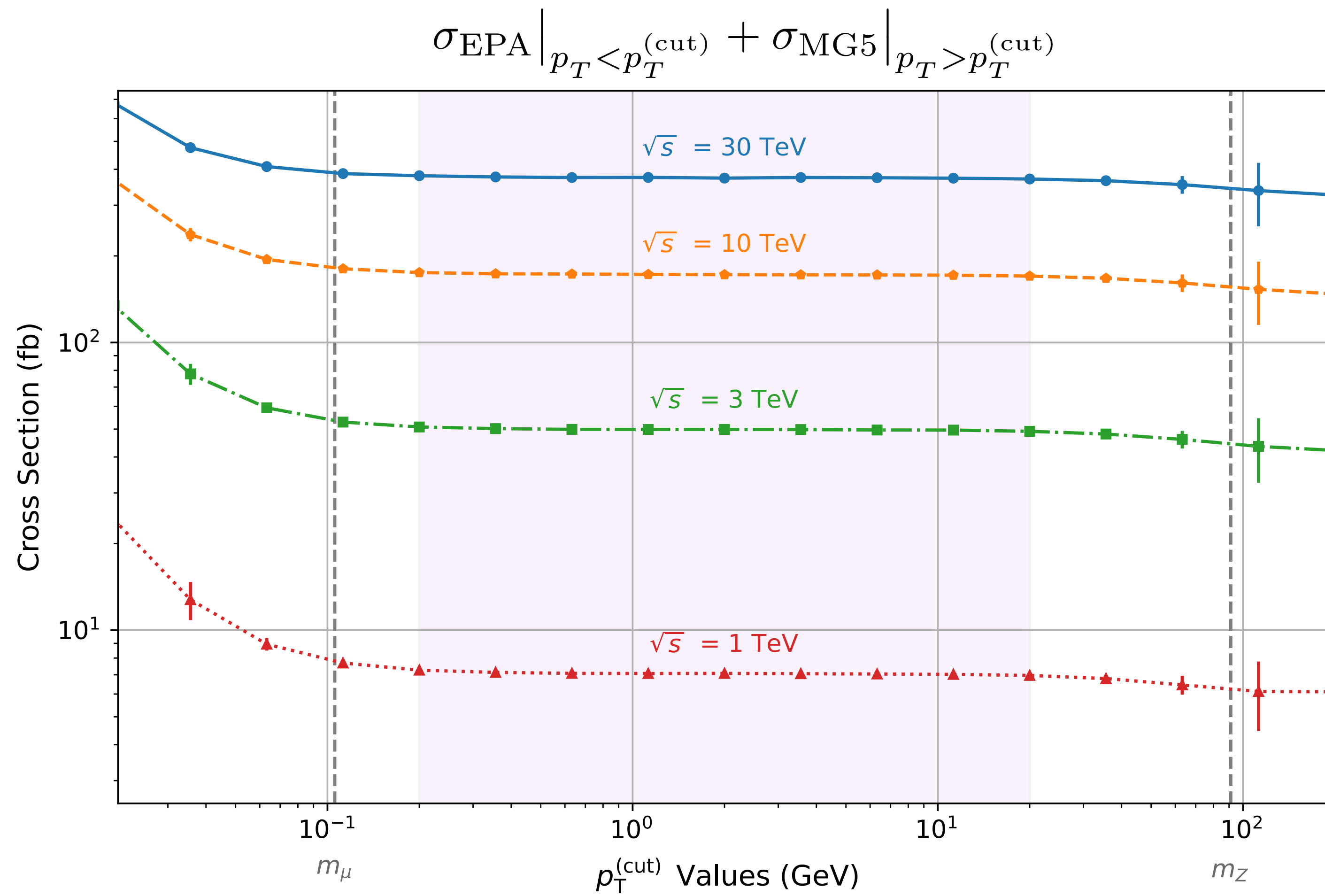
## EPA calculation ( $p_T < p_T^{\text{cut}}$ )



## MadGraph ( $p_T > p_T^{\text{cut}}$ )

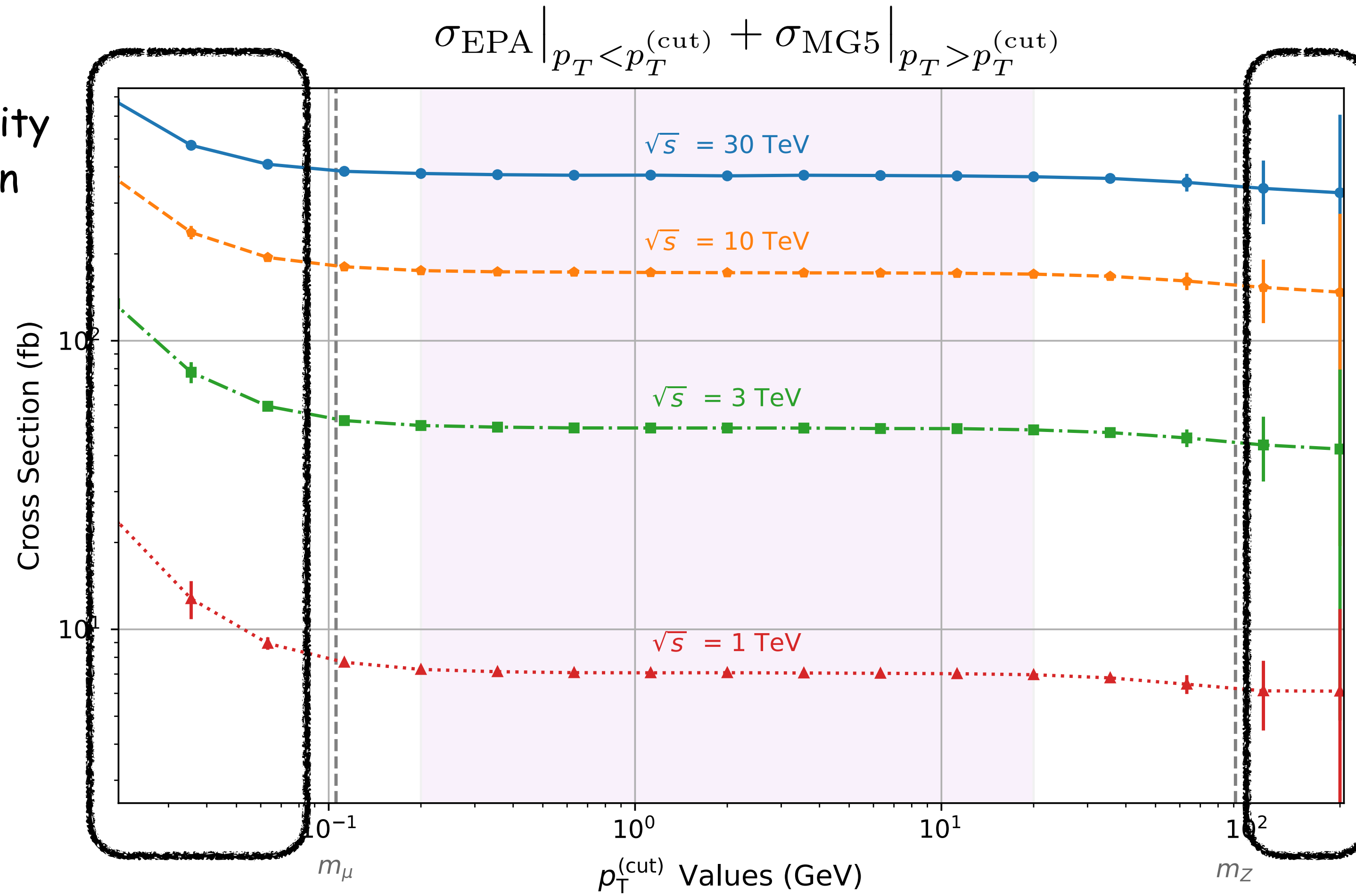


# Calculation in the pT-cut scheme



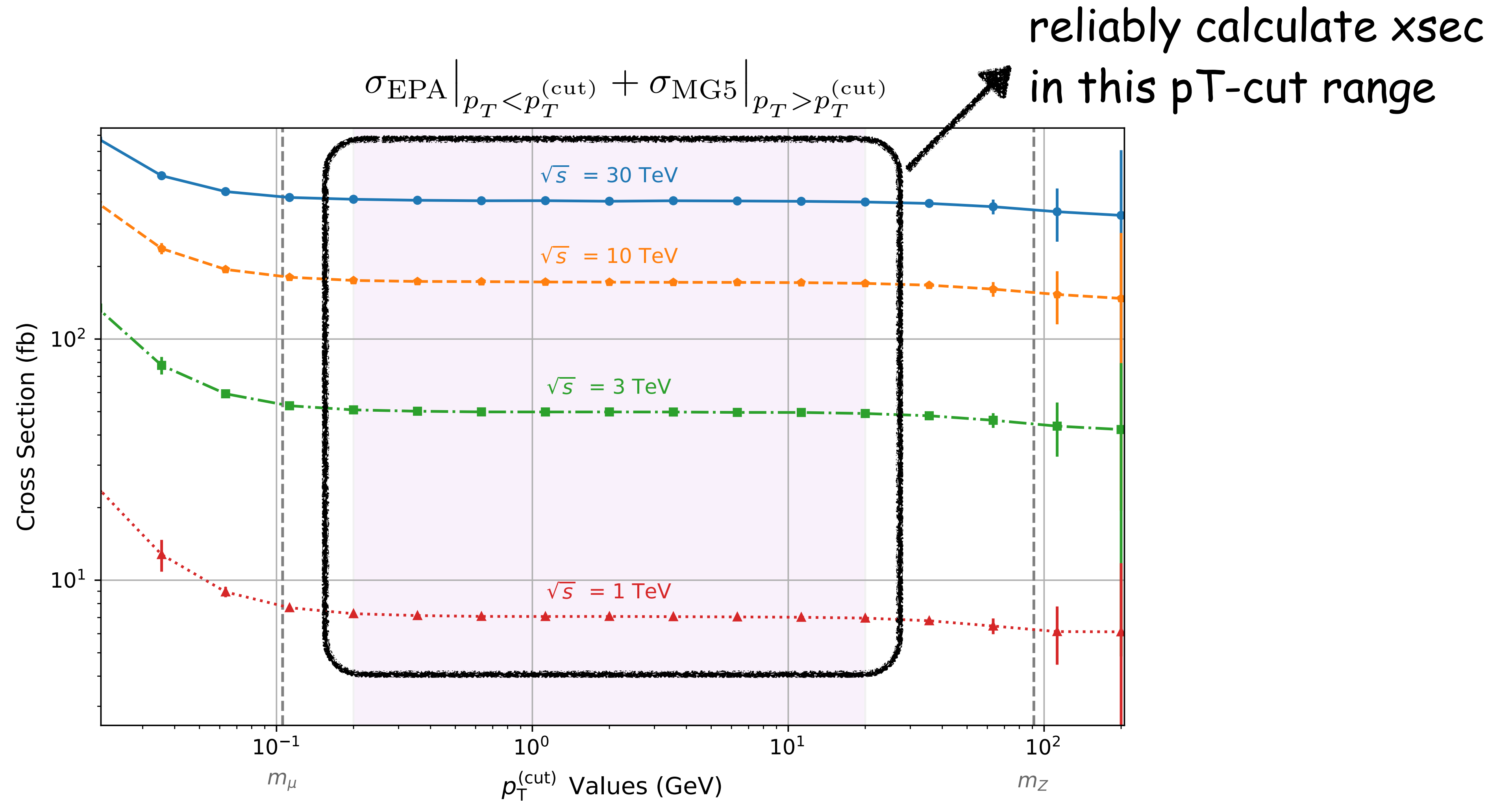
# Calculation in the pT-cut scheme

numerical instability  
in MG calculation



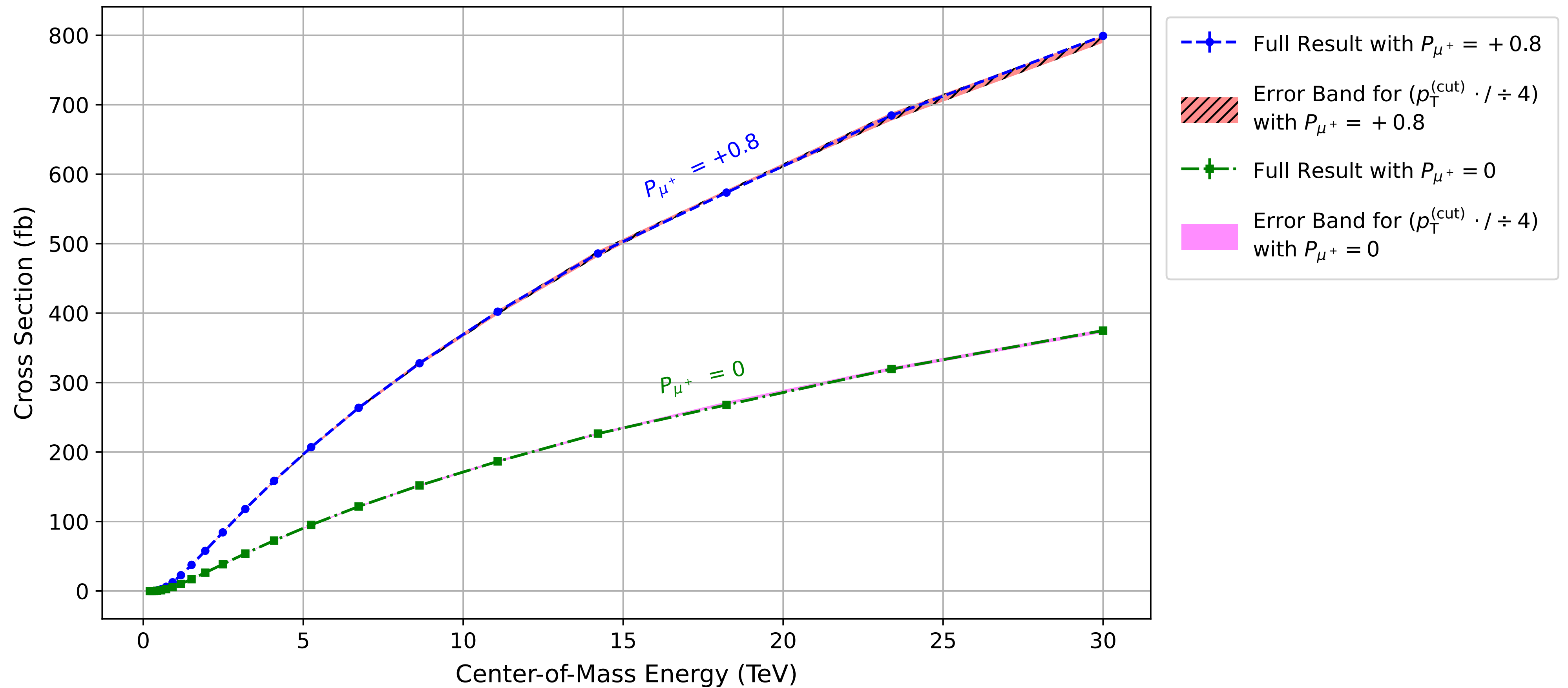
Z contribution  
non-negligible

# Calculation in the pT-cut scheme

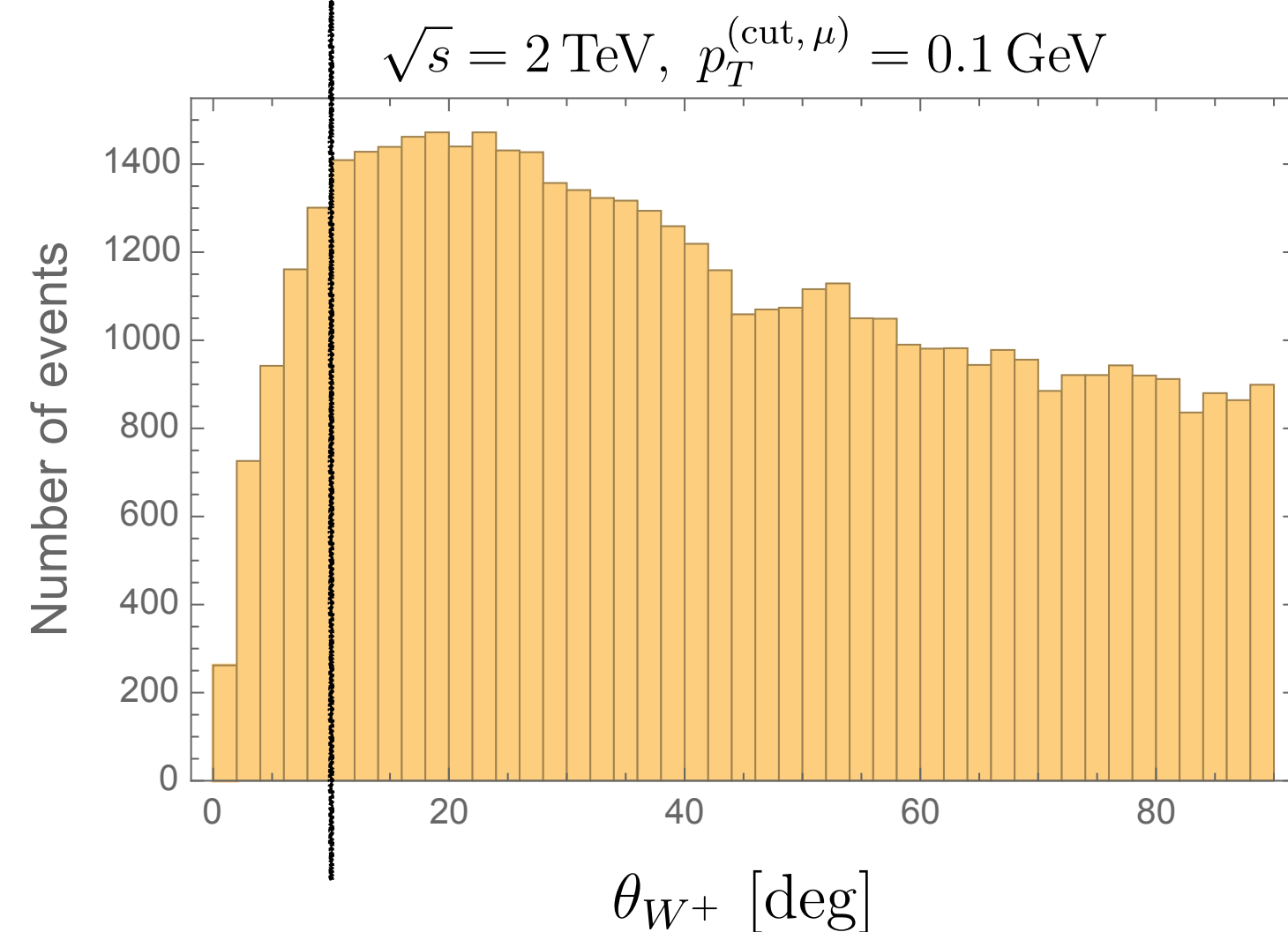
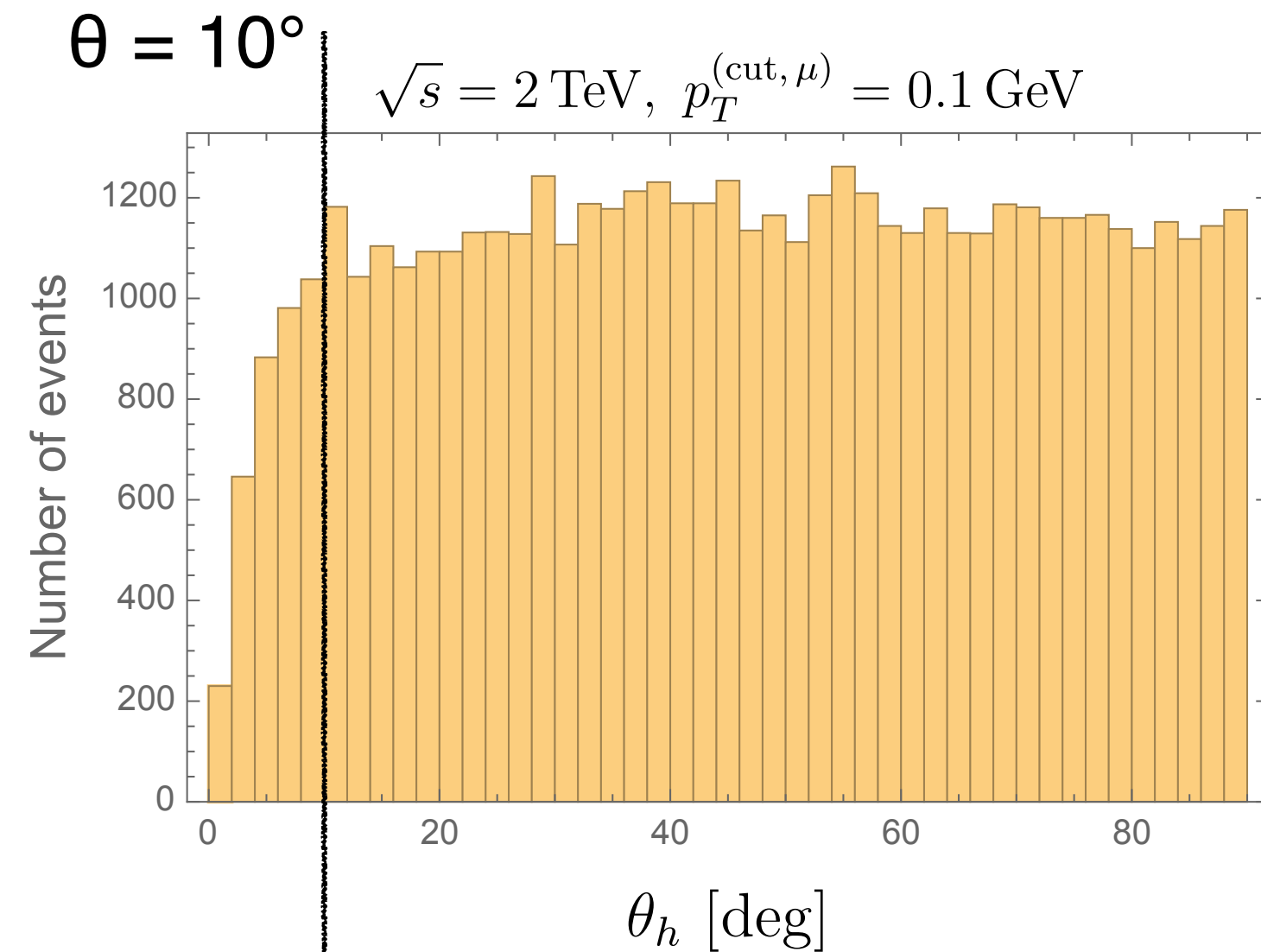


# Results for Higgs production

Cross Section vs. Center-of-Mass Energy for  $\mu^+ \mu^+ \rightarrow \mu^+ \bar{\nu}_\mu W^+ h$   
Full Result with Different Polarizations  $P_{\mu^+}$  for  $p_{\top}^{(\text{cut})} = \sqrt{m_\mu m_Z} \approx 3.10$  GeV with  $p_{\top}^{(\text{cut})}$  Error Bands



# Angular distributions for Higgs and W bosons



## Parton level analysis

- #(events generated) = 50,000
- hadron calorimeter coverage:  $\theta > 10^\circ$   
→ ~83% of events detectable!

background discrimination? ongoing, stay tuned!



# Summary

- Muon Colliders are a good Higgs factory.
- At high energy mu+ mu+ colliders,
  - $\mu^+\mu^+ \rightarrow \mu^+\bar{\nu}_\mu W^+h$  is the leading process
  - calculation not straightforward (pT-cut scheme)
- Corrections from collinear photon would be important at mu+ mu- colliders

Thanks for your attention

Back up

# Cross section comparison

Cross Section vs. Center-of-Mass Energy for Single Higgs Production via  $W$  Boson Fusion at  $\mu^+ \mu^+$  Colliders  
 Comparison of Different Processes, Calculation Methods, and Polarizations  $P_{\mu^+}$

