Making use of ν_e from muon storage ring

Brief review on T-violation and neutrino factory, and NuTRISTAN

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@Northwestern University

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YNU

- Three generation and CP violation
- Why T-violation instead of CP violation
- Review on NuFACT
- Towards “Nu”TRISTAN

1.Three generation and CP Violation

Long (long ?) ago, there was question whether neutrinos are massive and hence there is a lepton mixing.

Theory :

G321 (SM) ->G3221->G422 (Pati-Salam) -> SO (10) GUT

Neutrinos are massive since "RH neutrinos" are introduced.

Dirac mass with RH neutrino (SM singlet fermion) and

possibly very small masses due to seesaw mechanism

$$G_{321} \equiv SU(3)\times SU(2)_L \times U(1)_Y \rightarrow G_{3221} \equiv SU(3)\times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$(3, 2, \tfrac{1}{6}) \longrightarrow (3, 2, 1, \tfrac{1}{6})$$

u_L^c

d_L^c

$(\bar{3}, 1, -\tfrac{2}{3})$

$(\bar{3}, 1, \tfrac{1}{3})$

$$\longrightarrow Q_L^c = \begin{pmatrix} u_R^c \\ d_R^c \end{pmatrix}$$

$$\longrightarrow (\bar{3}, 1, 2, -\tfrac{1}{6})$$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$(1, 2, -\tfrac{1}{2}) \longrightarrow (1, 2, 1, -\tfrac{1}{2})$$

[redacted]

e_L^c

$(1, 1, 1)$

$$\longrightarrow L_L^c = \begin{pmatrix} \nu_L^c \\ e_L^c \end{pmatrix}$$

$$\longrightarrow (1, 1, 2, \tfrac{1}{2})$$

By noting that the ratio of U(1) is 1:−3 $SU(3)\times U(1) \rightarrow SU(4)$ $G_{3221} \rightarrow G_{422}$

$$\Rightarrow (4, 2, 1) \qquad\qquad \Rightarrow (\bar{4}, 1, 2)$$

Since $SU(4) \simeq SO(6)$ & $SU(2) \times SU(2) \simeq SO(4)$

$$G_{422} \subset SO(10)$$

All particle of one generation is unified into ONE PARTICLE

$$SO(10)\ 16$$

We have Standard Model singlet which can be interpreted as RH neutrino

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Experiment:

Solar Neutrino Deficit

Not enough neutrinos coming from the sun

& Atmospheric Neutrino Anomaly

Counts of Neutrino from atmosphere is strange

Both can be explained by Neutrino oscillation with

large lepton mixing ,,,

However …

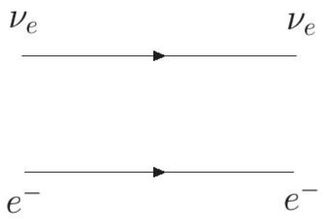
Pathological preconceptions that

Mixing MUST be small

though it may be natural because of GUT …

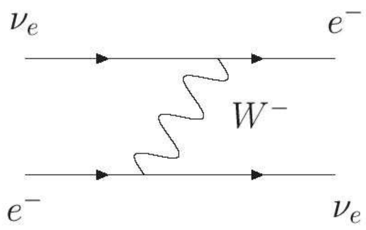
The Savior = Matter effect = MSW (Mikeyev Smirnov and Wolfenstein) effect

Refraction phenomeno when neutrino goes through matter like the sun and the earth



$\mathcal{A}_0$

Go through



$\mathcal{A}_1$

Forward Scattering

Propagating in matter
We cannot distinguish them
-> They interfere each other

$$\mathcal{M} = \mathcal{A}_0 + \mathcal{A}_1 \rightarrow |\mathcal{M}|^2 = \mathcal{A}_0^2 + 2\mathrm{Re}\left(\mathcal{A}_0\mathcal{A}_1^*\right)$$

$$\mathcal{A}_1 \simeq G_F(\bar{\psi}_e\gamma^\mu\psi_{\nu_e})(\bar{\psi}_{\nu_e}\gamma_\mu\psi_e) = G_F(\bar{\psi}_e\gamma^\mu\psi_e)(\bar{\psi}_{\nu_e}\gamma_\mu\psi_{\nu_e})$$

Non relativistic limit

$$\rightarrow G_F(\bar{e}\gamma^0 e)(\bar{\nu}_e\gamma_0\nu_e)$$

Modulation of **electron neutrino** energy

Framework of standard 3 flavor ν oscillation in matter

Here, matter density is constant.
$+$ for ν, $-$ for $\bar{\nu}$

$$i\frac{d}{dt}\begin{pmatrix}\nu_e(\bar{\nu}_e)\\ \nu_\mu(\bar{\nu}_\mu)\\ \nu_\tau(\bar{\nu}_\tau)\end{pmatrix} = \left[U\mathrm{diag}(0,\Delta E_{21},\Delta E_{31})U^\dagger + \mathrm{diag}(\pm A,0,0)\right]\begin{pmatrix}\nu_e(\bar{\nu}_e)\\ \nu_\mu(\bar{\nu}_\mu)\\ \nu_\tau(\bar{\nu}_\tau)\end{pmatrix}$$

$$= \tilde{U}^{(\pm)}\mathrm{diag}(\tilde{E}_1^{(\pm)},\tilde{E}_2^{(\pm)},\tilde{E}_3^{(\pm)})\tilde{U}^{(\pm)\dagger}\begin{pmatrix}\nu_e(\bar{\nu}_e)\\ \nu_\mu(\bar{\nu}_\mu)\\ \nu_\tau(\bar{\nu}_\tau)\end{pmatrix}$$

$$U = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{-i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}$$

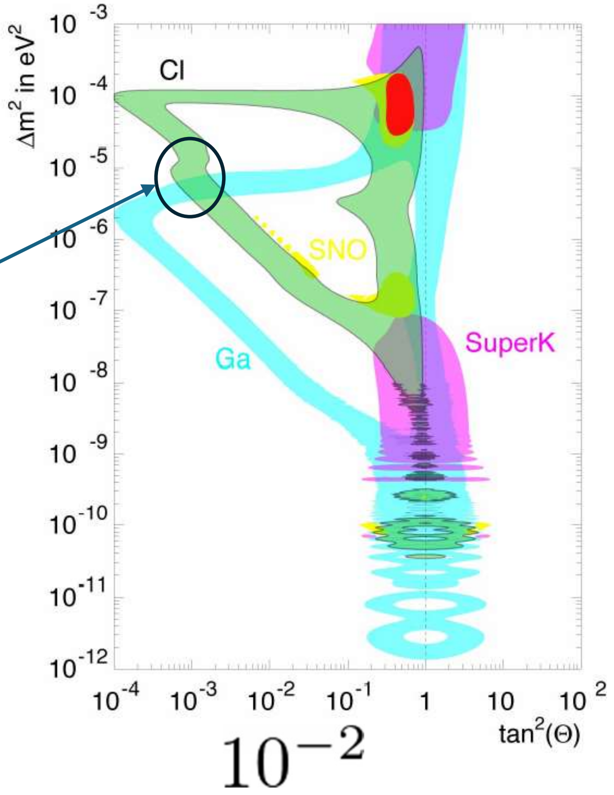
CP phase

Matter effect saved the small mixing partially.

Solar neutrino deficit could be explained by small mixing.
In the sun, with an appropriate parameter for neutrinos, even with small mixing, effectively they mix largely.

Fredman NOON2004

Large mixing also explains with matter effect appropriately …



At the beginning
Small mixing was prefered
even with Kamiokande.

However Atomospheric Neutrino anomaly was not …

Oscillation probability in matter

$$P\left(\nu_\alpha(\bar{\nu}_\alpha) \to \nu_\beta(\bar{\nu}_\beta)\right)$$
$$= \delta_{\alpha\beta} - 4\sum_{j>k} Re[\widetilde{U}^{(\pm)}{}_{\beta j}\widetilde{U}^{(\pm)}{}^*_{\alpha j}\widetilde{U}^{(\pm)}{}^*_{\beta k}\widetilde{U}^{(\pm)}{}_{\alpha k}]\ \sin^2\left(\frac{\Delta\tilde{E}^{(\pm)}_{jk}L}{2}\right)$$
$$+2\sum_{j>k} Im[\widetilde{U}^{(\pm)}{}_{\beta j}\widetilde{U}^{(\pm)}{}^*_{\alpha j}\widetilde{U}^{(\pm)}{}^*_{\beta k}\widetilde{U}^{(\pm)}{}_{\alpha k}]\sin\left(\Delta\tilde{E}^{(\pm)}_{jk}L\right)$$

CP-violation

$$P\left(\nu_\alpha \to \nu_\beta\right) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

Complicated contributions of matter effect and CP phase δ

T-violation

$$P\left(\nu_\alpha \to \nu_\beta\right) - P\left(\nu_\beta \to \nu_\alpha\right)$$

Not complicated contributions of matter effect and CP phase δ

2.CP Violation? T Violation?

Phys.Rev.D 55 (1997) 1653-1658 Arafune, JS

Phys.Rev.D 56 (1997) 3093-3099, Phys.Rev.D 60 (1999) 119905 (erratum) Arafune, Koike, JS

Nucl.Instrum.Meth.A 451 (2000) 36-41 JS

1996-

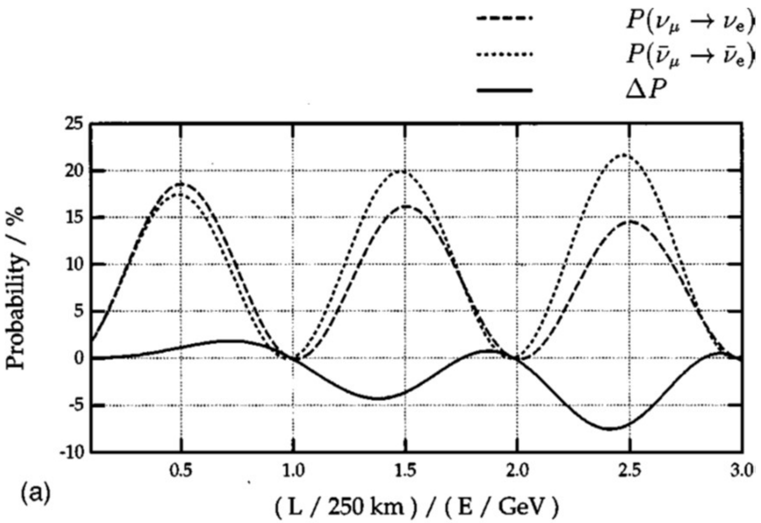
Theory :: Small mixing is believed

but Solar ν (θ_{12}) Small->Large , Atm (θ_{23}) Large

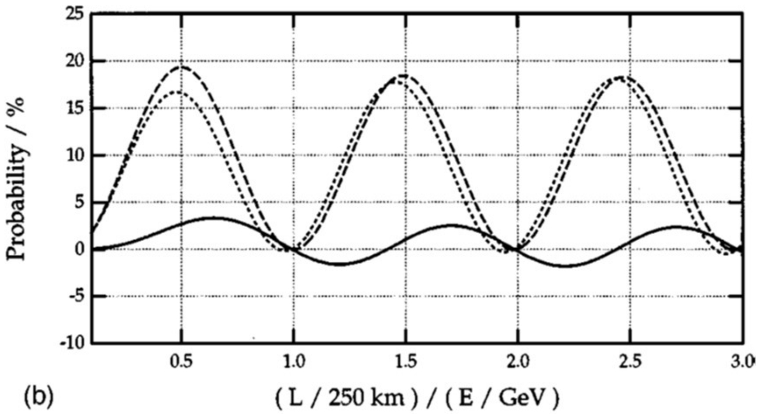
Last mixing (θ_{13}) upper bound ～ 0.1

Experiment :: K2K under construction

If both are large, CP violation can be observed in K2K !?

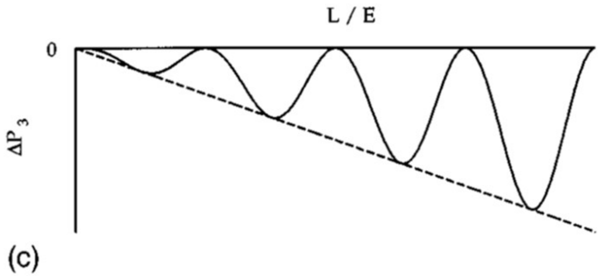


$$\delta = \frac{\pi}{2}$$

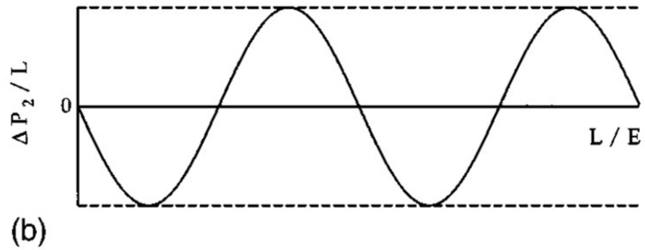


$$\delta = 0$$

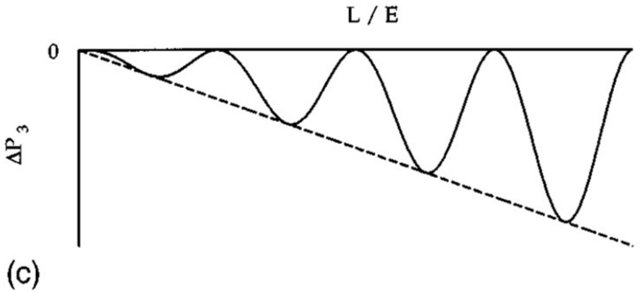
We discussed if we observe CPV effect in near future ecperiment with “Maximum parameter”



Includint to observe not only first maximum but this envelope



Matter effect



CPV

If we observe only first maximum can we distinguish them clearly ?

On the contrary T violation is very clear !!

Oscillation probability in matter

$$P\left(\nu_\alpha(\bar{\nu}_\alpha) \to \nu_\beta(\bar{\nu}_\beta)\right)$$

$$= \delta_{\alpha\beta} - 4\sum_{j>k} Re[\widetilde{U}^{(\pm)}{}_{\beta j}\widetilde{U}^{(\pm)}{}^*_{\alpha j}\widetilde{U}^{(\pm)}{}^*_{\beta k}\widetilde{U}^{(\pm)}{}_{\alpha k}]\ \sin^2\left(\frac{\Delta\tilde{E}^{(\pm)}_{jk}L}{2}\right)$$

$$+2\sum_{j>k} Im[\widetilde{U}^{(\pm)}{}_{\beta j}\widetilde{U}^{(\pm)}{}^*_{\alpha j}\widetilde{U}^{(\pm)}{}^*_{\beta k}\widetilde{U}^{(\pm)}{}_{\alpha k}]\sin\left(\Delta\tilde{E}^{(\pm)}_{jk}L\right)$$

CP-violation

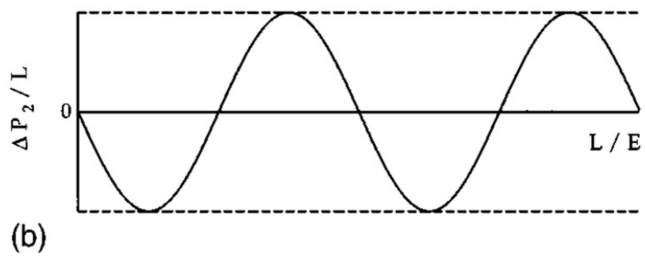
$$P\left(\nu_\alpha \to \nu_\beta\right) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

Complicated contributions of matter effect and CP phase δ

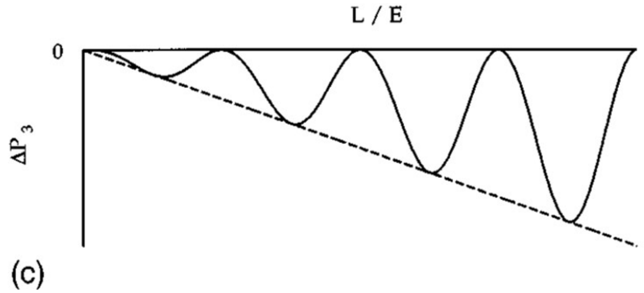
T-violation

$$P\left(\nu_\alpha \to \nu_\beta\right) - P\left(\nu_\beta \to \nu_\alpha\right)$$

Not complicated contributions of matter effect and CP phase δ



Matter effect



CPV

If we observe only first maximum can we distinguish them clearly ?

On the contrary T violation is very clear !!

However we need ν_e

Kuno-san told me it is easy to get !! It's from muon beam !!

First step for muon collider

3. Neutrino Factory

Phys.Rev.D57:6989-6997,1998; Erratum-ibid.D59:039903,1999 S.Geer

Neutrino Beams from Muon Storage Rings: Characteristics and Physics Potential

Basic Concept

大艦巨砲主義 policy of using large ships as a weapon

Game of number of events

1. Charged Current Cross Section $\sigma_{\nu(\bar{\nu})N} \sim 0.67(0.34) \times 10^{-38}\mathrm{cm}^2 E_{\nu(\bar{\nu})}[\mathrm{GeV}]$

Proportional to neutrino energy -> **Higher energy !** for high statistics

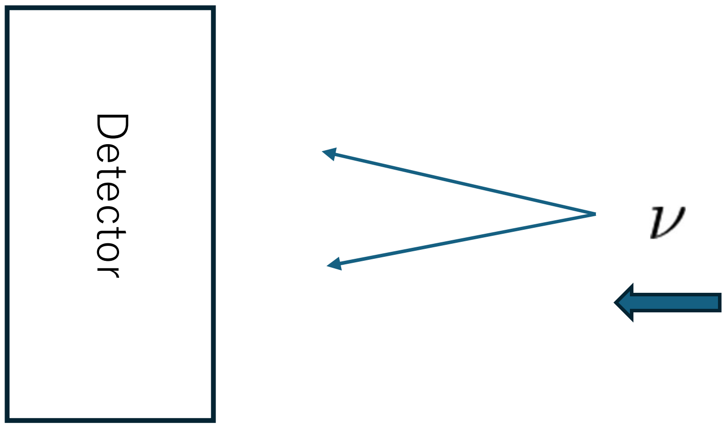
2.Oscillation Probability $\sim \sin^2 \frac{\delta m^2 L}{E}$

Higher energy = **Longer distance** to keep osc. max.

longer distance = smaller flux naively $flux \propto 1/L^2$

However neutrinos from high energy muon are focused forward

$$flux \propto E^2/L^2$$

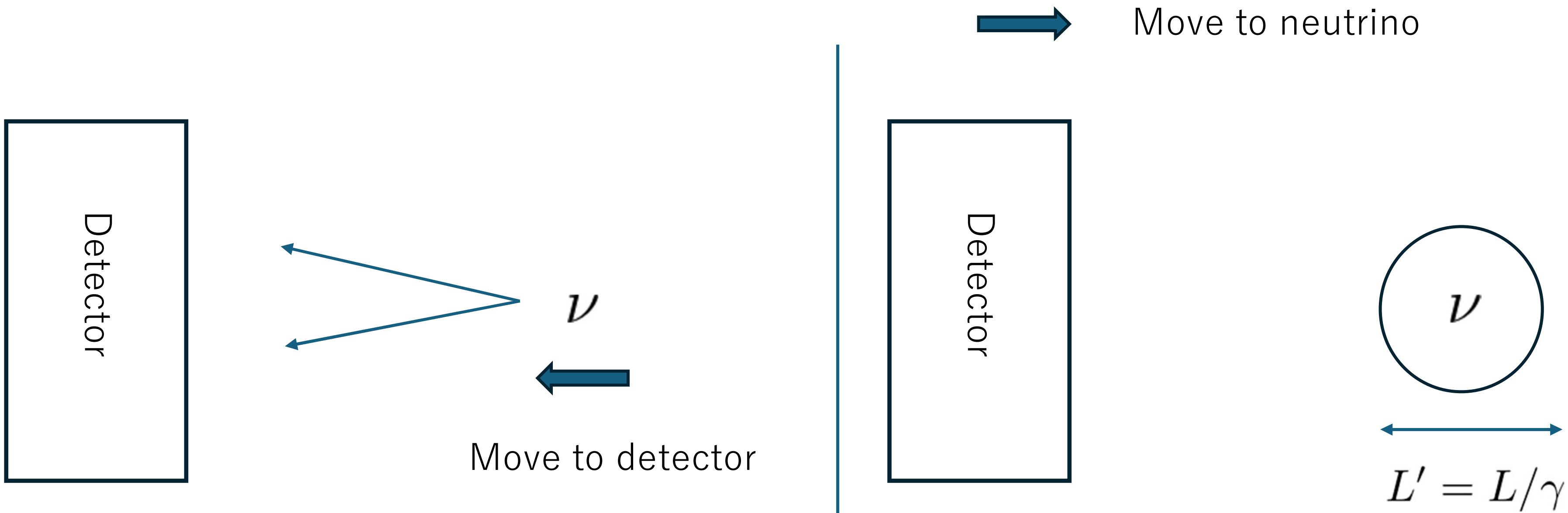


Lab. Frame :
Beam is forcused

longer distance = smaller flux naively $flux \propto 1/L^2$

However neutrinos from high energy muon are focused forward

$$flux \propto E^2/L^2$$



Diametor of
neutrino wave fn.

Lab. Frame :
Beam is forcused

Rest Frame of neutrino:
Distance is $L' = L/\gamma$

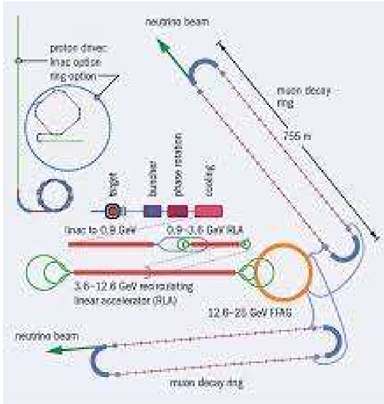
As a total, appearance event proportional to neutrino (parent muon) energy

-> longer baseline $\because L \propto E$ 30-50 GeV

3000km? 9000km?

Long decay volume

$\gamma \sim 300 - 500, \tau_\mu \sim 10^{-3}\mathrm{s}, d \sim 300\mathrm{km}$



CERN Courier

Wrong sign muon

signal muon (from oscillatated neutrino) has opposite charge

$$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$$

$$\nu_\mu \rightarrow \nu_\mu, \nu_\mu + N \rightarrow \mu^- + X$$ "Correct" sign = No osciilation

$$\bar{\nu}_e \rightarrow \bar{\nu}_\mu, \bar{\nu}_\mu + N \rightarrow \mu^+ + X$$ "Wrong" sign = Osciilation

Pros & Cons

Pros Number of events

Cons If making use of quasi elastic , # of event is constant of E

Mater effect is uncontrolled

Many Earth Model. PREM is just one of models

High energy and Longer baseline is really meaningful ?

Charge Identification ?

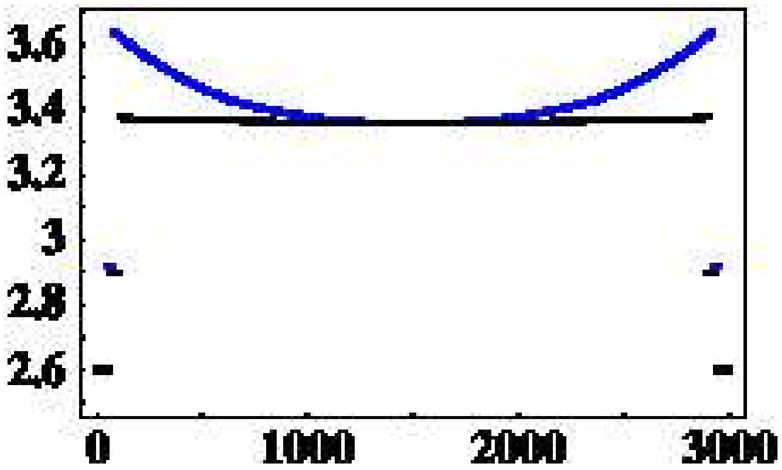
Several ideas …. E.g.

$$\mu^+ \to \bar{\nu}_\nu + \nu_e + e^+,\ e^+ + e^- \to 2\gamma\ (511\text{keV})$$

And others

Matter effect in very long baseline

Phys.Rev.D 67 (2003) 053003 • e-Print: hep-ph/0211095 [hep-ph]
Toshihiko Ota & JS



Matter density profile for 3000km baseline
Ak135-f (blue) vs PREM (black)

First Oscillation maximum
interferes strongly with
first Fourier mode of matter profile

Huge uncertainty for CP violation

Fatal to observe CPV

4. ν TRISTAN

- *To observe CPV phase, T-violation is robust against matter effect*
- *Low energy neutrino is welcome against matter effect*

➡ Oscillation of electron neutrino from μTRISTAN

ν TRISTAN

Pros

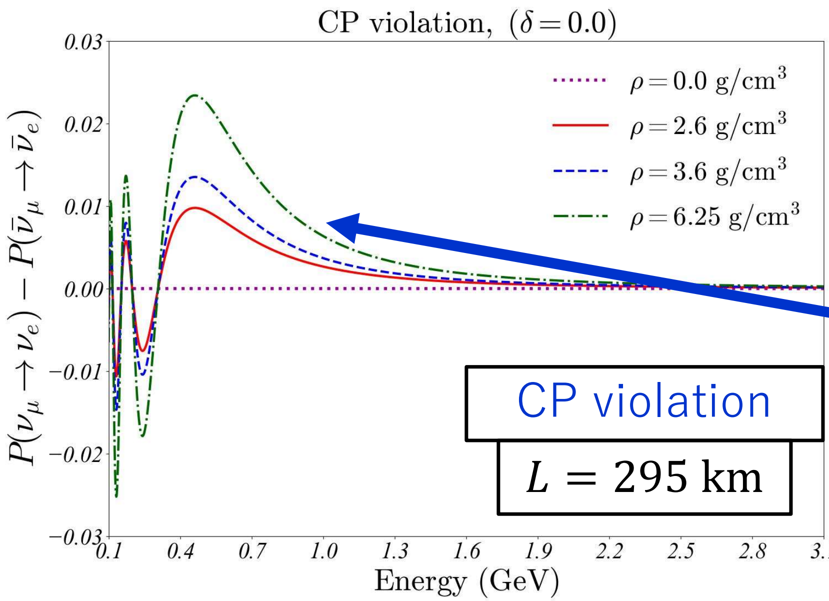
Electron neutrino

Same baseline with T2K

Not so long distance

Neutrino go through only crust

CP violation and T violation in matter



Assumption

$\boldsymbol{\delta = 0^\circ}$

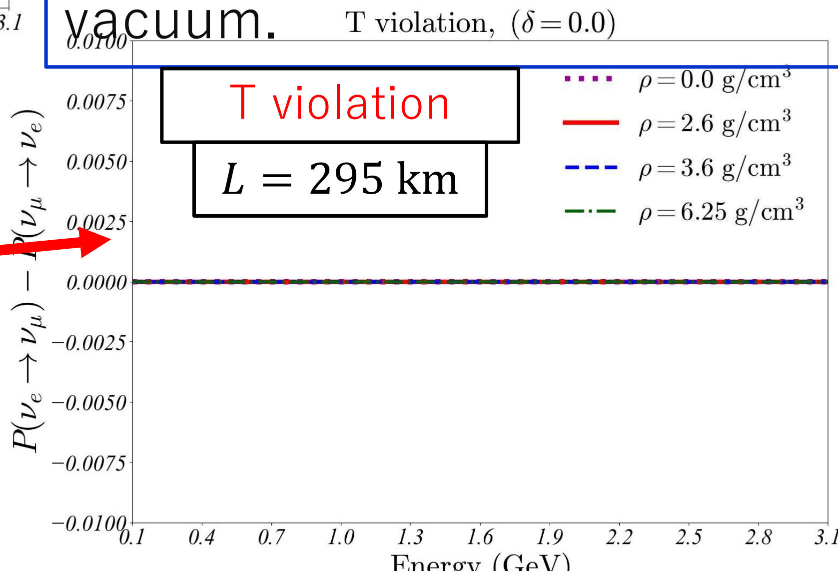
(No CP violation)

Normal Ordering

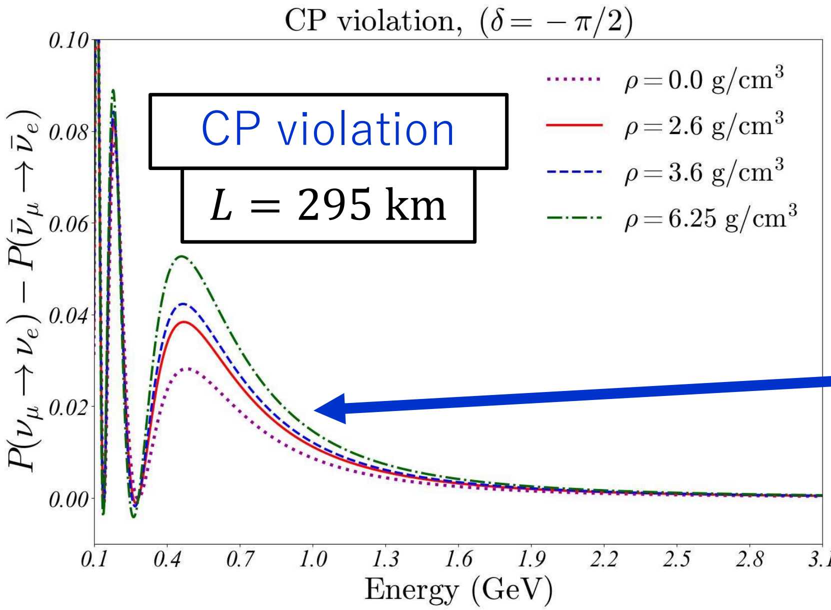
Fake CP violation exists

As long as the matter density is non-zero, i.e. not in a vacuum.

No fake T violation, regardless of matter effects.



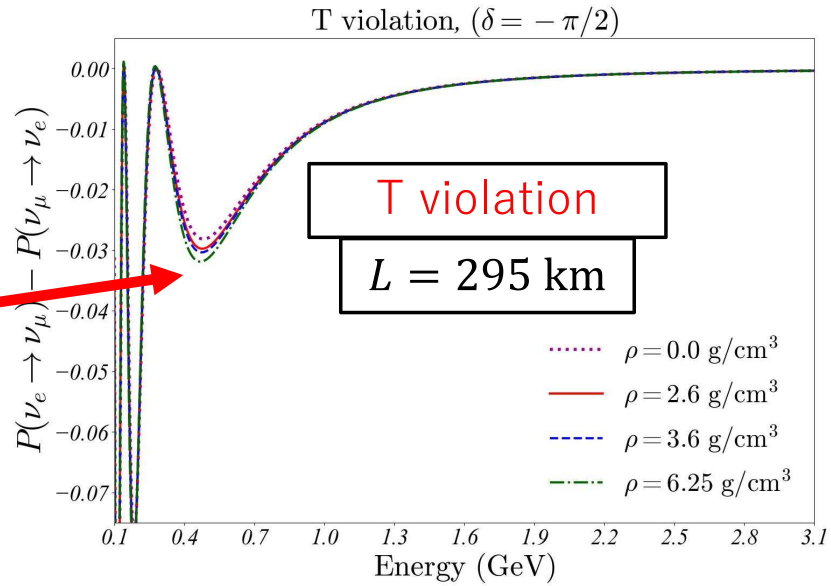
CP violation and T violation in matter



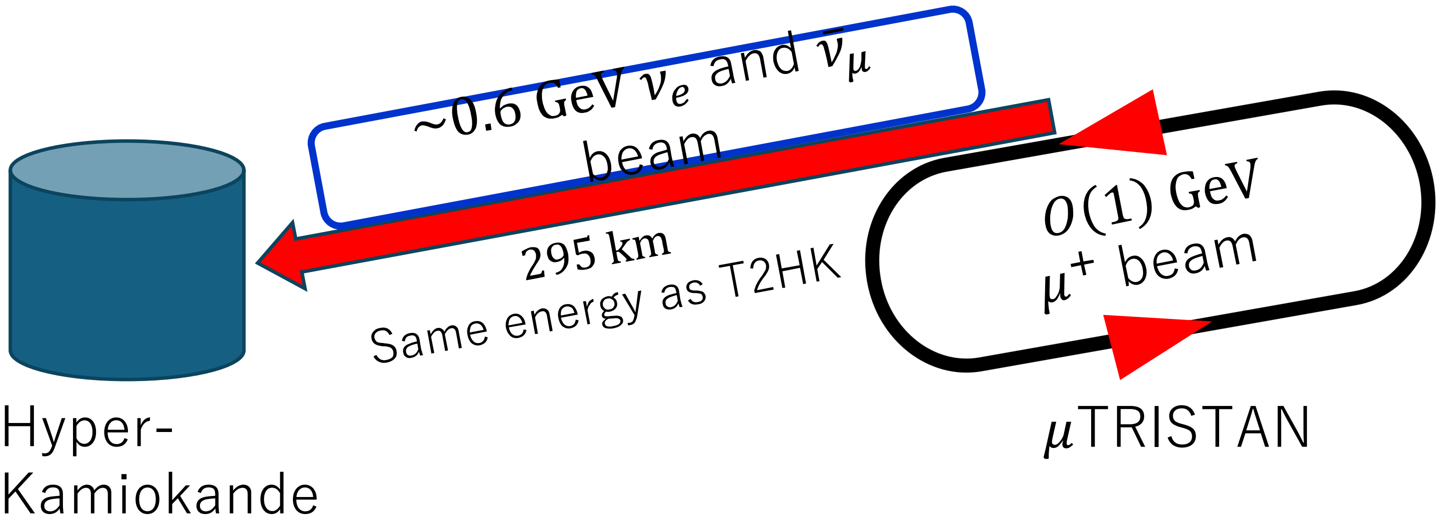
Assumption
$\delta = 270^\circ$
(Maximally CP violation)
Normal Ordering

Dependence on matter density is **large**

・Dependence on matter density is **tiny**



Future neutrino factory



$P(\nu_e \to \nu_\mu)$ from μTRISTAN

$P(\nu_\mu \to \nu_e)$ from T2HK

combine

T violation $P(\nu_e \to \nu_\mu) - P(\nu_\mu \to \nu_e)$

Robust test of CP phase free from matter effects!

Probability of $\nu_e \to \nu_\mu$

At the HK, in principle, ν_μ and $\bar{\nu}_\mu$ are distinguished by neutron tagging method.
We can define the oscillation probability $P(\nu_e \to \nu_\mu)$ as

Perfect charge identification

$$P(\nu_e \to \nu_\mu) = \frac{N_{\mathrm{far}}^{\nu_e \to \nu_\mu}}{N_{\mathrm{near}}^{\nu_e \to \nu_e}}$$

Non perfect charge identification

$$P(\nu_e \to \nu_\mu) = \frac{\left(N_{\mathrm{far}}^{\nu_e \to \nu_\mu} + N_{\mathrm{far}}^{\bar{\nu}_\mu \to \bar{\nu}_\mu}\right) - N_{\mathrm{far}}^{\bar{\nu}_\mu \to \bar{\nu}_\mu}\Big|_{\mathrm{T2HK}}}{N_{\mathrm{near}}^{\nu_e \to \nu_e}}$$

Statistical analysis

Definition of χ^2_{TV}

$$\chi^2_{\mathrm{TV}} \equiv \sum_j \frac{\left[P_j^{\mathrm{TV}}(\delta_0,\ \rho_0) - P_j^{\mathrm{TV}}(\delta^{\mathrm{test}},\ \rho^{\mathrm{test}})\right]^2}{\left(\Delta P_j{}^{\mathrm{TV}}\right)^2}$$

$$P_j^{\mathrm{TV}}(\delta, \rho) \equiv P_j(\nu_e \to \nu_\mu) - P_j(\nu_\mu \to \nu_e)\Big|_{\mathrm{T2HK}}$$

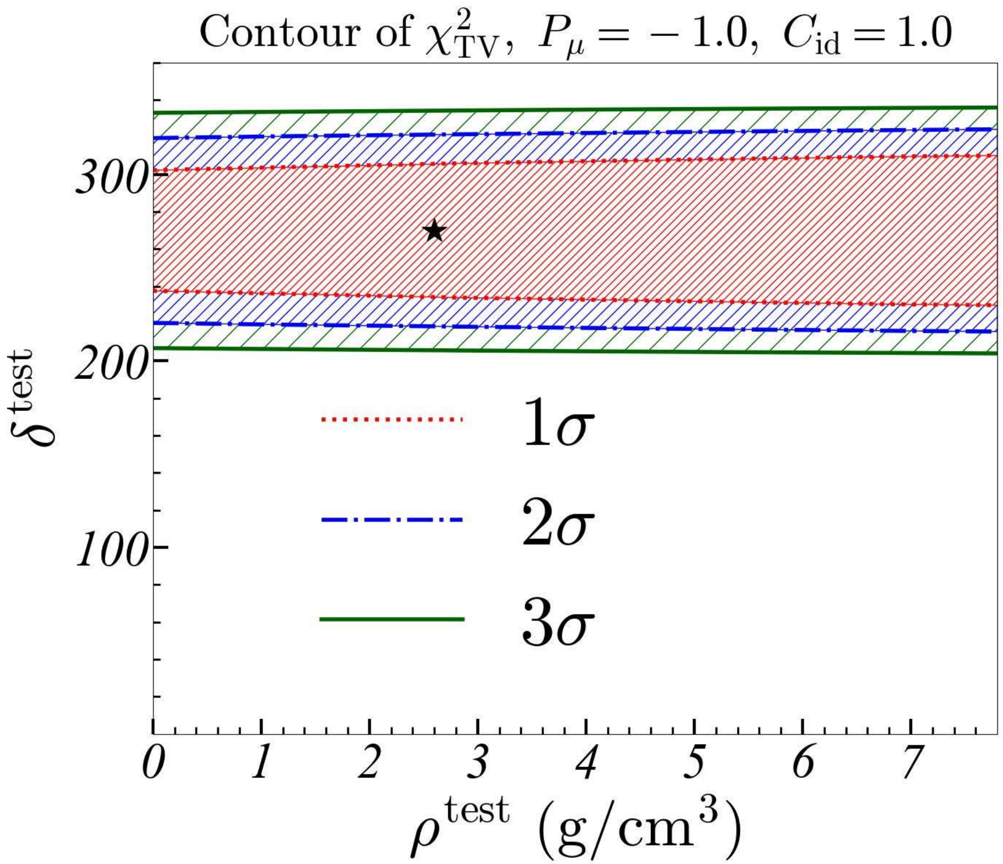
j runs over energy bins, ρ is matter density of the Earth
In this study, we consider only statistical error.
The ΔP_j^{TV} is obtained by modified oscillation probability

Reference values

$\Delta m_{21}^2/10^{-5}$ eV	$\Delta m_{31}^2/10^{-3}$ eV	θ_{12}	θ_{13}	θ_{23}
7.43	2.432	33.9°	8.49°	48.1°

The reference values are the arithmetic average of bfp in Particle Data Group 2022.
In this study, we consider only Normal Ordering.

Result of T violation

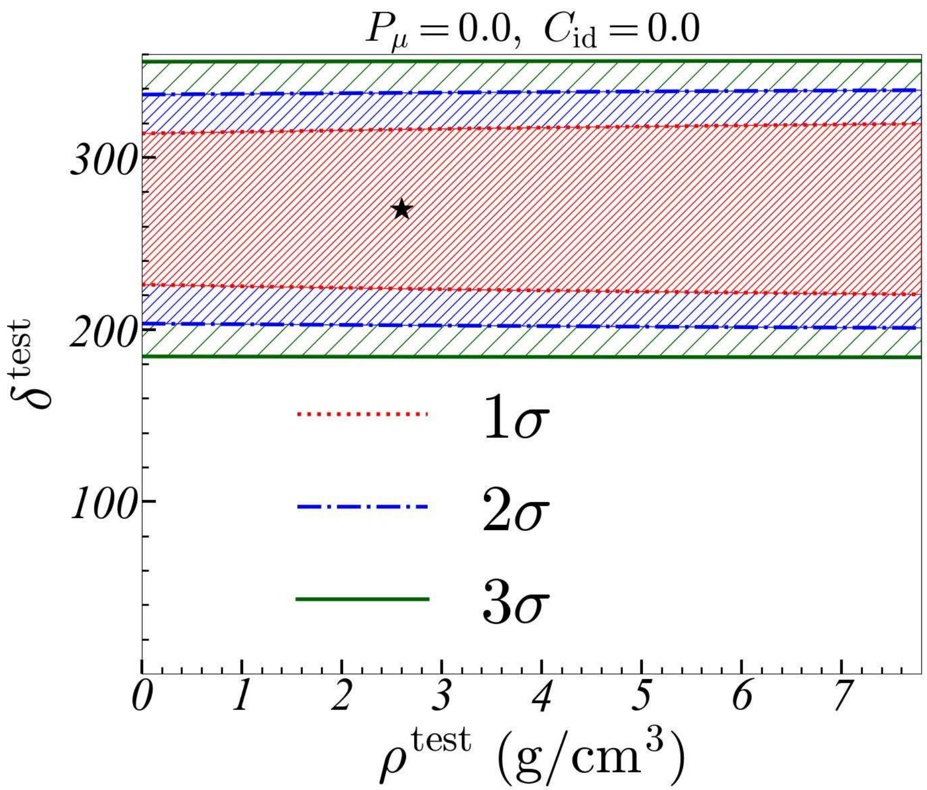


Free from matter density!

True value
$\delta_0 = 270^\circ$
$\rho_0 = 2.6\ \mathrm{g/cm^3}$

- $\chi^2_{\rm TV}$ only depends on δ.
- The confidence level as that for a 1 d.o.f., i.e., $\chi^2 = 1$ as 1σ.
- For the best case, perfect $C_{\rm id}$ and P_μ, the CP angle δ is determined to be about $\pm 30^\circ$.

Result of T violation



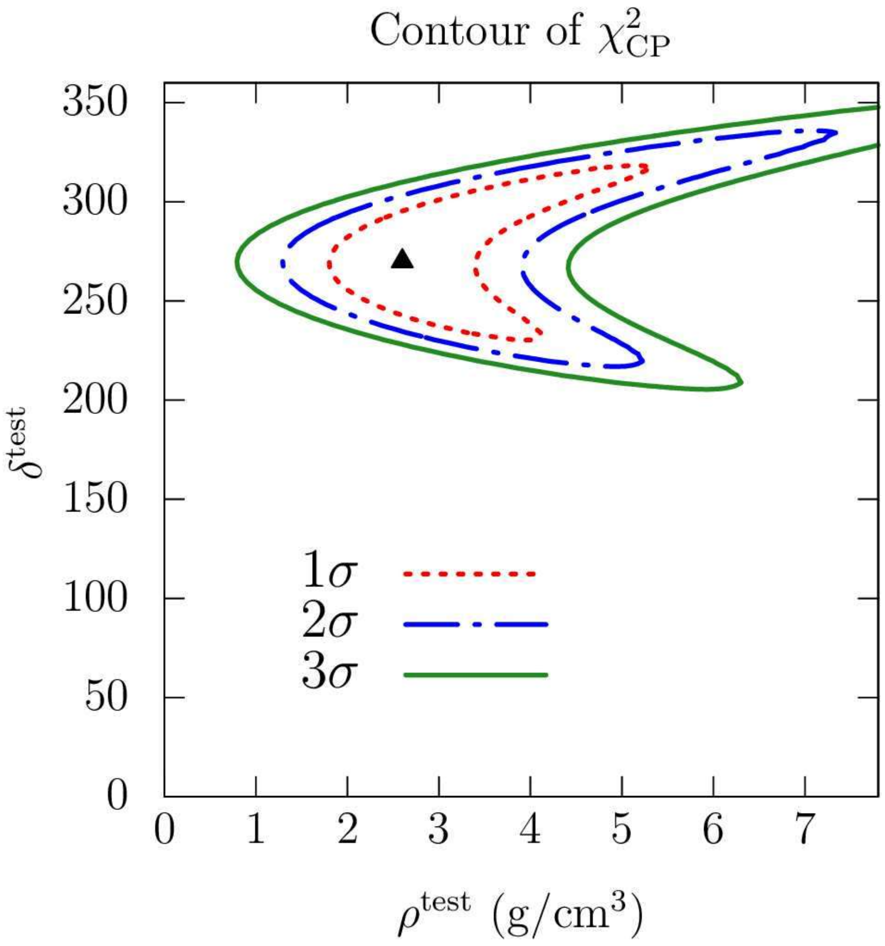
Free from matter density!

- $\chi^2_{\rm TV}$ only depends on δ.
  - For the worst case,

No $C_{\rm id}$ and P_μ,
CP (or T) conserving point,
$\delta = 0^\circ$ and 180° can be excluded at the level of 3σ.

  - With no ability of distinguish ν_μ and $\bar{\nu}_\mu$, the sensitivities do not change significantly.

Comparing with CP violation



- We define $\chi^2_{\mathbf{CP}}$ in a similar way as T violation.
- **Non-trivial $\boldsymbol{\rho}$ dependence.**
- A good knowledge of the matter density profile will be necessary.
- T violation will be an important additional information for the measurement of the CP angle δ.

Answer Fundamental Question

CPT Theorem holds ?

Comparison between CP and T Violation

Cons Charge Identification ?

5. Summary

- CP Phase is the last piece. Measurement itself is important
- To measure it T violation is ideal since robust against matter effect
- muTRISTAN offers an opportunity to measure T Violation. Baseline is same as T2K !
- Comparison with CP violation is CPT test too!

- CPV in lepton mixing can be a source for matter dominance.