On the Flux Vacua in F-theory Compactifications

Hajime Otsuka
(Waseda U.)

arXiv:1706.09417

with
Y. Honma (National Tsing-Hua U.)

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Outline

- **Introduction**

- **Flux compactification in type IIB string**

- **Flux compactification in F-theory**

i) F-theory
ii) Brane superpotential
iii) Flux compactification

- **Conclusion**

Introduction

The standard model of particle physics

Gauge group : SU(3) × SU(2) × U(1)

Matter content :

	spin1/2	$SU(3)_c,\ SU(2)_L,\ U(1)_Y$
quarks (×3 families)	$Q^i{=}(u_L,\ d_L)^i$	(3, 2, 1/6)
	u^i_R	($\bar{3}$, 2, -2/3)
	d^i_R	($\bar{3}$, 1, 1/3)
leptons (×3 families)	$L^i\ {=}(\nu,\ e_L)^i$	(1, 2, -1/2)
	e^i_R	(1, 1, 1)
	spin0	
Higgs	$H{=}(H^+,\ H^0)$	(1, 2, -1/2)

	spin1	$SU(3)_c$,$SU(2)_L$,$U(1)_Y$
gluon	g	(8, 1, 0)
W bosons	$W^{\pm}\quad W^0$	(1, 3, 0)
B boson	B^0	(1, 1, 0)

Introduction

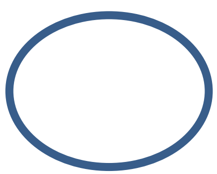
Problem:

No gravitational interaction in the standard model

String theory

A good candidate for the unified theory of the gauge and gravitational interactions

Closed string



Graviton: g_{MN}, B_{MN}, ϕ

Dp-brane

Ramond-Ramond field: C_{p+1}

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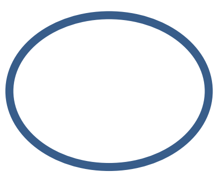

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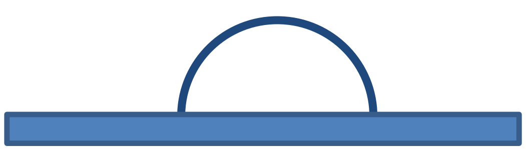


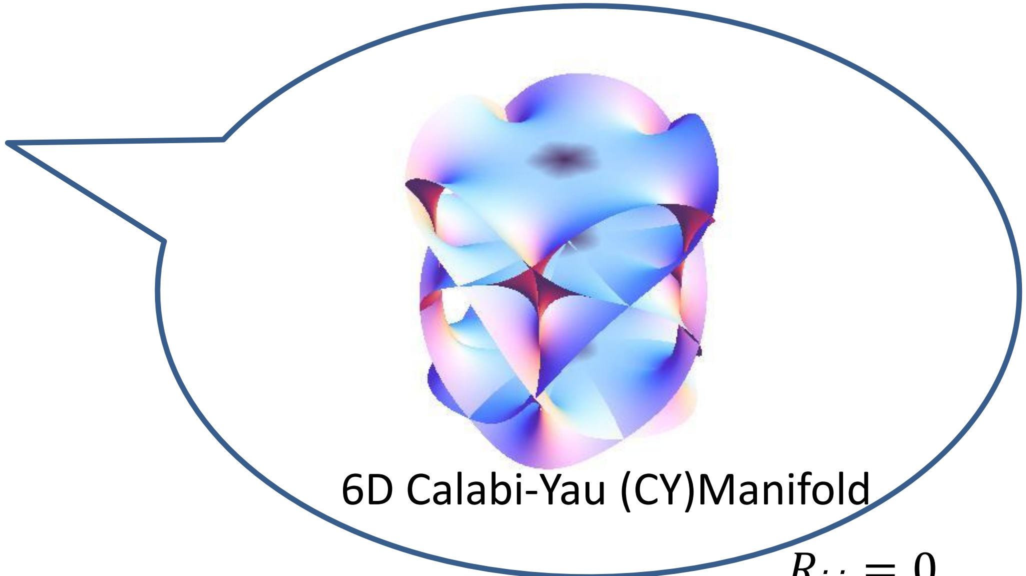
Dp-brane

Ramond-Ramond field: C_{p+1}

Gauge fields: A_{μ}

(Perturbative) superstring theory requires the extra 6 dimension.

10＝4＋6

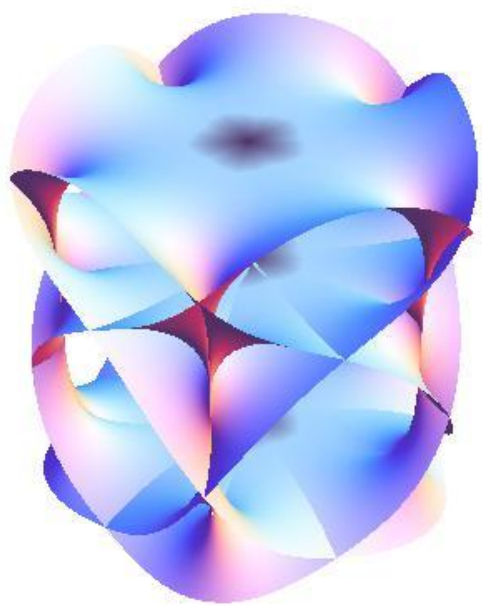


Extra 6D space should be compactified to be consistent with the observational and experimental data.

→ Stabilization of the extra dimensional space
<u>Moduli stabilization</u>

Two types of moduli fields (4D massless scalar fields) :

①Closed string moduli



i) Dilaton (τ)

$\langle \mathrm{Im}\tau \rangle = g_s^{-1}$

g_s: string coupling

ii) Kähler moduli (T)

Size of the internal cycles

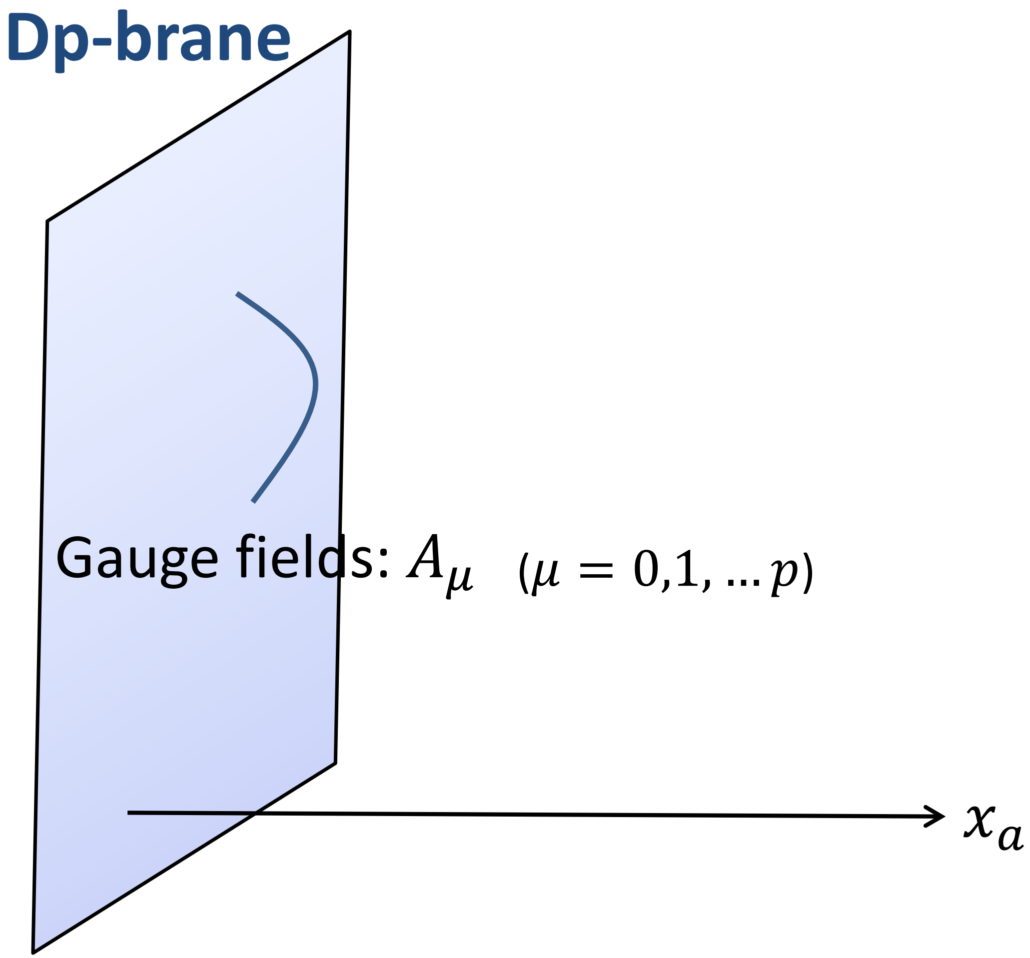
iii) Complex structure moduli (U)

Shape

<u>Moduli (axions) are ubiquitous in string compactifications</u>

→Inflation, SUSY breaking, Moduli problem

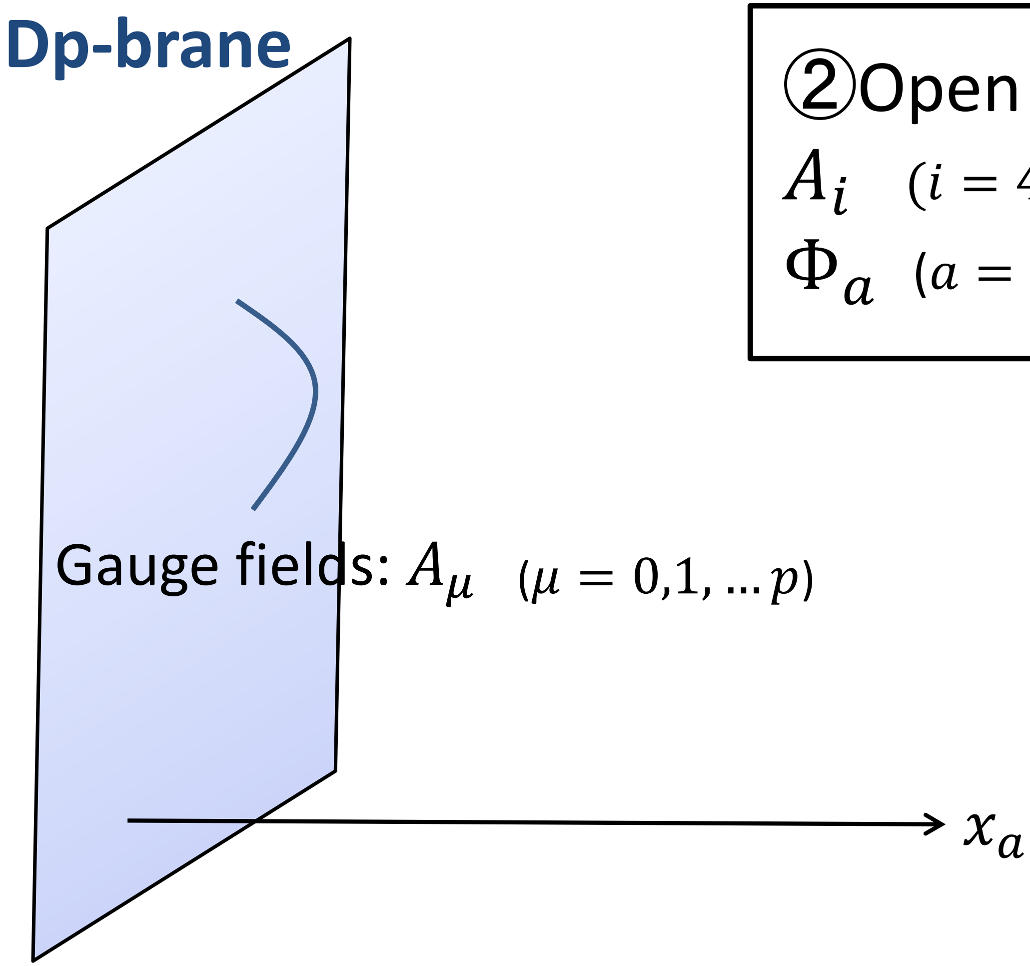
Two types of moduli fields (4D massless scalar fields) :



Gauge fields: A_μ $(\mu = 0,1, \ldots p)$

Scalar field : Φ_a $(a = p + 1, \ldots, 9)$

Two types of moduli fields (4D massless scalar fields) :



Scalar field : Φ_a $(a = p+1, \dots, 9)$

②Open string moduli

A_i $(i = 4,5, \dots, p)$

Φ_a $(a = p+1, \dots, 9)$

Two types of moduli fields (4D massless scalar fields):

①Closed string moduli
②Open string moduli

In this talk, we consider the stabilization of both the open and closed string moduli based on F-theory ("non-perturbative" description of IIB string).

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Flux compactification (Type IIB string on CY orientifold)

Low-energy effective theory based on 4D N=1 SUGRA

Kähler potential:

$$K = -\ln\left(i\int \Omega \wedge \overline{\Omega}\right) - \ln\left(-i(\tau - \bar{\tau})\right) - 2\ln(V)$$

$\Omega(U)$: hol. (3,0) form of CY
V : CY Volume

Flux induced potential=
"Gukov-Vafa-Witten (GVW) superpotential" [Gukov-Vafa-Witten '99]

$$W(\tau, U) = \int_{\mathrm{CY}} G_3(\tau) \wedge \Omega(U)$$

$G_3 = F_3 - \tau H_3$: three-form

Low-energy effective action based on 4D N=1 SUGRA

$$K = -\ln\left(i\int \Omega \wedge \overline{\Omega}\right) - \ln\left(-i(\tau - \bar{\tau})\right) - 2\ln(V(T))$$

$$W(\tau, U) = \int_{\mathrm{CY}} G_3(\tau) \wedge \Omega(U)$$

$$V = e^K \left(\sum_{I,J=\tau,U} K^{I\bar{J}} D_I W D_{\bar{J}} W + \underbrace{(K^{T\bar{T}} K_T K_{\bar{T}} - 3)}_{=\,\mathbf{0}} |W|^2 \right)$$

No-scale structure

$$D_I = \partial_I + K_I$$
$$K_I = \partial_I K$$

Dilaton and complex structure moduli are stabilized at

$$D_\tau W = D_U W = 0$$

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$= \mathbf{0}$

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$$W(\tau, z) = \int_{\mathrm{CY}} G_3(\tau) \wedge \Omega(U)$$

G_3 fluxes are constrained as the imaginary self-dual fluxes:

$$G_3 = i *_6 G_3$$

$G_3 = F_3 - \tau H_3$: three-form

Tadpole condition for C_4:

$$\int_{CY} H_3 \wedge F_3 + Q_{D3} = 0$$

Dilaton and complex structure moduli are stabilized at

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$G_3 = F_3 - \tau H_3$: three-form

Tadpole condition for C_4:

$$\int_{CY} C_4 \wedge G_3 \wedge \overline{G_3} \qquad \int C_4$$

$$\int_{CY} H_3 \wedge F_3 + Q_{D3} = 0$$

Moduli stabilization in type IIB string

The remaining Kähler moduli can be stabilized by the non-perturbative effects.

$$W = < W_{\text{flux}} > + Ae^{-aT}$$

- KKLT scenario ($< W_{\text{flux}} > \ll 1$) [Kachru-Kallosh-Linde-Trivedi ’03]
- LARGE volume scenario ($< W_{\text{flux}} > \sim O(1)$) [Balasubramanian-Berglund-Conlon-Quevedo ’05]

De Sitter vacua can be realized by introducing the anti D3-branes.

- Radiative moduli stabilization scenario

[Kobayashi-Omoto-Otsuka-Tatsuishi ’17]

Comments on F-term axion monodromy inflation

$$W(\tau, U) = \int_{\mathrm{CY}} G_3(\tau) \wedge \Omega(U) = \sum_\alpha (N_F^\alpha - \tau N_H^\alpha)\, \Pi_\alpha$$

Period vector:

S. Hosono, A. Klemm, S. Theisen and S. T. Yau ('95)

$$\Pi_\alpha = \int_{\gamma^\alpha} \Omega = \begin{pmatrix} 1 \\ U^i \\ \frac{1}{3!}\kappa_{ijk}U^iU^jU^k + \kappa_i U^i + \kappa_0 - \sum_\beta n_\beta^0 \left(\frac{2}{(2\pi i)^3}\mathrm{Li}_3(q^\beta) - \frac{d_i}{(2\pi i)^2}U^i \mathrm{Li}_2(q^\beta)\right) \\ -\frac{1}{2}\kappa_{ijk}U^jU^k - \kappa_{ij}U^j + \kappa_i - \frac{1}{(2\pi i)^2}\sum_\beta n_\beta^0 d_i \mathrm{Li}_2(q^\beta) \end{pmatrix}$$

$$q^{\beta_i} = e^{2\pi i d_i U^i}$$

Geometric corrections for U

➔ Non-trivial axion potential

Kobayashi-Oikawa-Otsuka '15

Phys.Rev. D**93** (2016) no.8, 083508.

Comments on F-term axion monodromy inflation

$$K(\tau,U) = -\ln\left(i\int \Omega\wedge\overline{\Omega}\right) - \ln\left(-i(\tau-\bar{\tau})\right)$$

$$W(\tau,U) = \int_{\mathrm{CY}} G_3(\tau)\wedge\Omega(U) = \sum_\alpha (N_F^\alpha - \tau N_H^\alpha)\,\Pi_\alpha$$

$(h^{1,2}-2)$ complex structure moduli and dilaton: z

Other complex structure moduli : U_1, U_2

At the leading level, we consider the following ansatz:

Hebecker-Mangat-Rompineve-Witkowski '15

$$K(\tau,U) = -\ln[f_0(\mathrm{Re}z, \mathrm{Re}\,U_1, \mathrm{Re}U_2)]$$

$$W(\tau,U) = g_0(z) + g_1(z)(U_2 + NU_1)$$

N : Integer (flux)

f_0 : Kähler potential at the LO

$g_{0,1}(z)$: flux-induced potential at the LO

Comments on F-term axion monodromy inflation

When we redefine the moduli, $\Psi \equiv U_2 + NU_1,$

$\Phi \equiv U_2,$

$$K(S,U) = -\ln\left[f_0\left(\mathrm{Re}\, z, (\mathrm{Re}\,\Psi - \mathrm{Re}\,\Phi)/N, \mathrm{Re}\,\Phi\right)\right],$$

$$W(S,U) = g_0(z) + g_1(z)\Psi,$$

$\mathrm{Re}\,\Phi, z, \Psi$ would be stabilized at supersymmetric minimum,

$$D_I W = 0 \text{ with } I = \Psi, z$$

$$K_\Phi = 0$$

$$D_I W = W_I + K_I W$$

$$W_I = \partial W / \partial \Phi^I$$

$$K_I = \partial K / \partial \Phi^I$$

- $\mathrm{Im}\,\Phi$ remains massless at this stage.
- Geometric corrections generate its potential.

Comments on F-term axion monodromy inflation

Period vector :

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$$\Pi_\alpha = \begin{pmatrix} 1 \\ U^i \\ \frac{1}{3!}\kappa_{ijk}U^iU^jU^k + \kappa_i U^i + \kappa_0 - \sum_\beta n^0_\beta\left(\frac{2}{(2\pi i)^3}\mathrm{Li}_3(q^\beta) - \frac{d_i}{(2\pi i)^2}U^i\mathrm{Li}_2(q^\beta)\right) \\ -\frac{1}{2}\kappa_{ijk}U^jU^k - \kappa_{ij}U^j + \kappa_i - \frac{1}{(2\pi i)^2}\sum_\beta n^0_\beta d_i \mathrm{Li}_2(q^\beta) \end{pmatrix}$$

It induces the geometric corrections to the Kähler potential and superpotential:

$$q^{\beta_i} = e^{2\pi i d_i U^i}$$

$$\Delta K \simeq -\frac{f_1^{(1)}}{\langle f_0\rangle}\left(\frac{2}{\pi} + \frac{\Psi + \bar{\Psi} - \Phi - \bar{\Phi}}{N}\right)\cos\left(-i\pi\frac{\Psi - \bar{\Psi} - \Phi + \bar{\Phi}}{N}\right)e^{-\pi\frac{\Psi+\bar{\Psi}-\Phi-\bar{\Phi}}{N}},$$

$$\Delta W \simeq \left(g_2^{(1)} + \frac{g_3^{(1)}}{N}(\Psi - \Phi)\right)e^{-2\pi\frac{\Psi-\Phi}{N}},$$

$$\Psi \equiv U_2 + NU_1,$$

$$\Phi \equiv U_2,$$

○ One can extract the potential of $\mathrm{Im}\,\Phi$

Comments on F-term axion monodromy inflation

Inflation potential depends on the Kähler moduli stabilization.

ϕ : Canonically normalized axion

① KKLT scenario

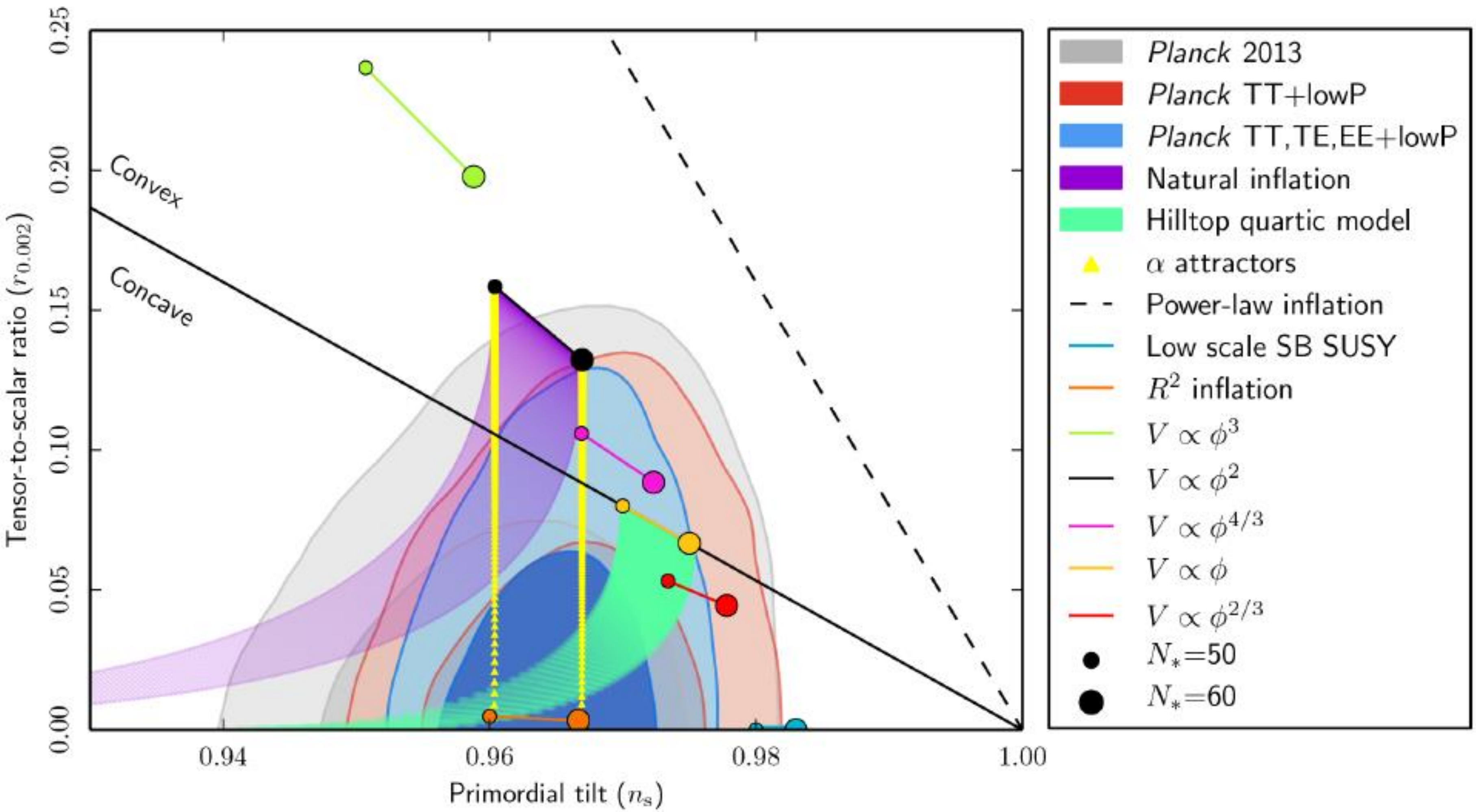
$$V_{\rm inf} \simeq \Lambda_1 \left(1 - \cos\frac{\phi}{M_1}\right) + \Lambda_2 \phi \sin\frac{\phi}{M_1}$$

$$M_1, M_3 \simeq N/2\pi$$

② LARGE volume scenario

$$V_{\rm inf} \simeq \Lambda_4 \phi^2 + \Lambda_5 \phi \sin\left(\frac{\phi}{M_3}\right) + \Lambda_6 \left(1 - \cos\left(\frac{\phi}{M_3}\right)\right)$$

Planck data

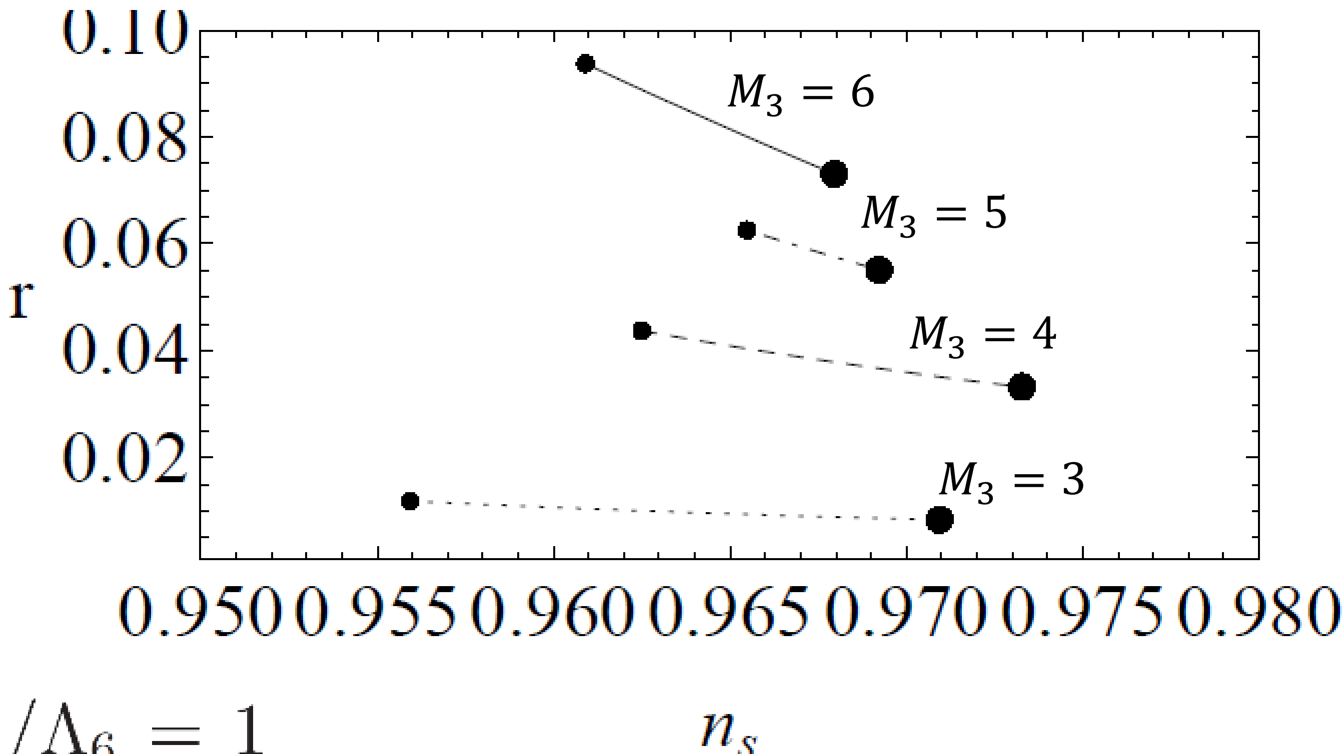

Tensor-to-scalar ratio ($r_{0.002}$)
Primordial tilt (n_s)
0.00
0.05
0.10
0.15
0.20
0.25
0.94
0.96
0.98
1.00
Convex
Concave
Planck 2013
Planck TT+lowP
Planck TT,TE,EE+lowP
Natural inflation
Hilltop quartic model
α attractors
Power-law inflation
Low scale SB SUSY
R^2 inflation
$V \propto \phi^3$
$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
N_*=50
N_*=60

Kobayashi-Oikawa-Otsuka '15
Phys.Rev. D**93** (2016) no.8, 083508.

② LARGE Volume Scenario

(Mixture of polynomial functions and sinusoidal functions)

$$V_{\rm inf} \simeq \Lambda_4 \phi^2 + \Lambda_5 \phi \sin\left(\frac{\phi}{M_3}\right) + \Lambda_6 \left(1 - \cos\left(\frac{\phi}{M_3}\right)\right)$$



$\Lambda_4/\Lambda_6 = 1$

$\Lambda_5/\Lambda_6 = 5$

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Type IIB action in Einstein frame(with other fields set to 0):

$$L_{IIB} = \sqrt{g}\left(R - \frac{|\partial\tau|^2}{2(\mathrm{Im}\,\tau)^2}\right)$$

where $\tau = C_0 + ie^{-\phi}$, ($\langle \mathrm{Im}\tau \rangle = g_s^{-1}$, g_s: string coupling)

This action is invariant under $SL(2,Z)$:

$$\tau \to \tau + 1\,, \quad \tau \to -1/\tau$$

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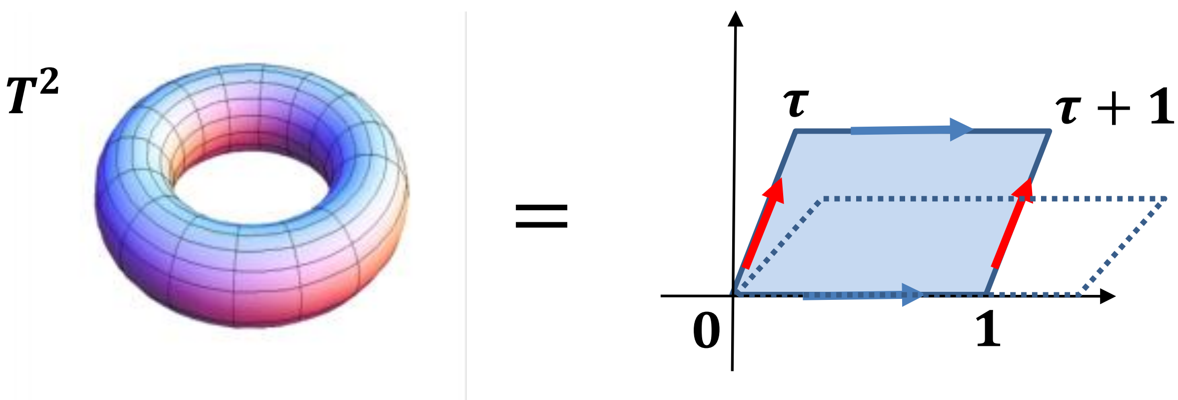
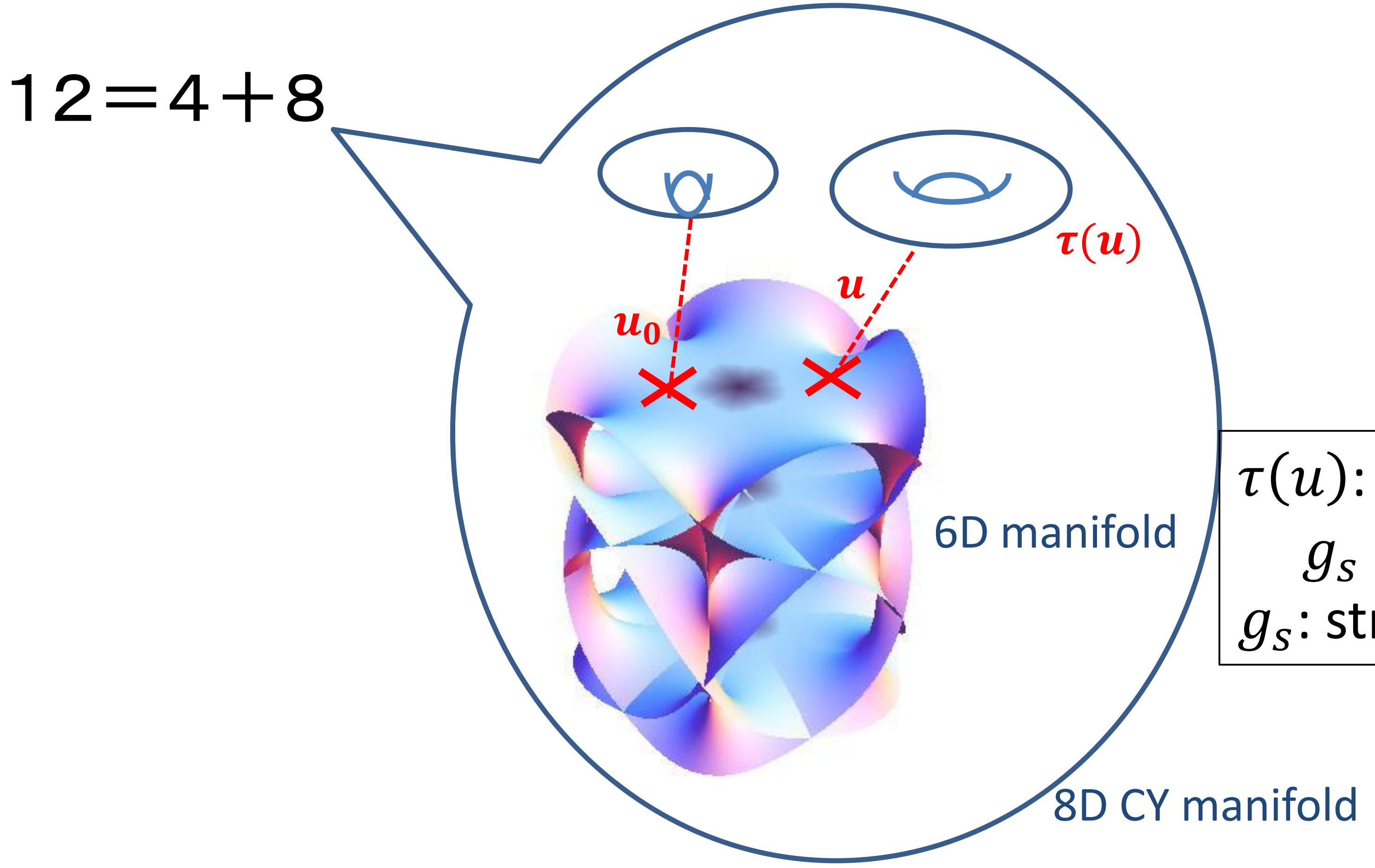
Interpret τ as complex structure of auxiliary torus T^2 (Vafa '96)



F-theory is defined in "12"D spacetime



$\tau(u)$: dilaton

$g_s = \langle \mathrm{Im}\,\tau \rangle^{-1}$

g_s: string coupling

String coupling can be taken as $g_s = \langle \mathrm{Im}\,\tau \rangle^{-1} > 1$.

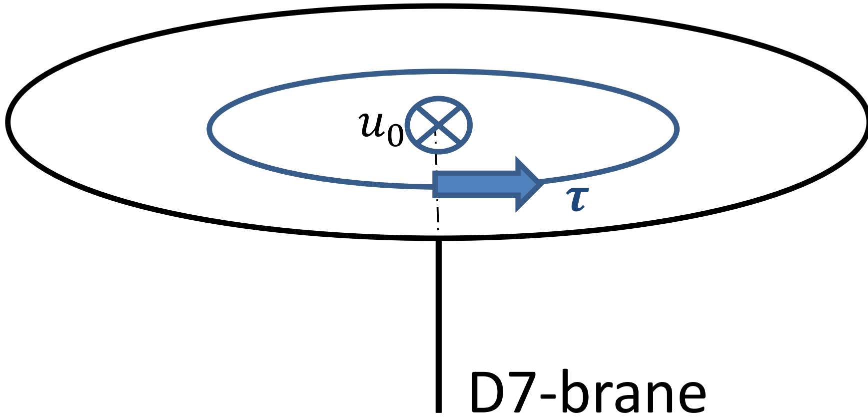
F-theory = "non-perturbative" description of type IIB

D7-brane looks like "cosmic string" in ambient space

(Greene, Shapere, Vafa, Yau, '89)

Metric:

$$ds_{10}^2 = \underbrace{-dt^2 + \sum_{i=1}^{7} dx_i^2}_{\text{D7}} + H(u,\bar{u})du d\bar{u}$$



D7-brane has magnetic charge under C_0

$$1 = \oint_{u=u_0} dC_0 = C_0\left(ue^{2\pi i}\right) - C_0(u)$$

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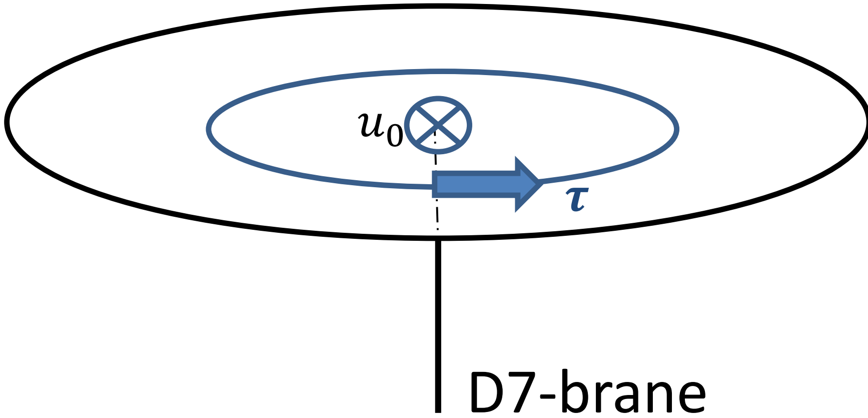

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$\mathrm{Re}\tau = C_0$

Near D7-brane : $\tau \simeq \dfrac{1}{2\pi i}\ln(u - u_0)$

D7-brane location : $\tau(u_0) \to i\infty$

(T^2 degenerate at $u = u_0$.)

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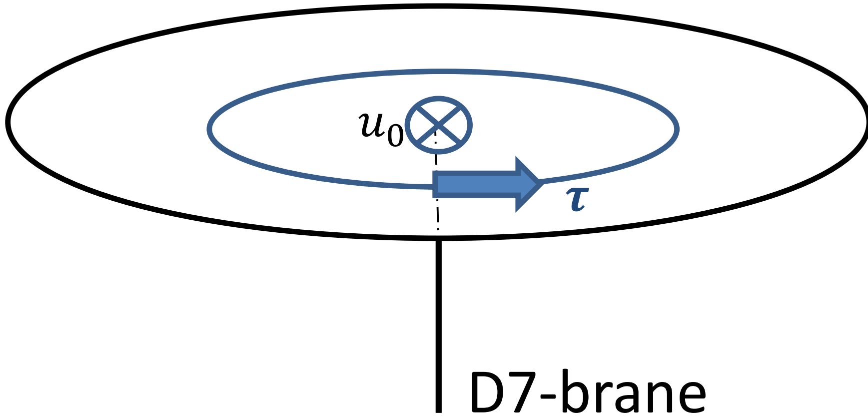

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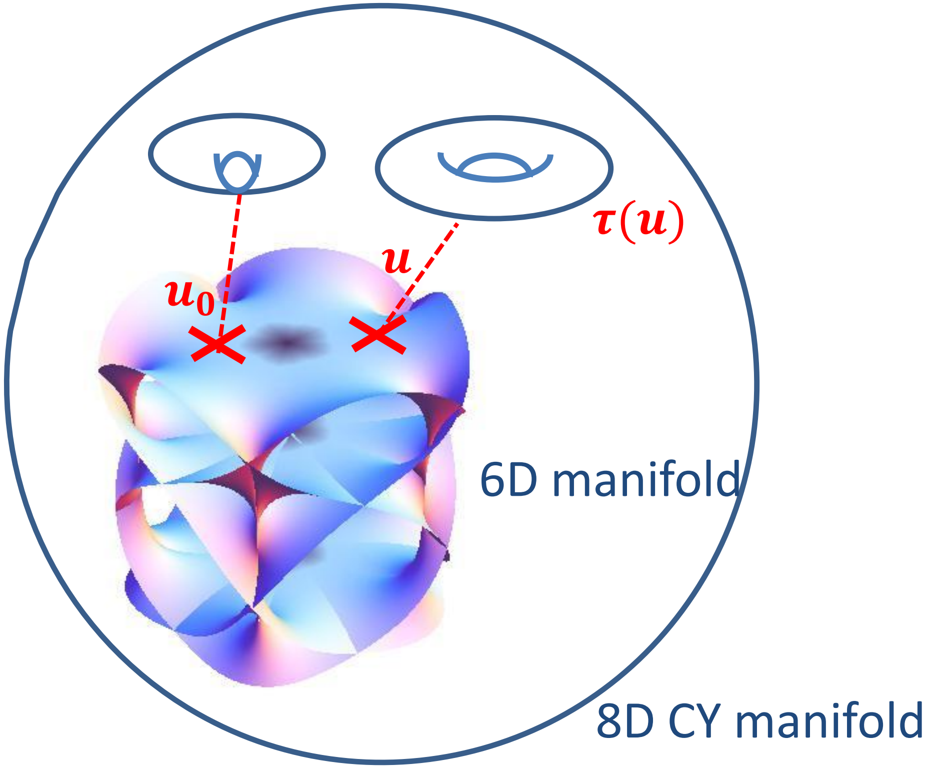


[1, 0]D7 brane ➔[p, q] 7-branes

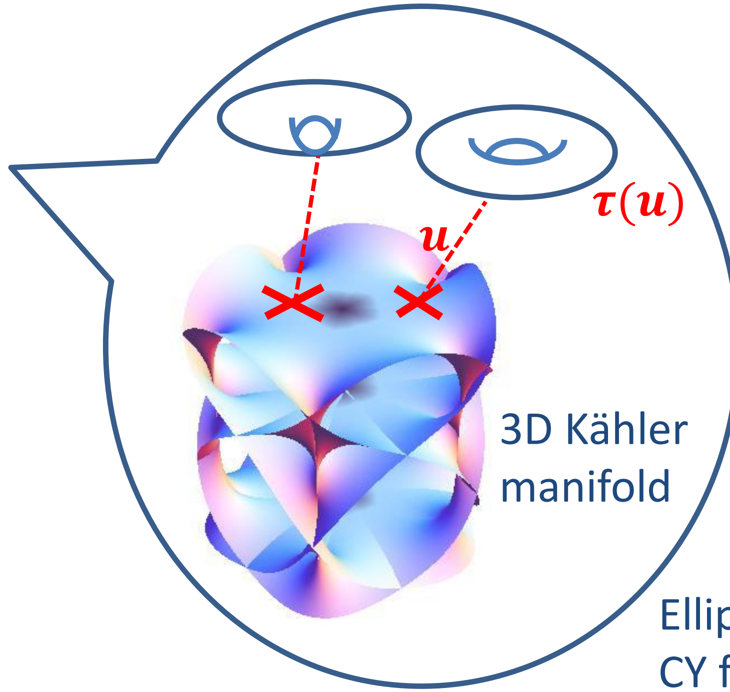
$$y^2 = x^3 + xf + g \qquad\qquad \Delta = 4f^3 + 27g^2$$

ord(f)	ord(g)	ord(Δ)	fiber type	singularity type
≥ 0	≥ 0	0	smooth	none
0	0	n	I_n	A_{n-1}
≥ 1	1	2	II	none
1	≥ 2	3	III	A_1
≥ 2	2	4	IV	A_2
2	≥ 3	$n+6$	I_n^*	D_{n+4}
≥ 2	3	$n+6$	I_n^*	D_{n+4}
≥ 3	4	8	IV^*	E_6
3	≥ 5	9	III^*	E_7
≥ 4	5	10	II^*	E_8

M. Bershadsky et al., arXiv:9605200.

F-theory is defined in “12”D spacetime

12＝4＋8



Elliptically fibered CY fourfold (CY4)

$\tau(u)$: dilaton

$g_s = \langle \mathrm{Im}\, \tau \rangle^{-1}$

g_s: string coupling

①7-branes exist at the singular limit of torus

②String coupling > 1
(“Non-perturbative” description of type IIB superstring)

③ Both open and closed string moduli are involved.

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Brane Superpotential:

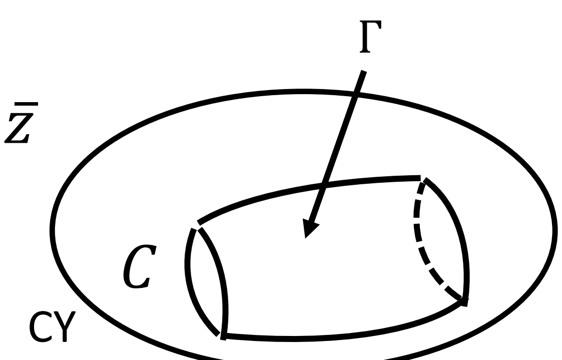
$$W_{\mathrm{brane}} = \widehat{N_b} \int_{\Gamma_b, \partial\Gamma_{\mathrm{b}} = \mathrm{C}} \Omega$$

For branes wrapping on the whole CY, open string partition function is given by holomorphic Chern-Simons theory [Witten '92]:

$$W = \int_{CY} \Omega \wedge \mathrm{Tr}[\mathrm{A} \wedge \bar{\partial} A + \frac{2}{3} A \wedge A \wedge A]$$

Lower dimensional branes wrapping on holomorphic submanifold C can be obtained by dimensional reduction $A \to \phi$ [Aganagic-Vafa '00]

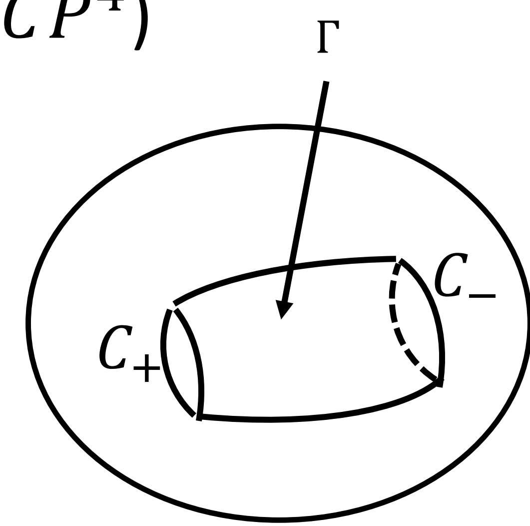
$$W_{\mathrm{brane}}(\psi, \phi) = \int_C \Omega_{ijz}\, \phi^i \bar{\partial}_z \phi^j dz d\bar{z}$$
$$= \int_{\Gamma, \partial\Gamma = \mathrm{C}} \Omega$$



Brane superpotential (Open mirror symmetry)

Mirror Quintic CY3 (degree 5 hypersurface in CP^4)

$$P(\psi) = \sum_{i=1}^{5} y_i^5 - 5\psi y_1 y_2\, y_3\, y_4\, y_5 = 0$$



Mirror Quintic CY3

Let us consider holomorphic 2-cycles where the brane wraps[Morrison-Walcher '07]

$$C_\pm : y_1 + y_2 = 0, y_3 + y_4 = 0, y_5^2 \pm \sqrt{5\psi} y_1 y_3 = 0$$

$$W = \int_\Gamma \Omega$$

No moduli dependence at fixed $C_\pm$!

Brane deformation : $\partial\Gamma$ into (geometrically non-holomorphic) curve surrounded by a holomorphic divisor

Brane superpotential (Open mirror symmetry)

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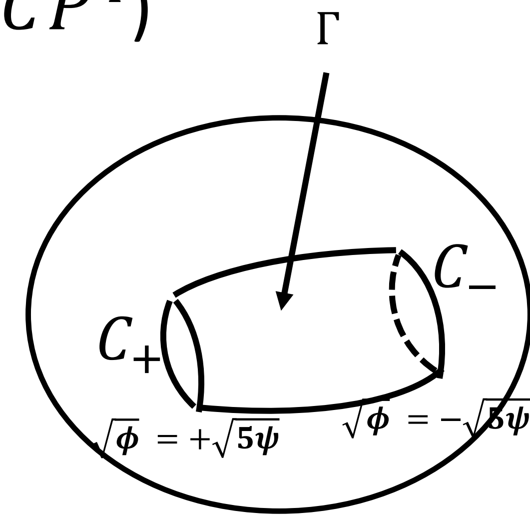


Mirror Quintic CY3

Continuous deformation of $C_{\pm}$:

(Hol. divisor defined by a degree 4 polynomial)

$$Q(\phi) = y_5^4 - 5\phi y_1 y_2 y_3 y_4 = 0$$

Brane deformation

Brane superpotential:

$$W_{\text{brane}}(\psi,\phi) = \int_{\Gamma} \Omega(\psi,\phi) = \int_{\hat{\Gamma},\partial\hat{\Gamma}=Q(\phi)} F \wedge \Omega$$

which is related to D7-brane with magnetic flux F

[Grimm-Ha-Klemm-Klevers '09]

CY3+brane →CY4 without brane

○In the toric language, the previous system corresponds to

A-model : Quintic CY3 over CP^1

[Berglund-Mayr '98,
Grimm-Ha-Klemm-Klevers '09,
Jockers-Mayr-Walcher '09]

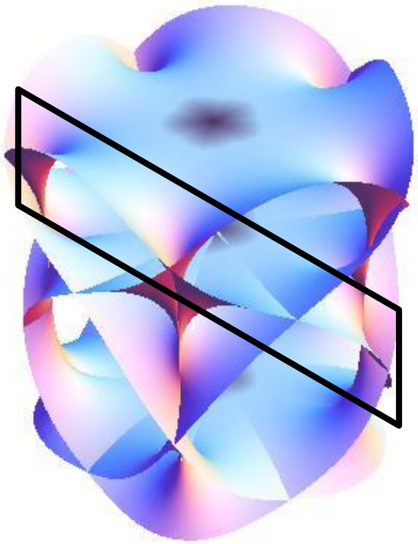
$$l_1 = (-4,0,1,1,1,1,-1,-1,0)$$
$$l_2 = (-1,1,0,0,0,0,1,-1,0)$$
$$l_3 = (0,-2,0,0,0,0,0,1,1)$$

$l_1 + l_2$: Quintic CY3
l_2: brane deformation
l_3: base CP^1

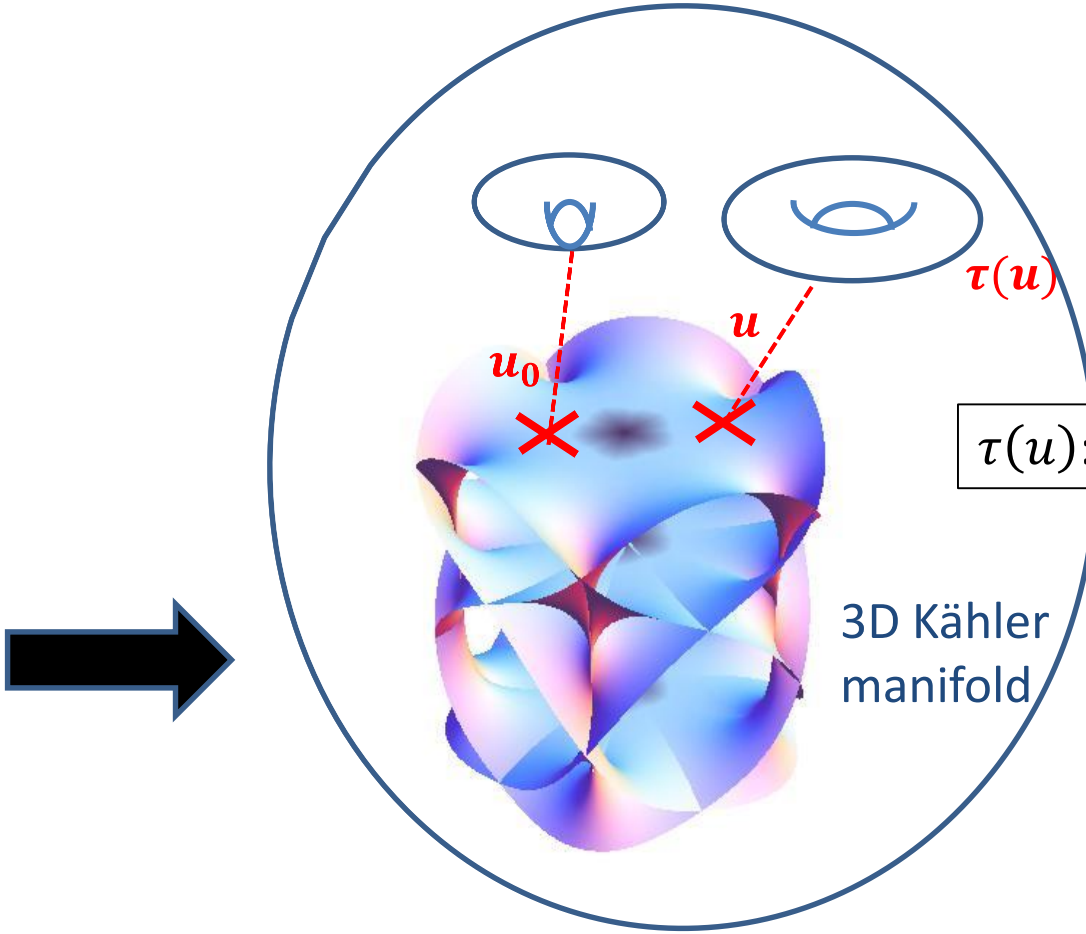
B-model : Elliptically fibered CY4

[Berglund-Mayr '98]

F-theory compactification on CY4



CY3+branes



$\tau(u)$: dilaton

Elliptically fibered CY4

Complex structure moduli of CY3
Dilaton
Open string (position) moduli

Complex structure moduli of CY4

Flux compactification in F-theory on CY4

○GVW superpotential + brane superpotential in type IIB
=G_4-flux superpotential in F-theory [Grimm-Ha-Klemm-Klevers '09,…]

$$W = \int_{\mathrm{CY4}} G_4 \wedge \Omega$$

○Imaginary self-dual three-form fluxes in type IIB
=correspond to self-dual G_4-fluxes [Gukov-Vafa-Witten '99]

$$G_4 = * G_4$$

○ Tadpole conditions

$$\frac{\chi}{24} = n_{D3} + \frac{1}{2}\int_{CY4} G_4 \wedge G_4$$

χ: Euler number of CY4
n_{D3}: # of D3

Flux compactification in F-theory on CY4

○The orientifold limit of F-theory

[Dasgupta-Rajesh-Sethi '99, Denef-Douglas-Florea-Grassi-Kachru '05]

○K3 × K3 background [Berglund-Mayr '13]

○Elliptically fibered CY4 in the large complex structure limit

Outline

- Introduction
- Flux compactification in type IIB string
- Flux compactification in F-theory

i) F-theory
ii) Brane superpotential
iii) Flux compactification

- **Conclusion**

F-theory on elliptically fibered CY4 → 4D N=1 supergravity

In 4D N=1 SUGRA

Kähler potential:

$$K = -\ln \int_{\mathrm{CY4}} \Omega \wedge \bar{\Omega} - 2\ln V$$
$$= -\ln(\Pi^i \eta_{ij} \bar{\Pi}^j) - 2\ln V$$

Superpotential:

$$W = \int_{\mathrm{CY4}} G_4 \wedge \Omega = n^i \eta_{ij} \Pi^j$$

$\Pi_i = \int_{\gamma^i} \Omega$: Fourfold periods

γ^i : Homology basis of $H_4^H(CY4, \boldsymbol{Z})$

η_{ij} : Topological intersection matrix

n^i : Quantized four-form fluxes

V : Volume of 3D Kähler base

F-theory compactification on elliptically fibered CY4

z: Complex structure modulus

S: Dilaton

z_1: Open string modulus

n_i: Quantized fluxes

$$\Pi_1 = 1,\ \Pi_2 = z,\ \Pi_3 = -z_1,\ \Pi_4 = S,$$
$$\Pi_5 = 5Sz,\ \Pi_6 = \frac{5}{2}z^2,\ \Pi_7 = 2z_1^2,\ \Pi_8 = -\frac{5}{2}Sz^2 - \frac{5}{3}z^3,$$
$$\Pi_9 = -\frac{2}{3}z_1^3,\ \Pi_{10} = -\frac{5}{6}z^3,\ \Pi_{11} = \frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4,$$

Kähler potential:

$$K = -\ln\left[-i(S-\overline{S})\right] - \ln\left[\frac{5i}{6}(z-\overline{z})^3 + \boxed{\frac{i}{S-\overline{S}}\left(-\frac{1}{6}(z_1-\overline{z_1})^4 + \frac{5}{12}(z-\overline{z})^4\right)}\right] - 2\ln\mathcal{V}$$

NLO in g_s correction

Superpotential:

$$W = n_{11} + n_{10}S + n_8 z + n_6 Sz + \frac{5}{2}\left(\frac{n_5}{5} + \frac{2n_6}{5}\right)z^2 - \frac{5n_4}{6}z^3 - n_2\left(\frac{5}{2}Sz^2 + \frac{5}{3}z^3\right) - n_9 z_1 - \frac{n_7}{2}z_1^2$$
$$- \frac{2n_3}{3}z_1^3 + n_1\left(\frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4\right)$$

F-theory compactification on elliptically fibered CY4

z: Complex structure modulus

S: Dilaton

z_1: Open string modulus

n_i: Quantized fluxes

$$\Pi_1 = 1,\ \Pi_2 = z,\ \Pi_3 = -z_1,\ \Pi_4 = S,$$
$$\Pi_5 = 5Sz,\ \Pi_6 = \frac{5}{2}z^2,\ \Pi_7 = 2z_1^2,\ \Pi_8 = -\frac{5}{2}Sz^2 - \frac{5}{3}z^3,$$
$$\Pi_9 = -\frac{2}{3}z_1^3,\ \Pi_{10} = -\frac{5}{6}z^3,\ \Pi_{11} = \frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4,$$

Kähler potential:

$$K = -\ln\left[-i(S-\overline{S})\right] - \ln\left[\frac{5i}{6}(z-\overline{z})^3 + \boxed{\frac{i}{S-\overline{S}}\left(-\frac{1}{6}(z_1-\overline{z_1})^4 + \frac{5}{12}(z-\overline{z})^4\right)}\right] - 2\ln\mathcal{V}$$

NLO in g_s correction

Superpotential:

$$W = n_{11} + n_6 Sz + \frac{5}{2}\left(\frac{n_5}{5} + \frac{2n_6}{5}\right)z^2 - \frac{n_7}{2}z_1^2 + n_1\left(\frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4\right)$$

The self-dual G_4 fluxes

Vacuum structure of F-theory

As a consequence of the self-dual condition to G_4 fluxes, all the moduli fields are stabilized at

$$D_S W = D_z W = D_{z_1} W = 0$$

z: CS modulus
S: Dilaton
z_1: Open string modulus
n_i: Quantized fluxes

VEVs

$$\mathrm{Re}z = \mathrm{Re}z_1 = \mathrm{Re}S = 0$$

$$\mathrm{Im}z = \left(\frac{6n_{11}}{5n_1}\right)^{1/4} \frac{2\sqrt{n_6}}{(8n_6(n_5+n_6)-5n_7^2)^{1/4}},$$

$$\mathrm{Im}z_1 = \left(\frac{30n_{11}}{n_1}\right)^{1/4} \frac{\sqrt{n_7}}{(8n_6(n_5+n_6)-5n_7^2)^{1/4}},$$

$$\mathrm{Im}S = \left(\frac{6n_{11}}{5n_1}\right)^{1/4} \frac{n_5}{\sqrt{n_6}(8n_6(n_5+n_6)-5n_7^2)^{1/4}}$$

Vacuum structure of F-theory

Although the fluxes are constrained by the tadpole condition,

$$\frac{\chi}{24} = n_{D3} + \frac{1}{2}\int_{CY4} G_4 \wedge G_4$$

χ=1860: Euler number of CY4
n_{D3}: # of D3

we find the consistent F-theory vacuum, e.g.,

$$n_1 = 1, n_5 = 15, n_6 = 10, n_7 = 2, n_{11} = 28$$

$$n_{\rm D3} = 0$$

All the moduli fields can be stabilized at the LCS point of CY fourfold

$$\mathrm{Re}z = \mathrm{Re}z_1 = \mathrm{Re}S = 0,$$
$$\mathrm{Im}z \simeq 2.28, \quad \mathrm{Im}z_1 \simeq 1.14, \quad \mathrm{Im}S \simeq 1.71$$

Conclusion

○Mirror symmetry techniques can be applied to the F-theory compactifications.

○We explicitly demonstrate the moduli stabilization around the large complex structure point of the F-theory fourfold.

○All the complex structure moduli can be stabilized at the Minkowski minimum.

Discussion

○Quantum corrections to the moduli potential
○Other CY4
○Stabilization of Kähler moduli
→LARGE volume scenario or KKLT