Quantification of the flavor diagonal hadronic CP violation

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Nara

CP violation

Discrete symmetries : P and C

Parity transformation (P):

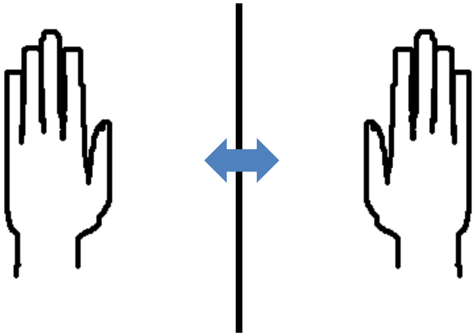
Invert all spatial coordinates

$$\vec{\mathbf{x}} \rightarrow -\vec{\mathbf{x}}$$

⇒ Mirror

For Dirac spinors,

$$\psi_L \leftrightarrow \psi_R$$



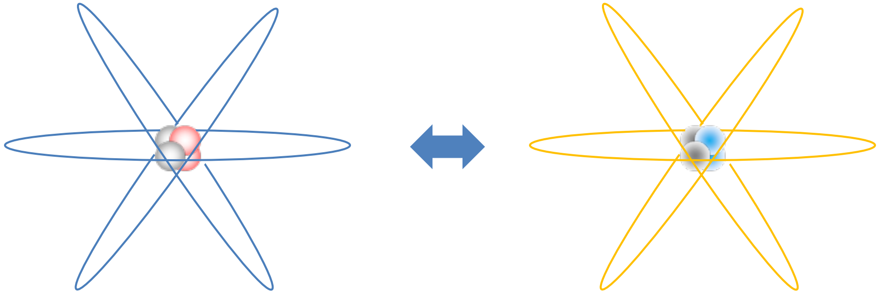
Charge conjugation (C):

Invert sign of all charges

$$\psi = e^{i\theta}|\psi|$$

$$\rightarrow \psi^* = e^{-i\theta}|\psi|$$

⇒ Invert complex phases



Matter (atom)

Antimatter (anti-atom)

CP symmetry

Fermion $\xleftrightarrow{\text{C}}$ Antifermion

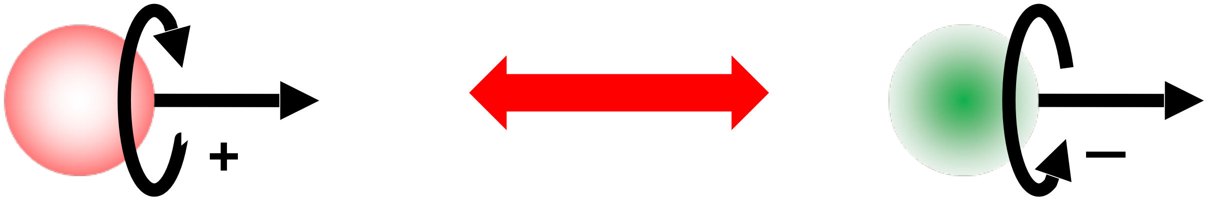
BUT !!

Fermions have spin (rotation of charge), opposite for antifermions

(antifermions have opposite P)

⇒ Left-handed quark and right-handed antiquark

have same properties if no interactions



Left-handed quark

Right-handed antiquark

⇒ This is the genuine symmetry between particles and antiparticles

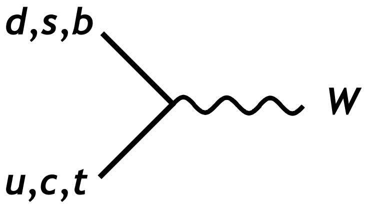
for massless and noninteracting fermions !

CP phases of Cabibbo-Kobayashi-Maskawa (CKM) matrix

CP violation arises when couplings have imaginary part, but not all are CP violating

Example of CKM matrix:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



3 x 3 complex elements ⇒ Naively, 18 free parameters . But,

Kobayashi and Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

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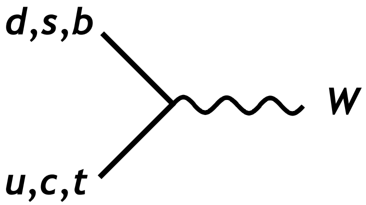

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Unitarity : $V_{\rm CKM}V^{\dagger}_{\rm CKM} = \hat{1}$ **9 equations, reduce to 9 free parameters**

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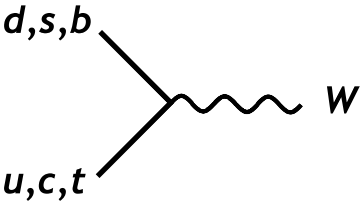

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Field redefinition : $\psi_q \to e^{i\theta_q} \psi_q$ Absorb 5 complex CP phases

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CP phases of Cabibbo-Kobayashi-Maskawa (CKM) matrix

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Example of CKM matrix:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\underline{e^{-i\delta}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\underline{e^{i\delta}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\underline{e^{i\delta}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\underline{e^{i\delta}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\underline{e^{i\delta}} & c_{23}c_{13} \end{pmatrix}$$

δ : CP violating phase

3 x 3 complex elements ⇒ Naively, 18 free parameters . But,

Unitarity : $V_{\mathrm{CKM}}V_{\mathrm{CKM}}^{\dagger} = \hat{1}$ **9 equations, reduce to 9 free parameters**

Field redefinition : $\psi_q \to e^{i\theta_q}\psi_q$ **Absorb 5 complex CP phases**

Remaining degree of freedom

3 mixing angles (flavor change)
1 complex phase : CP violation of SM !

Kobayashi and Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

A new BSM theory MUST violate CP

In SM, all CP phases have already been fixed (absorbed)

➡ If we add new particles and interactions in the SM, get nonzero relative phase, must violate CP !

(we call this “NATURALNESS”)

There is no reason for BSM couplings to have the same (aligned) complex phases as SM interactions, unless there is some physics behind

➡ We can look for new physics BSM by experimentally probing CP violation !

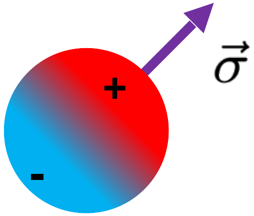
EDM, beta decay, T-violation in neutron scattering …

Electric dipole moment

Electric dipole moment (EDM)

Permanent polarization of internal charge of a particle.

$$\vec{d}_\psi = \sum_i \langle \psi | Q_i e \vec{r}_i | \psi \rangle$$



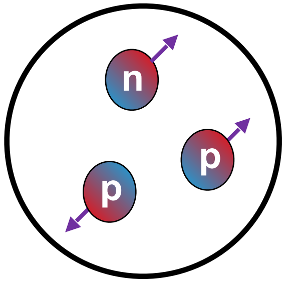
(The unit of EDM is e cm : effective distance of internal charge displacement)

- **Direction:** $\vec{d} \propto \vec{\sigma}$
- **Interaction:** $H_{\mathrm{EDM}} = -d \langle \vec{\sigma} \rangle \cdot \vec{E}$
- **Transformation properties:**
  - **Under parity tr.:** $\begin{cases} \vec{E} \xrightarrow{\mathrm{P}} -\vec{E} \\ \vec{\sigma} \xrightarrow{\mathrm{P}} \vec{\sigma} \end{cases}$ → H_{EDM} **is P-odd**
  - **Under time reversal:** $\begin{cases} \vec{E} \xrightarrow{\mathrm{T}} \vec{E} \\ \vec{\sigma} \xrightarrow{\mathrm{T}} -\vec{\sigma} \end{cases}$ → H_{EDM} **is CP-odd !**

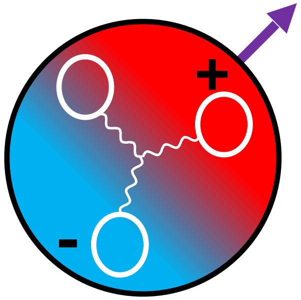
EDM of composite systems

EDM is often measured in composite systems (neutron, atoms, molecules, nuclei)

EDM is not only generated by the EDM of the components, but also by CP violating many-body interactions.

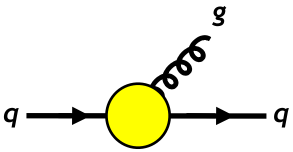


EDM of constituents

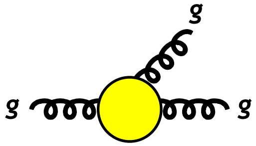


CP-odd many-body interaction

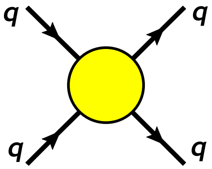
Example of QCD level many-body interactions inducing neutron EDM:



quark chromo-EDM



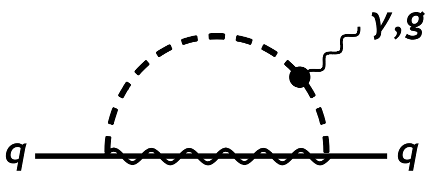
Weinberg operator



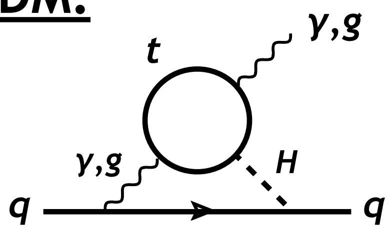
P, CP-odd 4-quark interaction

Elementary level CP violation from BSM physics (SMEFT)

- **Fermion EDM, quark chromo-EDM:**



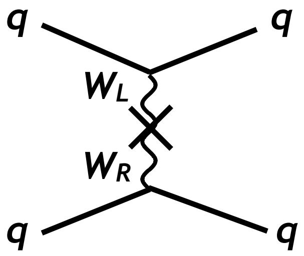
SUSY: 1-loop

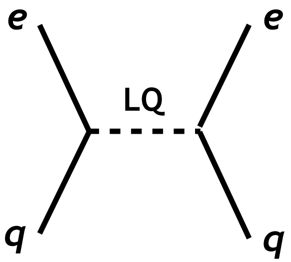


2-Higgs doublet: 2-loop

- **CP-odd 4-fermion interaction:**

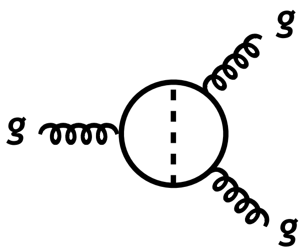
Tree level:
* Left-right symmetric model
* Scalar exchange (e.g. Higgs)
* Leptoquarks





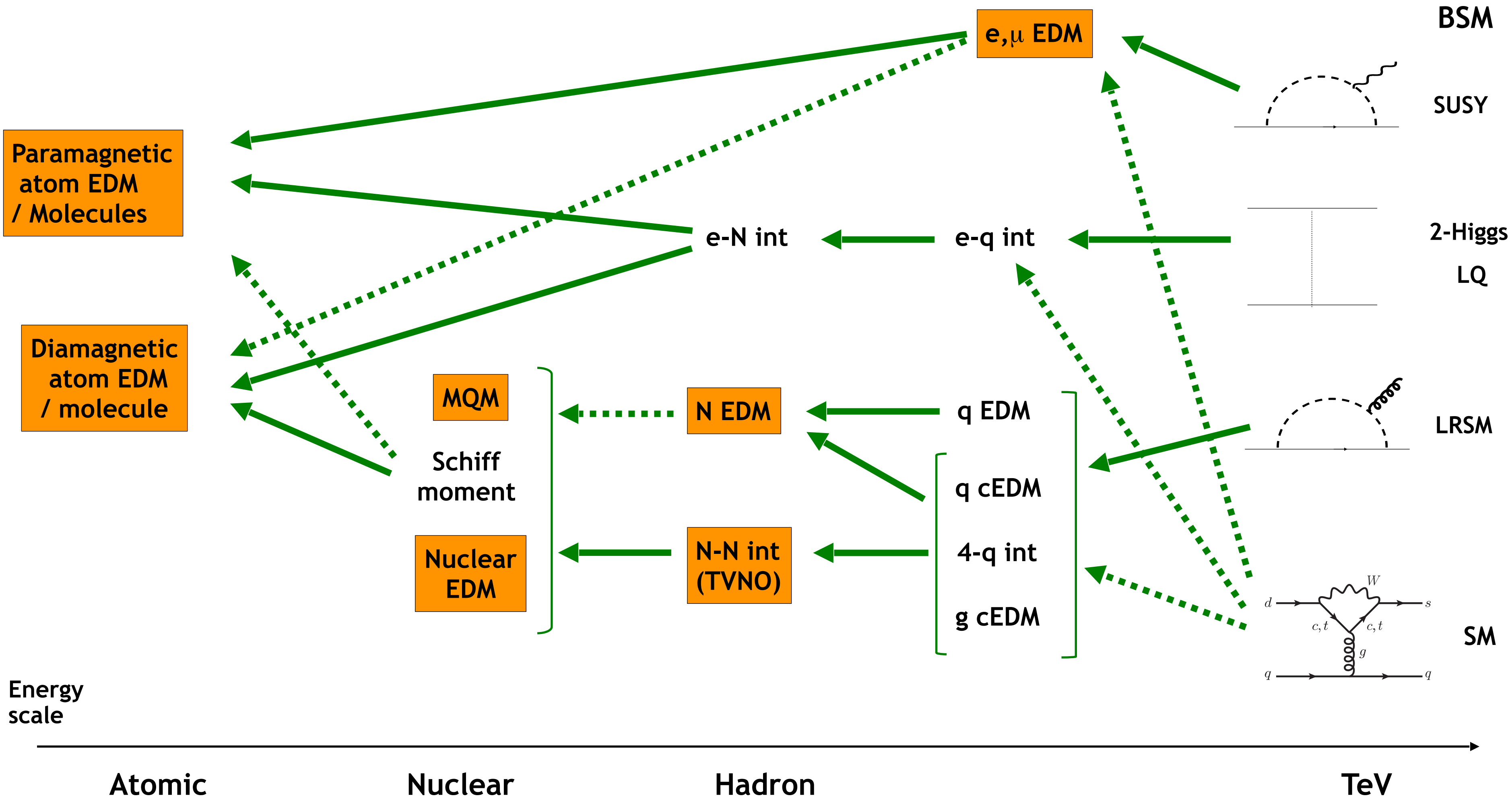
- **Weinberg operator:**

2-loop diagram:
* 2-Higgs doublet model
* Vectorlike quark model



CP violating interactions appear at scale below BSM particles (TeV?)

EDM from CPVBSM



BSM

e,μ EDM

SUSY

2-Higgs

LQ

LRSM

SM

Paramagnetic atom EDM / Molecules

Diamagnetic atom EDM / molecule

e-N int

e-q int

MQM

Schiff moment

Nuclear EDM

N EDM

N-N int (TVNO)

q EDM

q cEDM

4-q int

g cEDM

Energy scale

Atomic

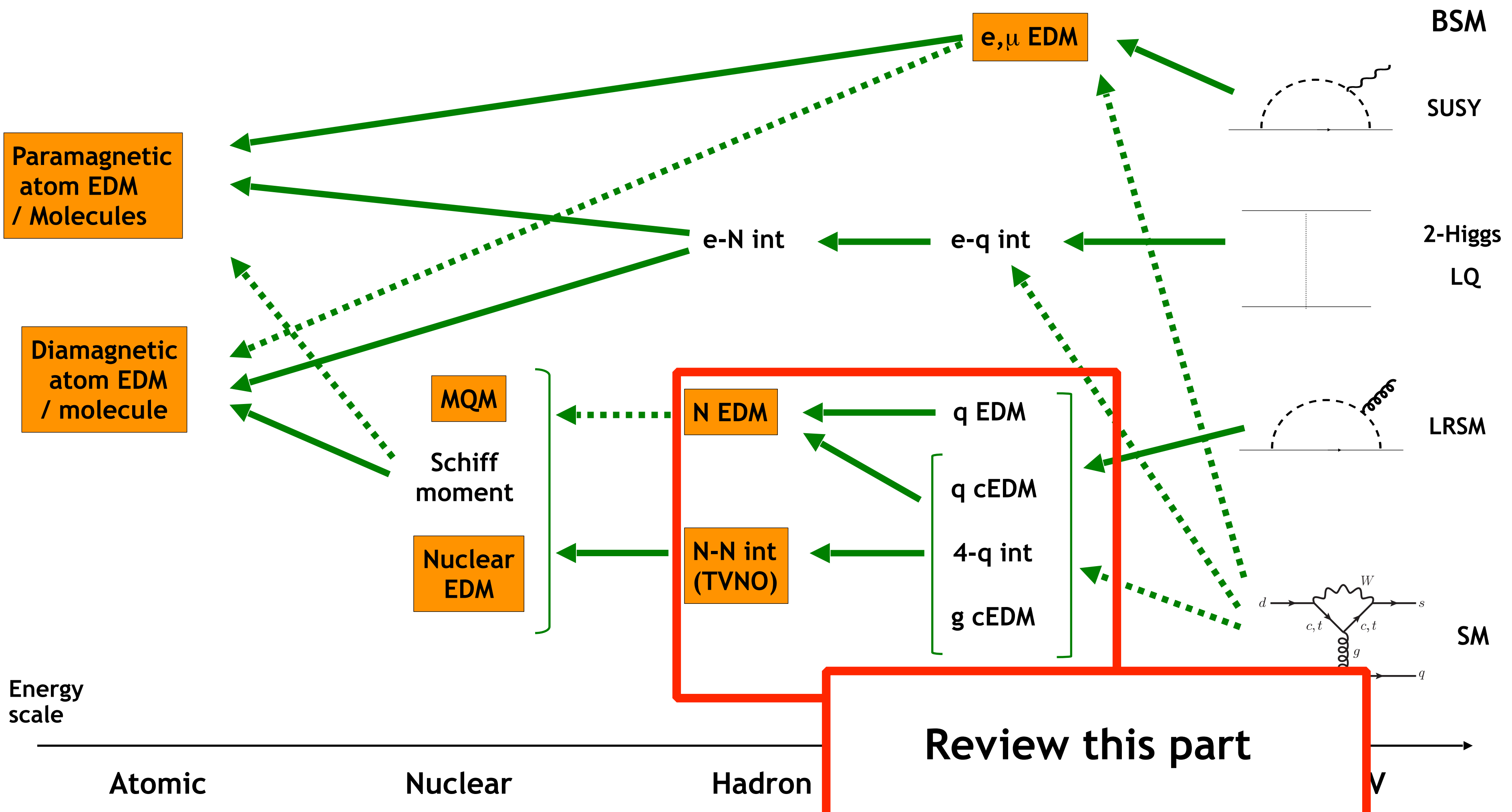
Nuclear

Hadron

TeV

observable	: Observable available at experiment
←	: Sizable dependence
⇠	: Weak dependence

EDM from CPVBSM



Review this part

(we assume that CPV interactions are already there at TeV)

observable : Observable available at experiment

← : Sizable dependence

←⋯ : Weak dependence

Hadron level CP violation

Hadron level CP violation

Hadronic CP violation has large uncertainty due to QCD nonperturbative physics

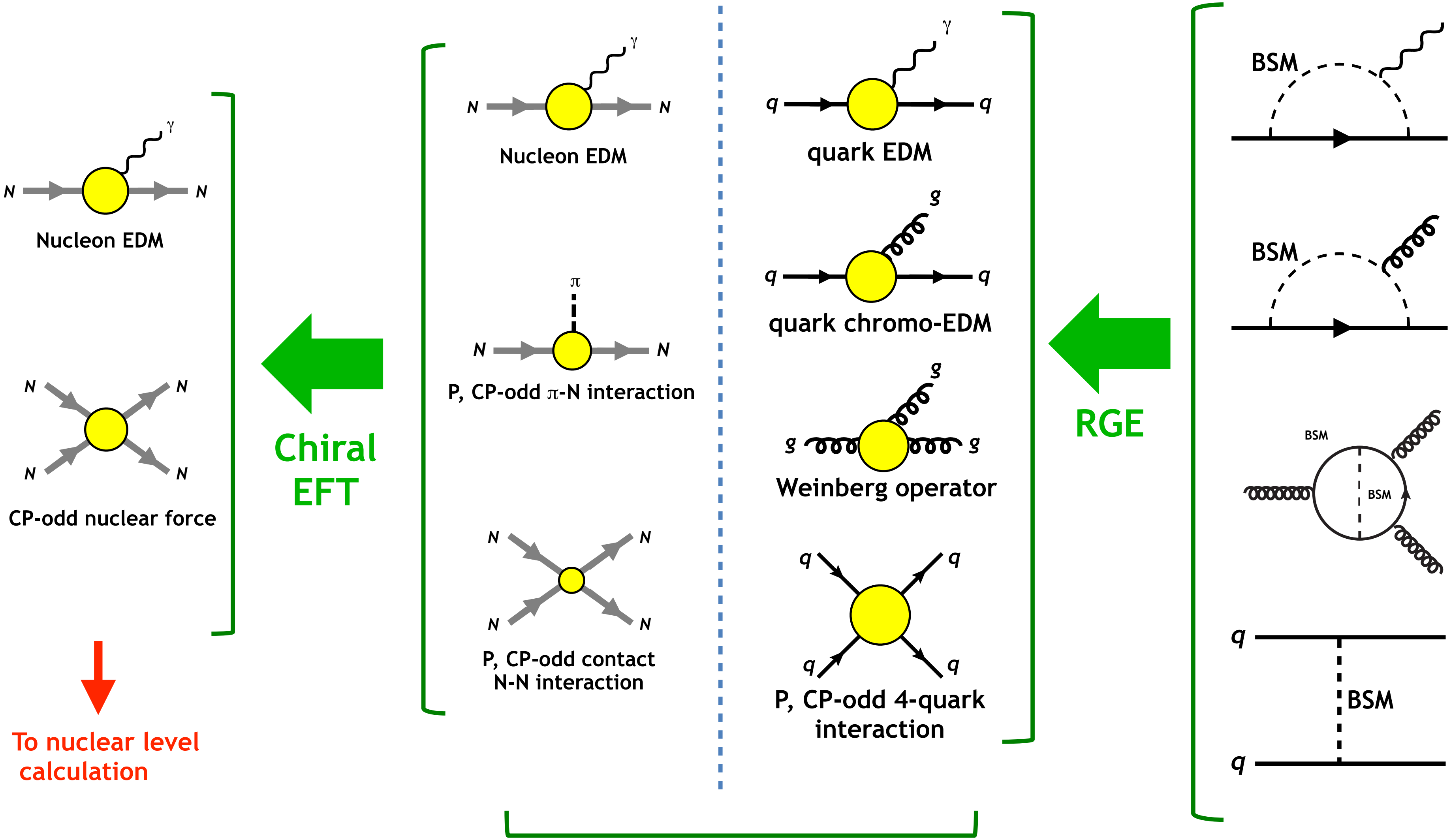
Ideally, calculated in lattice QCD, but quantified results are very limited

But recently, there were significant progresses in the quantification

In particular, the leading hadronic contribution has recently been identified

In this section, we review the hadronic CPV contributing to EDMs

Hadronic CPV : from TeV to hadron level



Nuclear level inputs

Nucleon EDM

CP-odd nuclear force

To nuclear level calculation

Chiral EFT

CPV hadron EFT

Nucleon EDM

P, CP-odd π-N interaction

P, CP-odd contact N-N interaction

GeV scale CPV QCD

quark EDM

quark chromo-EDM

Weinberg operator

P, CP-odd 4-quark interaction

RGE

TeV scale CPV

QCD calculations

E. Mereghetti et al., Phys. Lett. B **696** (2011) 97;
J. de Vries et al., Phys. Lett. B **766**, 254 (2017).

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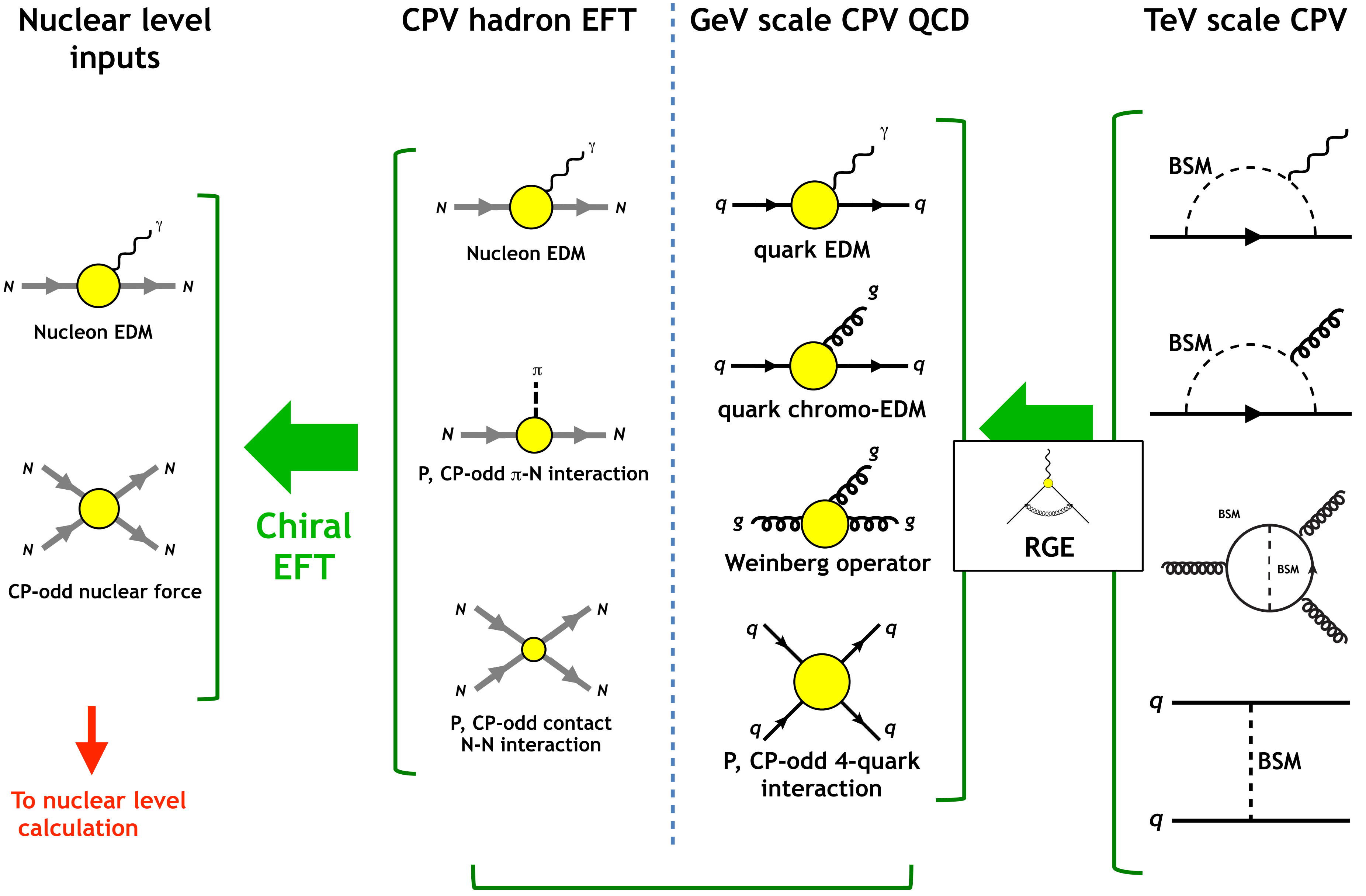

Nuclear level inputs

- Nucleon EDM
- CP-odd nuclear force

To nuclear level calculation

Chiral EFT

CPV hadron EFT

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- P, CP-odd π-N interaction
- P, CP-odd contact N-N interaction

GeV scale CPV QCD

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- quark chromo-EDM
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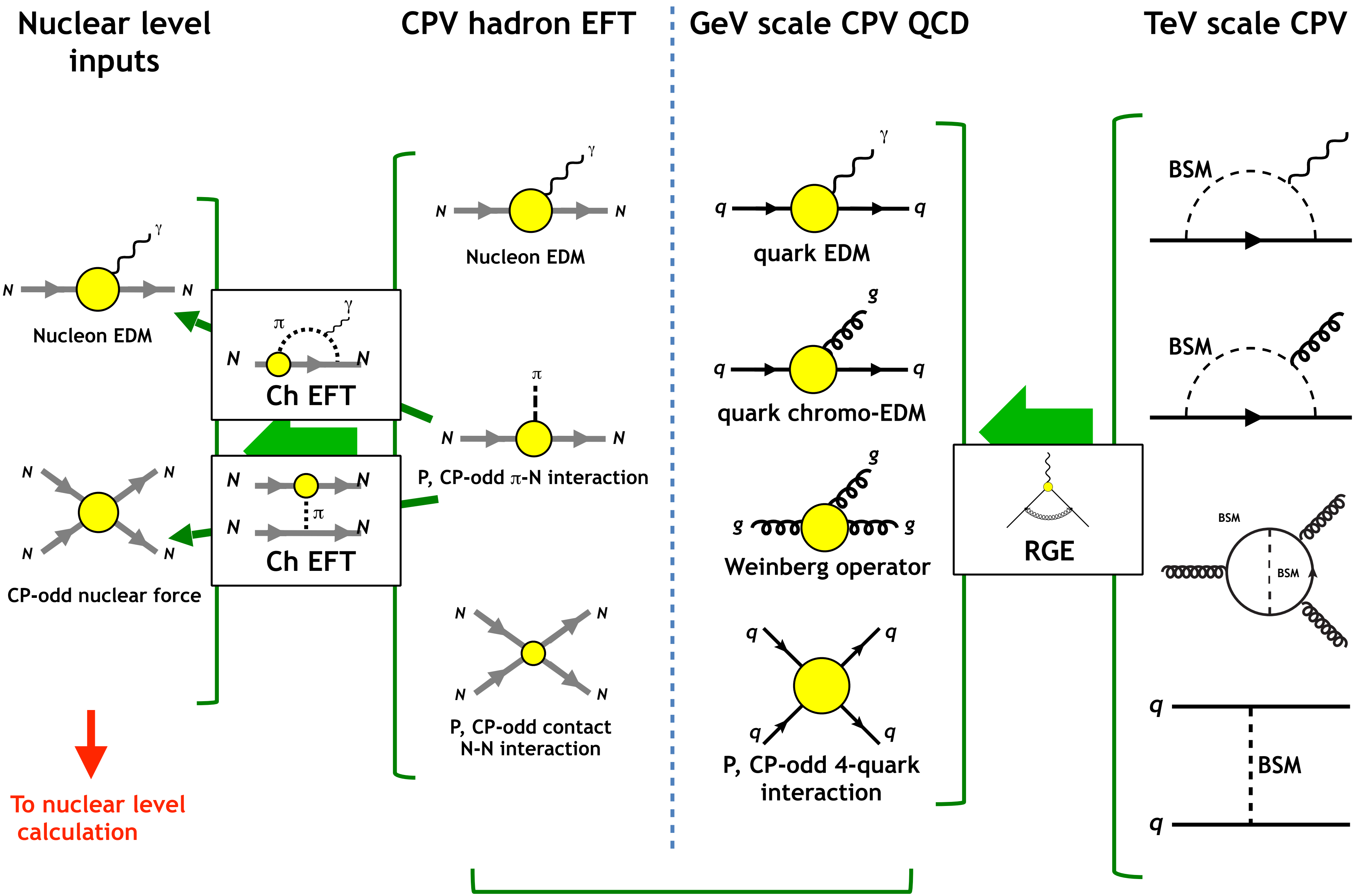

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CPV hadron EFT

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Ch EFT

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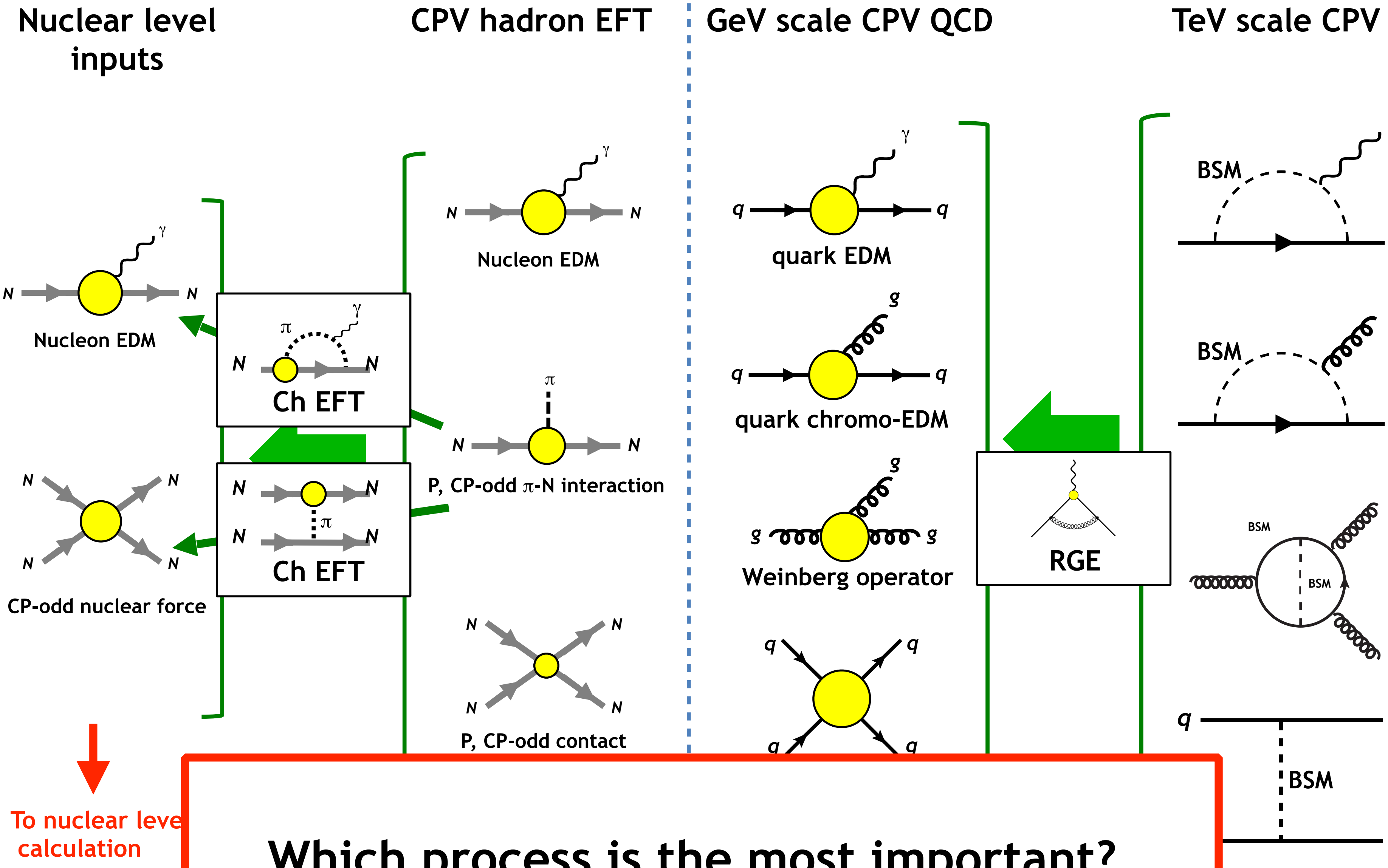

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RGE

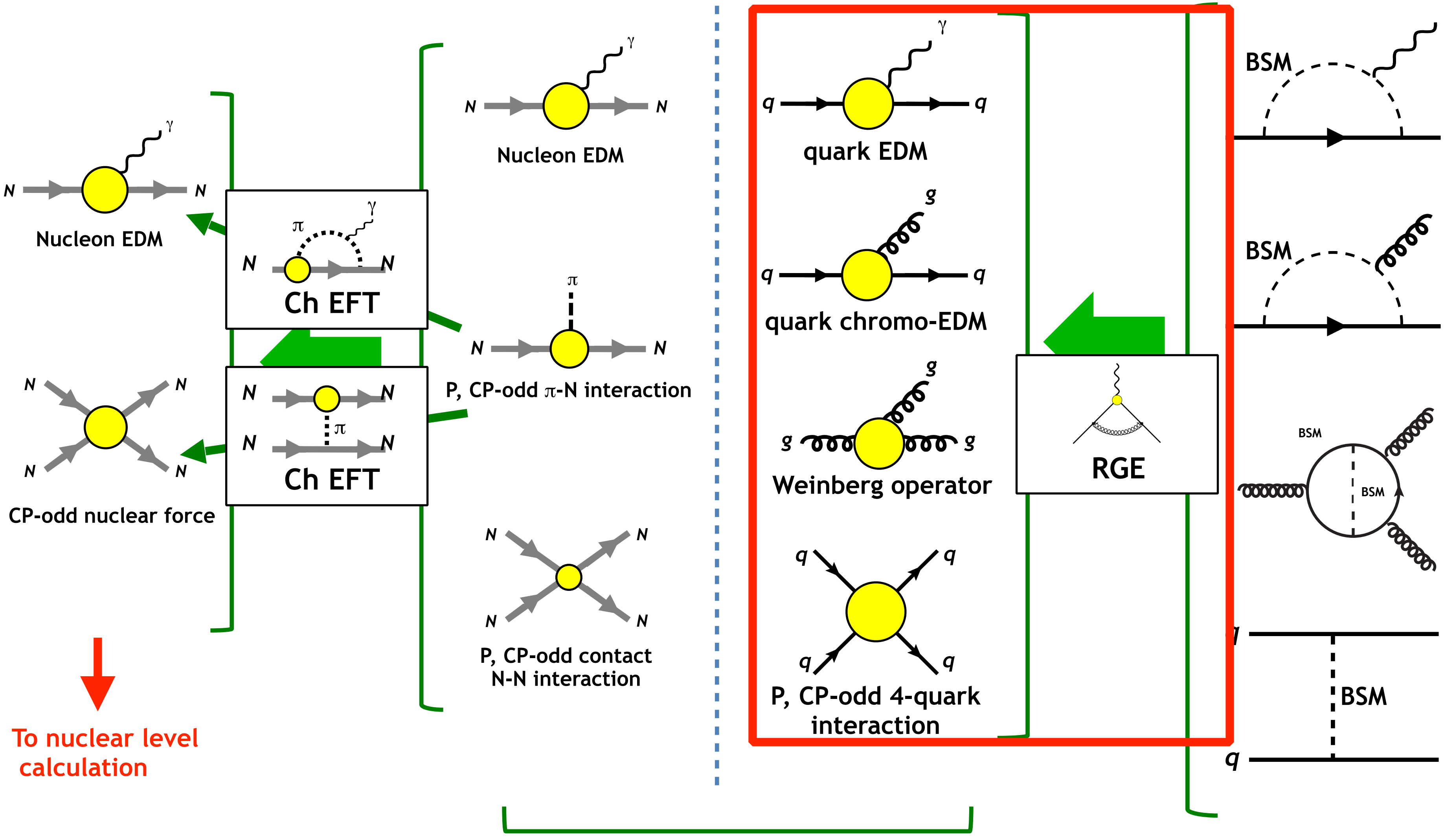
To nuclear level calculation

Which process is the most important?

t. B **696** (2011) 97;
J. de Vries et al., Phys. Lett. B **766**, 254 (2017).

Hadronic CPV : from TeV to hadron level

Nuclear level inputs | **CPV hadron EFT** | **GeV scale CPV QCD** | **TeV scale CPV**



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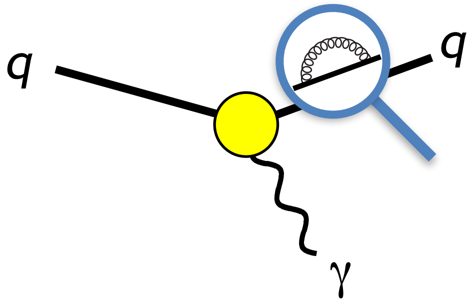
Renormalization group evolution of CPV QCD operators

Quark-gluon interactions change in scale due to quantum corrections

Calculated in perturbative QCD



- **Scalar density:**

  $\bar{q}q \rightarrow 2\,\bar{q}q$

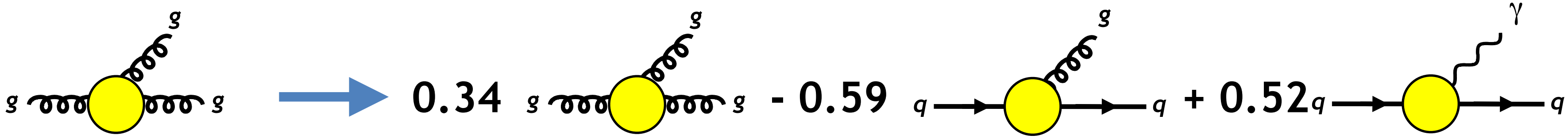
  μ = 1 TeV → μ = 1 GeV

- **Quark EDM:**

  $d_q \rightarrow 0.8\,d_q$

  μ = 1 TeV → μ = 1 GeV

- **Weinberg operator:**



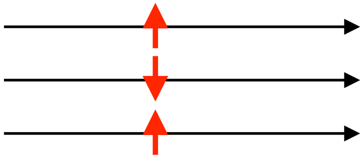
→ 0.34 - 0.59 + 0.52

μ = 1 TeV → μ = 1 GeV

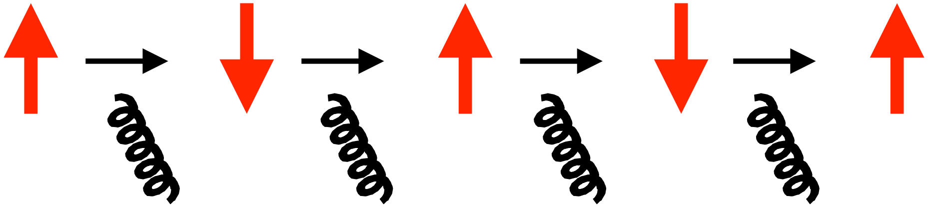
Roughly, scalar increases and spin decreases when scale goes down

Rough tendency: scalar and spin

- Spin : suppression

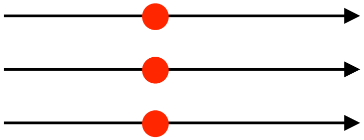


In composite states : Pairing of spin
⇒ No enhancement

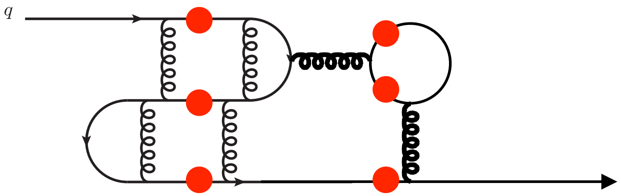


Qu. corrections: superposition
of spin flips after
gluon emissions/absorptions
⇒ Suppressed

- Scalar density ($\bar{\psi}\psi$) : enhancement



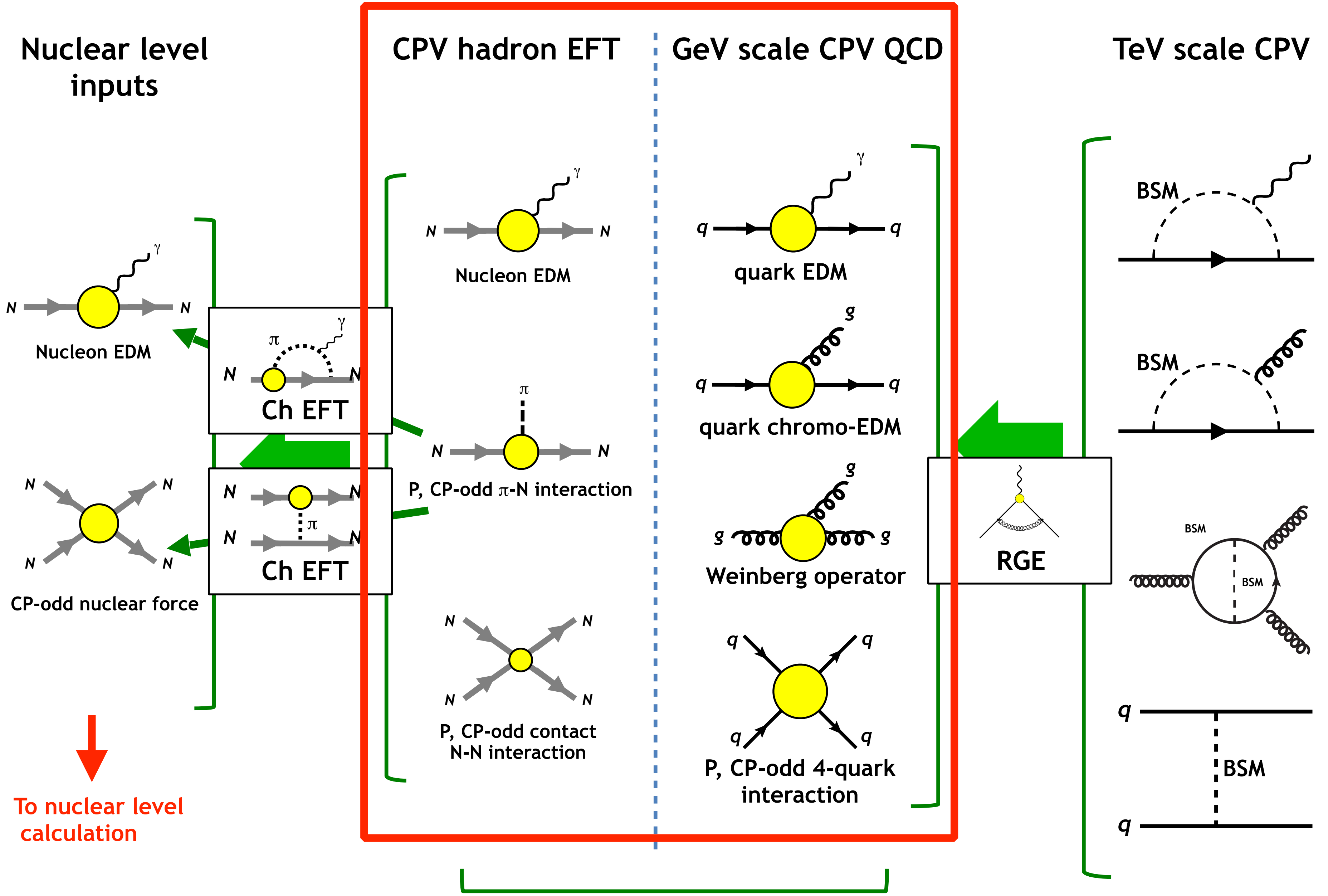
In composite states: coherent sum
⇒ Enhanced!



Qu. Corrections: $\bar{\psi}\psi$ of particles and
antiparticles have same sign
⇒ Enhanced by Z-graphs, loops

NY, T. M. Doi, S. Imai, H. Suganuma, PRD **88**, 074036 (2013); PRD **89**, 074017 (2014).

Hadronic CPV : from TeV to hadron level



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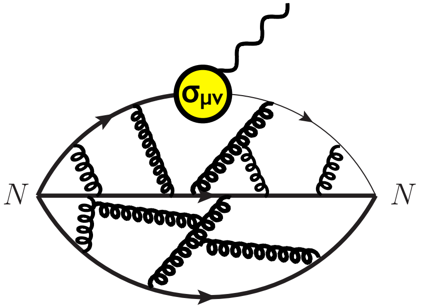
Quark EDM to nucleon EDM : lattice QCD



Quark EDM contribution to nucleon EDM is given by

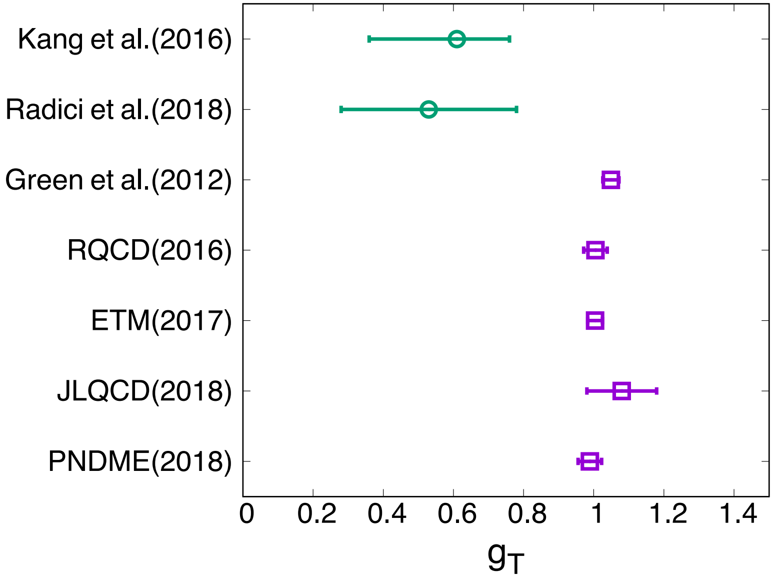
the tensor charge $\langle N|\bar{q}\sigma_{\mu\nu}q|N\rangle$

⇒ Lattice QCD results available

⇒ Accurate up to 10% error

(Little disagreement with pQCD)

⇒ No particular enhancement



Lattice calculations of other interactions are still very difficult though some first studies started to appear

T. Bhattacharya et al., PRD **108**, 074507 (2023); PoS LATTICE2024 (2025) 344.

Hadronic CPV : from TeV to hadron level

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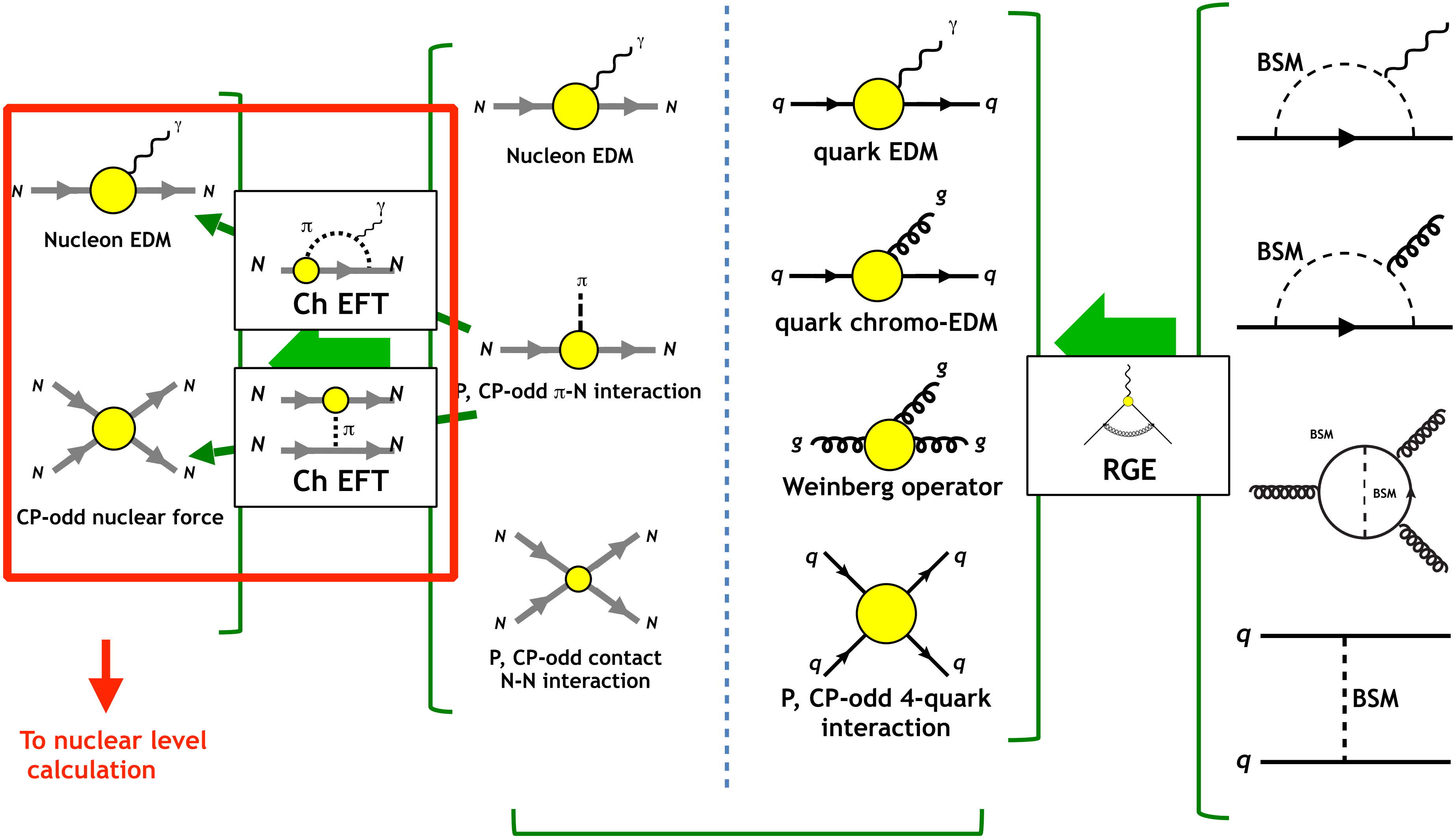
TeV scale CPV



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Chiral perturbation theory (Chiral EFT)

$$\mathcal{L}_{\rm QCD} = \underbrace{-\frac{1}{4}F_{\mu\nu,a}F^{\mu\nu}_{a} + \sum_{\psi=u,d,\cdots} \bar{\psi} i \not{D} \psi}_{\text{Chiral symmetric}} - \underbrace{m\bar{\psi}\psi}_{\text{Chiral symmetry breaking}}$$

QCD has small u, d masses (∼MeV) compared to $\Lambda_{\rm QCD}$ (∼200 MeV)

⇒ Expansion of hadron level interactions in $m_{u,d}/\Lambda_{\rm QCD}$ ∼0.01

⇒ Can organize and fit hadronic interactions with powers of $m_{u,d}/\Lambda_{\rm QCD}$

CPV hadronic interactions may also break chiral symmetry

E.g. $\mathcal{L}_{\pi NN} = \bar{g}^{(0)}_{\pi NN} \pi_a \bar{N} \tau_a N + \bar{g}^{(1)}_{\pi NN} \pi_0 \bar{N} N$ **⇒ Break chiral sym.**

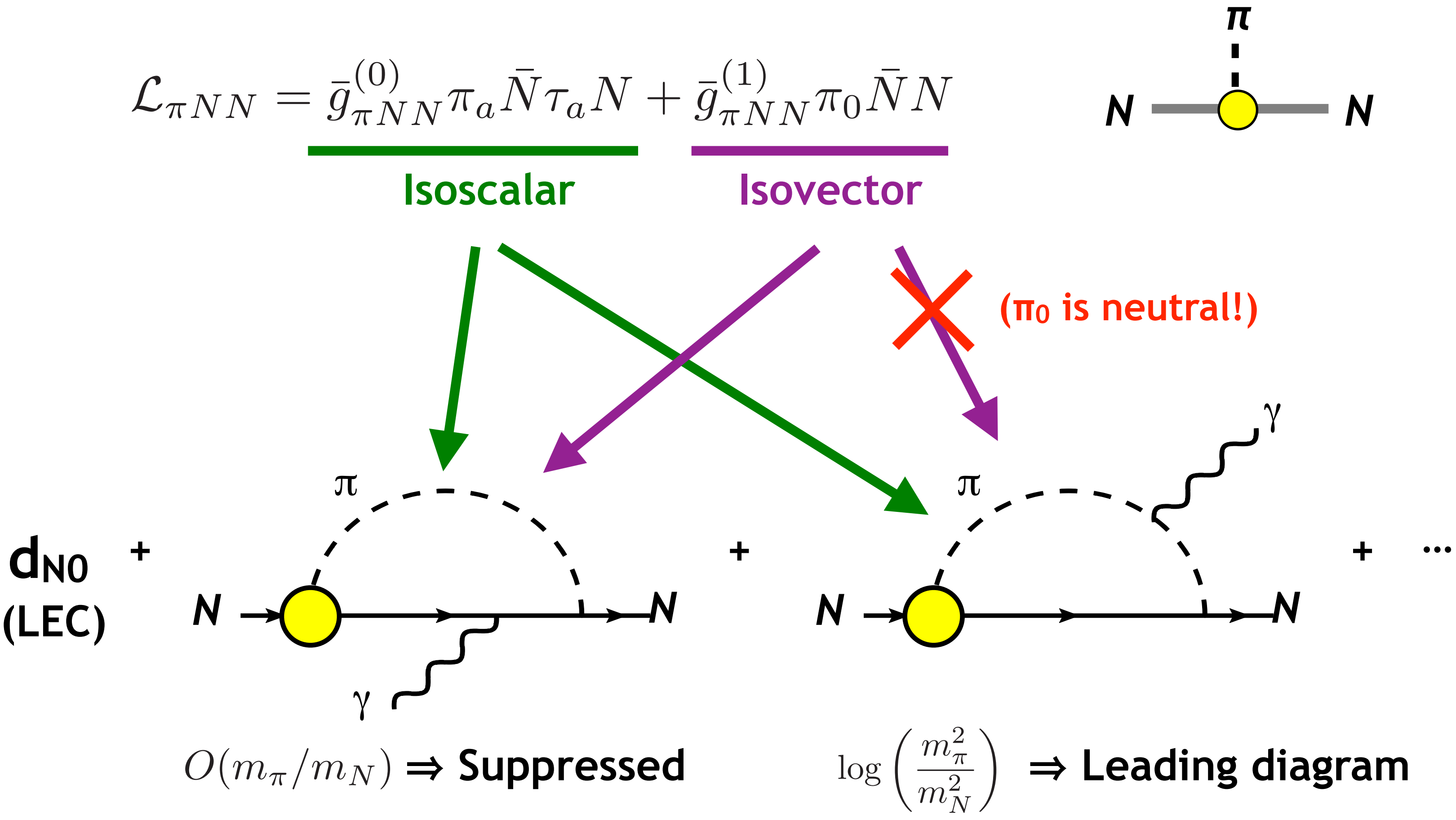
$\mathcal{L}_{NN} = \bar{C}_1 \bar{N} i\gamma_5 N \, \bar{N} N$ **⇒ Conserve chiral sym.**

➡ **Quantification using diagrams in terms of**

$m_{u,d}/\Lambda_{\rm QCD}$, isospin, CPV, … is possible ! (Chiral EFT)

Chiral EFT analysis of nucleon EDM

$$\mathcal{L}_{\pi NN} = \bar{g}^{(0)}_{\pi NN}\pi_a \bar{N}\tau_a N + \bar{g}^{(1)}_{\pi NN}\pi_0 \bar{N}N$$



$O(m_\pi/m_N)$ ⇒ **Suppressed**

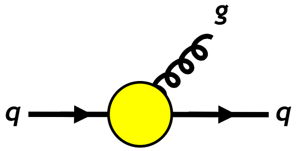
$\log\left(\frac{m_\pi^2}{m_N^2}\right)$ ⇒ **Leading diagram**

⇒ Essentially loop effect, no notable enhancement

⇒ Only $\bar{g}^{(0)}$ is important (sensitivity to specific BSM)

Chromo-EDM → CPV πNN interaction in chiral EFT

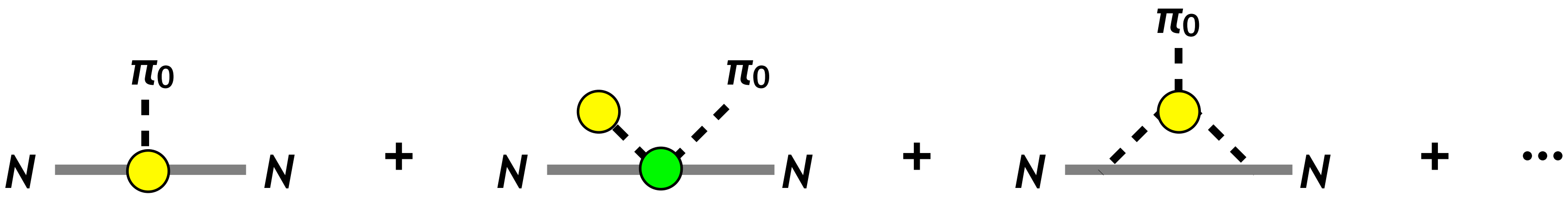
cEDM



is chiral sym. breaking, may contribute to both πNN

$$\mathcal{L}_{\pi NN} = \bar{g}^{(0)}_{\pi NN} \pi_a \bar{N} \tau_a N + \underline{\bar{g}^{(1)}_{\pi NN} \pi_0 \bar{N} N}$$

Chiral EFT analysis : isovector CPV πNN interaction is enhanced



Small ?
(Large error)

Enhanced by
πN sigma-term

Same sign as 2nd dia.
not small

Isovector CPV πNN interaction suppressed for nucleon EDM,

but large for, atomic/nuclear EDM (via CPV pion-exchange nuclear force)

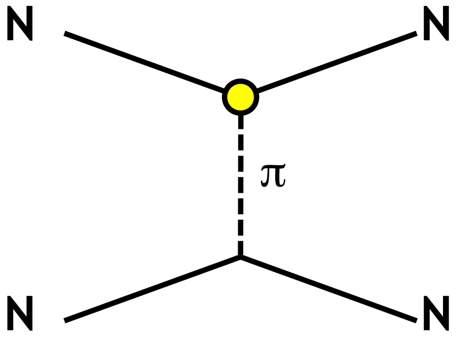
$$d_p \approx 1 \times e(d_u^c - d_d^c)$$

$$d_{\mathrm{He}} \approx 16 \times e(d_u^c - d_d^c)$$

← **Example of ³He EDM :**
nuclear level CPV is much larger

P, CP-odd nuclear force

P, CP-odd nuclear force : one-pion exchange most important



$$\sim \frac{1}{q^2 - m_\pi^2} \bar{N} N \; \bar{N} i \gamma_5 N$$

- P, CP-odd Hamiltonian (3-types):

$$H_{CPV} = - \frac{g_{\pi NN}}{8\pi m_N} \left[(\underbrace{\bar{g}^{(0)}_{\pi NN}}_{\text{Isoscalar}} \tau_a \cdot \tau_b + \underbrace{\bar{g}^{(2)}_{\pi NN}}_{\text{Isotensor}} (\tau_a \cdot \tau_b - 3 \tau_a^z \tau_b^z)) (\vec{\sigma}_a - \vec{\sigma}_b) + \underbrace{\bar{g}^{(1)}_{\pi NN}}_{\text{Isovector}} (\tau_a^z \vec{\sigma}_a - \tau_b^z \vec{\sigma}_b) \right] \cdot \vec{\nabla} \frac{e^{-m_\pi r}}{r}$$

Probed by atomic/nuclear EDM, CPV β decay, TV neutron optics, …

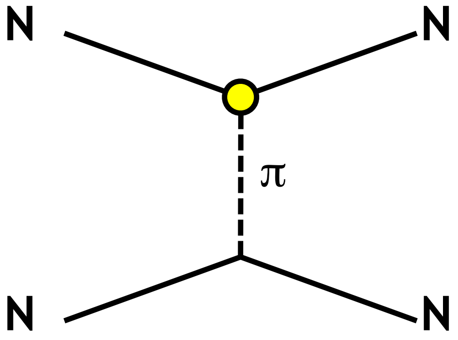
- Isovector CPVNN enhanced by nucleon number (via sigma-term)

⇒ The most sensitive process for quark-gluon CPV

- Enhancement by resonances, deformation at nuclear level

P, CP-odd nuclear force

P, CP-odd nuclear force : one-pion exchange most important



Isocalar CPV NN-int. :

Axion DM, θ-term,…

$i\gamma_5 N$

- **P, CP-odd Hamiltonian (3-types):**

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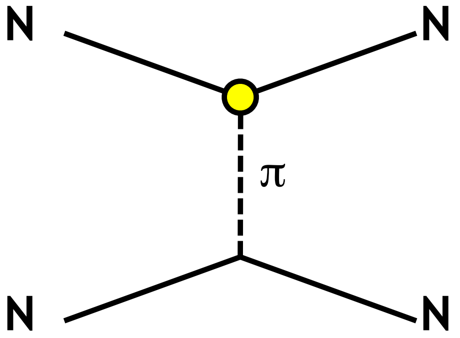

Isocalar CPV NN-int. :

Axion DM, θ-term,…

$i\gamma_5$

Isovector CPV NN-int. :

Higgs, SUSY, …

- P, CP-odd Hamiltonian (3-types):

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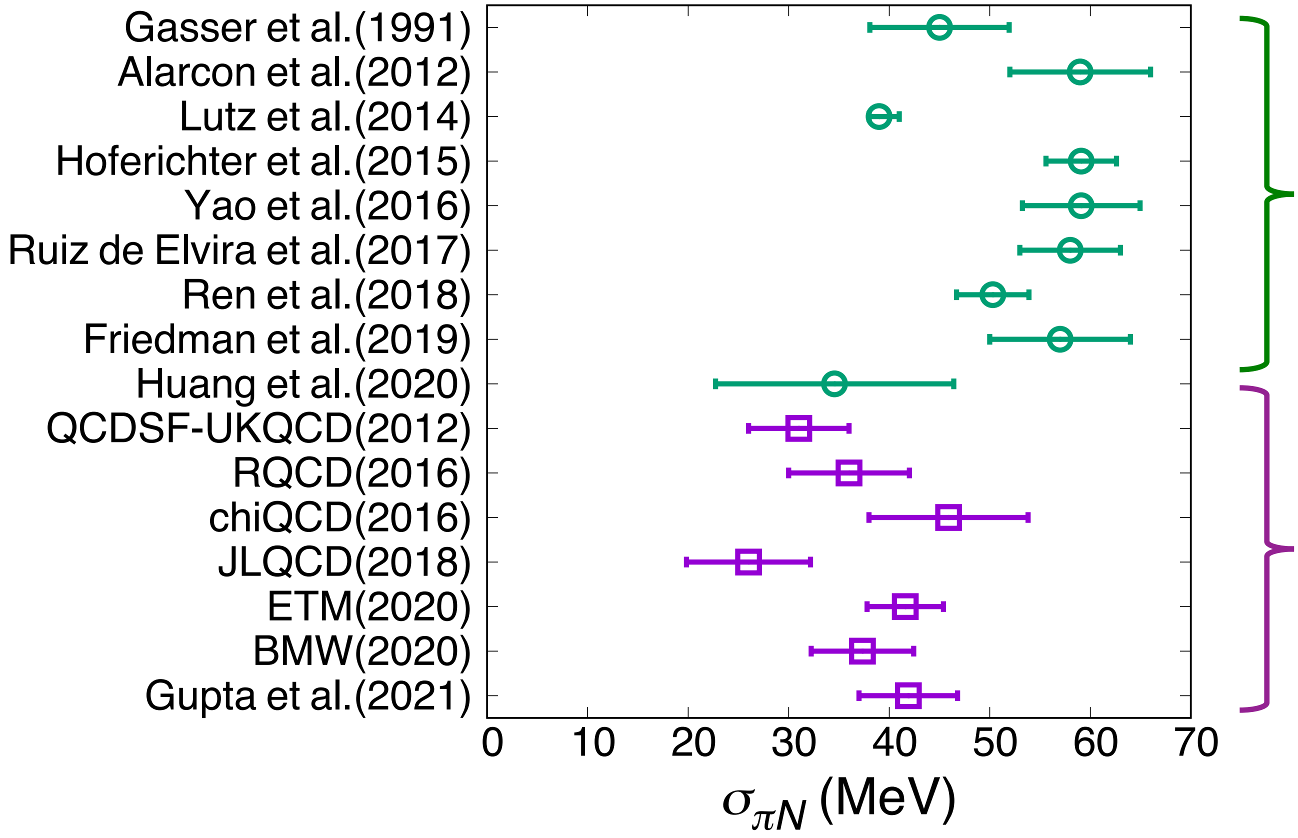
Present status of the calculations of πN sigma-term



Phenomenological extractions based on ChPT

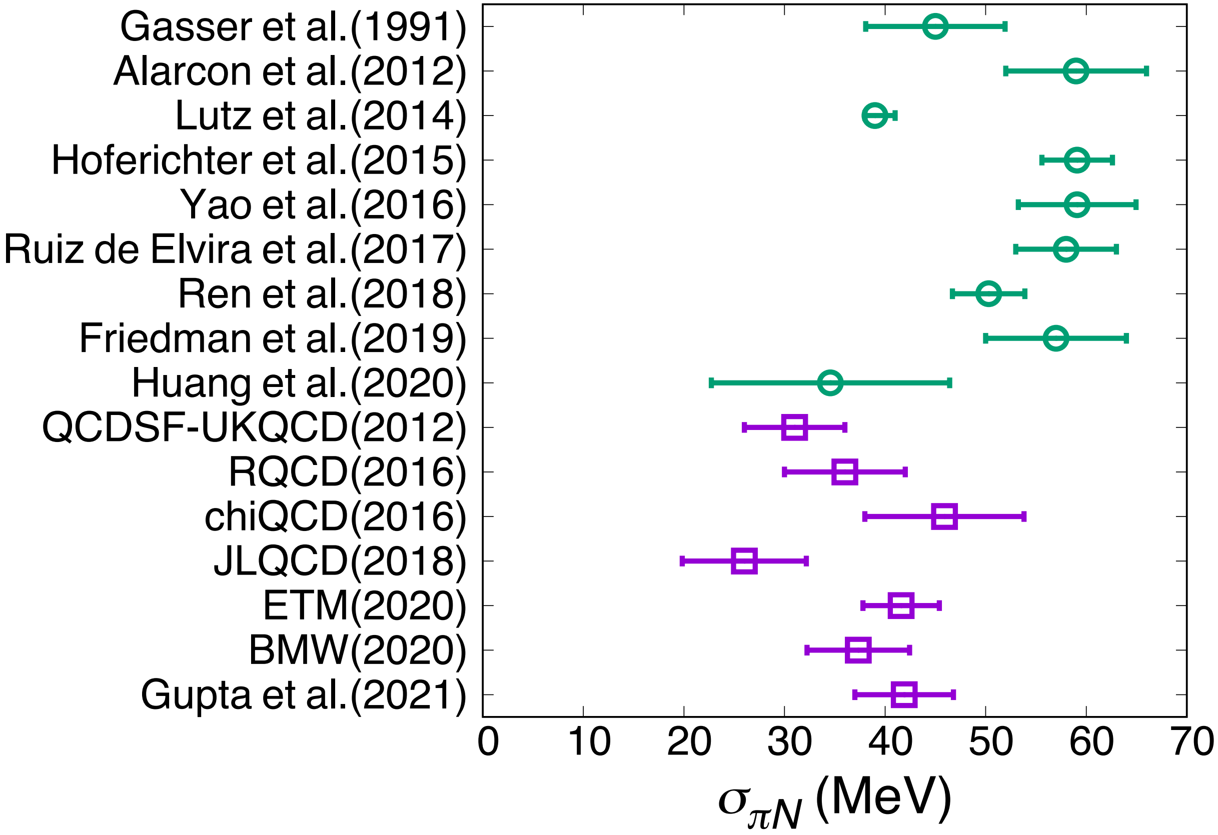
Direct lattice QCD calculations

Extractions of $\sigma_{\pi N} \equiv \frac{m_u + m_d}{2} \langle N|\bar{u}u + \bar{d}d|N\rangle$

suggests $\langle N|\bar{u}u + \bar{d}d|N\rangle \sim 10$ **(c.f. scalar charge)**

⇒ Enhancement of CPV nuclear force via $\sigma_{\pi N}$!

Present status of the calculations of πN sigma-term



Phenomenological extractions based on ChPT

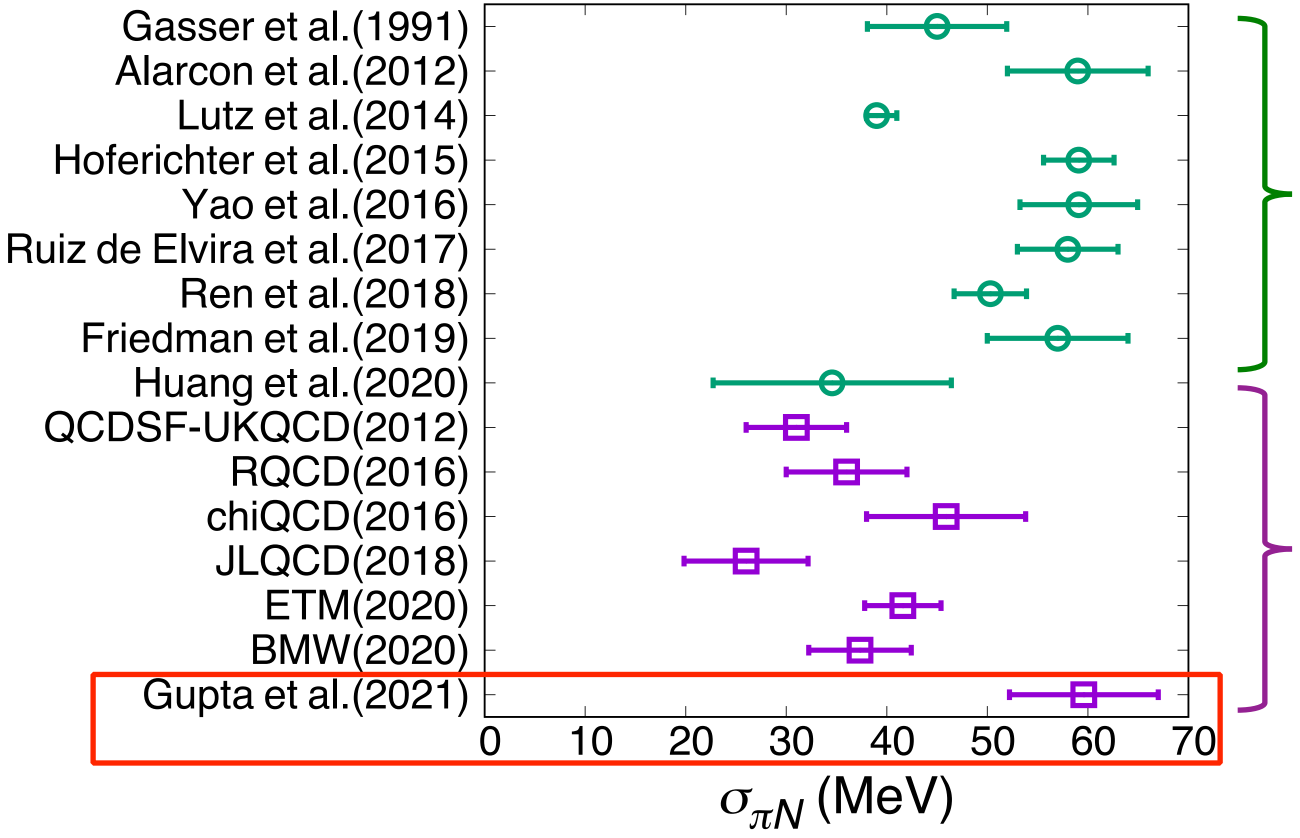
Direct lattice QCD calculations

Phenomenological extractions : $\sigma_{\pi N}$ ∼ 60 MeV

Lattice QCD calculations : $\sigma_{\pi N}$ ∼ 30 MeV

⇒ Visible disagreement

Present status of the calculations of πN sigma-term



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Direct lattice QCD calculations

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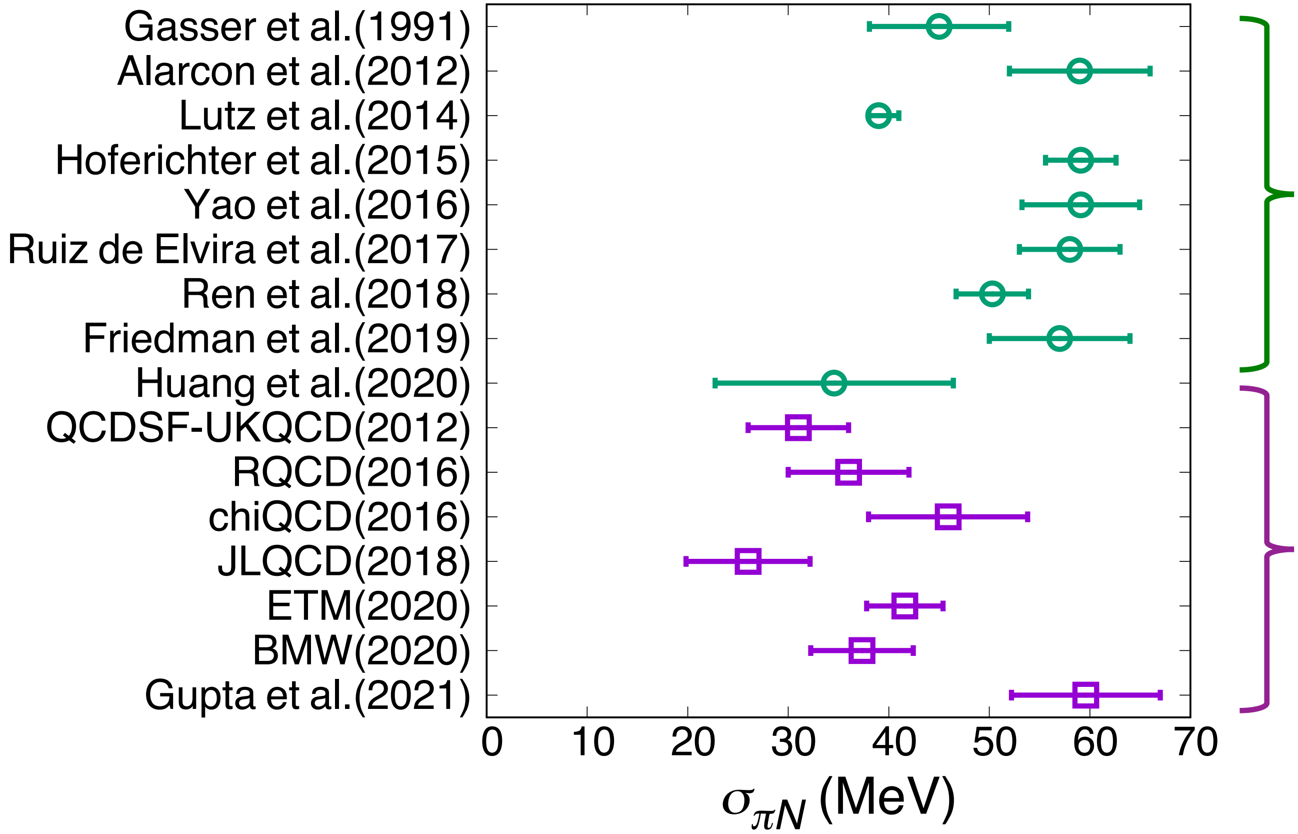
⇒ Gupta et al. are suggesting some potential resolution??

Present status of the calculations of πN sigma-term



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⇒ Visible disagreement

Homework:

We have to resolve this problem for quantifying EDMs

Summary of hadronic CPV

No notable enhancement for quark EDM

Weinberg operator suppressed by RGE and
hadron matrix elements

➡ Chromo-EDM contribution is the most enhanced!

(Some 4-quark interactions may also be enhanced,
but model specific, like left-right sym. models)

Enhanced by πN sigma-term , still large uncertainty

Theoretical challenge to resolve the Strong CP problem

Theta-term

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F_{\mu\nu,a} F_a^{\mu\nu} + \underline{\theta \frac{\alpha_s}{8\pi} F_{\mu\nu,a} \tilde{F}_a^{\mu\nu}}$$

$$\tilde{F}_a^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma}$$

θ-term is like E.B ⇒ Violate P and CP !

Θ-term contains topological information of QCD

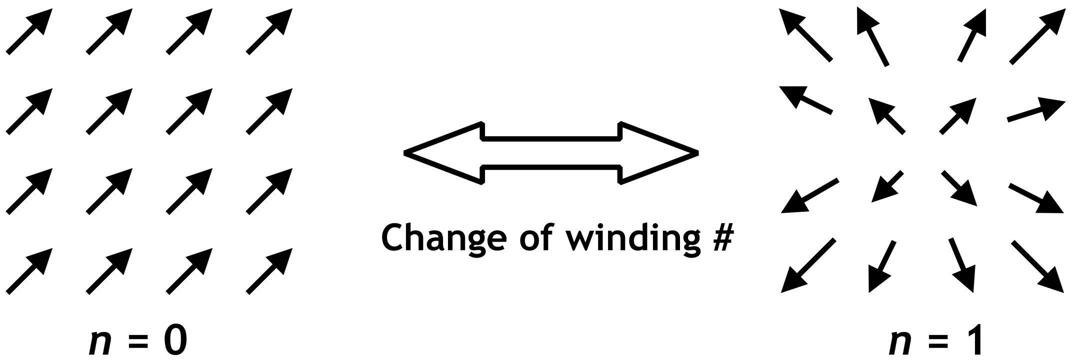
$$\int d^4x\, F_{\mu\nu,a} \tilde{F}_a^{\mu\nu} = \int d^4x\, \partial_\mu K^\mu \quad = \Delta n$$

Integer!

$$K_\mu \equiv \frac{\alpha_s}{8\pi} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu F_a^{\rho\sigma} - \frac{g_s}{3} f_{abc} A_a^\nu A_b^\rho A_c^\sigma \right]$$

Integral of total derivative, but nonzero!

⇒ Δ*n* = change of winding number of gauge configurations



Theta-term

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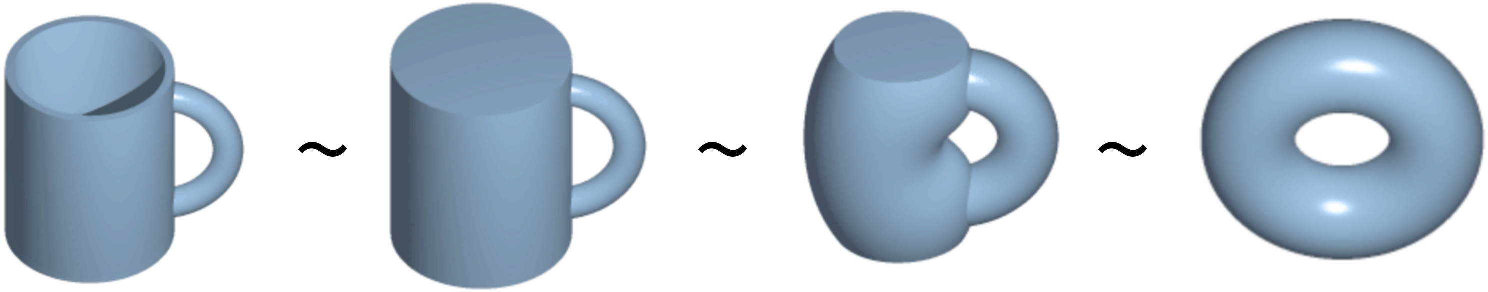
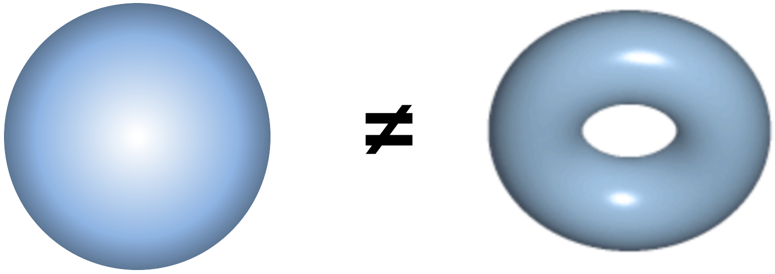
Θ-

$_cA_a^\nu A_b^\rho A_c^\sigma\Big]$

Topology:





Theta-term

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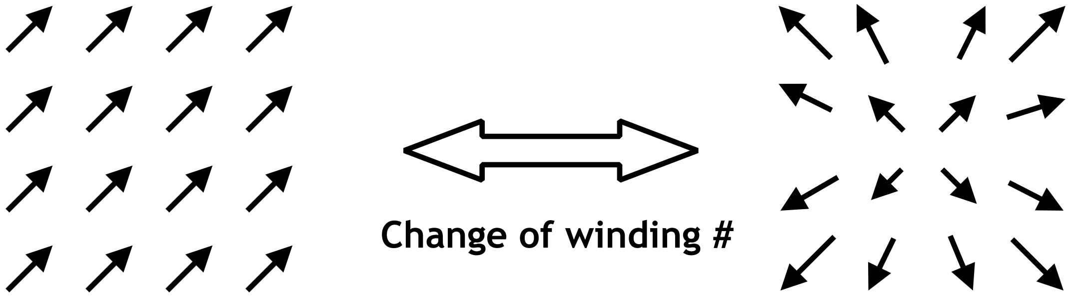


Change of winding #

***n* = 0**

***n* = 1**

Strong CP problem

θ-term : $\mathcal{L} = \theta \frac{g^2}{32\pi^2} F^{\mu\nu}_{,a} F^{\rho\sigma}_{,a} \epsilon_{\mu\nu\rho\sigma}$

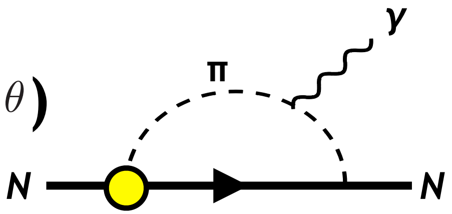
⇒ **A "natural" interaction term of QCD, θ should be O(1)**

Neutron EDM experimental data:

$|d_n| < 1.8 \times 10^{-26}$ e cm

C. Abel et al., Phys. Rev. Lett. **124**, 081803 (2020).

Converting to θ via chiral EFT analysis ($\bar{g}^{(0)}_{\pi NN} \approx 0.015\,\theta$)



$$|\theta| < 1 \times 10^{-10}$$

…Why so small??

➡ **Strong CP problem**

R. G. Crewther et al., Phys. Lett. B **88** (1979) 123;
E. Mereghetti et al., Phys. Lett. B **696** (2011) 97.

Resolutions proposed for the Strong CP problem

Conventional resolutions:

- Massless up quark
- Spontaneous CP breaking
- Axion mechanism

Need BSM

Resolutions proposed for the Strong CP problem

Conventional resolutions:

- Massless up quark
- Spontaneous CP breaking
- Axion mechanism

} Need BSM

Recently, several resolutions within QCD were also proposed:

- CP conserving correlations under instanton background

  W.-Y. Ai, J. S. Cruz, B. Garbrecht, C. Tamarit, Phys. Lett. B **822**, 136616 (2021).

- Color field screening by finite θ

  Y. Nakamura and G. Schierholz, Nucl. Phys. B **986**, 116063 (2023).

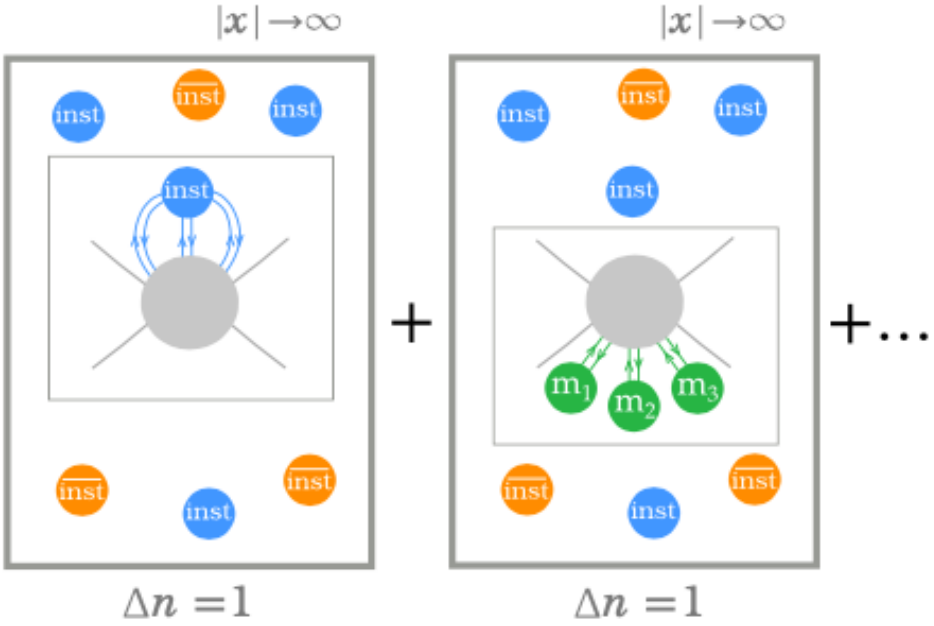
- Unphysical topological charge

  NY, arXiv:2212.10994 [hep-th]; arXiv:2212.11820 [hep-ph].

CP conserving correlations under instanton background

Basic idea :

Calculate quark correlators with CP-odd mass within dilute instanton gas



Take first infinite volume (take all instantons)

Then, sum over topology

⇒ Effect of θ cancels with fermion mass complex phase!

⇒ Effective θ is zero!

W.-Y. Ai, J. S. Cruz, B. Garbrecht, C. Tamarit, Phys. Lett. B **822**, 136616 (2021).

Color field screening by finite θ

Basic idea : Witten effect

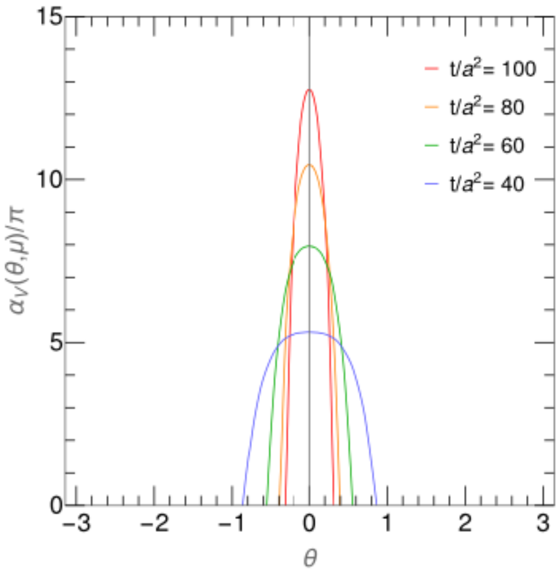
Dual superconductor : magnetic monopole condenses in color confining vacuum

Magnetic monopoles get electric charge under finite θ

E. Witten, Phys. Lett. B **86** (1979) 283.

⇒ Color screening, no monopole condensation,

deconfined colored particles with |θ| > 0 !



Calculation of α_s scale evolution
In lattice :

α_s damps for |θ| > 0
with decreasing scale!
(α_s only grows for θ = 0)

⇒ Confinement only at θ = 0 !

Y. Nakamura and G. Schierholz, Nucl. Phys. B **986**, 116063 (2023).

Unphysical topological charge

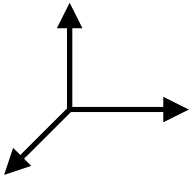
Topological charge is a triple product of gauge fields

$$\frac{\alpha_s}{8\pi}\int d^4x\, F_{\mu\nu,a}\tilde{F}^{\mu\nu}_a =$$

$$\frac{ig_s\alpha_s}{24\pi}\int d^3\vec{x}\, f_{abc}\epsilon_{ijk}A_{ia}(\vec{x})A_{jb}(\vec{x})A_{kc}(\vec{x})\Big|^{t=+\infty}_{t=-\infty}$$

Time is frozen, restricted to 3-dimensional space

ε_{ijk} means



⇒ Covers all 3-dimensional directions!

⇒ Also covers the unphysical gauge freedom direction!

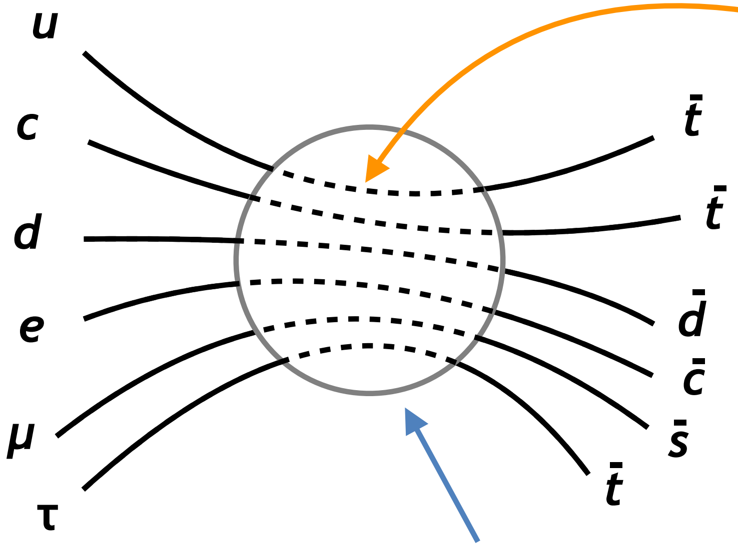
$$(\vec{A}_a \to \vec{A}_a + \vec{\nabla}\chi_a)$$

➡ Topological charge is unphysical !!

NY, arXiv:2212.10994 [hep-th]; arXiv:2212.11820 [hep-ph].

Implication to sphaleron process

’t Hooft vertex : multi-fermion vertex generated by chiral Dirac zero-modes



Fermion propagators
(chiral zero-modes)

Area with topological charge

G. ’t Hooft, PRL **37**, 8 (1976), PRD **14**, 3432 (1976).

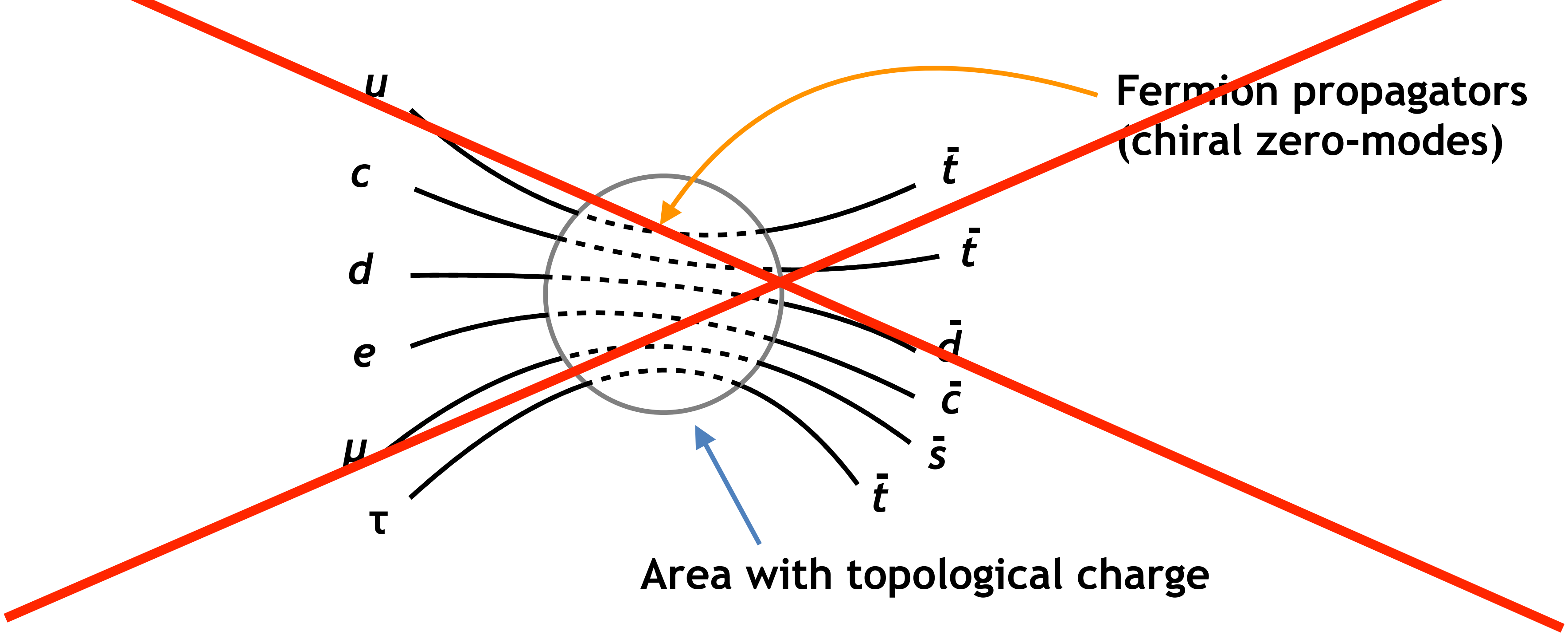
(Nonlocal) B+L violating 12-fermion interaction

⇒ Baryon and lepton numbers may be generated with weak interaction!

V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985), 36;
M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.

Implication to sphaleron process

~~'t Hooft vertex : multi-fermion vertex generated by chiral Dirac zero-modes~~



Fermion propagators
(chiral zero-modes)

Area with topological charge

G. 't Hooft, PRL **37**, 8 (1976), PRD **14**, 3432 (1976).

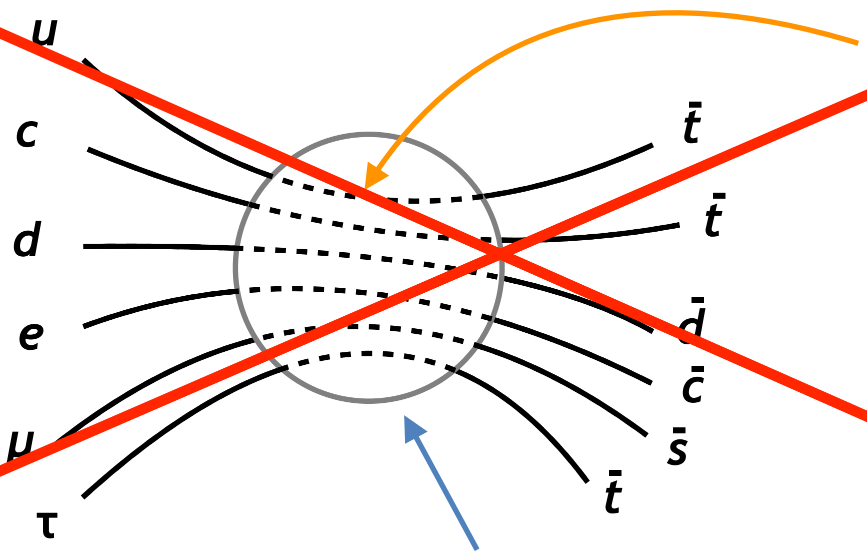
Atiyah-Singer theorem : topological charge = # of chiral Dirac zero-modes

⇒ 't Hooft vertex is unphysical !

since 't Hooft vertex is generated by the propagation of zero-modes

Implication to sphaleron process

~~’t Hooft vertex : multi-fermion vertex generated by chiral Dirac zero-modes~~



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Area with topological charge

G. ’t Hooft, PRL **37**, 8 (1976), PRD **14**, 3432 (1976).

No matter - antimatter asymmetry generation at TeV scale

⇒ Baryogenesis happened at GUT ?

Quiz to physicists

Topological objects spread across infinite space(-time)

(just imagine infinite cosmic string)

(Fig. from BBC Science)

⇒ Their observations provide the information of infinitely large distance

⇒ Zero energy-momentum resolution !

Very fine low energy detector needed due to uncertainty principle

⇒ Impossible to observe ?

Please try to answer this question

Conjecture:

All topological quantities cannot be observed ?

Summary

- Non-removable phases of theory after field redefinition generates CPV.
- General BSM physics brings new phases: CP violation is a direct probe of BSM physics.
- Review of qualitative mechanisms of hadronic CP violation: chromo-EDM → CPV nuclear force is the most sensitive process.
- Review of Strong CP problem: theoretical resolutions without axions?

Future prospects:

- Homework : resolve the systematics of πN sigma-term.
- No sphaleron induced baryogenesis : new scenario in early Universe ?

End