Chiral symmetry of QCD and some of its implications

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Chiral symmetry

- Symmetry of QCD in limit of massless quarks
- Broken
  - Explicitly
  - Spontaneously
- Impact on low-energy hadrons and interactions

QCD Lagrangian

- Lagrangian

$$\mathcal{L}_{\text{QCD}} = \sum_{f=\substack{u,d,s,\\c,b,t}} \bar{q}_f \left(i\not{D} - m_f\right) q_f - \frac{1}{4}\mathcal{G}_{a\mu\nu}\mathcal{G}_a^{\mu\nu}$$

- Light quarks $m_u, m_d, m_s \ll 1$ GeV, heavy quarks $m_c, m_b, m_t \gtrsim 1$ GeV
- Low energies $\ll 1$ GeV $\rightarrow$ ignore heavy quarks as d.o.f.
- Simple quark model: $M_p \gg 2m_u + m_d \rightarrow$ approximate $m_u = m_d = 0\,(= m_s)$

Chiral limit

QCD in the chiral limit

- QCD Lagrangian in chiral limit

$$\mathcal{L}_0 = (\bar{q}_R i \not{D} q_R + \bar{q}_L i \not{D} q_L) - \frac{1}{4}\mathcal{G}_{a\mu\nu}\mathcal{G}_a^{\mu\nu}$$

- Right- and left-handed quark fields

$$q_R = \frac{1}{2}(\mathbb{1} + \gamma_5)q, \quad q_L = \frac{1}{2}(\mathbb{1} - \gamma_5)q$$

where

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \quad \text{or} \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

Chiral symmetry

$$\mathcal{L}_0 = (\bar{q}_R i \not{D} q_R + \bar{q}_L i \not{D} q_L) - \frac{1}{4}\mathcal{G}_{a\mu\nu}\mathcal{G}_a^{\mu\nu}$$

- Invariant under independent global $U(N_l)$ transformations

$$q_R \mapsto \tilde{U}_R q_R, \quad q_L \mapsto \tilde{U}_L q_L$$

- Consider $U(2) \simeq SU(2) \times U(1)$
- $U(1)_A$ broken by anomaly $\Rightarrow SU(2)_R \times SU(2)_L \times U(1)_V$

Chiral symmetry: Invariance of $\mathcal{L}_0$ under global $SU(2)_R \times SU(2)_L$

Explicit chiral symmetry breaking

- Quark masses nonzero

$$\mathcal{L}_{\mathcal{M}} = -(\bar{q}_R \mathcal{M} q_L + \bar{q}_L \mathcal{M} q_R)$$

where

$$\mathcal{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

- Not invariant under $SU(2)_R \times SU(2)_L$

Explicit chiral symmetry breaking

- For $m_u = m_d = \hat{m} \Rightarrow \mathcal{M} = \hat{m}\mathbb{1}$

$\rightarrow$ Invariant under $U_R = U_L \in SU(2)_V$: isospin

Expected consequences of chiral symmetry

- $SU(2)_R \times SU(2)_L$
  - Approximately mass degenerate hadron multiplets
  - Opposite parity (“parity doubling”)
- Observed hadron spectrum
  - Isospin multiplets $\rightarrow SU(2)_V$
  - No parity doubling
  - Pions much lighter than other nonstrange hadrons

Spontaneous breaking of chiral symmetry

Chiral symmetry spontaneously broken

$$SU(2)_R \times SU(2)_L \to SU(2)_V$$

- Pions = Nambu-Goldstone bosons
- Nonzero quark masses $\Rightarrow$ pions massive, but light
- Assumption: No proof for spontaneous chiral symmetry breaking

Chiral perturbation theory

- Effective field theory of low-energy QCD
- Formulated in terms of observed hadrons (pions, nucleons)
- Chiral symmetry
  - Nonlinearly realized
  - Ward-Takahashi identities: relations between different processes
- Systematic expansion in terms of q/Λ_χ and M_π/Λ_χ with $\Lambda_\chi \approx$ 1 GeV$(?)$
- Power counting: determine importance of individual contributions

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Weinberg (1979); Gasser, Leutwyler (1984,85)

Example 1: $\pi\pi$ scattering

- At threshold two S-wave scattering lengths $a_0^{I=0}$, $a_0^{I=2}$
- ChPT predicts scattering lengths at LO
- Calculated up to two-loop order, e.g.,

$$
\begin{aligned}
a_0^{I=0} &= 0.16 && \text{(LO)} \\
a_0^{I=0} &= 0.20 \pm 0.01 && \text{(NLO)} \\
a_0^{I=0} &= 0.217 \pm 0.009 && \text{(NNLO)}
\end{aligned}
$$

- $|a_0^{I=0} - a_0^{I=2}|$ in agreement with measurement in pionium

Weinberg (1966); Gasser, Leutwyler (1984); Bijnens et al. (1996); Adeva et al. (2011)

Precision tool in meson sector

- π and K form factors
- $\pi^0 \to \gamma\gamma$
- (Semi-)leptonic K decays $\to |V_{us}|$
- Rare decays
- …

Baryon ChPT

- Include nucleons as degrees of freedom
- Nucleon mass m_N does not vanish in chiral limit: "This complicates life a lot."
- New approaches for consistent power counting:
  - Heavy-baryon ChPT (HBChPT): expansion in $1/m_N$
  - Infrared regularization (IR)
  - Extended On-Mass-Shell (EOMS) scheme

Gasser, Sainio, Švarc (1988); Jenkins, Manohar (1991); Becher, Leutwyler (1999); Gegelia, Japaridze (1999); Fuchs et al. (2003)

Example 2: Nucleon axial radius

Axial form factor of nucleon $G_A(q^2)$

- Parameterized as

$$G_A(q^2) = \frac{g_A}{(1-\frac{q^2}{M_A^2})^2} = g_A\left(1+\frac{1}{6}\langle r_A^2\rangle q^2 + \cdots\right)$$

- M_A: axial mass, $\langle r_A^2\rangle = \frac{12}{M_A^2}$: axial radius

- Determined from
  - (Quasi-)elastic (anti-)neutrino scattering: $M_{A,\nu} = (1.001 \pm 0.020)$ GeV
  - Pion electroproduction $\gamma^* + p \to n + \pi^+$ at threshold: $M_{A,\pi} = (1.068 \pm 0.017)$ GeV

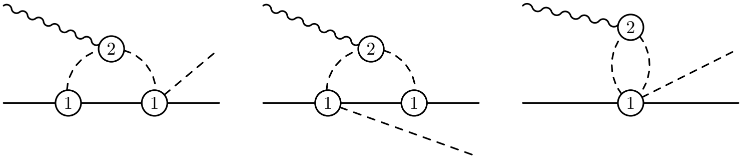
- In pion electroproduction axial radius determined from amplitude

$$E_{0+}^{(-)}(k^2) = \frac{e g_A}{8\pi F_\pi}\underbrace{[1 + \frac{k^2}{4m_N^2}\left(\kappa_V + \frac{1}{2}\right) + \frac{k^2}{6}r_A^2}_{\text{old}} + \cdots]$$

Bernard, Kaiser, Meißner (1992)

- In pion electroproduction axial radius determined from amplitude

$$E_{0+}^{(-)}(k^2) = \frac{eg_A}{8\pi F_\pi}[\underbrace{1 + \frac{k^2}{4m_N^2}\left(\kappa_V + \frac{1}{2}\right) + \frac{k^2}{6}r_A^2}_{\text{old}} + \underbrace{\frac{M^2}{8\pi^2 F_\pi^2} f\left(\frac{k^2}{M^2}\right)}_{\text{new: ChPT}} + \cdots]$$



- $\Delta M_A = -0.056$ GeV
- $M_{A,\pi} = (1.068 \pm 0.017)$ GeV $\leftrightarrow M_{A,\nu} = (1.001 \pm 0.020)$ GeV

Bernard, Kaiser, Meißner (1992)

BChPT applications

- Mass
- Magnetic moment
- Form factors
- Compton scattering
- πN scattering
- Pion photoproduction
- …

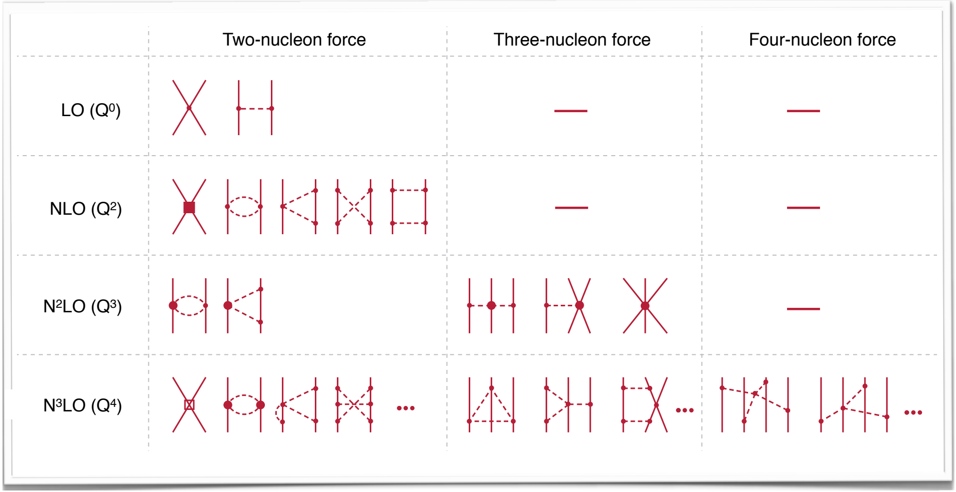
Nucleon-nucleon interactions

- Long range: one-pion exchange
- Intermediate range: two-pion exchange
- Short range: three-pion exchange, heavy meson exchanges,…?
- Impact of chiral symmetry?

Systematic approach to nuclear forces

Weinberg (1990,91); Ordoñez, Ray, van Kolck (1996)

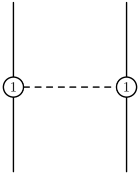
Chiral Effective Field Theory



Source: E. Epelbaum

Chiral Effective Field Theory

- One-pion exchange

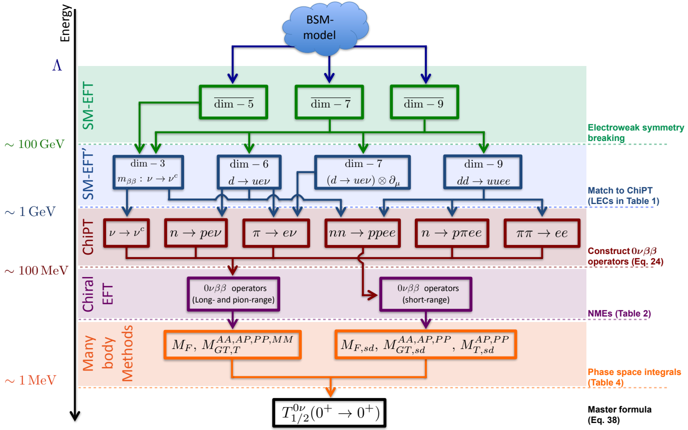


- Vertices given by πN Lagrangian of BChPT
- Also holds for two-, three-,…pion exchange
- Long- and intermediate-range physics determined from pion-pion and pion-nucleon results
- Also applies to electromagnetic and weak probes $\rightarrow$ two-body currents

Chiral symmetry and BSM physics

- Nucleons used in searches for beyond-the-standard-model (BSM) physics
- BSM models formulated in terms of quark fields
- Map chiral symmetry properties of quark operators onto nucleon level
  $\Rightarrow$ constraints on allowed operators
- Relations between different operators from chiral Ward identities

Chiral symmetry and BSM physics



Cirigliano et al., JHEP 12 (2018)

Neutrinoless double beta decay

- Renormalization analysis of light Majorana exchange in two-nucleon sector $\Rightarrow nn \to ppe^-e^-$ contact terms at LO
- Determine from
  - Data?
  - Lattice QCD?
- Related through chiral symmetry to charge independence breaking (CIB) NN interaction
- Use constraints on CIB to constrain $0\beta nn$ contribution

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Cirigliano et al. (2018); Cirigliano et al. (2021);Richardson, MRS, Pastore, Springer (2021)

Chiral extrapolations

- Lattice QCD calculations performed at various pion masses
- Need to extrapolate to physical pion mass
- Chiral perturbation theory: M_π expansion of observables
- Use ChPT techniques to account for finite volume and/or finite lattice spacing

Conclusions

Chiral symmetry

- Approximate symmetry of QCD
- Spontaneously and explicitly broken
- Determines low-energy hadron interactions
- Highly successful in meson, one-baryon, and NN, NNN, …sectors
- Important guide in constructing hadronic level operators for BSM physics
- Chiral extrapolations of lattice QCD results