The Electron g–2 as a Precision Test of QED and a Probe of New Physics

Makiko Nio (RIKEN & Saitama U)

SSP2025, Nara, Japan

September 23 – 28, 2025

Based on

long-term collaboration of AHKN: T. Aoyama, M. Hayakawa, T. Kinoshita(Cornell U), and M. Nio

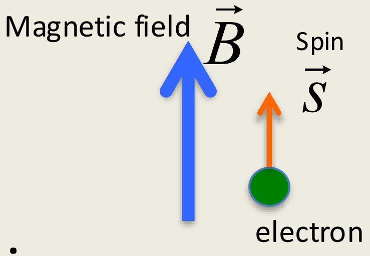
Phys. Rev. D **111**, L031902, arXiv:2412.06473

w/ T. Aoyama(U Tokyo), M. Hayakawa(Nagoya U & RIKEN), A. Hirayama(Saitama U)

The QED section of the muon g-2 in the SM: an update, White Paper 2025, arXiv:2025.21476

w/ T. Aoyama, M. Hayakawa, and S. Volkov(MPI)

Electron g-2



Magnetic moment of a point-like Dirac fermion

$$H = -\vec{\mu} \cdot \vec{B} \qquad \vec{\mu} = g \frac{e\hbar}{2m} \frac{\vec{s}}{\hbar}$$

dimensionless constant

Relativistic QM Dirac equation

$$(i\gamma^\mu \mathcal{D}_\mu - m)\psi = 0$$

$$g = 2$$

QED

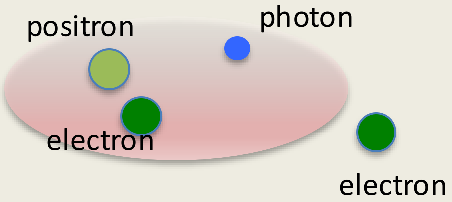
$$\mathcal{L}_{\text{QED}} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi$$

$$g = 2(1 + 0.00116\ldots)$$

$$a \equiv \frac{g-2}{2}$$

Anomalous magnetic moment

Particles can exist
in a very short time period
Fluctuation of vacuum



Precision tests of g-2 in 1948

New physics was QED !

P. Kusch

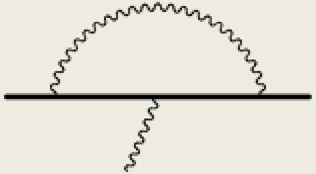
Experiment: Ga and Na atom hfs spectra

$$a_e = 0.001\ 19 \pm 0.000\ 05 \qquad 4.2\%$$

if the orbital g factor $g_l = 1$

J. Schwinger

Theory: QED 2nd-order (1 loop)



$$a_e = \frac{\alpha}{2\pi}$$

$$= 0.001\ 162 \cdots$$

with the fine-structure constant

$$1/\alpha = 137.035 \ldots$$

Electron g-2 Penning Trap measurement in 2022

arXiv: 2209.13084, PRL130,071801(2023)

Fan, Mayers, Sukra, and Gabrielse,   Northwestern U

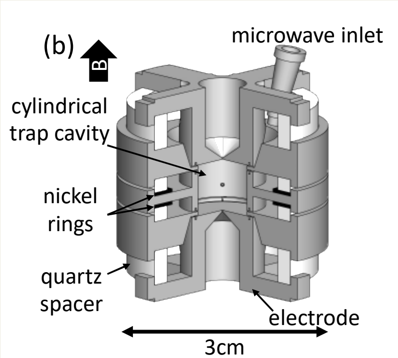
$$a_e(\mathrm{NW22}) = 1\,159\,652\,180.59\,(13) \times 10^{-12}, \quad \underline{0.11\mathrm{ppb}}$$

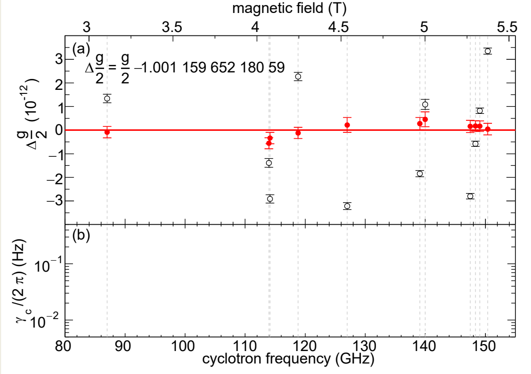
$$a_e(\mathrm{HV08}) = 1\,159\,652\,180.72\,(28) \times 10^{-12}, \quad 0.24\mathrm{ppb}$$

2.2 times better than before

different values of magnetic fields

less systematic uncertainty





Electron g-2 in Standard Model $a_e = +0.00116\cdots$

0.11ppb

Standard-Model prediction of the electron g-2

$$a_e(\text{SM}) = a_e(\text{QED}) + a_e(\text{hadron}) + a_e(\text{weak})$$

The electron is the lightest charged lepton, 0.51 MeV/c^2

	Particles involved	Percentage to the total contribution	
QED	photon (γ), electron(e), muon(μ), tau-lepton(τ)	$999\,999\,998.48 \times 10^{-9}$	dominant
Hadronic	e, γ quarks and gluons as hadrons,	1.49×10^{-9}	
Weak	e, γ, quarks or hadrons, Higgs weak bosons ($W^{\pm}$, Z^0)	0.03×10^{-9}	
SM		1	

QED contribution to g-2

precision required < 0.1 ppb

QED contribution to the electron g-2

g-2 is a dimensionless number, and mass appears in the form of a ratio

$$a_e(\mathrm{QED}) = A_1 + A_2\left(\frac{m_e}{m_\mu}\right) + A_2\left(\frac{m_e}{m_\tau}\right) + A_3\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right)$$

QED perturbation theory as a power series of the fine-structure constant α:

$$A_i = \left(\frac{\alpha}{\pi}\right) A_i^{(2)} + \left(\frac{\alpha}{\pi}\right)^2 A_i^{(4)} + \left(\frac{\alpha}{\pi}\right)^3 A_i^{(6)} + \cdots$$

Need up to the 10th-order, since $(\alpha/\pi)^5/a_e \sim 0.058 \times 10^{-9}$

Status of the QED electron g-2 calculations

Order of perturbation 2n	$A_1^{(2n)}$	$A_2^{(2n)}(m_e/m_\mu)$	$A_2^{(2n)}(m_e/m_\tau)$	$A_3^{(2n)}$
2	analytic 1948	0	0	0
4	analytic 1957-1958	analytic 1966	analytic 1966	0
6	analytic 1996	analytic 1993	analytic 1993	analytic 1999
8	almost analytic 2017	analytic 2013	analytic 2013	analytic 2013
10	numerical 2012 – partially double-checked	numerical 2012 – partially double-checked	not yet Too small	not yet Too small

$A_1^{(10)}$ is the only relevant but unchecked term until 2024

Analytic results:
Schwinger('48),Petermann('57), Sommerfield('58), Elend('66), Laporta and Remiddi('93,'96), Czarnecki & Skrzypek ('99) Laporta('17), Kurz, Liu, Marquard, and Steinhauser ('13)

10^{th}-order QED g-2 $A_1^{(10)}$

AHKN's calculation of $A_1^{(10)}$

- arXiv:2012.5368, 2012.5380 Phys. Rev. Lett.109, 11708 & 11809 (2012)
- several updates on Set V latest is AHKN2019 Atoms 7(1), 28 (2019)

S. Volkov's calculation of $A_1^{(10)}$

- arXiv:1909.08015 Phys. Rev. D 100, 096004 (2019)

  Diagrams of Set V without fermion loop

  discrepancy 4.8σ from the AHKN 2019 result

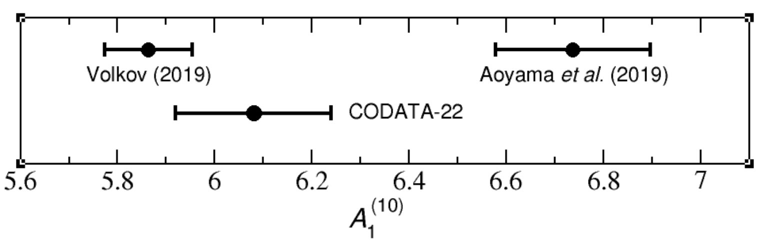


Fig.2, CODATA22, 2024

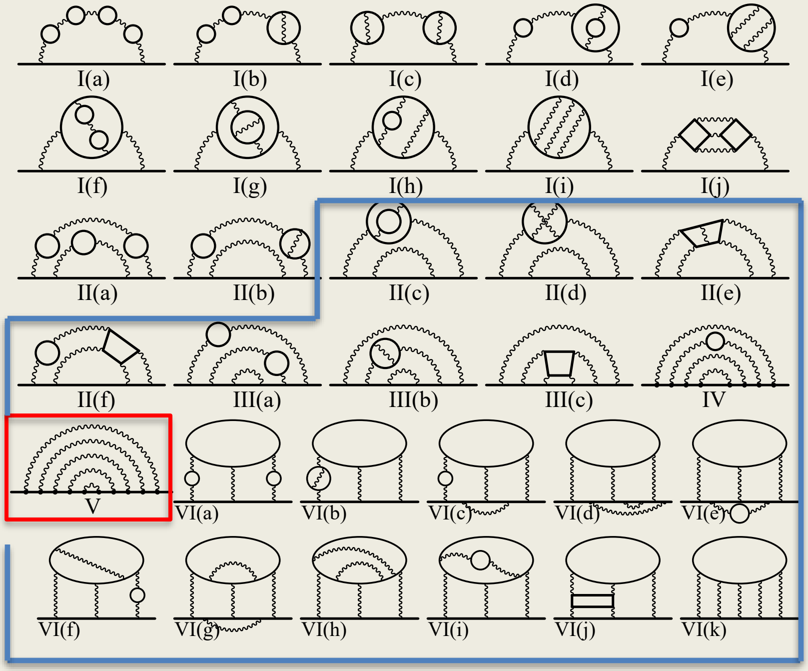
- arXiv:2404.00649 Phys. Rev. D 110, 036001 – Published 2 August 2024

  Diagrams w/ fermion loop agree with the AHKN 2012 results

  Set V discrepancy 4.6σ from the AHKN 2019 result

See the poster at the entrance hall

10th-order diagrams



Double-checked in 2024

- 12,672 vertex diagrams over 32 Sets

- 31 Sets w/ fermion loop (6,318 diagrams)

  I(a-j), II(a,b) were already confirmed

  S. Lapota (1994)

  P. A. Baikov, A. Maier, and P. Marquard (2013)

  agreement between AHKN and Volkov

  AHKN2018 Volkov2024

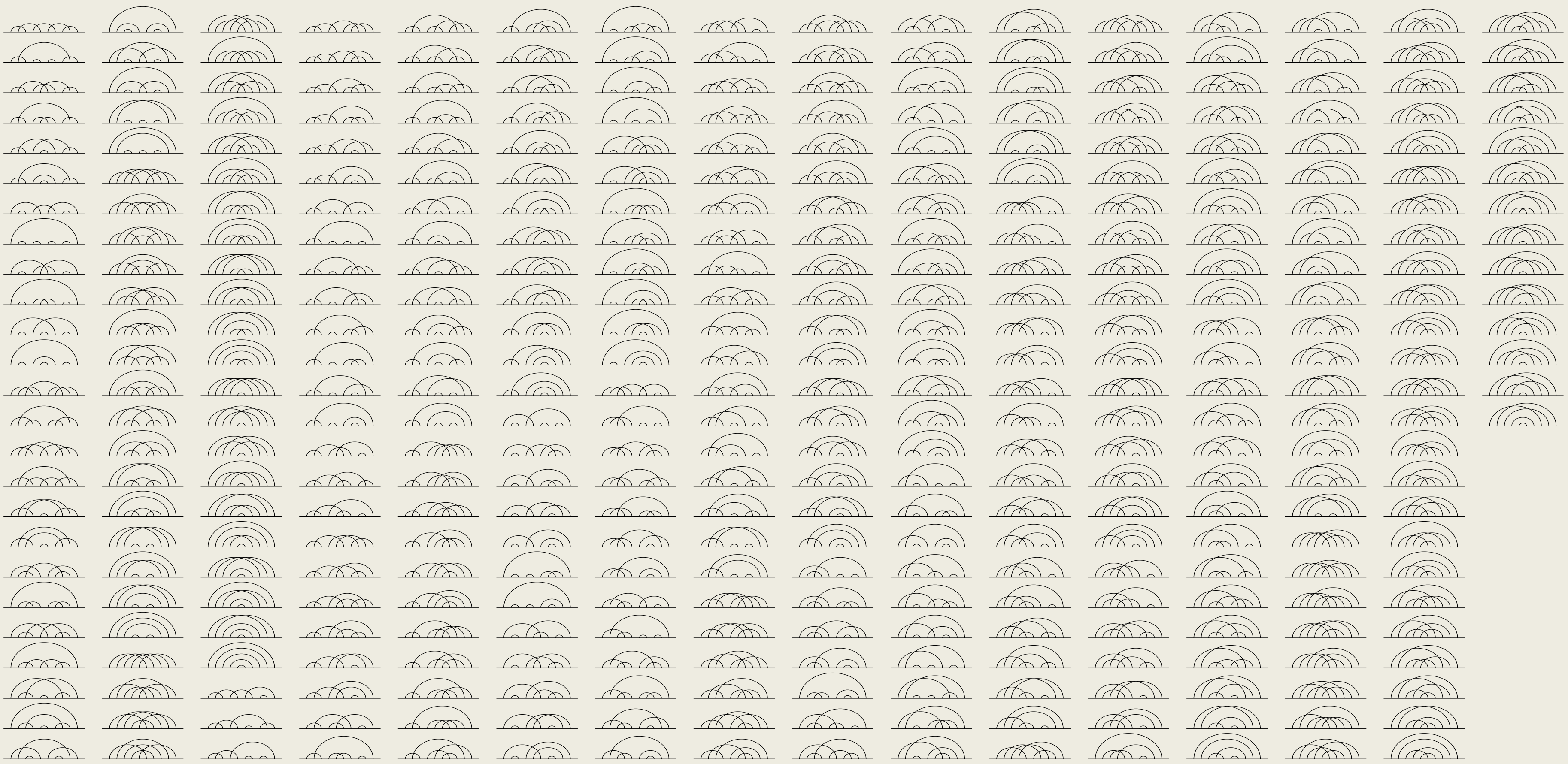
$$-0.9304\,(36) - \{-0.9377\,(35)\} = 0.0073\,(50)$$

- Set V w/o fermion loop (6,354 diagrams)

  AHKN2019 Volkov

$$7.668\,(159) - \begin{cases} 6.793\,(90) & = 0.875\,(182), \quad 4.8\sigma\,(2019) \\ 6.857\,(81) & = 0.811\,(178), \quad 4.6\sigma\,(2024) \end{cases}$$

Set V 6,354 vertex diagrams represented by 389 self-energy diagrams



Set V: Ward-Takahashi sum v.s. Vertices

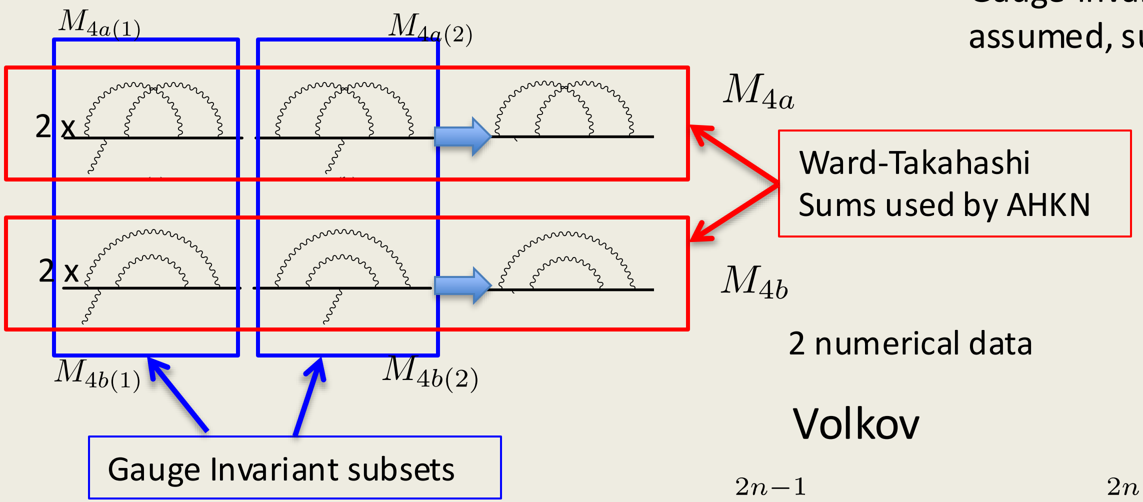
AHKN: 389 Ward-Takahashi sums of v-diagrams

Thanks to the U(1) gauge invariance of QED

Volkov: 3,213 vertex diagrams

Gauge-invariant UV and IR regularizations are assumed, such as dimension *D*

4th-order



2 numerical data

Every vertices are calculated by Volkov
4 numerical data

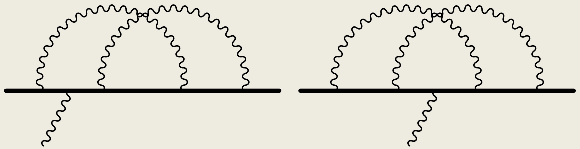
Volkov   AHKN

$$\sum_{i=1}^{2n-1} \Lambda^{\nu}_{\mathcal{G}(i)}(p,q) \simeq -q_{\mu} \sum_{i=1}^{2n-1} \left[\frac{\partial \Lambda^{\mu}_{\mathcal{G}(i)}(p,q)}{\partial q_{\nu}} \right]_{q=0} - \frac{\partial \Sigma_{\mathcal{G}}(p)}{\partial p_{\nu}}$$

The WT-sum of Volkov's integrals does not match the AHKN integral.   Why not?

Connection b/w Volkov and AHKN

Volkov's integrals

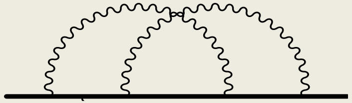


$$2\Delta M_{4a(1)} \equiv 2(M_{4a(1)} - LV_2\, M_2)$$

$$\Delta M_{4a(2)} \equiv M_{4a(2)}$$

finite bare UV renorm.

AHKN integral



$$\Delta M_{4a} \equiv M_{4a} - 2LK_2\, M_2$$

finite bare UV renorm.

With gauge-invariant regularization, the WT-identity guarantees

$$2M_{4a(1)} + M_{4a(2)} = M_{4a}$$

Thus, we have the connection

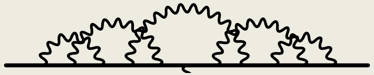
$$\Delta M_{4a} - (2\Delta M_{4a(1)} + \Delta M_{4a(2)}) = 2\ \delta L_2\ M_2$$

AHKN' s integral Volkov's integrals gap equation

where $$\delta L_2 \equiv LV_2 - LK_2$$

difference of renormalization constants

X001 as an example



abacbdcede

gap equation

$$\Delta M_{X001} - \sum_{i=1}^{9} \Delta M_{X001(i)} = \Delta M_2 \left(-3(\delta L_{4a1})^2 - 6\delta L_2 \delta L_{6f1} + 12(\delta L_2)^2 \delta L_{4a1} - 5(\delta L_2)^4 + 2\delta L_{01v1}\right)$$
$$+ \Delta M_{01} \left(2\delta L_2\right)$$
$$+ \Delta M_{6f} \left(2\delta L_{4a1} - 3(\delta L_2)^2\right)$$
$$+ \Delta M_{4a} \left(2\delta L_{6f1} - 6\delta L_2 \delta L_{4a1} + 4(\delta L_2)^3\right)$$

AHKN

Volkov2019

Numerical integrals

Symbolic expression

$$\begin{aligned}
\text{l.h.s} &= -0.16083\ (334) - 0.58095\ (534) \\
&= -0.74178\ (630) \\
\text{r.h.s} &= -0.73854\ldots \\
\text{l.h.s} - \text{r.h.s} &= -0.00324\ (630)
\end{aligned}$$

Substitute numerical values for lower-order symbols

A. Hirayama, JPS 2021 spring meeting

X001 safely passes the numerical check

Both AHKN and Volkov correctly calculated X001

All X001 – X389 pass the check

almost 0

Diagram $\mathcal{G}$	Expression	$\Delta M_{\mathcal{G}} - \sum_i \Delta M_{\mathcal{G}(i)}$	Gap Equation	Difference
X001	*abacbdcede*	−0.7418	−0.7385	−0.0033 (63)
X002	*abaccddebe*	8.0130	8.0253	−0.0123 (139)
X003	*abacdbcede*	2.0226	2.0221	0.0006 (29)
X004	*abacdcdebe*	−6.5041	−6.5146	0.0104 (130)
X005	*abacddbece*	−0.2680	−0.2789	0.0110 (106)
X006	*abacddcebe*	0.5522	0.5522	−0.0000 (125)
X007	*abbcadceed*	−0.2365	−0.2250	−0.0115 (128)
X008	*abbccddeea*	−3.8164	−3.8115	−0.0050 (168)
X009	*abbcdaceed*	0.1962	0.2050	−0.0089 (69)
X010	*abbcdcdeea*	−1.4020	−1.4014	−0.0007 (137)
X011	*abbcddaeec*	−0.6609	−0.6645	0.0035 (110)
X012	*abbcddceea*	−0.4561	−0.4717	0.0156 (129)
X013	*abcabdecde*	1.7891	1.7879	0.0012 (14)
X014	*abcacdedbe*	0.2015	0.2018	−0.0003 (32)
X015	*abcadbecde*	1.1125	1.1141	−0.0016 (6)
X016	*abcadcedbe*	−0.4574	−0.4567	−0.0007 (5)
X017	*abcaddebce*	−0.7935	−0.7966	0.0030 (15)
X018	*abcaddecbe*	−0.4154	−0.4194	0.0040 (17)
X019	*abcbadeced*	2.3984	2.3968	0.0016 (31)
X020	*abcbcdedea*	7.6668	7.6799	−0.0131 (131)
X021	*abcbdaeced*	0.3403	0.3408	−0.0005 (16)
X022	*abcbdcedea*	−1.1144	−1.1238	0.0094 (114)
X023	*abcbddeaec*	−0.4987	−0.5074	0.0087 (58)
X024	*abcbddecea*	4.5369	4.5595	−0.0226 (131)
X025	*abccadeebd*	2.4091	2.4333	−0.0242 (119)
X026	*abccbdeeda*	1.2078	1.2042	0.0035 (110)
X027	*abccdaeebd*	−0.2238	−0.2272	0.0034 (46)

and 262 more 0's

Both AHKN and Volkov correctly formulated the Set V integrals.

Why did the discrepancy arise?

- The differences range from -0.03 to $+0.03$
- Not randomly distributed → Bias in numerical integration

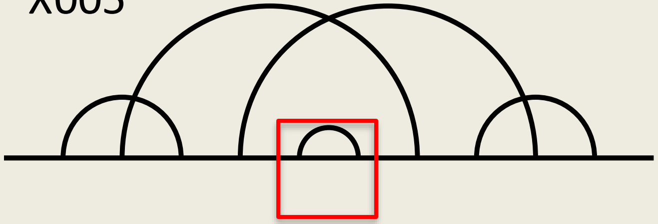
  # of negative differences << # of positive differences

- We divided 389 self-energy diagrams into 4 classes

Diagrams w/o a self-energy subdiagram	XL	135	
Diagrams w/ one 2^{nd}-order self-energy subd	XB1B2	98	**The culprit**
Diagrams w/ two 2^{nd}-order self-energy subd	XB2B2	33	
Others		123	

XB1B2 98 SE diagrams

X005



2^{nd}-order s.e. subdiagram

Differences are positive for 90 diagrams

negative for 8 diagrams

$$\sum_G \left(\Delta M_G - \sum_{i=1}^{9} \Delta M_{G(i)}\right) = -49.095\,(73)$$

$$\sum_G \text{the gap equation for } G = -50.089$$

$$\sum_G \text{differences} = +0.994\,(73)$$

AHKN is bigger than the answer

OR

Volkov is smaller than the answer

This accounts for the discrepancy between AHKN and Volkov of Set V

$$7.668\,(159) - \begin{cases} 6.793\,(90) & = 0.875\,(182), \quad 4.8\sigma\,(2019) \\ 6.857\,(81) & = 0.811\,(178), \quad 4.6\sigma\,(2024) \end{cases}$$

Re-evaluation of the 98 integrals of AHKN

Numerical calculations, December 2023 – April 2024 @HOKUSAI-BW2, RIKEN
during the 4-month test operation period

- 12dim. Monte-Carlo integration with the algorithm VEGAS used for previous works
- Quasi double-double-precision calculation
- Statistics of sampling points increased from O(10^9) to O(10^10)
- ~ 3.2 x 10^7 core-hours (133 days with 10,000 cores)

old new shift

$$\sum_G \Delta M_G = 57.806\,(64) \longrightarrow 57.002\,(33), \quad \boxed{-0.803\,(72)}$$

Updated Set V result

If 98 integrals of XB1B2 are replaced by the new results ,

Aoyama,Hayakawa,Hirayama,Nio 2024

AHKN Volkov

$$6.800\,(128) - \underline{6.828\,(60)} = 0.028\,(141), \qquad 0.2\sigma$$

The discrepancy has been resolved

weighted average of Volkov2019 and Volkov2024

- The 98 integrals of XB1B2 are relatively easy to evaluate
- Calculated around 2008 - 2012, more than a decade ago
- The # of samplings for Monte Carlo integration was not sufficient

Summary of the QED $A_1^{(10)}$

	AHKN	Volkov
$A_1^{(10)}$ [SetV]	$6.800\,(128)$	$6.828\,(60)$
$A_1^{(10)}$ [others]	$-0.9304\,(36)$	$-0.9377\,(35)$
$A_1^{(10)}$ [all]	$5.870\,(128)$	$5.891\,(61)$

Set V	6,354 diagrams with no fermion loop
others	6,318 diagrams with at least one fermion loop
all	sum of the above, 12,672 diagrams

AHKN and Volkov have agreed to present the weighted average

$$A_1^{(10)} = 5.877\,(55)$$

Aoyama, Hayakawa, Nio, and Volkov, in the QED section of the muon g-2 WP25

Theory of Electron g-2

$$a_e = +0.00116\cdots$$

0.11ppb

Standard-Model prediction of the electron g-2

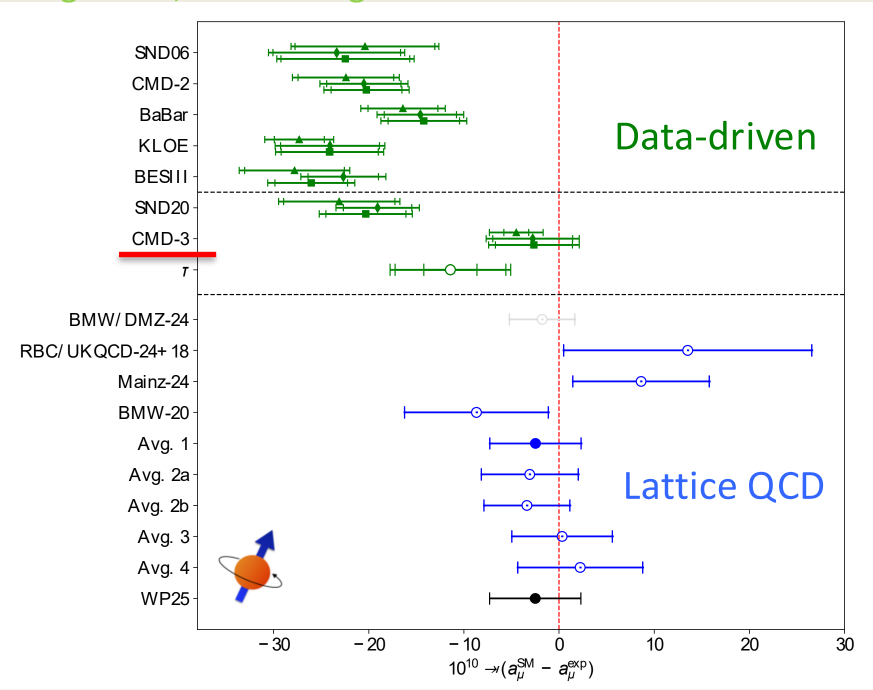
$$a_e(\text{SM}) = a_e(\text{QED}) + a_e(\text{hadron}) + a_e(\text{weak})$$

The electron is the lightest charged lepton, 0.51 MeV/c^2

	Particles involved	Percentage to the total contribution
QED	photon (γ), electron(e), muon(μ), tau-lepton(τ)	$999\,999\,998.48 \times 10^{-9}$
Hadronic	e, γ quarks and gluons as hadrons,	1.49×10^{-9}
Weak	e, γ, quarks, Higgs(H) weak bosons ($W^{\pm}$, Z^0)	0.03×10^{-9}
SM		1

Hadronic contribution to the muon g-2

Figure 40, the muon g-2 WP25

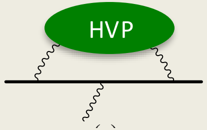


The same HVP data can be used to calculate $a_e^{\text{HVP, LO}}$

$$a_l^{\text{HVP, LO}} = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{ds}{s} R(s) K_l(s)$$

HVP data

lepton-mass dependence



Leading order hadronic vacuum-polarization (HVP, LO) contribution to the muon g-2 $a_\mu^{\text{HVP, LO}}$

HVP, LO to the electron g-2

Three calculations so far

$$a_e^{\text{HVP, LO}} = \begin{cases} 1.861\,(7) \times 10^{-12} \\ 1.920\,(9) \times 10^{-12} \\ 1.893\,(62) \times 10^{-12} \end{cases}$$

5σ (between (1) and (2))

(1) Data-driven w/o CMD-3
Keshavarzi, Nomura, Teubner 2019

(2) Data-driven w/ CMD-3 ππ channel
Di Luzio, Keshavarzi, Masiero, Paradisi 2024

(3) Lattice QCD
Budapest-Marseille-Wuppertal（BMW) 2017

El-Khadra and Hoferichter, personal communication, 2025

Suggestions from the QCD researchers of the muon g-2 WP25:

- Average (1) and (2), and assign half of the difference to the uncertainty
- The uncertainty of Lattice QCD is too large. It is just a reference

We took

$$a_e^{\text{HVP, LO}} = 1.89\,(3) \times 10^{-12}$$

The fine-structure constant α from the electron g-2

Assume that the SM is correct

$$a_e^{\rm exp}({\rm NW22}) = a_e^{\rm SM}[\alpha]$$

Experiment α is the only unknown in SM theory

Solve α

$$\alpha^{-1}(a_e) = 137.035\,999\,163\,(15) \quad 0.11\,{\rm ppb}$$

Experiment NW22 only

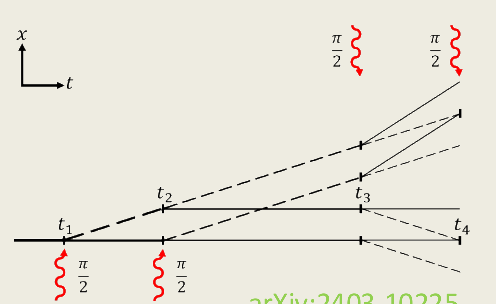
cause of change	shift in α^{-1}	uncertainty
QED 10^{th}-order $A_1^{(10)}$	-6.8×10^{-9}	0.44×10^{-9}
Hadron VP $a_e^{\rm HVP,LO}$	$+3.4 \times 10^{-9}$	3.6×10^{-9}

non-QED values of the fine-structure constant α

- h/M of Cs or Rb using an atom interferometer

$$\alpha = \left[\frac{h}{M} \times \frac{A_r(M)}{A_r(m_e)} \times \frac{2R_\infty}{c}\right]^{1/2}$$

least precise



arXiv:2403.10225

$h/M_{\mathrm{Cs}} = 3.002\,369\,4721\,(12) \times 10^{-9}\,\mathrm{m^2s^{-1}}$ 0.40 ppb — Parker et al, UC Berkley 2018

$h/M_{\mathrm{Rb}} = 4.591\,359\,258\,90\,(65) \times 10^{-9}\,\mathrm{m^2s^{-1}}$ 0.14 ppb — Morel et al, LKB Paris 2020

$R_\infty = 10\,973\,731.568\,157(12)\,\mathrm{m}^{-1}$ 1.1 ppt

$A_r(e) = 5.485\,799\,090\,441\,(97) \times 10^{-4}\,\mathrm{u}$ 18 ppt

$A_r(M_{\mathrm{Cs}}) = 132.905\,451\,9585\,(86)\,\mathrm{u}$ 65 ppt

$A_r(M_{\mathrm{Rb}}) = 86.909\,180\,5291\,(65)\,\mathrm{u}$ 75 ppt

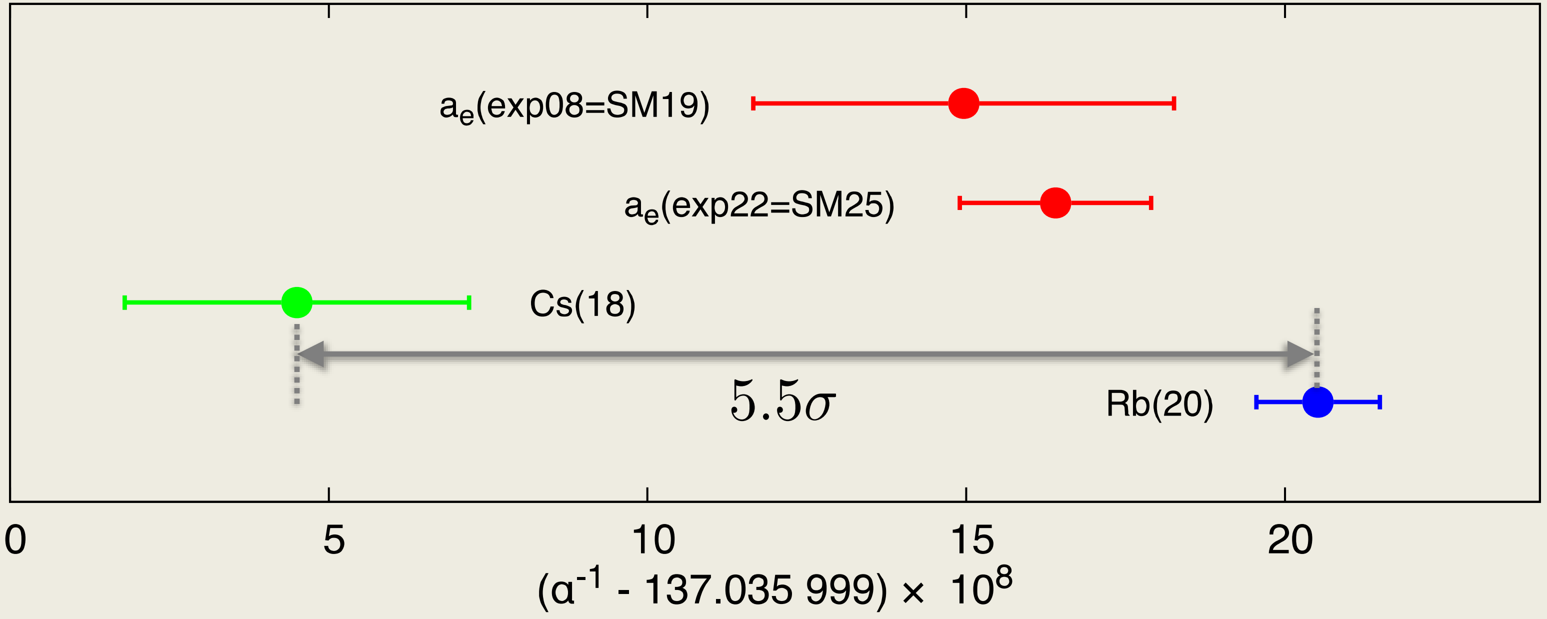
CODATA2022, 2024

$\alpha^{-1}(\mathrm{Cs}) = 137.035\,999\,045\,(27)$ 0.20 ppb

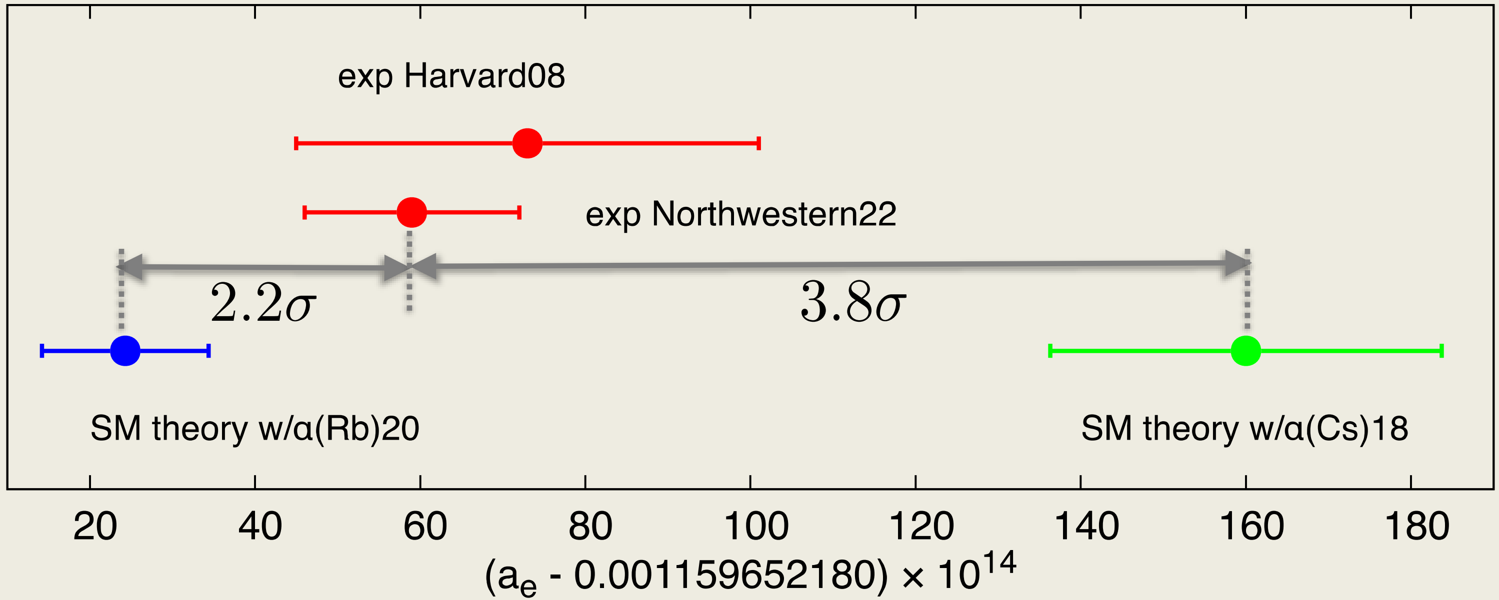
$\alpha^{-1}(\mathrm{Rb}) = 137.035\,999\,2052\,(97)$ 0.071 ppb

$\alpha^{-1}(a_e) = 137.035\,999\,163\,(15)$ 0.11 ppb — as a reference

Three values of the fine-structure constant α



Electron g-2 Theory v.s. Experiment



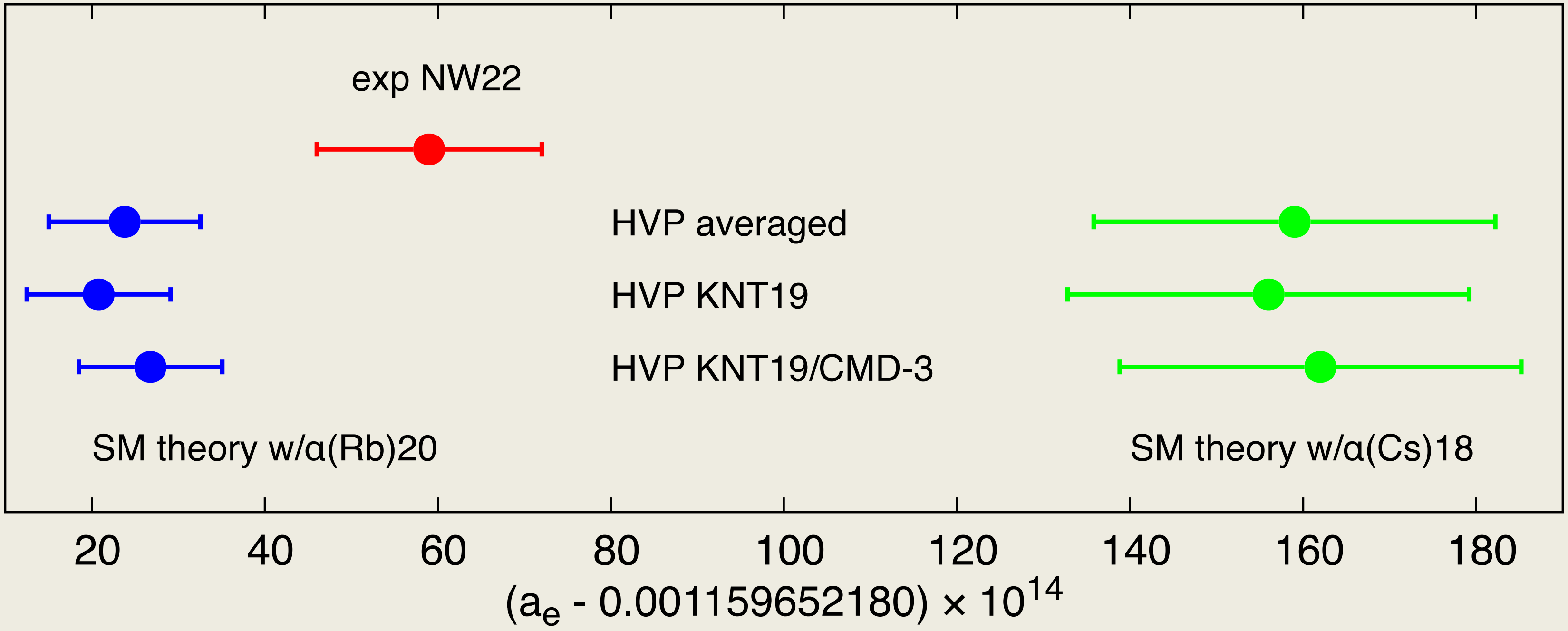
$$a_e^{\mathrm{exp}}(\mathrm{NW22}) = \quad 1\,159\,652\,180.59\,(13) \times 10^{-12} \qquad 0.11\,\mathrm{ppb}$$

$$a_e^{\mathrm{SM}}[\alpha(\mathrm{Cs})] = \quad 1\,159\,652\,181.59\,(23)(0)(3)[23] \times 10^{-12} \qquad 0.20\,\mathrm{ppb}$$

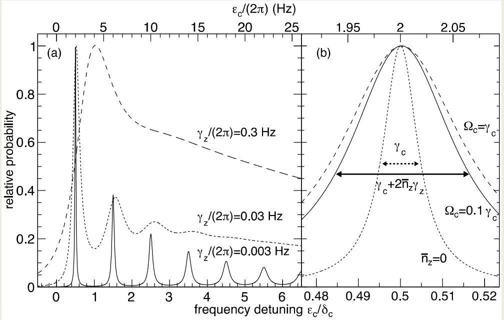
$$a_e^{\mathrm{SM}}[\alpha(\mathrm{Rb})] = \quad 1\,159\,652\,180.238\,(82)(4)(30)[87] \times 10^{-12} \qquad 0.075\,\mathrm{ppb}$$

α QED hadron total

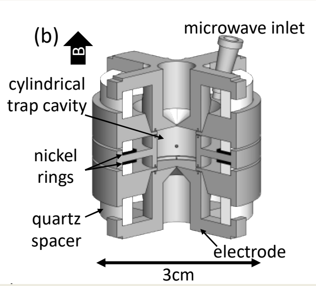
Effects of the HVP, LO $a_e^{\mathrm{HVP,LO}}$ on the SM theory

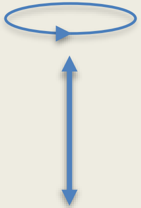


Prospects on the electron g-2 experiment



- positron g-2 measurement
- electron g-2 measurement

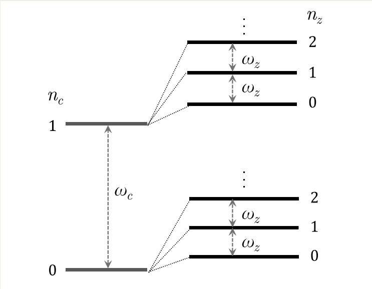




Cyclotron motion is in the ground state

Axial motion is NOT in the ground state

→ Source of the systematic error



- cooling axial motion, quantum measurement

  could not be realized in the NW22 measurement

X. Fan and G. Gabrielse
arXiv:2008.01898,PRL 2021

A factor 10 ~ 20 improvement can be expected

Prospects on the electron g-2 theory

- The crucial input number, the fine-structure constant α

  need to be improved from 0.1 ppb to 0.01 ppb

  h/m_X, X=Cs (UC Berkley), Rb (or Sr) (LKB Paris), and Sr and/or Yb (Oxford U, 2024)

  A_r(m_Rb) … its uncertainty is just half of h/m_Rb

  75 ppt 140 ppt

- QED calculations

  The discrepancy in the universal term $A_1^{(10)}$ has been resolved

  Further independent checks are needed for the mass-dependent term $A_2^{(10)}$

  Preparation of the 12^{th}-order QED is in progress 202,770 Feynman diagrams

  quenched lattice QED Kitano et al. 2021,2022, Kitano 2024

  diagrams with fermion loops Yamazaki, master thesis, Saitama University 2023

- Hadronic contributions ???

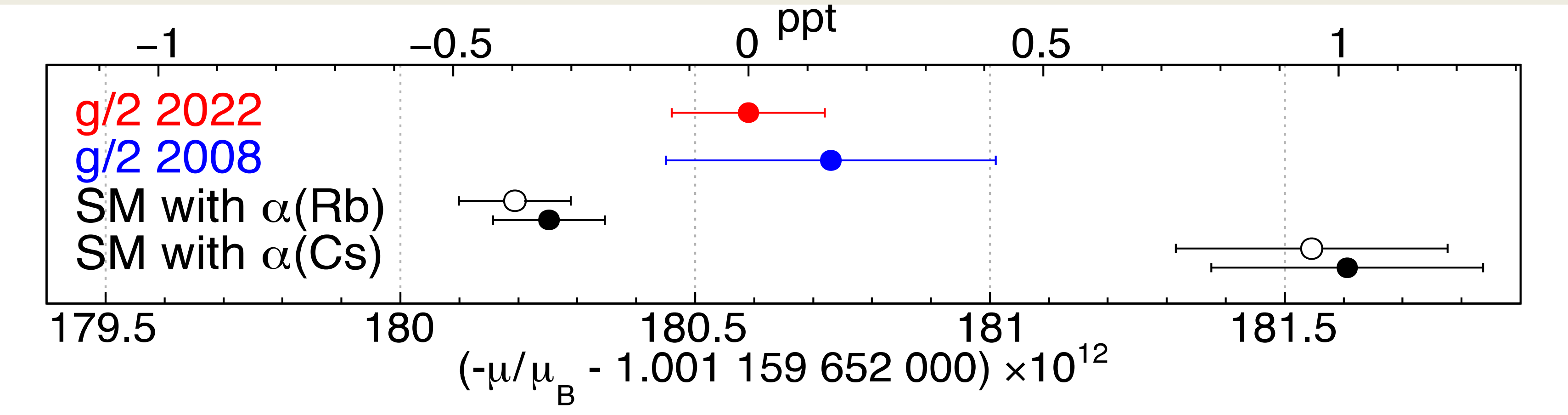
  Open questions both experimentally and theoretically

Thank you for your attention

backup

Effects of two QED results of the 10th-order term

X. Fan, et. al. arXiv: 2209.13084,PRL130,071801(2023)



○ QED 10th-order term from AHKN 2019

● QED 10th-order term from S. Volkov 2019

The difference is **not crucial** right now, but must be resolved.

Other non-QED but SM corrections

$$a_e^{\mathrm{HVP,\ NLO}} = -0.2263\,(35) \times 10^{-12}$$

Keshavarzi, Nomura, Teubner 2019
Di Luzio, Keshavarzi, Masiero, Paradisi 2024

$\propto \alpha^3$

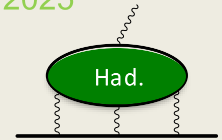
$$a_e^{\mathrm{HVP,NNLO}} = 0.027\,99\,(17) \times 10^{-12}$$

Jegerlehner 2017

$\propto \alpha^4$

$$a_e^{\mathrm{HLbL}} = 0.0351\,(23) \times 10^{-12}$$

Hoferichter, Stoffer, Zillinger 2025



$$a_e^{\mathrm{EW}} = 0.030\,53\,(23) \times 10^{-12}$$

Jegerlehner 2017

Corrections involving weak bosons Z and W

Different renormalization schemes

On-shell renormalization constants for a self-energy diagram *G*:

$L_{G(i)}$ for vertex renormalization

B_G for wave-function renormalization

Volkov: IR-free, Ward-Takahashi identity holds:

$$BV_G + \sum_{i=1}^{2n-1} LV_{G(i)} = 0$$

AHKN: IR free, easy-to-determine, but breaking WT-identity:

$$BK_G + \sum_{i=1}^{2n-1} LK_{G(i)} + \Delta LB_G = 0$$

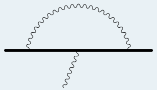
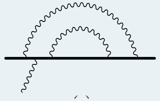
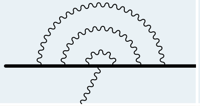
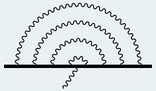
Finite renormalization

Restore the gauge invariance

Numerical calculation of $\delta L_{G(i)}$

Difference of vertex renormalization constants
numerically calculated for n=1, 2, 3, 4 loops

(#) # of independent diagrams

Order 2n	2	4	6	8
# of vertex diagrams	1	6 (4)	50 (38)	518 (269)
dimension of integrals	1dim 	4 dim 	7 dim 	10 dim 

$$\delta L_{G(i)} = \left(1 + \sum_f \left[\prod_{S_i \in f} (-\mathbb{K}_{S_i})\right]\right) (LV_{G(i)} - LK_{G(i)})$$

UV subtraction w/ AHKN's K-operation

269 x 1 hour x 40 core ~ 10,000 core x hours
easy calculation compared to the 10^{th}-order g-2

To-do list for a diagram-by-diagram comparison

To compare AHKN and Volkov's numerical results of integrals,

- Obtain the symbolic expressions of the gap equations
  expressed by $\delta L \text{ and } \Delta M_G$ of the 2nd ~ 8th order quantities

- Calculate the values of δL
  ΔM_G are known from AHKN's old publications.

- The difference of numerical integrals is compared to the numerical values of the gap equation.