Effective Theory of Muon-to-Electron Conversion

- Nucleon level theory: What can and cannot be learned about charged lepton flavor violation (CLFV) from elastic muon-to-electron conversion
- Connecting low-energy CLFV measurements to UV enemies: the EFT tower
- Magnifying the impact of Mu2e and COMET: the fortuitous case of 27Al and its excited states

SPS 2025: Nara, Japan Wick Haxton Sept 27, 2025

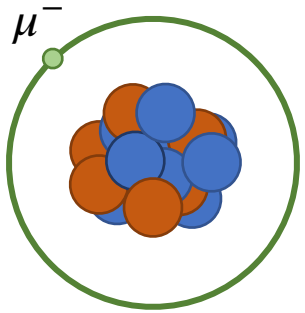
Talk based on collaborative work:

Evan Rule, WH, Ken McElvain, Phys. Rev. Lett. 130 (2023) 131901
WH, Evan Rule, Ken McElvain, M. J. Ramsey-Musolf, Phys. Rev. C 107 (2023) 035504
WH, Evan Rule, Phys. Rev. Lett. 133 (2024) 261801
WH, Ken McElvain, Tony Menzo, Evan Rule, Jure Zupan, JHEP 11 (2024) 76
WH, Evan Rule, Phys. Rev. C111 (2025) 025501

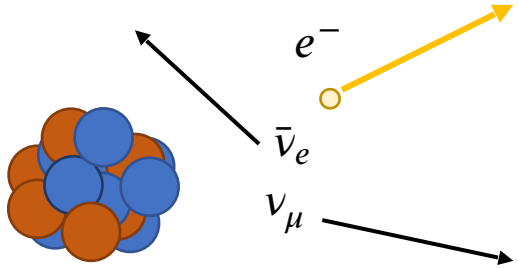
Radioactive Muonic Atoms

Formed when muons produced in pion decay are stopped in a metal target

- Quickly cascade into the 1s atomic orbit around the target nucleus



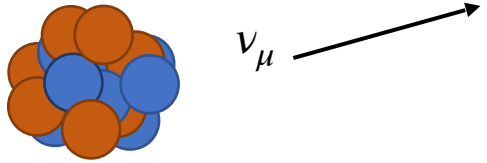
SM:
free decay



$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

(free decay)

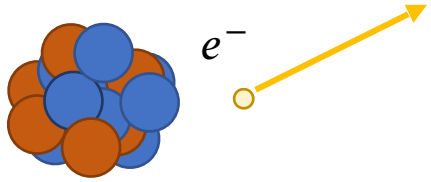
SM:
muon capture



$$\mu^- + (A, Z) \to \nu_\mu + (A, Z-1)$$

(dominant in heavier nuclei)

CLFV



$$\mu^- + (A, Z) \to e^- + (A, Z)$$

(exotic: flavor changing)

Radioactive Muonic Atoms

Formed when muons produced in pion decay are stopped in a metal target

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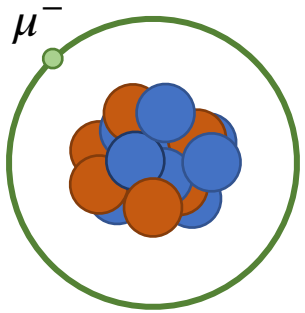
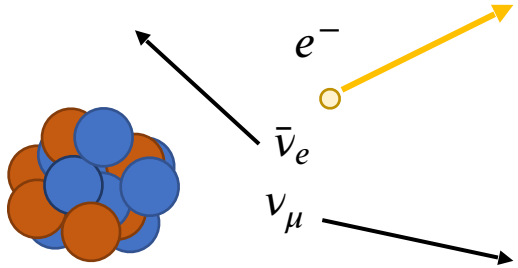
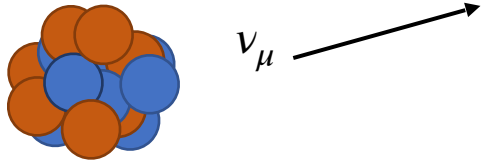




$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

(free decay)

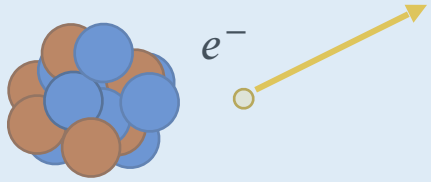


$$\mu^- + (A, Z) \to \nu_\mu + (A, Z-1)$$

(dominant in heavier nuclei)

Simple kinematics:

$$E_e \sim m_\mu - E_\mu^{\text{binding}}$$



$$\mu^- + (A, Z) \to e^- + (A, Z)$$

(exotic: flavor changing)

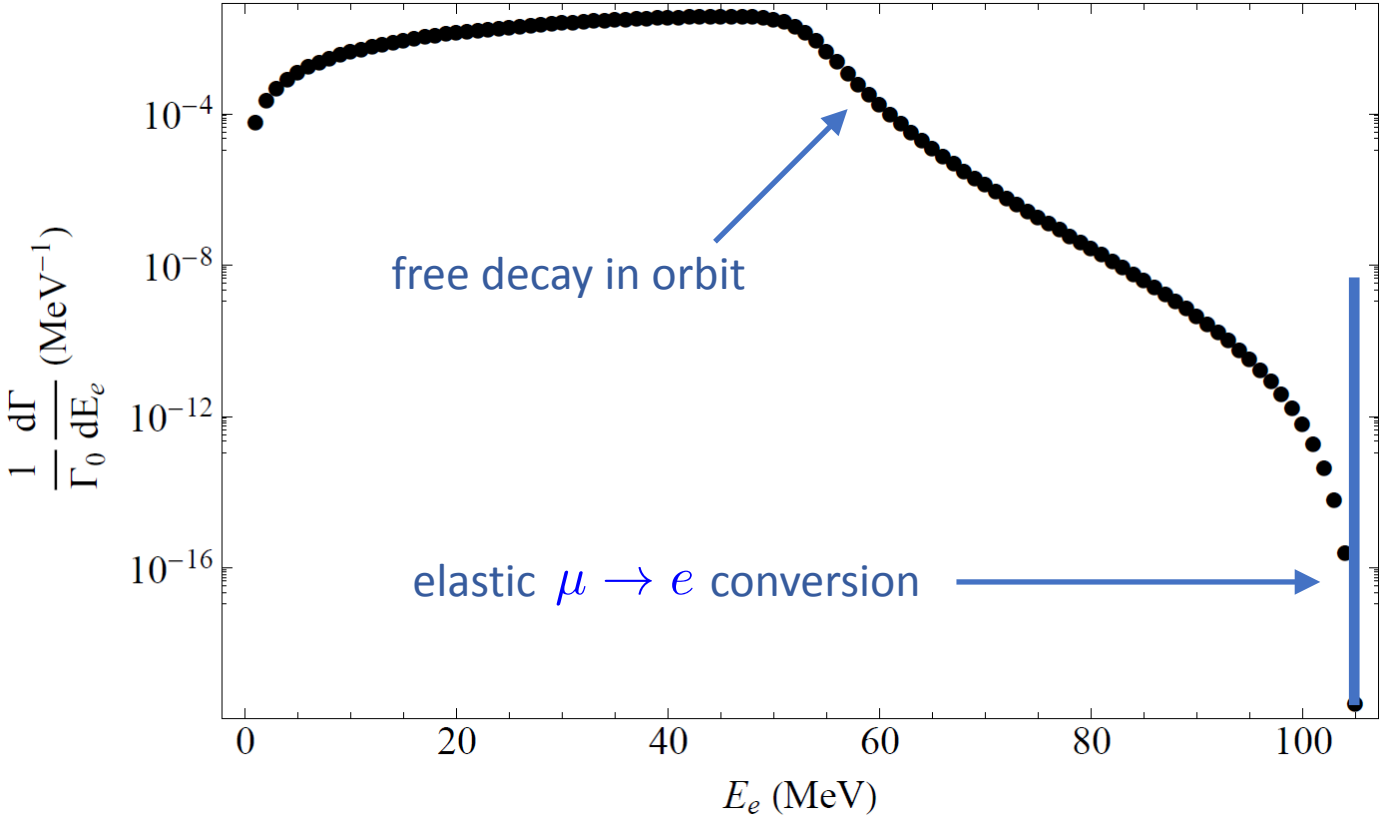
Electron Spectrum

Figure: Czarnecki, Garcia i Tormo, & Marciano, Phys. Rev. D **84**, 013006 (2011)

Detecting $\mu \to e$ conversion:

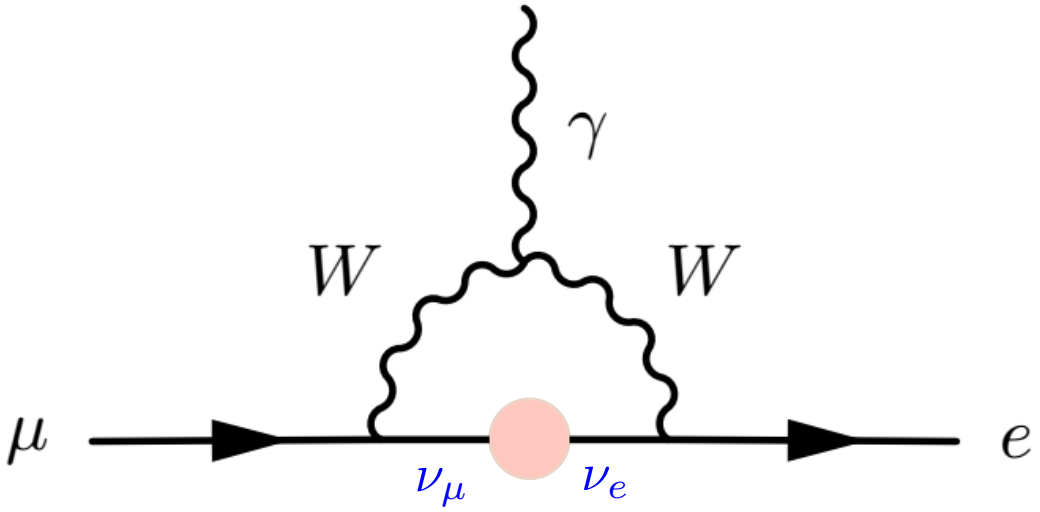
Experimental Requirements

- intense muon beams
- good endpt. energy resolution



$$\mathrm{BR}\left(\mu^- + (A,Z) \to e^- + (A,Z)\right) \equiv \frac{\Gamma\left(\mu^- + (A,Z) \to e^- + (A,Z)\right)}{\Gamma\left(\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right)}$$

massive neutrinos & flavor mixing

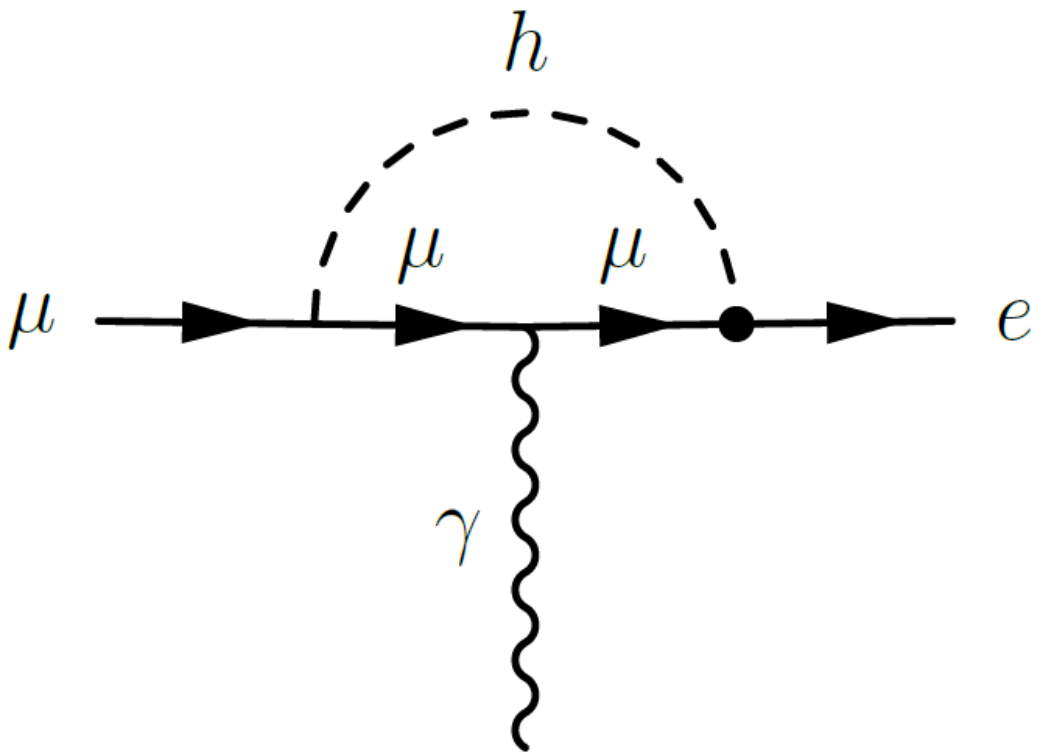


$$\left(\frac{m_\nu}{M_W}\right)^2$$

branching ratio suppression $\sim \left(\frac{m_\nu}{M_W}\right)^4 \sim 10^{-50}$

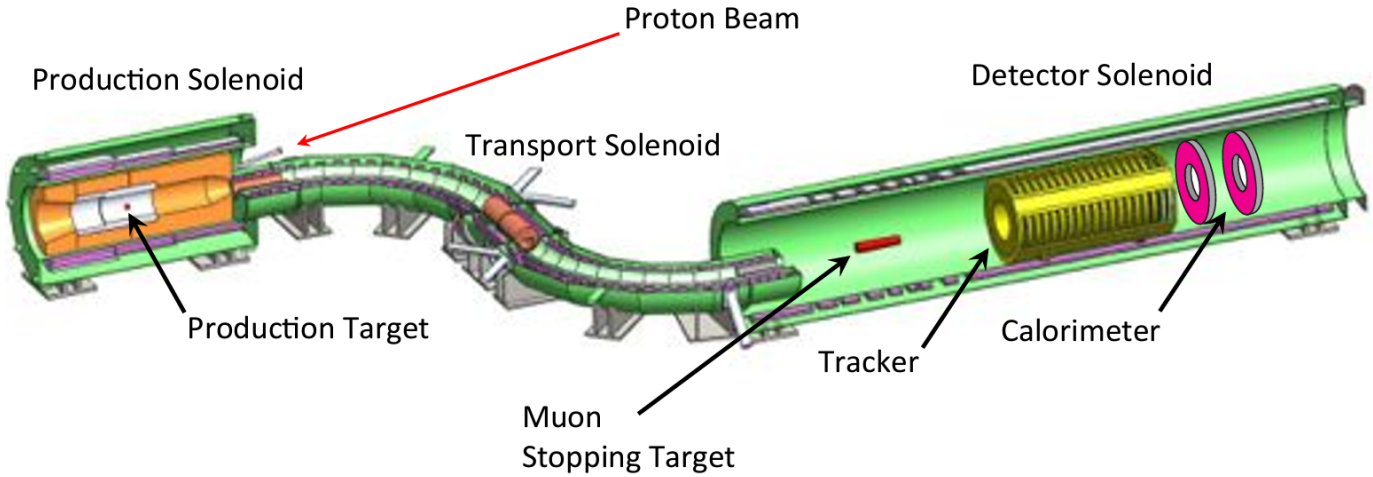
nevertheless, establishes that CLFV occurs

but many other BSM possibilities other than those induced by neutrinos



CLFV a probe off new sources of flavor mixing

NRET: motivation: New experiments



- Huge fluences: 10^{18} muons will be captured
- Clean CLFV signal, free of SM backgrounds
- Expected to improve branching ratio limits by $\sim 10^{4}$ to $\sim 10^{-17}$
- ^{27}Al target — targets can be varied to obtain complimentary constraints

Motivation for developing an EFT formulation

- Of $\sim$ two dozen papers in the literature, 90% focused on one operator, the coherent vector charge, 10% on spin-flip

- Unclear what kinds or how many specific constraints on BSM sources of CLFV can be extracted from Mu2e, COMET, and other low-energy experiments

Construction of the nucleon-level NRET

- Begin by enumerating the available charges and currents

  charges: $1_N \quad \vec{\sigma}_N \cdot \vec{v}_N$ currents: $\vec{\sigma}_N \quad \vec{v}_N \quad \vec{\sigma}_N \times \vec{v}_N$

- Include the possibility of curls and gradients of these quantities

- The response of the nucleus $\sum_i e^{i\vec{q}\cdot\vec{r}_i}$

- Identify the hierarchy of small parameters governing the physics

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad |\vec{v}_N| \quad > \quad |\vec{v}_\mu| \quad > \quad |\vec{v}^{\,\text{recoil}}_{\text{nucleus}}| \sim 0$$

- One can use this hierarchy to construct interactions that are successively more complete

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- One can use this hierarchy to construct interactions that are successively more complete

Significant angular moment transfer
from the leptons to the nucleus:
needed multiple electron partial waves

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad |\vec{v}_N| \quad > \quad |\vec{v}_\mu| \quad > \quad |\vec{v}^{\,\text{recoil}}_{\text{nucleus}}| \sim 0$$

- The momentum transfer between leptons and nucleus is sufficient that a partial wave expansion is needed, accounting for the angular momentum transfer

- One can use this hierarchy to construct interactions that are successively more complete

Currents:
generates new kinds of coherence
plays a key role in selection rules

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad \boxed{|\vec{v}_N|} \quad > \quad |\vec{v}_\mu| \quad > \quad |\vec{v}^{\,\text{recoil}}_{\text{nucleus}}| \sim 0$$

- In the NRET, this is a bound-state (Jacobi) velocity — the inter-nucleon velocity

- One can use this hierarchy to construct interactions that are successively more complete

Generates the muon's lower component:
plays no role in selection rules:
amounts to a nuclear form factor
of about 5% for Al

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad |\vec{v}_N| \quad > \quad \boxed{|\vec{v}_\mu|} \quad > \quad |\vec{v}^{\,\text{recoil}}_{\text{nucleus}}| \sim 0$$

- In NRET also a bound-state velocity, the muon velocity w.r.t. the nuclear center of mass

- One can use this hierarchy to construct interactions that are successively more complete

contributes in the amplitude to order

$$\frac{m_\mu}{M_T} < 1\%$$

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad |\vec{v}_N| \quad > \quad |\vec{v}_\mu| \quad > \quad |\vec{v}^{\,\text{recoil}}_{\text{nucleus}}| \sim 0$$

- Neglected

Effective Theory Variations

Coherent charge operator

Coherent Response

$1_N, 1_L, \hat{q}, \vec{\sigma}_L$

2 Nucleon-level Operators

1 Response function

M_0

$\mathcal{O}_1 = 1_L 1_N$

$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L \, 1_N$

- Even multipoles of the charge operator: A single response

Effective Theory Variations

"Allowed" operators

Coherent Response	Allowed Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N$
2 Nucleon-level Operators	6 Nucleon-level operators
1 Response function	3 Response functions
M_0	$M_0, \Sigma_1', \Sigma_1''$

$\mathcal{O}_1 = 1_L 1_N$

$\mathcal{O}_4 = \vec{\sigma}_L \cdot \vec{\sigma}_N$

$\mathcal{O}_6 = i\hat{q} \cdot \vec{\sigma}_L \, i\hat{q} \cdot \vec{\sigma}_N$

$\mathcal{O}_9 = \vec{\sigma}_L \cdot \left(i\hat{q} \times \vec{\sigma}_N \right)$

$\mathcal{O}_{10} = 1_L \, i\hat{q} \cdot \vec{\sigma}_N$

$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L \, 1_N$

- Most general CLFV response of point-like nucleus

  Distinct transverse and longitudinal spin responses

Effective Theory Variations

Nucleon velocity, three-momentum transfer: yields the general form

Coherent Response	Allowed Response	General Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N, \vec{v}_N$
2 Nucleon-level Operators	6 Nucleon-level operators	16 Nucleon-level operators
1 Response function	3 Response functions	6 Response functions
$\mathbf{M_J}$	$M_J, \Sigma'_J, \Sigma''_J$	$M_J, \Sigma'_J, \Sigma''_J, \Delta_J, \tilde{\Phi}'_J, \Phi'_J$

- All nuclear responses allowed by P and T symmetries

$$\mathcal{O}_1 = 1_L 1_N$$
$$\mathcal{O}'_2 = 1_L\, i\hat{q} \cdot \vec{v}_N$$
$$\mathcal{O}_3 = 1_L\, i\hat{q} \cdot \left(\vec{v}_N \times \vec{\sigma}_N\right)$$
$$\mathcal{O}_4 = \vec{\sigma}_L \cdot \vec{\sigma}_N$$
$$\mathcal{O}_5 = \vec{\sigma}_L \cdot \left(i\hat{q} \times \vec{v}_N\right)$$
$$\mathcal{O}_6 = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_7 = 1_L\, \vec{v}_N \cdot \vec{\sigma}_N$$
$$\mathcal{O}_8 = \vec{\sigma}_L \cdot \vec{v}_N$$
$$\mathcal{O}_9 = \vec{\sigma}_L \cdot \left(i\hat{q} \times \vec{\sigma}_N\right)$$
$$\mathcal{O}_{10} = 1_L\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L\, 1_N$$
$$\mathcal{O}_{12} = \vec{\sigma}_L \cdot \left(\vec{v}_N \times \vec{\sigma}_N\right)$$
$$\mathcal{O}'_{13} = \vec{\sigma}_L \cdot \left[i\hat{q} \times \left(\vec{v}_N \times \vec{\sigma}_N\right)\right]$$
$$\mathcal{O}_{14} = i\hat{q} \cdot \vec{\sigma}_L\, \vec{v}_N \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{15} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \left(\vec{v}_N \times \vec{\sigma}_N\right)$$
$$\mathcal{O}'_{16} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{v}_N$$

Effective Theory Variations

Coherent Response	Allowed Response	General Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N, \vec{v}_N$
2 Nucleon-level Operators	6 Nucleon-level operators	16 Nucleon-level operators
1 Response function	3 Response functions	6 Response functions
M_0	$M_J, \Sigma'_J, \Sigma''_J$	$M_J, \Sigma'_J, \Sigma''_J, \Delta_J, \tilde{\Phi}'_J, \Phi'_J$

- All nuclear responses allowed by P and T symmetries

This theory is complete for nuclear analysis: 6 observables in $\mu \to e$ conversion

$$\mathcal{O}_1 = 1_L 1_N$$
$$\mathcal{O}'_2 = 1_L\, i\hat{q} \cdot \vec{v}_N$$
$$\mathcal{O}_3 = 1_L\, i\hat{q} \cdot \left(\vec{v}_N \times \vec{\sigma}_N\right)$$
$$\mathcal{O}_4 = \vec{\sigma}_L \cdot \vec{\sigma}_N$$
$$\mathcal{O}_5 = \vec{\sigma}_L \cdot \left(i\hat{q} \times \vec{v}_N\right)$$
$$\mathcal{O}_6 = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_7 = 1_L\, \vec{v}_N \cdot \vec{\sigma}_N$$
$$\mathcal{O}_8 = \vec{\sigma}_L \cdot \vec{v}_N$$
$$\mathcal{O}_9 = \vec{\sigma}_L \cdot \left(i\hat{q} \times \vec{\sigma}_N\right)$$
$$\mathcal{O}_{10} = 1_L\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L\, 1_N$$
$$\mathcal{O}_{12} = \vec{\sigma}_L \cdot \left(\vec{v}_N \times \vec{\sigma}_N\right)$$
$$\mathcal{O}'_{13} = \vec{\sigma}_L \cdot \left[i\hat{q} \times \left(\vec{v}_N \times \vec{\sigma}_N\right)\right]$$
$$\mathcal{O}_{14} = i\hat{q} \cdot \vec{\sigma}_L\, \vec{v}_N \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{15} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \left(\vec{v}_N \times \vec{\sigma}_N\right)$$
$$\mathcal{O}'_{16} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{v}_N$$

$$\mathcal{L}_{\text{eff}} \sim \sqrt{2} G_F \sum_{\tau=0,1} \sum_{i=1}^{16} \tilde{c}_i^{\tau} \mathcal{O}_i t^{\tau}$$

- Any UV Lagrangian will reduce at low energy to the NRET form

$$\mathcal{L}_{\rm eff} = \sqrt{2} G_F \sum_{\tau=0,1} \sum_{i=1}^{16} \tilde{c}_i \mathcal{O}_i t^\tau \qquad t^0 = 1,\ t^1 = \tau_3$$

and thus can be matched, if a consistent counting is employed

<u>What happens when we embed this in a nucleus?</u>

1. An additional operator $\sim \sum_{i=1}^{A} e^{i\vec{q}\cdot\vec{r}_i}$ that transfers linear and angular momentum to the nucleus

2. Selection rules imposed by P and T, that reduce how much can be learn about the nucleon-level CLFV Lagrangian: only 6 observables

How do we evaluate the rate?

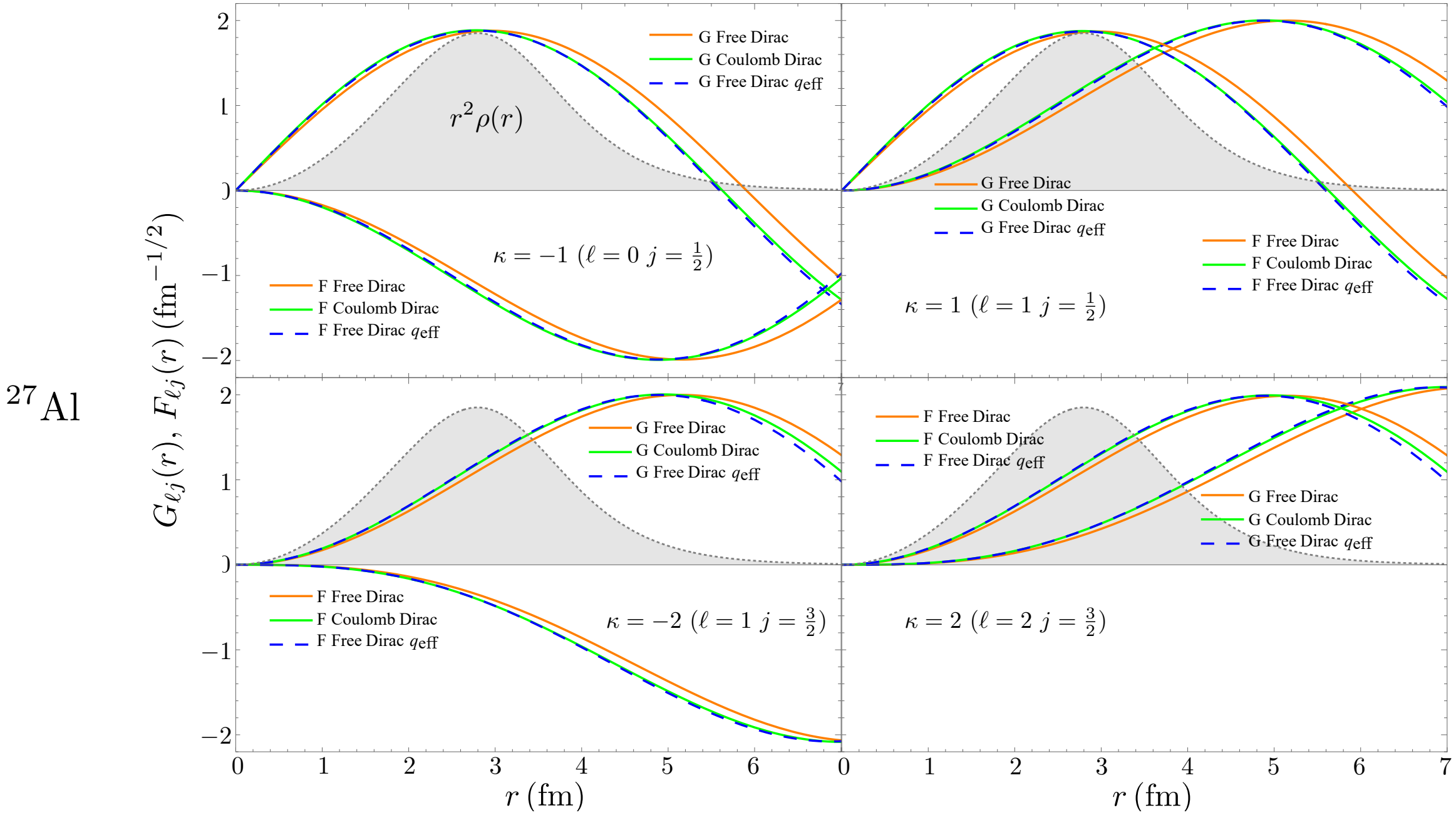
- The plane wave is distorted by the nuclear Coulomb field: important for heavier nuclei. Past investigators using simple CLFV operators have overly simplified this physics … retaining only the Dirac waves $\kappa = \pm 1/2$

- With 16 single-nucleon operators and distorted waves — seems difficult

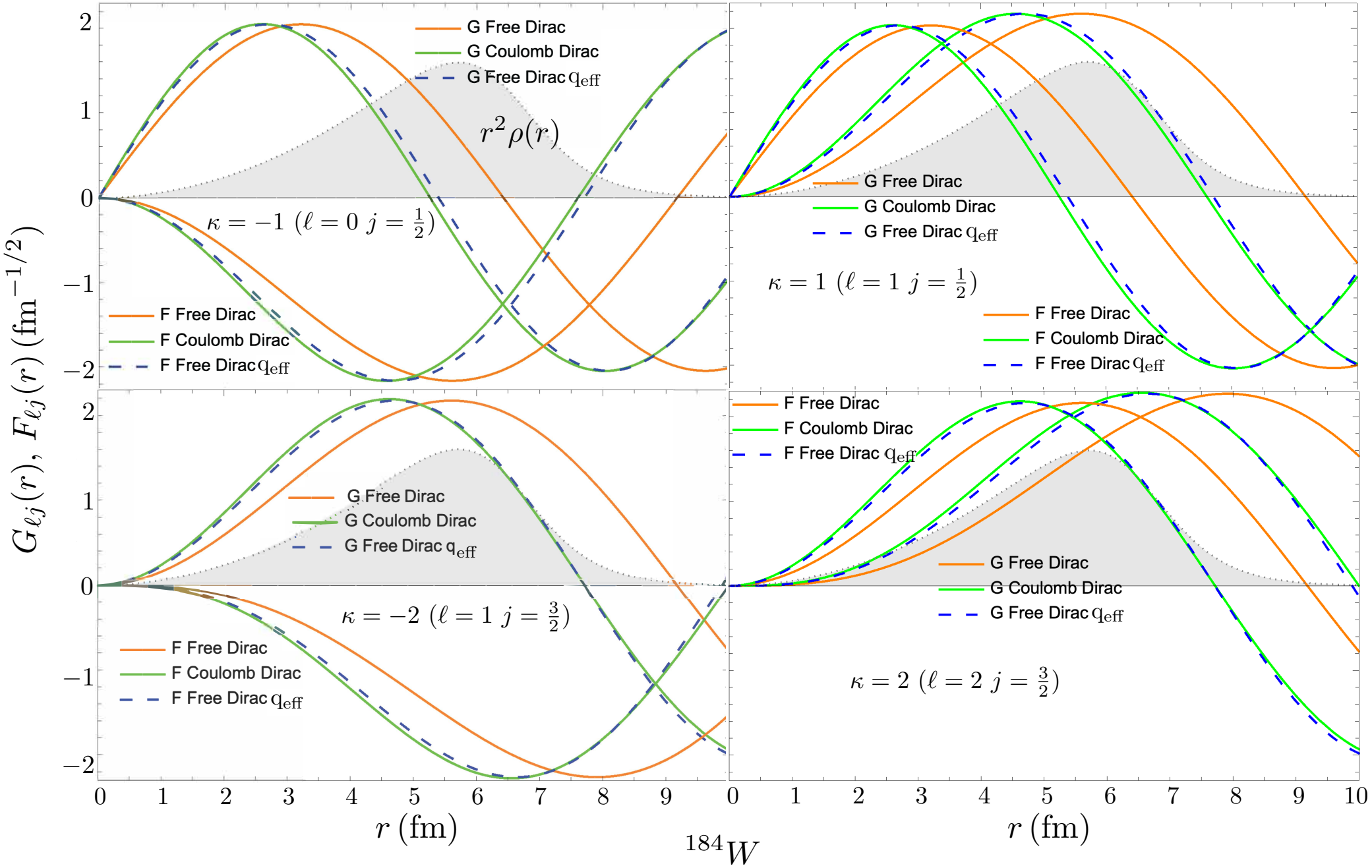
- But there is a nice trick,

$$U(q,s)e^{i\vec{q}\cdot\vec{r}} \to \frac{q_{\rm eff}}{q}\sqrt{\frac{E_e}{2m_e}}\begin{pmatrix} \xi_s \\ \vec{\sigma}_L\cdot\hat{q}\;\xi_s \end{pmatrix} e^{i\vec{q}_{\rm eff}\cdot\vec{r}}$$

using a local electron momentum obtained from averaging the Coulomb potential over a nuclear volume: yields an effective plane wave, to which all the technology of spherical Bessel vector harmonics can be applied


^{27}Al
$G_{\ell j}(r)$, $F_{\ell j}(r)$ (fm$^{-1/2}$)
$r^2\rho(r)$
G Free Dirac
G Coulomb Dirac
G Free Dirac $q_{\rm eff}$
F Free Dirac
F Coulomb Dirac
F Free Dirac $q_{\rm eff}$
$\kappa = -1$ ($\ell = 0$ $j = \frac{1}{2}$)
$\kappa = 1$ ($\ell = 1$ $j = \frac{1}{2}$)
$\kappa = -2$ ($\ell = 1$ $j = \frac{3}{2}$)
$\kappa = 2$ ($\ell = 2$ $j = \frac{3}{2}$)
r (fm)

^{184}W


G Free Dirac
G Coulomb Dirac
G Free Dirac $q_{\rm eff}$
$r^2\rho(r)$
$\kappa = -1$ $(\ell = 0$ $j = \frac{1}{2})$
F Free Dirac
F Coulomb Dirac
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$\kappa = 1$ $(\ell = 1$ $j = \frac{1}{2})$
$\kappa = -2$ $(\ell = 1$ $j = \frac{3}{2})$
$\kappa = 2$ $(\ell = 2$ $j = \frac{3}{2})$
$G_{\ell j}(r),\ F_{\ell j}(r)\ (\mathrm{fm}^{-1/2})$
r (fm)
^{184}W

CLFV Decay Rate

(Master Formula)

Conceptually simple

$$\omega = \frac{G_F^2}{\pi} \frac{q_{\rm eff}^2}{1+\frac{q}{M_T}} |\phi_{1s}^{Z_{\rm eff}}(\vec{0})|^2 \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ \left[\tilde{R}_M^{\tau\tau'} \; W_M^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma''}^{\tau\tau'} \; W_{\Sigma''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma'}^{\tau\tau'} \; W_{\Sigma'}^{\tau\tau'}(q_{\rm eff})\right]$$

$$+\frac{q_{\rm eff}^2}{m_N^2} \left[\tilde{R}_{\Phi''}^{\tau\tau'} \; W_{\Phi''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\tilde{\Phi}'}^{\tau\tau'} \; W_{\tilde{\Phi}'}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta}^{\tau\tau'} \; W_{\Delta}^{\tau\tau'}(q_{\rm eff})\right]$$

$$-\frac{2q_{\rm eff}}{m_N} \left[\tilde{R}_{\Phi''M}^{\tau\tau'} \; W_{\Phi''M}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'} \; W_{\Delta\Sigma'}^{\tau\tau'}(q_{\rm eff})\right] \Bigg\}$$

$R_i^{\tau\tau'}(q_{\rm eff}) \leftrightarrow$ "CLFV particle physics"

$$\tilde{R}_M^{\tau\tau'} = \tilde{c}_1^{\tau}\tilde{c}_1^{\tau'*} + \tilde{c}_{11}^{\tau}\tilde{c}_{11}^{\tau'*}$$

$$\tilde{R}_{\Phi''}^{\tau\tau'} = \tilde{c}_3^{\tau}\tilde{c}_3^{\tau'*} + (\tilde{c}_{12}^{\tau} - \tilde{c}_{15}^{\tau})\left(\tilde{c}_{12}^{\tau'*} - \tilde{c}_{15}^{\tau'*}\right)$$

$$\tilde{R}_{\Phi''M}^{\tau\tau'} = {\rm Re}\left[\tilde{c}_3^{\tau}\tilde{c}_1^{\tau'*} - (\tilde{c}_{12}^{\tau} - \tilde{c}_{15}^{\tau})\,\tilde{c}_{11}^{\tau'*}\right]$$

$$\tilde{R}_{\tilde{\Phi}'}^{\tau\tau'} = \tilde{c}_{12}^{\tau}\tilde{c}_{12}^{\tau'*} + \tilde{c}_{13}^{\tau}\tilde{c}_{13}^{\tau'*}$$

$$\tilde{R}_{\Sigma''}^{\tau\tau'} = (\tilde{c}_4^{\tau} - \tilde{c}_6^{\tau})(\tilde{c}_4^{\tau'*} - \tilde{c}_6^{\tau'*}) + \tilde{c}_{10}^{\tau}\tilde{c}_{10}^{\tau'*}$$

$$\tilde{R}_{\Sigma'}^{\tau\tau'} = \tilde{c}_4^{\tau}\tilde{c}_4^{\tau'*} + \tilde{c}_9^{\tau}\tilde{c}_9^{\tau'*}$$

$$\tilde{R}_{\Delta}^{\tau\tau'} = \tilde{c}_5^{\tau}\tilde{c}_5^{\tau'*} + \tilde{c}_8^{\tau}\tilde{c}_8^{\tau'*}$$

$$\tilde{R}_{\Delta\Sigma'}^{\tau\tau'} = {\rm Re}\left[\tilde{c}_5^{\tau}\tilde{c}_4^{\tau'*} + \tilde{c}_8^{\tau}\tilde{c}_9^{\tau'*}\right]$$

$W_i^{\tau\tau'}(q_{\rm eff}) \leftrightarrow$ "Nuclear dials"

vary these by picking the right nuclear targets

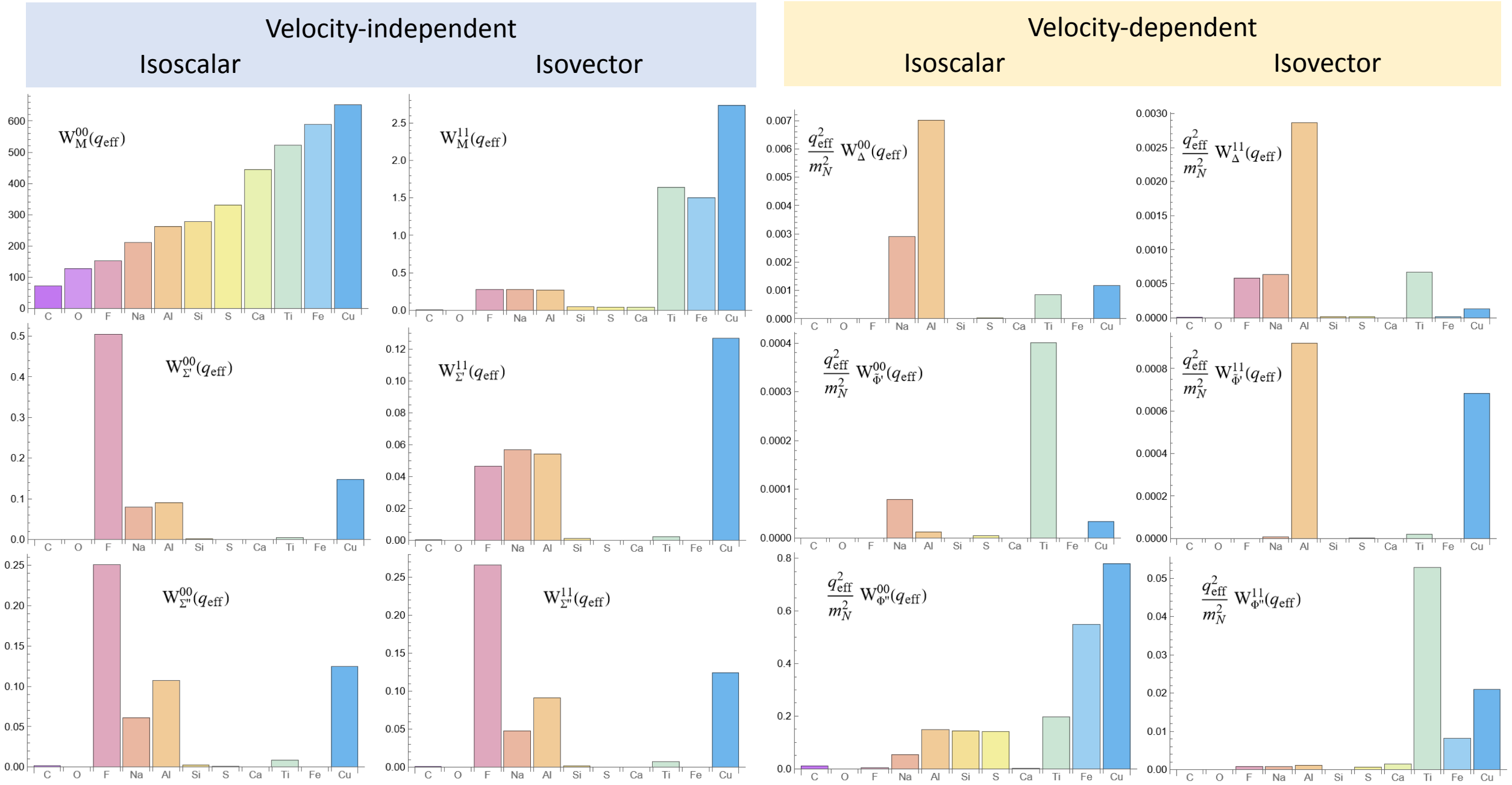
to determine these

- We find a form with six response functions: a result we also deduce from symmetry arguments alone (constraints of good nuclear P, T)

- The experimental goal is thus most efficiently expressed at the nuclear scale: the extraction of the six $\tilde{R}_i^{\tau\tau'}$

- This in principle can be done by carefully selecting targets with the requisite properties

- Once extracted, these “nuclear LECs” become universal constraints on all higher level theories of CLFV


Velocity-independent
Isoscalar
Isovector
Velocity-dependent
Isoscalar
Isovector
$W_M^{00}(q_{eff})$
$W_M^{11}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_\Delta^{00}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_\Delta^{11}(q_{eff})$
$W_{\Sigma'}^{00}(q_{eff})$
$W_{\Sigma'}^{11}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_{\tilde{\Phi}'}^{00}(q_{eff})$
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$W_{\Sigma''}^{11}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_{\Phi''}^{00}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_{\Phi''}^{11}(q_{eff})$
C O F Na Al Si S Ca Ti Fe Cu

Nucleon-level NRET

Coherent (super-allowed)	Allowed	v_{N}-associated	Target Recoil
	c_1^1, c_{11}^1 W_M^{11}		
	c_4, c_6, c_{10} $W_{\Sigma''}$		
	c_4, c_9 $W_{\Sigma'}$		
		$c_3^1, c_{12}^1, c_{15}^1$ $\frac{q^2}{m_N^2} W_{\Phi''}^{11}$	
		c_{12}, c_{13} $\frac{q^2}{m_N^2} W_{\Phi'}$	
		c_5, c_8 $\frac{q^2}{m_N^2} W_{\Delta}$	

<u>altered by</u>
<u>nuclear embedding</u>
(elastic channel)

	Coherent	Allowed	v_N-associated	Target Recoil
Coherence	c_1^0, c_{11}^0 $A_{eff}^2 W_M^{00}$	c_1^1, c_{11}^1 W_M^{11}		
Siegert			c_2^0, c_{16}^0 $\frac{A_{eff}^2 q^2}{A^2 m_N^2} W_M^{00}$	c_2^1, c_{16}^1 $\frac{q^2}{A^2 m_N^2} W_M^{11}$
		c_4, c_6, c_{10} $W_{\Sigma''}$		c_7, c_{14} $\frac{q^2}{A^2 m_N^2} W_{\Sigma''}$
		c_4, c_9 $W_{\Sigma'}$		
Coherence		$c_3^0, c_{12}^0, c_{15}^0$ $\frac{A_{eff}^2 q^2}{m_N^2} W_{\Phi''}^{00}$	$c_3^1, c_{12}^1, c_{15}^1$ $\frac{q^2}{m_N^2} W_{\Phi''}^{11}$	
			c_{12}, c_{13} $\frac{q^2}{m_N^2} W_{\Phi'}$	
			c_5, c_8 $\frac{q^2}{m_N^2} W_{\Delta}$	

<u>altered by</u>
<u>nuclear embedding</u>
(elastic channel)

Enhanced vector charge operator

Enhanced sensitivity to tensor mediators

	Coherent (super-allowed)	Allowed	v_N-associated	Target Recoil
Coherence	c_1^0, c_{11}^0: $A_{eff}^2 W_M^{00}$	c_1^1, c_{11}^1: W_M^{11}		
		c_4, c_6, c_{10}: $W_{\Sigma''}$		
		c_4, c_9: $W_{\Sigma'}$		
Coherence		$c_3^0, c_{12}^0, c_{15}^0$: $\frac{A_{eff}^2 q^2}{m_N^2} W_{\Phi''}^{00}$	$c_3^1, c_{12}^1, c_{15}^1$: $\frac{q^2}{m_N^2} W_{\Phi''}^{11}$	
			c_{12}, c_{13}: $\frac{q^2}{m_N^2} W_{\Phi'}$	
			c_5, c_8: $\frac{q^2}{m_N^2} W_{\Delta}$	
Siegert			c_2^0, c_{16}^0: $\frac{A_{eff}^2 q^2}{A^2 m_N^2} W_M^{00}$	c_2^1, c_{16}^1: $\frac{q^2}{A^2 m_N^2} W_M^{11}$
				c_7, c_{14}: $\frac{q^2}{A^2 m_N^2} W_{\Sigma''}$

(axial charge)

Suppressed axial charge operator

Coherent vector charge operator

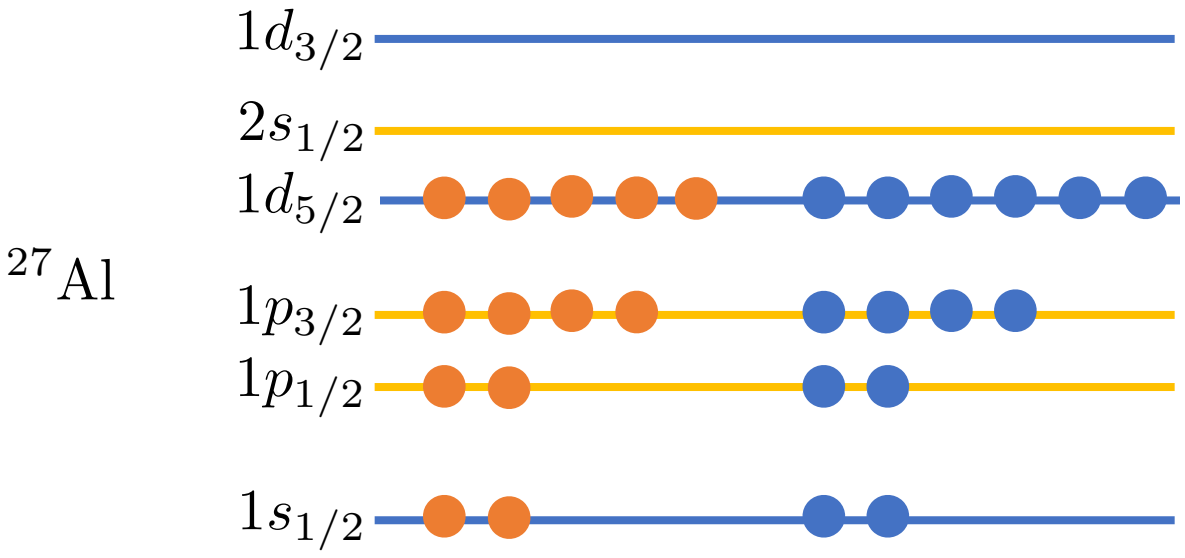
$$M_{00}(0) = \frac{1}{\sqrt{4\pi}} \sum_{i=1}^{A} 1_N(i) \sim \frac{1}{\sqrt{4\pi}} A$$

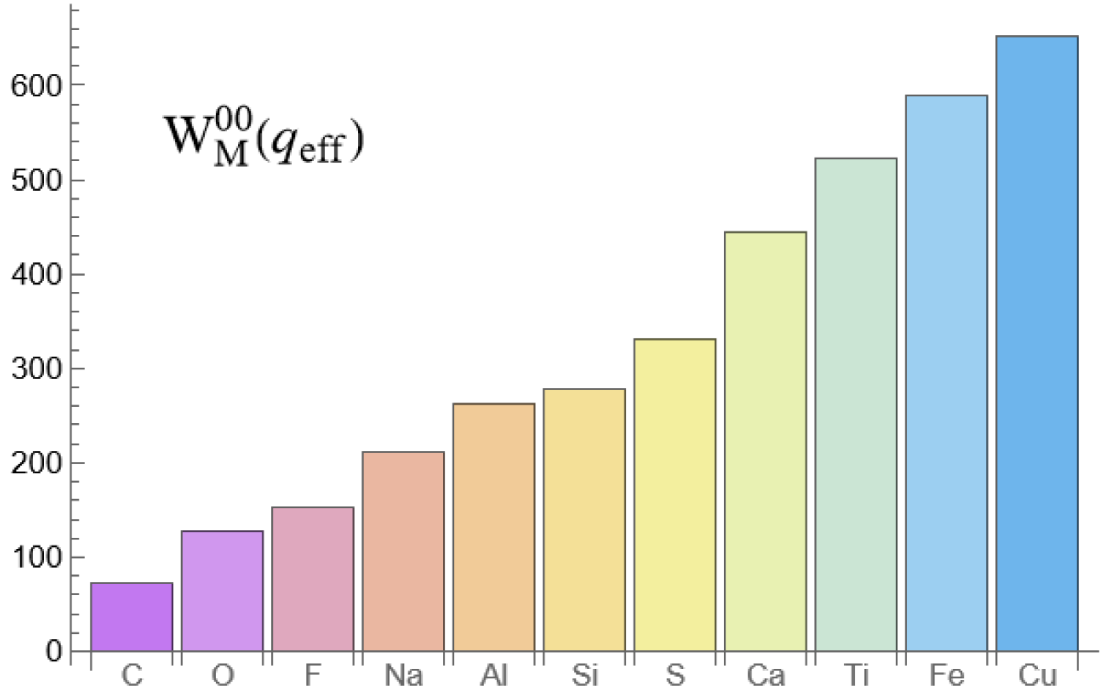
Isoscalar charge operator in the q=0 limit
sums coherently over the nucleus

Easily isolated: J=0 target

vector mediator
$\bar{\chi}_e \gamma_\mu \chi_\mu \;\; \bar{N} \gamma_\mu N$

distinctive nuclear pattern





Second scalar coherent operator

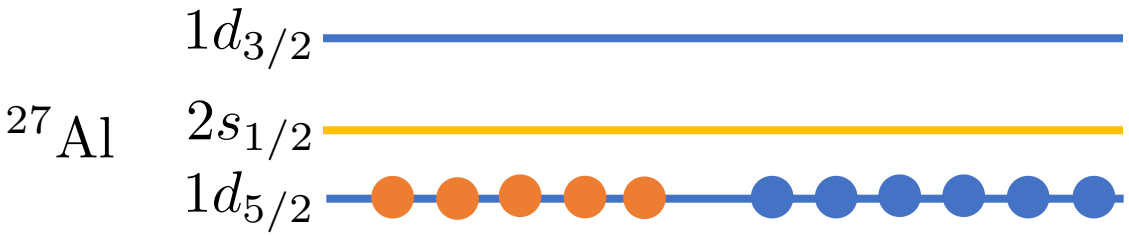
$$\Phi''_{00}(0) = -\frac{1}{6\sqrt{\pi}} \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \vec{\ell}(i)$$

Sums coherently over $j = \ell + \frac{1}{2}$ and $j = \ell - \frac{1}{2}$ subshells but vanishes when both subshells are filled

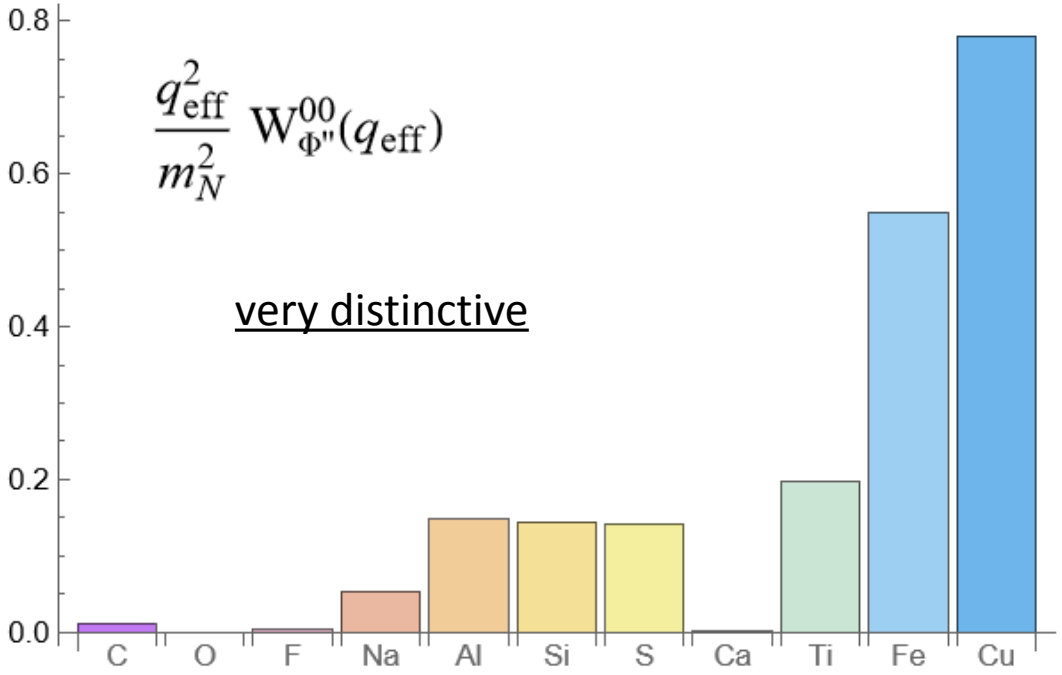
$$\bar{\chi}_e i\sigma^{\mu\nu}\gamma^5 \chi_\mu \bar{N} i\sigma_{\mu\nu}\gamma^5 N$$

$$\omega \propto \left\langle \frac{q}{2m_N} M_{00} - \frac{q_{\rm eff}}{m_N} \Phi''_{00} \right\rangle^2$$

- Velocity-dependent contribution same order as charge monopole
- Generated by tensor-mediated interactions



spin-orbit paired core



Provides a nice example of experimental forensics

- Suppose a $\mu \to e$ conversion signal were found in a J=0 nucleus like ^{28}Si

- One would do a second measurement in, say, ^{40}Ca
  - If a somewhat stronger signal is seen, the vector charge operator is the source of the CLFV
  - If no signal is seen, one can deduce that the second scalar NRET operator is responsible, and consequently that the operator is tensor mediated

Limits on LECs

- Using our expression for the decay rate, we can constrain LECs using existing and future branching ratio limits for various nuclei
- We can also estimate the energy scale probed by each operator
- Assume only one operator is responsible for CLFV
- Mathematica + Python scripts to compute $\mathrm{B}(\mu \to e)$ in terms of $\tilde{c}_i^{\tau}$ for a selection of nuclear targets

	Al		Ti	
Coupling	LEC Limit	~ Scale Probed	LEC Limit	~ Scale Probed
i=1,11 ; 0	4.0E-10	12,000 TeV	7.4E-08	910 TeV
i=1,11; 1	1.2E-08	2,200 TeV	1.3E-06	210 TeV
i=3,15; 0	1.6E-08	1,900 TeV	3.8E-06	130 TeV
i=3,15;1	1.9E-07	570 TeV	7.3E-06	91 TeV
i=4; 0	1.4E-08	2,100 TeV	1.5E-05	63 TeV
i=4; 1	1.7E-08	1,900 TeV	1.7E-05	59 TeV
i=5,8; 0	7.8E-08	880 TeV	5.8E-05	32 TeV
i=5,8; 1	1.2E-07	720 TeV	6.5E-05	30 TeV
i=6,10; 0	2.0E-08	1,800 TeV	1.8E-05	59 TeV
i=6,10; 1	2.2E-08	1,700 TeV	2.0E-05	55 TeV
i=9; 0	2.1E-08	1,700 TeV	2.8E-05	47 TeV
i=9; 1	2.8E-08	1,500 TeV	3.4E-05	42 TeV
i=12; 0	1.6E-08	1,900 TeV	3.8E-06	130 TeV
i=12; 1	1.4E-07	660 TeV	7.3E-06	91 TeV
i=13; 0	1.8E-06	180 TeV	8.4E-05	27 TeV
i=13; 1	2.1E-07	540 TeV	3.7E-04	13 TeV

†: P. Wintz, Proc. 1st Int. Symp. on Lepton and Baryon Number Violation

NRET Status

- The NRET defines in a crisp way the low-energy CLFV information available in elastic $\mu \to e$ experiments; six responses that in principle can be extracted from a program of measurements on suitably chosen targets

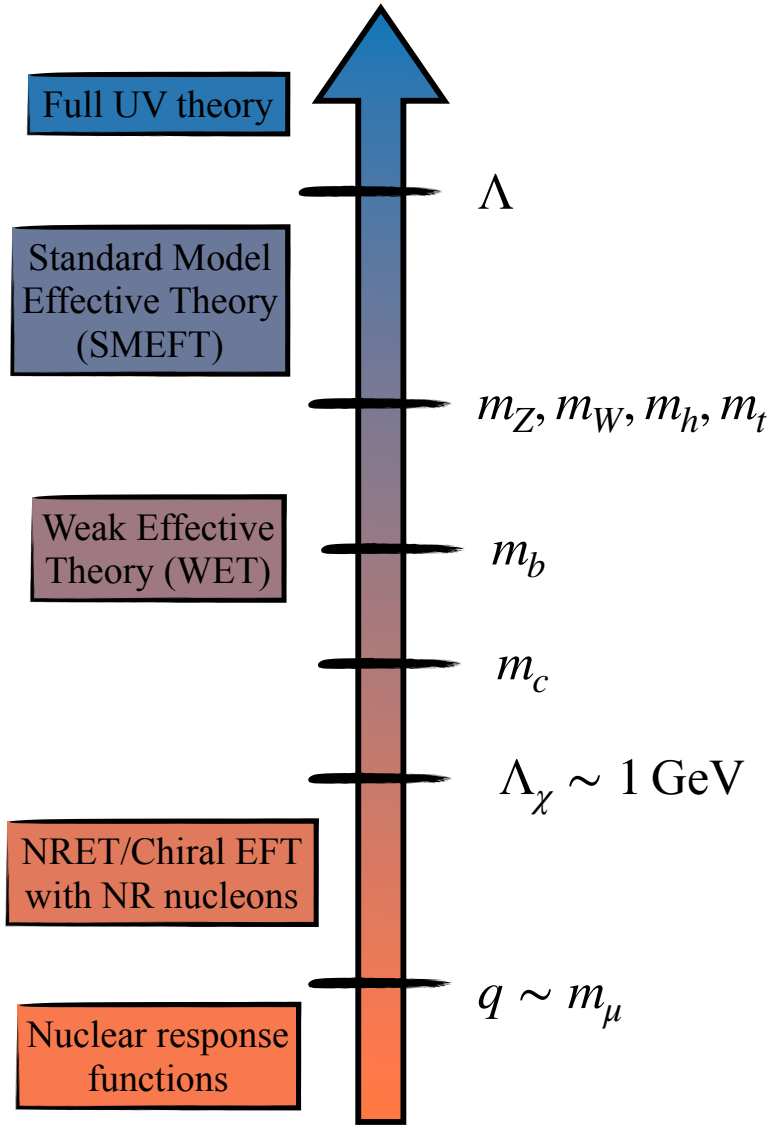
- Publicly-available Python & Mathematica codes created for $\mu \to e$ conversion, using the NRET and the best available NP; extended to include the effects of the muon's lower component ($o(v_\mu)$) — additional NRET operators

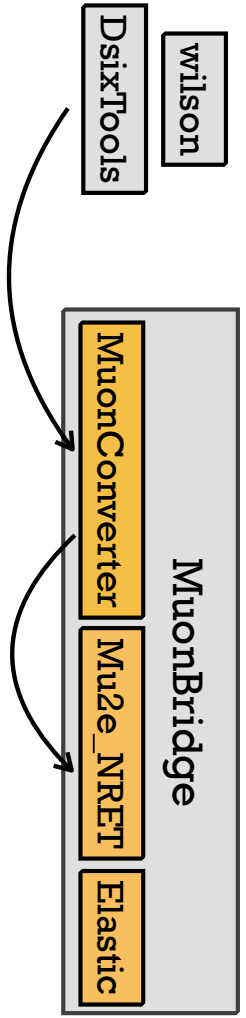
- State-of-the-art nuclear form factors via a cloud library, for almost all targets of interest

\

- Nucleon-level input can be either NR or covariant
  - 4 scalar- and 16 vector-mediated interactions $\to$ 12 NRET operators
  - 12 tensor-mediated interactions needed to match the light-quark CLFV operators of dimension $\leq$ 7 to the NRET basis (the remain 4 NRET operators then needed)

Connecting to higher scales





interfaces built to existing community tools

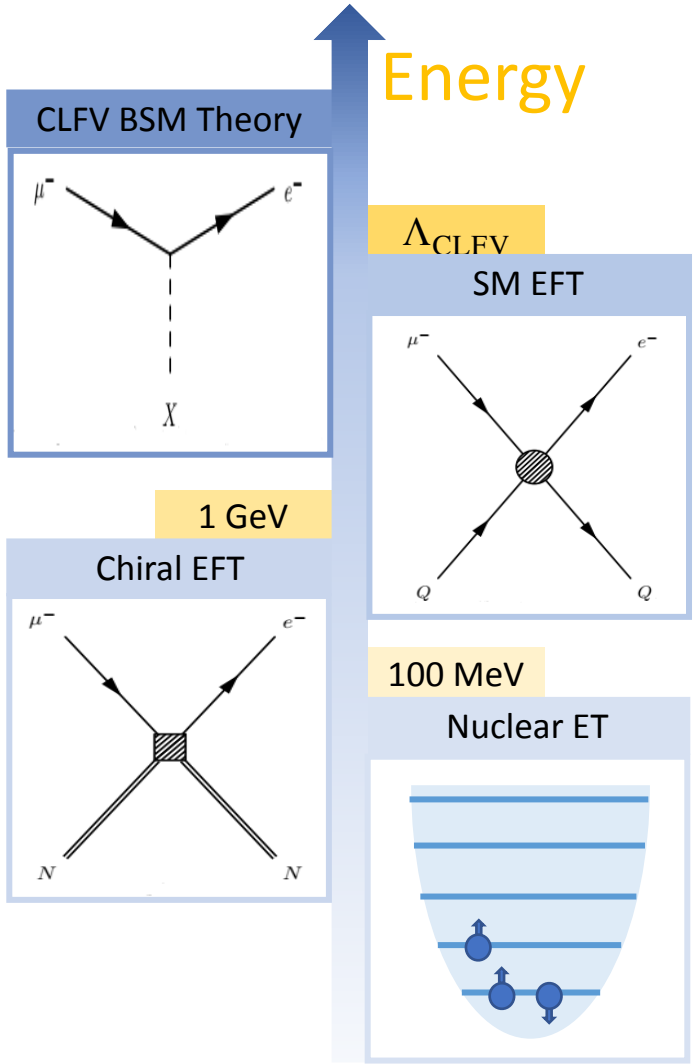
WET

$\mu = 2$ GeV

$u,\ d\ ,s$

gluons, γ s

shared with DM



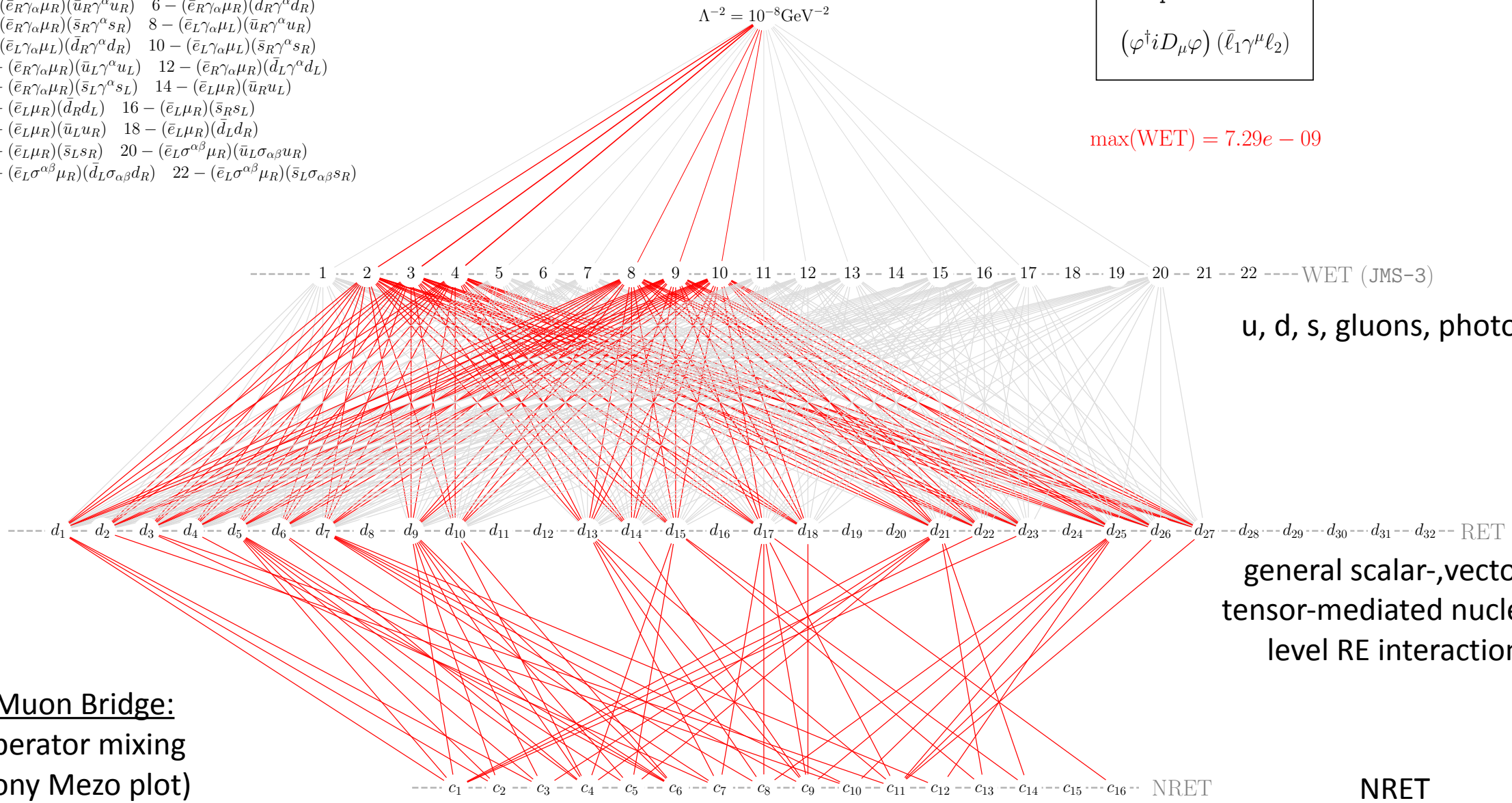
WH, Menzo, McElvain, Rule, Zupan

$1 - \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu}$ $2 - (\bar{e}_L \gamma_\alpha \mu_L)(\bar{u}_L \gamma^\alpha u_L)$
$3 - (\bar{e}_L \gamma_\alpha \mu_L)(\bar{d}_L \gamma^\alpha d_L)$ $4 - (\bar{e}_L \gamma_\alpha \mu_L)(\bar{s}_L \gamma^\alpha s_L)$
$5 - (\bar{e}_R \gamma_\alpha \mu_R)(\bar{u}_R \gamma^\alpha u_R)$ $6 - (\bar{e}_R \gamma_\alpha \mu_R)(\bar{d}_R \gamma^\alpha d_R)$
$7 - (\bar{e}_R \gamma_\alpha \mu_R)(\bar{s}_R \gamma^\alpha s_R)$ $8 - (\bar{e}_L \gamma_\alpha \mu_L)(\bar{u}_R \gamma^\alpha u_R)$
$9 - (\bar{e}_L \gamma_\alpha \mu_L)(\bar{d}_R \gamma^\alpha d_R)$ $10 - (\bar{e}_L \gamma_\alpha \mu_L)(\bar{s}_R \gamma^\alpha s_R)$
$11 - (\bar{e}_R \gamma_\alpha \mu_R)(\bar{u}_L \gamma^\alpha u_L)$ $12 - (\bar{e}_R \gamma_\alpha \mu_R)(\bar{d}_L \gamma^\alpha d_L)$
$13 - (\bar{e}_R \gamma_\alpha \mu_R)(\bar{s}_L \gamma^\alpha s_L)$ $14 - (\bar{e}_L \mu_R)(\bar{u}_R u_L)$
$15 - (\bar{e}_L \mu_R)(\bar{d}_R d_L)$ $16 - (\bar{e}_L \mu_R)(\bar{s}_R s_L)$
$17 - (\bar{e}_L \mu_R)(\bar{u}_L u_R)$ $18 - (\bar{e}_L \mu_R)(\bar{d}_L d_R)$
$19 - (\bar{e}_L \mu_R)(\bar{s}_L s_R)$ $20 - (\bar{e}_L \sigma^{\alpha\beta} \mu_R)(\bar{u}_L \sigma_{\alpha\beta} u_R)$
$21 - (\bar{e}_L \sigma^{\alpha\beta} \mu_R)(\bar{d}_L \sigma_{\alpha\beta} d_R)$ $22 - (\bar{e}_L \sigma^{\alpha\beta} \mu_R)(\bar{s}_L \sigma_{\alpha\beta} s_R)$

"phil1_12"

$(\varphi^\dagger i D_\mu \varphi)(\bar{\ell}_1 \gamma^\mu \ell_2)$

$\max(\text{WET}) = 7.29e - 09$



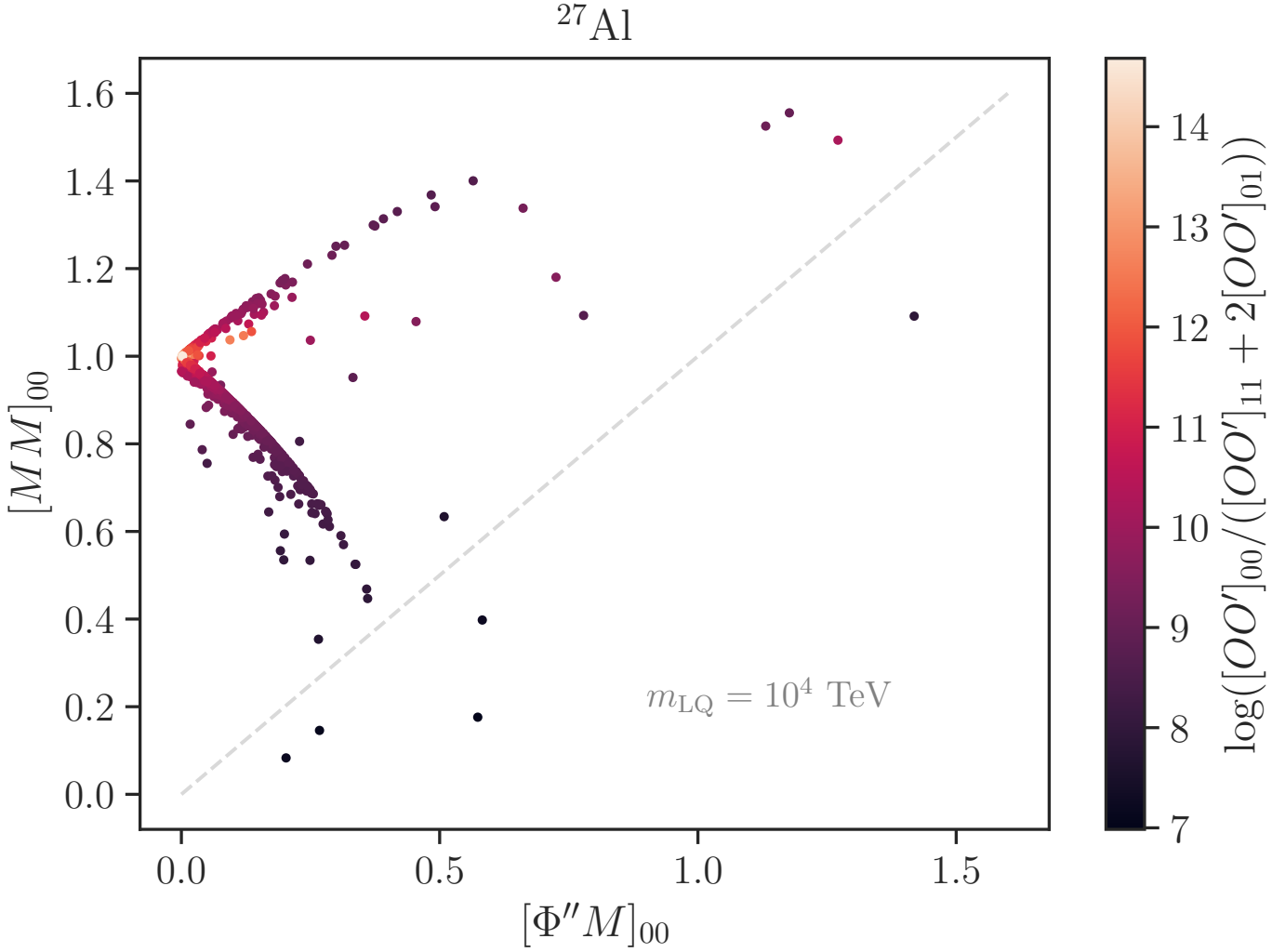
u, d, s, gluons, photons

general scalar-,vector-, tensor-mediated nucleon-level RE interaction

NRET

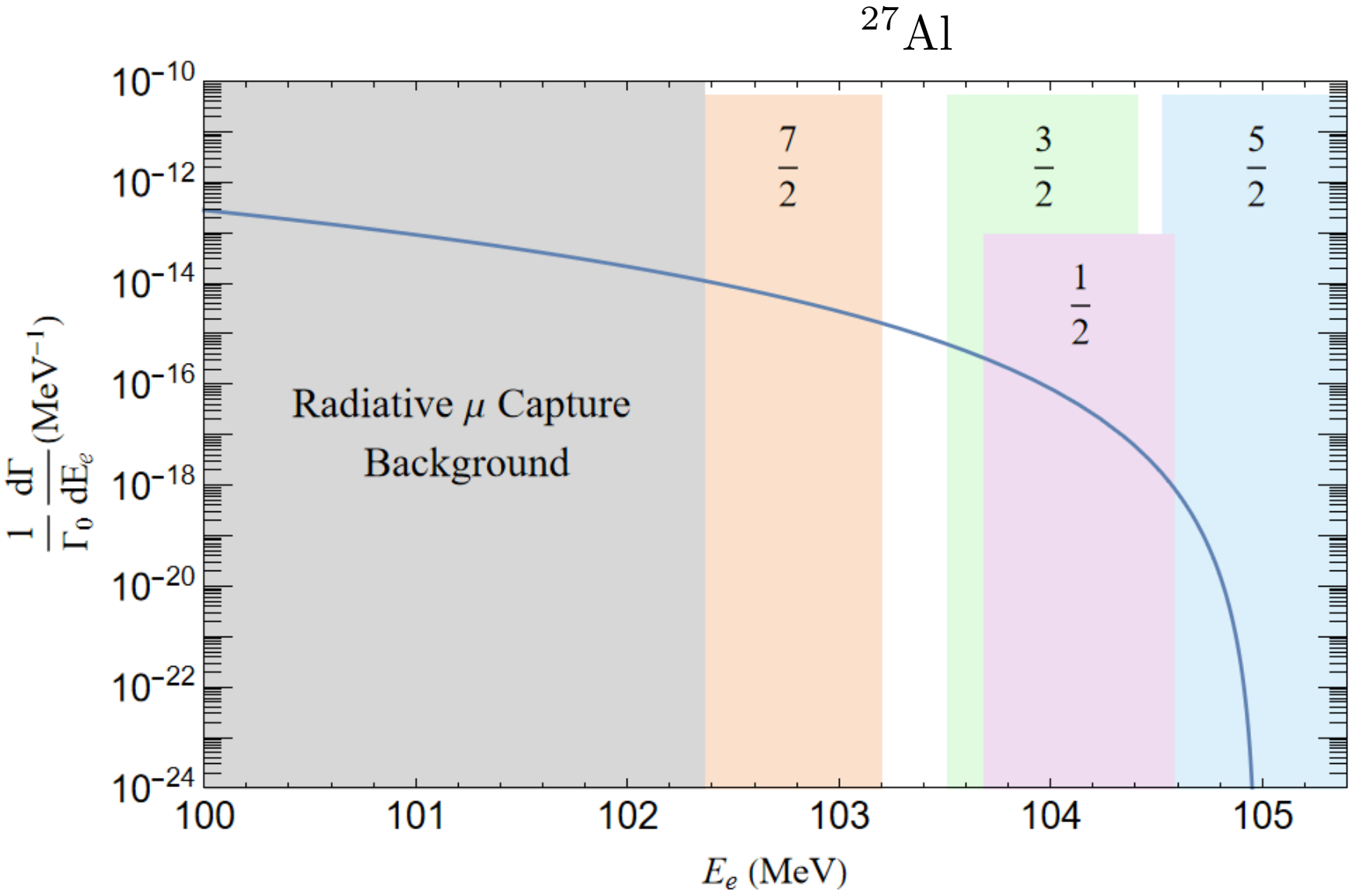
<u>Muon Bridge:</u>
operator mixing
(Tony Mezo plot)

What low-energy interactions will stand out?



Allows one to do MC parameter studies: leptoquark example, generates through tensor-mediate interactions the two scalar operators shown previously. Plot by T. Menzo.

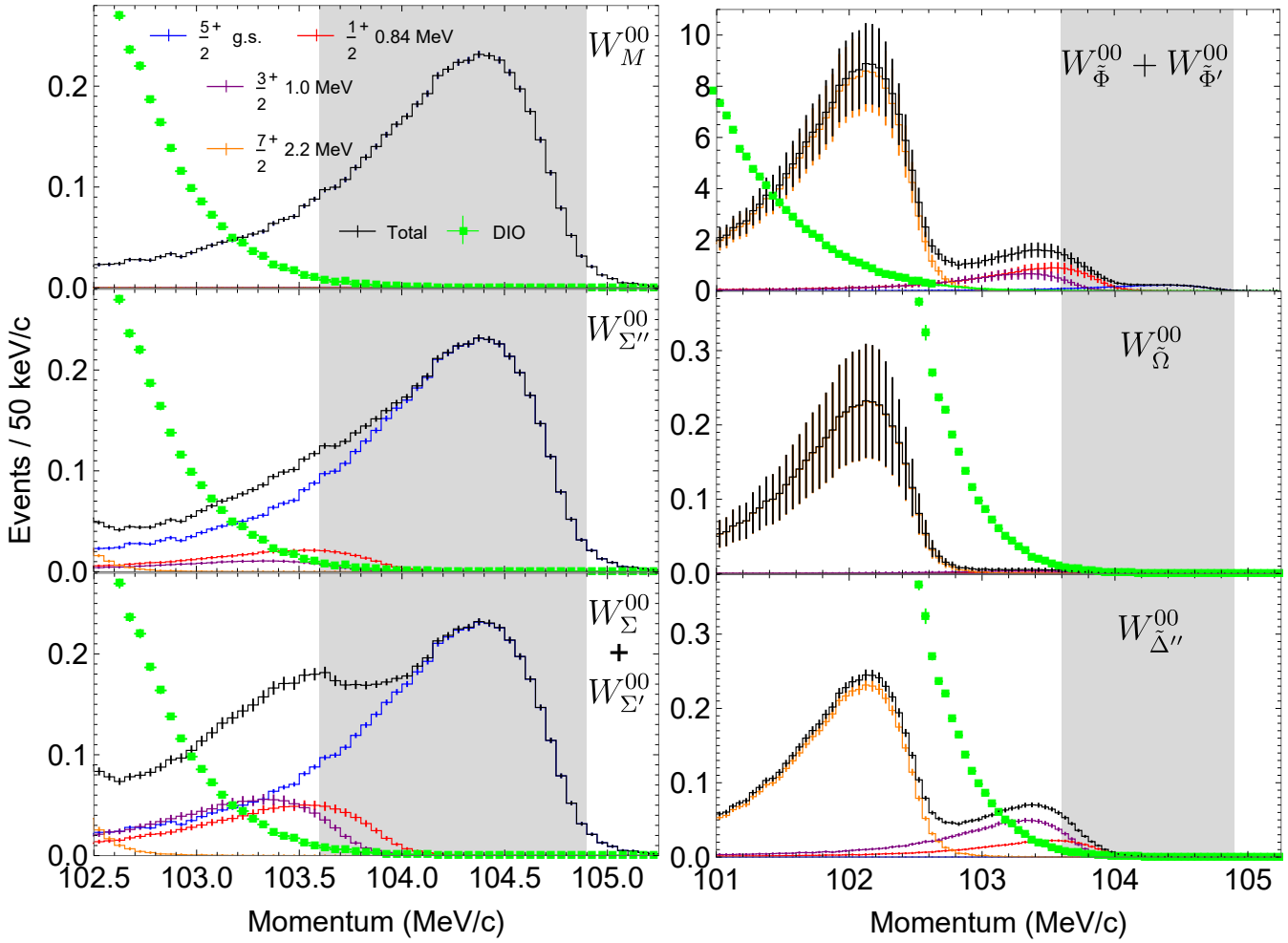
Inelastic conversion



vector charge

longitudinal spin

transverse spin



spin-velocity

axial charge

vector current

$^{27}\mathrm{Al}$

- Previously the experimentalists thought they would just get one number, a rate, for the Mu2e and COMET experiments

- Were they to see a signal, they would know CLFV occurs

- But they would learn nothing about the operator source of the CLFV — one can dial the unknown coefficient of any operator to account for the signal

- More information would require a new target with new sensitivities, and five years

- Not the case: 27Al is a fortuitous choice, yielding several independent numbers because the ground state and three excited states and resolvable

- This information will constrain the operator source of CLFV

Summary

- Decades ago major efforts to probe CLFV began at the meson facilities: LAMPF, TRIUMF, SIN/PSI

- Since that time we have learned that CLFV must occur — but at a largely unknown level

- The new experiments Mu2e and COMET will stop unprecedented numbers of muons, pushing limits by four orders of magnitude

- EFT allows one to connect in a fully general way those rates/limits, to the high-energy BSM theories ultimately responsible for signals

Thanks!