



OPTIMAL EXPERIMENTAL DESIGN OF MEASUREMENT CONFIGURATIONS FOR GEODETIC METROLOGY

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INTRODUCTION

- » Alignment accuracy depends not only on the instruments, but on the measurement geometry — where fiducials and instrument stations are placed. That geometry has to be fixed before any measurement is taken. We use a CERN open-source software (LGC) [1] to construct the least-squares problem
- » Today these choices follow mechanical constraints and experience and are validated a posteriori: a candidate setup is simulated, its covariance inspected, the design accepted or adjusted by hand — a trial-and-error loop that explores a small fraction of the configuration space.
- » The covariance of a least-squares estimate depends only on the Jacobian at the nominal configuration and on the stochastic model. Any candidate design can therefore be graded from a **single Jacobian evaluation**, before any measurement — the basis of **Optimal Experimental Design (OED)**.

Stochastic model

$$\ell = f(\tilde{x}) + e, \quad e \sim \mathcal{N}(0, P^{-1})$$

Least-squares problem

$$\hat{x} = \underset{x \in \mathbb{R}^{n_x}}{\operatorname{argmin}} \| \ell - f(x) \|_P^2$$

$$\|v\|_P^2 = v^T P v,$$

OED FORMULATION

- » **Quantities of interest.** The unknowns split into the **Qol** θ — e.g. the six pose parameters of a component frame — and **nuisances** ν (station positions, orientation unknowns).

- » **Design map pipeline:**

$$\textcircled{1} x = \begin{pmatrix} \theta \\ \nu \end{pmatrix} \quad \textcircled{2} A = (A_\theta \quad A_\nu) \quad \textcircled{3} N = A^T P A \quad \Sigma = \begin{bmatrix} \Sigma_\theta & [N^{-1}]_{12} \\ [N^{-1}]_{21} & [N^{-1}]_{22} \end{bmatrix}$$

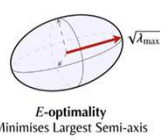
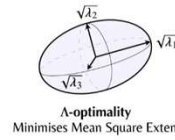
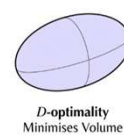
$$\textcircled{4} \Sigma = N^{-1} \quad \textcircled{5} \Sigma_\theta := \sigma_0 [N^{-1}]_{11}$$

- » **Optimality criteria.** To evaluate what the Qol we use 3 well know optimality criteria: **D-optimality**, **A-optimality** and **E-optimality**

$$\Phi_D(\Sigma_\theta) := \det \Sigma_\theta$$

$$\Phi_A(\Sigma_\theta) := \operatorname{tr}(\Sigma_\theta)$$

$$\Phi_E(\Sigma_\theta) := \lambda_{\max}(\Sigma_\theta)$$



CASE STUDY I — MAGNET FIDUCIALISATION

- » **Continuous placement.** Fiducial and station coordinates are free inside some boxes, solved by gradient-based optimization.
- » One quadrupole with two beam pipes and two sextupoles on a girder: the functional axes of the sextupoles must be aligned with the corresponding channels of the quadrupole, by estimating each component frame from targets on its surface.
- » Simulated Leica AT930 with precision: 0.25 mgon horizontal angle, 0.30 mgon zenith distance, 0.01 mm distance, 0.5 ppm scale.
- » **Stage I** optimizes the fiducial layout on the quadrupole. **Stage II** mounts it on the girder and jointly optimizes the sextupoles fiducials and the station positions inside their feasible boxes, the sextupoles frame being defined relative to the quadrupole frame.

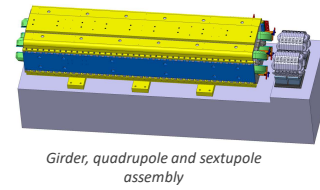
Table 1 — Quadrupole frame

Parameter	Unit	4 fid. (1 face)	8 fid. (3 faces)	12 fid. (3 faces)
TX	mm	0.0147	0.0080	0.0067
TY	mm	0.0109	0.0080	0.0069
TZ	mm	0.0110	0.0085	0.0073
RX	cc	4.691	3.561	2.766
RY	cc	23.941	12.605	9.615
RZ	cc	4.408	3.372	2.679

- » The tabulated uncertainties are the direct output of the optimizer, which places the stations optimally for each fiducial configuration. **RY (roll)** — critical for magnetic-axis alignment — is the parameter that gains most.

$$\min_{w \in S} \Phi(\Sigma_\theta(w)) \quad S = \prod_{j=1}^m [l_j, u_j] \subset \mathbb{R}^m, \quad l_j < u_j,$$

Mathematical formulation Feasible set



Girder, quadrupole and sextupole assembly

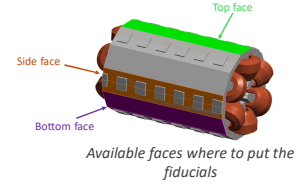
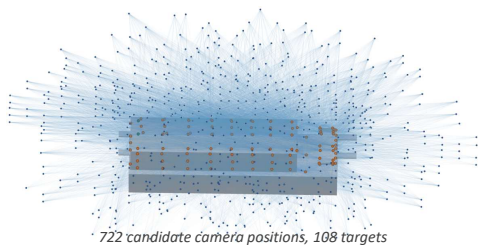


Table 2 — Sextupole frame

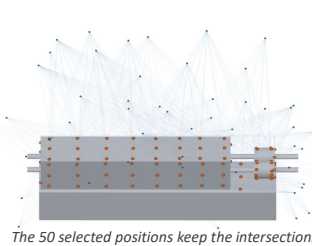
Parameter	Unit	3 fid. (1 face)	4 fid. (2 faces)	6 fid. (3 faces)
TX	mm	0.0309	0.0098	0.0085
TY	mm	0.0090	0.0072	0.0062
TZ	mm	0.0106	0.0094	0.0090
RX	cc	19.685	13.759	11.039
RY	cc	109.754	27.868	21.056
RZ	cc	16.889	13.329	11.529

CASE STUDY II — PHOTOGRAMMETRY

- » **Discrete selection.** The **geometry is frozen** and the most informative observation groups are chosen under a budget K. Information adds up across groups; greedy selection permits us to obtain a solution.
- » To address this problem, an initial configuration comprising 722 well-distributed camera positions around an object covered with 108 targets was considered.



722 candidate camera positions, 108 targets

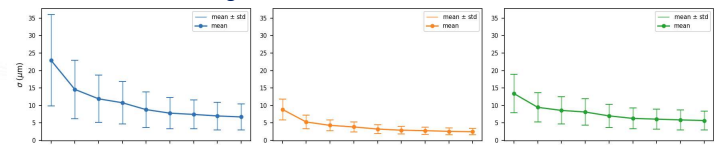


The 50 selected positions keep the intersections

$$\min_{S \subseteq \Omega, |S|=K} \Phi(\Sigma_\theta(S)) \quad \text{subj to } N(S) > 0$$

Mathematical formulation

- » The primary goal is to determine the position of the targets with the lowest possible uncertainty, thereby minimizing the associated uncertainty to achieve accuracy levels comparable to those obtained with laser trackers.
- » It is crucial to analyse the standard deviations along the three spatial components (**SX, SY, SZ**), which represent the respective measurement uncertainties along the coordinate axes



Comparison of the average standard deviations (SX, SY, SZ) across all targets as a function of different optimized camera configurations.

CONCLUSION

- » OED turns configuration design from a posteriori validation into a quantitative decision-support tool for the design phase.
- » This approach is flexible to multiple possible setups, and these two case studies are just representative of its potential

References

- [1] G. Kautzmann, J. Gutekunst, F. Klumb, R. Singh, and P. Elson, "LGC: CERN's geodetic network adjustment software goes open source", in Proc. FIGCongress 2026, Cape Town, South Africa, 2026, 2026, https://fig.net/resources/proceedings/fig_proceedings/fig2026/papers/ts08d/ts08d_kautzmann_gutekunst_et_al_13858.pdf



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