The 8th International School on Beam Dynamics and Accelerator Technology (ISBA25)

Circular Accelerator and Synchrotron

焦毅, Yi JIAO
中国科学院高能物理研究所
Institute of High Energy Physics, Beijing, China

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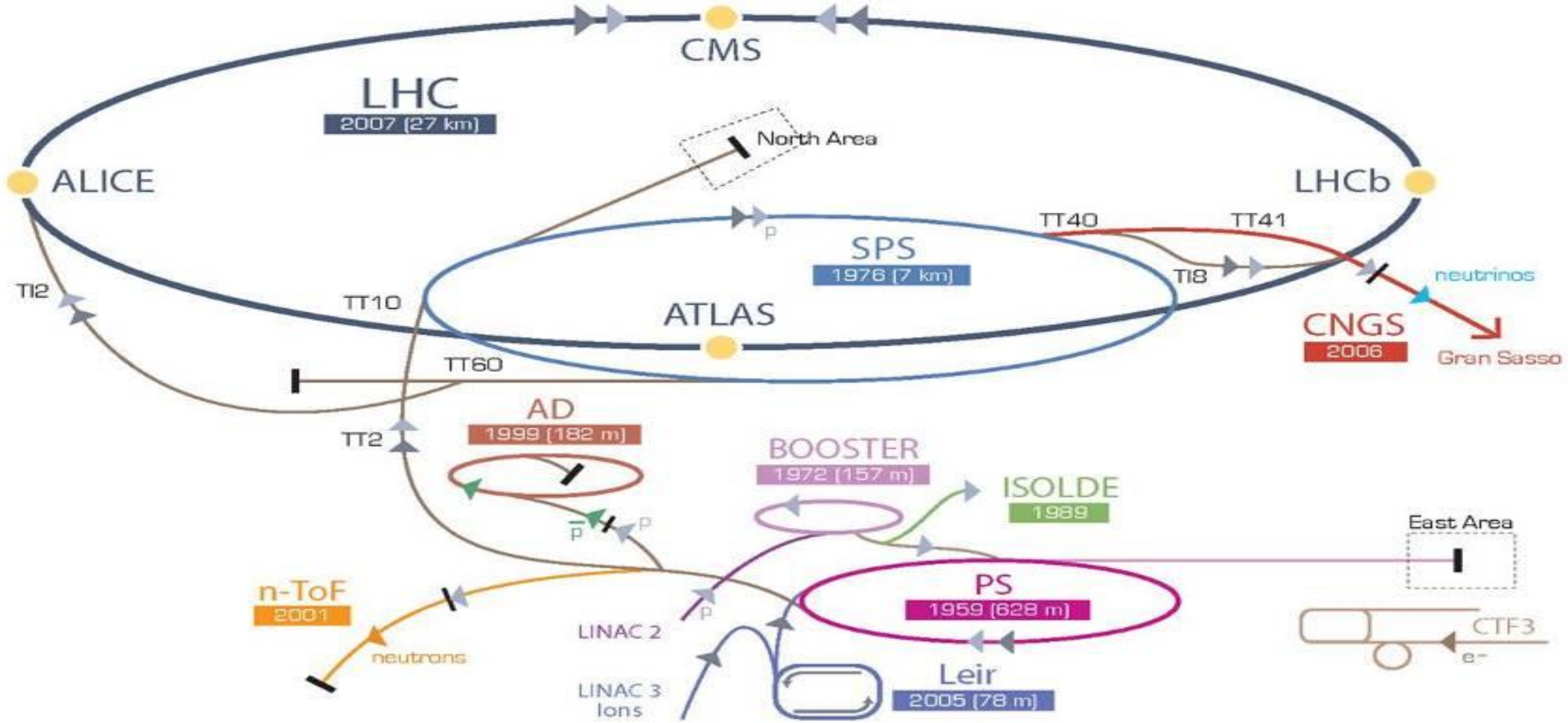
Outline

- Circular Accelerator
  - Based on accelerator development history
- Synchrotron
  - Basic physics concept and issues

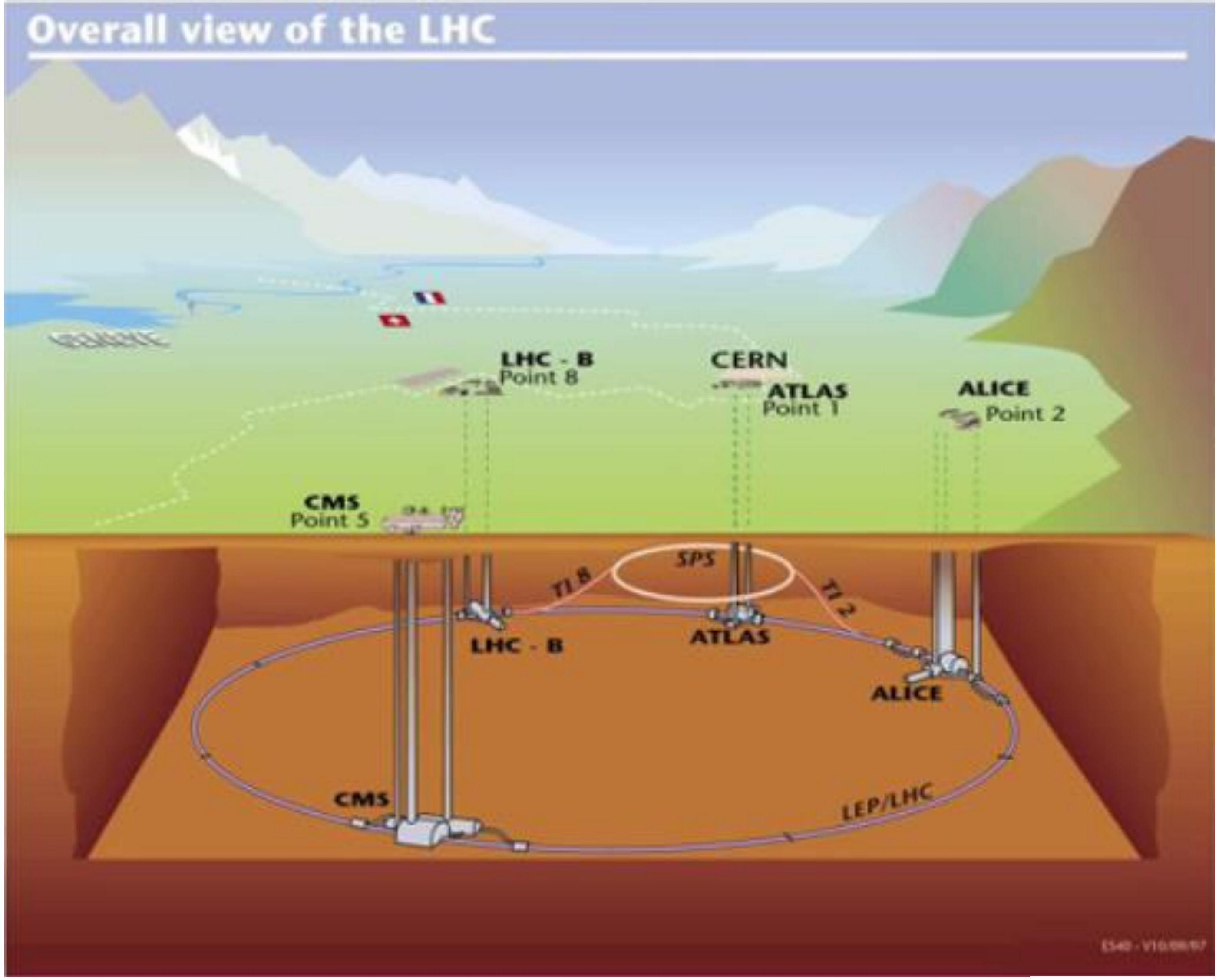
Circular Accelerator

Based on accelerator development history

World's largest circular accelerator： Large Hadron Collider，LHC， 28 km



World's largest circular accelerator: Large Hadron Collider, LHC, 28 km



Located in Geneva, Switzerland, the European Organization for Nuclear Research (CERN) straddles the border between France and Switzerland. It can accelerate protons to energy of 7 TeV (teraelectronvolts), achieving a collision energy of 14 TeV.

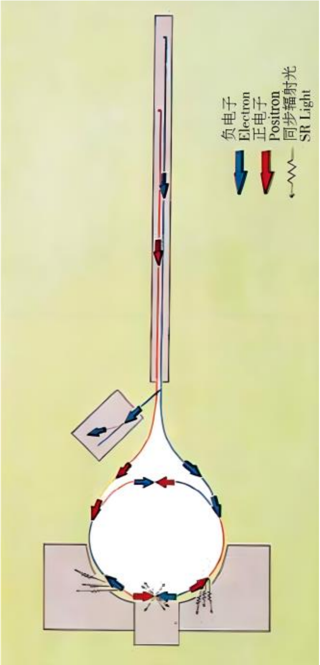
http://home.web.cern.ch,2015

Circular accelerator: one major type of accelerators in the world

Lepton (electron), hadron (proton or ion),
Collider, synchrotron radiation light source,
Circumferences cover 10 m order to 10 km order

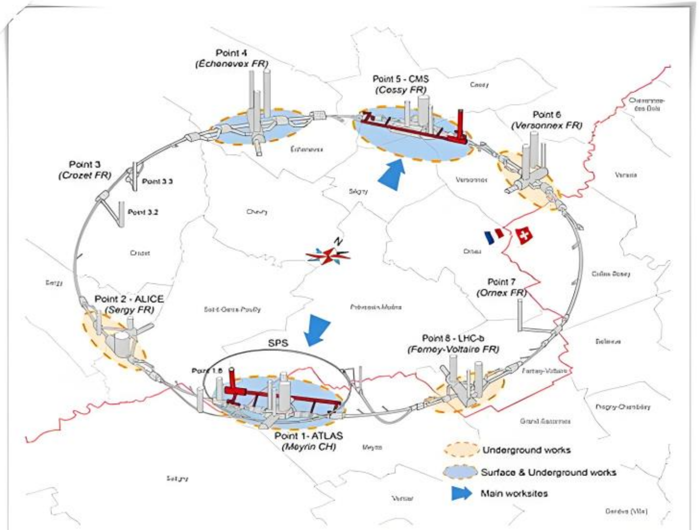
HLS, Hefei

BEPC, Beijing



HEPS, Beijing

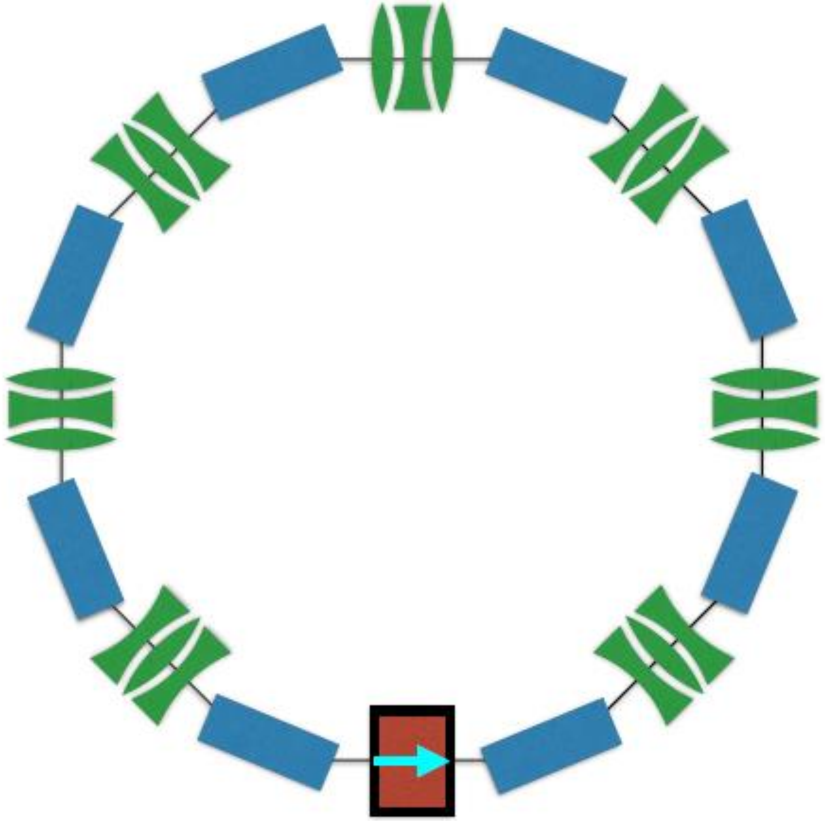
LHC, Geneva



10 m | 100 m | 1 km | 10km | 100 km

Circular accelerator

Nowadays, circular accelerator ≌ synchrotron

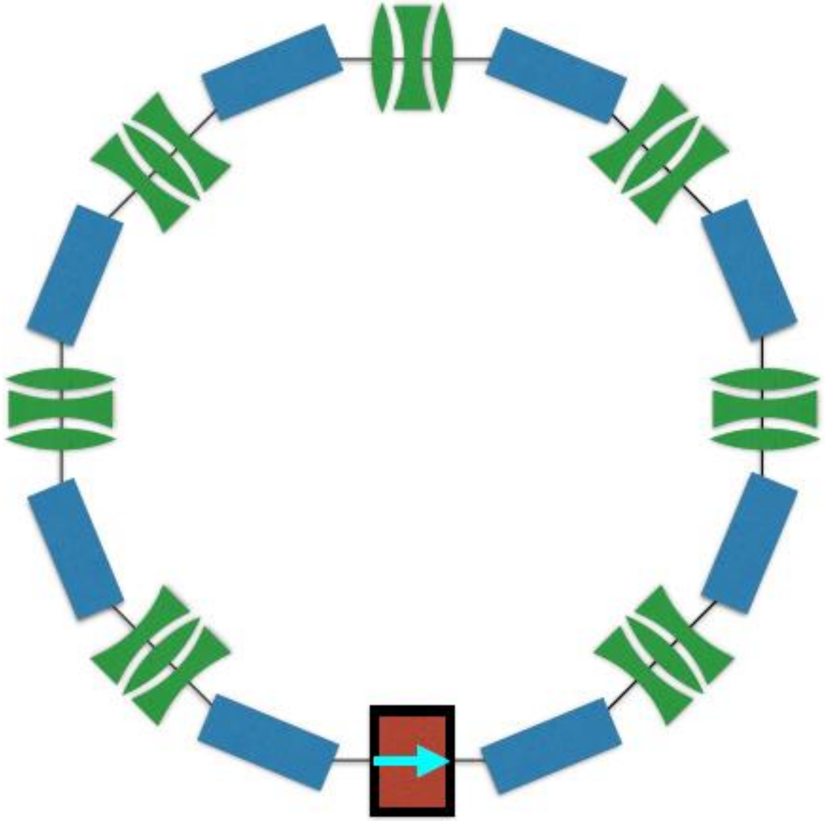


synchrotron

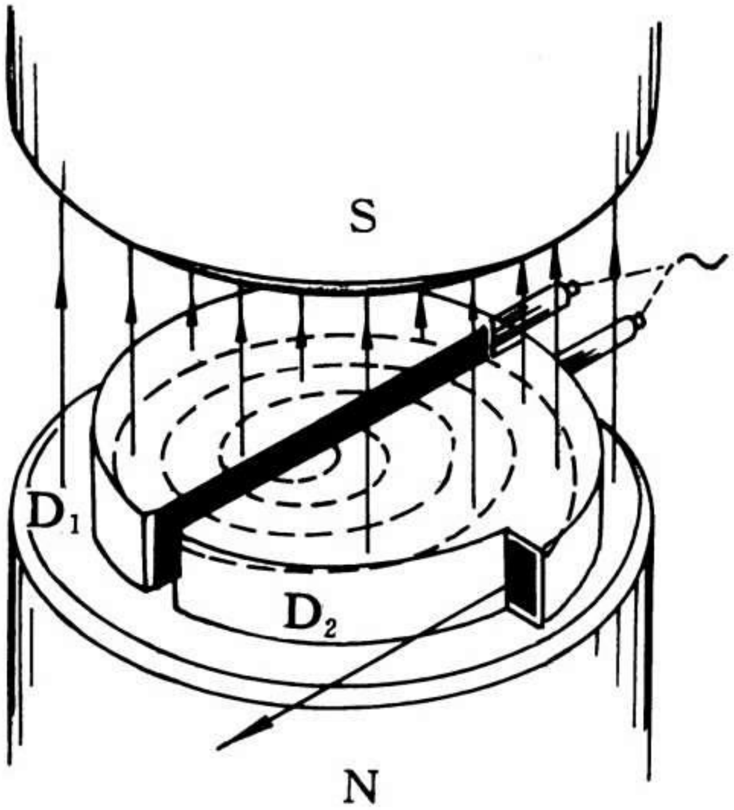
Separate acceleration and guiding components

Circular accelerator

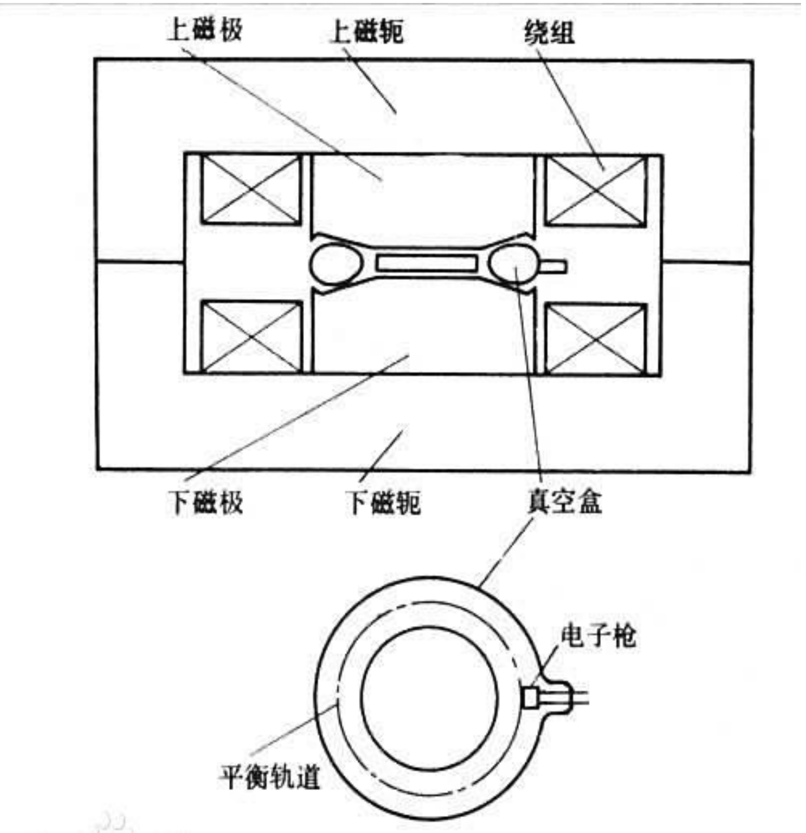
In history, circular accelerator ≠ synchrotron



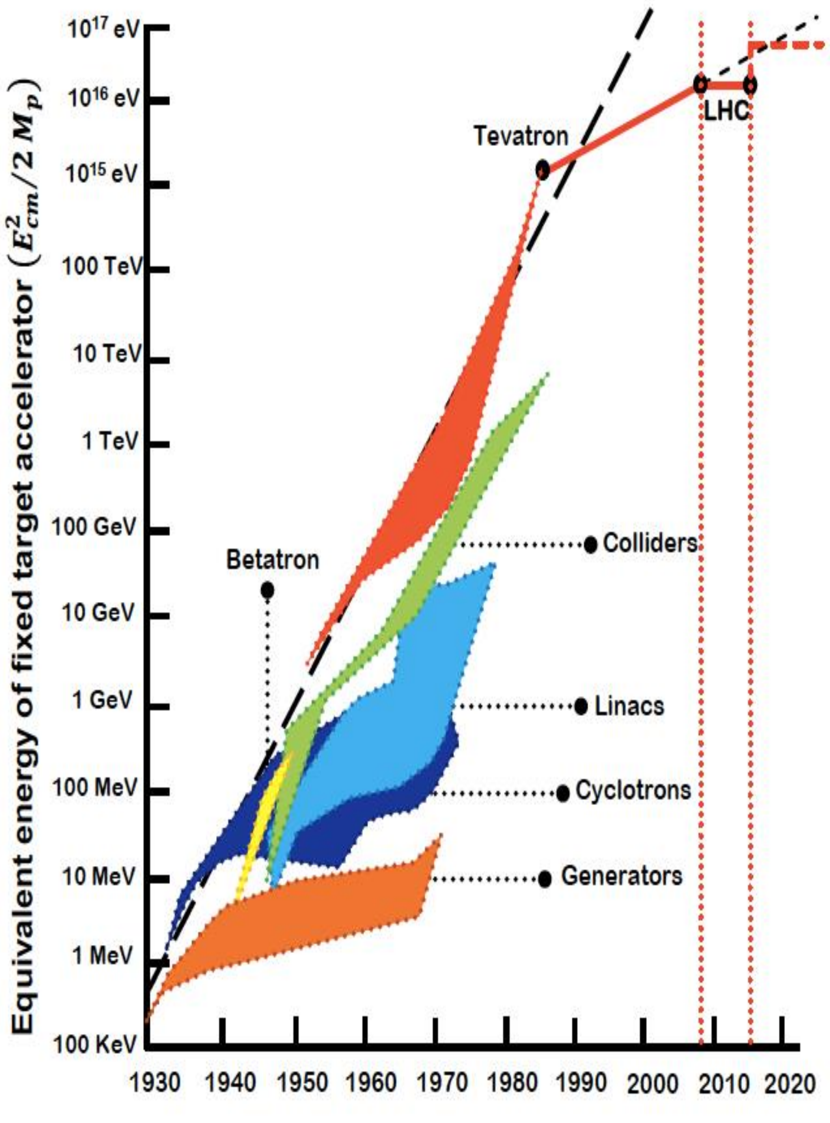
synchrotron



cyclotron



betatron



Livingston Chart

Rutherford's plea for particle accelerators, and the start point of accelerators — 1919

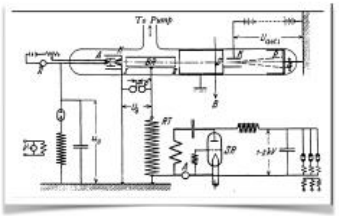
G. Ising, and Rolf Wiederoe: **Linear accelerator** — 1924

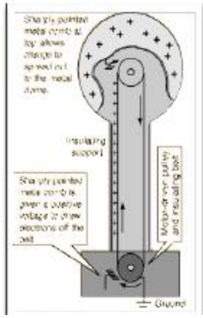
Van De Graaff: **Tandem** — 1929

E.T. Lawrence: **Cyclotron** — 1930

Cockcroft and Walton: **Voltage Multiplying Accelerator** — 1932

D. Kerst: **Betatron** — 1940





1932 by M. S. Livingston
Proton: 80keV, 1kV

A.W. Hull: Magnetron — 1921

Rusell and Sigurd Varain and William Hansen: Klystron — 1937

Synchrotron — 1940

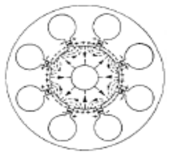
Vladimir Veksler, Edwin McMillan: Phase stability — 1943

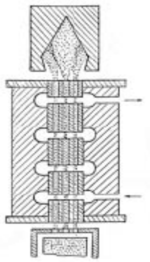
Electron Linac — 1944

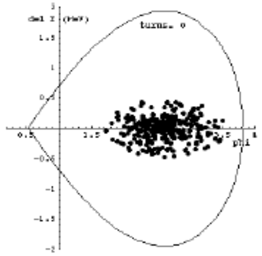
E.Courant, S.Livingston and H. Synder: Strong Focusing-AG — 1946

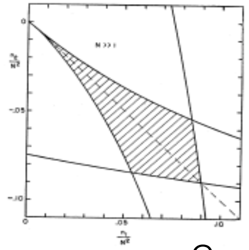
SLAC 3km Linac: 50GeV electron, with 240 Klystrons, 64MW, and SLED — 1966

LHC: 7TeV Proton Synchrotron. Begin operation 2012: discovered higgs boson — 2008

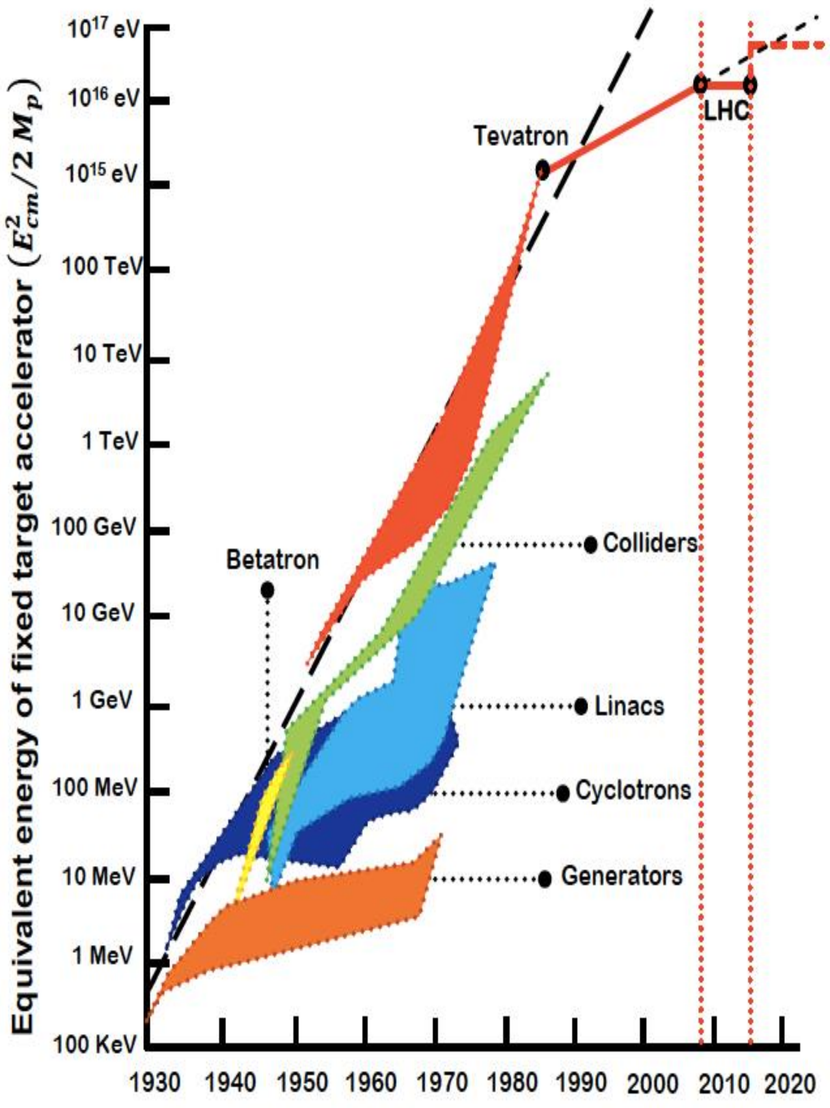








Courtesy of Chuanxiang Tang



Livingston Chart

1930s-today：

- (Equivalent) accelerated energy increased by ~10 order of magnitudes
- Cost of unit energy increase reduced by ~ 4 order of magnitudes

Many kinds of accelerators

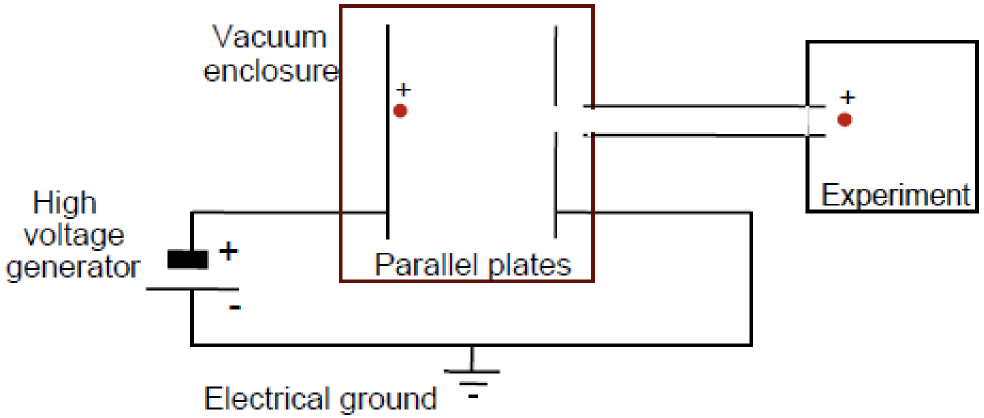
- DC high-voltage accelerator、Cyclotron、Linac、Synchrotron、Betatron……

Important principles/concept

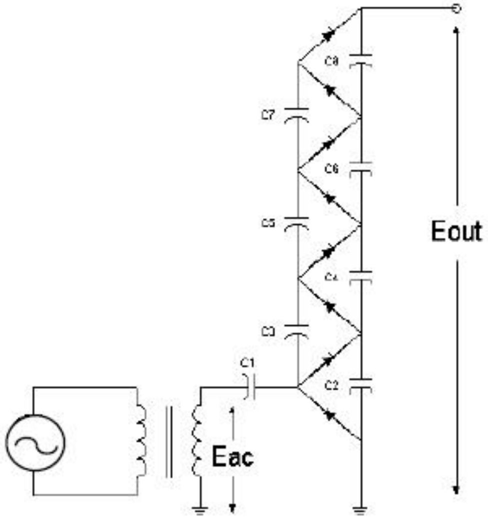
- Phase stability, strong focusing, collider

DC electro-static accelerator（Max.~10 MeV）

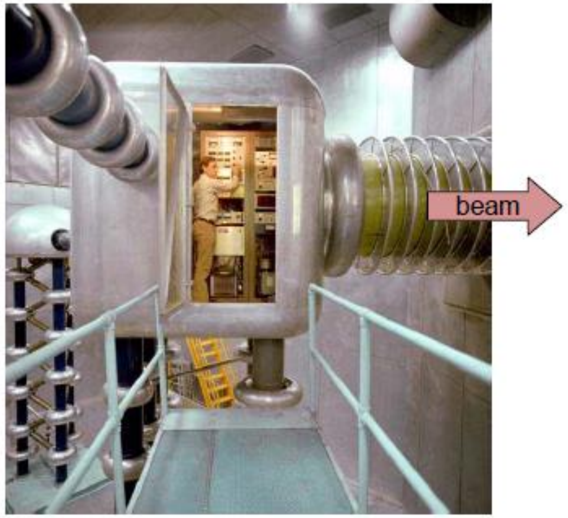
1932， J.D.Cockcroft, U.S. & E.T.S.Walton, Ireland， 1933， R.J.van de Graaff， U.S.



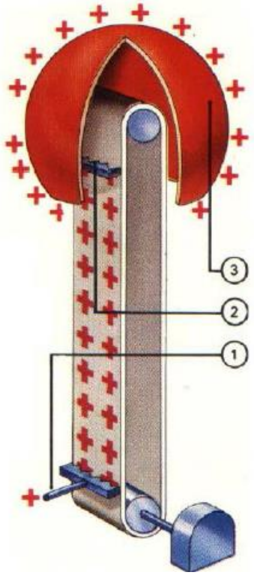
High-voltage breakdown limits the acceleration energy

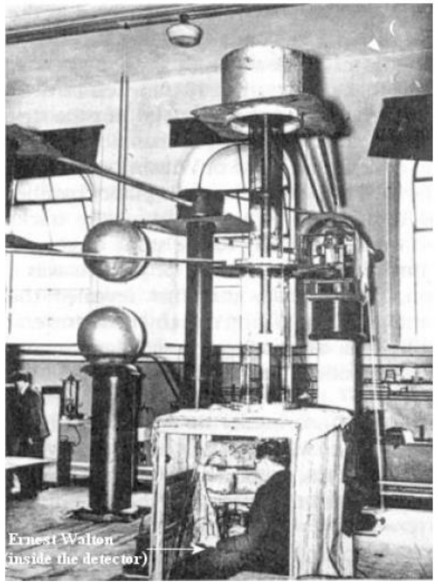


$E_{out} = N_{stage}\ E_{ac}$



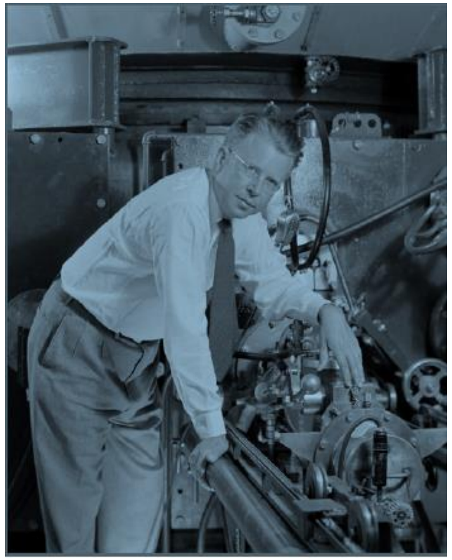
Crockroft-Walton at FNAL accelerates H- to 750keV



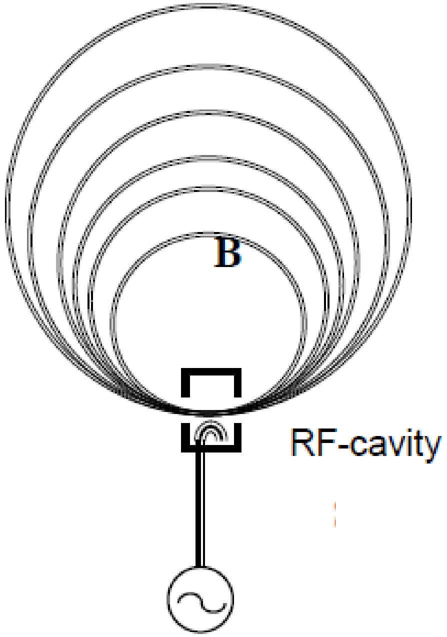


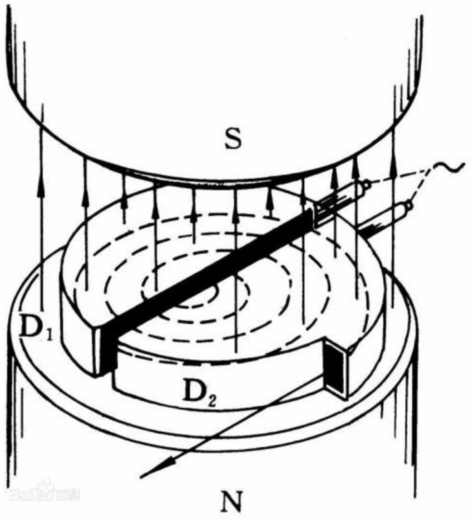
Cyclotron

Lawrence built the first cyclotron in 1932 and used it to produce artificial radioactive isotopes, for which he was awarded the 1939 Nobel Prize in Physics.

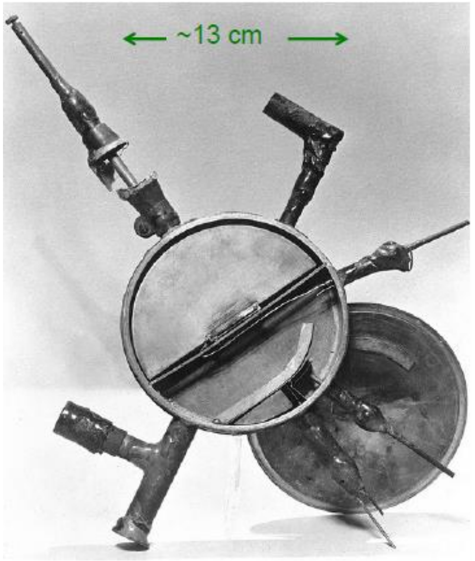


E.O. Lawrence
（1901-1958）





Repeatedly passing through the acceleration gap (alternating voltage),Continuously obtaining energy



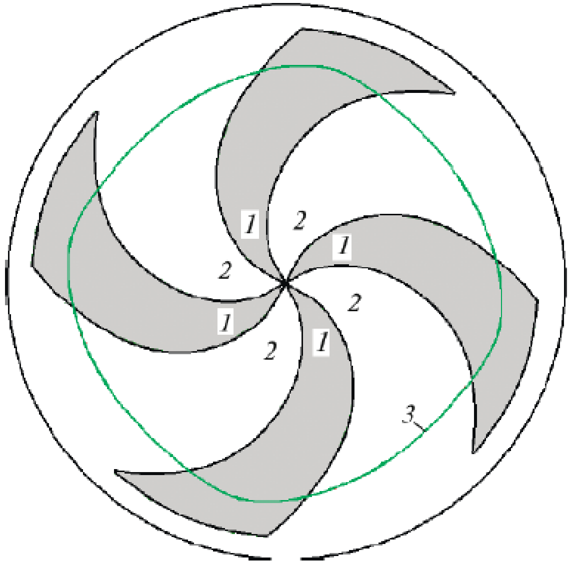
1931，Demonstration, ~13 cm，80 keV

~70 cm，5 MeV

As the energy increases, the period of particle motion gradually no longer matches the acceleration voltage period, thus limiting the energy, to 10 MeV order

Cyclotron

- Based on synchronous acceleration and other technologies, cyclotron can accelerate the energy to higher values, e.g., up to several hundred MeV, mainly used for the production of various medical isotopes, proton or ion therapy.

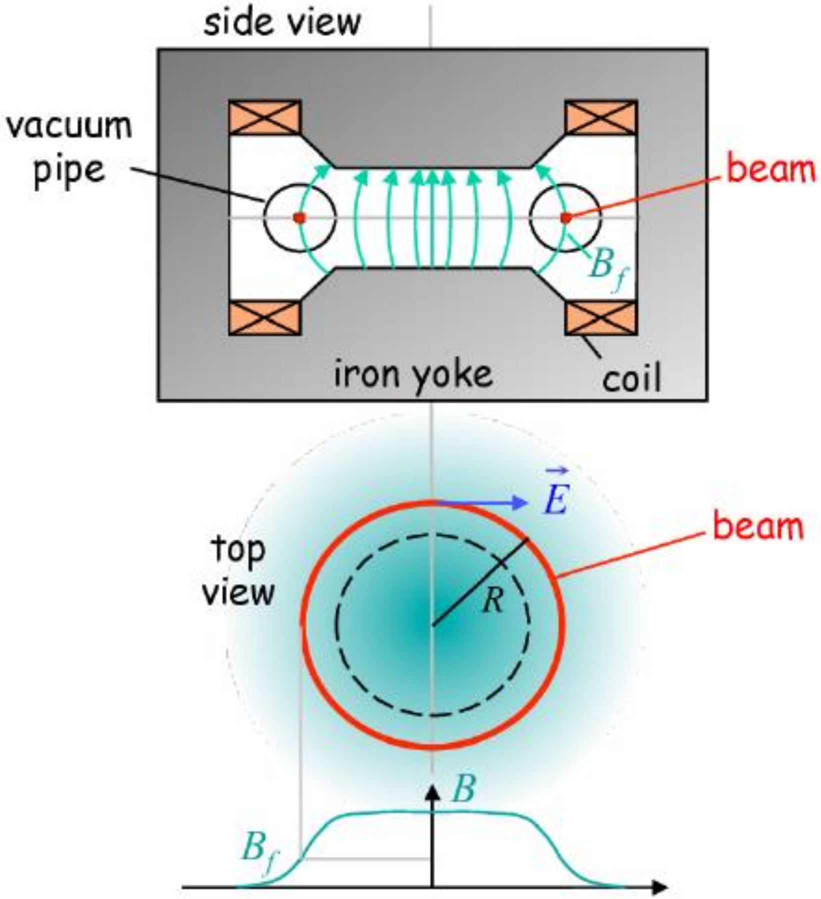


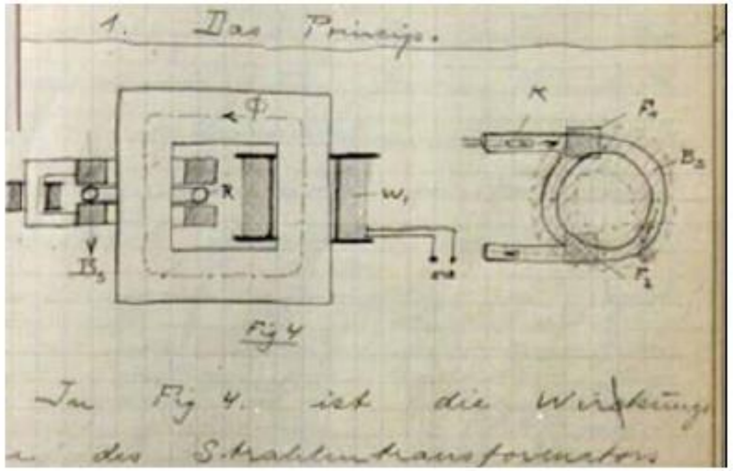
Spiral sector cyclotron

Superconducting cyclotron

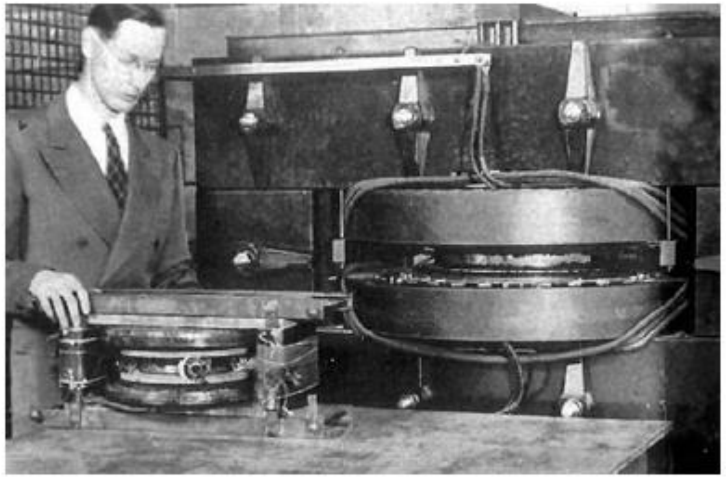
Cyclotron for proton therapy

Betatron



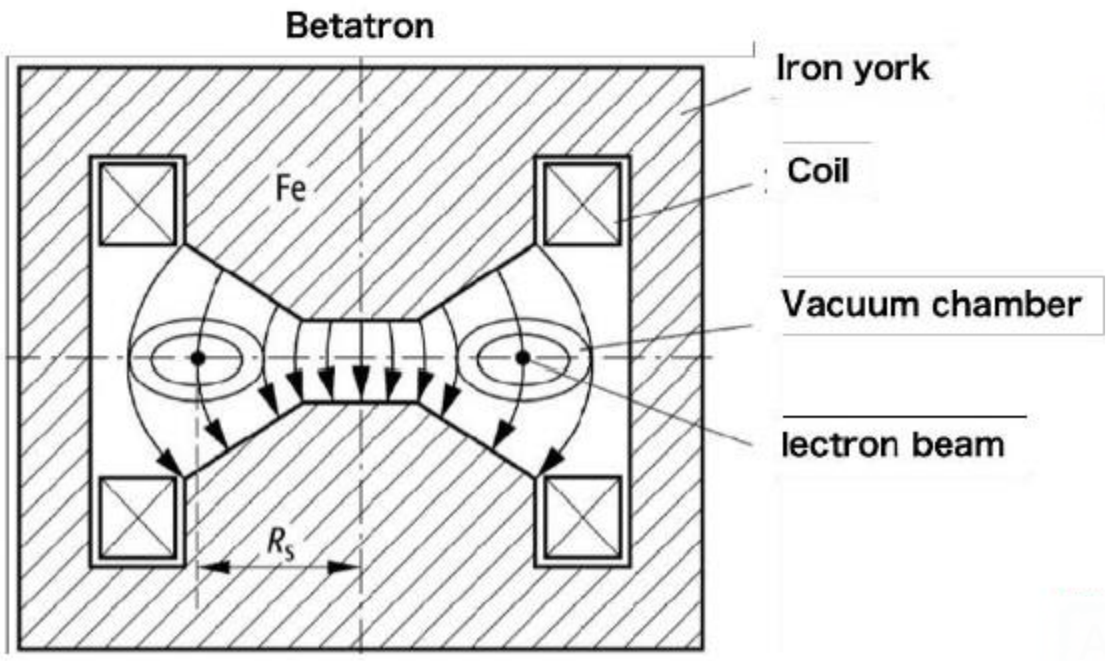


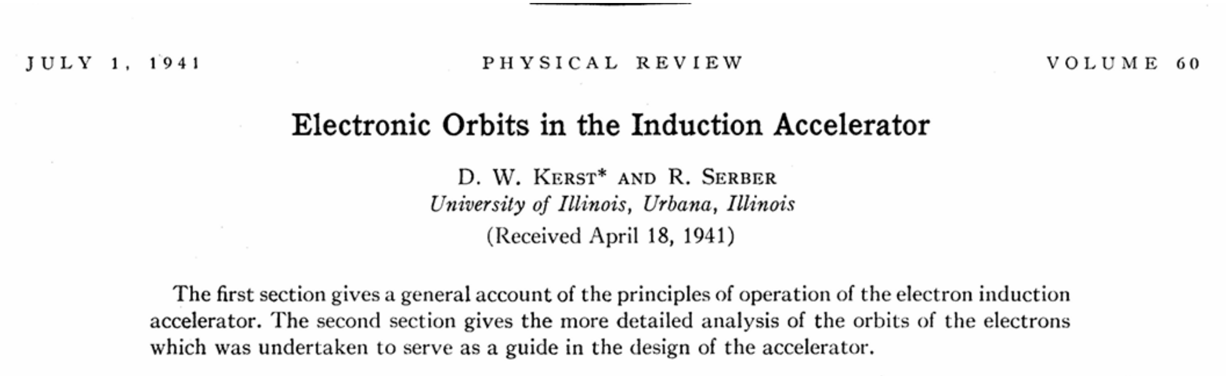
The original idea is given by Wiederoe's ray transformer in 1920s



Donald Kerst built the world's first magnetic induction accelerator at the University of Illinois in 1940

Betatron





JULY 1, 1941 PHYSICAL REVIEW VOLUME 60

Electronic Orbits in the Induction Accelerator

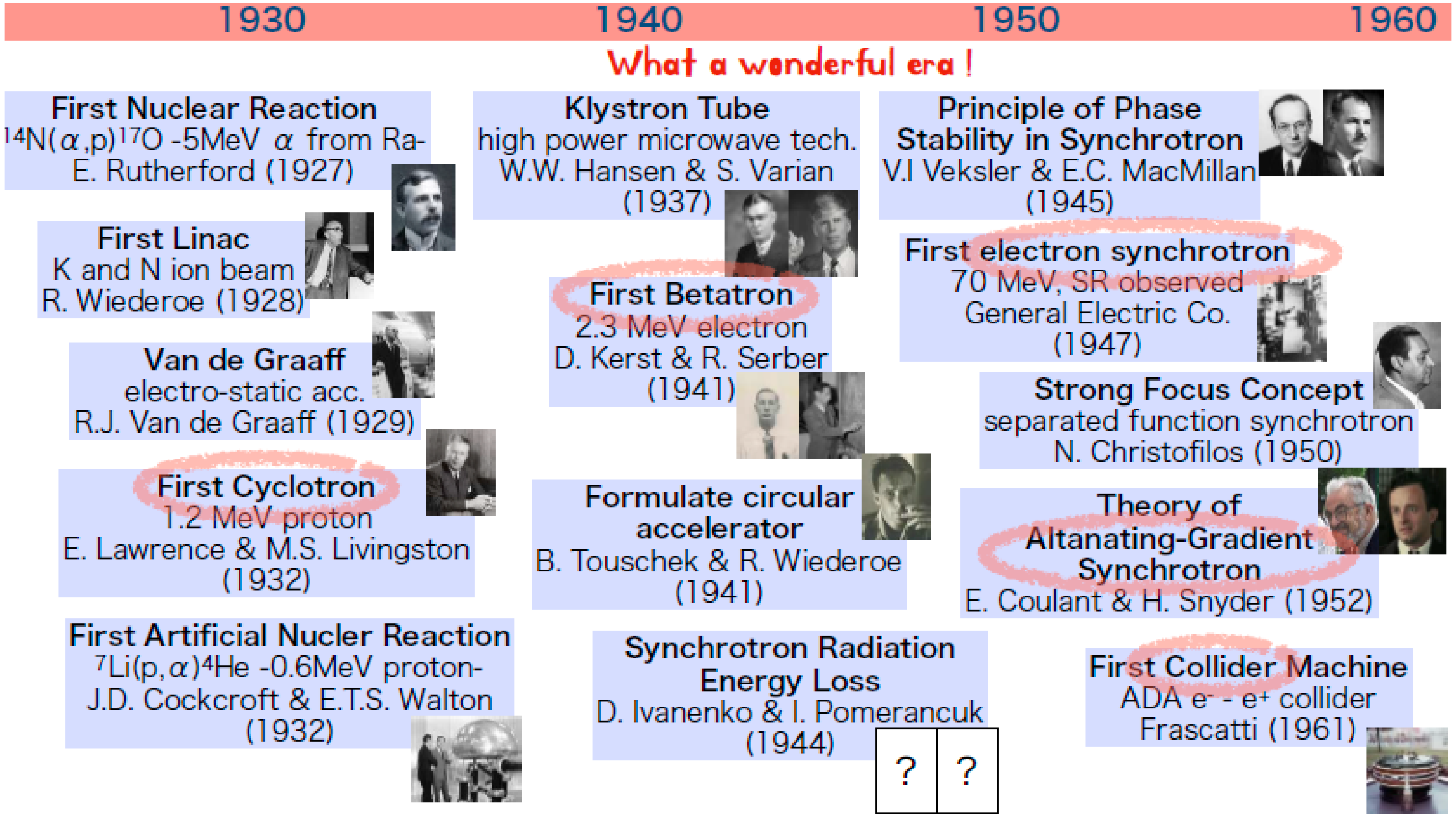
D. W. KERST* AND R. SERBER
University of Illinois, Urbana, Illinois
(Received April 18, 1941)

The first section gives a general account of the principles of operation of the electron induction accelerator. The second section gives the more detailed analysis of the orbits of the electrons which was undertaken to serve as a guide in the design of the accelerator.

- **Basic principle:**
  - Varying magnetic field induces electric field
- **Main characteristics:**
  - Appropriate only for accelerating electrons
  - To keep the electrons on the orbit, magnetic field of the orbit and average magnetic field insider the orbit must satisfy the 2:1 condition.
- **Others:**
  - The maximum energy for accelerating electrons is 100 MeV order. Thus, it did not become a primary type of accelerator used in the field of high-energy physics.
  - But it played a significant role in understanding beam dynamics and in early applications.
  - For the transverse motion of electrons, Kerst and Serber developed the analysis and named it "betatron motion." Today, the term "betatron motion (or oscillation)" is used to describe the transverse oscillations of particles in strong-focusing circular accelerators (as well as in linear accelerators/beam transport lines)

Road to modern synchrotron accelerator

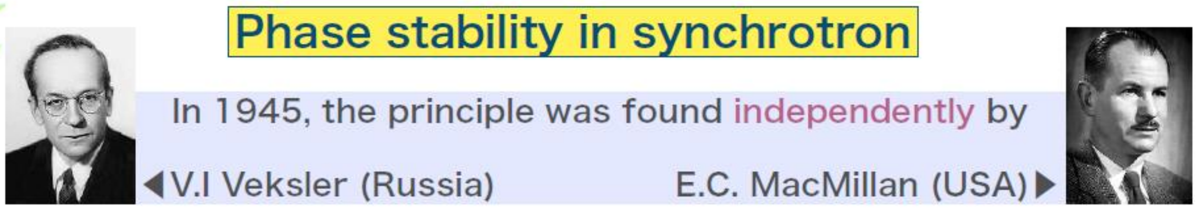
Several revolutionary principles proposed by scientists in the 1940s and 1960s.

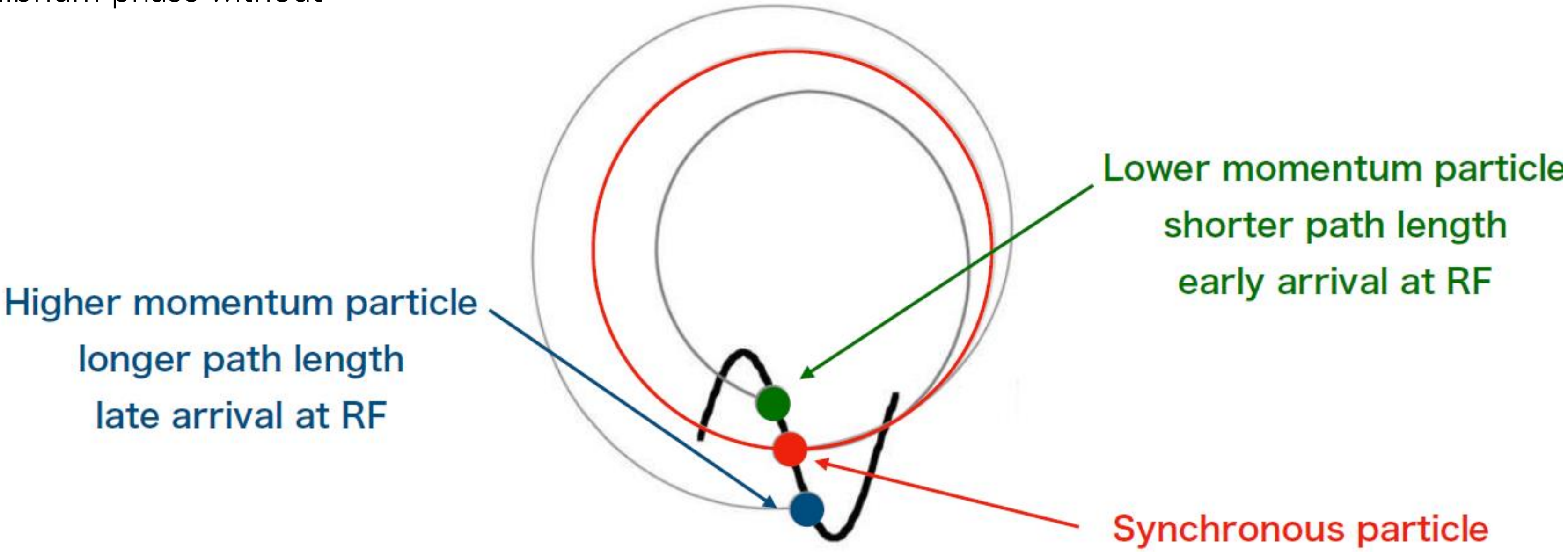


Courtesy of Hiroyuki Hama

Principle of phase stability

In a suitable alternating accelerating electric field, it is not required all particles arrive at the accelerating gap at the "perfect" moment. **There exists a stable phase region (equilibrium phase).** Particles that arrive earlier or later than the "ideal" particle will experience a self-correcting force, causing their motion to oscillate around this equilibrium phase without being lost.

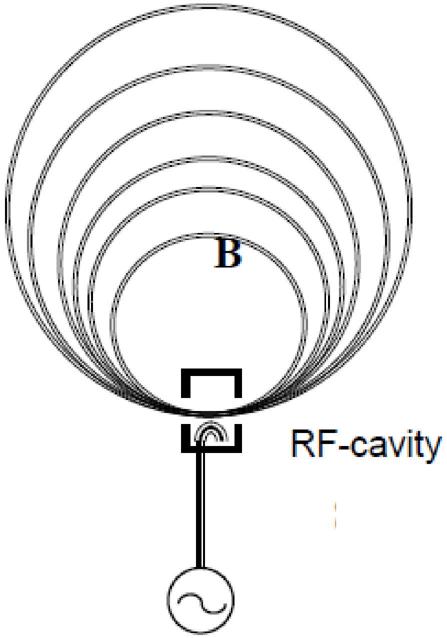




Courtesy of Hiroyuki Hama

Principle of phase stability

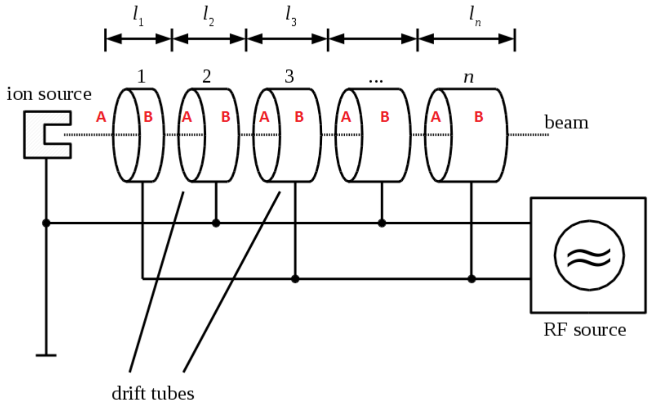
1945， V.I.Veksler， Russia & E.M.McMillan， USA

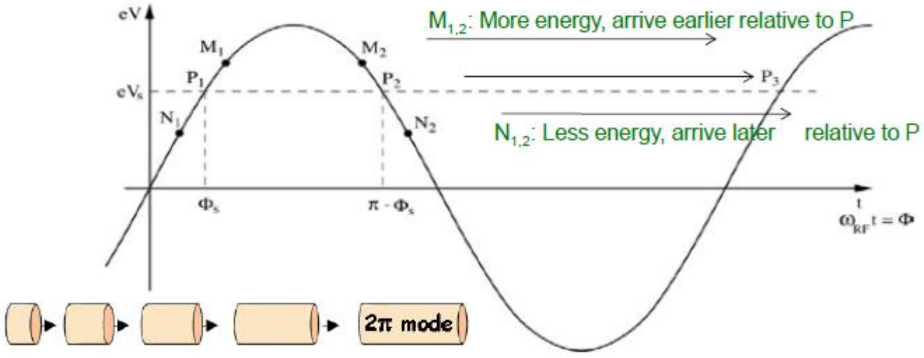


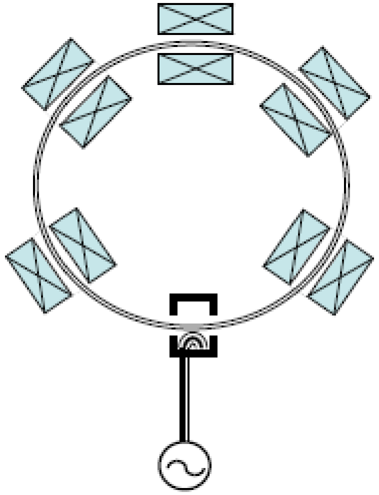
As energy increases, change the period or frequency of accelerating field

Synchro-cyclotron

Synchro-Linac







Keep the period or frequency of accelerating field, and change magnetic filed as energy increases to keep the orbit unchanged

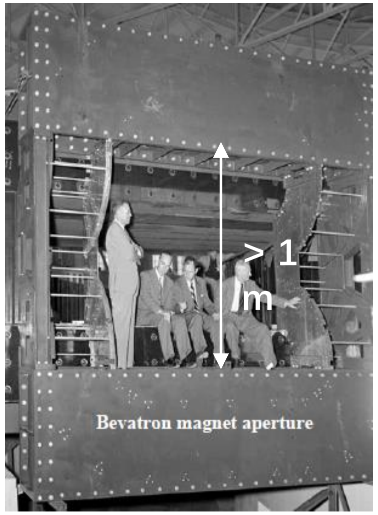
Synchrotorn

Synchrotron with weak focusing

GeV order, but very expensive!

Weak focusing: $B_z(r) = B_z(r_s)\frac{1}{r^n}, \quad 0 < n < 1$

Bevatron（1954）, weak focusing，6.2 GeV



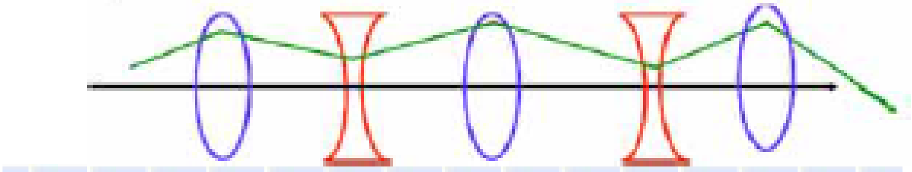
The total weight of the magnets is 10000 tons; The aperture of the vacuum chamber exceeds 1 meter!

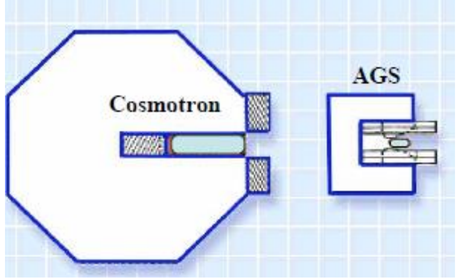
Synchrotron with strong focusing（> 10 GeV）

1952，E.D.Courant、M.S.Livingston, H.S.Schneider, USA

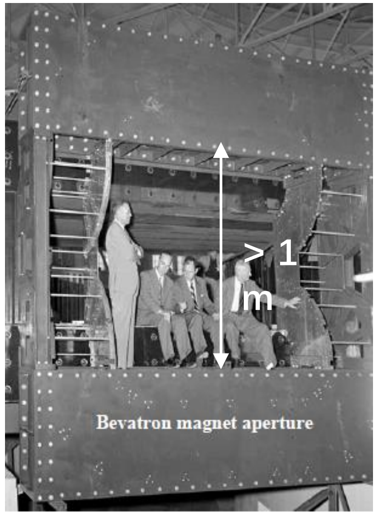
Weak focusing: $B_z(r) = B_z(r_s)\dfrac{1}{r^n}, \quad 0 < n < 1$

Strong focusing: $n(\theta) \gg 1, \quad \text{and}, \quad n(\theta) \ll -1$ alternatively.





Bevatron（1954）, weak focusing，6.2 GeV

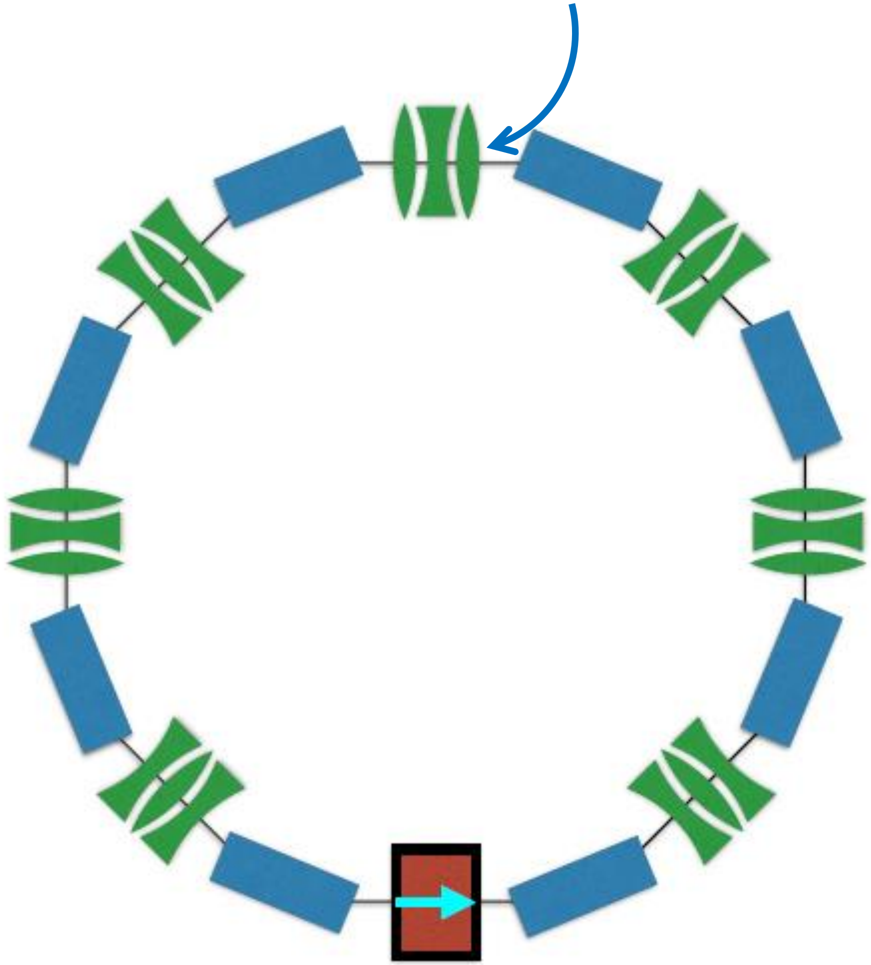


AGS（1960）, strong focusing，33 GeV

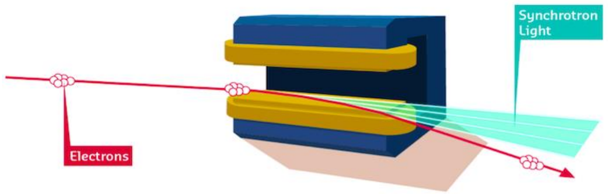
The total weight of the magnets is 10000 tons; The aperture of the vacuum chamber exceeds 1 meter!

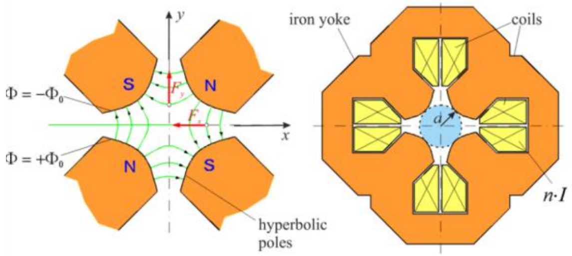
The total weight of the magnets is 4000 tons, Vacuum chamber aperture reduced significantly!

Modern circular accelerator

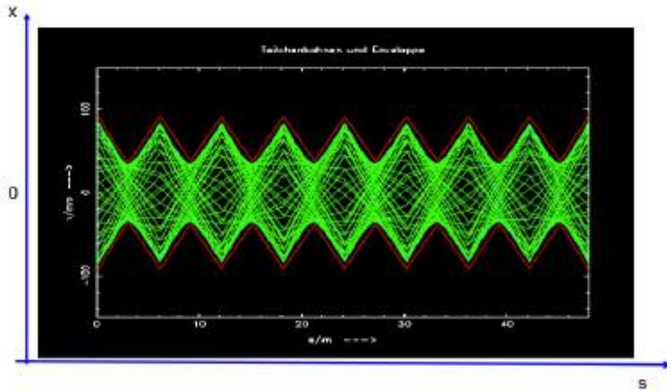


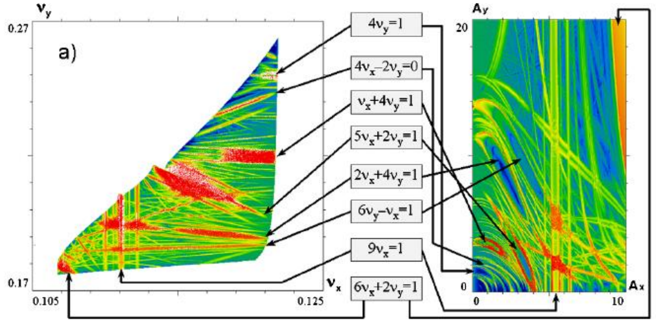
Separate bending and focusing magnets





Linear optics and nonlinear dynamics



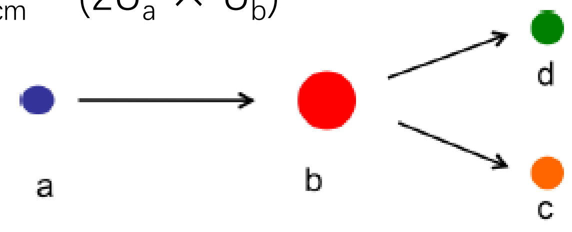


Concept of Collider（equivalent energy ~TeV）

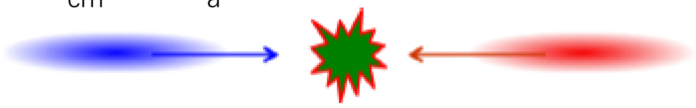
1960，B. Touschek, Italy

When high-energy particles collide with a stationary target (particle), only **the energy in the center-of-mass frame** is the effective energy for particle interaction

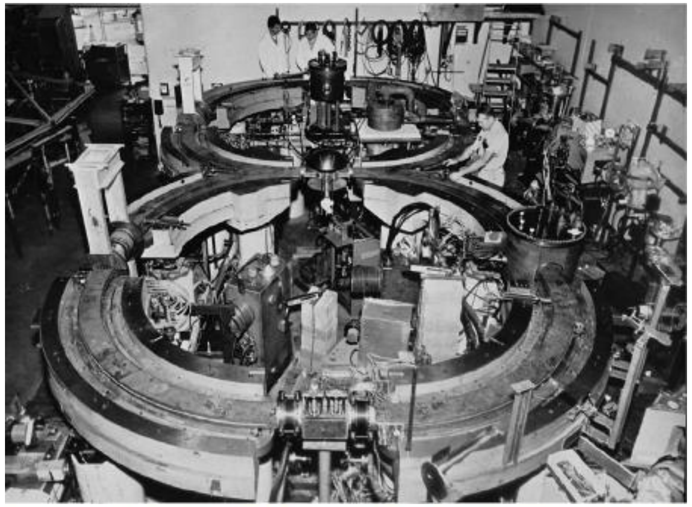
stationary target，
$U_{cm} \sim (2U_a \times U_b)^{1/2}$

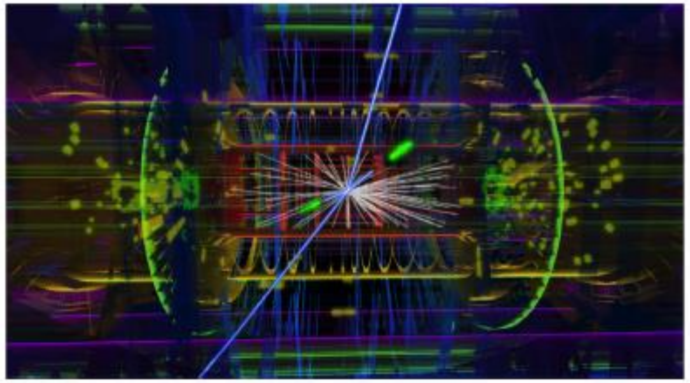


Head-on collision (same energy),
$U_{cm} \sim 2U_a$



A head-on collider with **two 100 GeV accelerator**, is equivalent to
A stationary target collider with **one 20000 GeV accelerator**





LHC, 27 km, 7 TeV, CERN

On July 4, 2012, CERN announced that the CMS and ATLAS detectors on the LHC had discovered a new boson with a mass of 126.5 GeV, which is likely to be the long-awaited Higgs boson for scientists.

Belgian physicist François Englert and British physicist Peter Higgs, who described the Standard Model of particle physics and predicted the fundamental particle—the Higgs boson, which was experimentally discovered on the LHC, were awarded the 2013 Nobel Prize in Physics for their work.

Phase stability, strong focusing, collider concept

Synchrotron Accelerator

Let us see how to design an electron synchrotron, during which we will go through main physics concept and issues. At last we will talk about some special issues in proton and ion synchrotrons.

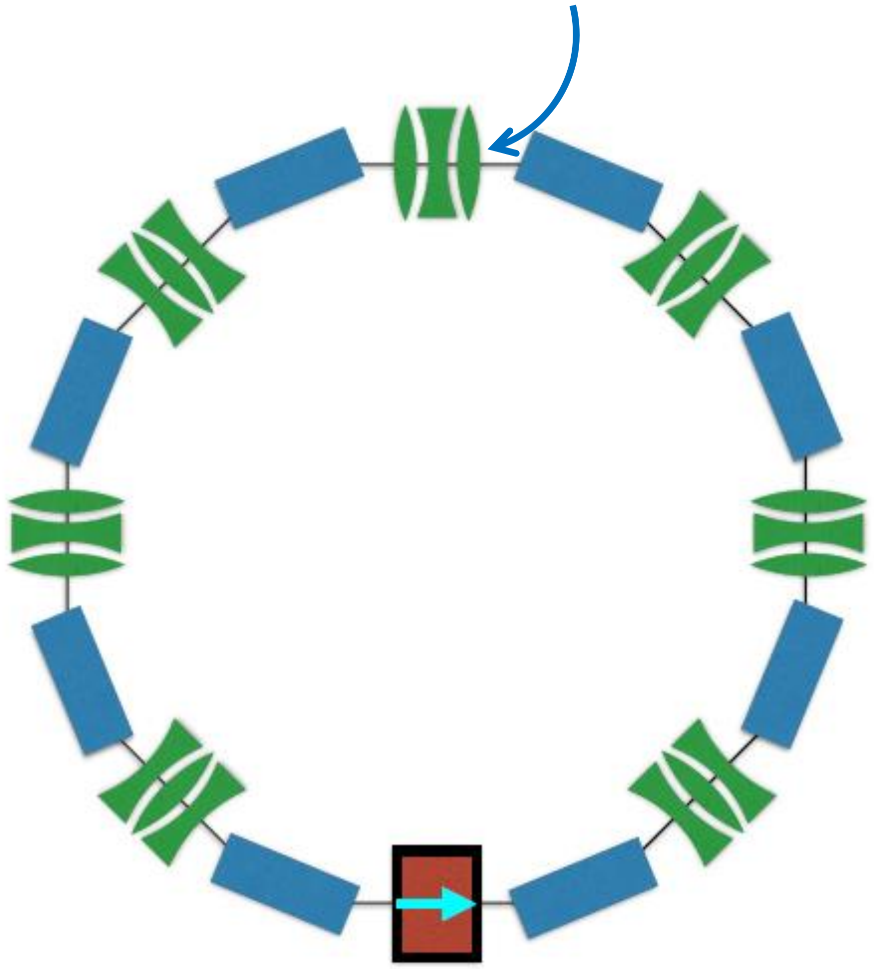
Synchrotron Physics & design

Transverse optics (design)

Energy → Dipole + Quadrupole

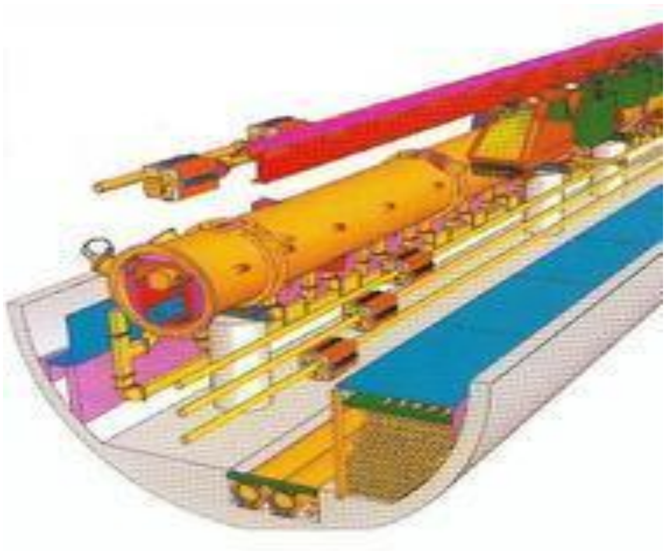
Energy, Dipole → RF cavity

Longitudinal dynamics (design)



Energy, velocity, momentum

Mass-energy equivalence



$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \sqrt{1-1/\gamma^2}$$

$$E = \gamma m_0 c^2 = \gamma E_0 \qquad p = \gamma m_0 \beta c = \beta E / c \qquad T = E - E_0 = (\gamma - 1) m_0 c^2$$

$$1\ \mathrm{eV} = (1.602 \times 10^{-19}\ \mathrm{C})(1\ \mathrm{V}) = 1.602 \times 10^{-19}\ \mathrm{J}$$

$$1\ \mathrm{MeV} = 1.602 \times 10^{-13}\ \mathrm{J}$$

$$1\ \mathrm{GeV} = 1.602 \times 10^{-10}\ \mathrm{J}$$

e- rest mass energy：0.511 MeV （0.511 × 10^6 eV）

Beijing electron positron collider（BEPC）energy：2.5 GeV（2.5 × 10^9 eV），v = 0.999999979 c, γ = 4892

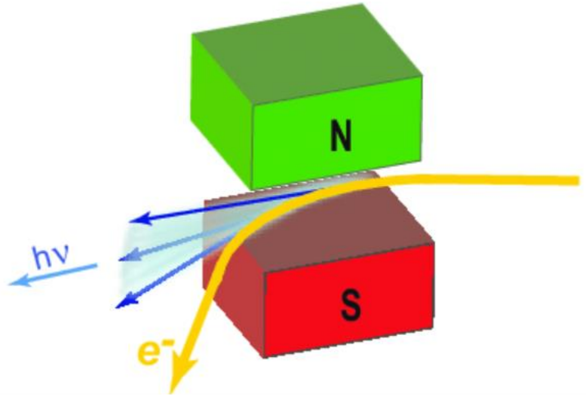
Bending magnet and magnetic rigidity

- For a synchrotron, which comprises a large number of magnets, the most fundamental type of magnet is the **bending magnet or dipole**, used to bend the beam.
- From the beam dynamics' perspective, an important concept is **magnetic rigidity** of the central reference trajectory.
  - It is defined as the ratio of momentum (p) to charge (e); and it reflects how easily a charged particle can be deflected

$$B\rho[T\cdot m]=\frac{p}{e}=\beta\frac{E[GeV]}{0.29979}$$

where β represents the normalized velocity, and E denotes the energy of the charged particle.

From the equation, if the particle's energy is known, Bρ can be calculated; and higher energy corresponds to a larger Bρ. In other words, to accelerate or store particle of a specific energy, a sufficiently large Bρ is required.



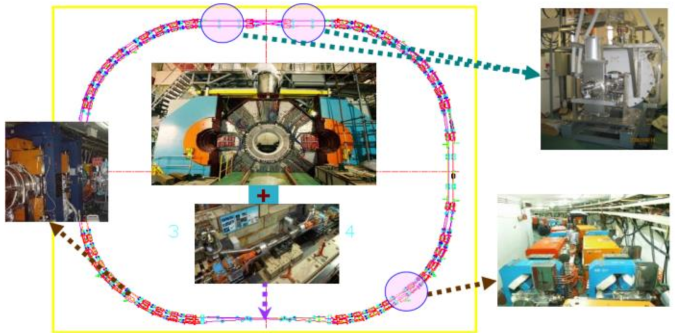
Bending magnet and magnetic rigidity

- Consider a particle passing through a dipole magnet, B_0 is the bending field strength, and the bending radius of the central trajectory ρ_0 is given by

$$\rho_0 = B\rho / B_0 = \beta E\,[\mathrm{GeV}]/0.29979/\ B_0$$

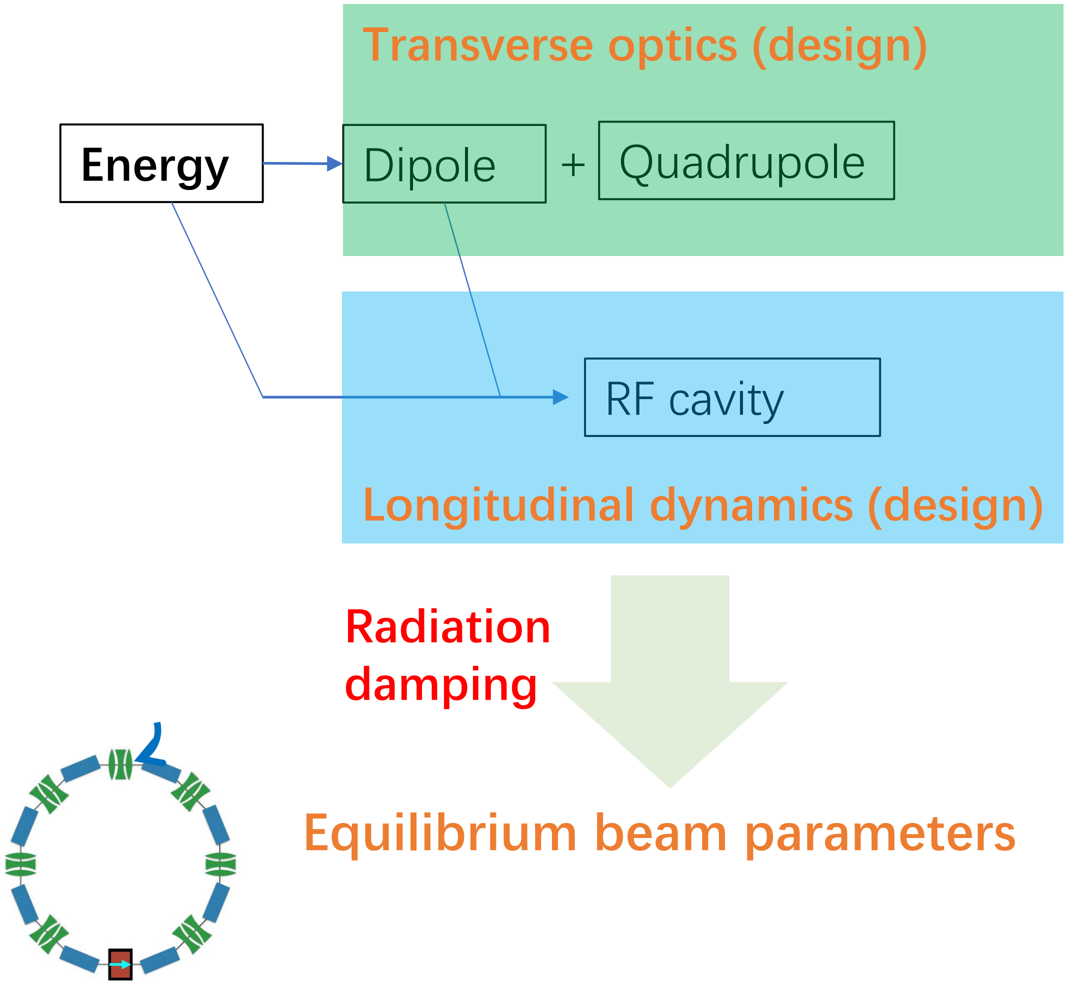
- From this equation, one can **roughly estimate the circumference required to build a synchrotron**
  - (1) In a ring accelerator one must bend the beam by 360 degrees, this requires dipole magnets with a total length of approximately $\rho_0 \times \pi = L_{B,\ total}$.
  - (2) A synchrotron cannot consist solely of dipole; the dipole must be divided into multiple shorter dipole magnets, with quadrupoles separated with a specific distance placed between dipoles to focus the beam in transverse plane.
  - (3) For a reasonable estimation, the circumference needs to be 5 ~ 10 times of $L_{B,\ total}$.

Taking BEPC as an example, for an energy of 2.5 GeV:
$B\rho$ = 2.5 [GeV]/0.29979 = 8.34 T·m;
Practically used bending field is typically close to but less than 1 T, consider B_0 = 0.83 T; we get ρ_0 ~ 10 m; and $L_{B,\ total}$ ~ 31 m. According, the circumference would be roughly 155–310 m.



BEPC storage ring circumference is 240 m

Equilibrium beam parameters



Once a layout of dipole and quadrupole magnets is established, the magnetic focusing structure formed by such an arrangement is referred to as a **lattice**. With this lattice, we can proceed to analyze and design the transverse optics (linear transverse beam dynamics) and longitudinal dynamics.

In fact, in electron synchrotron, synchrotron radiation plays a significant role. As the beam circulates continuously in the accelerator, the emission of synchrotron radiation causes the beam emittance—regardless of its initial value—to converge toward a specific equilibrium value. This equilibrium emittance is commonly known as the **natural emittance**.

Given a lattice of an electron synchrotron, many beam parameters can be basically determined: natural emittance, transverse beam size along the ring, bunch length, energy spread, radiation energy loss per turn, etc.

Note: this is only correct for electron synchrotron.

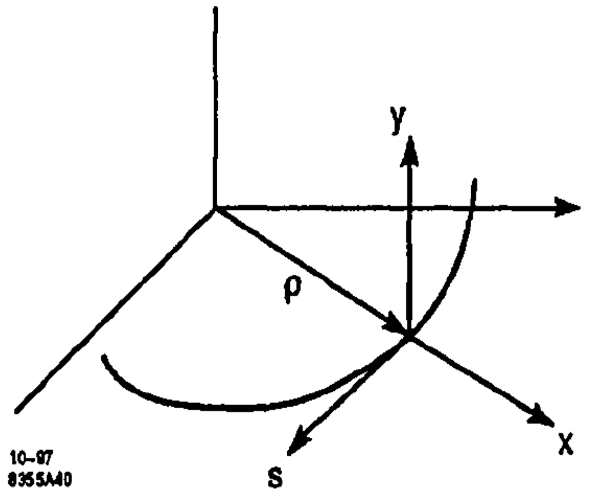
Particle motion

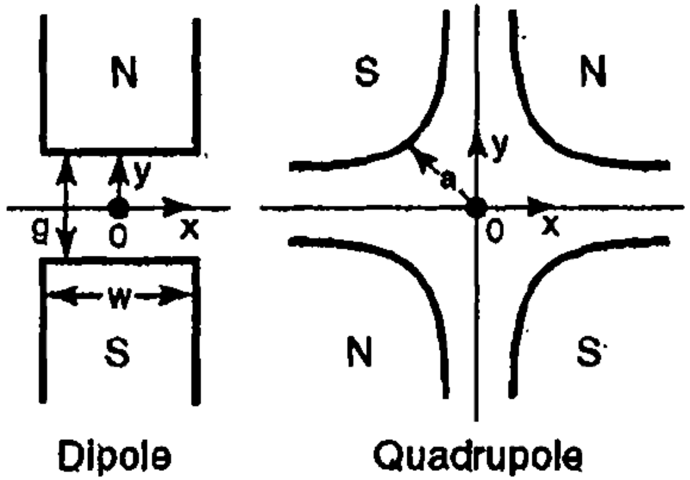
- Transverse dynamics is to study the particle motion at the xy plane. x is the radial direction, and y is the axial direction, s is the longitudinal direction (along the circular orbit in the ring)
- At any position *s* measured along a reference trajectory, a charged particle is represented by a vector (single column matrix) X(s) with X(s) = (x(s),x'(s),y(s),y'(s),z(s),δ).
- When a particle starts from s0 to s1, by passing through an element (dipole or quadrupole), the vector will be transformed from X(s0) to X(s1), described by

$$X(s1) = RX(s0)$$

where R is 6X6 **transfer matrix** characterizing each magnet or drift between two adjacent magnets.

- We assume the transverse and longitudinal motion is decoupled from each other, then we can deal with them separately.

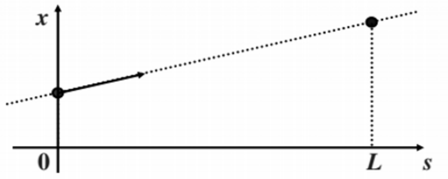




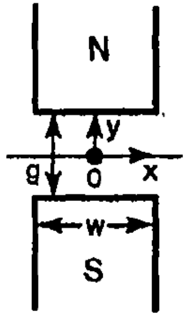
Transfer matrix

$$\mathbf{R}_{\text{drift}} = \begin{bmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

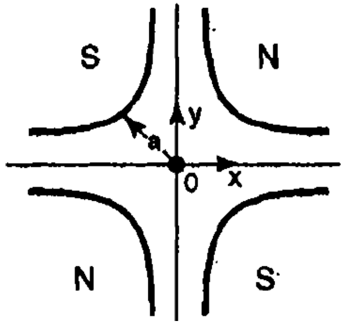
Drift
L is the drift length



Drift



Dipole



Quadrupole

$$\mathbf{R}_{\text{dip}} = \begin{bmatrix} \cos\theta & \rho\sin\theta & 0 & 0 \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & \rho\theta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Dipole
ρ is the bending radius, θ is the bending angle

$$\mathbf{R}_{\text{QF}} = \begin{bmatrix} \cos\sqrt{k_1}L & \frac{\sin\left(\sqrt{k_1}L\right)}{\sqrt{k_1}} & 0 & 0 \\ -\sqrt{k_1}\sin\left(\sqrt{k_1}L\right) & \cos\left(\sqrt{k_1}L\right) & 0 & 0 \\ 0 & 0 & \cosh\sqrt{k_1}L & \frac{\sinh\left(\sqrt{k_1}L\right)}{\sqrt{k}} \\ 0 & 0 & \sqrt{k_1}\sinh\left(\sqrt{k_1}L\right) & \cosh\sqrt{k_1}L \end{bmatrix}$$

Focusing quadupole (in x plane, thick length)
L is the quadrupole length, $k_1 = G/B\rho$, *G* is the quadrupole gradient (in unit of T/m)

$$\mathbf{R}_{\text{thin QF}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & 1 \end{bmatrix}$$

Focusing quadupole (in x plane, thin-length approx.)
$1/f = k_1 L = GL/B\rho$

One-turn transfer matrix, Twiss parameters, tune

- Starting from an arbitrary location of the ring, sequentially multiplying the matrices of all elements encountered during the one-turn circulation, one can obtain the **one-turn transfer matrix** (consider only one plane), that can be written in terms of the Twiss parameters,

$$\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \cos\Delta\psi_C + \alpha\sin\Delta\psi_C & \beta\sin\Delta\psi_C \\ -\gamma\sin\Delta\psi_C & \cos\Delta\psi_C - \alpha\sin\Delta\psi_C \end{pmatrix}$$

- From one-to-one correspondence of the above equation, one can obtain the Twiss or Courant-Snyder parameters (β, α, γ) of this location, and also the one-turn phase advance (tune*2π).

$$\beta = \frac{b}{\sin\Delta\psi_C} \qquad \alpha = \frac{a-d}{2\sin\Delta\psi_C} \qquad \gamma(s) \equiv \frac{1+\alpha^2}{\beta} \qquad \cos\Delta\psi_C = \frac{1}{2}(a+d) = \frac{1}{2}\mathrm{Tr}\,\mathbf{M}$$

Note: there are also transfer matrices of Twiss parameters for different elements, that can be derived based on the transfer matrix of coordinates

Beta function

- The transverse motion (betatron motion) of can be expressed as

$$x(s) = A\sqrt{\beta(s)}\cos[\psi(s)+\delta]$$

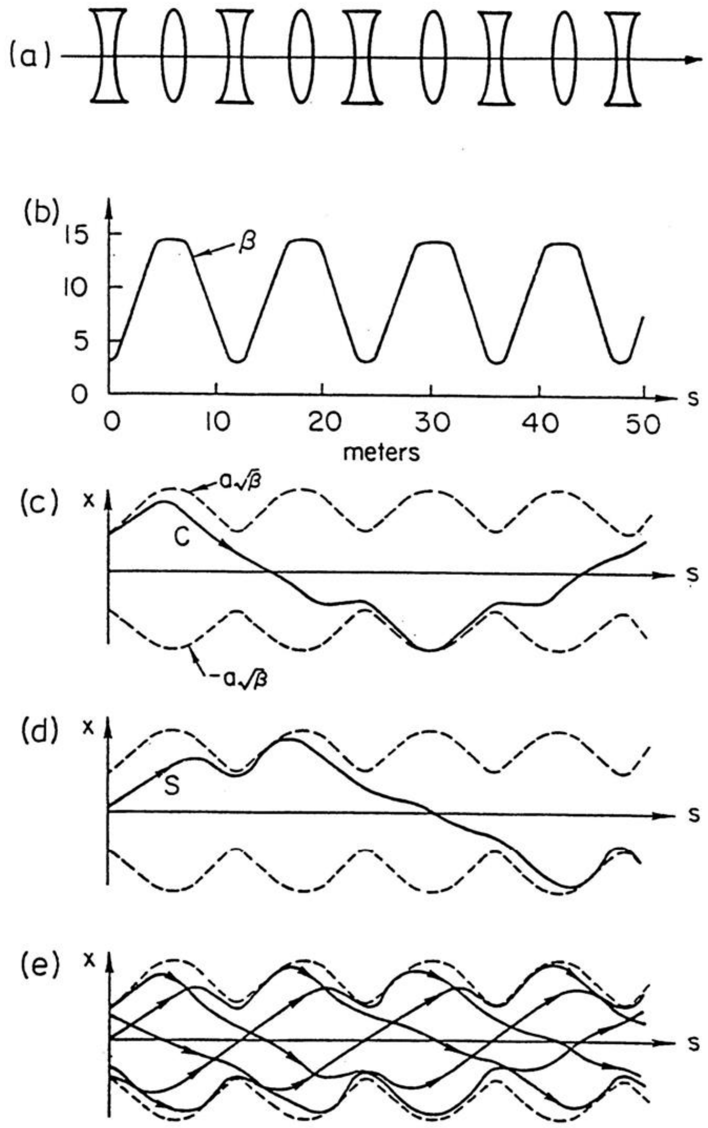
- **The beta function is referred to as the beam envelope function**, which characterizes the maximum amplitude of transverse particle oscillations at different positions for a specific lattice. This function is one of the fundamental parameters in single-particle beam dynamics.

The **transverse beam size** at any point can be expressed as a constant multiplied by the square root of the beta function at that location, $\boldsymbol{\sigma = A\beta^{1/2}}$.

The phase advance per revolution (or **the tune**, i.e., the number of transverse oscillation periods per turn) is given by the integral of the inverse of the beta function along the longitudinal direction s, divided by 2π.

$$\nu \equiv \frac{1}{2\pi}\oint \frac{ds}{\beta(s)}$$

A smaller beta function results in a smaller beam size and a larger phase advance, indicating stronger transverse focusing.



Betatron motion in a FODO lattice

Dispersion function

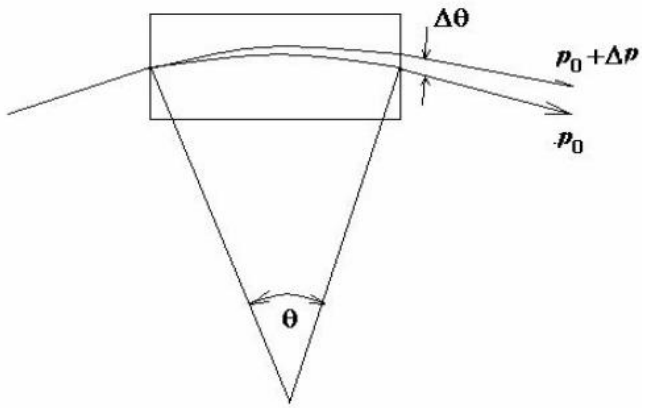
- In a beam composed of a group of particles, not all the particles are exactly at the design energy. For the particle with small energy deviation, their trajectory differs slightly from that of reference particles. The trajectory deviation is proportional to the energy deviation, and the ratio between the trajectory deviation and the energy deviation is named dispersion.

$$x(s)=\sqrt{\beta_x(s)\varepsilon}\cos(\psi+\delta)+D_x(s)\frac{\Delta p}{p}$$

- The dispersion function is from the bending of the particles in dipole magnets.

Consider a particle with energy slightly higher than the design energy. When bent by a dipole magnet, it is more difficult to deflect and therefore undergoes a smaller bending angle. As a result, its transverse position at the exit of the dipole magnet will differ from that of an on-energy particle.

We typically construct the ring in a horizontal plane and primarily use horizontal-bending dipole magnets to bend the beam. As a result, in a idealized lattice we **usually have only non-zero dispersions in horizontal plane**.



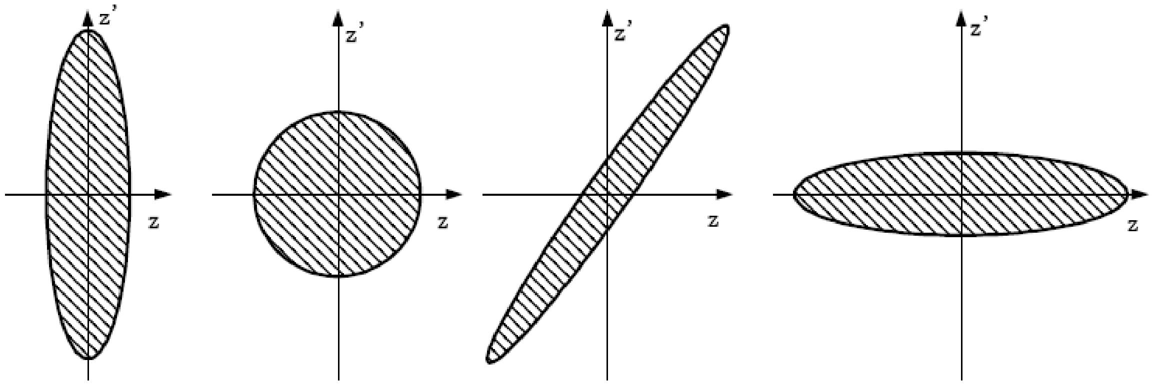
Note: there are also transfer matrices of dispersion functions for different elements, that can be derived based on the transfer matrix of coordinates.

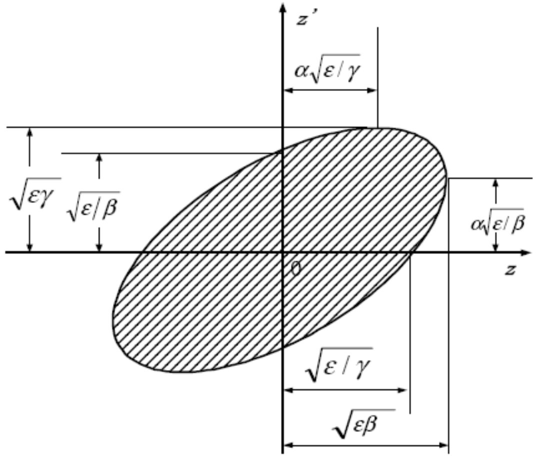
Emittance

- In the transverse motion equation, the constant parameter A has physical meaning,

$$x(s) = A\sqrt{\beta(s)}\cos[\psi(s)+\delta]$$

- The transverse beam motion in phase space is in an ellipse. This ellipse varies in both profile and orientation at different locations along the ring, but the ellipse area remains constant, $S = \pi A^2$. The **emittance** is defined as $\varepsilon = S/\pi = A^2$.





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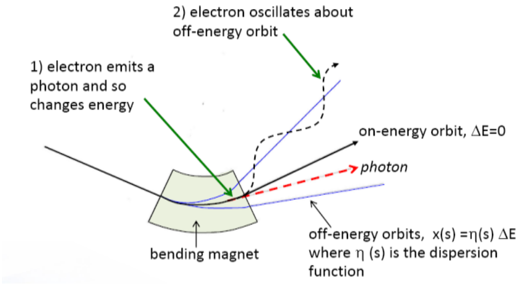
- **In an electron ring accelerator**, as the result of the balance between quantum excitation and synchrotron radiation damping, the beam emittance will reach an equilibrium value, which is called **natural emittance**.
- The natural emittance is only related to the dispersion function (& beta) inside the dipole.

Simple explanation

- Emittance is driven by randomness of photon emission in presence of dispersive (energy-dependent) orbits
- Breaking up dipoles and putting focusing (quadrupoles) between the parts allows tightly controlling the magnitude of dispersive orbits



$$\varepsilon_x = C_q \frac{\gamma^2}{J_x} \frac{\oint H(s)/\rho(s)^3\, ds}{\oint 1/\rho(s)^2\, ds}$$

$$H = \gamma_x \eta_x + 2\alpha_x \eta_x \eta'_x + \beta_x \eta'^2_x$$

Minimum natural emittance of different style lattice

Lattice style	Minimum emittance	Conditions/comments
90° FODO	$\varepsilon_0 \approx 2\sqrt{2} C_q \gamma^2 \theta^3$	$\frac{f}{L} = \frac{1}{\sqrt{2}}$
137° FODO	/2.36 $\varepsilon_0 \approx 1.2 C_q \gamma^2 \theta^3$	minimum emittance FODO
DBA	/18.6 $\varepsilon_0 \approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3$	$\eta_{x,0} = \eta_{px,0} = 0$ $\beta_{x,0} \approx \sqrt{12/5}L \quad \alpha_{x,0} \approx \sqrt{15}$
TME	/3 $\varepsilon_0 \approx \frac{1}{12\sqrt{15}} C_q \gamma^2 \theta^3$	$\eta_{x,\min} \approx \frac{L\theta}{24} \quad \beta_{x,\min} \approx \frac{L}{2\sqrt{15}}$

Longitudinal motion

- In each revolution in the ring, particle loses a certain amount of energy due to synchrotron radiation, denoted by U_0, which is then compensated by the radio-frequency (RF) cavity (its voltage is V_{rf}) at a specific location in the ring to maintain the beam energy.

$$e\, V_{rf} \sin(\varphi_s) = U_0$$

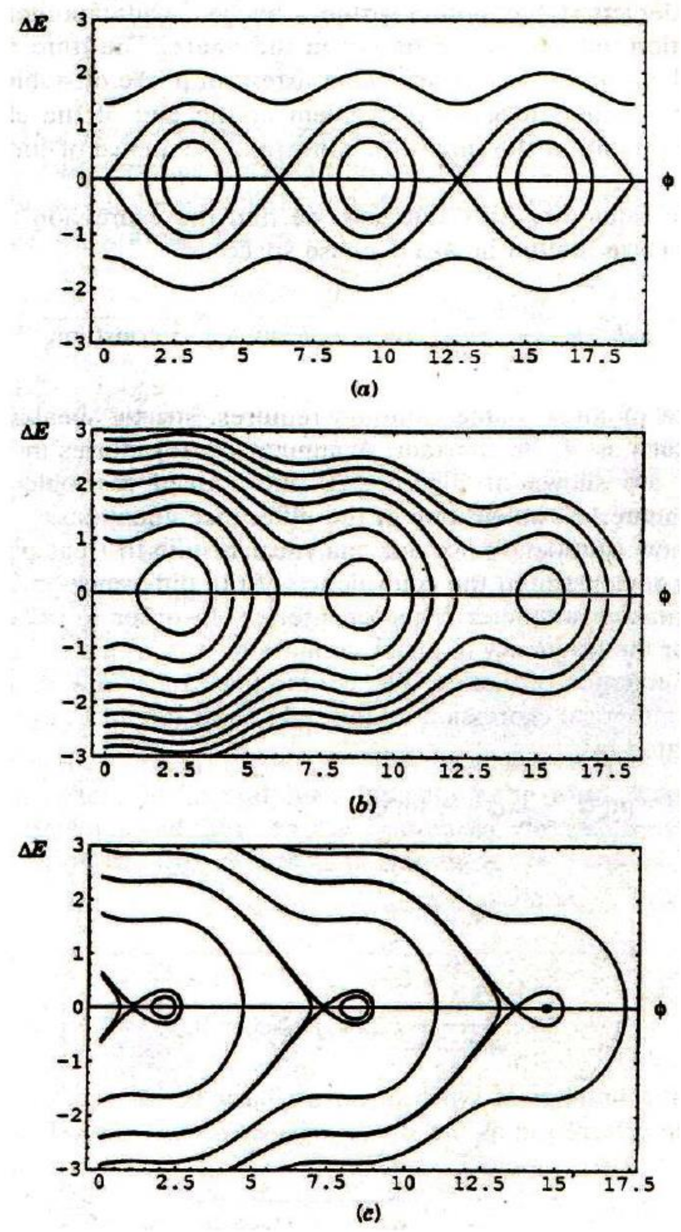
where φ_s is the **synchronous phase**, the RF phase seen by the idealized synchronous particle as it traverse the cavity.

- For an electron ring that has a uniform bending field, the energy loss per turn is given by

$$U_0\,[\mathrm{keV}] = 88.5\, E^4[\mathrm{GeV}]/\rho_0\,[\mathrm{m}]$$

- If a particle does not arrive at the RF cavity at the exact specified time, its longitudinal position (RF phase φ at the arrive time) and energy deviation ΔE will oscillate.
- For small deviations, this oscillation is stable and periodic. The stable area is called **bucket**.

Note: U_0 is related only to dipole parameters.



Stable area (bucket) with $\varphi_s = \pi$, $5\pi/6$, and $2\pi/3$

Longitudinal motion

- For small-amplitude longitudinal oscillation, the tune is given by

$$\nu_s = \sqrt{-\frac{\eta heV\cos\phi_s}{2\pi E_s}} \approx \sqrt{-\frac{\alpha_p heV\cos\phi_s}{2\pi E_s}}$$

The approximation is correct for high-energy electron synchrotron

- The longitudinal tune is determined by RF relevant parameters, RF voltage (V), synchronous phase (φ_s), and harmonic number (ratio between RF frequency and revolutionary frequency), and also the momentum compaction factor α_p.
- The momentum compaction factor α_p describes the difference in path length between particles with momentum (or energy) deviations and the design orbit. Its specific value is determined by the lattice and transverse beam dynamics.

$$\alpha_0 = \frac{1}{C_0}\oint \frac{\eta_0}{\rho_0}\,ds$$

where η_0 is the dispersion

- In electron ring accelerators, due to combined effects of radiation damping and quantum excitation, the bunch length and energy spread will reach equilibrium values.

$$\sigma_\varepsilon = \frac{\sigma_E}{E_0} = \gamma\sqrt{\frac{C_q}{J_\varepsilon \rho_0}}, \qquad \sigma_l = \frac{\alpha_p \overline{R}\sigma_\varepsilon}{\nu_s}$$

where Cq = 3.832×10^{-13} m, Je ~2 is the longitudinal partition number, the bending field is assumed to be constant, and $\overline{R}$ is average ring radius.

Radiation integrals

- Particle beam parameters in a electron storage ring are modified by the emission process of synchrotron radiation. These effects are governed by **radiation integrals**.
- Many beam parameters can be obtained with these integrals

$$\mathcal{I}_1[\mathrm{m}] = \oint \left(\frac{D_x}{\rho_x} + \frac{D_y}{\rho_y}\right) ds$$

$$\mathcal{I}_2[\mathrm{m}^{-1}] = \oint \left(\frac{1}{\rho_x^2} + \frac{1}{\rho_y^2}\right) ds$$

$$\mathcal{I}_3[\mathrm{m}^{-2}] = \oint \left(\frac{1}{|\rho_x|^3} + \frac{1}{|\rho_y|^3}\right) ds$$

$$\mathcal{I}_{4u}[\mathrm{m}^{-1}] = \begin{cases} \oint \frac{D_u}{\rho_u^2}\left(1 \pm 2\rho_u^2 k\right) ds \\ \pm \oint 2\frac{D_u k}{\rho_u} ds \end{cases}$$

$$\mathcal{I}_{5u}[\mathrm{m}^{-1}] = \oint \frac{\mathcal{H}_u}{|\rho_u|^3} ds$$

$$\mathcal{I}_{6u}[\mathrm{m}^{-1}] = \oint k^2 D_u^2 \, ds$$

$$\mathcal{H}_u(s) = \beta_u D_u'^2 + 2\alpha_u D_u D_u' + \gamma_u D_u^2, \; u = x \text{ or } y$$

Momentum compaction $\alpha_p = \frac{I_1}{C}$,

Energy loss per turn $U_0 = \frac{2r_e E_0^4}{3(m_0c^2)^3} I_2$,

Equilibrium parameters

$$\left(\frac{\sigma_\varepsilon}{E_0}\right)^2 = C_q\gamma^2 \frac{I_3}{2I_2 + I_4} = \frac{C_q\gamma^2}{J_\varepsilon} \cdot \frac{I_3}{I_2},$$

$$\varepsilon_{x0} = C_q\gamma^2 \frac{I_5}{I_2 - I_4} = \frac{C_q\gamma^2}{J_x} \cdot \frac{I_5}{I_2},$$

Damping partition numbers

$$J_x = 1 - \frac{\mathcal{I}_{4x}}{\mathcal{I}_2}, \quad J_y = 1 - \frac{\mathcal{I}_{4y}}{\mathcal{I}_2}$$

$$J_s = 2 + \frac{\mathcal{I}_{4x} + \mathcal{I}_{4y}}{\mathcal{I}_2}$$

Handbook of Accelerator Physics and Engineering, 3rd printing, Section 3.1, edited by A.W. Chao, M. Tigner
Note: nowadays, many accelerator modeling programs can calculate these integrals and equilibrium parameters quickly.

Synchrotron Physics & design

Energy

Transverse optics (design)

Dipole + Quadrupole

RF cavity

Longitudinal dynamics (design)

Modification

Injection (design)

Sextupoles
Nonlinear dynamics (optimization)

Essentially required measures

Error effects

Collective effects

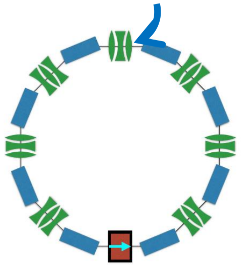
Interaction w/ environment

Self-effects
IBS/space charge

Perturbation

Though undesirable, should be addressed.

Radiation damping



Equilibrium beam parameters

Through construction of a linear lattice and the calculation of its associated parameters, we can gain a general understanding of the beam motion and beam parameters in a synchrotron under first-order approximation.

As a complex system, an accelerator requires the consideration of necessary **modification** terms and **perturbation** terms to accurately model the actual machine.

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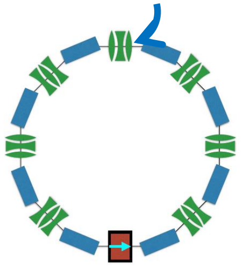

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Next we first talk about one “modification” term, Chromaticity correction and nonlinear dynamics

Chromaticity correction

- As mentioned earlier, energy deviation affects the transverse trajectory of particles (orbit). To account for this, the dispersion function was introduced.

- Similarly, since energy deviation also influences the transverse oscillation frequency (tune) of particles, we need to introduce the concept of **chromaticity**.It is defined as the ratio of the change in tune to the relative energy deviation:

$$\xi = \Delta\upsilon \,/\, (\Delta E/E)$$

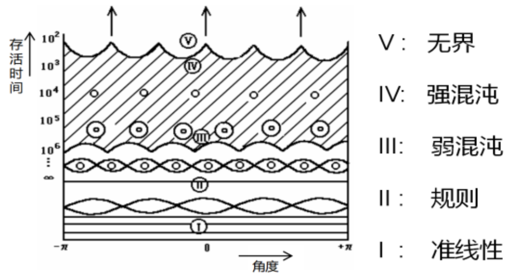
- The inherent chromaticity of a linear lattice (with only dipoles & quads), known as the **natural chromaticity**, **is always negative**. Generally, the stronger the focusing, the greater the absolute value of the natural chromaticity.

- A negative natural chromaticity can lead to beam instabilities (e.g., the head-tail instability). Therefore, it must be corrected to a slightly positive value.
- To achieve this, chromaticity correction **sextupole magnets** are introduced—at least two families, one focusing and one defocusing.

Nonlinear dynamics

- Introducing sextupoles for chromaticity correction is necessary but has side effects. The kick the particle experienced is not linear but quadratic with the oscillation amplitude, which introduces nonlinearity to the particle motion.
  - Sextupoles can excite third-order or even higher-order **resonances**, and lead to a reduction in the **dynamic aperture**.

Dynamic aperture: in presence of nonlinear perturbations, the motion of particles exhibits linear or periodic behavior within only a limited range. Beyond a certain boundary, the motion becomes unbounded, with rapidly increasing amplitude. This boundary is referred to as the dynamic aperture or, more generally, the dynamic acceptance.



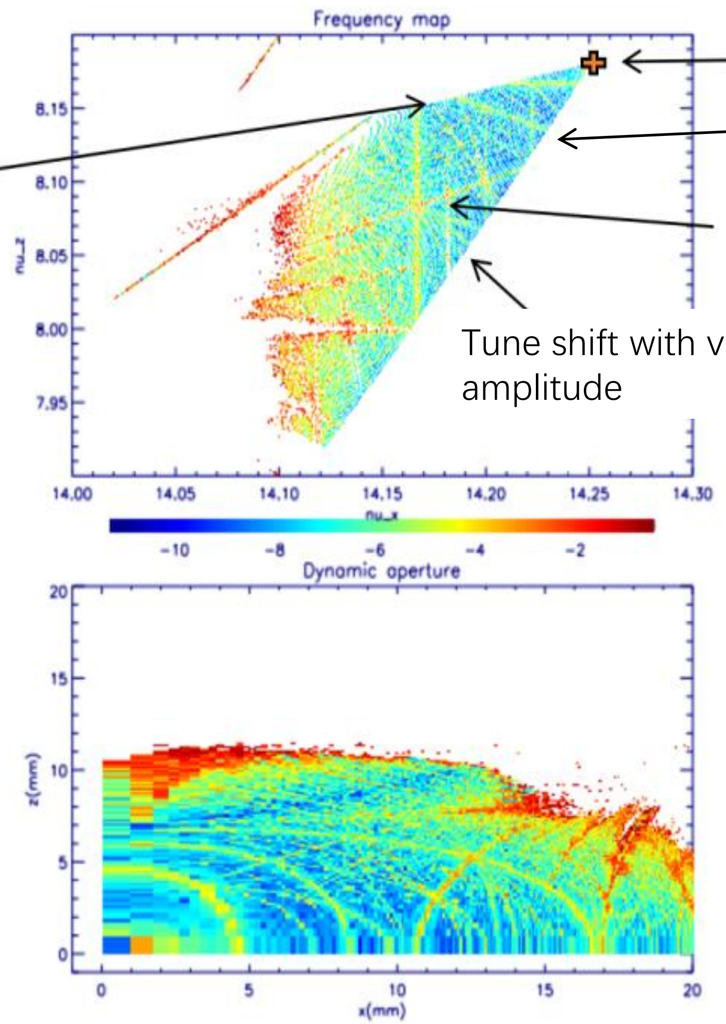
- With the pursuit of higher beam quality **in modern accelerators**—for example, the fourth-generation synchrotron radiation light sources—the strong nonlinearities induced by sextupoles have made **nonlinear dynamics analysis and optimization a critical issue in accelerator physics design and studies**.

Note: In a linear lattice, dipole and quadrupole magnetic field errors excite first-order (integer) and second-order (half-integer) resonances, respectively.

Nonlinear dynamics analysis and optimization

- Related research can generally be divided into analytical and numerical approaches.
- Analytical approach
  - Over the past few decades, accelerator physics has extended Courant-Snyder analysis of the linear dynamics to nonlinear dynamics, by introducing perturbation theory from classical mechanics, Lie algebra from mathematics, and differential algebra into nonlinear beam dynamics analysis.
  - With **Lie Algebra**, one can derive the **one-turn map** including the nonlinear perturbations for analysis of the nonlinearity of a lattice (and for particle tracking in 1980 to 1990s). With the one-turn map and **normal form analysis**, one can compute higher-order chromatic terms, tune shifts with amplitudes, resonance driving terms, and other nonlinear driving terms.
- Numerical approach
  - The primary approach involves numerical simulation of particle motion to extract information about dynamical stability. A successful example is **frequency map analysis** (FMA).
  - The main methodology of FMA is based on an improved Fourier analysis technique, which precisely calculates the fundamental frequencies of motion over a finite time interval and establishes a mapping from amplitude space to frequency space. This allows studying the stability of conservative dynamical systems in frequency space rather than in amplitude space.

Frequency map analysis (FMA)



Nominal tune

Tune footprint of the particle motion in tune space

Tune shift with horizontal amplitude

resonances

Tune shift with vertical amplitude

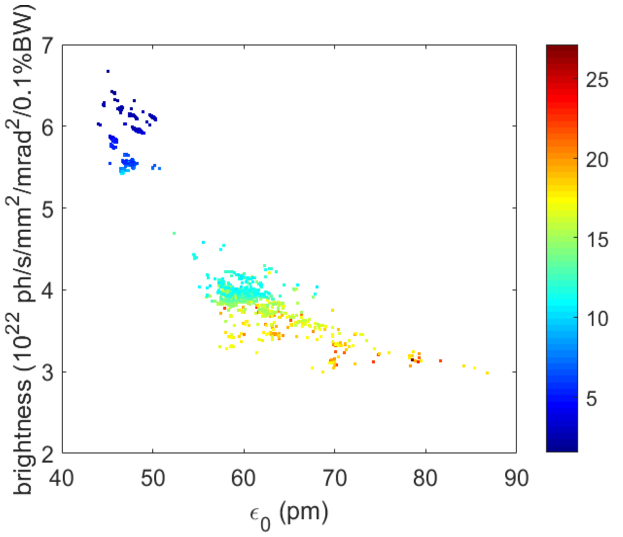
The FMA method provides clear and intuitive visualization of single-particle transverse dynamics in frequency space, including the nominal tune, amplitude-dependent tune shift, and resonance effects.

Based on the obtained information, one can evaluate the stability of the dynamical system, identify factors limiting beam dynamics, and look for ways of optimizing the dynamic aperture.

Nonlinear dynamics optimization

- Analytical approach
  - By selecting an appropriate phase advance between sextupoles of the same family, the nonlinear driving terms can be partially or completely canceled to a certain extent. This approach enables us to obtain a good enough nonlinear performance.
- Numerical approach
  - Accelerator experts have developed numerical optimization methods that integrate numerical simulations, parallel computing, and stochastic optimization algorithms (e.g., multi-objective genetic algorithm, MOGA). This approach can incorporate complex models including errors and synchrotron radiation effects, and enables precise simulation of various performance parameters such as dynamic aperture and beam lifetime. The results obtained through this method is more closely reflect the actual performance of the machine.

Optimization example:
For a light source, using MOGA to obtain many candidate designs with different natural emittance (x axis), brightness (y axis), and dynamic aperture (marked in different color).



Note: a new tendency is to incorporate **machine learning** to improve the optimization efficiency and performance.

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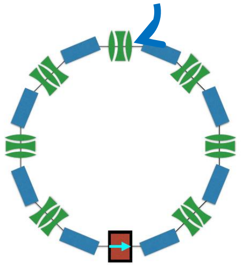
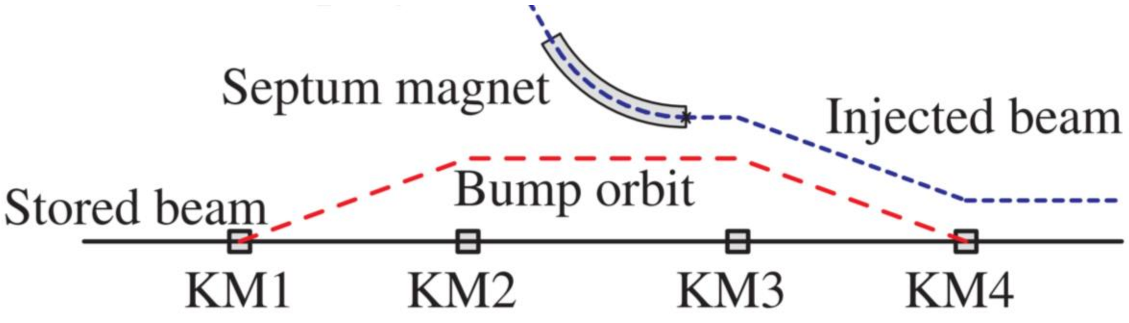


Equilibrium beam parameters

Next we will talk about another "modification" term, injection

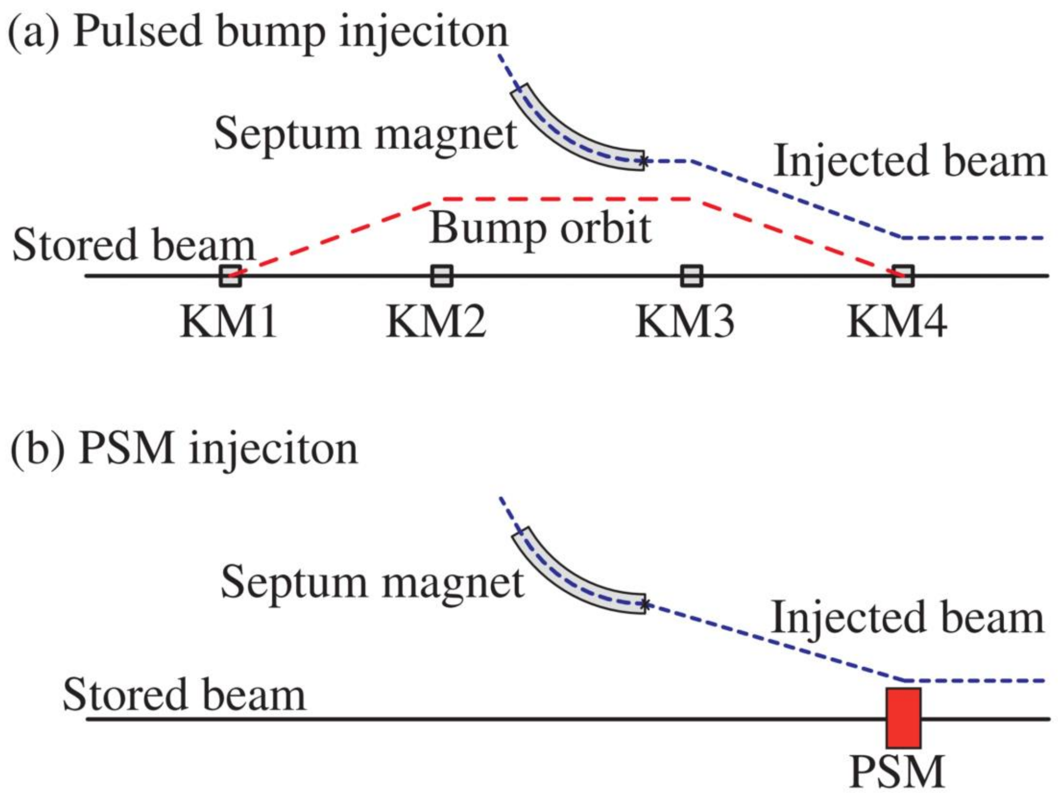
Injection to the synchrotron

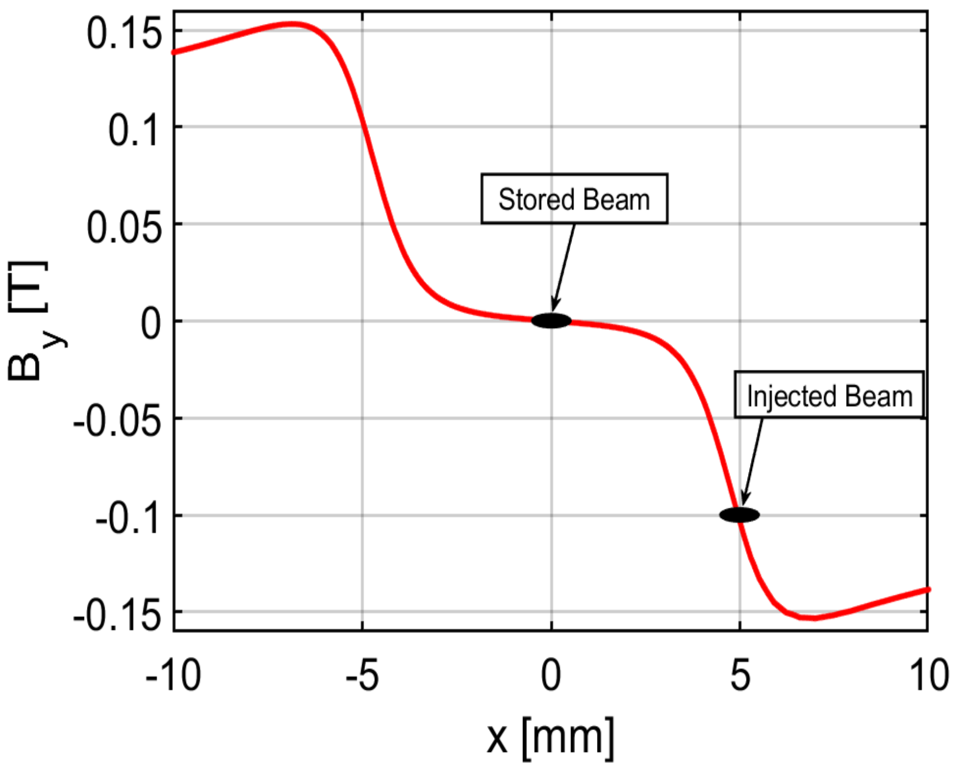
- Synchrotrons typically require an injector and a carefully designed injection scheme to continuously send particle beams into the ring.
  - The fundamental requirements are to ensure the highest possible injection efficiency while minimizing perturbations to the stored beam.
  - For electron synchrotron, the injection schemes can be roughly divided to two kinds: off-axis and on-axis injection.
- For electron synchrotrons, the most commonly used method is **pulsed bump injection**.
  - This scheme employs three or four pulsed magnets to generate a transient orbit bump, which guides the injected beam into the dynamic acceptance of the storage ring.
  - After injection, the pulsed bump disappear, allowing the injected beam to be inside the chamber of the storage ring.
  - With the help of radiation damping effects, the injected beam gradually merges with the stored beam.



A drawback of this injection scheme is that non-ideal design of the pulsed magnets can induce non-negligible residual oscillations of the stored beam.

Off-axis with nonlinear kicker

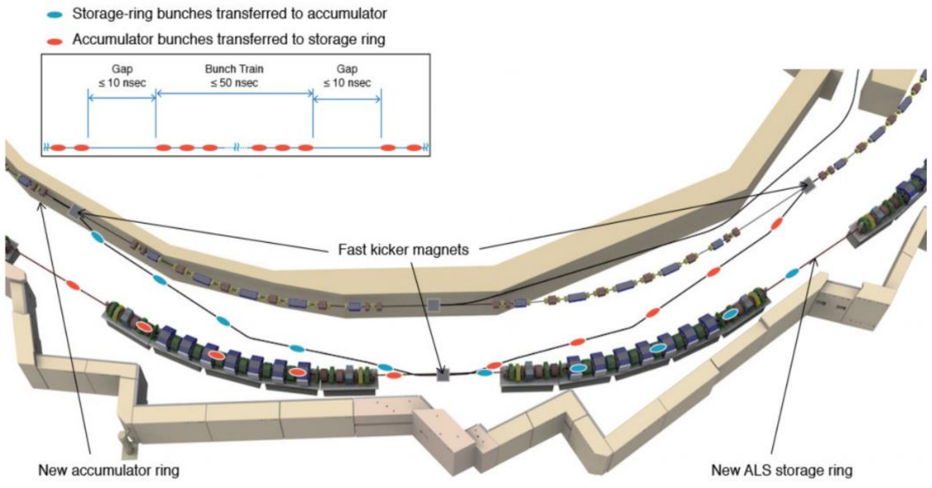


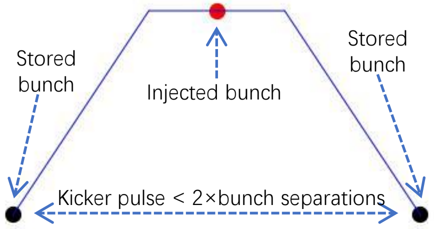


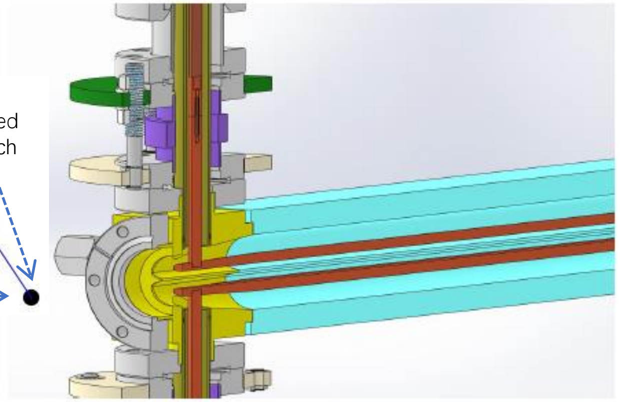
Pulsed nonlinear kicker (e.g., pulsed sextupole) injection
The injected beam is injected at an off-axis position where it experiences a strong kick, directing it into the dynamic acceptance of the storage ring. While the stored beam pass the center of the kicker and experience a "zero" kick and thus a reduced perturbation to the stored beam.

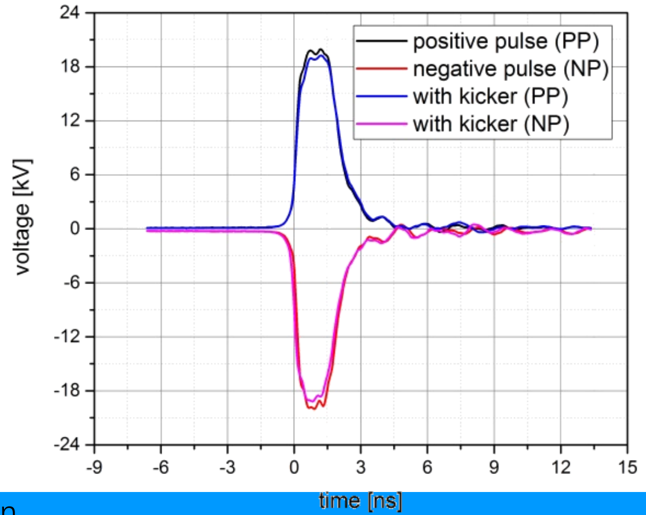
On-axis injection

- For off-axis injection, sufficiently large DA (> 5mm or 10 mm) is essentially required.
- In past few years, experts developed on-aixs schemes that is suitable for injection with small DA (1 mm order).









➢**On-axis, transverse (swap-out)**

- Perturbation is low.
- Non-accumulation, swap-out
- **Fast kicker（duration < 10ns）**
- Full-charge requirement for the injector

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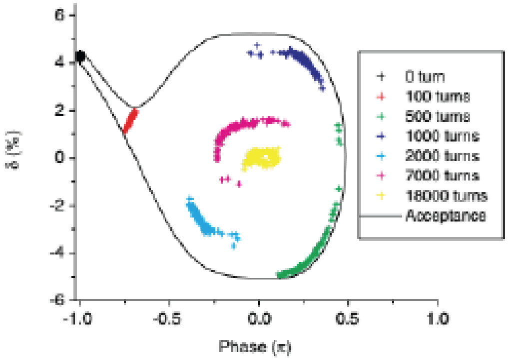
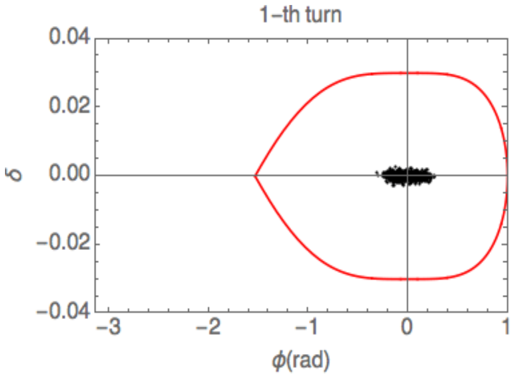
Single RF frequency

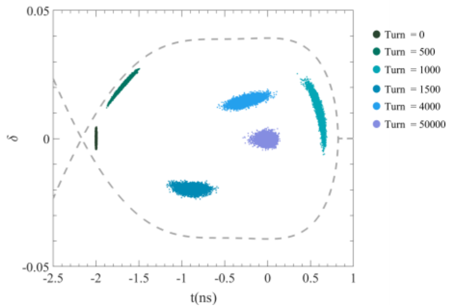


Double RF frequency &
RF gymnastics

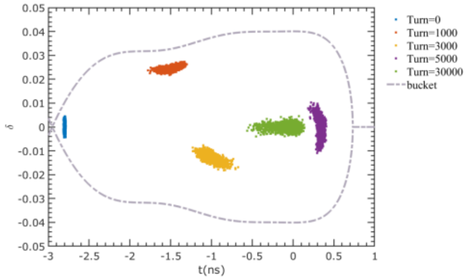


➢**On-axis, transverse (swap-out)**
- Perturbation is low.
- Non-accumulation, swap-out
- **Fast kicker（duration < 10ns)**
- Full-charge requirement for the injector

➢**On-axis, longitudinal**
- **Capable of accumulation**
- Requires large MA
- Even faster kicker (< 5ns)



Double RF frequency



Triple RF frequency

Synchrotron Physics & design

Energy

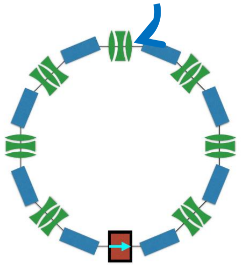
Transverse optics (design)

Dipole + Quadrupole

RF cavity

Longitudinal dynamics (design)

Radiation damping



Equilibrium beam parameters

Modification

Injection (design)

Sextupoles
Nonlinear dynamics (optimization)

Essentially required measures

Error effects

Collective effects

Interaction w/ environment

Self-effects
IBS/space charge

Perturbation

Though undesirable, should be addressed.

Next we will talk about one "perturbation" term, error effects

Error effects

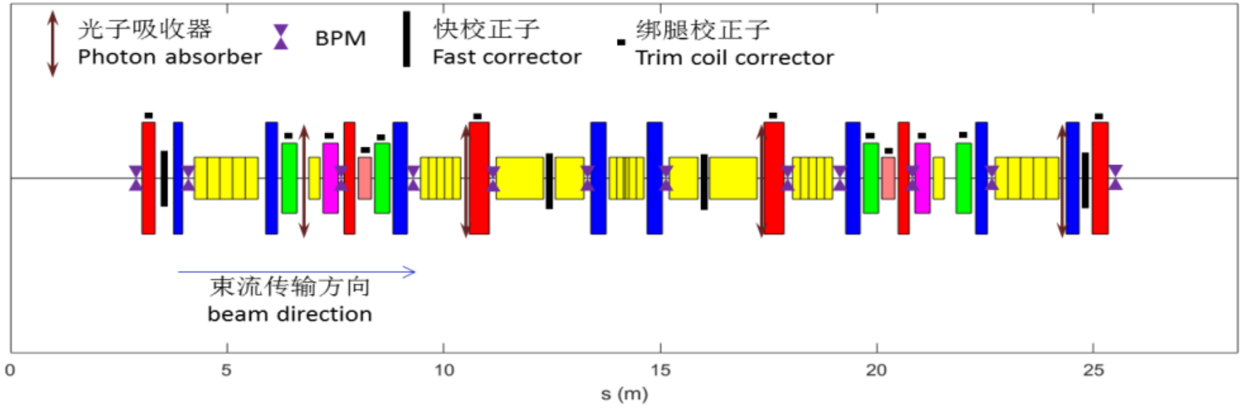
- Compare with the design where all components are assumed to be ideal , **practical accelerators inevitably exhibit various electrical and magnetic errors** due to mechanical, manufacturing, material, and installation imperfections. In the worst case, these errors can significantly degrade beam quality and machine performance.
  - On one hand, control the error source, and on the other hand, correct the error effects.
- These errors can be roughly categorized into **static** and **dynamic** types.
  - Static errors: magnet position/magnetic field deviations caused in process of magnet fabrication, installation, due to precision of driven power supply, and so on.
  - Dynamic errors: ground vibration and induced magnet vibration (1-200 Hz) due to **human activities** around accelerators, as well as **operational equipment** such as air conditioning systems, vacuum pumps, and cooling water, and also **high-frequency ripple in current of magnet-driven power supplies** (in China, 50 Hz and its harmonics).

Error effects and correction

- Static error effects:
  - Actual beam orbit to deviate from the designed one. When the deviation is so large that the amplitude exceeds the physical aperture (e.g., the vacuum chamber aperture), beam will get lost.
  - Discrepancies between the actual Twiss parameters/dispersion functions and their design values. This may cause tune deviation, growth in emittance and beam size in some locations.
- Correction of static error effects:
  - Typically, install correctors between existing magnets (dipole/quadrupoles) to correct orbit distortions.
  - Fine-tuning the currents of quadrupole power supplies to bring the actual optical parameters closer to their design values.
  - The widely adopted method is to measure the Jacobian matrix (also known as the **response matrix**) between the actual orbit and corrector strengths.
  - Perform global orbit correction based on **Singular Value Decomposition (SVD)** of response matrix, while use **LOCO (Linear Optics from Closed Orbit)** algorithm to determine optimal adjustments for quadrupole magnet strengths.

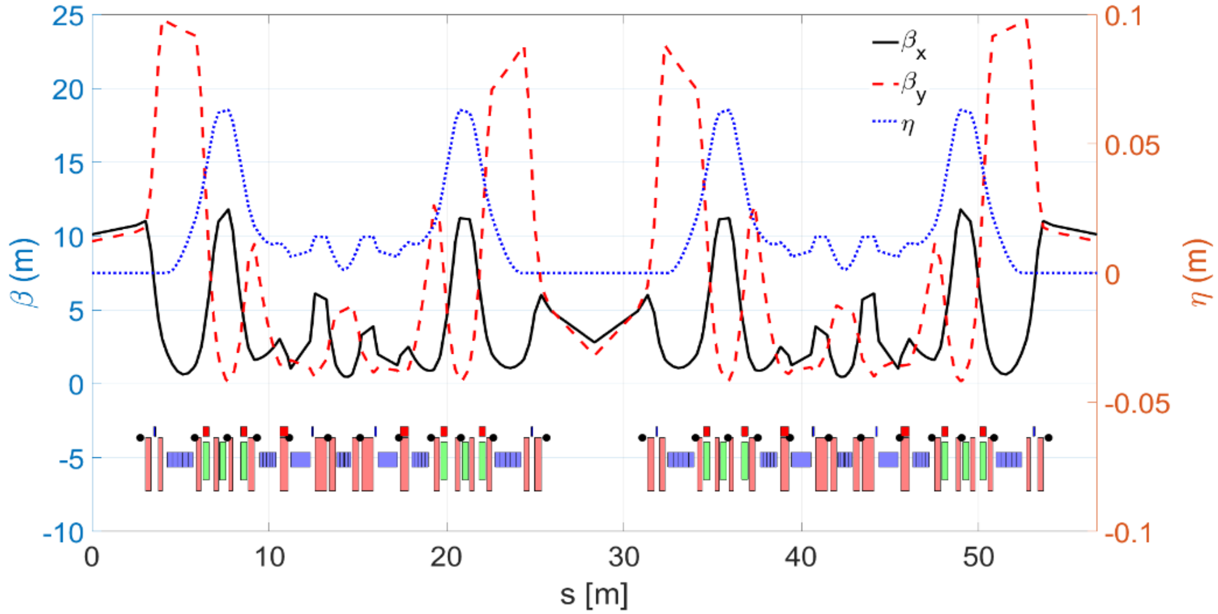
Note: In recent years, accelerator experts have begun developing beam-based correction methods to address nonlinear dynamics.

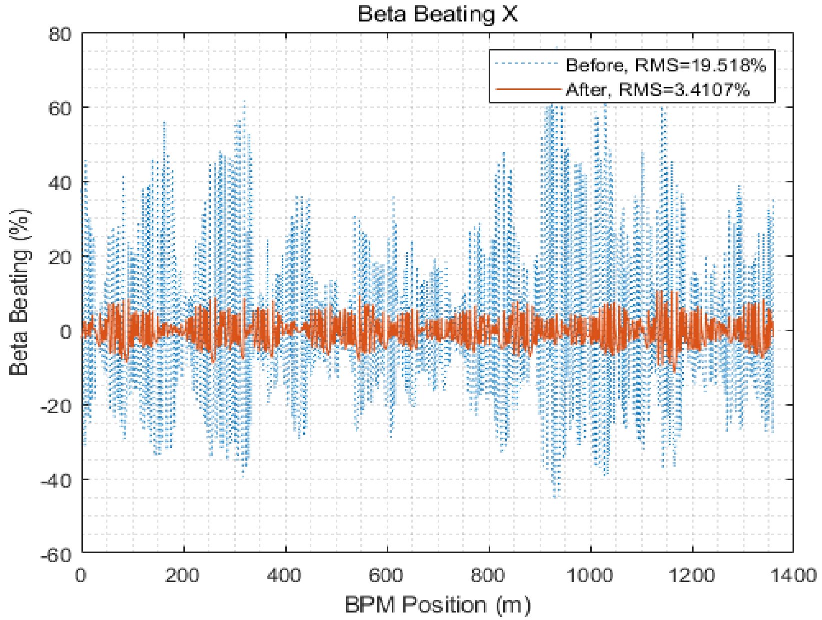
Corrector layout



Corrector and BPM (beam position monitor) layout in HEPS storge ring:

Black dots: BPMs
Read blocks: correctors (some are trim coils mounted to quadrupole/sextupole magnets)

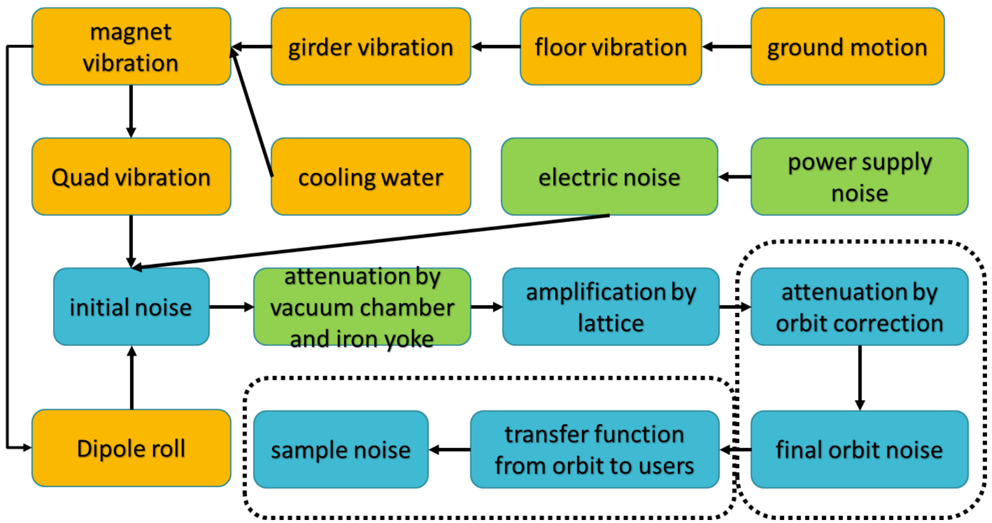




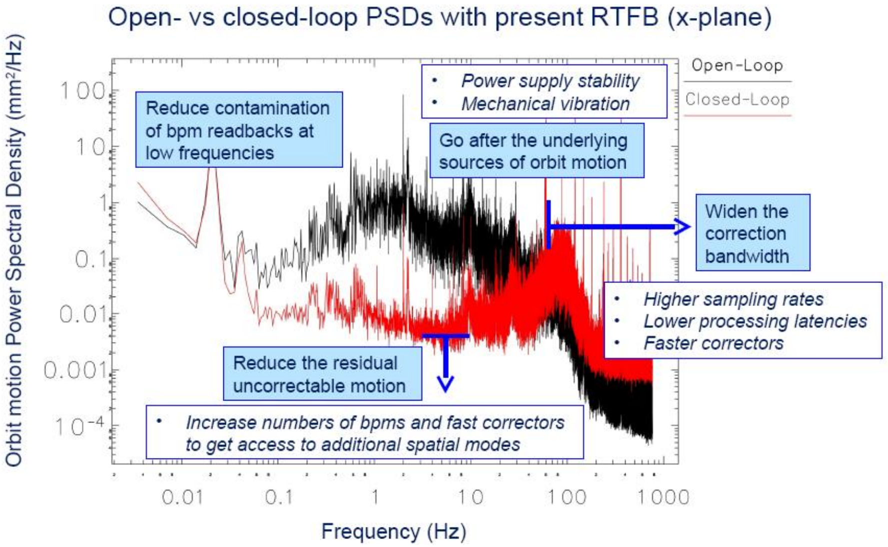
HEPS beta function deviation before and after orbit and optics correction

Orbit feedback

- For dynamic errors, due to their rapidly time-varying nature, high-frequency measurement and correction are essential—a approach generally termed **feedback**, like **fast orbit feedback (FOFB).**
- State-of-the-art FOFB system now operates at very high frequencies (on the order of 1 kHz or 10 kHz). Within a broad bandwidth, they effectively suppress orbit fluctuations caused by dynamic errors, ensuring the beam fluctuations to levels below 10% of the transverse beam size.



Schematic Diagram of Error Sources and Propagation



APS-U light source proposed a FOFB design with effective bandwidth of up to 1000 Hz

Synchrotron Physics & design

Energy

Transverse optics (design)

Dipole + Quadrupole

RF cavity

Longitudinal dynamics (design)

Modification

Injection (design)

Sextupoles
Nonlinear dynamics (optimization)

Essentially required measures

Error effects

Collective effects

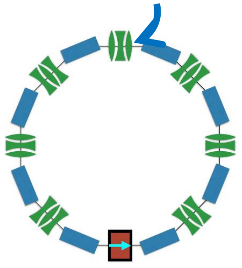
Interaction w/ environment

Self-effects
IBS/space charge

Perturbation

Though undesirable, should be addressed.

Radiation damping



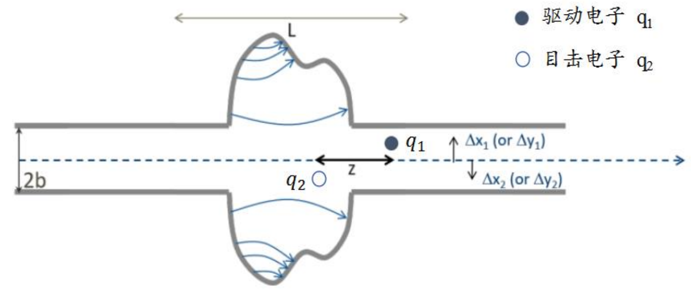
Equilibrium beam parameters

Next we will talk about another “perturbation” term, collective effects

Collective effects

- The above-mentioned linear beam optics and nonlinear dynamics are called single-particle dynamics. In an actual machine, if the number of particles in a bunch is small, the motion of individual particles closely follows single-particle dynamics. However, when the particle number increases beyond a certain threshold, collective effects should be considered as at least a "perturbation" to single-particle dynamics (in the worst case, it may dominate the beam dynamics).
- The collective effects can be roughly divided as two types:
  - Beam-environment interaction effects (**impedance effects**);
  - Beam self-effects (e.g., **intra-beam scattering, space charge effects**).

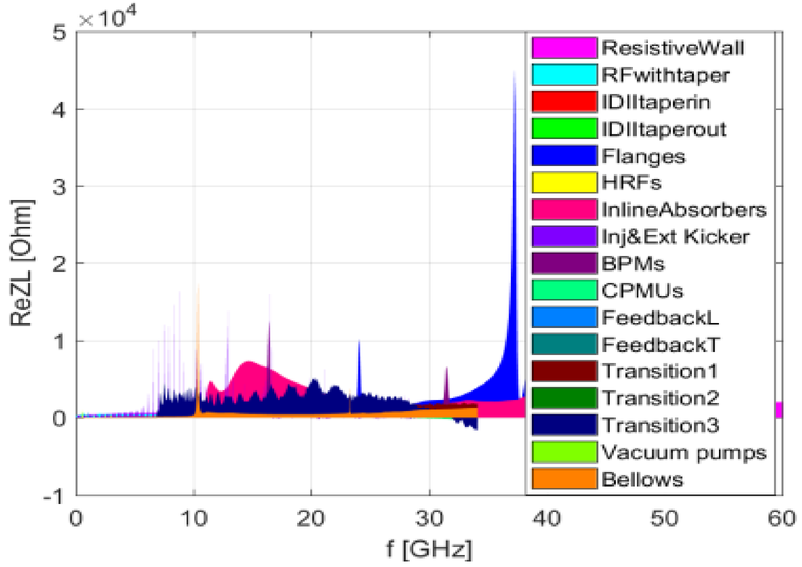
Beam impedance arises from non-ideal conductive vacuum chambers and non-smooth or discontinuous structures in the ring accelerator, which alter the electromagnetic fields of the beam. This impedance can cause deviations in energy loss (in addition to radiation energy loss), tune (from the design value) and beam distribution (from ideal Gaussian distribution), and in severe cases, may lead to exponential growth in the oscillation amplitudes of partial or all particles, resulting in particle loss.



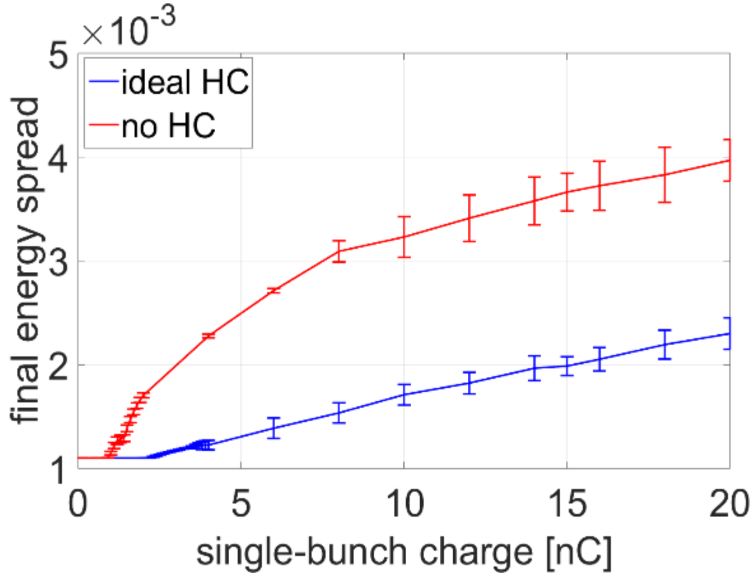
$$\vec{F}_{\text{wake}} = \vec{F}_{\text{wake}}(x_1, y_1, s_1, x_2, y_2, s_2, t)$$
$$= q_2\left[\vec{E}(x_1, y_1, s_1, x_2, y_2, s_2, t) + v_z\vec{e}_z \times \vec{B}(x_1, y_1, s_1, x_2, y_2, s_2, t)\right]$$

Impedance and instability

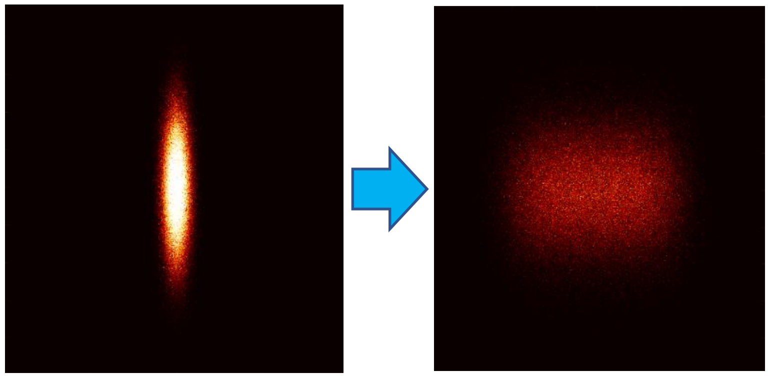
- Measures to deal with impedance effects
  - Detailed and accurate modeling of ring impedance
  - Optimizing the impedance of critical vacuum components
  - Assessment of collective beam instabilities, and proposing suppression methods to minimize collective effects on beam quality



Longitudinal Impedance budget for the HEPS storage ring



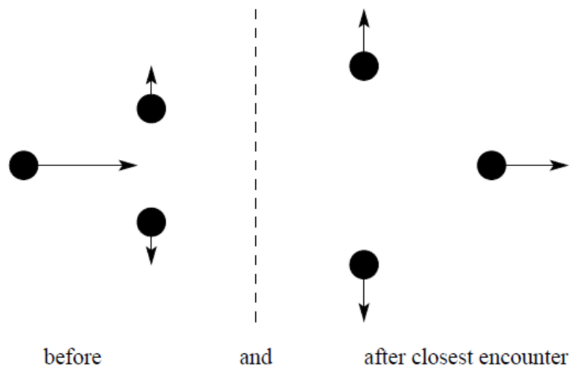
Energy spread growth driven by longitudinal impedance of the HEPS storage ring



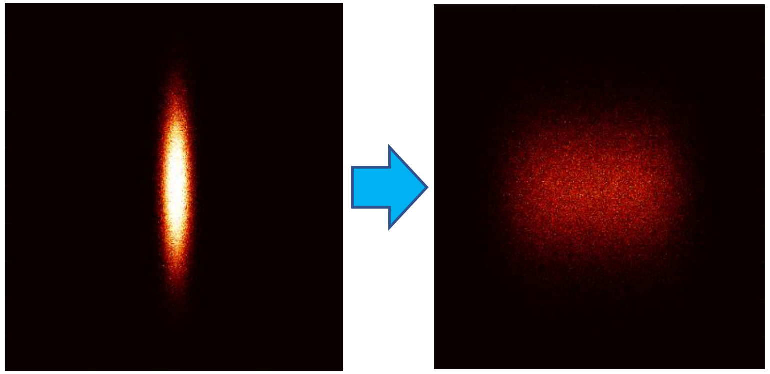
Lengthen the bunch to mitigate collective effects by use of harmonic RF cavities

Intra-beam scattering (IBS)

- **Intra-beam scattering (IBS)** is due to multiple Coulomb scattering of charged particles in a stored beam, which causes expansion of 6D beam distributions, increase in both emittance and energy spread.
- For IBS, there is no straightforward mitigation method. Fortunately, in electron accelerators, thanks to radiation damping the 6D phase space parameters will stabilize at a new equilibrium values slightly larger than the “zero-current approximation” values (e.g., natural emittance).
- Another similar effect is **Touschek effect**, which is a relativistic effect (the change in long. momentum is amplified by γ), is a single scattering effect where only the energy transfer from transverse to longitudinal plays a role and leads to particle loss. This effect is usually measured with Touschek lifetime, which is usually a dominate effect to the beam lifetime in an electron ring accelerator.
- For the Touschek effect, if electron density cannot be reduced, nonlinear dynamics optimization can be employed to increase the momentum acceptance, thereby achieving a reasonably practical beam lifetime.



Sketch of mechanism of dynamic friction for repelling force between particles



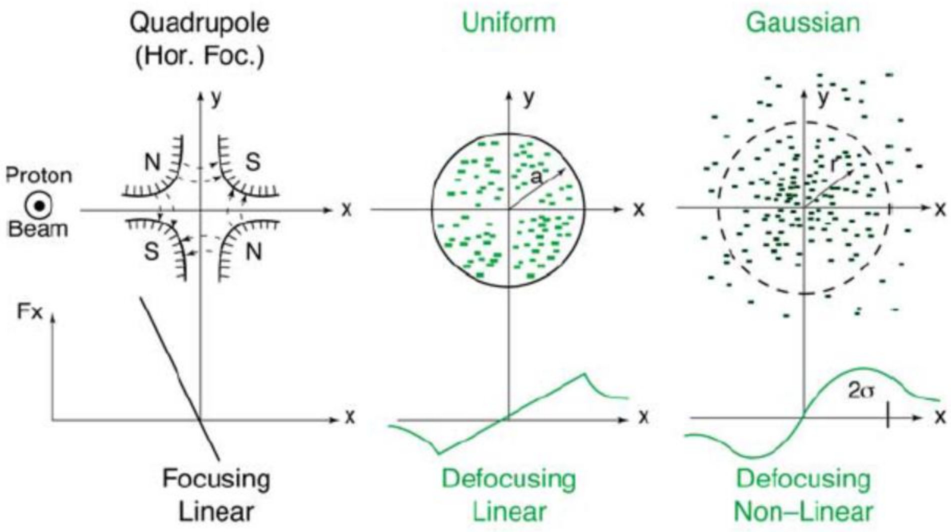
Lengthen the bunch to mitigate IBS and other collective effects by introducing harmonic RF cavities

Space charge effect

- The Coulomb forces between the charged particles of a high-intensity beam in an accelerator create a self-field which acts on the particles inside the beam like a distributed lens, defocusing in both transverse planes.
- **The direct space charge force is proportional to γ^2, and the direct space charge effect is nonrelativistic in nature**.

$$F_r = e(E_r - \beta B_\theta) = \frac{2\lambda e^2}{a^2\gamma^2} r.$$

where λ is the line charge density,
a is the transverse beam size



Space charge effect is completely negligible in high-energy accelerator (e.g., BEPC, E = 2.5 GeV, γ ~ 4892).

While space charge effect is often the dominate effect in ion synchrotron and high-intensity proton synchrotron.

Synchrotron Physics & design

Energy

Transverse optics (design)

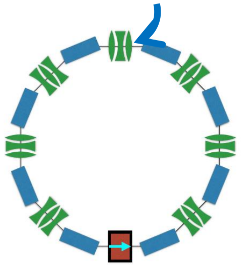
Dipole + Quadrupole

RF cavity

Longitudinal dynamics (design)

Radiation damping

Equilibrium beam parameters



Modification

Injection (design)

Sextupoles
Nonlinear dynamics (optimization)

Error effects

Collective effects

Interaction w/ environment

Self-effects
IBS/space charge

Perturbation

For proton/ion synchrotron

—the design strategy is fundamentally the same

—due to the fact that the γ factor in proton/ion machines is much lower than that achievable in electron accelerators.

(1) space charge effects often cannot be ignored.

(2) proton/ion synchrotrons lack a natural damping mechanism such as synchrotron radiation.

Special issues in proton/ion synchrotrons

- The issues include beam dynamics analysis, injection, and measures to deal with IBS effect.
- Beam dynamics:
  - The space charge acts on the beam like a defocusing lens in addition to external focusing forces. This results in **beam envelop equation for proton/ion beams**,

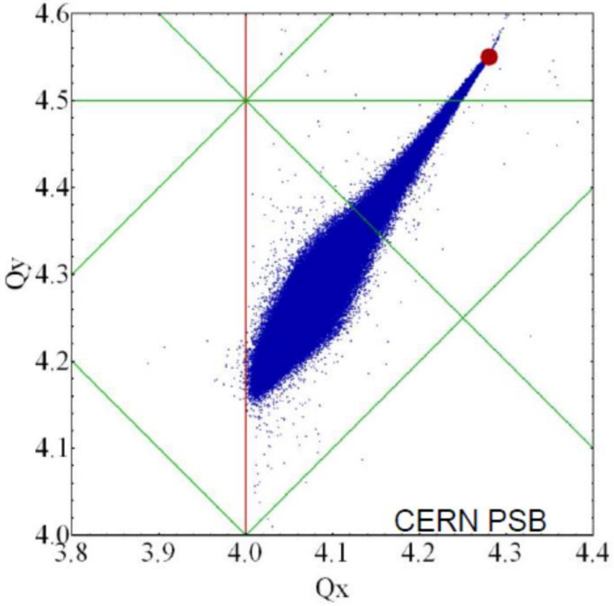
$$\frac{d^2 a_x}{ds^2} + K_x(s) a_x - \frac{\varepsilon_x^2}{a_x^3} - \frac{2\lambda r_0}{\gamma^3 (a_x + a_y)} = 0$$

where a_x and a_y are beam half-widths

  - Another space charge effect is an incoherent tune shift.

$$\Delta Q_{x,y} \propto \frac{N}{\gamma^2 \beta \varepsilon_N}$$

Selection of an appropriate working point is very important in proton/ion synchrotrons, in order to minimize or avoid crossing resonances that could lead to an increase in transverse beam size and even particle loss.



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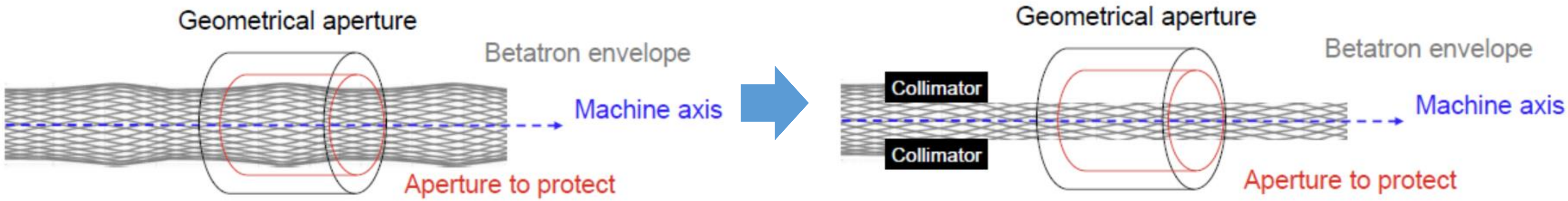
  - For proton/ion rings, the DA is defined as the aperture within which the undamped particles survive for the entire length of an injection cycle. This cycle can last for **10 million turns**. (while in electron rings, it needs only to track about 1000 turns to obtain the DA, due to presence of radiation damping).

Special issues in proton/ion synchrotrons

- The issues include beam dynamics analysis, injection, and measures to deal with IBS effect.
- Injection:
  - The injection process must account for space charge effects (such as emittance growth and beam loss). To mitigate or avoid these effects, it is common practice to **increase the injection energy as much as possible or to rapidly ramp the beam energy right after the injection**.
  - Due to absence of radiation damping, particular attention must be paid to **controlling beam loss during injection**, for example, by using **collimators** to remove particles with excessively large amplitudes.



Special issues in proton/ion synchrotrons

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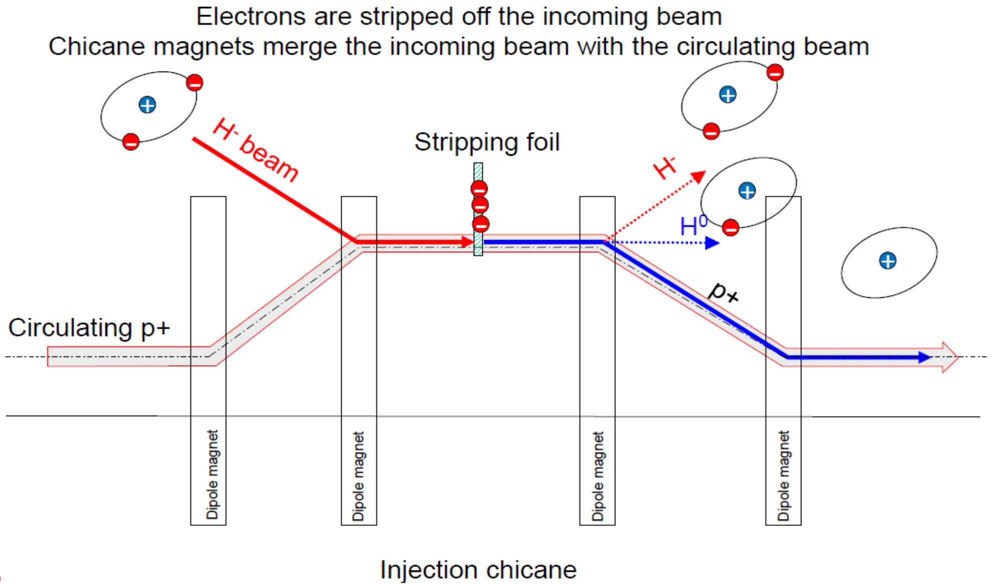
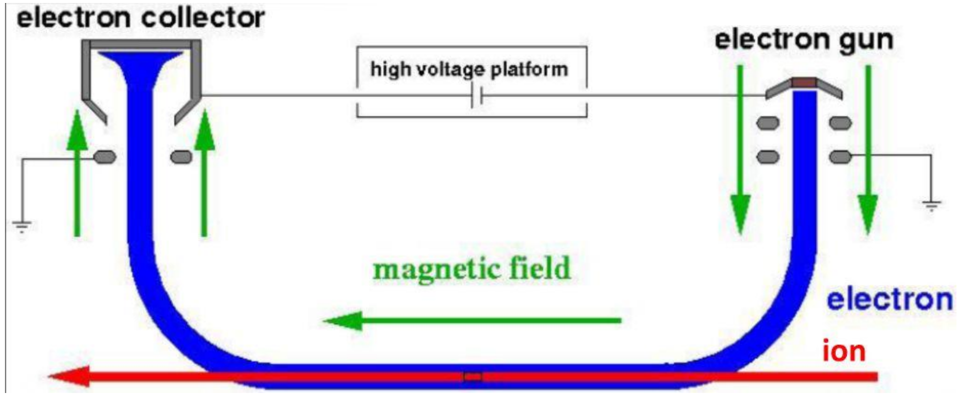
- Injection:
  - The injection process must account for space charge effects (such as emittance growth and beam loss). To mitigate or avoid these effects, it is common practice to **increase the injection energy as much as possible or to rapidly raise the beam energy after injection**.
  - Due to absence of radiation damping, particular attention must be paid to **controlling beam loss during injection**, for example, by using **collimators** to remove particles with excessively large amplitudes.
  - Special injection schemes have been proposed to address these challenges, such as **charge exchange injection**, which enables multi-turn injection with low injection loss.



Special issues in proton/ion synchrotrons

- The issues include beam dynamics analysis, injection, and measures to deal with IBS effect.
- Measures to mitigate IBS effect:
  - At sufficiently high energies, space charge effects will not be the primary concern. Instead, attention shifts to the Intrabeam Scattering (IBS) effect, particularly in storage rings and colliders where beams must be stored for a long time. IBS gradually increases beam emittance and energy spread, ultimately impacting machine performance.
  - To counter this, artificial "damping" mechanisms must be introduced. Among these, **electron cooling** stands out as the most effective method to compensate for IBS effects.

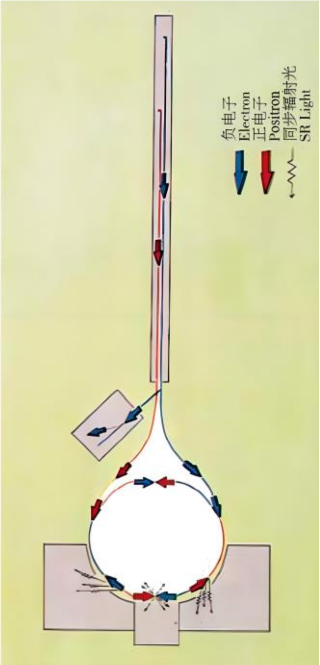
Electron cooling:
Electrons are significantly lighter than ions. When electrons and ions move at the same velocity, electrons exhibit a much lower "temperature" (energy spread). The fundamental principle of electron cooling is merging an electron beam with an ion beam at the same average velocity. Through Coulomb collisions, the continuously refreshed electron beam "cools" the ion beam, thereby reducing the momentum/energy spread and emittance of the stored ion beam.



Perspective

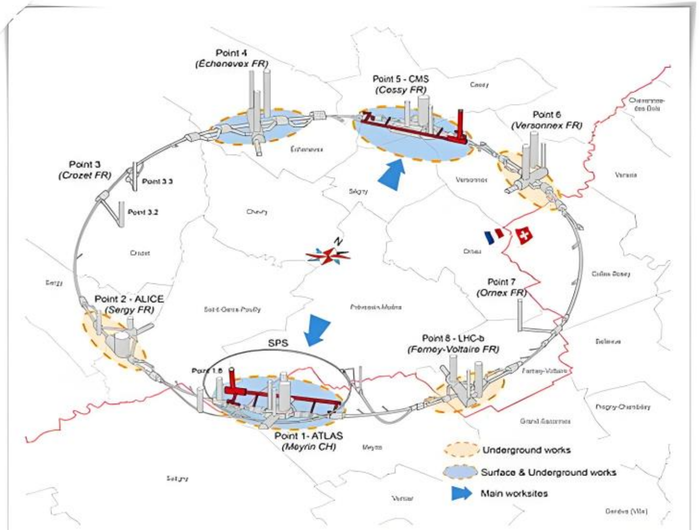
HLS， Hefei

BEPC， Beijing



HEPS， Beijing

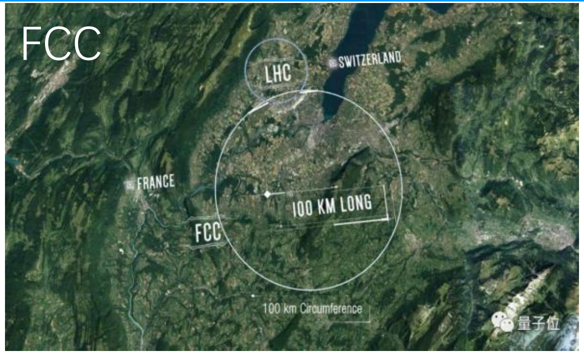
LHC， Geneva

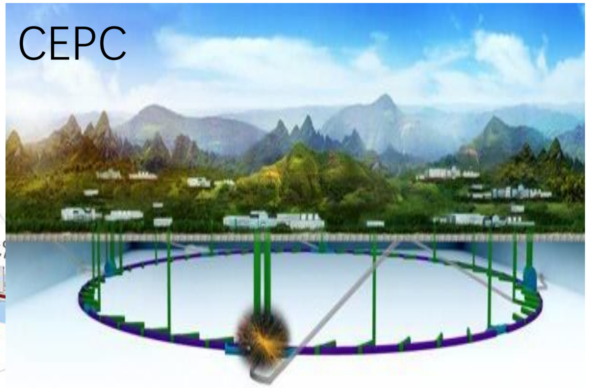


10 m　100 m　1 km　10k m　100 km

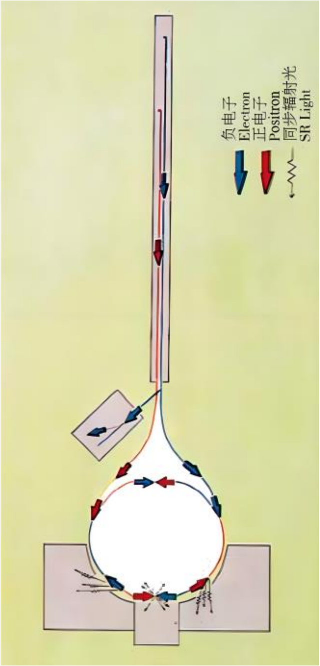
Perspective

100 km order synchrotron accelerator?

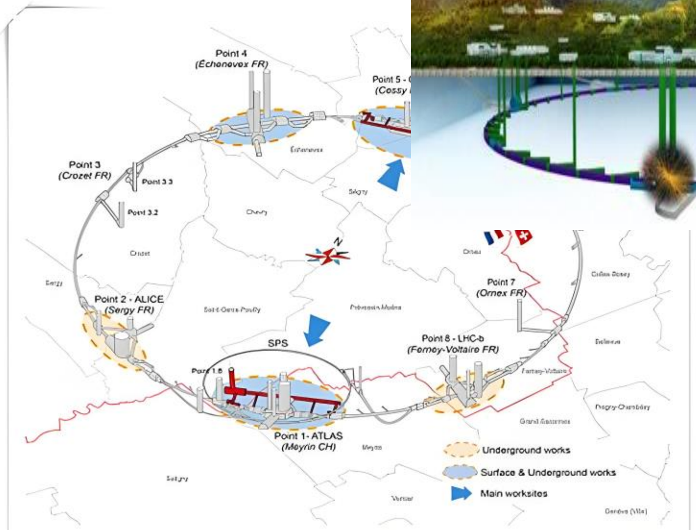




BEPC, Beijing



LHC, Geneva



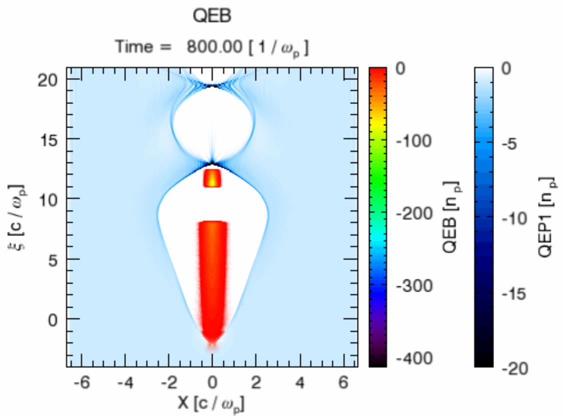
HEPS, Beijing

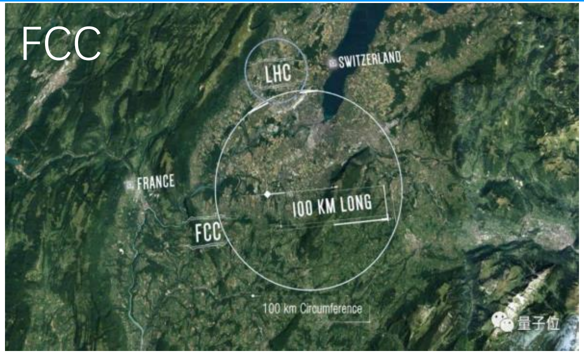
HLS, Hefei

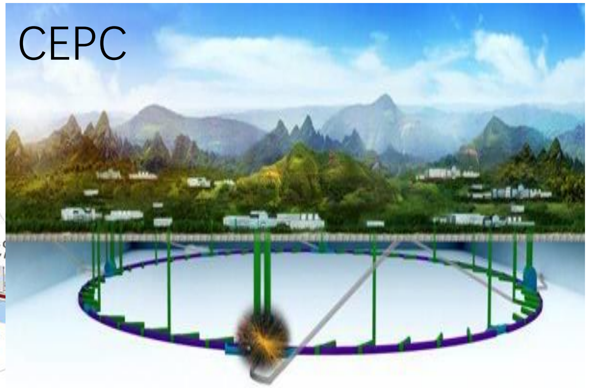
10 m — 100 m — 1 km — 10k m — 100 km

Perspective

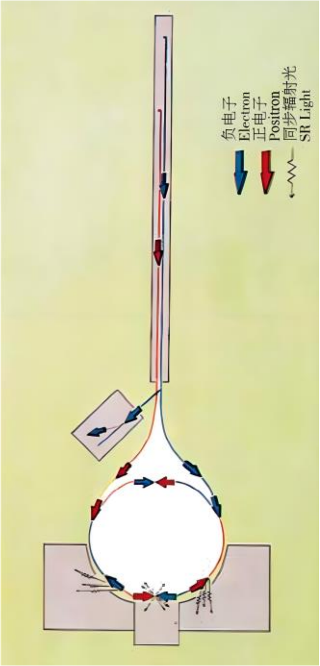
100 km order synchrotron accelerator?
Other effective ways to accelerate the beam?





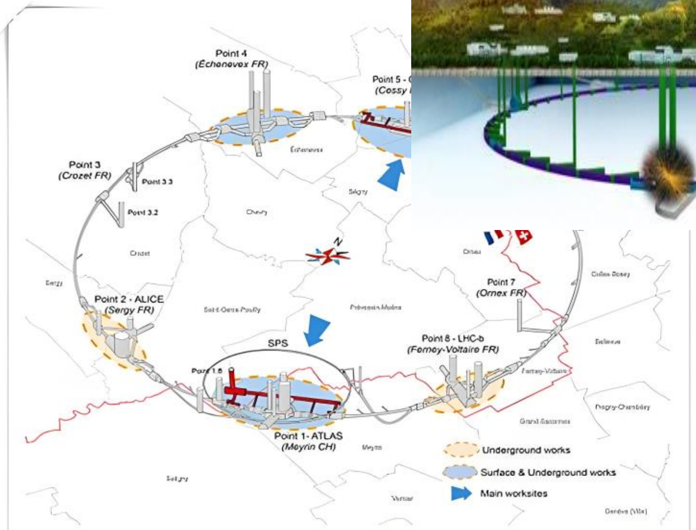


BEPC, Beijing

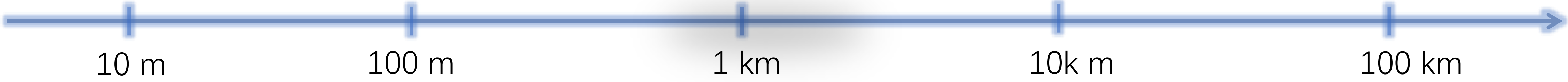


HEPS, Beijing

LHC, Geneva



HLS, Hefei




Thanks for your attention！

Physics & design

Energy → Dipole + Quadrupole; Energy → RF cavity

Transverse optics (design)
- Dipole + Quadrupole

Longitudinal dynamics (design)
- RF cavity

Modification
- Injection (design)
- Sextupoles
  Nonlinear dynamics (optimization)

Perturbation
- Error effects
- Collective effects
  - Interaction w/ environment
  - Self-effects
    IBS/space charge

Radiation damping

Equilibrium beam parameters