Advanced Beam Dynamics on Linear accelerators and Linear Colliders

Jie Gao

gaoj@ihep.ac.cn

Institute of High Energy Physics, CAS, China

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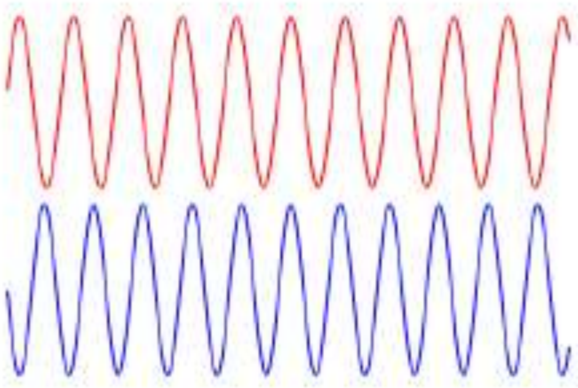
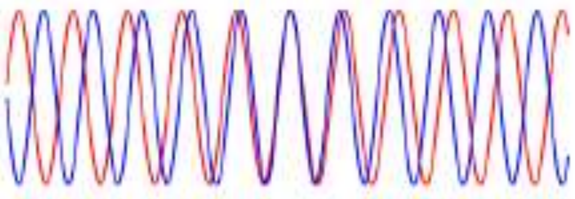
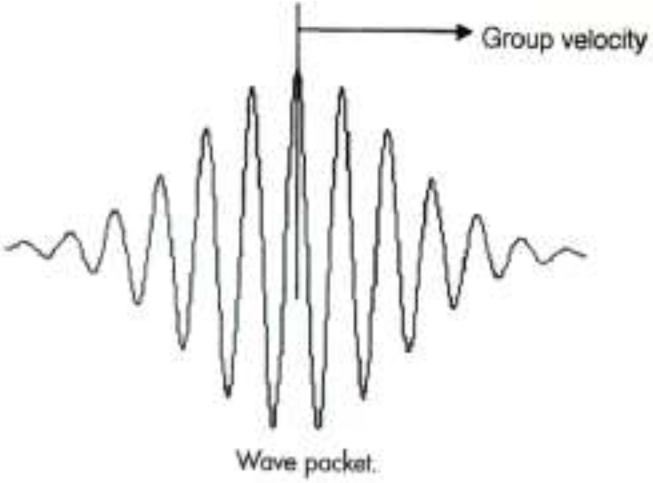
- Analytical design of radio frequency electron gun
- Analytical treatment of the emittance growth in linacs
- Analytical estimates of halo current loss rates in space charge dominated beams
- Analytical formulae for precise rf field measurements
- Linear collider design

Wave Propagation and Signal







$$V(z,t) = e^{j(\omega_1 t - k_1 z)} + e^{j(\omega_2 t - k_2 z)}$$

$$= 2\cos\frac{(\omega_1 - \omega_2)t - (k_1 - k_2)z}{2} e^{j[(\omega_1+\omega_2)t-(k_1+k_2)z]/2}$$

$$v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} \rightarrow \frac{d\omega}{dk}$$

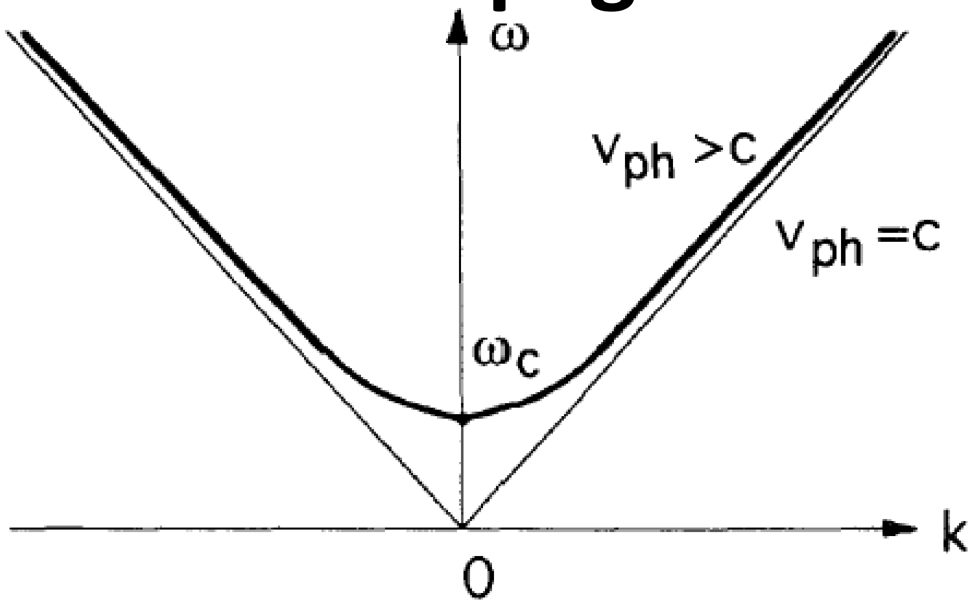
$$v_E = \frac{P}{U} = v_g$$

Group Velocity

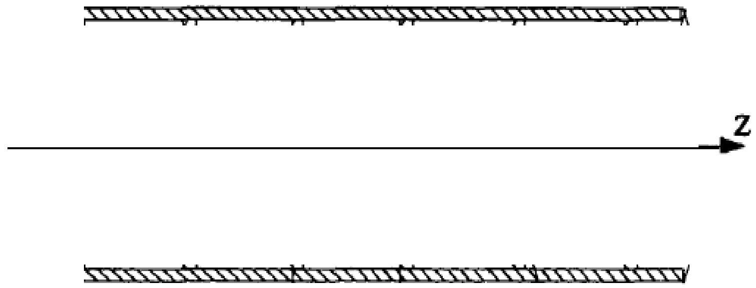
$$v_p = \frac{\omega_1 + \omega_2}{k_1 + k_2} = \frac{\overline{\omega}}{\overline{k}}$$

Phase Velocity

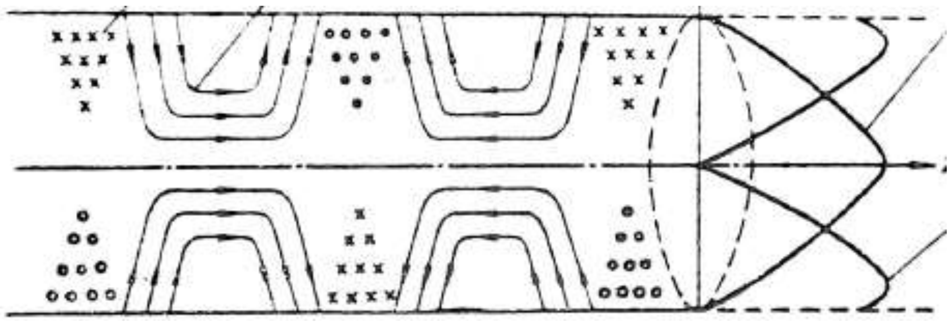
Wave Propagation in a Cylindrical Waveguide



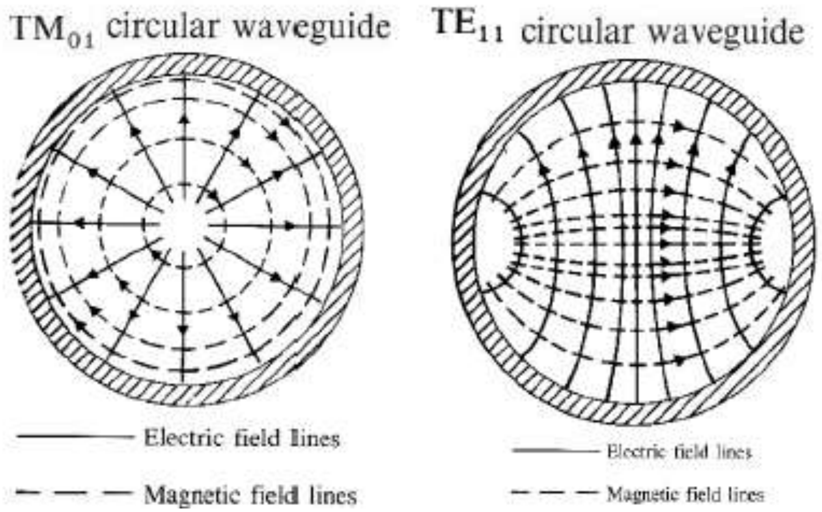
Dispersion curve



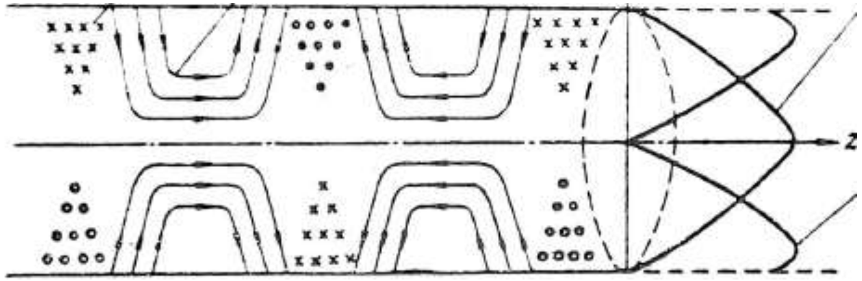
Cylindrical waveguide

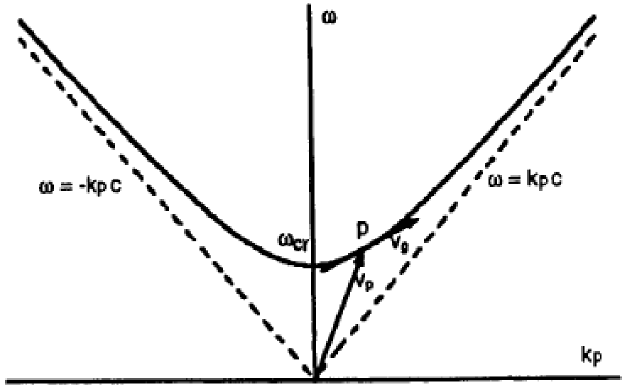


TM_{01} mode EM fields distribution



TM_{01} and TE_{11} modes





$$E_z(r,z,t) = E_0 J_0(k_c r) e^{j(\omega t - k_p z)}$$

$$E_r(r,z,t) = jE_0[1-(\frac{\omega_{cr}}{\omega})^2]^{1/2} J_1(k_c r) e^{j(\omega t - k_p z)}$$

$$B_\theta(r,z,t) = j\mu_0 E_0 J_1(k_c r) e^{j(\omega t - k_p z)}$$

$$E_z(r,z,t) = \underbrace{E_0 J_0(k_c r)}\, e^{\underbrace{j(\omega t - k_p z)}}$$

TM$_{01}$ mode EM fields distribution

$$k^2 = k_c^2 + k_p^2$$

$$k = \frac{\omega}{c}$$

$$k_c = \frac{\omega_{cr}}{c}$$

$$k_p = \frac{\omega}{v_p}$$

$$k_c R = \frac{\omega_{cr}}{c} \cdot R = 2.405$$

$$\omega^2 = \omega_{cr}^2 + \left(\frac{\omega}{v_p} c\right)^2$$

$$k_p^2 = k^2 - k_c^2 = \left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega_{cr}}{c}\right)^2$$

Dispersion relation

$$\left(\frac{\omega}{c}\right)^2 - k_p^2 = \left(\frac{\omega_{cr}}{c}\right)^2$$

$$v_g = \frac{d\omega}{dk_p}$$

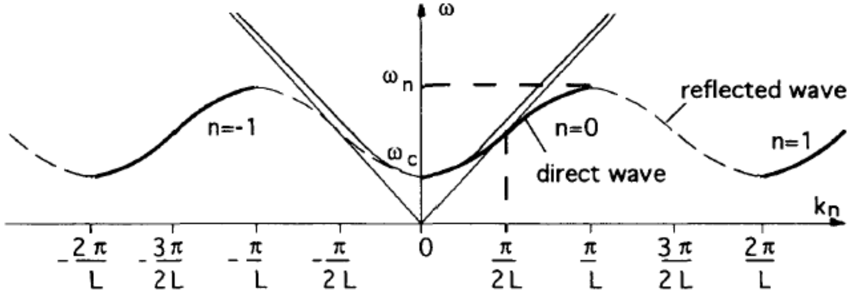
Phase velocity

$$v_p = \frac{\omega}{k_p} \qquad v_p = \frac{\omega}{k_p} = \frac{c}{\sqrt{1-\left(\frac{\omega_{cr}}{\omega}\right)^2}} > c$$

Dispersion relation

$$\omega^2 = \omega_{\pi/2}^2(1 - k\cos(\beta_0 D))$$

Periodic structures of electrically and magnetically coupled systems



Periodic structures of electrically coupled system (TM010)

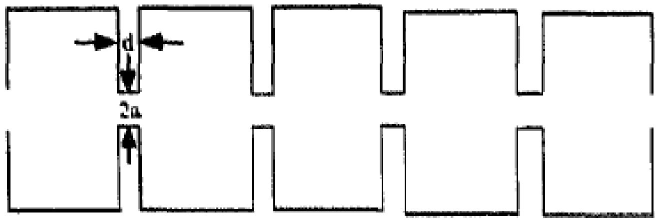
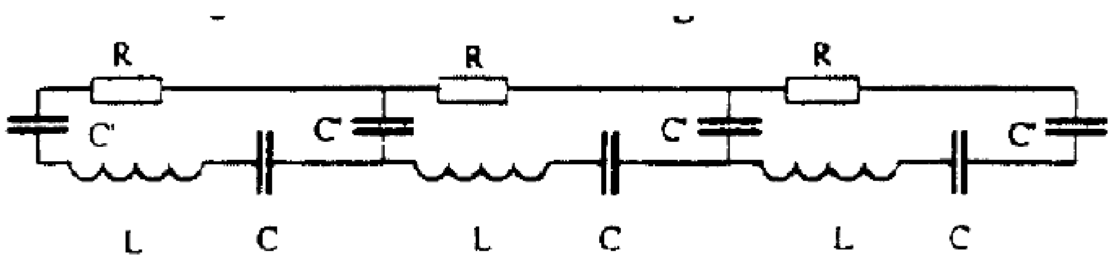
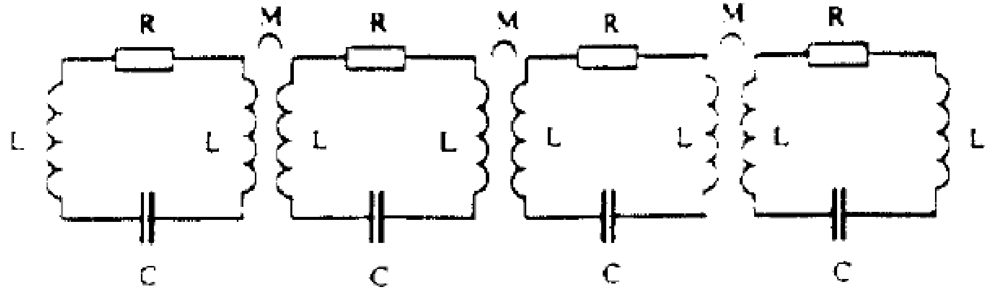


Fig. 4. Electrically coupled slow wave structure.





Dispersion relation

$$\omega^2 = \frac{\omega_{\pi/2}^2}{(1 - k\cos(\beta_0 D))} \approx \omega_{\pi/2}^2(1 + k\cos(\beta_0 D))$$

Periodic structures of magnetically coupled system (TE110)

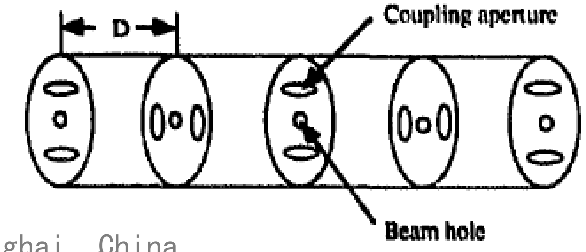
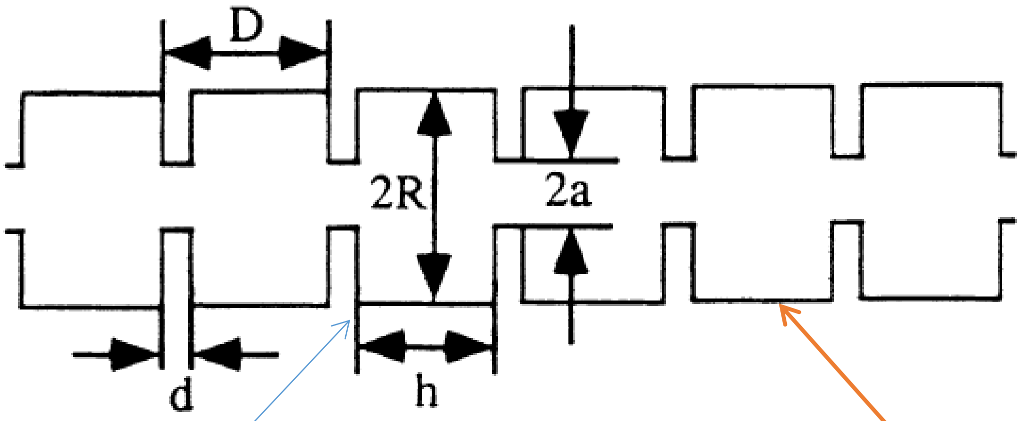


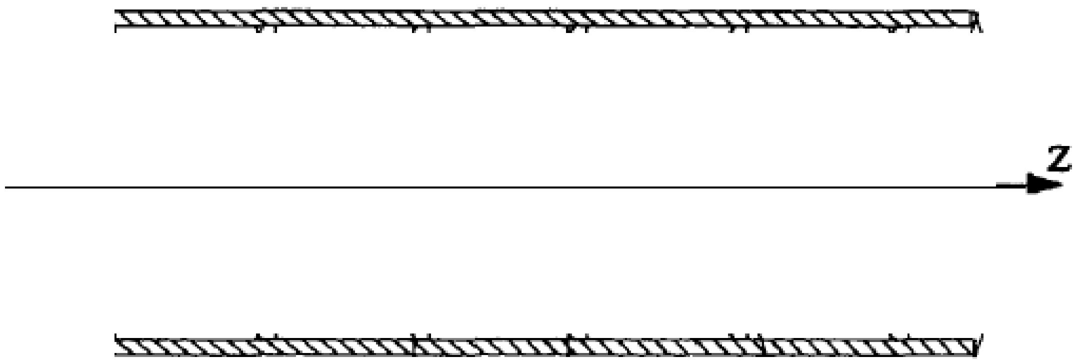
Fig. 6. Magnetically coupled slow wave structure.

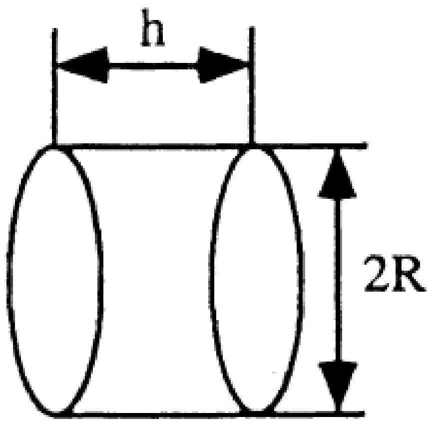


Two ways to study a disc-loaded slow wave structre:

1) Traditionally, starts from a waveguide (all books and lecteurs)

2) New point of view, start from a single cavity (this lecteur by Jie GAO)





General Accelerating Cavity and Structure Theories

Nuclear Instruments and Methods in Physics Research A311 (1992) 437–443
North-Holland

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH
Section A

Nucl. Instr. and Meth. A311 (1992) 437-443.

Analytical formulas for the resonant frequency changes due to opening apertures on cavity walls

J. Gao

Laboratoire de L'Accélérateur Linéaire, 91405 Orsay cedex, France

Received 30 May 1991 and in revised from 27 August 1991

Cited by: T.P. Wangler, RF Linear Accelerators, John Wiley& Sons, Inc. 1998.

Slater Perturbation Theory

$$\omega^2 = \omega_0^2 \left[1 + \frac{1}{2U} \int_{\Delta v} (\mu_0 H^2 - \epsilon_0 E^2) \mathrm{d}v \right]$$

$$\omega^2 = \omega_0^2 \left[1 + \frac{2}{U} (\Delta W_{\mathrm{m}} - \Delta W_{\mathrm{e}}) \right]$$

The frequency change due to energy change in a cavity

where ΔW_e and ΔW_m are the time-average electric and magnetic energies stored in the perturbation volume.

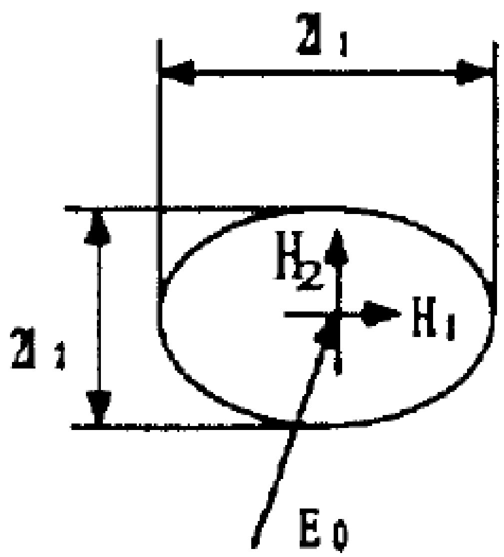
H. Bethe and R.E. Collin Theories on Equivalent Dipoles

$$P = -\frac{\pi l_1^3 (1 - e_0^2)}{3E(e_0)} \epsilon_0 E_0$$

$$M_1 = \frac{\mu_0 \pi l_1^3 e_0^2}{3[K(e_0) - E(e_0)]} H_1$$

$$M_2 = \frac{\mu_0 \pi l_1^3 e_0^2 (1 - e_0^2)}{3[E(e_0) - (1 - e_0^2) K(e_0)]} H_2$$

$$e_0 = \left(1 - \frac{l_2^2}{l_1^2}\right)^{\frac{1}{2}}$$



where ϵ_0 is the permittivity of vacuum, μ_0 is the permeability of vacuum, P and M_1, M_2 are the electric and magnetic dipole moments, respectively. E_0 is the electric field perpendicular to the surface of the ellipse. H_1 and H_2 are the magnetic fields parallel to the major and minor axis of this ellipse. l_1 and l_2 are the lengths of demi- major and minor axis, respectively (see Fig. 1). $K(e_0)$ and $E(e_0)$ are complete elliptic integrals of the first and second kinds [6].

$$K(e_0) = \frac{\pi}{2}\left[1 + \left(\frac{1}{2}\right)^2 e_0^2 + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 e_0^4 + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 e_0^6 + \left(\frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}\right)^2 e_0^8 + \cdots\right]$$

$$E(e_0) = \frac{\pi}{2}\left[1 - \left(\frac{1}{2}\right)^2 e_0^2 - \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 \frac{e_0^4}{3} - \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 \frac{e_0^6}{5} - \left(\frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}\right)^2 \frac{e_0^8}{7} - \cdots\right]$$

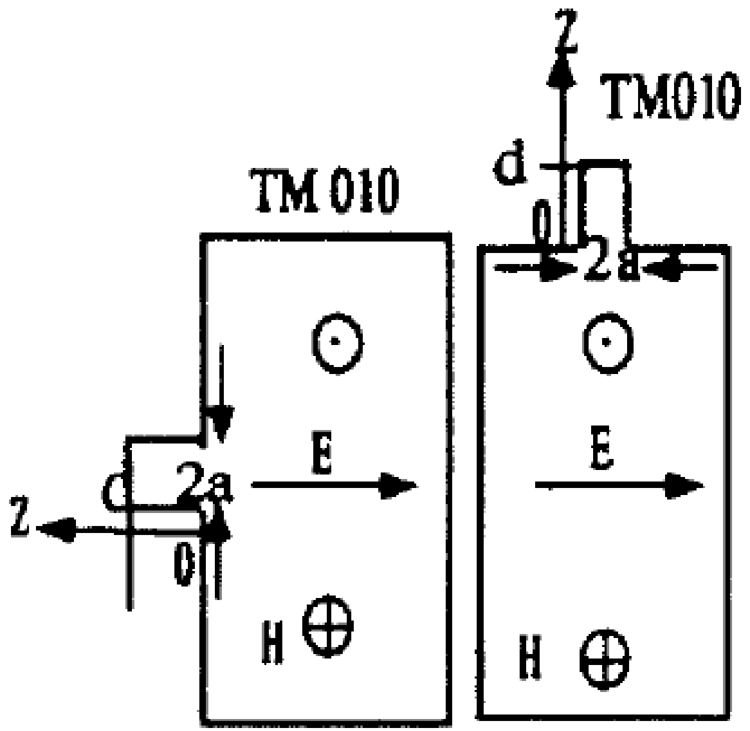
$$e_0 = \left(1 - \frac{l_2^2}{l_1^2}\right)^{\frac{1}{2}}$$

$$\Delta U_e = -\frac{1}{2}\boldsymbol{P}\cdot\boldsymbol{E}' = \frac{\pi l_1^3(1-e_0^2)}{12E(e_0)}\epsilon_0 E_0^2 = -\Delta W_e$$

$$\Delta U_m = \Delta U_{m,1} + \Delta U_{m,2} = -\Delta W_m$$

$$\Delta U_{m,1} = \frac{1}{2}\boldsymbol{M}_1\cdot\boldsymbol{H}'_1 = \frac{\mu_0\pi l_1^3 e_0^2}{12[K(e_0)-E(e_0)]}H_1^2$$

$$\Delta U_{m,2} = \frac{1}{2}\boldsymbol{M}_2\cdot\boldsymbol{H}'_2 = \frac{\mu_0\pi l_1^3 e_0^2(1-e_0^2)}{12[E(e_0)-(1-e_0^2)K(e_0)]}H_2^2$$

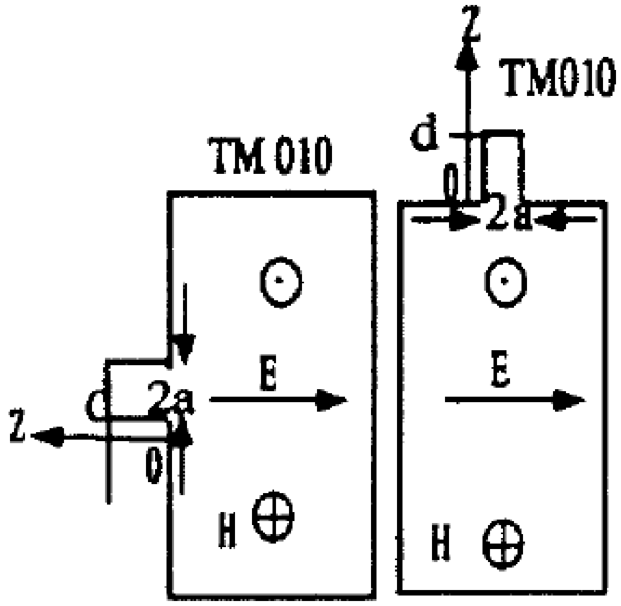


Electric coupling

$$\omega^2 = \omega_0^2\left[1 + \frac{2\Delta U_e}{U}(1 - e^{-2\alpha_1 z})\right]$$

$$\delta\omega = \omega_0 \frac{a^3 \epsilon_0 E_0^2}{6U}(1 - e^{-2\alpha_1 z})$$

$$\frac{d\omega}{dz} = \omega_0 \frac{a^3 \epsilon_0 \alpha_1 E_0^2}{3U} e^{-2\alpha_1 z}$$



Magnetic coupling

$$\omega^2 = \omega_0^2\left[1 - \frac{2\Delta U_m}{U}(1 - e^{-2\alpha_2 z})\right]$$

$$\delta\omega = -\omega_0 \frac{a^3 \mu_0 H_0^2}{3U}(1 - e^{-2\alpha_2 z})$$

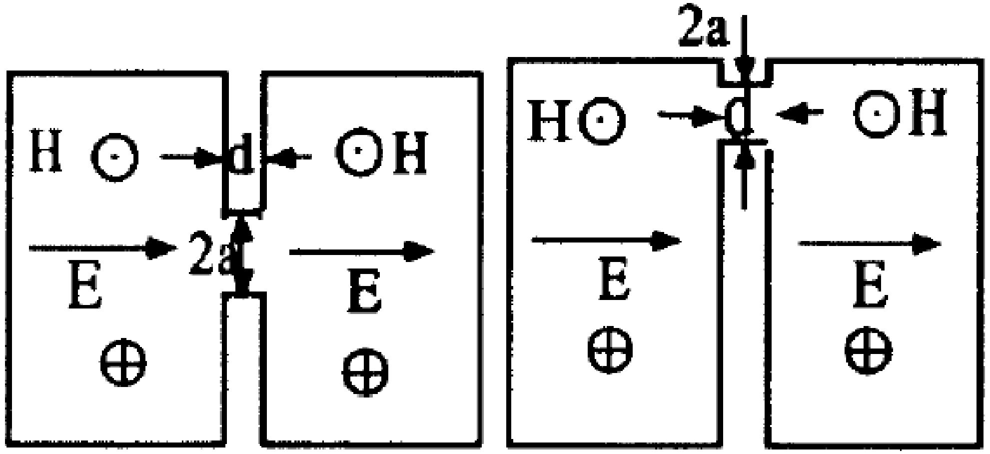
$$\frac{d\omega}{dz} = -2\omega_0 \frac{a^3 \mu_0 \alpha_2 H_0^2}{3U} e^{-2\alpha_2 z}$$

$$\alpha_1 = \frac{2\pi}{\lambda}\left[\left(\frac{\lambda}{\lambda_{c1}}\right)^2 - 1\right]^{\frac{1}{2}}$$

$$\alpha_2 = \frac{2\pi}{\lambda}\left[\left(\frac{\lambda}{\lambda_{c2}}\right)^2 - 1\right]^{\frac{1}{2}}$$

$\lambda_{c1} = 2.62a$; TM_{01}

$\lambda_{c2} = 3.41a$; TE_{11}



$$\Delta W_{e,1} = \frac{1}{2} \boldsymbol{P}_1 \cdot \boldsymbol{E}'_1 - \frac{1}{2} \boldsymbol{P}_1 \cdot \boldsymbol{E}'_2$$

where $\mathbf{P_1}$ is the dipole moment corresponding to first cavity, $\mathbf{E}'_1 = 1/2\mathbf{E}_1$, $\mathbf{E}'_2$ is the electric field of the second cavity seen by the electric dipole of the first cavity, with $\mathbf{E}'_2 = 1/2e^{-\alpha_1 d}\mathbf{E}_2$, d is the thickness of the common wall where the aperture is located, and $\mathbf{E}_1$, $\mathbf{E}_2$ are the electric fields at the center of the aperture in the two cavities when the aperture is replaced by an ideal metallic boundary. Therefore according to eq. 2, one can get the frequency change of the first cavity as

$$\omega_1^2 = \omega_{0,1}^2\left(1 - \frac{2\Delta W_{e,1}}{U}\right)$$

$$= \omega_{0,1}^2\left(1 + \frac{1}{3}a^3\epsilon_0\frac{\mathbf{E_1}\cdot\mathbf{E_1}}{U} - \frac{1}{3}a^3\epsilon_0\frac{\mathbf{E_1}\cdot\mathbf{E_2}}{U}e^{-\alpha_1 d}\right)$$

$$= \omega_{0,1}^2\left(1 + \frac{1}{3}a^3\epsilon_0\frac{E_1^2}{U} - \frac{1}{3}a^3\epsilon_0\frac{E_1E_2 cos\theta}{U}e^{-\alpha_1 d}\right)$$

where θ is the phase difference between $\mathbf{E_1}$ and $\mathbf{E_2}$.

$$\omega_1^2 = \omega_{0,1}^2\left(1 + \frac{2\Delta U_m}{U}\right)$$

$$= \omega_{0,1}^2\left(1 - \frac{2}{3}a^3\mu_0\frac{\mathbf{H_1}\cdot\mathbf{H_1}}{U} + \frac{2}{3}a^3\mu_0\frac{\mathbf{H_1}\cdot\mathbf{H_2}}{U}e^{-\alpha_2 d}\right)$$

$$= \omega_{0,1}^2\left(1 - \frac{2}{3}a^3\mu_0\frac{H_1^2}{U} + \frac{2}{3}a^3\mu_0\frac{H_1H_2 cos\theta}{U}e^{-\alpha_2 d}\right)$$

where θ is the phase difference between $\mathbf{H_1}$ and $\mathbf{H_2}$.

DISPERSION RELATION OF SLOW WAVE STRUCTURE

According to Floquet's theorem it is known that

$$F(r, z+D) = F(r, z)e^{j\beta_0 D}$$

Mode in a periodic structure

$\theta = \beta_0 D$, where β_0 is the foundamental wave number, and D is the space periodicity of the periodic structure.

$$\omega^2 = \omega_0^2\left(1 + \frac{N}{3}a^3\epsilon_0\frac{E_1^2}{U} - \frac{N}{3}a^3\epsilon_0\frac{E_1E_2cos(\beta_0 D)}{U}e^{-\alpha_1 d}\right)$$

$$\omega_{\pi/2}^2 = \omega_0^2\left(1 + \frac{N}{3}a^3\epsilon_0\frac{E_1^2}{U}\right)$$

where N is the number of the same type coupling apertures in each cavity

$$\omega^2 = \omega_{\pi/2}^2\left(1 - \frac{N}{3}a^3\epsilon_0\frac{E_1E_2cos(\beta_0 D)}{U}e^{-\alpha_1 d}\right)$$

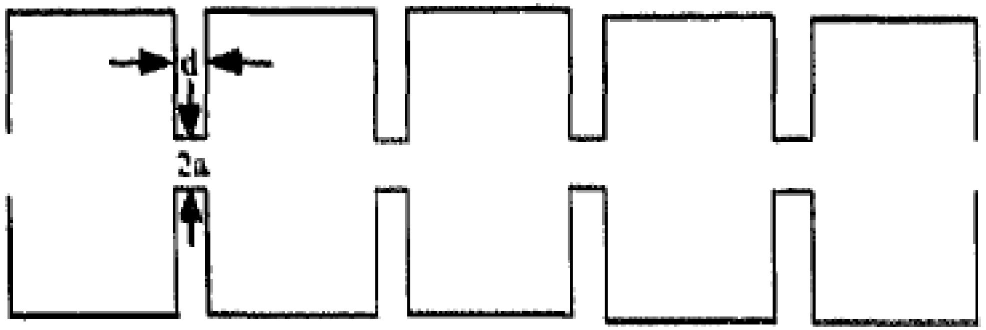


Fig. 4. Electrically coupled slow wave structure.

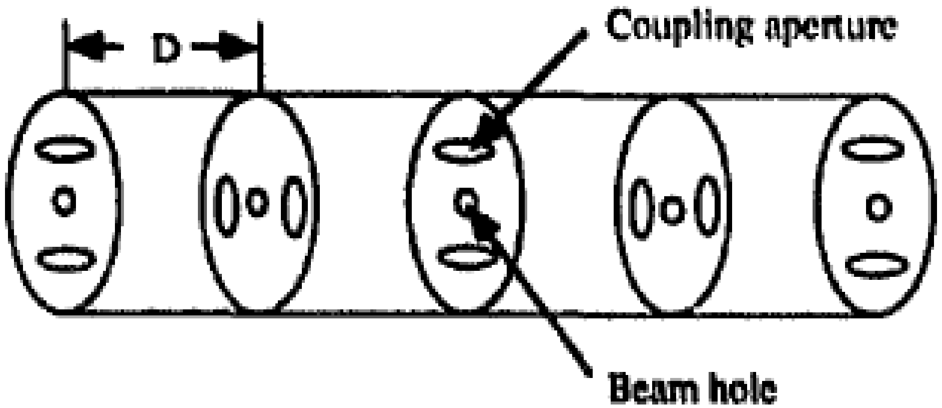
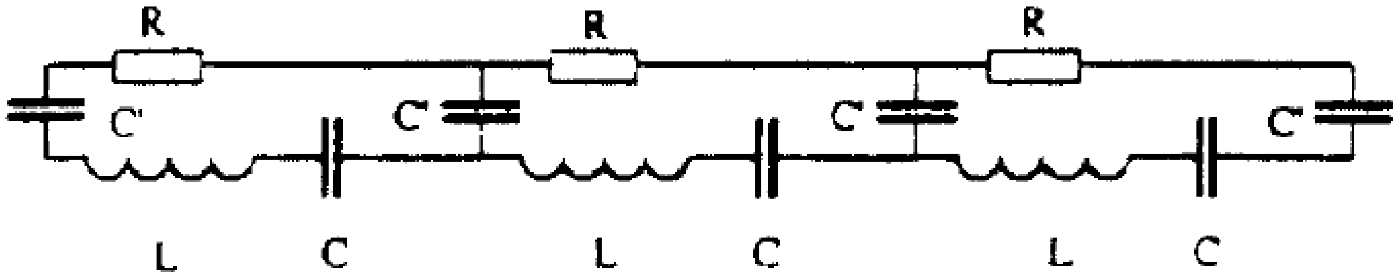
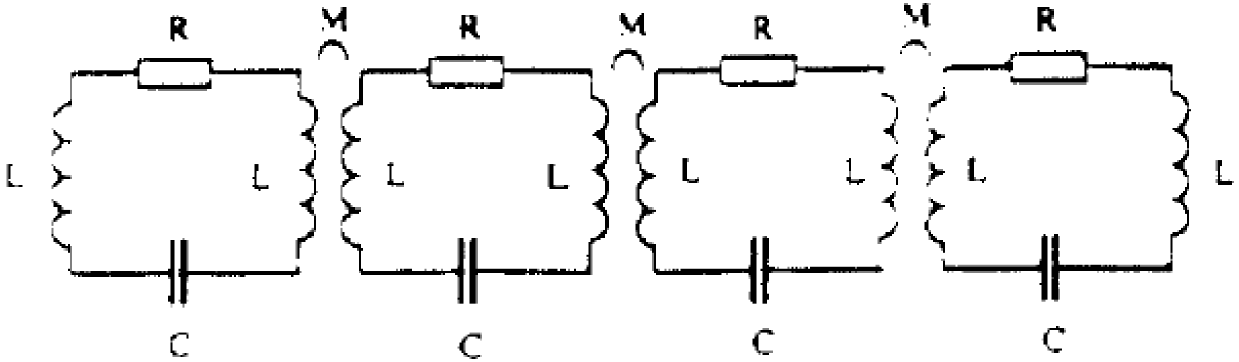
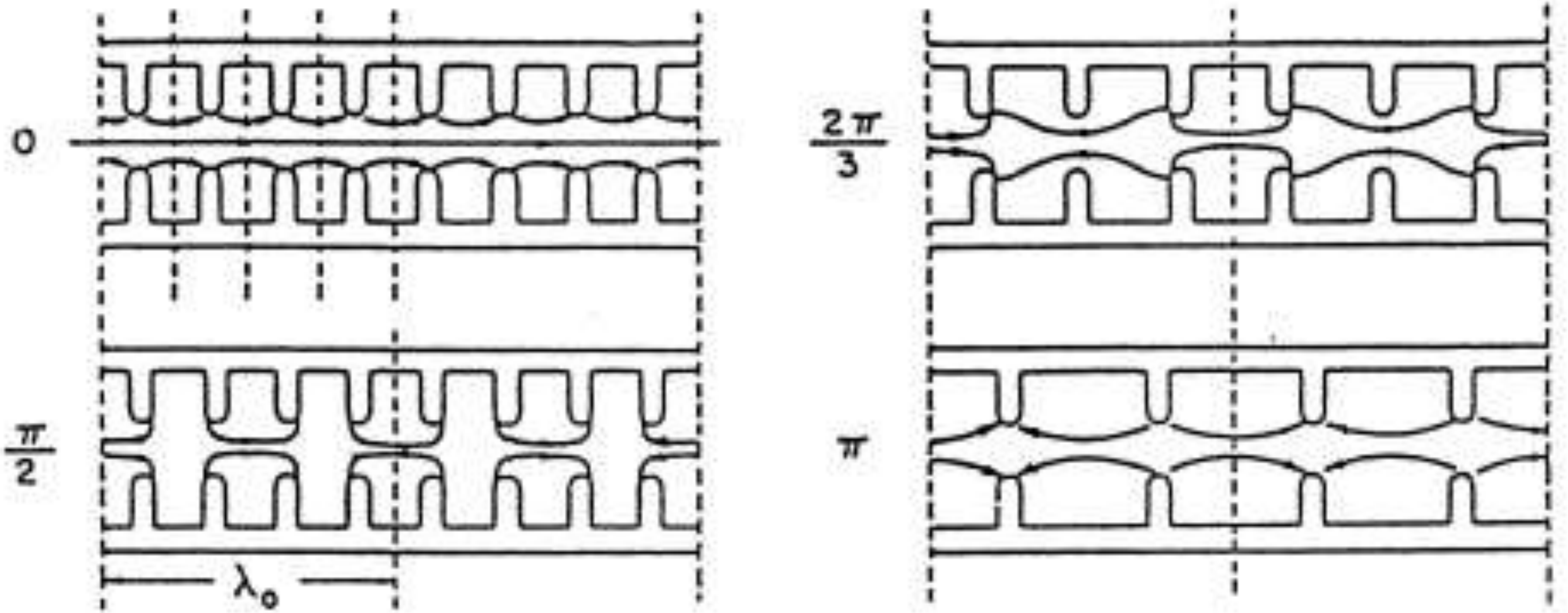


Fig. 6. Magnetically coupled slow wave structure.





Electric Field Distributions for Different Modesθ (it is possible to chose other modes)



θ=0, 2π/3, π/2, π

$$\omega^2 = \omega_{\pi/2}^2(1 - k\cos(\beta_0 D)) \qquad k = \frac{N}{3}a^3\epsilon_0\frac{E_1E_2}{U}e^{-\alpha_1 d}$$

$$v_g = \frac{d\omega}{d\beta_0} = \omega_{\pi/2}\frac{N}{6}a^3\epsilon_0\frac{\alpha_e D E_1^2 \sin(\beta_0 D)}{U}e^{-\alpha_1 d}$$

$$\omega^2 = \omega_{\pi/2}^2(1 + \frac{2N}{3}a^3\mu_0\frac{H_1H_2\cos(\beta_0 D)}{U}e^{-\alpha_2 d})$$

$$\omega_{\pi/2}^2 = \omega_{0,1}^2(1 - \frac{2N}{3}a^3\mu_0\frac{H_1^2}{U}) \qquad k = \frac{2N}{3}a^3\mu_0\frac{H_1H_2}{U}e^{-\alpha_2 d}$$

$$\omega^2 = \frac{\omega_{\pi/2}^2}{(1 - k\cos(\beta_0 D))} \approx \omega_{\pi/2}^2(1 + k\cos(\beta_0 D))$$

$$v_g = \frac{d\omega}{d\beta_0} = -\omega_{\pi/2}\frac{N}{3}a^3\mu_0\frac{\alpha_m D H_1^2 \sin(\beta_0 D)}{U}e^{-\alpha_2 d}$$

Disk-loaded Accelerating Structure Theory (Application of the general theory)

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ANALYTICAL APPROACH AND SCALING LAWS IN THE DESIGN OF DISK-LOADED TRAVELLING WAVE ACCELERATING STRUCTURES

J. GAO

Laboratoire de L'Accélérateur Linéaire, IN2P3-CNRS et Université de Paris-Sud, Centre d'Orsay, F-91405 Orsay cedex, France

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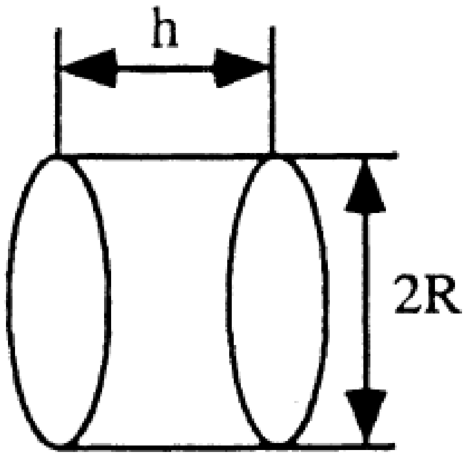
Starting from a single resonant rf cavity, disk-loaded travelling (forward or backward) wave accelerating structures' properties, such as the coupling coefficient K in the dispersion relation, group velocity v_g, shunt impedance R, wake potential W (longitudinal and transverse), the coupling coefficient β of the coupler cavity and the coupler cavity axis shift δ_r which is introduced to compensate the asymmetry caused by the coupling aperture, can be determined by rather simple analytical formulae.

KEY WORDS: Travelling wave, accelerating structures and perturbation methods

Cited by: T.P. Wangler, RF Linear Accelerators, John Wiley& Sons, Inc. 1998.



A closed pill box cavity as the starting point

Panofsky and Wenzel[1] proved a theorem which says that if a charged particle at the speed of light passing through a closed cavity of arbitrary shape containing *em* fields, the transverse kick experienced by this particle can be expressed as:

$$\vec{p}_\perp = \frac{iq}{\omega_0} \int_0^h dz [\vec{\nabla}_\perp E_z(z,t)]_{t=z/c}$$

where q is the electrical charge and ω_0 is the angular frequency of the mode corresponding to $E_z(z,t)$. It is obvious that TE_{mnl} modes have no influences on the particle either longitudinally or transversely if this particle crosses the cavity along the z axis. Our attention, therefore, will put on TM_{mnl} modes only.

TM_{mnl} modes' EM fields in a closed pill box

In the cylindrical coordinate system the em field distributions of TM_{mnl} modes are:

$$E_r = -\frac{\varepsilon_0 l\pi R}{u_{mn}h} J'_m\left(\frac{u_{mn}}{R}r\right)\cos(m\phi)\sin\left(\frac{l\pi z}{h}\right) \tag{2}$$

$$E_\phi = \frac{\varepsilon_0 l\pi m R^2}{u_{mn}^2 h r} J_m\left(\frac{u_{mn}}{R}r\right)\sin(m\phi)\sin\left(\frac{l\pi z}{h}\right) \tag{3}$$

$$E_z = \varepsilon_0 J_m\left(\frac{u_{mn}}{R}r\right)\cos(m\phi)\cos\left(\frac{l\pi z}{h}\right) \tag{4}$$

$$H_r = -j\omega_{mnl}\epsilon_0 \frac{\varepsilon_0 m R^2}{u_{mn}^2 r} J_m\left(\frac{u_{mn}}{R}r\right)\sin(m\phi)\cos\left(\frac{l\pi z}{h}\right) \tag{5}$$

$$H_\phi = -j\omega_{mnl}\epsilon_0 \frac{\varepsilon_0 R}{u_{mn}} J'_m\left(\frac{u_{mn}}{R}r\right)\cos(m\phi)\cos\left(\frac{l\pi z}{h}\right) \tag{6}$$

$$H_z = 0 \tag{7}$$

$$m = 0,1,2,\cdots, n = 0,1,2,\cdots, l = 0,1,2,\cdots \tag{8}$$

where u_{mn} is the nth root of the Bessel function $J_m(x)$. The resonant angular frequencies of the TM_{mnl} modes are determined by:

TM_{mnl} modes' frequncyies in a closed pill box

$$\omega_{mnl} = c\left(\left(\frac{u_{mn}}{R}\right)^2 + \left(\frac{l\pi}{h}\right)^2\right)^{1/2} \tag{9}$$

According to Reference 2 the power dissipation P_{mnl}, stored energy U_{mnl} and quality factor $Q_{0,mnl}$ are expressed as:

$$P_{mnl} = \frac{R_{s,m}\omega_{mnl}^2\epsilon_0^2\varepsilon_0^2\pi R^3 J_{m+1}^2(u_{mn})}{2\xi u_{mn}^2}\left(R + \frac{h}{2\delta}\right) \tag{10}$$

$$U_{mnl} = \frac{\omega_{mnl}^2\epsilon_0^2\mu_0 h\varepsilon_0^2\pi R^4 J_{m+1}^2(u_{mn})}{8\delta\xi u_{mn}^2} \tag{11}$$

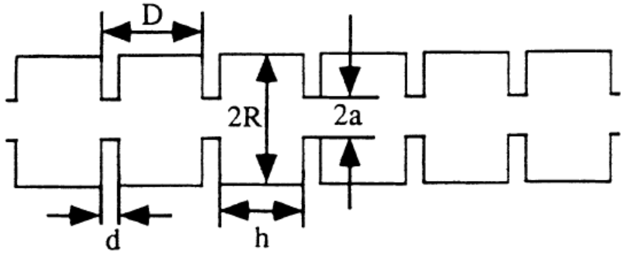
$$Q_{0,mnl} = \frac{Z_0 R\left((\frac{u_{mn}}{R})^2 + (\frac{l\pi}{h})^2\right)^{1/2}}{2R_{s,m}\left(1 + \frac{2R\delta}{h}\right)} \tag{12}$$

where

$$\delta = \{^{1, l\neq 0}_{1/2, l=0} \tag{13}$$

$$\xi = \{^{1, m\neq 0}_{1/2, m=0} \tag{14}$$

$R_{s,m}$ and Z_0 are the metal surface resistance and vacuum impedance, respectively.



$$P = -\frac{2}{3}a^3\epsilon_0 E_0$$

$$M_1 = M_2 = \frac{4}{3}a^3\mu_0 H_{1,2}$$

$$P = -\frac{\pi l_1^3(1-e_0^2)}{3E(e_0)}\epsilon_0 E_0$$

$$M_1 = \frac{\pi l_1^3 e_0^2}{3(K(e_0)-E(e_0))}\mu_0 H_1$$

$$M_2 = \frac{\pi l_1^3 e_0^2(1-e_0^2)}{3(E(e_0)-(1-e_0^2)K(e_0))}\mu_0 H_2$$

$$e_0 = \left(1-\left(\frac{l_2}{l_1}\right)^2\right)^{1/2}$$

$$\omega^2_{\theta_0,e} = \omega^2_{\pi/2,e}\left(1-\frac{4a^3\cos(\theta_0)}{3\pi h R^2 J_1^2(u_{01})}e^{-\alpha_e d}\right)$$

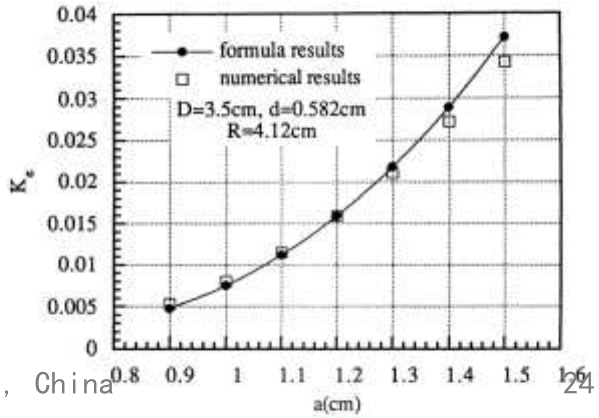
$$\omega^2_{\pi/2,e} = \omega^2_{010}\left(1+\frac{4a^3}{3\pi h R^2 J_1^2(u_{01})}\right)$$

$$\alpha_e = \frac{2\pi}{\lambda}\left(\left(\frac{\lambda}{2.62a}\right)^2-1\right)^{1/2}$$

Dispersion relation
$$\omega^2_{\theta_0,e} = \omega^2_{\pi/2,e}(1-K_e\cos(\theta_0))$$

Coupling coefficient
$$K_e = \frac{4a^3}{3\pi h R^2 J_1^2(u_{01})}e^{-\alpha_e d}$$

Group velocity
$$\frac{v_{g,e}}{c} = \frac{1}{c}\frac{d\omega_{\theta_0,e}}{d\beta_0} = \frac{\omega^2_{\pi/2,e}K_e D\sin(\theta_0)}{2c\omega_{\theta_0,e}}$$



TM_{110} Dispersion relation

$$\omega^2_{\theta_0,h} = \omega^2_{\pi/2,h}\left(1 + \frac{2Na^3\mu_0 H_0^2\cos(\theta_0)}{3U_{110}}e^{-\alpha_h d}\right)$$

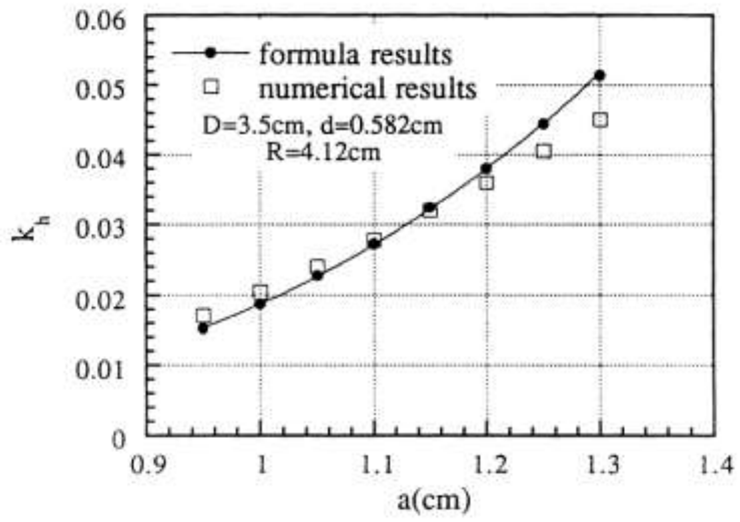
$$\omega^2_{\pi/2,h} = \omega^2_{110}\left(1 - \frac{2Na^3\mu_0 H_0^2}{3U_{110}}\right)$$

$$\omega^2_{\theta_0,h} = \omega^2_{\pi/2,h}\left(1 + \frac{4a^3\cos(\theta_0)}{3\pi h R^2 J_2^2(u_{11})}e^{-\alpha_h d}\right)$$

$$\omega^2_{\pi/2,h} = \omega^2_{110}\left(1 - \frac{4a^3}{3\pi h R^2 J_2^2(u_{11})}\right)$$

$$\alpha_h = \frac{2\pi}{\lambda}\left(\left(\frac{\lambda}{3.41a}\right)^2 - 1\right)^{1/2}$$

$$\omega^2_{\theta_0,h} = \omega^2_{\pi/2,h}(1 + K_h\cos(\theta_0))$$



Coupling coefficient

$$K_h = \frac{4a^3}{3\pi h R^2 J_2^2(u_{11})}e^{-\alpha_h d}$$

Group velocity

$$\frac{v_{g,h}}{c} = \frac{1}{c}\frac{d\omega_{\theta_0,h}}{d\beta_0} = -\frac{\omega^2_{\pi/2,h}K_h D\sin(\theta_0)}{2c\omega_{\theta_0,h}}$$

SHUNT IMPEDANCES AND WAKE POTENTIALS

The shunt impedance for the accelerating passband is defined as

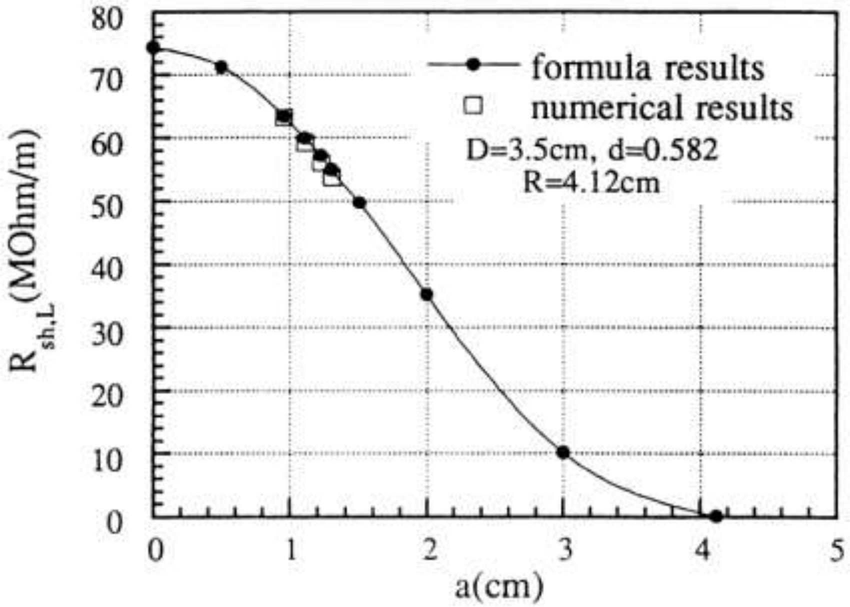
$$R_{sh,L} = \frac{E_{s,z}^2}{dP/dz}$$

Fondamental longgitudinal mode shunt impedance

$$R_{sh,L} = R_{M,T} = \frac{E_{s,z}^2 D}{P_{010}}$$

$$= \frac{D\eta_{\theta_0}^2 Z_0^2}{\pi R_{s,0} R J_1^2(u_{01})(R+h)}$$

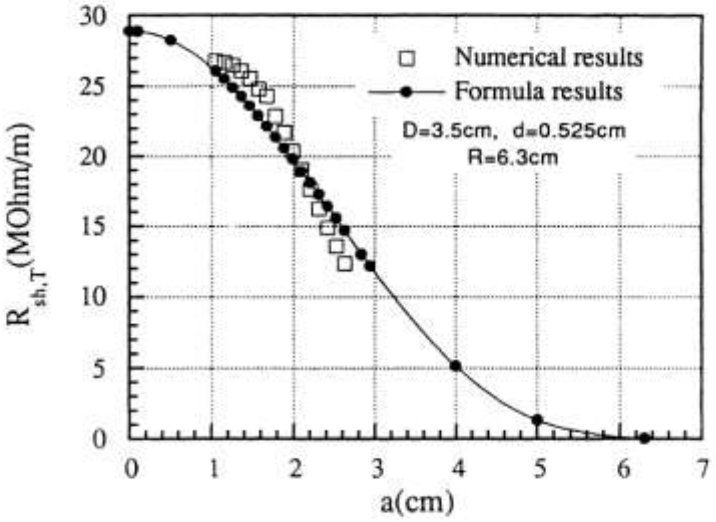
$$\eta_{\theta_0} = \frac{2}{\theta_0}\sin(\frac{\theta_0 h}{2D})$$



Fundamental transverel mode shunt impedance

$$R_{sh,L} = R_{M,T} = \frac{E_{s,z}^2 D}{P_{010}}$$

$$= \frac{D\eta_{\theta_0}^2 Z_0^2}{\pi R_{s,0} R J_1^2(u_{01})(R+h)}$$



$$R_{sh,L}(a) = R_{M,L} J_0^2(\frac{u_{01}}{R} a) = R_{M,L} J_0^2(2\pi \frac{a}{\lambda})$$

$$R_{sh,T} = \frac{(\frac{\partial E_{s,z}}{\partial x})^2}{k^2 dP/dz} = \frac{(\frac{E_{s,z}}{a})^2}{k^2 dP/dz}$$

$$R_{sh,T}(a) = \frac{2DZ_0^2 \eta_{\theta_0}^2 J_1^2(\frac{u_{11}}{R} a)}{\pi R_{s,1} a^2 k^2 J_2^2(u_{11}) R(R+h)}$$

$$= R_{M,T} \left(\frac{2R}{au_{11}} J_1 \left(\frac{u_{11}}{R} a \right) \right)^2 = R_{M,T} \left(\frac{\lambda}{a\pi} J_1 \left(2\pi \frac{a}{\lambda} \right) \right)^2$$

where

$$R_{M,T} = \frac{DZ_0^2 u_{11}^2 \eta_{\theta_0}^2}{2\pi R_{s,1} k^2 J_2^2(u_{11}) R^3 (R+h)}$$

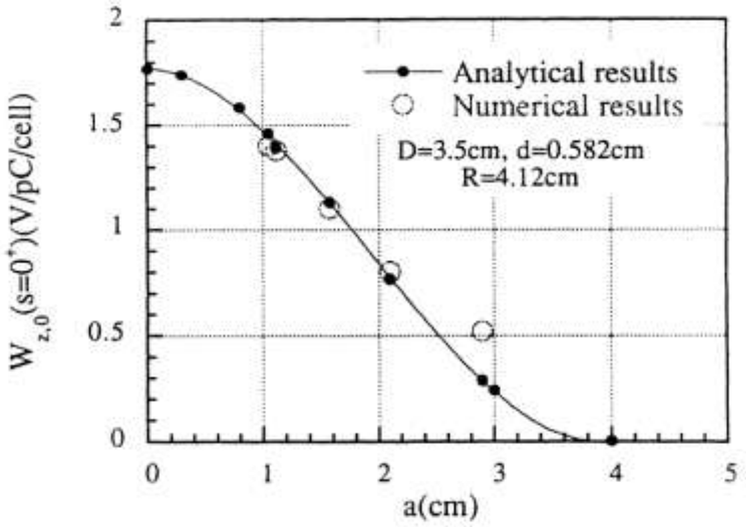
Fudamental mode loss factor

The loss factor of the fundamental mode passband is defined as[8]

$$k_0(a) = \frac{[E_{s,z}(r=a)]^2}{4dU/dz} \tag{55}$$

where dU/dz is the stored energy per unit length. Similar to getting $R_{sh,L}$, one obtains

$$k_0(a) = \frac{D\eta_{\theta_0}^2 J_0^2(\frac{u_{01}}{R}a)}{2\epsilon_0 \pi h R^2 J_1^2(u_{01})} \tag{56}$$



mental mode wake potential is

$$W_{z,0}(a,s) = 2k_0(a)\cos\left(\frac{\omega_{\theta_0,e}}{c}s\right) \tag{57}$$

the distance between the driving charge and the test charge.

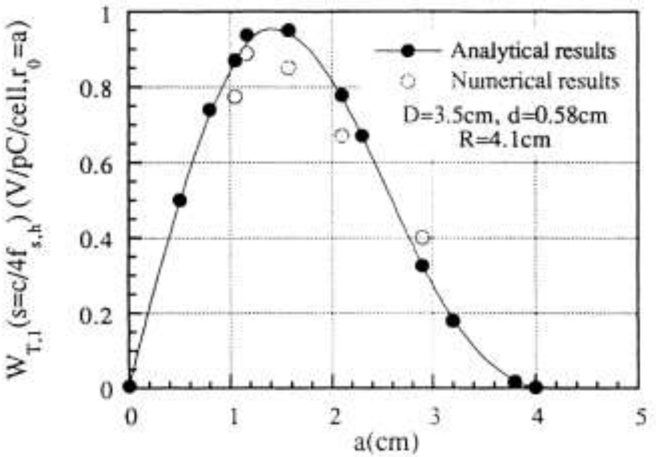
Transverse mode loss factor

The loss factor of the TM_{110} mode passband is defined as[8]

$$k_1(a) = \frac{[E_{s,t}(r=a)]^2}{4dU/dz} \tag{58}$$

Similarly one gets

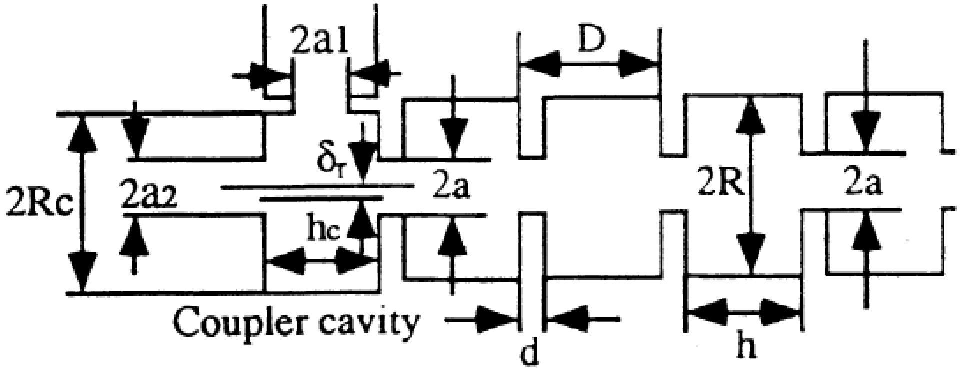
$$k_1(a) = \frac{a^2 u_{11}^2 D \eta_{\theta_0}^2}{4\pi\epsilon_0 h R^4 J_2^2(u_{11})}\left(\frac{2R}{au_{11}} J_1\left(\frac{u_{11}}{R}a\right)\right)^2 \tag{59}$$



he dipole wake potential is expressed as

$$W_{T,1}(a,s) = \frac{2cr_0 k_1(a)}{\omega_{\theta_{s,h}} a^2} \sin\left(\frac{\omega_{\theta_{s,h}}}{c}s\right)(\vec{r}\cos(\vartheta) - \vec{\vartheta}\sin(\vartheta)) \tag{60}$$

here r_0 is the driving charge's transverse deviation from the axis and $\theta_{s,h}$ is the nchronous frequency at which the test charge moves at the same velocity as that of ııe *em* wave. $\vec{r}$ and $\vec{\vartheta}$ are unit vectors, and the driving charge is assumed to be at $\vartheta = 0$.



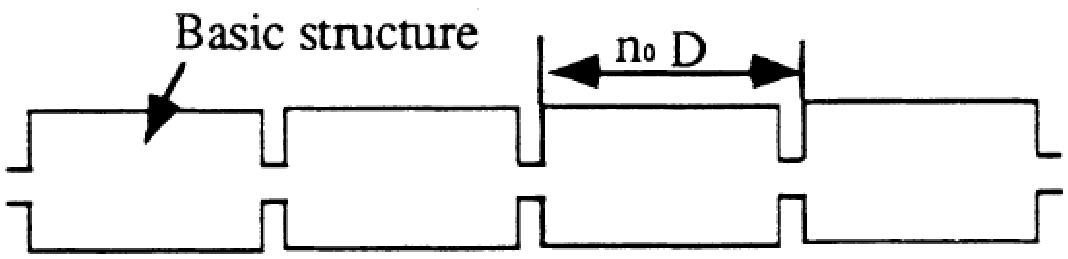
Coupler axis shift due to coupling aperture

$$\delta_r = \frac{16 a_1^3 J_1(u_{01})}{3\pi u_{01} R_c^2}$$

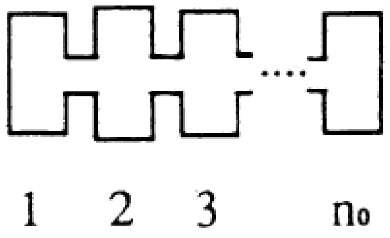
$$\delta_{r,1} = \frac{4 l_1^3 e_0^2 J_1(u_{01})}{3(K(e_0) - E(e_0)) u_{01} R_c^2}$$

$$\delta_{r,2} = \frac{4 l_1^3 e_0^2 (1 - e_0^2) J_1(u_{01})}{3(E(e_0) - (1 - e_0^2) K(e_0)) u_{01} R_c^2}$$

MULTI-PERIODIC STRUCTURE

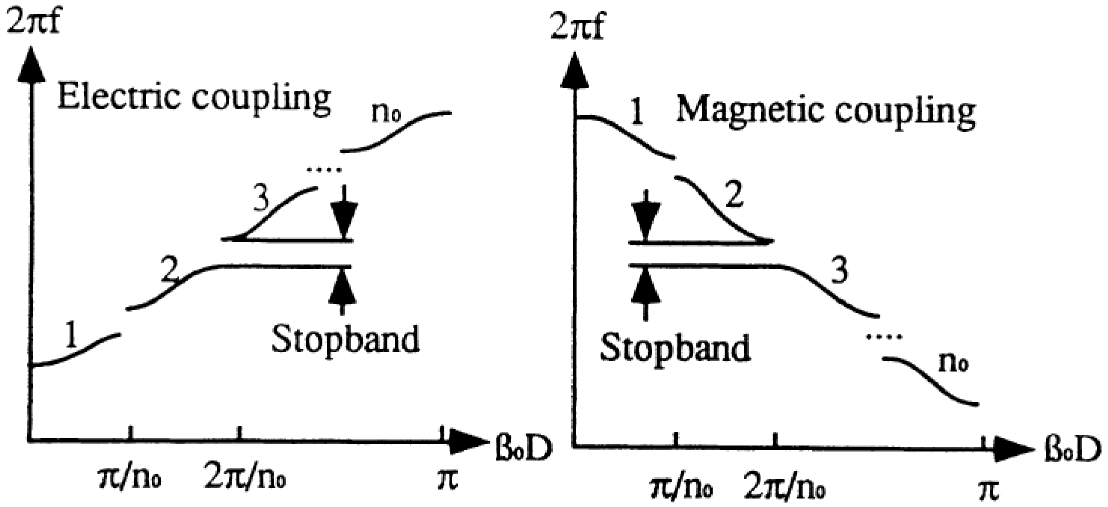


Multi-periodic structure



The coupling apertures between basic structures are closed in order to calculate $E^*E/U_{mnl,n_0}$ and $H^*H/U_{mnl,n_0}$

The detail of the basic structure



Coupling Coefficent of a Coupler Cavity

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

Analytical formula for the coupling coefficient β of a cavity–waveguide coupling system

J. Gao

Laboratoire de L'Accélérateur Linéaire, 91405 Orsay cedex, France

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INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

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Analytical determination of the coupling coefficient of waveguide cavity coupling systems

J. Gao*

Laboratoire de L'Accélérateur Linéaire, IN2P3-CNRS et Université de Paris-Sud, B.P. 34, 91898 Orsay Cedex, France

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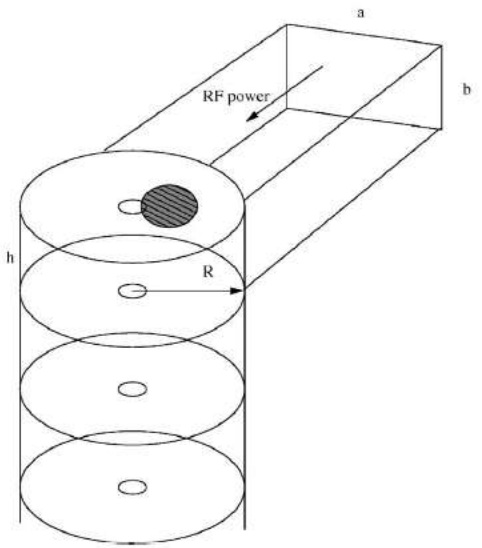
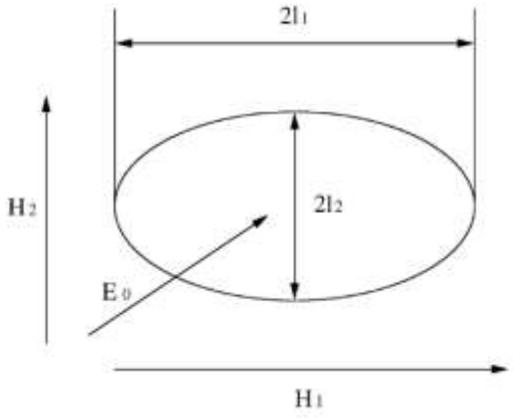


Fig 2. A waveguide cavity coupling system type II.



$$\mathbf{P} = -\frac{\pi l_1^3(1-e_0^2)}{3E_0(e_0)}\varepsilon_0 \mathbf{E}_0$$

$$\mathbf{M}_1 = \frac{\pi l_1^3 e_0^2}{3(K(e_0)-E_0(e_0))}\mu_0 \mathbf{H}_1$$

$$\mathbf{M}_2 = -\frac{\pi l_1^3 e_0^2(1-e_0^2)}{3(E_0(e_0)-(1-e_0^2)K(e_0))}\mu_0 \mathbf{H}_2$$

$$K(e_0) = \frac{\pi}{2}\left(1 + \left(\frac{1}{2}\right)^2 e_0^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 e_0^4 + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 e_0^6 + \cdots\right)$$

$$E(e_0) = \frac{\pi}{2}\left(1 - \left(\frac{1}{2}\right)^2 e_0^2 - \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \frac{e_0^4}{3} - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \frac{e_0^6}{5} - \cdots\right)$$

Coupling coefficient between waveguide and cavity

$$\beta = \frac{P}{P_0^*}$$

β=1， matching condition, no reflection

$$e_0 = \left(1 - \frac{l_2^2}{l_1^2}\right)^{1/2}$$

where P is the power radiated into the waveguide from the cavity through the coupling aperture

$P_0^* = P_0 + Uv_g/h$, P_0 is the power dissipated on the coupler cavity wall,

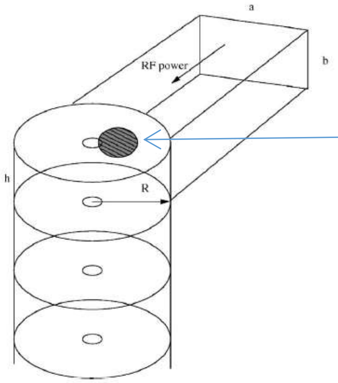


Fig. 2. A waveguide cavity coupling system type II.

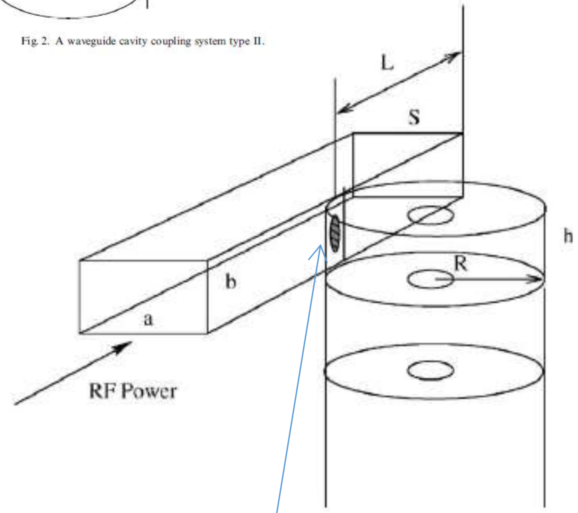


Fig. 1. A waveguide cavity coupling system type I.

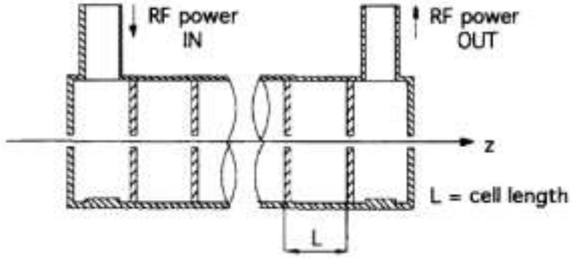
$$\beta = \frac{P}{P_0^*}$$

$$\beta = \frac{2\pi^2 k_0 k_{c,11}^2 (1-e_0^2)^2 l_1^6 \, \mathrm{e}^{-2\alpha d}}{9abZ_0 k_{11} E(e_0)^2} \times \frac{E_0^2}{P_0 + P_b + (U/L)v_g},$$

where

$$k_{c,11} = \pi\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^{1/2},$$

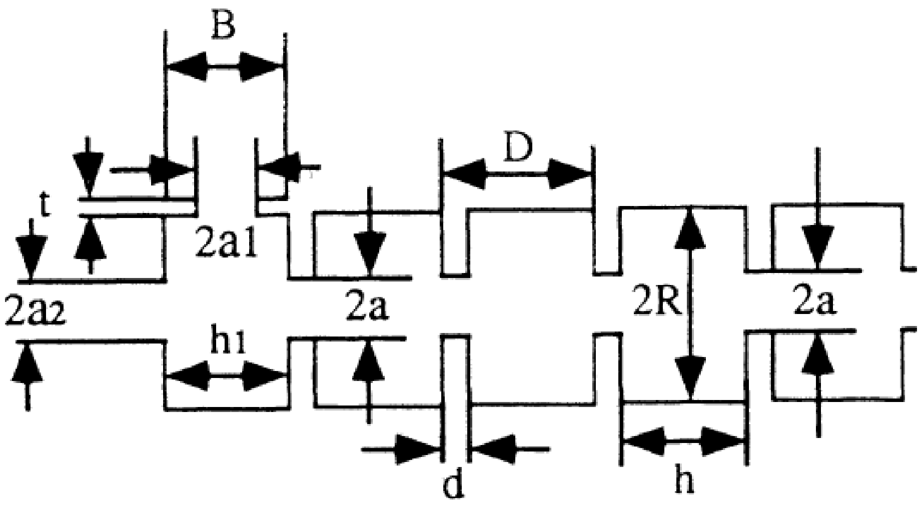
$$k_{11} = (k_0^2 - k_{c,11}^2)^{1/2}.$$

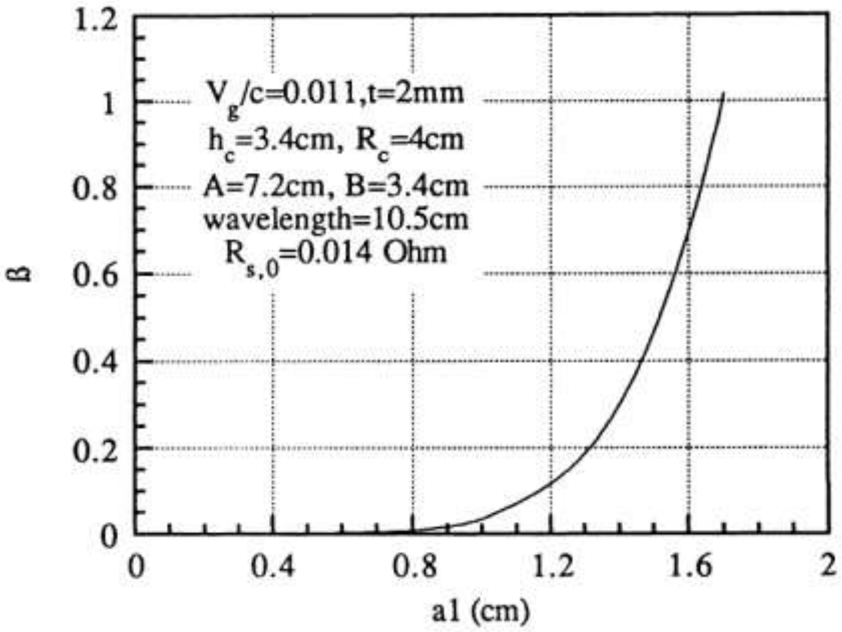


$$\beta = \frac{N\pi Z_0 k_0 \Gamma_{10} l_1^6 e_0^4 \mathrm{e}^{-2\alpha d} \sin^2(2\pi L/\lambda_{g,10})}{9abRR_s(R+h)(K(e_0)-E(e_0))^2(1+(Z_0R/2R_s(R+h))(v_g/c))}\left(\frac{\pi}{a\Gamma_{10}}\right)^2 \quad (36)$$

where R_s is the metal surface resistance. If the aperture is circular with radius r the attenuation coefficient should be expressed as $\alpha = (2\pi/\lambda)((\lambda/3.41r)^2 - 1)^{1/2}$.

$$\Gamma_{10} = k_0\left(1 - \left(\frac{\lambda}{2a}\right)^2\right)^{1/2}.$$





Coupling coefficient of coupler

$$\beta(a_1) = \frac{16 Z_0 k k_{10} a_1^6 e^{-2\alpha_c t}}{9\pi A B R_c R_{s,0}(R_c + h_c)(1 + \frac{Z_0 R_c}{2R_{s,0}(R_c+h_c)}(\frac{v_g}{c}))}$$

$$\alpha_c = \frac{2\pi}{\lambda}((\frac{\lambda}{3.41 a_1})^2 - 1)^{1/2}$$

$$k_{10} = k(1 - (\lambda/2A)^2)^{1/2}$$

Coupler cavity frequency

$$\omega_c^2 = \omega_{\theta_0}^2 = \omega_{c,010}^2 \left(1 + \frac{1}{3} a_2^3 \epsilon_0 \frac{\varepsilon_{c,0}^2}{U_{c,010}} + \frac{1}{3} a^3 \epsilon_0 \frac{\varepsilon_{c,0}^2}{U_{c,010}} - \frac{2}{3} a_1^3 \mu_0 \frac{H_c^2}{U_{c,010}} - \frac{1}{3} a^3 \epsilon_0 \frac{\varepsilon_{c,0}^2 \cos(\theta_0) e^{-\alpha_e d}}{U_{c,010}}\right)$$

Wakefields of Periodic Structures-I

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

Letter to the Editor

Analytical formulae and the scaling laws for the loss factors and the wakefields in disk-loaded periodic structures

Jie Gao
Laboratoire de L'Accélérateur Linéaire, IN2P3-CNRS et Université de Paris-Sud, 91405 Orsay cedex, France

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A
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Analytical formulae for the wakefields produced by the nonrelativistic charged particles in periodic disk-loaded structures

J. Gao*
Laboratoire de L'Accélérateur Linéaire IN2P3-CNRS et, Université Paris-Sud, B.P. 34, F-91898 Orsay cedex, France

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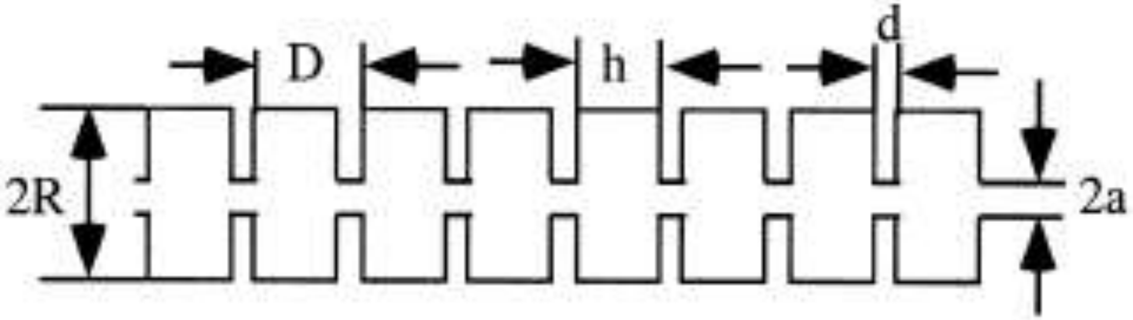


Fig. 1. Disk-loaded accelerating structure.

$$W_z(\tau) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} W_{z,mnl}(\tau)$$

$$W_r(\tau) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} W_{r,mnl}(\tau)$$

$$W_\phi(\tau) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} W_{\phi,mnl}(\tau)$$

where

$$W_{z,mnl}(\tau) = 2k_{mnl}\left(\frac{r}{a}\right)^m \left(\frac{r_q}{a}\right)^m \cos(m\phi)\cos(\omega_{mnl}\tau)$$

$$W_{r,mnl}(\tau) = 2m\frac{ck_{mnl}}{\omega_{mnl}a}\left(\frac{r}{a}\right)^{m-1}\left(\frac{r_q}{a}\right)^m \cos(m\phi)\sin(\omega_{mnl}\tau)$$

$$W_{\phi,mnl}(\tau) = -2m\frac{ck_{mnl}}{\omega_{mnl}a}\left(\frac{r}{a}\right)^{m-1}\left(\frac{r_q}{a}\right)^m \sin(m\phi)\sin(\omega_{mnl}\tau)$$

$$\omega_{mnl}^2 = c^2\left(\left(\frac{u_{mn}}{R}\right)^2 + \left(\frac{l\pi}{h}\right)^2\right)$$

$$W_{G,z}(\tau) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} W_{z,mnl}(\tau) \exp\left(-\frac{\omega_{mnl}^2 \sigma_t^2}{2} \right) \tag{12}$$

$$W_{G,r}(\tau) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} W_{r,mnl}(\tau) \exp\left(-\frac{\omega_{mnl}^2 \sigma_t^2}{2} \right) \tag{13}$$

$$W_{G,\phi}(\tau) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} W_{\phi,mnl}(\tau) \exp\left(-\frac{\omega_{mnl}^2 \sigma_t^2}{2} \right). \tag{14}$$

For the mth mode the total loss factor of a Gaussian bunch will be

$$K_m(\sigma_t) = \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} k_{mnl}(\sigma_t) = \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} k_{mnl} \exp(-\omega_{mnl}^2 \sigma_t^2). \tag{15}$$

The general expression of the loss factor k_{mnl} corresponding to the mnlth passband [4] is generalized as

$$k_{mnl} = \frac{2\xi h u_{mn}^2 J_m^2((u_{mn}/R)a)}{((u_{mn}/R)^2 + (l\pi/h)^2)\varepsilon_0 D\pi R^4 J_{m+1}^2(u_{mn})} \times \left(\frac{S(x_1)^2 + S(x_2)^2}{4}\right), \tag{16}$$

where

$$\xi = \begin{cases} 1, m \neq 0 \\ \frac{1}{2}, m = 0 \end{cases} \quad (17)$$

$$S(x) = \frac{\sin(x)}{x} \quad (18)$$

and

$$x_1 = \frac{h}{2\beta}\left(\left(\left(\frac{u_{mn}}{R}\right)^2 + \left(\frac{l\pi}{h}\right)^2\right)^{1/2} - \frac{l\pi}{h}\right) \quad (19)$$

$$x_2 = \frac{h}{2\beta}\left(\left(\left(\frac{u_{mn}}{R}\right)^2 + \left(\frac{l\pi}{h}\right)^2\right)^{1/2} + \frac{l\pi}{h}\right). \quad (20)$$

When $\beta = 1$, by setting $m = 0, n = 1$, and $l = 0$, one gets from Eq. (16) the point charge fundamental mode loss factor of a disk-loaded structure as obtained before in Ref. [5]:

$$k_{010} = \frac{2J_0^2((u_{0n}/R)a)\sin^2(u_{01}h/2R)}{\varepsilon_0 \pi h D J_1^2(u_{0n})u_{01}^2}. \quad (21)$$

Obviously when $a = 0$ and $h = D$, Eq. (21) gives the point charge fundamental mode loss factor of a closed pill-box cavity, and when $a = R$ one gets $k_{mnl} \equiv 0$, which corresponds to a round beam pipe without resistive losses.

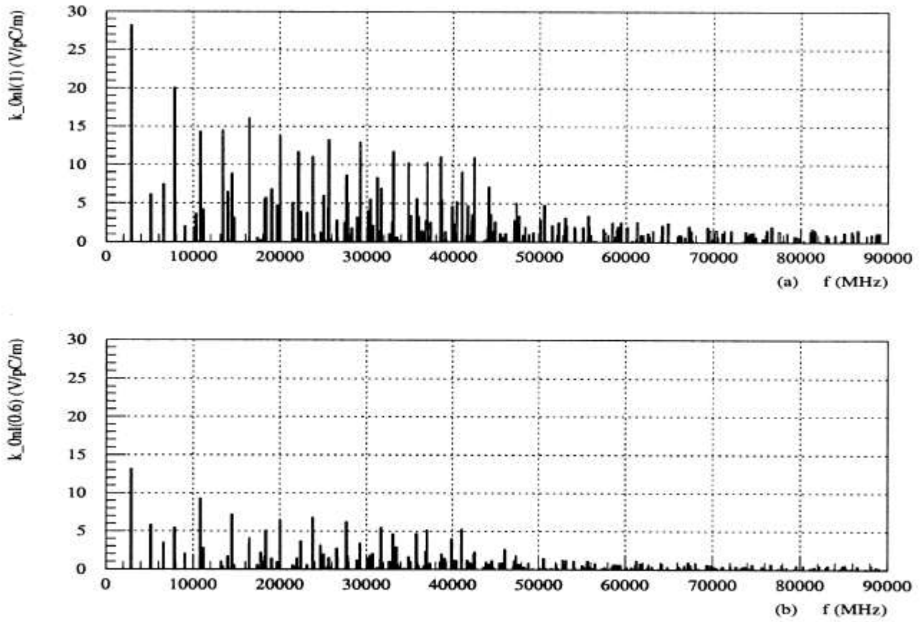


Fig. 4. A closed pill-box case: $a = 0$, $h = D = 0.035$ m and $\sigma_z = 0.01$ m. The monopole mode loss factors versus the frequency: (a) $\beta = 1$ and (b) $\beta = 0.6$.

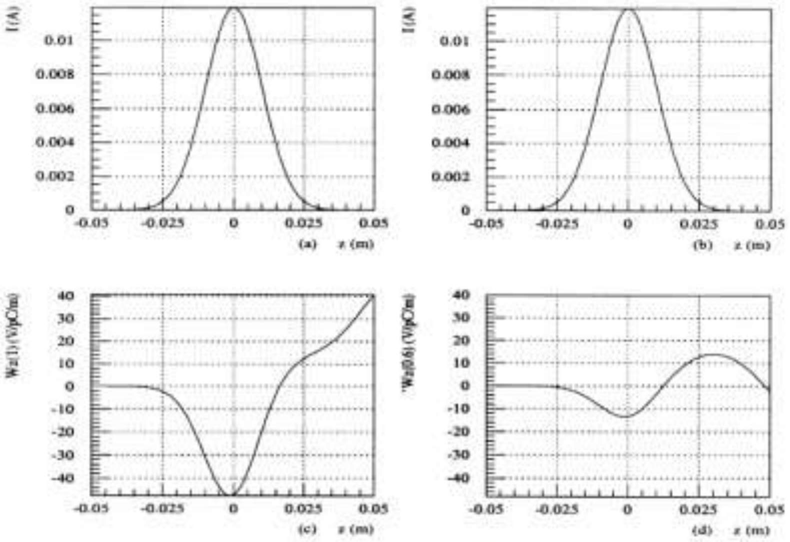
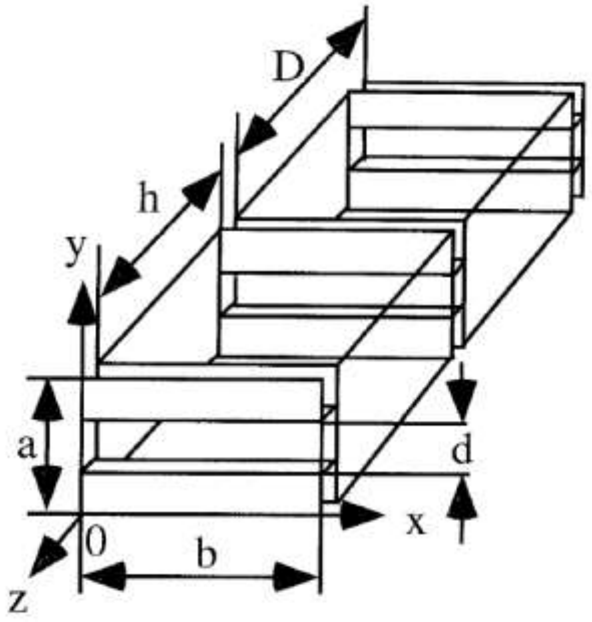


Fig. 5. A closed pill-box case: $a = 0$, $h = D = 0.035$ m and $\sigma_z = 0.01$ m. (a) and (b) are the Gaussian bunch current distributions of a total charge of 1 pC. The short range longitudinal wakefields at (c) $\beta = 1$, and (d) $\beta = 0.6$.

Wakefields of Periodic Structures-II

Analytical formulae for the loss factors and wakefields of a rectangular accelerating structure

Proceedings of EPAC 96, 1996



$$W_x(\tau) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\sum_{l=0}^{\infty} W_{x,mnl}(\tau) \quad (6.14)$$

$$W_y(\tau) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\sum_{l=0}^{\infty} W_{y,mnl}(\tau) \quad (6.15)$$

$$W_z(\tau) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\sum_{l=0}^{\infty} W_{z,mnl}(\tau) \quad (6.16)$$

where $W_{x,mnl}$, $W_{y,mnl}$ and $W_{z,mnl}$ are the wakefields corresponding to the mnlth synchronous mode. To find out the expressions of $W_{x,mnl}$, $W_{y,mnl}$ and $W_{z,mnl}$, one has to use the generalized Panofsky-Wenzel theorem derived in ref. 3. We know therefore that in a cartesian coordinate system

$$W_{x,mnl}(s) = Z_l(s)\frac{\partial T_{mn}(x,y)}{\partial x} \quad (6.17)$$

$$W_{y,mnl}(s) = Z_l(s)\frac{\partial T_{mn}(x,y)}{\partial y} \quad (6.18)$$

$$W_{z,mnl}(s) = T_{mn}(x,y)\frac{dZ_l(s)}{ds} \quad (6.19)$$

Genrerlized Panofsky-Wenzel theorem

where $s = \tau c$, c is the velocity of light in vacuum and s is the distance between the exciting charge and a test charge. $T_{mn}(x,y)$ and $Z_l(s)$ satisfy the following equations:

$$Z_l(s)\frac{\partial^2 T_{mn}(x,y)}{\partial x^2} + Z_l(s)\frac{\partial^2 T_{mn}(x,y)}{\partial y^2} - T_{mn}(x,y)\frac{d^2 Z_l(s)}{dz^2} = 0 \quad (6.20)$$

It is found that

$$W_{G,z}(\tau) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\sum_{l=0}^{\infty} W_{z,mnl}(\tau)\exp\left(-\frac{\omega_{mnl}^2\sigma_t^2}{2}\right)$$

$$W_{G,x}(\tau) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\sum_{l=0}^{\infty} W_{x,mnl}(\tau)\exp\left(-\frac{\omega_{mnl}^2\sigma_t^2}{2}\right)$$

$$W_{G,y}(\tau) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\sum_{l=0}^{\infty} W_{y,mnl}(\tau)\exp\left(-\frac{\omega_{mnl}^2\sigma_t^2}{2}\right)$$

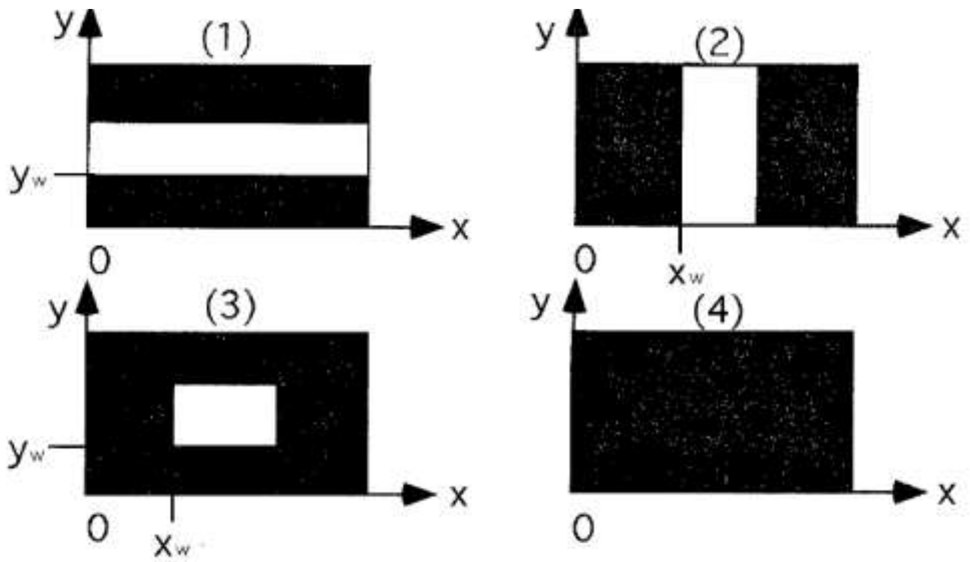


Figure 6.2: Four types of coupling apertures.

$$W_{z,mnl}(\tau) = 2k_{mnl}\frac{\sin\left(\frac{m\pi x}{b}\right)\sin\left(\frac{n\pi y}{a}\right)}{\sin\left(\frac{m\pi x_w}{b}\right)\sin\left(\frac{n\pi y_w}{a}\right)}\times$$

$$\frac{\sin\left(\frac{m\pi x_q}{b}\right)\sin\left(\frac{n\pi y_q}{a}\right)}{\sin\left(\frac{m\pi x_w}{b}\right)\sin\left(\frac{n\pi y_w}{a}\right)}\cos(\omega_{mnl}\tau)$$

$$W_{x,mnl}(\tau) = 2m\frac{c\pi k_{mnl}}{\omega_{mnl}b}\frac{\cos\left(\frac{m\pi x}{b}\right)\sin\left(\frac{n\pi y}{a}\right)}{\sin\left(\frac{m\pi x_w}{b}\right)\sin\left(\frac{n\pi y_w}{a}\right)}\times$$

$$\frac{\sin\left(\frac{m\pi x_q}{b}\right)\sin\left(\frac{n\pi y_q}{a}\right)}{\sin\left(\frac{m\pi x_w}{b}\right)\sin\left(\frac{n\pi y_w}{a}\right)}\sin(\omega_{mnl}\tau)$$

$$W_{y,mnl}(\tau) = 2n\frac{c\pi k_{mnl}}{\omega_{mnl}a}\frac{\sin\left(\frac{m\pi x}{b}\right)\cos\left(\frac{n\pi y}{a}\right)}{\sin\left(\frac{m\pi x_w}{b}\right)\sin\left(\frac{n\pi y_w}{a}\right)}\times$$

$$\frac{\sin\left(\frac{m\pi x_q}{b}\right)\sin\left(\frac{n\pi y_q}{a}\right)}{\sin\left(\frac{m\pi x_w}{b}\right)\sin\left(\frac{n\pi y_w}{a}\right)}\sin(\omega_{mnl}\tau)$$

$$W_{z,mnl}(\tau) = 2k_{mnl,i}\sin\left(\frac{m\pi x}{b}\right)\sin\left(\frac{n\pi y}{a}\right)\times$$

$$\sin\left(\frac{m\pi x_q}{b}\right)\sin\left(\frac{n\pi y_q}{a}\right)\cos(\omega_{mnl}\tau)$$

$$W_{x,mnl}(\tau) = 2m\frac{c\pi k_{mnl,i}}{\omega_{mnl}b}\cos\left(\frac{m\pi x}{b}\right)\sin\left(\frac{n\pi y}{a}\right)\times$$

$$\sin\left(\frac{m\pi x_q}{b}\right)\sin\left(\frac{n\pi y_q}{a}\right)\sin(\omega_{mnl}\tau)$$

$$W_{y,mnl}(\tau) = 2n\frac{c\pi k_{mnl,i}}{\omega_{mnl}a}\sin\left(\frac{m\pi x}{b}\right)\cos\left(\frac{n\pi y}{a}\right)\times$$

$$\sin\left(\frac{m\pi x_q}{b}\right)\sin\left(\frac{n\pi y_q}{a}\right)\sin(\omega_{mnl}\tau)$$

$$k_{mnl,1} = k^*_{mnl}\sin^2\left(\frac{n\pi y_w}{a}\right)$$

$$k_{mnl,2} = k^*_{mnl}\sin^2\left(\frac{m\pi x_w}{b}\right)$$

$$k_{mnl,3} = k^*_{mnl}\sin^2\left(\frac{n\pi y_w}{a}\right)\sin^2\left(\frac{m\pi x_w}{b}\right)$$

$$k_{mnl,4} = k^*_{mnl}$$

$$k_{mnl} = \frac{E_{s,z}^{mnl}(x = x_w, y = y_w)^2 D}{4U_{mnl}}$$

$$= \frac{4h\left((m\pi/b)^2 + (n\pi/a)^2\right)\sin^2\left(\frac{m\pi x_w}{b}\right)\sin^2\left(\frac{n\pi y_w}{a}\right)}{\epsilon_0 abD\left((m\pi/b)^2 + (n\pi/a)^2 + (l\pi/h)^2\right)}\left(\frac{S(x_1)^2 + S(x_2)^2}{2}\right)$$

$$= k_{mnl}^* \sin^2\left(\frac{m\pi x_w}{b}\right)\sin^2\left(\frac{n\pi y_w}{a}\right)$$

where

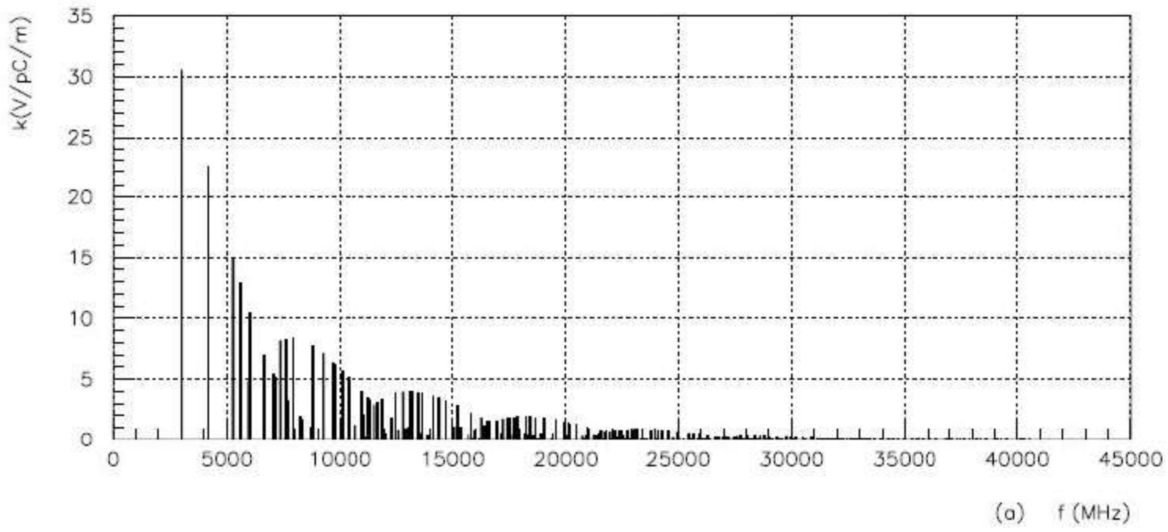
$$k_{mnl}^* = \frac{4h\left((m\pi/b)^2 + (n\pi/a)^2\right)}{\epsilon_0 abD\left((m\pi/b)^2 + (n\pi/a)^2 + (l\pi/h)^2\right)}\left(\frac{S(x_1)^2 + S(x_2)^2}{4}\right)$$

$$S(x) = \frac{\sin(x)}{x}$$

and

$$x_1 = \frac{h}{2}\left(\left(\left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2 + \left(\frac{l\pi}{h}\right)^2\right)^{1/2} - \frac{l\pi}{h}\right)$$

$$x_2 = \frac{h}{2}\left(\left(\left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2 + \left(\frac{l\pi}{h}\right)^2\right)^{1/2} + \frac{l\pi}{h}\right)$$



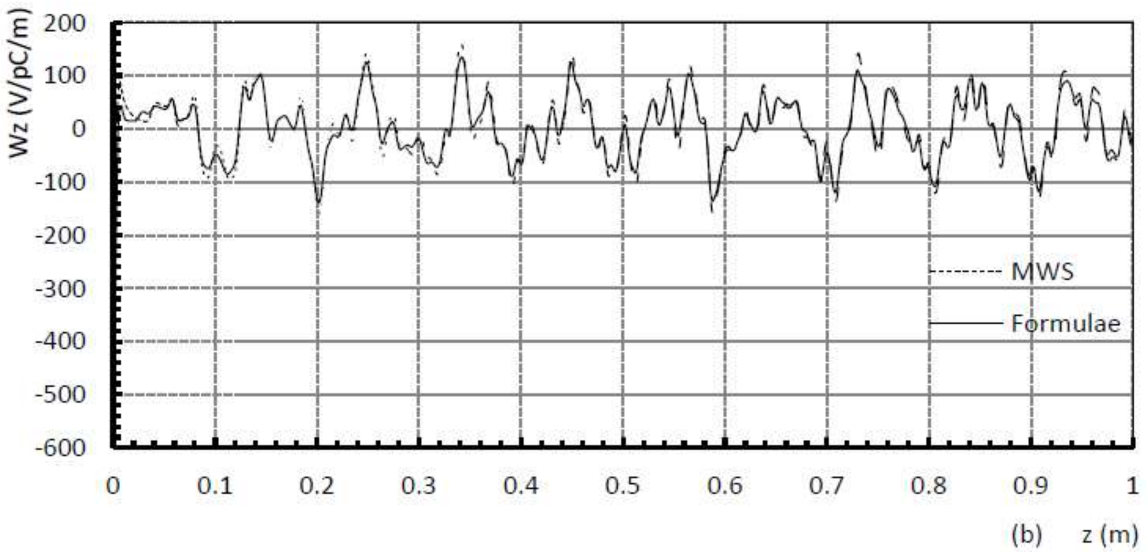


Figure 3: (a) Loss factors $k_{mnl,4}(\sigma_z)$ vs frequency, and (b) $W_{G,z}$ (V/pC/m) vs distance calculated by formulae and CST-MWS. For both figures σ_z=2.5 mm, $x=x_q= b/2$, and $y=y_q= a/2$. The dimension of the structure: D=h=2.92 cm, a=6 cm, and b=9 cm.

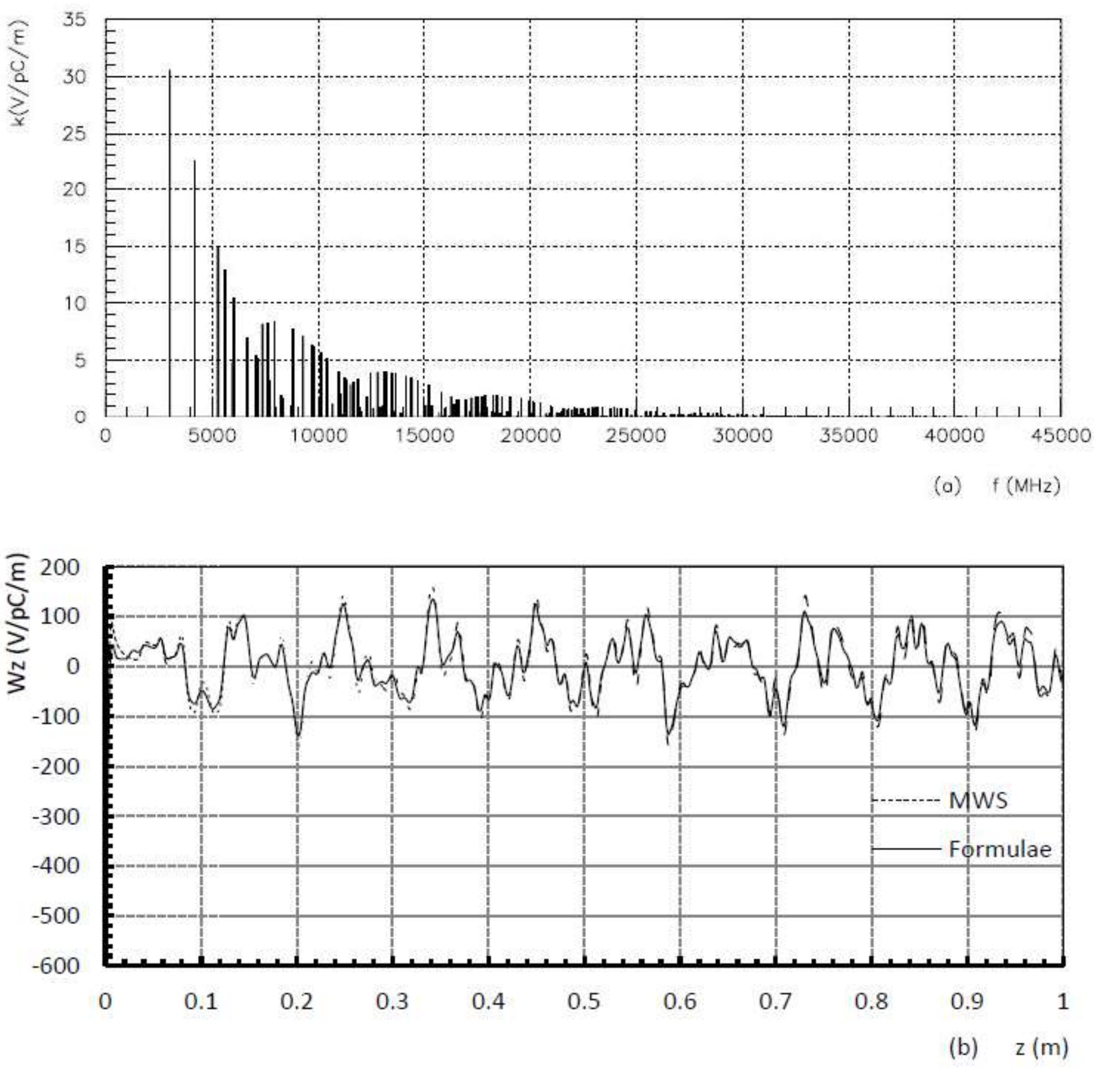


Figure 3: (a) Loss factors $k_{mnl,4}(\sigma_z)$ vs frequency, and (b) $W_{G,z}$ (V/pC/m) vs distance calculated by formulae and CST-MWS. For both figures σ_z=2.5 mm, $x=x_q= b/2$, and $y=y_q= a/2$. The dimension of the structure: D=h=2.92 cm, a=6 cm, and b=9 cm.

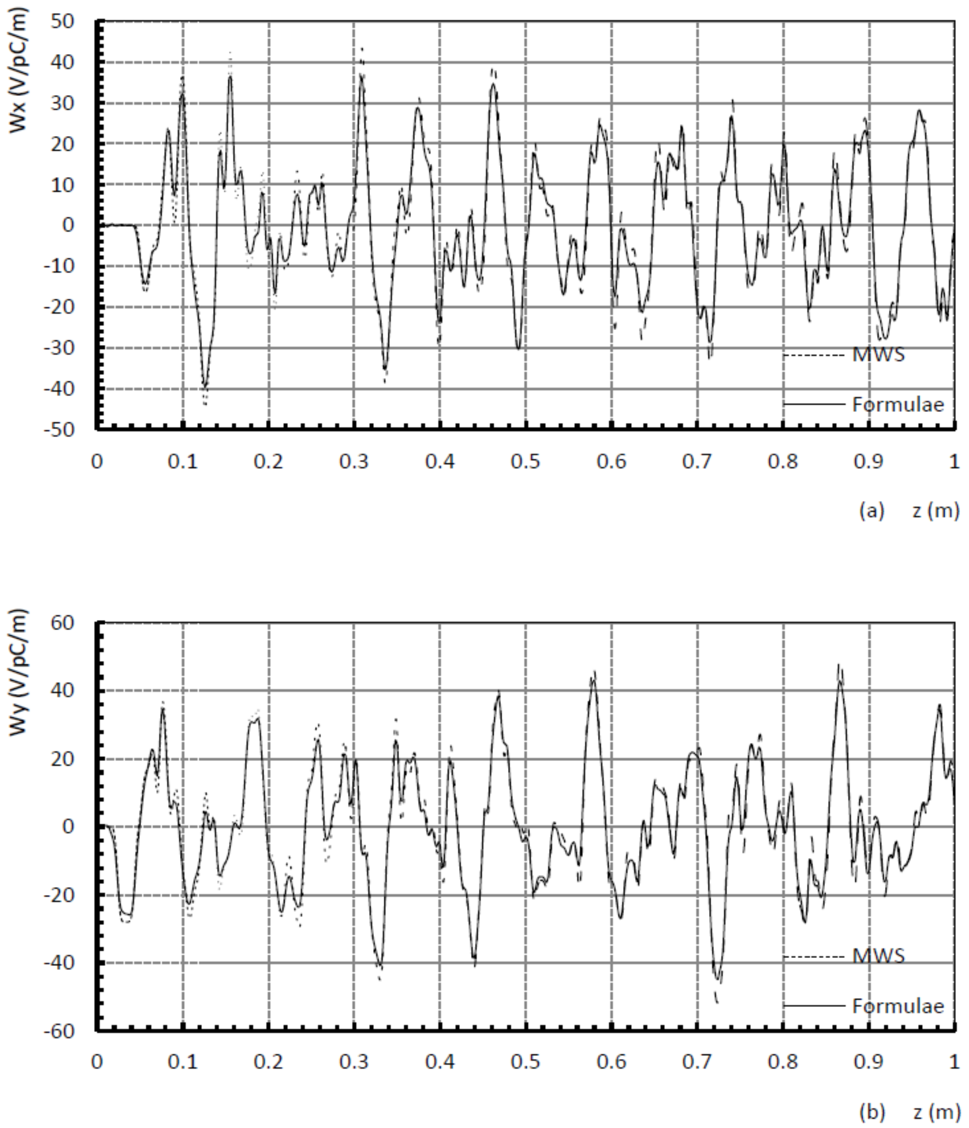


Figure 4: (a) $W_{G,x}$ (V/pC/m) vs distance calculated by formulae and CST-MWS, and (b) $W_{G,y}$ (V/pC/m) vs distance calculated by formulae and CST-MWS. For both figures σ_z=2.5 mm, $x=x_q= b/2$+0.8 cm, and $y=y_q= a/2$+0.8 cm. The dimension of the structure: D=h=2.92 cm, a=6 cm, and b=9 cm.

Coupling between Cavities with Losses

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

Letter to the Editor

The criterion for the coupling states between cavities with losses

Jie Gao

Laboratoire de L'Accélérateur Linéaire, IN2P3-CNRS et Université de Paris-Sud, 91405 Orsay cedex, France

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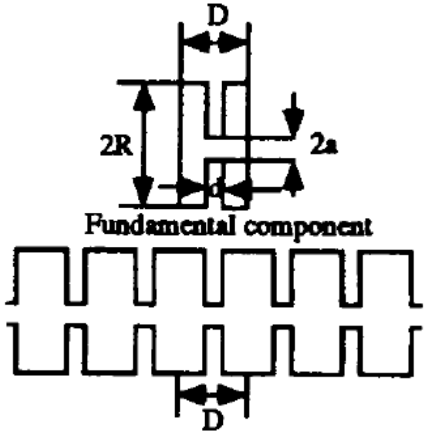


Fig. 1. A two coupled cavity system is the fundamental component of the linear accelerator structure.

$$\frac{dW}{dt} = -\frac{\omega}{Q_0}W + \frac{W}{L}v_g = 0$$

It is necessary to mention that for a travelling wave structure one has always

$$v_g > \frac{\omega}{Q_0}L \tag{3}$$

and the power out of the structure is

$$P_{out} = \left(\frac{v_g}{L} - \frac{\omega}{Q_0}\right)W. \tag{4}$$

$$1 - k\cos(\theta) = \frac{kQ_0}{2}|\sin(\theta)|, \tag{6}$$

where $\theta = \beta D$. Since, usually, $k \ll 1$ and $Q_0 \gg 1$, Eq. (6) can be simplified as

$$1 = \frac{kQ_0}{2}|\sin(\theta)|, \tag{7}$$

$$\theta_1 = \arcsin\left(\frac{kQ_0}{2}\right), \tag{8}$$

$$\theta_2 = \pi - \theta_1. \tag{9}$$

$$1 - k\cos(N\theta) = \frac{kQ_0}{2}|\sin(N\theta)|, \tag{13}$$

$$1 = \frac{kQ_0}{2}|\sin(N\theta)|. \tag{14}$$

If $N = 2$ the solution for θ can be found in Fig. 4. If $kQ_0 \approx \infty$ there will be only three solutions: $\theta_1 = 0$, $\theta_2 = \pi/2$ and $\theta_3 = \pi$; otherwise there are four solutions of θ

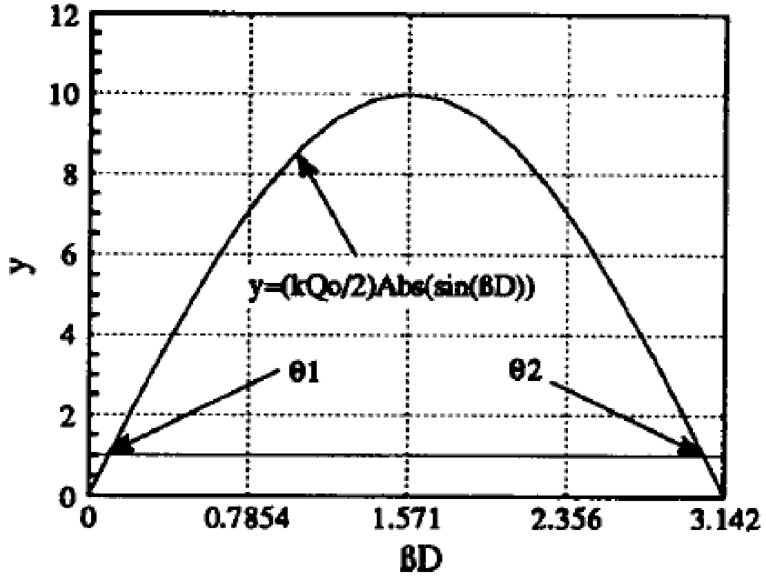


Fig. 2. Solutions of θ.

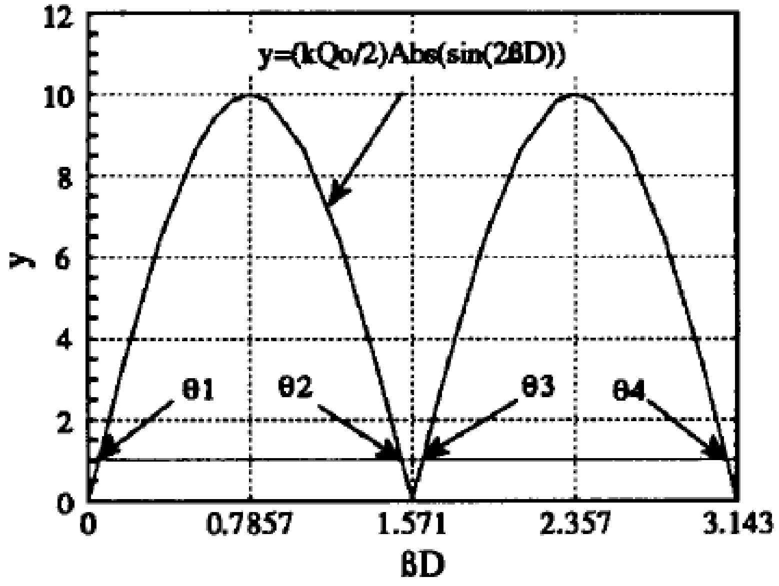


Fig. 4. Solutions of θ.

This paper gives a practical criterion to determine the coupling state of a multicavity standing wave structure: coupled ($kQ_0 > 2$), critically coupled ($kQ_0 = 2$) or uncoupled ($kQ_0 < 2$). For a coupled multicavity standing wave structure the resonant modes (θ) can be found by solving Eq. (13) and the corresponding resonant frequency can be found from Eq. (5). The relation between the resonant frequency and kQ_0 is well established. This information is very important for the building of damped or heavily beam loaded multicavity standing wave structures.

Radio Frequency Electron Gun Theory

CPC(HEP & NP), 2009, **33**(4): 306—310 Chinese Physics C Vol. 33, No. 4, Apr., 2009

On the theory of photocathode rf guns

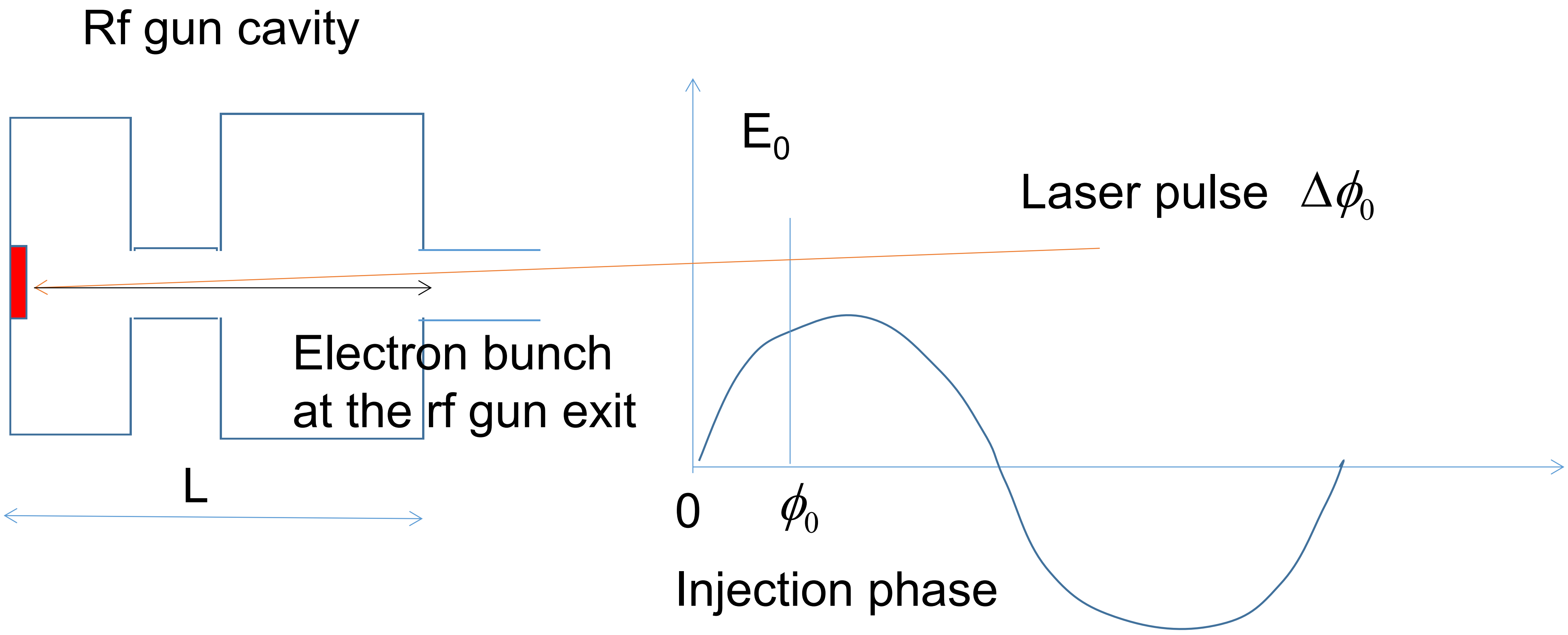
GAO Jie(高杰)
(Institute of High Energy Phyics, CAS, Beijing 100049, China)

Abstract In this paper we give a set of analytical formulae to describe the characteristics of photocathode rf guns at any rf frequencies, such as energy, energy spread, bunch length, out going current, and emittance etc. as functions of the laser injection phase, which are useful in the design and practical operation of rf guns.

Key words rf gun, photocathode, microwave rf gun

PACS 29.25.Bx

Radio Frequency Electron Gun Theory



Electric field inside the Rf gun cavity

$$E_z(z,t) = E_0 \cos(kz)\sin(\omega t + \phi_0)$$

Electron energy at the exit of the Rf gun

$$\gamma(\phi_0) = 1 + \frac{\alpha}{2}\left(kL\sin\phi_f + \frac{1}{2}\left(\cos\phi_f - \cos(\phi_f + 2kL)\right)\right)$$

$$\phi = \frac{1}{\alpha\sin(\phi_0 + \delta\phi)}\left((\Gamma^2 - 1)^{1/2} - (\Gamma - 1)\right) + \phi_0$$

Final phase of the electron

$$\phi_f(\phi_0) = \frac{1}{\alpha\sin\left(\phi_0 + \frac{\sqrt{2}\pi}{6\sqrt{\alpha}}\right)} + \phi_0 + \frac{2\pi}{15\alpha}$$

$$\Gamma = 1 + \alpha\sin(\phi_0 + \delta\phi)kz \ ,$$

$$\alpha = \frac{qE_0}{m_0c^2k} \ ,$$

$$\alpha = \frac{qE_0}{m_0c^2k}$$

Maximum electric field on cathod surface

Final energy spread of the electron bunch at the exit of rf gun

$$\Delta W(\phi_0) = m_0 c^2 \frac{\mathrm{d}\gamma(\phi_0)}{\mathrm{d}\phi_0} \Delta\phi_0$$

Final relative energy spread of the electron bunch at the exit of rf gun

$$\frac{\Delta W(\phi_0)}{W(\phi_0)} = \frac{1}{\gamma(\phi_0)} \frac{\mathrm{d}\gamma(\phi_0)}{\mathrm{d}\phi_0} \Delta\phi_0$$

Final electron bunch length at the exit of rf gun

$$\Delta\phi_{\mathrm{f}}(\phi_0) = \frac{\mathrm{d}\phi_{\mathrm{f}}(\phi_0)}{\mathrm{d}\phi_0} \Delta\phi_0$$

Final electron bunch charge at the exit of rf gun

$$Q_0(\phi_0) = eQE_0 \frac{W_1}{h\nu} \exp\left(\frac{e}{kT_{\mathrm{e}}} \sqrt{\frac{eE_0 \sin\phi_0}{4\pi\epsilon_0}}\right)$$

Cathode Quantum efficiency

$$QE(\phi_0) = QE_0 \exp\left(\frac{e}{kT_{\mathrm{e}}} \sqrt{\frac{eE_0 \sin\phi_0}{4\pi\epsilon_0}}\right)$$

Final electron bunch current at the exit of rf gun

$$I(\phi_0) = \frac{2.35\omega Q_0(\phi_0)}{\sqrt{2\pi}\Delta\phi_{\mathrm{f}}(\phi_0)}$$

Normalized emittance due to rf phase variation

$$\epsilon_{\mathrm{n,rf}}(\pi\mathrm{m}\cdot\mathrm{rad}) = 4(\langle p_{\mathrm{rf}}^2\rangle\langle r^2\rangle - \langle p_{\mathrm{rf}} r\rangle^2)^{1/2} = \frac{\alpha k \sigma_{\mathrm{r}}^2}{2\pi}\left|\cos(\phi_{\mathrm{f}}(\phi_0))\frac{\mathrm{d}\phi_{\mathrm{f}}(\phi_0)}{\mathrm{d}\phi_0}\right|\Delta\phi_0 ,$$

Normalized emittance due to space charge effect

$$\epsilon_{\mathrm{n,sp}}(\pi\mathrm{m}\cdot\mathrm{rad}) = \frac{\pi I_{\mathrm{av}}}{2\alpha k I_{\mathrm{A}}\sin(\phi_{\mathrm{f}}(\phi_0))}\left(\frac{1}{3\frac{\sigma_{\mathrm{r}}}{\sigma_{\mathrm{z}}}+5}\right), \tag{22}$$

where I_{A} is the so-called Alfvén current, $I_{\mathrm{A}} = 4\pi\epsilon_0 m_0 c^3/e = 17000$ A, σ_{r} and σ_{z} are bunch transverse and longitudinal rms dimensions, and $I_{\mathrm{av}} = I(\phi_0)/2$ (I_{av} is the full bunch length current). How-

Normalized emittance due to cathode temperature

$$\epsilon_{\mathrm{n},T}(\pi\mathrm{m}\cdot\mathrm{rad}) = \frac{\sigma_{\mathrm{r}}}{2}\sqrt{\frac{kT_{\mathrm{e}}}{m_0 c^2}}$$

Normalized emittance due to all effects of the electron bunch

$$\epsilon_{\mathrm{n,total}} = \sqrt{\epsilon_{\mathrm{n,rf}}^2 + \epsilon_{\mathrm{n,sp}}^2 + \epsilon_{\mathrm{n,T}}^2}$$

S-band 1.6Cell BNL RF Gun Example

Now we apply the formulae given above to make analytical estimations on the performances of an S-band 1.6 cell BNL type rf gun. In the analytical model we take $f = 2856$ MHz, and cavity rf gun $L = 0.8c/f$. For $E_0 = 100$ MV/m and laser FWHM pulse length of 10 ps.

Laser pulse (wavelength of 266 nm, $h\nu = 4.6$ eV) of 1 μJ illuminating a photocathode located inside the rf gun, for $QE_0 = 4 \times 10^{-5}$, $kT_e = 0.22$ eV, and $E_0 = 100$ MV/m

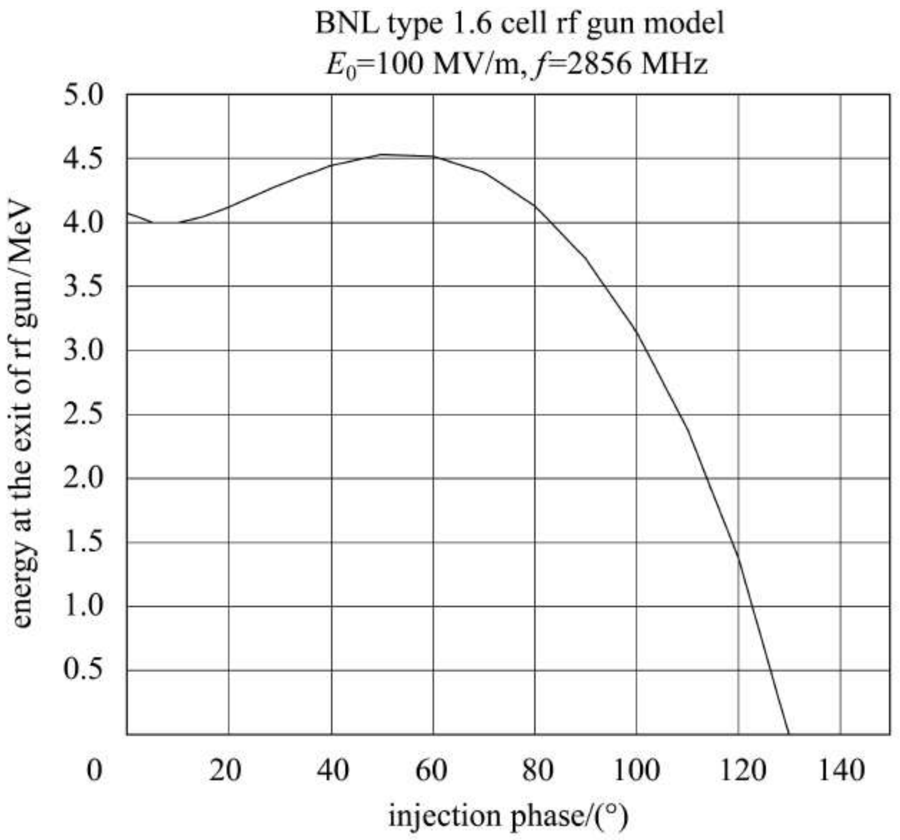


Fig. 1. Output energy vs the injection phase.

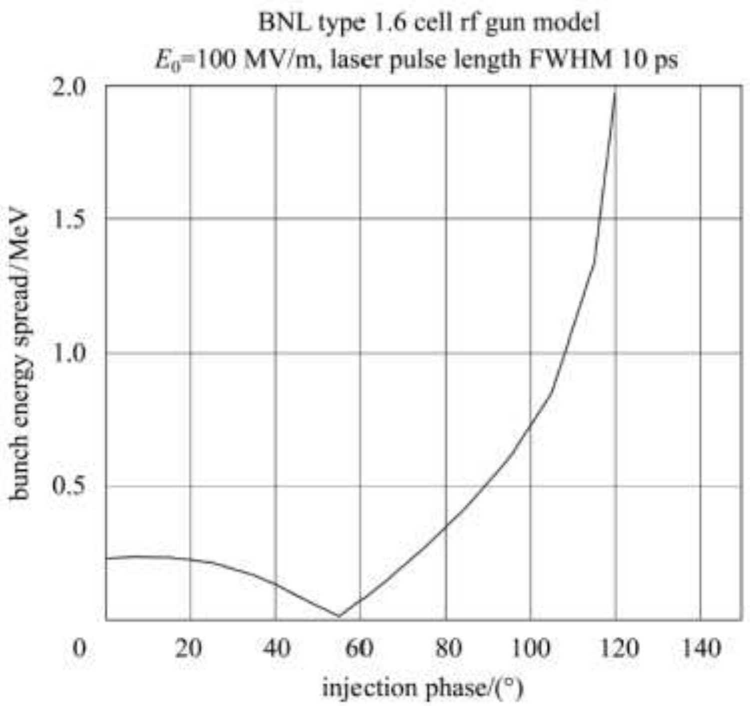


Fig. 2. Bunch energy spread FWHM vs the injection phase.

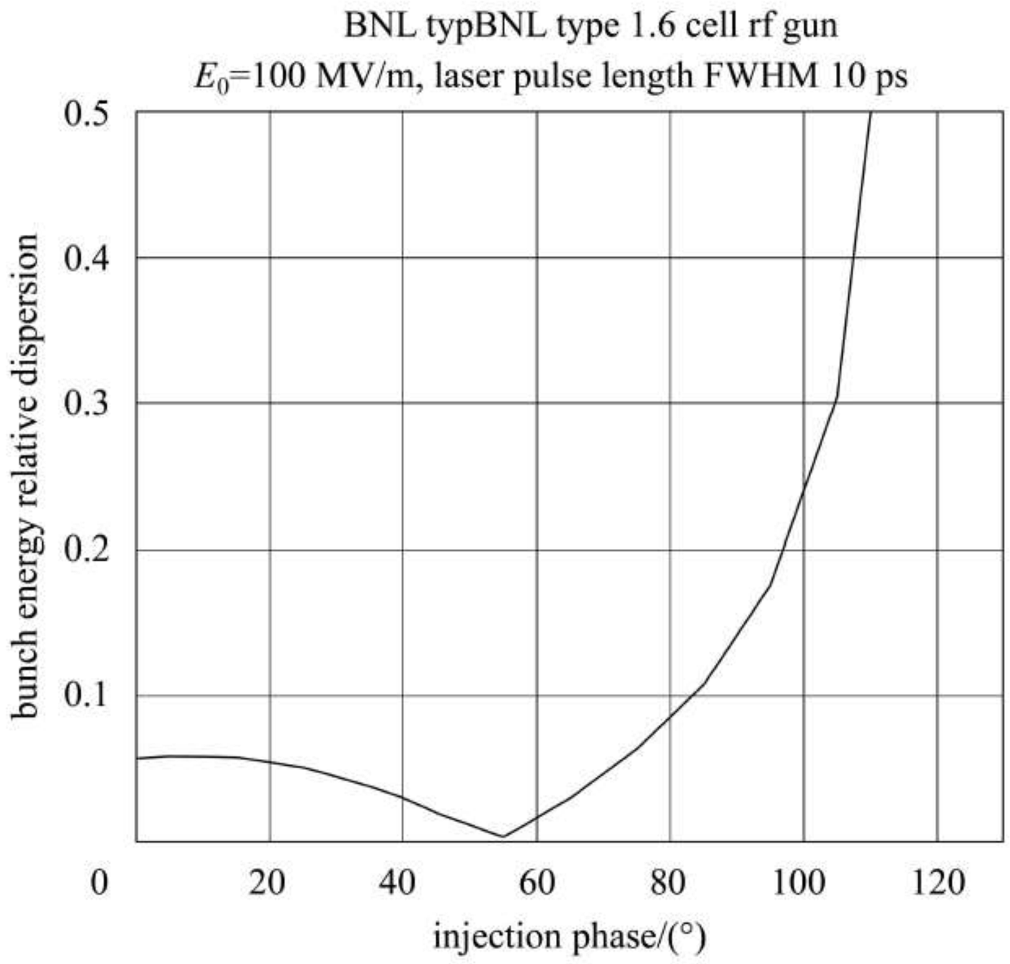


Fig. 3. Bunch relative energy dispersion vs the injection phase.

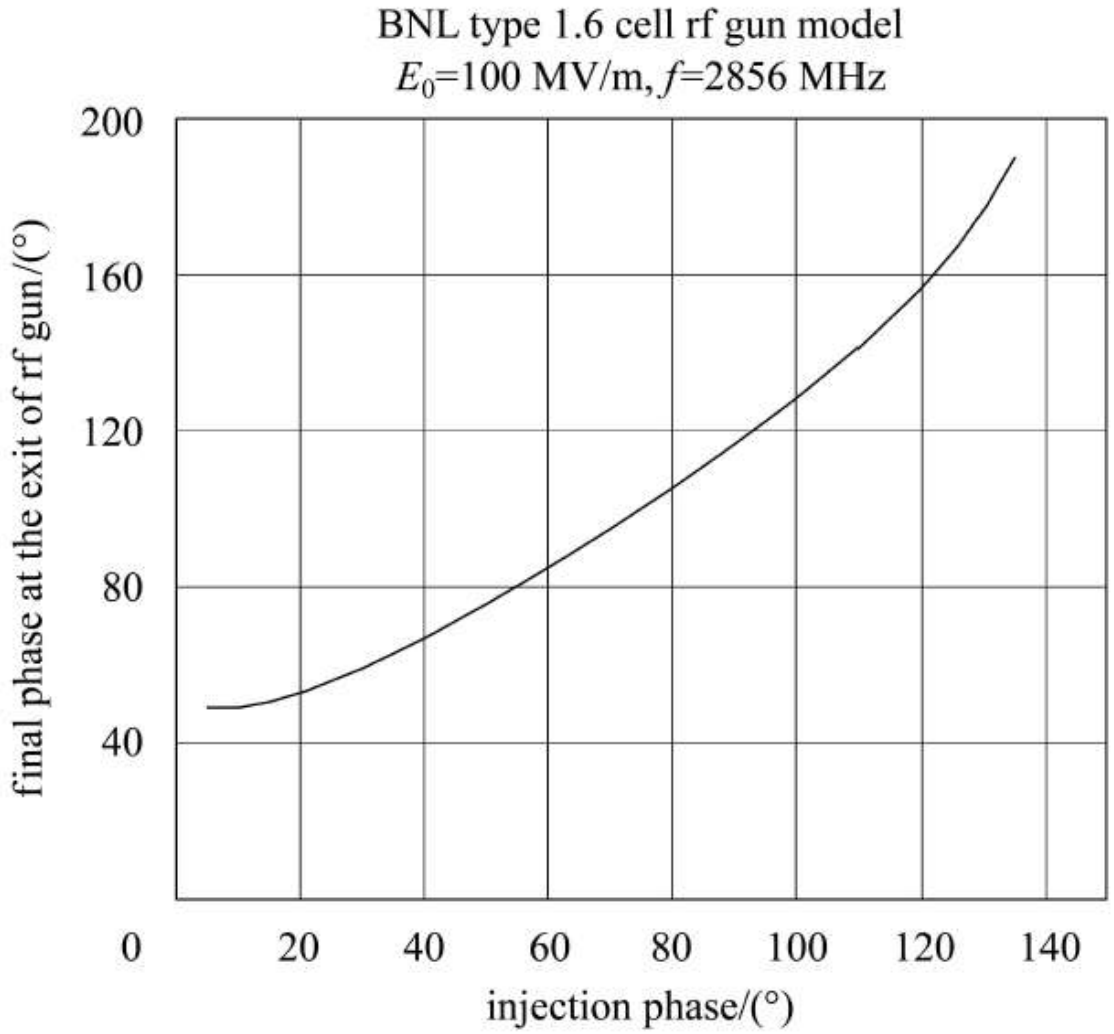


Fig. 4. Final phase ϕ_f vs the injection phase.

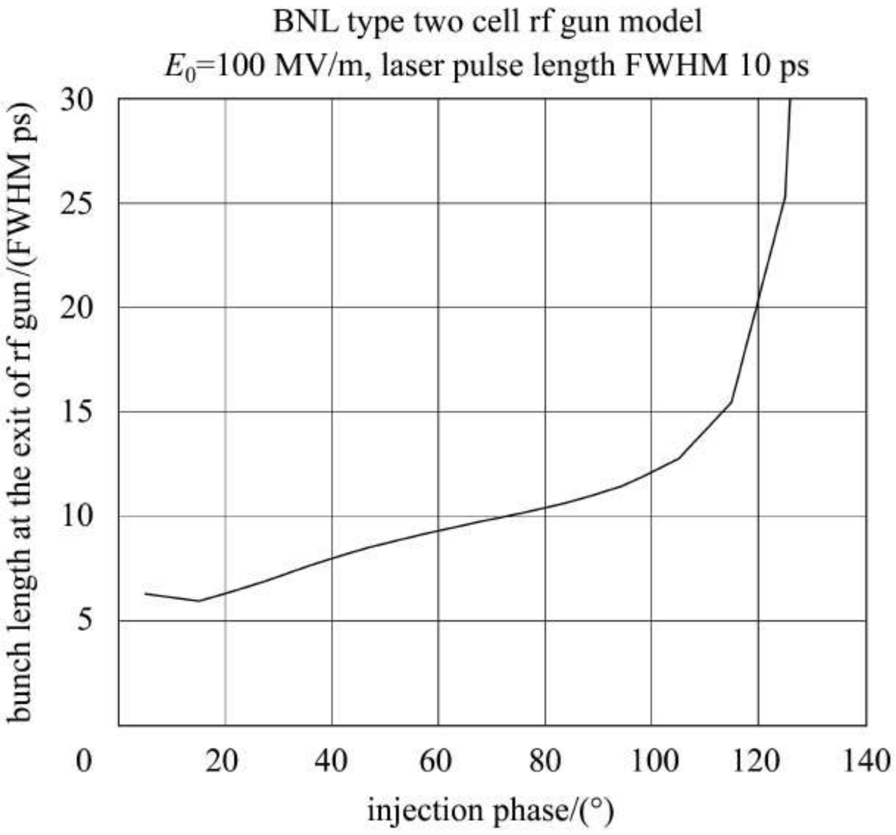


Fig. 5. Bunch length vs the injection phase with laser pulse FWHM 10 ps.

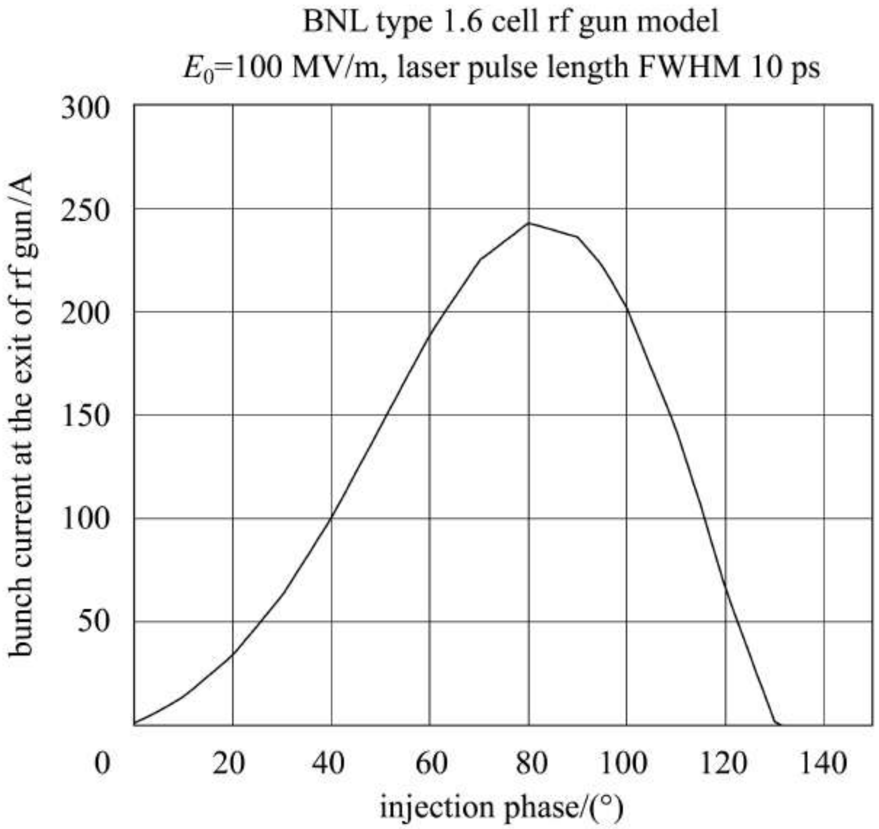


Fig. 6. Bunch current at the exit of the rf gun vs the injection phase.

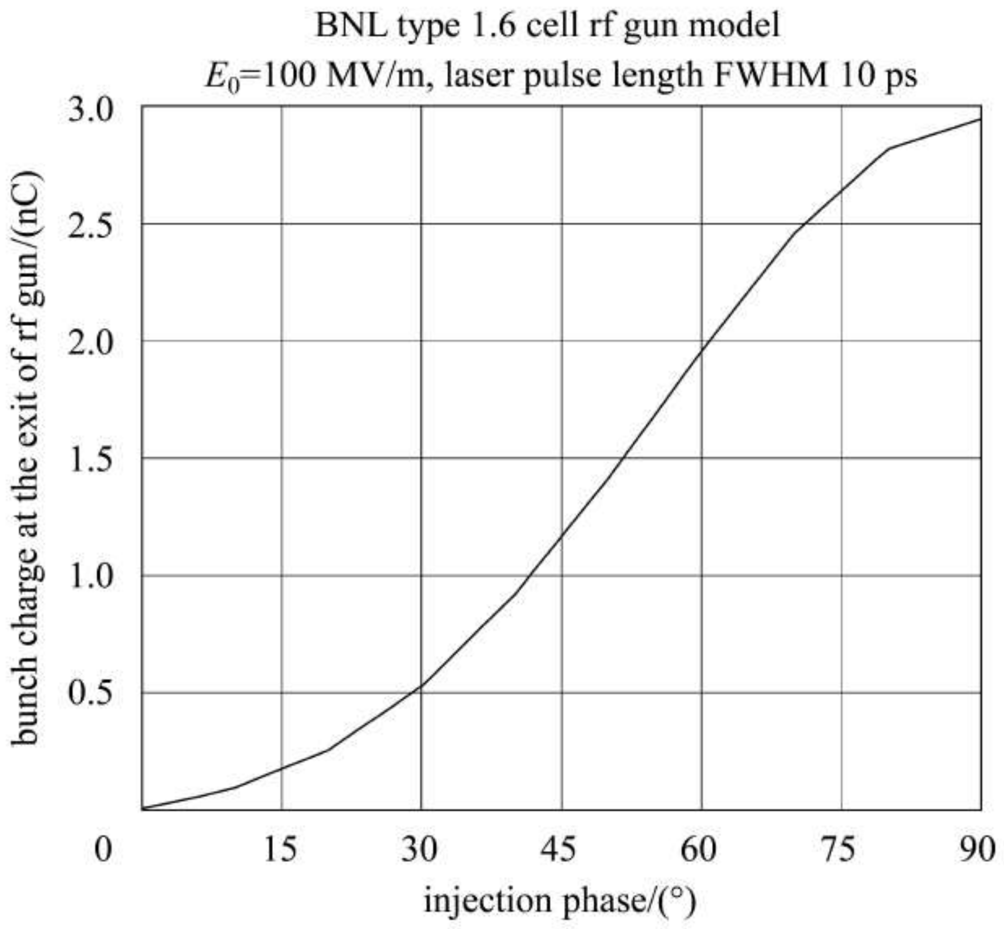


Fig. 7. Bunch charge out of the cathode vs the injection phase.

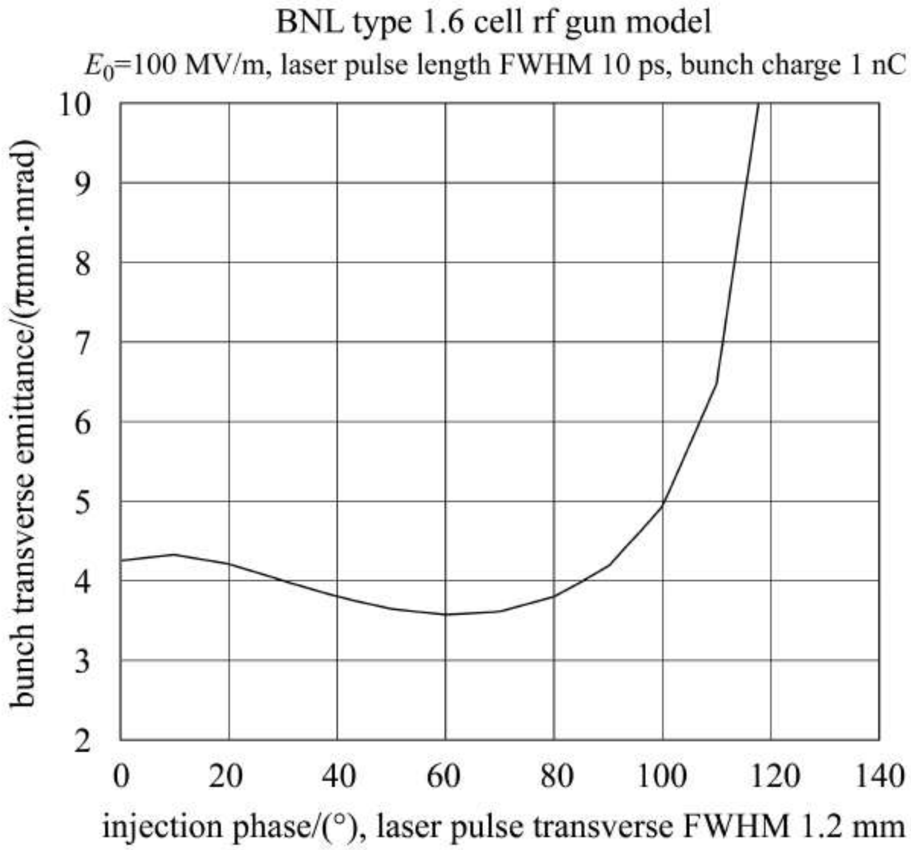


Fig. 8. Bunch transverse emittance at the exit of the rf gun vs the injection phase corresponding to the experimental situation in Ref. [10].

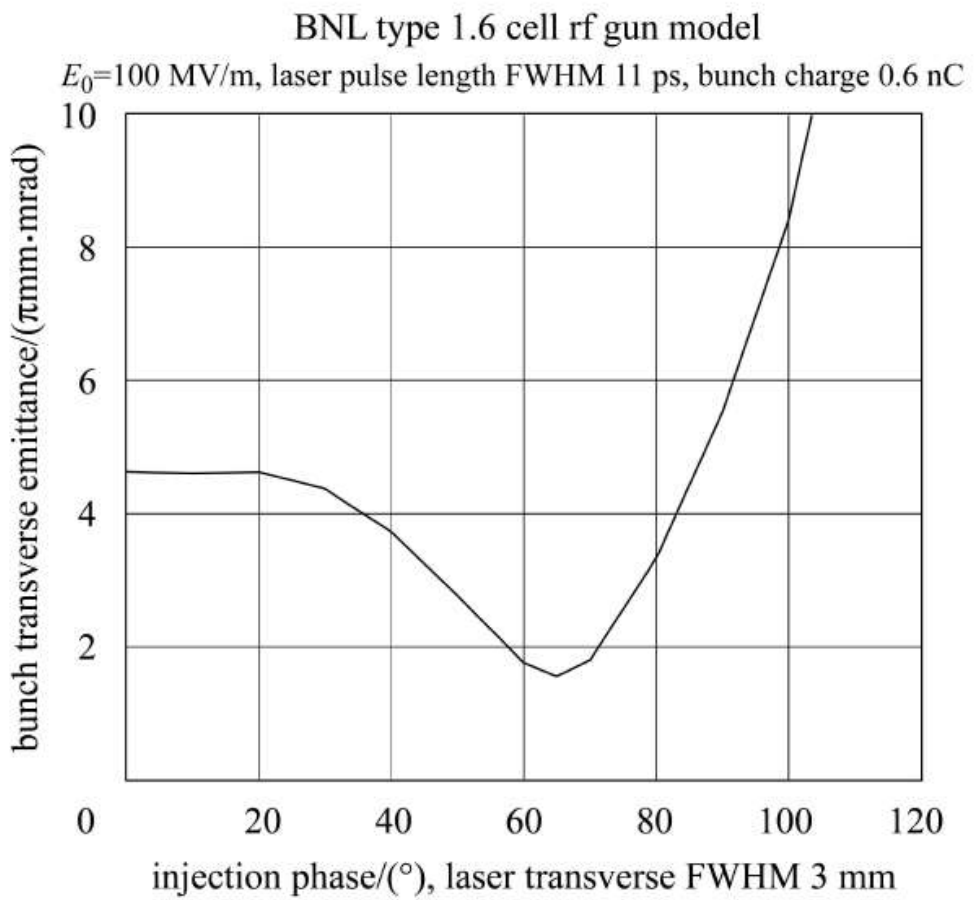


Fig. 9. Bunch transverse emittance at the exit of the rf gun vs the injection phase corresponding to the experimental situation in Ref. [11].

Analytical Treatment of Emittance Growth in Linacs

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A
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Analytical treatment of the emittance growth in the main linacs of future linear colliders

J. Gao*

Laboratoire de L'Accélérateur Linéaire, IN2P3-CNRS et Université de Paris-Sud, B.P. 34, 91898 Orsay Cedex, France

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Abstract

In this paper the single bunch emittance growth in the main linac of a linear collider is analytically treated in analogy to the Brownian motion of a molecule, and the analytical formulae for the emittance growth due to accelerating structure misalignment errors are obtained by solving Langevin equation. The same set of formulae are derived also by solving directly Fokker–Planck equation. Comparison with numerical simulation result is made and the agreement is quite well. The problem of the single bunch emittance growth in an electron storage ring is also discussed.

J. Gao, "Analytical treatment of the emittance growth in the main linacs of future linear colliders", **Nucl. Instr. and Meth., A 441** (2000) 314-319

2. Equation of transverse motion

The differential equation of the transverse motion of a bunch with zero transverse dimension is given as

$$\frac{d^2y(s,z)}{ds^2}+\frac{1}{\gamma(s,z)}\frac{d\gamma(s,z)}{ds}\frac{dy(s,z)}{ds}+k(s,z)^2y(s,z)$$
$$=\frac{1}{m_0c^2\gamma(s)}e^2N_e\int_z^{\infty}\rho(z')\mathscr{W}_{\perp}(s,z'-z)y(s,z')\,dz' \quad (2)$$

where $k(s,z)$ is the instantaneous betatron wave number at position s, z denotes the particle longitudinal position inside the bunch, and $\int_{-\infty}^{\infty}\rho(z')\,dz'=1$. Now, we rewrite Eq. (2) as follows:

$$\frac{d^2y(s,z)}{ds^2}+\Gamma\frac{dy(s,z)}{ds}+k(s,z)^2y(s,z)=\Lambda \quad (3)$$

where $\Gamma=\gamma(0)G/\gamma(s,z)$, $G=eE_z/m_0c^2\gamma(0)$, E_z is the effective accelerating gradient in the linac,

$$\Lambda=\frac{e^2N_eW_{\perp}(s,z)y(s,0)}{m_0c^2\gamma(s,z)},$$

$$W_{\perp}(s,z)=\int_z^{\infty}\rho(z')\mathscr{W}_{\perp}(s,z'-z)\,dz'$$

and $y(s,0)$ is the deviation of the bunch head with respect to accelerating structures center. In this

Asymptotic Emittance Growth Formulae

The emittance growth of a bunch go through a long linac of many accelerating structures of section length l_s, N_e is the particle number in a bunch, σ_y is the misalignment errors.
When the linac is very long, for the asymptotic values of emittance growth,

one gets

$$\varepsilon_{\mathrm{rms}} = \frac{\sigma_y^2 l_{\mathrm{s}}}{2\gamma(s,z)\gamma(0)Gk(s,z)}\left(\frac{e^2 N_{\mathrm{e}} W_{\perp}(z)}{m_0 c^2}\right)^2 \qquad (18)$$

and

$$\varepsilon_{\mathrm{n,rms}} = \frac{\sigma_y^2 l_{\mathrm{s}}}{2\gamma(0)Gk(s,z)}\left(\frac{e^2 N_{\mathrm{e}} W_{\perp}(z)}{m_0 c^2}\right)^2. \qquad (19)$$

Analytical Estimates of Halo Current Loss Rates in Space Charge Dominated Beams

Nuclear Instruments and Methods in Physics Research A 484 (2002) 27–35

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

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Analytical estimates of halo current loss rates in space charge dominated beams

J. Gao*

Laboratoire de L'Accélérateur Linéaire, IN2P3-CNRS et Université de Paris-Sud, BP 34, 91898 Orsay Cedex, France

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Abstract

In this paper, we investigate the dynamical behaviours of a continuous intense charged particle beam in a transport system. It is assumed that fermion particles, such as electron and proton, in the beam follow Fermi–Dirac statistics in the equilibrium state. The halo particles executing stochastic motions due to the envelope oscillation induced nonlinear resonances are investigated. Analytical formulae for the halo current and halo current loss rate on the beam pipe wall are established.

J. Gao, "Analytical estimates of halo current loss rates in space charge dominated beams", **Nucl. Instr. and Meth. A 484** (2002) 27–35

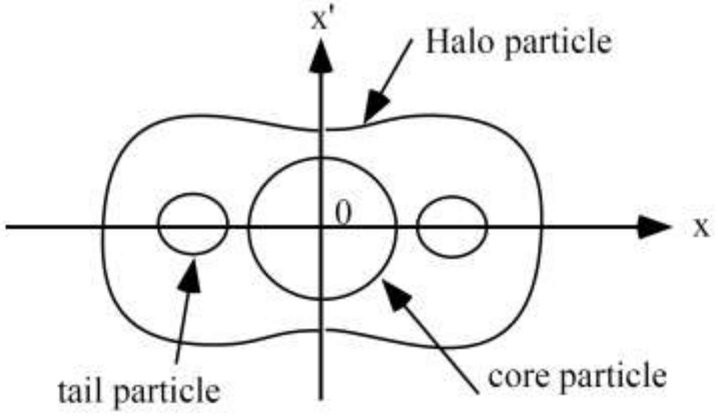


Fig. 1. Schematic illustration of three types of stroboscopic trajectories in phase space.

Halo current loss rate formula (A/meter):

$$\mathscr{R} = \frac{4 I_b f}{L}\left(\frac{\Delta R_0}{R_m}\right)^2 \frac{\ln\left(\dfrac{1+\exp\left(\frac{2\Delta x_{\max} R_0+\Delta x_{\max}^2}{\Delta R_0 R_0}\right)}{\exp\left(\frac{2\Delta x_{\max} R_0+\Delta x_{\max}^2}{\Delta R_0 R_0}\right)}\right)}{\ln\left(\dfrac{1+\exp(-R_0/\Delta R_0)}{\exp(-R_0/\Delta R_0)}\right)} \qquad (32)$$

Theory of RF Field Measurement by Perturbation Methods

618 IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT, VOL. 40, NO. 3, JUNE 1991

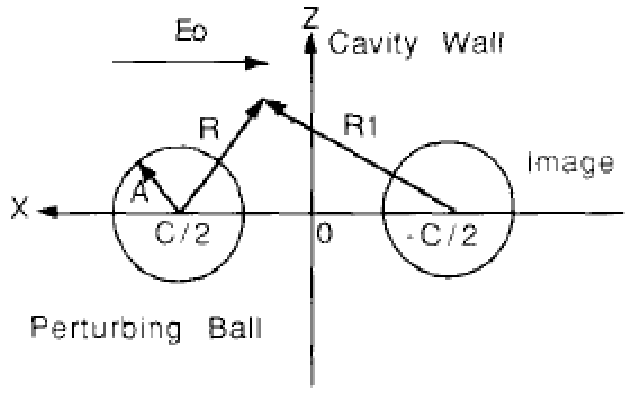
Effects of the Cavity Walls on Perturbation Measurements

J. Gao

$$(f_o^2 - f^2)/f_o^2 = (4/3)3\pi E_o^2 A^3 \quad (1)$$

Uncorrected perturbation formula

$$E_0^2 = \frac{\epsilon_0 E^2}{2W} \quad (2)$$



where f_o is the unperturbed resonant frequency of the cavity, f is the perturbed resonant frequency of the cavity, E_o, and E are the normalized and the real electric fields where the small sphere is located, respectively, A is the radius of the sphere, and W is the energy stored in the cavity.

Corrected perturbation formula by α , taking into accound the image charge effect due to perturbation object approching the cavity wall

$$(f_o^2 - f^2)/f_o^2 = \frac{4\pi}{3} 3E_o^2 A^3(1 + \alpha) \quad (A15)$$

where

$$\alpha = 4(A/C)^3 + 16(A/C)^6 + 55.636(A/C)^8 + 32(A/C)^9 + 226.3(Ak)^{11}. \quad (A16)$$

Linear Collider Design-1

第30卷增刊1 高能物理与核物理 Vol. 30, Supp. I
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Parameter Choice for International Linear Collider (ILC)*

GAO Jie[1)]

(Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China)

Abstract In this paper a general procedure to determine linear collider parameters is given. As an example, a parameter list is proposed for ILC with very low bunch charge. The main aim of this paper is to demonstrate the beam parameter relations with the constraints from the interaction point and damping ring. It is suggested that the energy of the damping ring should be 7GeV instead of 5GeV if a 17km damping ring is to be used. However, if 6km damping ring (which is preferable) is adopted, 5GeV damping ring energy is reasonable.

Key words linear collider, ILC, parameter choice, damping ring

J. Gao, "Parameter Choice for International Linear Collider (ILC)", **HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS**, Vol. 30, Supp. Feb., 2006

2 Beam parameter relations

The luminosity of two Gaussian head-on colliding beams is given by:

$$L=\frac{f_{\rm rep}N_{\rm b}N_{\rm e}^2}{4\pi\sigma_x\sigma_y}H_{\rm D} \quad (1)$$

where $f_{\rm ref}$ is the repetition frequency of the bunch train, $N_{\rm b}$ is the number of bunches in the train, $N_{\rm e}$ is the number of particles per bunch, $\sigma_x=\sqrt{\varepsilon_x\beta_x}$, $\sigma_y=\sqrt{\varepsilon_y\beta_y}$, $\beta_{x,y}$ and $\varepsilon_{x,y}$ are the values of the beta functions at the IP and the emittances, respectively, and $H_{\rm D}$ is the pinch enhancement factors which are functions of the so-called disruption parameters $D_{x,y}$ of a bunch. In the following we will express the luminosity and colliding beam parameters as the function of constrains from IP (flat beam case).

$$L=f_{\rm rep}N_{\rm b}\left(\frac{N_{\rm had}}{n_\gamma^2\sigma_{\gamma\gamma\to\rm had}}\right), \quad (2)$$

$$\sigma_x=\frac{\pi r_{\rm e}^3 H_{\rm had}}{2.6\delta_{\rm B}\alpha H_{\rm D}n_\gamma\sigma_{\gamma\gamma\to\rm had}}, \quad (3)$$

$$\sigma_y=\frac{r_{\rm e}n_\gamma^3}{41.5\delta_{\rm B}\alpha^3}, \quad (4)$$

$$\sigma_z=\frac{r_{\rm e}n_\gamma^2\gamma}{4.6\delta_{\rm B}\alpha^2}, \quad (5)$$

$$R=\frac{\sigma_x}{\sigma_y}=\frac{16\pi\alpha^2 r_{\rm e}^2 N_{\rm had}}{H_{\rm D}n_\gamma^4\sigma_{\gamma\gamma\to\rm had}}, \quad (6)$$

$$\beta_x=\frac{3.5\pi\gamma r_{\rm e}^3 N_{\rm had}}{\delta_{\rm B}H_{\rm D}\sigma_{\gamma\gamma\to\rm had}n_\gamma^2}, \quad (7)$$

$$\beta_y=\sigma_z/0.75, \quad (8)$$

$$\gamma\varepsilon_x=\frac{\pi r_{\rm e}^3 N_{\rm had}}{23.4\delta_{\rm B}H_{\rm D}\alpha^2\sigma_{\gamma\gamma\to\rm had}}, \quad (9)$$

$$\gamma\varepsilon_y=\frac{0.75 n_\gamma r_{\rm e}^3}{374\delta_{\rm B}\alpha^4}, \quad (10)$$

$$N_{\rm e}=\frac{\pi r_{\rm e}^2 N_{\rm had}}{5.2\delta_{\rm B}H_{\rm D}\alpha^2\sigma_{\gamma\gamma\to\rm had}}, \quad (11)$$

$$\theta_x=\theta_y=\frac{n_\gamma}{\alpha\gamma}, \quad (12)$$

$$f_{\rm rep}N_{\rm b}=\frac{Ln_\gamma^2\sigma_{\gamma\gamma\to\rm had}}{N_{\rm had}}, \quad (13)$$

$$P_{\rm b}=\frac{\pi e W_{\rm cm}r_{\rm e}^2 n_\gamma^2 L}{10.4H_{\rm D}\delta_{\rm B}\alpha^2}, \quad (14)$$

where $r_{\rm e}=2.82\times10^{-15}$m is the classical electron radius, α is the fine structure constant, γ is the ratio of the colliding particle energy to its rest energy, $\sigma_{\gamma\gamma\to\rm had}=4.2\times10^{-35}{\rm m}^2$ is the $\gamma\gamma$ to hadron total cross section, $\delta_{\rm B}$ is the "beamstrahlung" energy spread, n_γ is the average photon number emitted per incident particle, $N_{\rm had}$ is number hadron produced per crossing, and $H_{\rm D}$ is about 1.5 with $D_y=9$ which is used later in this paper. In addition to constraints at IP, in

3 Very low charge case

Given the designed beam energy of 250GeV, the luminosity after pinch effect, $L=2\times10^{34}{\rm cm}^{-2}\cdot{\rm s}^{-1}$, and the constraints shown in Table 1, one gets the beam parameters shown in Table 2.

Table 1. Constrain parameters from IP.

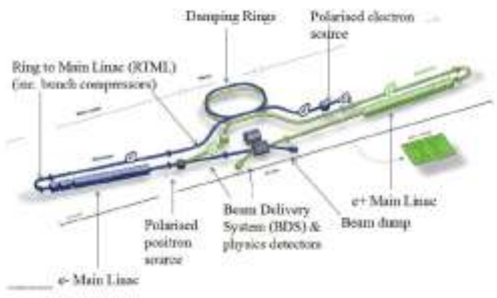
$\delta_{\rm B}$	n_γ	$N_{\rm had}$	D_y	$H_{\rm D}$
0.03	0.8	0.125	9	1.5

Table 2. Beam parameters at IP.

σ_x/μm	σ_y/nm	σ_z/μm	$N_{\rm e}(\times10^{10})$	$\theta_{x,y}$/rad
0.31	3	125	0.6	0.000224
β_x/m	β_y/m	$\gamma\varepsilon_x$/μm	$\gamma\varepsilon_y$/μm	$f_{\rm rep}N_{\rm b}$
0.012	0.00016	3.74	0.0272	43010

Linear Collider Design-2

ILC 250GeV parameter comparison-1



ILC@250GeV	Yokoya	IHEP	IHEP-2
E_{cm} (GeV)	250	250	250
Ne	2.0×10^{10}	2.0×10^{10}	2.0×10^{10}
F_{rep} (Hz)	5	5	5
N_b	1312	1312	1312
Bunch separation (ns)	554	554	554
I_b (mA)	5.8	5.8	5.8
P_b (MW)/beam	2.65	2.62	2.62
β_x (mm)	13.0	11.0	9.0
β_y (μm)	410	464	469
$\gamma\varepsilon_x$ (μm)	5.0	5.05	5.0
$\gamma\varepsilon_y$ (nm)	35	37.5	37.5
σ_x/σ_y (nm)	515.5/7.66	476.5/8.4	428.9/8.5
σ_z (μm)	300	317.8	328
δ_B	0.024	0.0264	0.0315
n_γ	1.62	1.7	1.88
Dy	34.5	35.8	40.8
H_D	2.43	2.84	3.39
Disruption angle θ (rad)	0.00088	0.00095	0.00105
N_{had}	2.1	2.72	4.4
θx/θy (urad)	39.7/18.7	43.3/18.2	47.6/18.1
L_0 ($cm^{-2}s^{-1}$)	1.285×10^{34}	1.475×10^{34}	1.946×10^{34}

Example of application of the previous analytical method to make a linear collider design

J. Gao, "Parameter Choice for International Linear Collider (ILC)", **HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS**, Vol. 30, Supp. Feb., 2006
D. Wang, J. Gao," ILC high luminosity study at 250GeV", Asian Linear Collider Workshop, May 28 to June 1, 2018, Fukuoka, Japan

Some Fundamental Property Formulae

Following equations you could find from general linear accelerator lectures or books

Conductivity of copper

$$\sigma = 1.7\times10^{-8}\ \mathrm{S}\cdot m$$

Skin depth

$$\delta = \sqrt{\frac{2}{\sigma\mu_0\omega}}$$

Surface resistance

$$R_s = 1/\sigma\delta$$

Surface resistance

$$R_s = \sqrt{\mu_0\omega/2\sigma}$$

Surface dissipated power

$$\frac{R_s}{2}\int_s H^2 dS$$

Quality factor

$$Q = \frac{\omega U}{P_w}$$

Shunt impedance

$$Z_s = \frac{E_a^2}{P_w}$$

Shunt impedance over Q

$$Z_s / Q = E_a^{\ 2} / \omega U$$

Phase shift due to frequency change

$$\Delta\varphi = Q\frac{\Delta f}{f}$$

Constant Impedance Structure Property Formulae

$$\frac{dP}{dz} = -P_w = -\frac{\omega U}{Q} = -\frac{\omega P}{Q v_g} = -2\alpha_0 P$$

$$P = P_0 e^{-2\int_0^L \alpha_0(z)dz}$$

$$\alpha_0 = \frac{\omega}{2Qv_g}$$

$$\frac{1}{P}\frac{dP}{dz} = -2\alpha_0$$

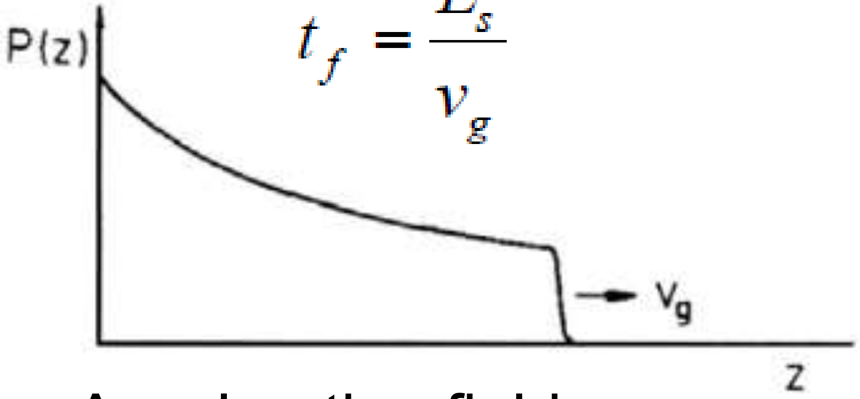
$$P(z) = P_0 e^{-2\alpha_0 z}$$

$$\tau_0 = \alpha_0 L_s$$

$$E_a(z) = E_0 e^{-\alpha_0 z}$$

$$\tau_0 = \int_0^L \alpha_0(z)dz$$

$$t_f = \frac{L_s}{v_g}$$



Filling time of the structure

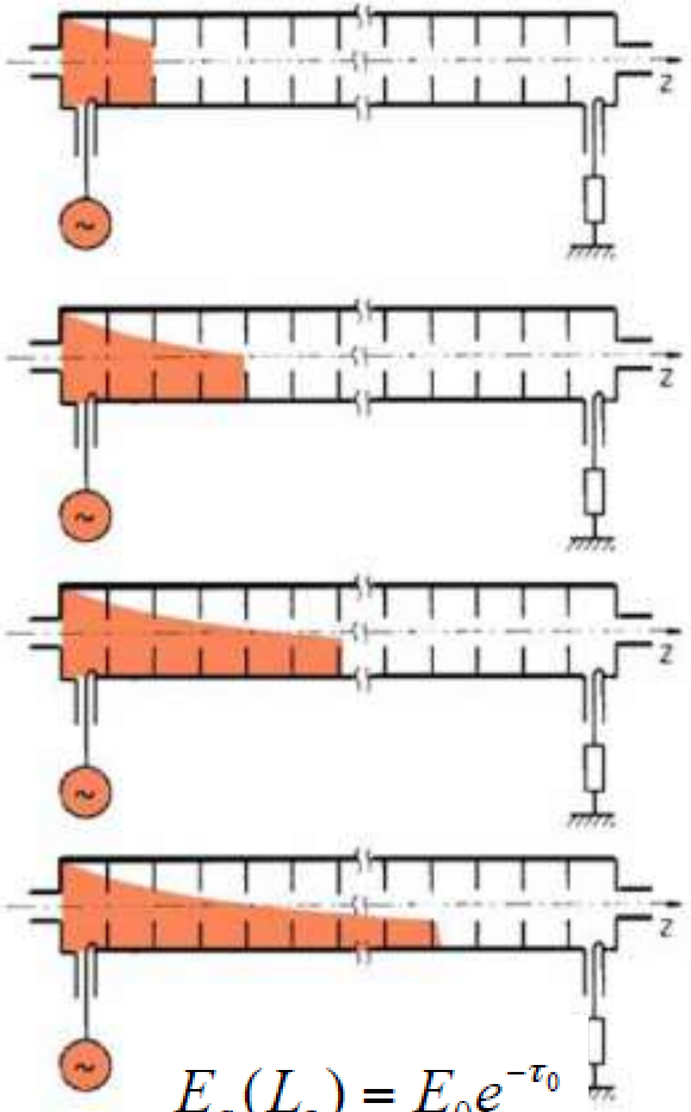
$$t_f = \frac{L_s}{v_g} = \frac{2Q\tau_0}{\omega}$$

Accelerating field

$$E_a^2 = 2\alpha_0 Z_s P$$

Energy gain in the structure

$$\Delta W = e\sqrt{2Z_s P_{in} L_s}\cdot\left(\frac{1-e^{-\tau_0}}{\sqrt{\tau_0}}\right)$$



$$E_a(L_s) = E_0 e^{-\tau_0}$$

$$P(L_s) = P_0 e^{-2\tau_0}$$

Constant Gradient Structure Property Formulae

$E_a = \text{const.} \Rightarrow dP/dz = \text{const.}$
$\Rightarrow \alpha_0(z) \neq \text{const}$

Accelerating field

$$v_g(z) = \frac{\omega L_s}{Q} \cdot \frac{1 - \frac{z}{L_s}(1 - e^{-2\tau_0})}{1 - e^{-2\tau_0}}$$

$$E_a^2 = -Z_s \frac{dP}{dz}$$

$$\int_{P_0}^{P_{Ls}} \frac{dP}{P} = -2\int_0^{L_s} \alpha(z)\,dz$$

$$\tau_0 = \int_0^{L_s} \alpha_0(z)\,dz$$

$$\frac{dP}{dz} = -2\alpha_0(z)P$$

$$P_{Ls} = P_0 e^{-2\tau_0}$$

Filling time of the structure

$$t_f = \int_0^{L_s} \frac{dz}{v_g(z)} = \frac{Q}{\omega L_s}(1 - e^{-2\tau_0})\int_0^{L_s} \frac{dz}{1 - \frac{z}{L_s}(1 - e^{-2\tau_0})} = \frac{2Q\tau_0}{\omega}$$

$$P(z) = P_0 + \frac{P_{L_s} - P_0}{L_s} z = P_0\left[1 - \frac{1 - e^{-2\tau_0}}{L_s} z\right]$$

$$\alpha_0(z) = \frac{1}{2L_s} \cdot \frac{1 - e^{-2\tau_0}}{1 - \frac{z}{L_s}(1 - e^{-2\tau_0})}$$

$$\frac{dP}{dz} = -\frac{P_0}{L_s}(1 - e^{-2\tau_0})$$

$$v_g(z) = \frac{\omega L_s}{Q} \cdot \frac{1 - \frac{z}{L_s}(1 - e^{-2\tau_0})}{1 - e^{-2\tau_0}}$$

Energy gain in the structure

$$\Delta W = e\sqrt{Z_s P_0 L_s (1 - e^{-2\tau_0})}$$

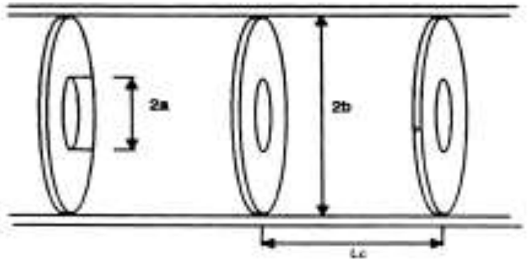
Some Scaling Laws

$Z_s \sim f_0^{1/2}$, (Skin-depth $\sim f_0^{-1/2}$) $Q \sim f_0^{-1/2}$ $P_w \sim f_0^{-1/2}$

$Z_s / Q \sim f_0$ $U \sim f_0^{-2}$ $t_f \sim f_0^{-3/2}$ $(t_f = L_c / v_g)$ a 、 $b \sim f_0^{-1}$

Longitudinal wakefield $\sim f_0^2$ Transverse wakefield $\sim f_0^3$

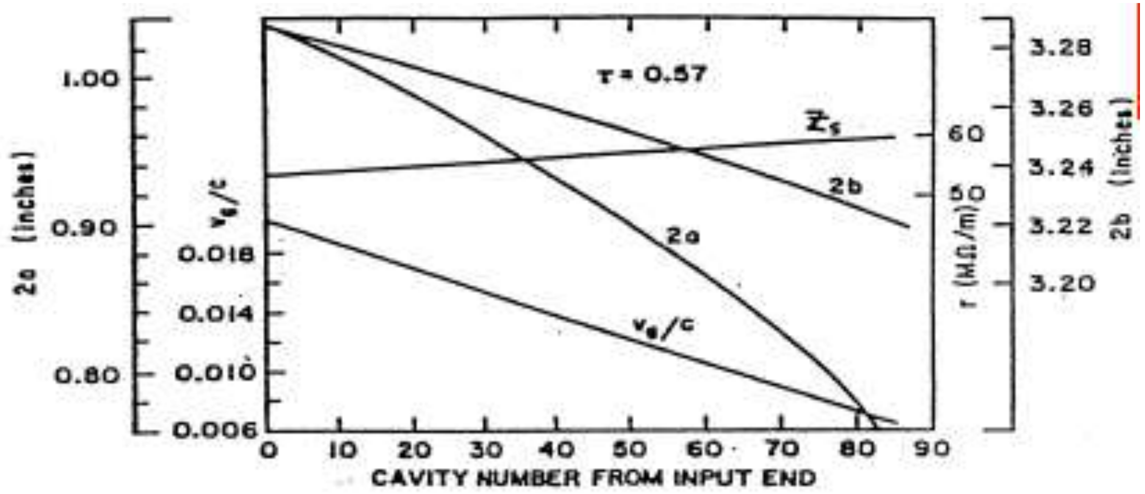
SLAC Type 2π/3 Mode Constance Impedance Accelerating Structure



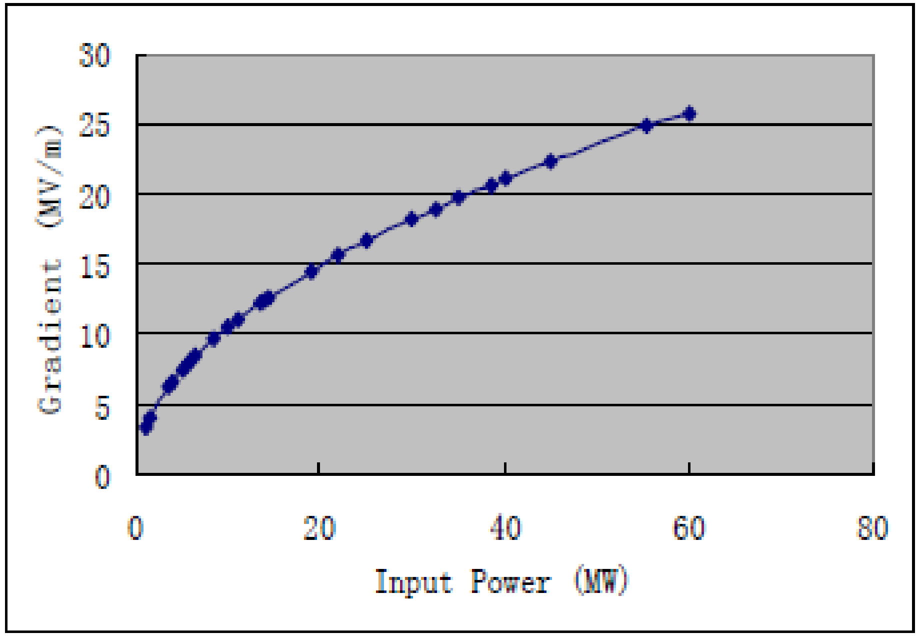
$$2b \approx 8.4 - 8.2 \text{ cm}, \quad 2a \approx 2.6 - 1.9 \text{ cm},$$

$$v_g/c \approx 0.021\text{-}0.007, \quad \tau_0 = 0.57, \quad L_s = 3.05 \text{ m}$$

$$< Z_s > = 57 \text{ M}\Omega / \text{m}, \quad t \approx 5.84 \text{ ms}.$$



BEPC-II type 2π/3 Mode Constance Impedance Accelerating Structure



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