Hadron tomography for pion and its gravitational form factors

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Outline

Generalized distribution amplitude (GDA) of pion

- Motivation
- GDA in two-photon process
- GDA analysis for Belle data
Generalized Parton Distributions (GPDs) provide information on $\Delta L$ to solve the proton puzzle!

Generalized Distribution Amplitudes (GDAs) $\leftrightarrow$ s-t crossing of GPDs

Pion GDAs are investigated.

GDA carry many important physical quantities of the hadron, such as distribution amplitudes (DAs) and timelike form factors.
**Generalized distribution amplitude for pion**

In the process $\gamma\gamma^* \rightarrow h\bar{h}$, an hard part describing the process $\gamma\gamma^* \rightarrow q\bar{q}$ with produced collinear and on-shell quark, and a soft part describing the production of the hadron $h$ pair from a $q\bar{q}$. This soft part is called Generalized Distribution Amplitude (GDA).

GDA is an important quantity of hadron, it is defined as

$$
\Phi^q(z,\xi,W^2) = \int \frac{dx^-}{2\pi} e^{-ix^+z} \langle h(p)\bar{h}(p')|\bar{q}(x^-)\gamma^+q(0)|0 \rangle 
$$

$$
z = \frac{k^+}{p^+}, \quad \xi = \frac{p^+}{p^+}, \quad s = W^2 = (p+p')^2 = P^2
$$

GDA is closely related to generalized parton distribution (GPD) by the s-t crossing, so GDA could provide another way to obtain GPD information.

\[ \Phi^q(z, \xi, W^2) \leftrightarrow H^q \left( x = \frac{1-2z}{1-2\xi}, \xi = \frac{1}{1-2\xi}, t = W^2 \right) \]

GPD can be used to study the proton spin puzzle!

\[ \int \frac{dx^-}{2\pi} e^{-iz(P^-x)} \left\langle h(p_2) | \bar{q}(x^-) \gamma^+ q(0) | h(p_1) \right\rangle \]

\[ = \frac{1}{2\bar{P}^+} \left[ H^q(x, \xi, t) \bar{u}(p_2) \gamma^+ u(p_1) + E^q(x, \xi, t) \bar{u}(p_2) \frac{i\sigma^{+\alpha} \Delta^\alpha}{2m} u(p_1) \right] \]

\[ \bar{P} = (p_1 + p_2) / 2, \ \Delta = p_2 - p_1, \ x = \frac{-q_1^2}{2p_1q_1}, \ \xi = \frac{\Delta^+}{p_1^+ + p_2^+} \]

The cross section of process $\gamma^* \gamma \rightarrow \pi^0 \pi^0$

$$d\sigma = \frac{1}{4} \frac{1}{4\sqrt{(q_1 q_2)^2 - q_1^2 q_2^2}} \sum_{\lambda_1 \lambda_2} | -iT^{\mu\nu}(q_1)\varepsilon^\nu(q_2)|^2 d\Phi_2$$

$$\gamma^* (q_1 \lambda_1)$$

$$\gamma (q_2 \lambda_2)$$

$$A_{\lambda_1 \lambda_2}$$ is the helicity amplitude, and there are 3 independent helicity amplitudes, they are $A_{++}, A_{0+}$ and $A_{+-}$. The leading-twist amplitude $A_{++}$ has a close relation with the generalized distribution amplitude (GDA) $\Phi^q(z, \xi, W^2)$.

$$A_{\lambda_1 \lambda_2} = T^{\mu\nu}(\lambda_1)\varepsilon^\nu(\lambda_2)/e^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z - 1}{z(1-z)} \Phi^q(z, \xi, W^2)$$

Higher-twist and higher order contributions

Higher-twist contribution $A_{0+}$ requires a helicity flip along the fermion line, and it decreases as $1/Q$. Higher-order contribution $A_{+-}$ contributes with the GDA of gluon, since $A_{+-}$ indicates the angular momentum $L_z = 2$. Therefore $A_{+-}$ is suppressed by running coupling constant $\alpha_s$.

\[ \gamma^*(q_1 \lambda_1) \rightarrow \pi^0(p_1) \]
\[ \gamma(q_2 \lambda_2) \rightarrow \pi^0(p_2) \]

**GDA expression**

At very high energy $Q^2$, we can have the asymptotic form of the GDA

$$
\sum_q \Phi^+_{q}(z, \xi, W^2) = 18n_f z(1 - z)(2z - 1)[B_{10}(W) + B_{12}(W)P_2(2\xi - 1)]
$$

$$
= 18n_f z(1 - z)(2z - 1)[\tilde{B}_{10}(W) + \tilde{B}_{12}(W)P_2(\cos \theta)]
$$

The GDAs are related to the energy-momentum form factor in the timelike region.

$$
\int dz(2z - 1)\Phi^+_{q}(z, \xi, W^2) = \frac{2}{(P^+)^2} \langle \pi^+(p_1)\pi^-(p_2) | T^{++}(0) | 0 \rangle
$$

where the energy-momentum form factor for quarks is defined as

$$
\langle \pi^0(p_1)\pi^0(p_2) | T^{\mu\nu}(0) | 0 \rangle = \frac{1}{2} \left[ (s g^{\mu\nu} - P^\mu P^\nu) \Theta_1 + \Delta^\mu \Delta^\nu \Theta_2 \right]
$$

$$
P = p_1 + p_2, \Delta = p_1 - p_2
$$

By using this sum rule we can obtain

$$
B_{12}(0) = \frac{5R_\pi}{9}
$$

where $R_\pi$ is the momentum fraction carried by quarks in the pion.

In 2016, the Belle Collaboration released the measurements of differential cross section for $\gamma^*\gamma \rightarrow \pi^0\pi^0$. The GDAs can be obtained by analyzing the Belle data.

In these figures, the resonance $f_2(1270)$ is clearly seen around $W = 1.25$ GeV, however, other resonances are not clearly seen due to the large errors.

M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.
The scale dependence of the Belle data. We have red color for $W = 0.525$ GeV, blue color for $W = 0.975$ GeV, and green color for $W = 1.550$ GeV.

The scaling violation of the GDAs is not so obvious in the Belle data on account of the large errors, so that the $Q^2$-independent GDAs could be used in analyzing the Belle data.
$\Phi^q(z, \xi, W^2) = 18n_f z(1-z)(2z-1)\left[B_{10}(W) + B_{12}(W)P_2(2\xi-1)\right]$ 

$= 18n_f z(1-z)(2z-1)\left[\tilde{B}_{10}(W) + \tilde{B}_{12}(W)P_2(\cos\theta)\right]$ 

$\tilde{B}_{10}(W) = B_{10}(W)e^{i\delta_0}, \tilde{B}_{12}(W) = B_{12}(W)e^{i\delta_2}$

In the above equation $\delta_0$ and $\delta_2$ and are the $\pi\pi$ elastic scattering phase shifts in the isospin=0 channel (see the figure). Above the KK threshold, the additional phase is introduced for S-wave

The S wave and D-wave $\pi\pi$ scattering phase shifts.

Resonance effects

In the process $\gamma^*\gamma \to \pi^0\pi^0$, the $\pi^0\pi^0$ can be produced through intermediate meson state $h$. The $q\bar{q} \to h$ amplitude should be proportional to the decay constant $f_h$ or the distribution amplitude (DA), and the $h \to \pi^0\pi^0$ amplitude can be expressed by the coupling constant $g_{h\pi\pi}$. These resonance contributions read

$$\bar{B}_{12}(W) = \beta^2 \frac{10 g_{f_2\pi\pi} f_{f_2} M^2_{f_2}}{9 \sqrt{2} \sqrt{(M^2_{f_2} - W^2)^2 - \Gamma_{f_2}^2 M^2_{f_2}}}$$

$$\bar{B}_{10}(W) = \frac{5 g_{f_0\pi\pi} f_{f_0}}{3 \sqrt{2} \sqrt{(M^2_{f_0} - W^2)^2 - \Gamma_{f_0}^2 M^2_{f_0}}}$$

The resonance effects play an important role in the resonance regions.
We adopt a simple expression of GDA to analyze Belle data, here resonance effects of $f_0(500)$ and $f_2(1270)$ are introduced.

$$\Phi_q^+(z, \xi, W^2) = N_h z^\alpha (1 - z)^\alpha (2z - 1)[\tilde{B}_{10}(W) + \tilde{B}_{12}(W)P_2(\cos \theta)]$$

$$\tilde{B}_{10}(W) = \left[\frac{-3 + \beta^2}{2} \frac{5R_\pi}{9} F_h(W^2) + \frac{5g_{f_0\pi\pi}f_{f_0}}{3\sqrt{2}\sqrt{(M_{f_0}^2 - W^2)^2 - \Gamma_{f_0}^2 M_{f_0}^2}}\right] e^{i\delta_0}$$

$$\tilde{B}_{12}(W) = \left[\beta^2 \frac{5R_\pi}{9} F_h(W^2) + \beta^2 \frac{10g_{f_2\pi\pi}f_{f_2} M_{f_2}^2}{9\sqrt{2}\sqrt{(M_{f_2}^2 - W^2)^2 - \Gamma_{f_2}^2 M_{f_2}^2}}\right] e^{i\delta_2}$$

$$F_h(W^2) = \frac{1}{\left[1 + \frac{W^2 - 4m_\pi^2}{\Lambda^2}\right]^{n-1}}$$

The function $F_h(W^2)$ is the form factor of the quark part of the energy-momentum tensor, and the parameter $\Lambda$ is the momentum cutoff in the form factor. The parameter $n$ is predicted as $n = 2$ at very high energy, because we have $d\sigma/d|\cos \theta|/\sim 1/W^6$ by the counting rule. In the asymptotic limit, $\alpha = 1$. 
Results

By analyzing the Belle data, the values of parameters are obtained

<table>
<thead>
<tr>
<th></th>
<th>Set 1</th>
<th>Set 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$0.801 \pm 0.042$</td>
<td>$1.157 \pm 0.132$</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>$1.602 \pm 0.109$</td>
<td>$1.928 \pm 0.213$</td>
</tr>
<tr>
<td>$a$</td>
<td>$3.878 \pm 0.165$</td>
<td>$3.800 \pm 0.170$</td>
</tr>
<tr>
<td>$b$</td>
<td>$0.382 \pm 0.040$</td>
<td>$0.407 \pm 0.041$</td>
</tr>
<tr>
<td>$f_{f_0}$</td>
<td>------</td>
<td>$0.0184 \pm 0.034$</td>
</tr>
</tbody>
</table>

$$\frac{\chi^2}{NOF} = 1.22$$  $$\frac{\chi^2}{NOF} = 1.09$$

Set 1 is the analysis without the resonance effect $f_0(500)$, in Set 2 the resonance effect $f_0(500)$ is included.

The $W$ dependence of the differential cross section (in units of nb), and in comparison with Belle data.
The $W$ dependence of the differential cross section (in units of nb), and in comparison with Belle data.
By considering the following sum rule, we can also obtain the energy-momentum form factors for pion.

\[
\int dz (2z - 1) \Phi_q^+(z, \xi, W^2) = \frac{2}{(P^+)^2} \langle \pi^0(p_1) \pi^0(p_2) | T^{++}(0) | 0 \rangle
\]

\[
\langle \pi^0(p_1) \pi^0(p_2) | T^{\mu\nu}(0) | 0 \rangle = \frac{1}{2} \left[ (g^{\mu\nu} - P^\mu P^\nu) \Theta_1 + \Delta^\mu \Delta^\nu \Theta_2 \right]
\]

\[
\Theta_1 = \frac{3}{5} (\tilde{B}_{12} - 2 \tilde{B}_{10}), \quad \Theta_2 = \frac{9}{5 \beta^2} \tilde{B}_{12}
\]

\(\Theta_1 \rightarrow \text{Mechanical (pressure and shear force)}\)

\(\Theta_2 \rightarrow \text{Mass}\)

The timelike form factors \(\Theta_1\) and \(\Theta_2\)

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Timelike form factor \( \rightarrow \) Spacelike form factor (pion radius) : dispersion relation

\[
F(t) = \int_{4m^2}^{\infty} \frac{ds \text{ Im}(F(s))}{\pi(s-t-i\varepsilon)}
\]

The spacelike form factors \( \Theta_1 \) and \( \Theta_2 \)

Fourier Transform of \( \Theta_1 \) and \( \Theta_2 \)

Radius can be obtained by the following equation

\[ <r^2> = 6 \int_{4m^2}^{\infty} \frac{\text{Im}(F(s))}{s^2} \]

\[ \sqrt{\langle r^2 \rangle} = 0.69 \text{ fm for } \Theta_2 \text{ Mass radius} \]

\[ \sqrt{\langle r^2 \rangle} = 1.45 \text{ fm for } \Theta_1 \text{ Mechanical radius (pressure and shear force)} \]

In our analysis we introduce the additional phase for S-wave above the KK threshold. However, the additional phase could be add to D-wave phase above the threshold, in this case we have

Mass radius: 0.56-0.69 fm, Mechanical radius: 1.45-1.56 fm

Summary

◆ By analyzing the Belle data the pion GDAs are obtained, and the obtained GDAs can also give a good description of experimental data.

◆ The energy-momentum form factors for pion are calculated from the GDA of pion.

◆ This is the first finding on gravitational radii of hadrons from actual experimental measurements: The mass radius (0.56-0.69fm) is obtained.

Thank you very much