

# Relaxing Cosmological Constraints on the $U(1)_{L_\mu-L_\tau}$ Model with One-Loop Corrections

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# Intro: Zeros in neutrino mass matrix

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The SM can not explain neutrino mass

- Adding Majorana mass term  $m_\nu$
- # observed parameter  $\theta_{12}, \dots < \# m_\nu$  parameter
- Assuming **some zeros** in  $m_\nu \rightarrow m_1, m_2, m_3$  can be determined  
without UV parameters

$$\text{case } B_5: \mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix},$$

$$\text{case } B_6: \mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix},$$

$$\text{case } D: \mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix},$$

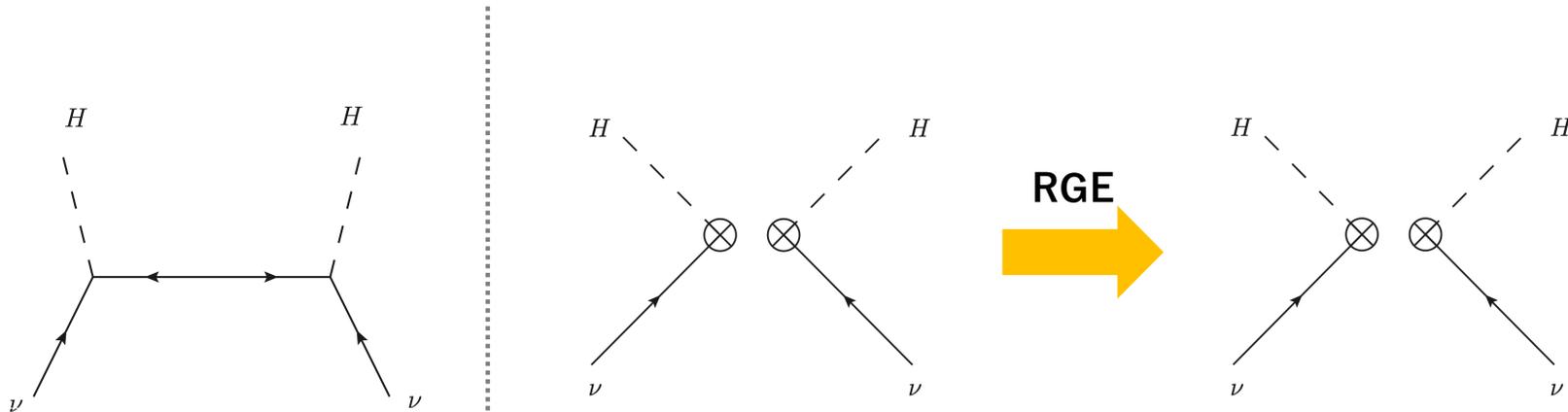
$U(1)_{L_\mu - L_\tau}$  is case D

[L. Lavoura 0411232](#)

# Intro: Type-I SeeSaw

Adding Right-handed neutrino in the SM

$$\mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$$



RGE(SM) keeps zero minors

$$- \kappa_\ell L_\ell \tilde{H} \bar{N}_\ell$$

$\sim 10^{14} \text{ GeV}$

$$C_{\ell\ell'} (L_\ell \tilde{H})(L_{\ell'} \tilde{H})$$

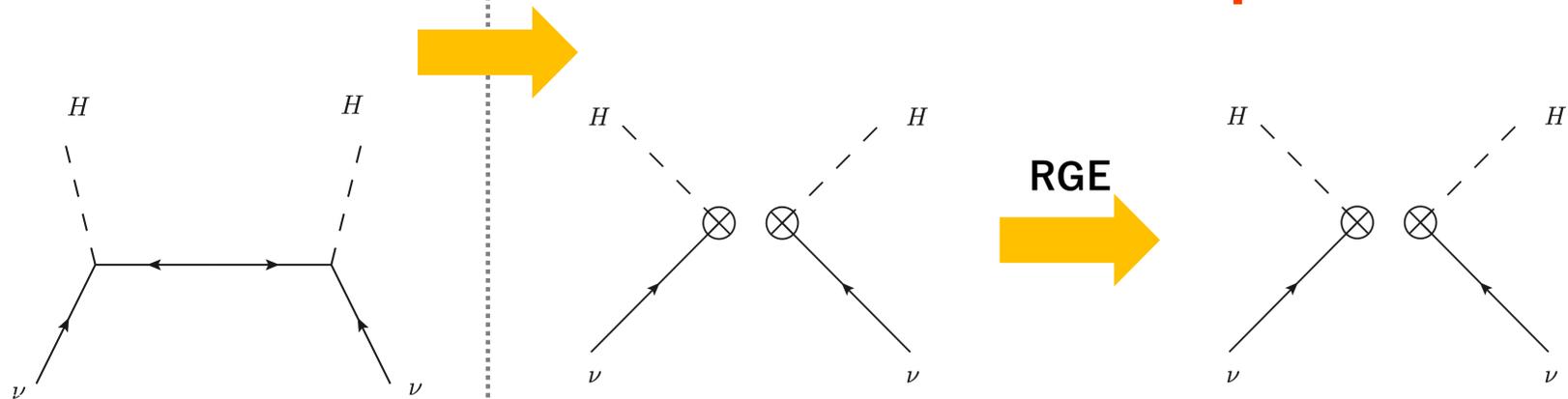
$$C_{\ell\ell'} (L_\ell \tilde{H})(L_{\ell'} \tilde{H})$$

After EW SSB turns to Majorana mass

$\sim 10^2 \text{ GeV}$

# Intro: Type-I SeeSaw

How about Threshold Effect ( One Loop ) ?  $\mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$



RGE(SM) keeps zero minors

$$- \kappa_\ell L_\ell \tilde{H} \bar{N}_\ell$$

$\sim 10^{14} \text{ GeV}$

$$C_{\ell\ell'} (L_\ell \tilde{H})(L_{\ell'} \tilde{H})$$

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# Our Result

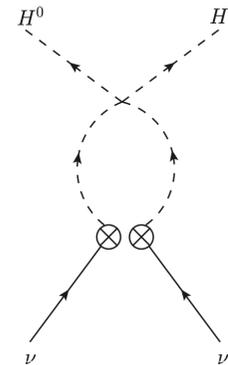
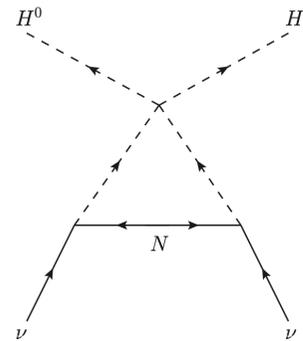
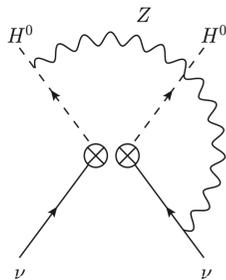
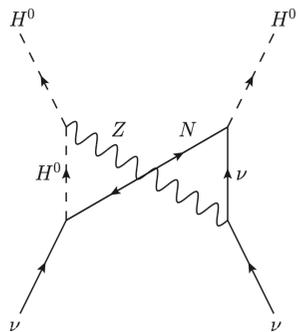
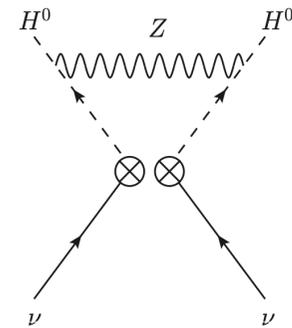
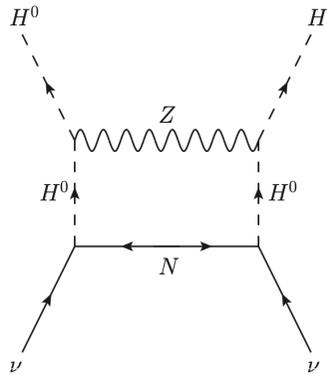
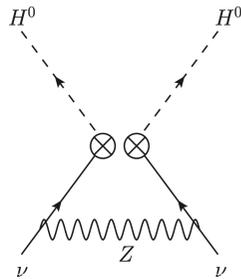
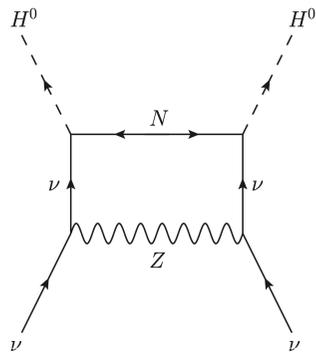
1. Threshold Effects (**SM** one loop) generally breaks minor structure; this is not specific to  $U(1)_{L_\mu-L_\tau}$
2. Additional **BSM** correction can also break the minor structure in  $U(1)_{L_\mu-L_\tau}$

# Part I

## 1. Threshold effects (SM one loop) breaks minor structure

One loop matching of Wilson coefficient  
of Weinberg Operator

# Diagrams



# Results

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$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i N_\ell^\dagger \bar{\sigma}^\mu \partial_\mu N_\ell - \kappa_\ell L_\ell \tilde{H} \bar{N}_\ell - \frac{1}{2} M_{\ell\ell'} N_\ell N_{\ell'} + \text{h.c.}$$

Type-I seesaw

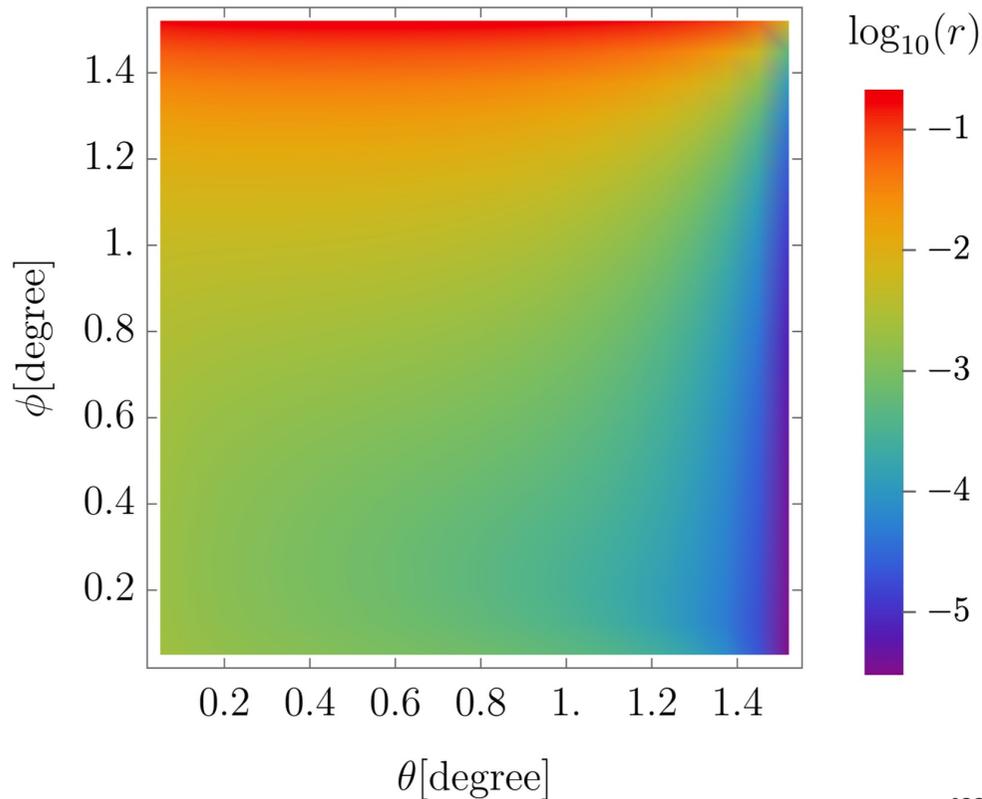
$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + C_{\ell\ell'} (L_\ell \tilde{H})(L_{\ell'} \tilde{H}) \quad C^{\text{tree}} = Y^D \frac{1}{\hat{M}} (Y^D)^T = \kappa M^{-1} \kappa^T$$

$$C(\mu_{\text{UV}}^2)^{\text{one-loop}} = \frac{m_Z^2}{16\pi^2 v_{\text{EW}}^2} Y^D \frac{1}{\hat{M}} \left( -3 \ln \frac{\hat{M}^2}{\mu_{\text{UV}}^2} + 1 \right) (Y^D)^T + \frac{m_h^2}{16\pi^2 v_{\text{EW}}^2} Y^D \frac{1}{\hat{M}} \left( -\ln \frac{\hat{M}^2}{\mu_{\text{UV}}^2} + 1 \right) (Y^D)^T$$

**At NLO, there is no need of Right-handed neutrino mass  $M$ !!**

# Breaking down of minors

$$r = (C^{-1})_{\mu\mu} / (C^{-1})_{ee}$$



$r = 0$  , if minor is hold

$$(\kappa_e, \kappa_\mu, \kappa_\tau) = \kappa(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$$

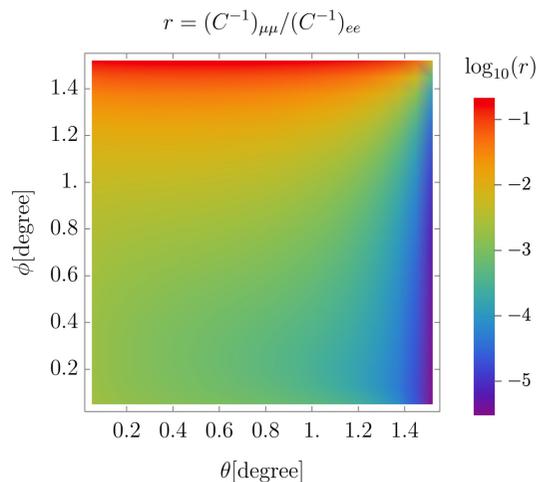
$$m_\nu = -v_{\text{EW}}^2 Y^D \hat{M}^{-1} (Y^D)^T - v_{\text{EW}}^2 C(v_{\text{EW}}^2; \hat{M})^{\text{one-loop}}$$

# Short summary I

Zero minors are broken by SM One Loop Threshold Effects

The deviation can be evaluated by Yukawa couplings as new parameter ( No need of RH neutrino mass )

The correlation with Leptogenesis may be interesting



$$C(\mu_{UV}^2)^{\text{one-loop}}$$
$$= \frac{m_Z^2}{16\pi^2 v_{EW}^2} Y^D \frac{1}{\hat{M}} \left( -3 \ln \frac{\hat{M}^2}{\mu_{UV}^2} + 1 \right) (Y^D)^T$$
$$+ \frac{m_h^2}{16\pi^2 v_{EW}^2} Y^D \frac{1}{\hat{M}} \left( -\ln \frac{\hat{M}^2}{\mu_{UV}^2} + 1 \right) (Y^D)^T$$

# Part II

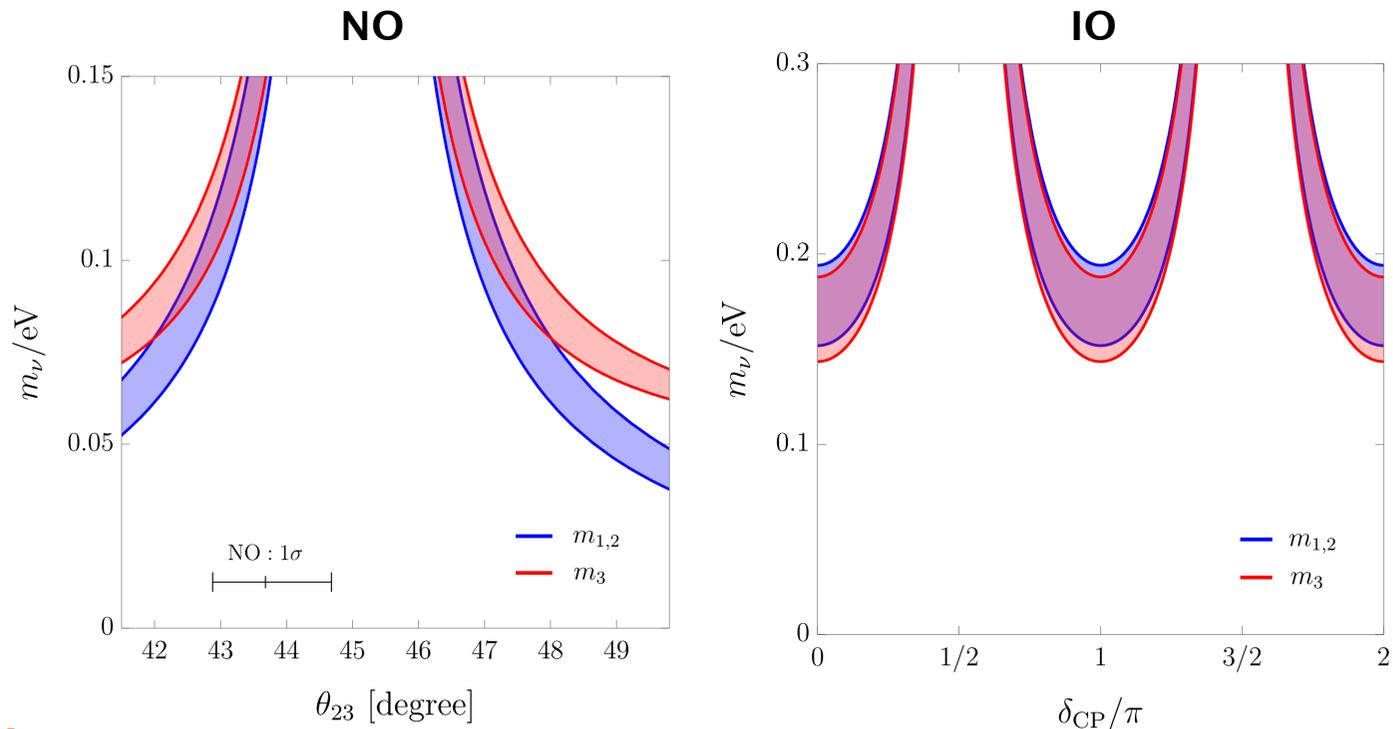
2. Additional BSM correction also breaks in  $U(1)_{L_\mu-L_\tau}$

**SM couplings are small**

**So, BSM loop contribution is important!!**

(Much more diagrams contribute)

# Motivation



**But...**

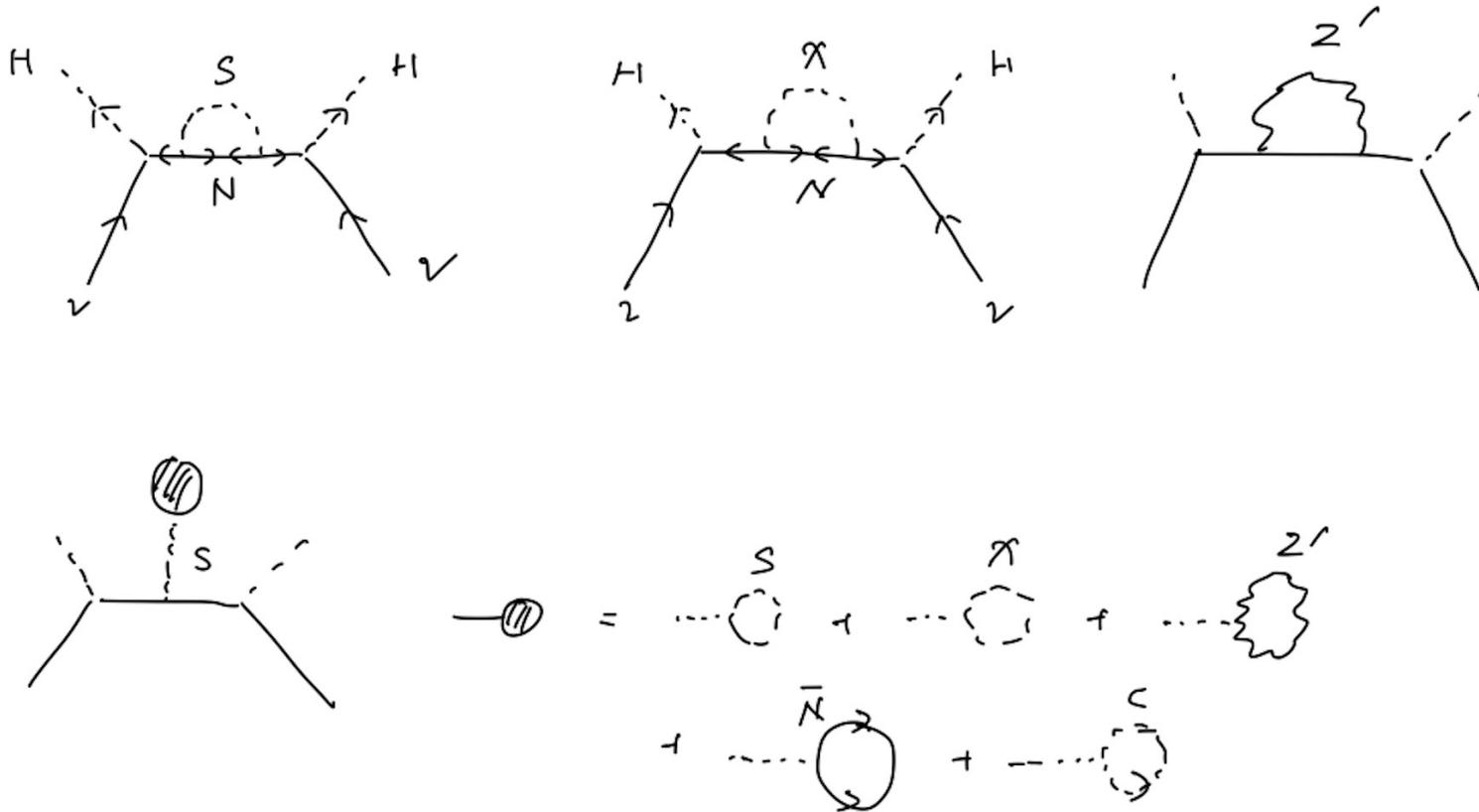
$U(1)_{L_\mu-L_\tau}$   $\sum m_\nu$  at least  $\sim$  **0.3 eV**, Latest cosmology bound  $\sum m_\nu <$  **0.2 eV**

# Modification of two-zero minor

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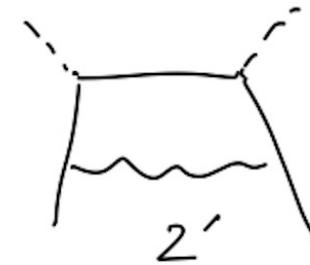
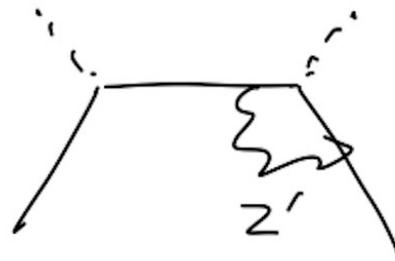
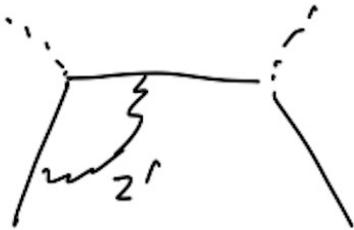
$$\begin{array}{l}
 U_{\mu 1}^2 + \frac{U_{\mu 2}^2}{r_{21}} + \frac{U_{\mu 3}^2}{r_{31}} = 0, \\
 U_{\tau 1}^2 + \frac{U_{\tau 2}^2}{r_{21}} + \frac{U_{\tau 3}^2}{r_{31}} = 0.
 \end{array}
 \quad
 \begin{array}{c}
 m_{\nu}^{-1} = -m_D^{T-1} (M + \Delta M) m_D^{-1} \\
 \xrightarrow{\text{modified radiatively}}
 \end{array}
 \quad
 \begin{array}{l}
 U_{\mu 1}^2 + \frac{U_{\mu 2}^2}{r_{21}} + \frac{U_{\mu 3}^2}{r_{31}} = \epsilon e^{2i\eta_1}, \\
 U_{\tau 1}^2 + \frac{U_{\tau 2}^2}{r_{21}} + \frac{U_{\tau 3}^2}{r_{31}} = \epsilon' e^{2i\eta_1},
 \end{array}$$

# Diagrams



# Diagrams

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$Z'$ :  $U(1)_{B-L}$  gauge boson

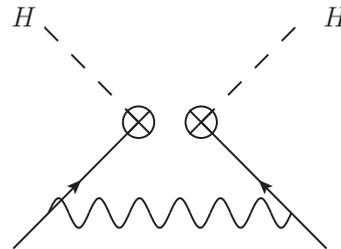
$$\phi = \frac{1}{\sqrt{2}} (V_{M-2} + S + iX)$$

# Results $\Delta M_{22}$ and $\Delta M_{33}$

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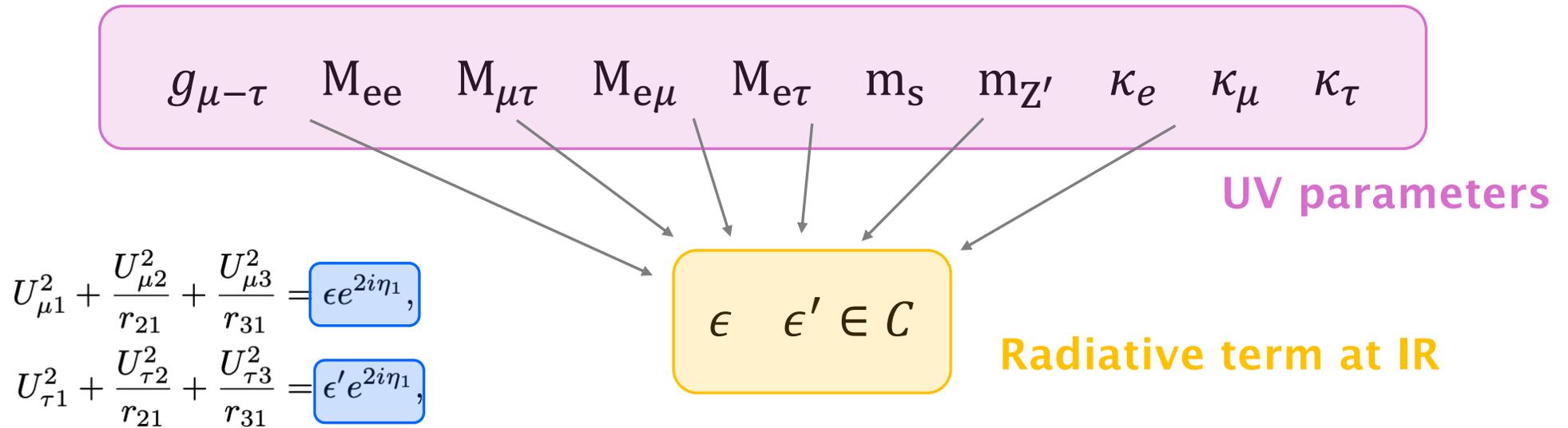
$$\begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ M_{e\tau} & 0 & 0 \end{pmatrix} U_R \left( \frac{m_s^2}{m_{Z'}^2} \frac{\hat{M}}{\hat{M}^2 - m_s^2} \ln \frac{\hat{M}^2}{m_s^2} \right) U_R^T \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ M_{e\tau} & 0 & 0 \end{pmatrix} \\ + 3 \begin{pmatrix} 0 & M_{e\mu} & -M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ -M_{e\tau} & 0 & 0 \end{pmatrix} U_R \left( \frac{\hat{M}}{\hat{M}^2 - m_{Z'}^2} \ln \frac{\hat{M}^2}{m_{Z'}^2} \right) U_R^T \begin{pmatrix} 0 & M_{e\mu} & -M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ -M_{e\tau} & 0 & 0 \end{pmatrix}$$

Renormalization scale  $\mu$  free, No gauge dependence  $\xi$   
**Second term also arises from EFT side !!**



$$U_R^T M U_R = \hat{M} = \text{diag}(M_1, M_2, M_3)$$

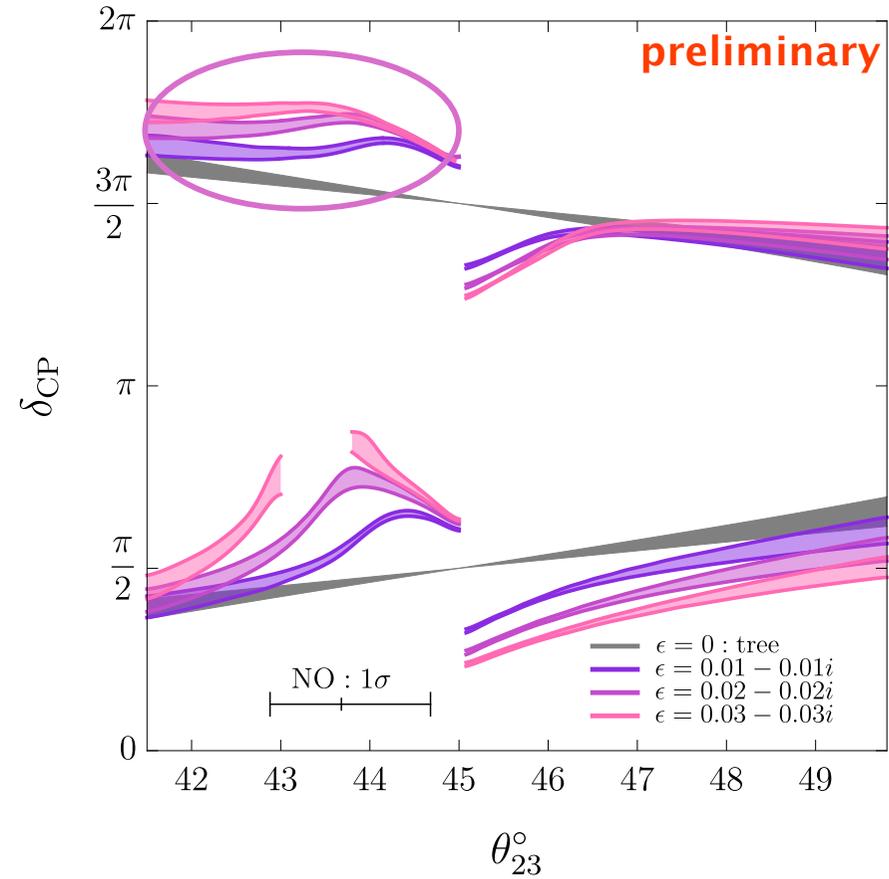
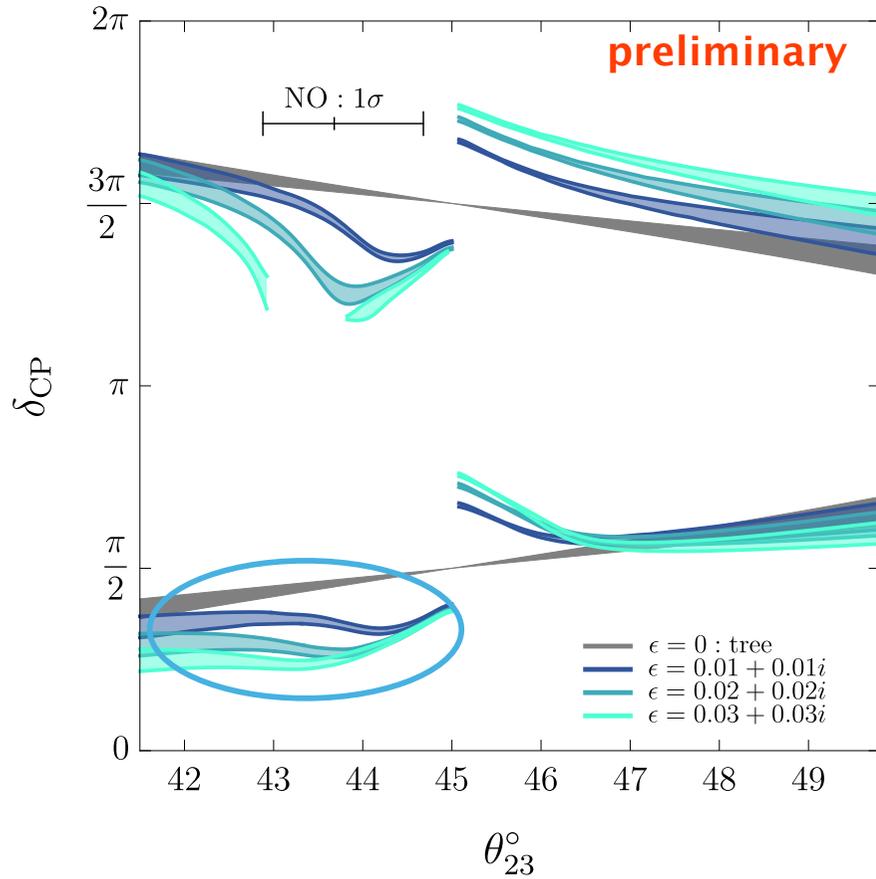
# UV parameters degeneration



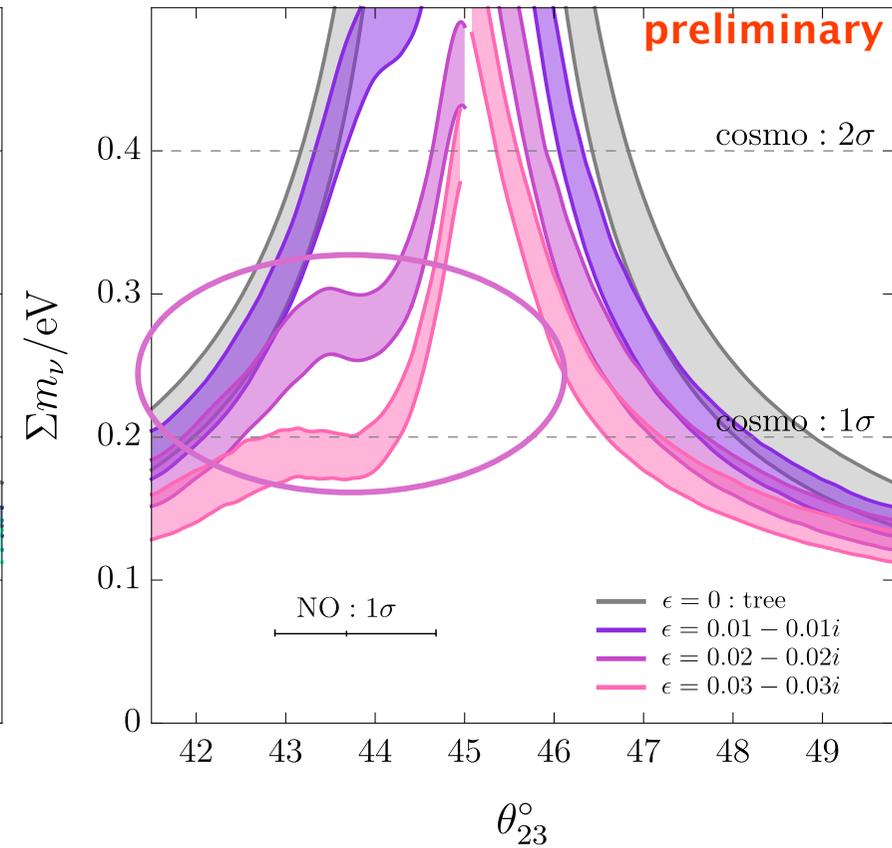
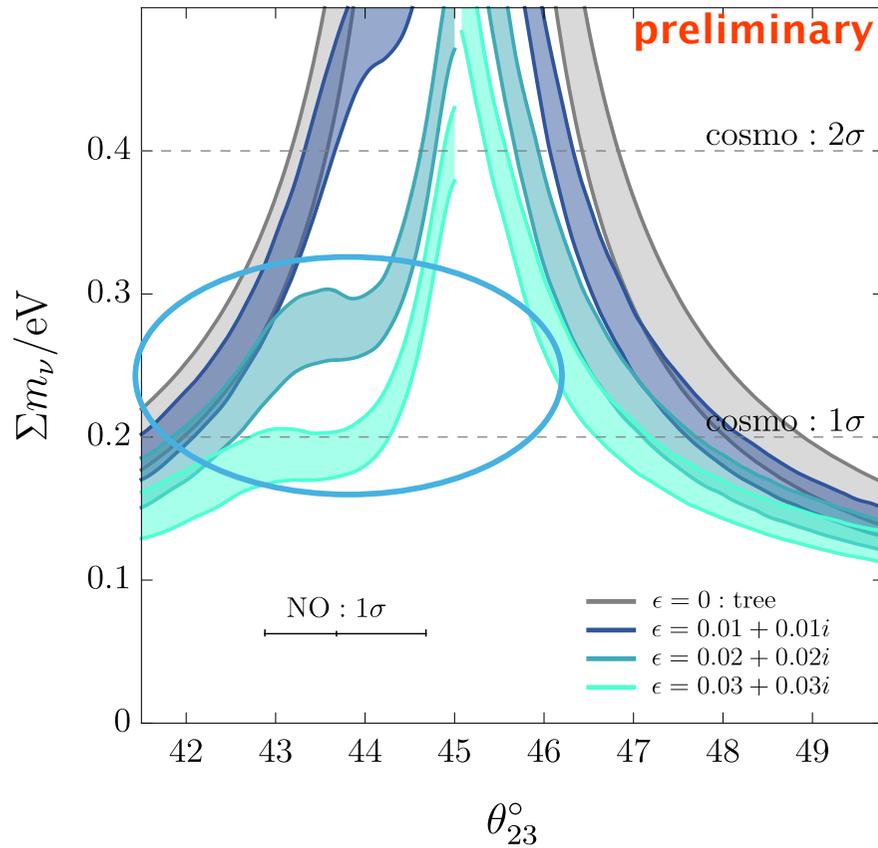
$\epsilon/\epsilon'$  is determined only IR observables  $\theta_{23}$  etc.

Test using  $\epsilon$  as 2 parameters UV parameters independently

# UV parameters independent predictions I

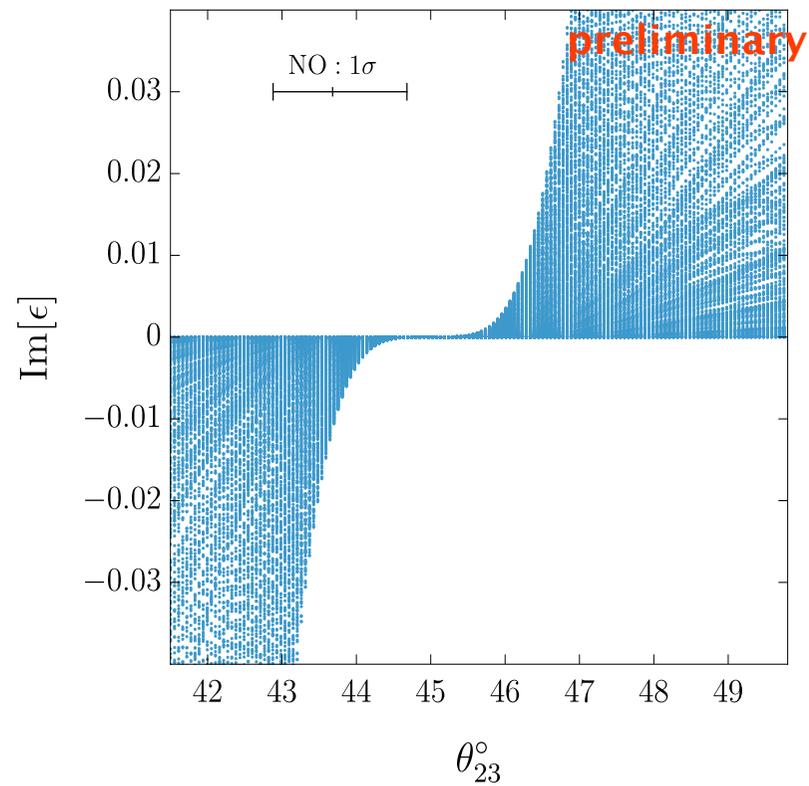
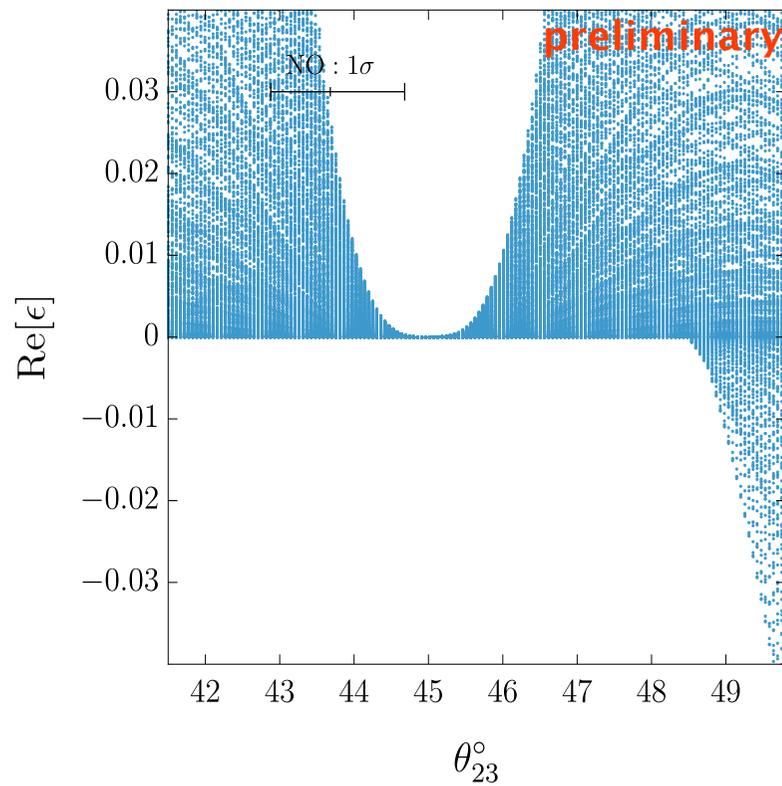


# UV parameters independent predictions II



# UV parameters $\Rightarrow \epsilon$

Taking  $g_{\mu-\tau} = 1$ ,  $\lambda = 1$



# Conclusion and messages

- Threshold corrections can break zero minor even if SM loop contribution
- The impact might be not negligible (but loop suppressed)
- All UV parameters are not needed at NLO because of zero minor holds at LO
- Correlation with Leptogenesis may be interesting since Yukawa couplings are sensitive to the NLO prediction.

