International Workshop on Future Linear Colliders, 2025 10/22

Higgs production in association with a Z boson at TeV-scale lepton colliders

(**Work in progress**)

@Sora Kobayashi

Hiroyuki Furusato (Iwate University)

Kentarou Mawatari
Satsuki Hosoya
Shouta Suzuki

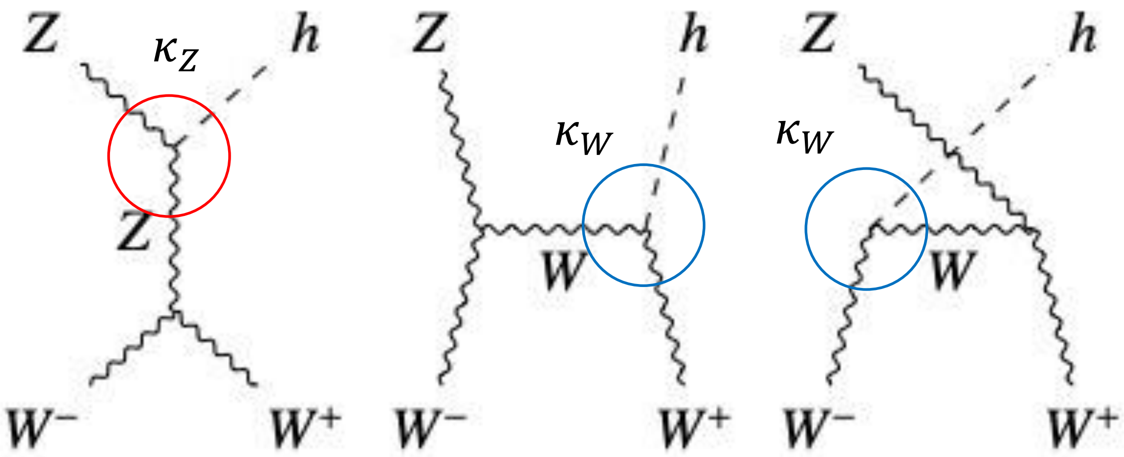
Contents

1. Introduction
2. $l^- l^+ \to \nu_l \bar{\nu}_l Z h$
3. FD gauge
4. Summary

Contents

1. Introduction

There is much to understand about the Higgs boson.
→ **The origin of "Spontaneous Electroweak Symmetry Breaking"**
We focus on the
Vector-boson and higgs (Vh) production via Vector Boson Scattering (VBS).

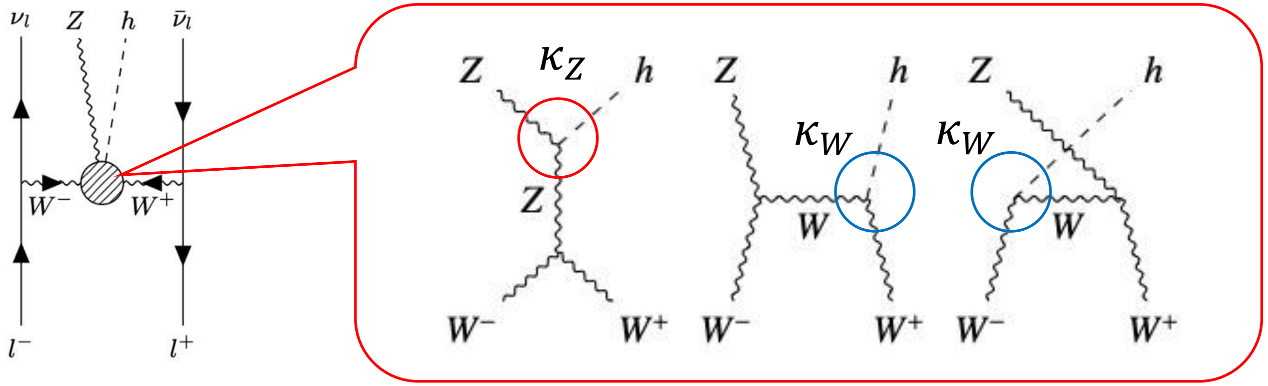


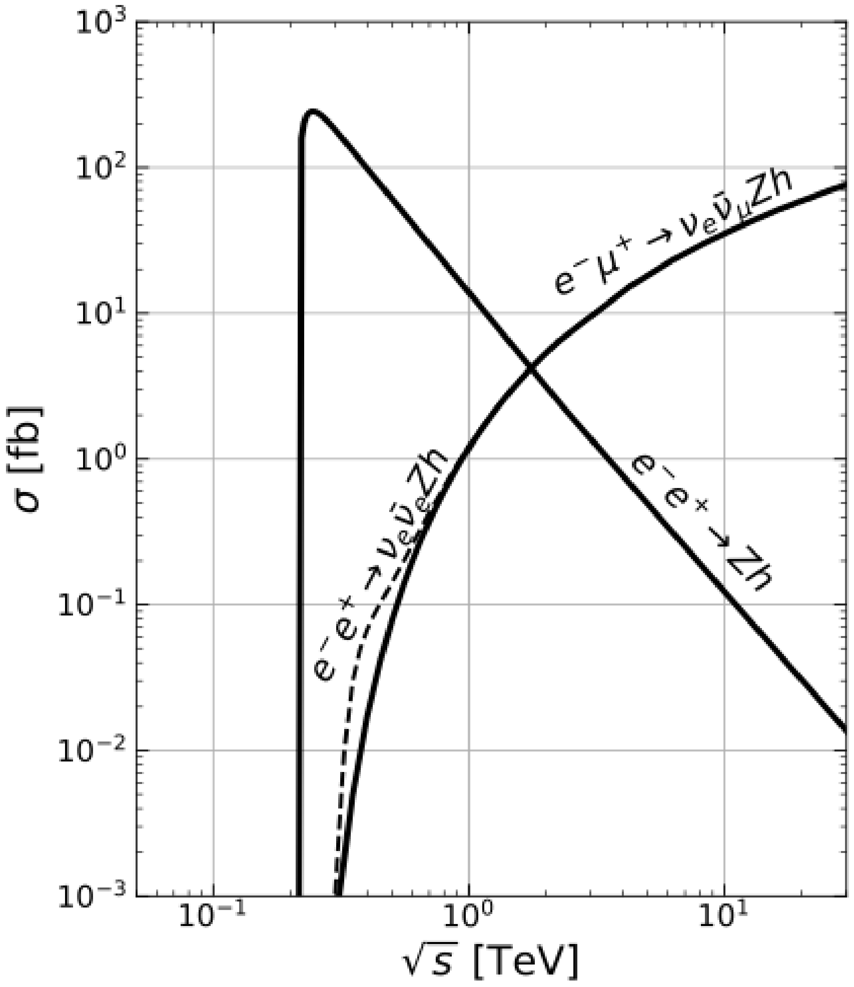
This is an important process where the hZZ and hWW couplings can be determined simultaneously.

1. Introduction

We focus on the $l^- l^+ \to \nu\bar{\nu}Zh$ process.

- At high energies, the cross section of $l^- l^+ \to \nu\bar{\nu}Zh$ is larger than the cross section of $l^- l^+ \to Zh$.
- The Vh production via vector boson scattering is included.





Here, we consider $e^-\mu^+$ scattering.

$l^- l^+ \to \nu\bar{\nu}Zh$ is important at high-energy lepton collider experiments to study Higgs coupling to the weak gauge bosons.

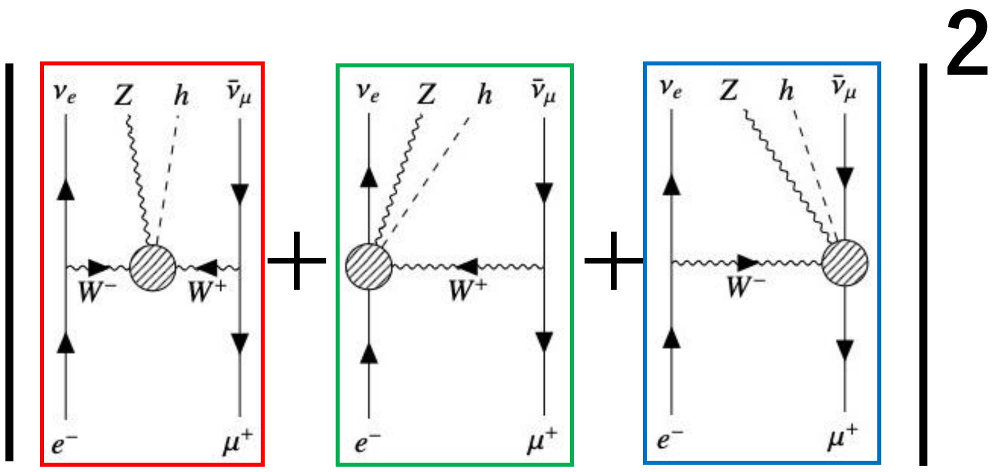
Contents

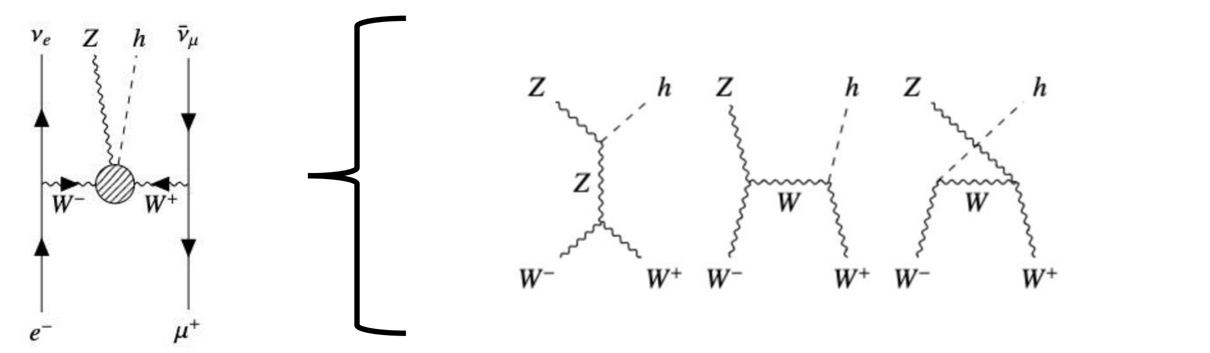
2. $l^{-}l^{+} \rightarrow \nu_l \bar{\nu}_l Zh$

We divide $e^{-}\mu^{+} \rightarrow \nu\bar{\nu}Zh$ into 3 groups.

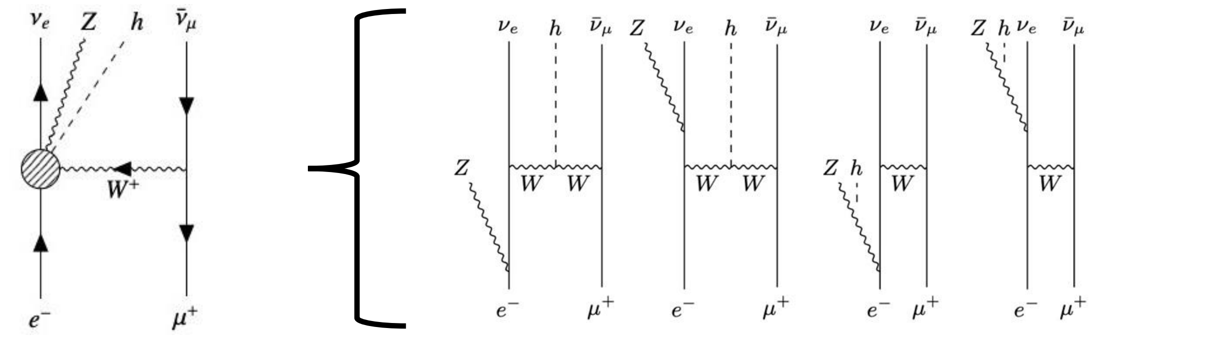
$\boldsymbol{\sigma} \propto$

Total

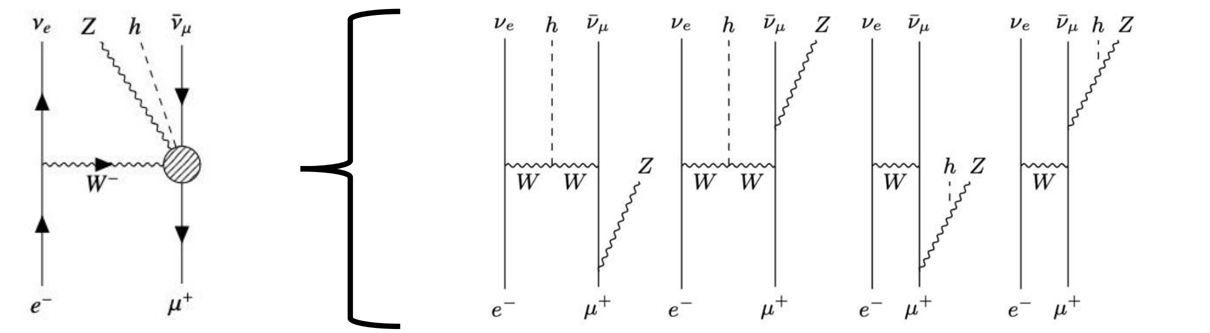




(a) VBS



(b) $e^{-}W^{+}$



(c) $W^{-}\mu^{+}$

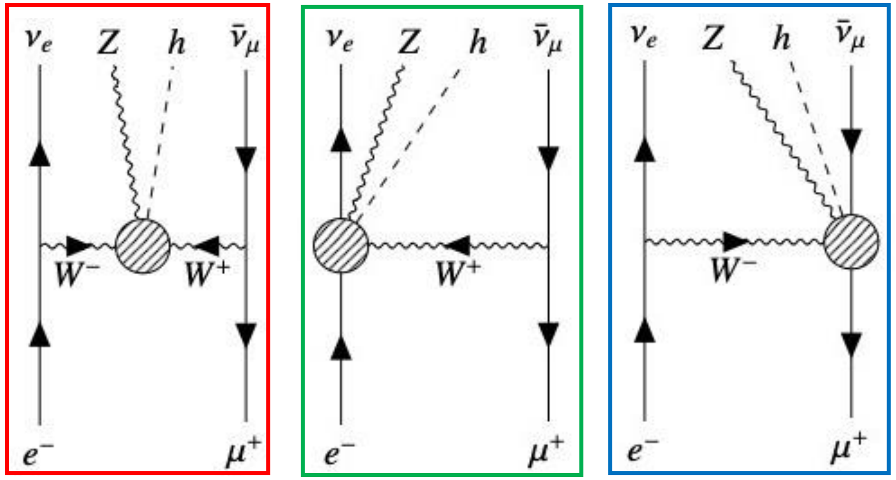
Gauge invariant : total $\propto |(\mathrm{a}) + (b) + (c)|^2$,

dependent : $(\mathrm{a}) \propto |M_{(a)}|^2, (b) \propto |M_{(b)}|^2, (c) \propto |M_{(c)}|^2$

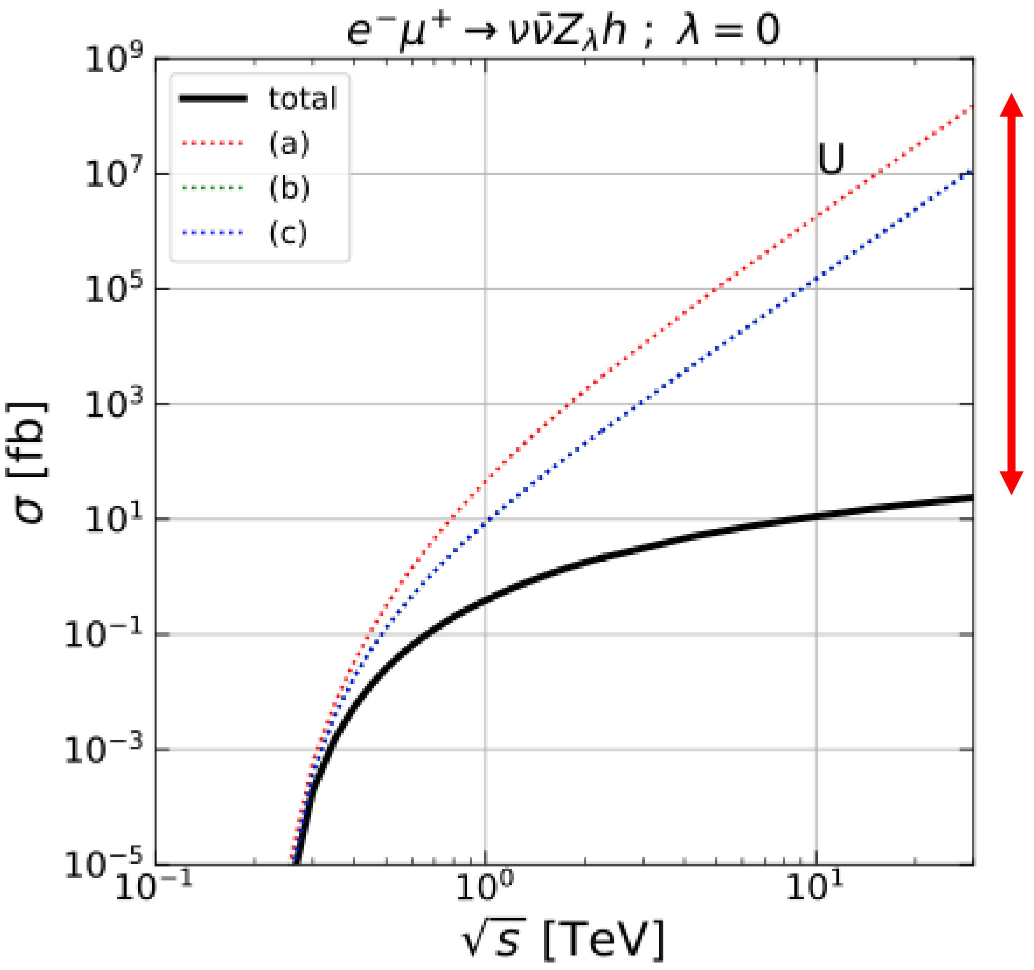
2. $l^-l^+ \to \nu_l\bar{\nu}_l Zh$

In the longitudinal massive gauge boson scattering, Gauge cancellation (energy growth)
→loss of significance

This is a major obstacle for future high-energy experiments.



(a) (b) (c)



total$\propto |(a) + (b) + (c)|^2$,

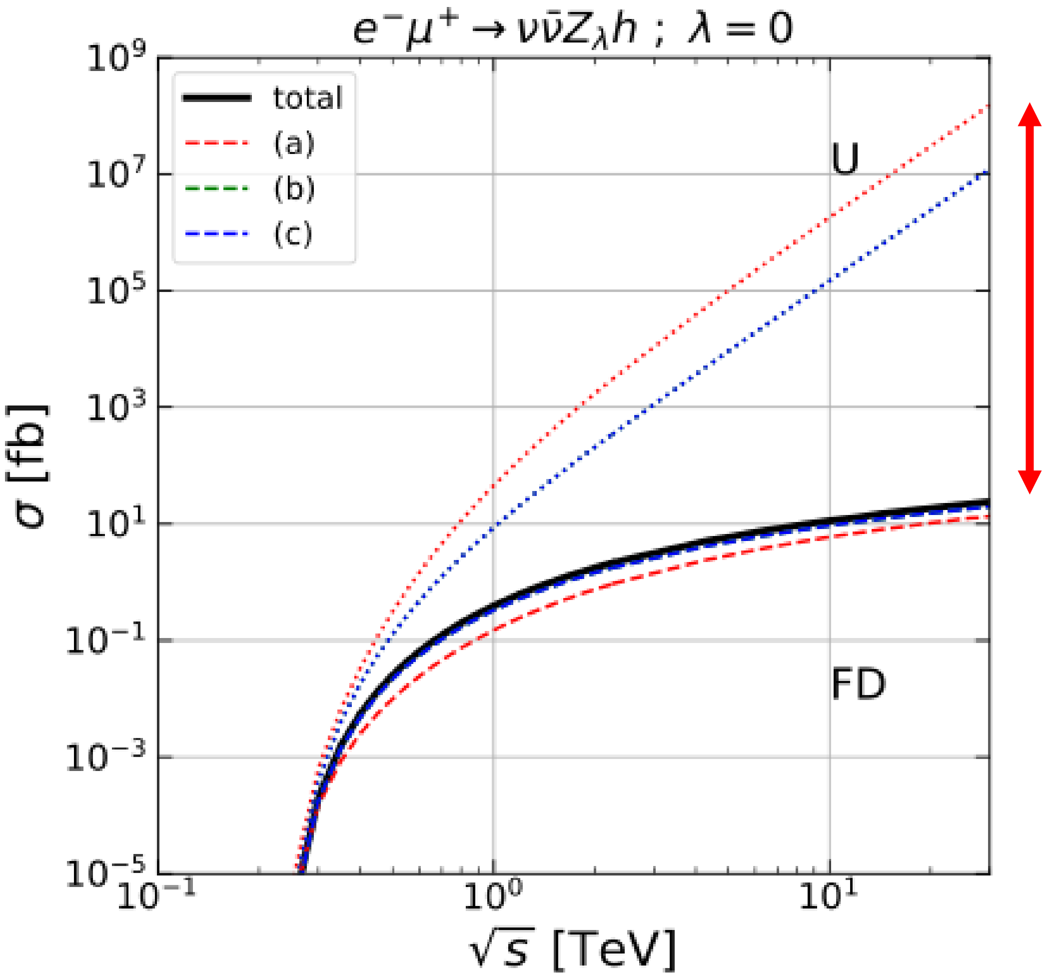
$(a) \propto |M_{(a)}|^2 > (b) \propto |M_{(b)}|^2 \approx (c) \propto |M_{(c)}|^2$

2. $l^-l^+ \to \nu_l\bar{\nu}_l Zh$

In the longitudinal massive gauge boson scattering, Gauge cancellation (energy growth)
→loss of significant digits

This is a major obstacle for future high-energy experiments.

A new gauge fixing was proposed and it indicates gauge cancellation among the interfering amplitudes can be avoided!!



Feynman-Diagram (FD) gauge [1],[2],[3],[4]

$\text{total} \propto |(a) + (b) + (c)|^2$,

$(a) \propto |M_{(a)}|^2 > (b) \propto |M_{(b)}|^2 \approx (c) \propto |M_{(c)}|^2$

[1] J. Chen, K. Hagiwara, J. Kanzaki and K. Mawatari, "Helicity amplitudes without gauge cancellation for electroweak processes", Eur.Phys.J.C 83 (2023) 10, 922.
[2] J. Chen, K. Hagiwara, J. Kanzaki, K. Mawatari and Y. Zheng , "Helicity amplitudes in light-cone and Feynman-diagram gauges", Eur.Phys.J.Plus 139 332 (2024)
[3] K. Hagiwara, J. Kanzaki, O. Mattelaer, K. Mawatari and Y. Zheng, "Automatic generation of helicity amplitudes in Feynman-Diagram gauge", Phys. ReV. D 110, 056024 (2024)
[4] H.Furusato, K.Mawatari, Y.Suzuki, Y.Zheng, "W-boson pair production at lepton colliders in the Feynman-Diagram gauge", *Phys.Rev.D 110 (2024) 5, 053005*

Contents

U gauge vs FD gauge

U gauge (Covariant R_ξ gauge with $\xi \to \infty$)

Only weak gauge bosons (Nambu-Goldstone bosons are decoupled.)

$$\beta = \frac{q}{E} = \sqrt{1 - \frac{m^2}{E^2}},\quad \gamma = \frac{E}{m} = \frac{1}{\sqrt{1-\beta^2}}.$$

- propagator $(\mu, \nu = 0, 1, 2, 3)$

$$P^{\mathrm{U}}_{\mu\nu} = \frac{i}{q^2 - m^2 + i\epsilon}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2}\right)$$

- polarization vector

$$\epsilon^\mu(q, 0) = \frac{1}{m}(q, 0, 0, E) = \gamma\,(\beta, 0, 0, 1)$$

FD gauge (Non-covariant light-cone gauge with $n^\mu = \left(\mathrm{sgn}(q^0), -\frac{\vec{q}}{|\vec{q}\,|}\right)$

Superposition of weak gauge bosons and Nambu-Goldstone bosons

- 5×5 propagator $(M, N = 0, 1, 2, 3, 4)$

$$P^{\mathrm{FD}}_{MN} = \frac{i}{q^2 - m^2 + i\epsilon}\begin{pmatrix} -\tilde{g}_{\mu\nu} & im\frac{n_\mu}{n\cdot q} \\ -im\frac{n_\nu}{n\cdot q} & 1 \end{pmatrix}$$

$$-\tilde{g}_{\mu\nu} = -g_{\mu\nu} + \frac{n_\mu q_\nu + q_\mu n_\nu}{n\cdot q}$$

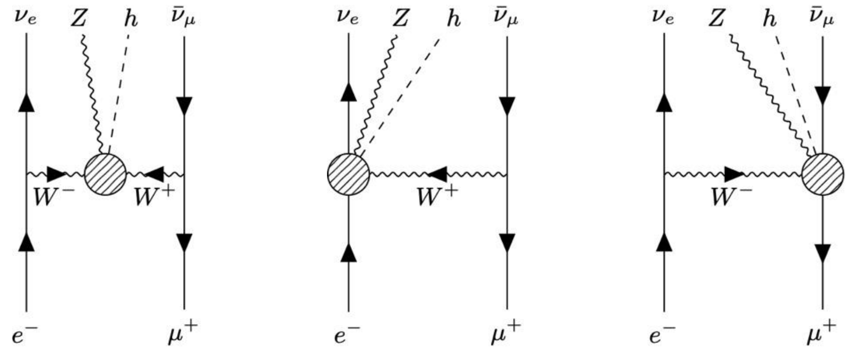
- 5 component polarization vector

$$\epsilon^M(q, 0) = (\tilde{\epsilon}^\mu(q, 0),\ i)$$

$$\tilde{\epsilon}^\mu(q, 0) = \epsilon^\mu(q, 0) - \frac{q^\mu}{Q} = \frac{1}{\gamma(1+\beta)}(-1, 0, 0, 1)$$

3. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$

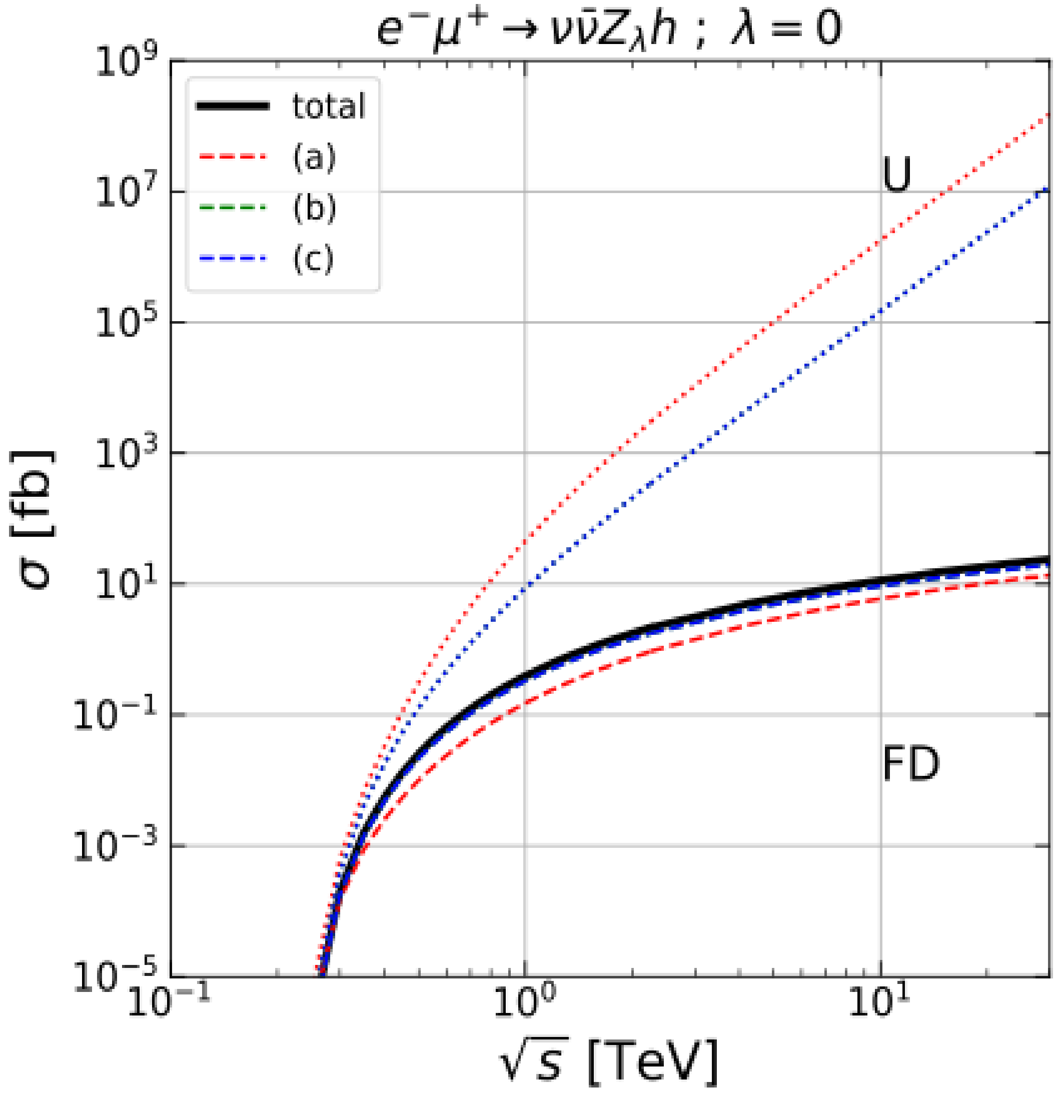
- **<u>The solid black line</u>** is the physical observable.
  total$\propto$ |(a) + (b) + (c)|^2



- U gauge ⋯⋯ , FD gauge - - - -

U gauge:
- Each category of the amplitudes has energy growth.
- Large cancellation among ((a),(b),(c)) at high energies.

FD gauge :
- No such energy growth of each category at all.



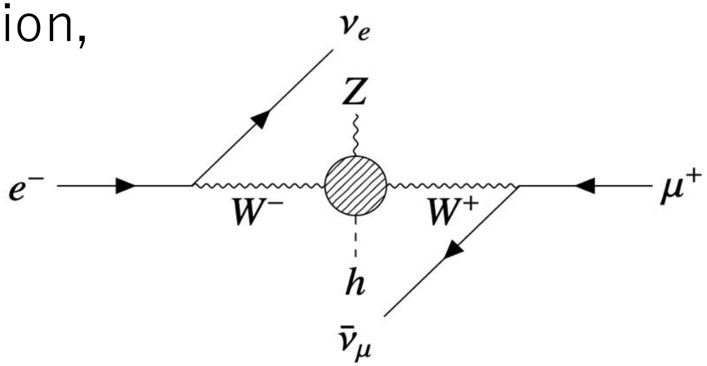
3. $e^-\mu^+ \rightarrow \nu_e \bar{\nu}_\mu Zh$

U gauge:

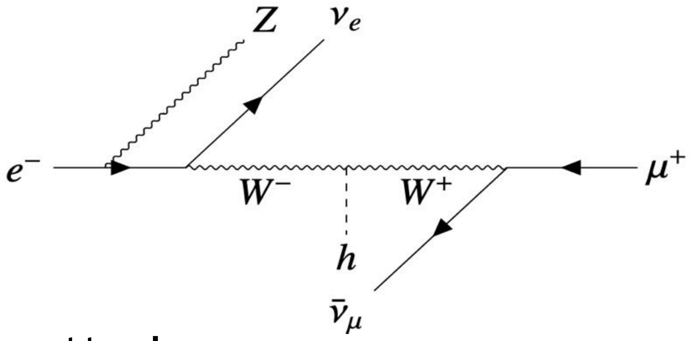
- No clue for the physical distribution from each contribution.

FD gauge :

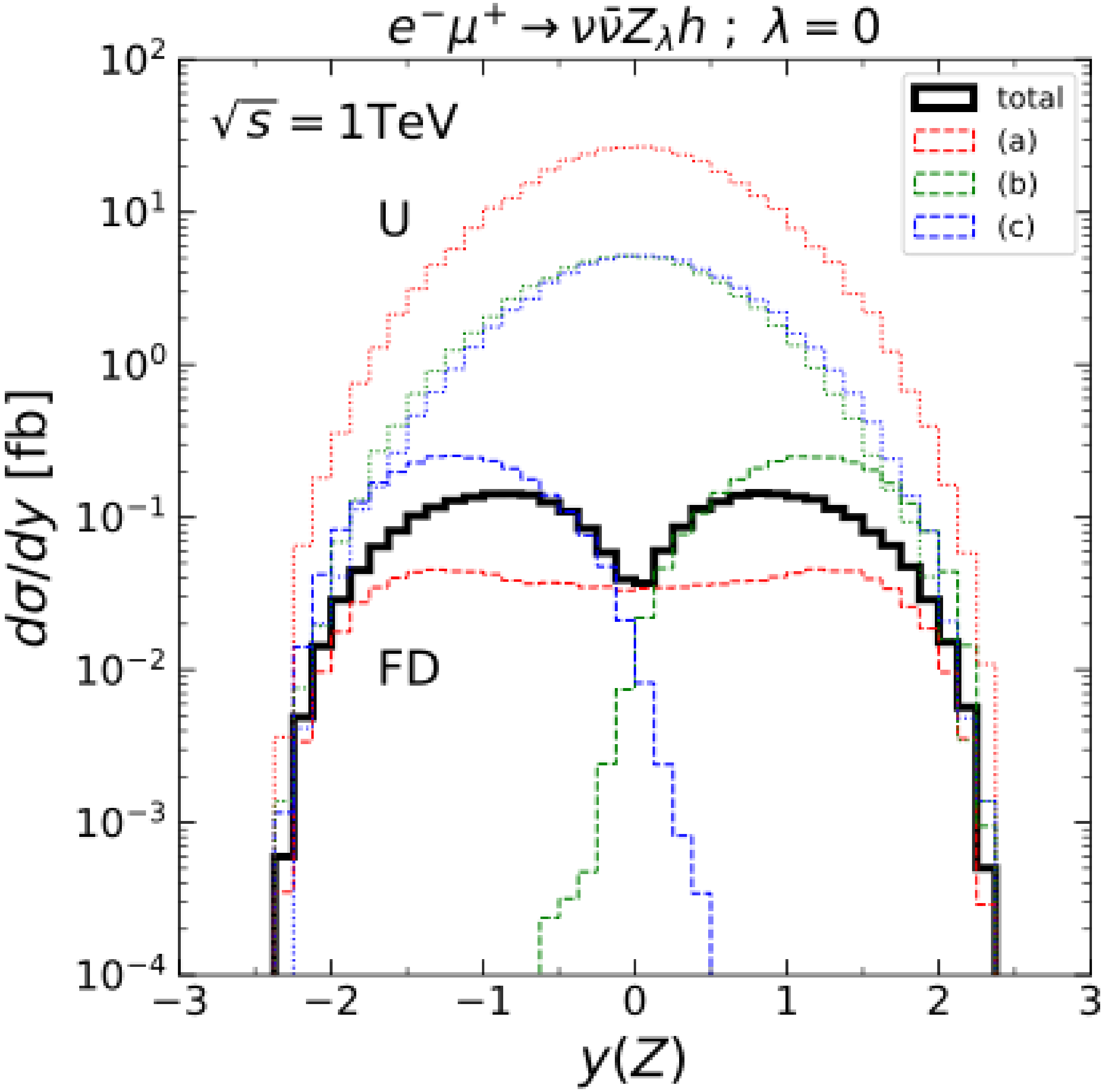
- (a) : Central region,



(b) : Forward scattering,

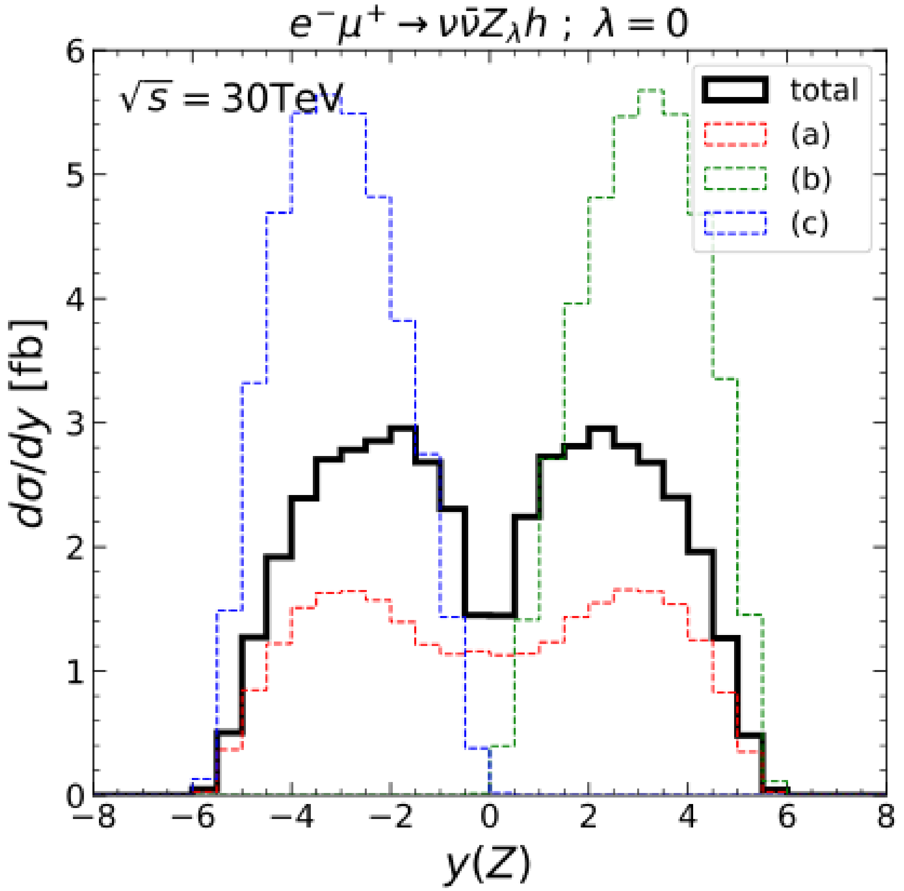


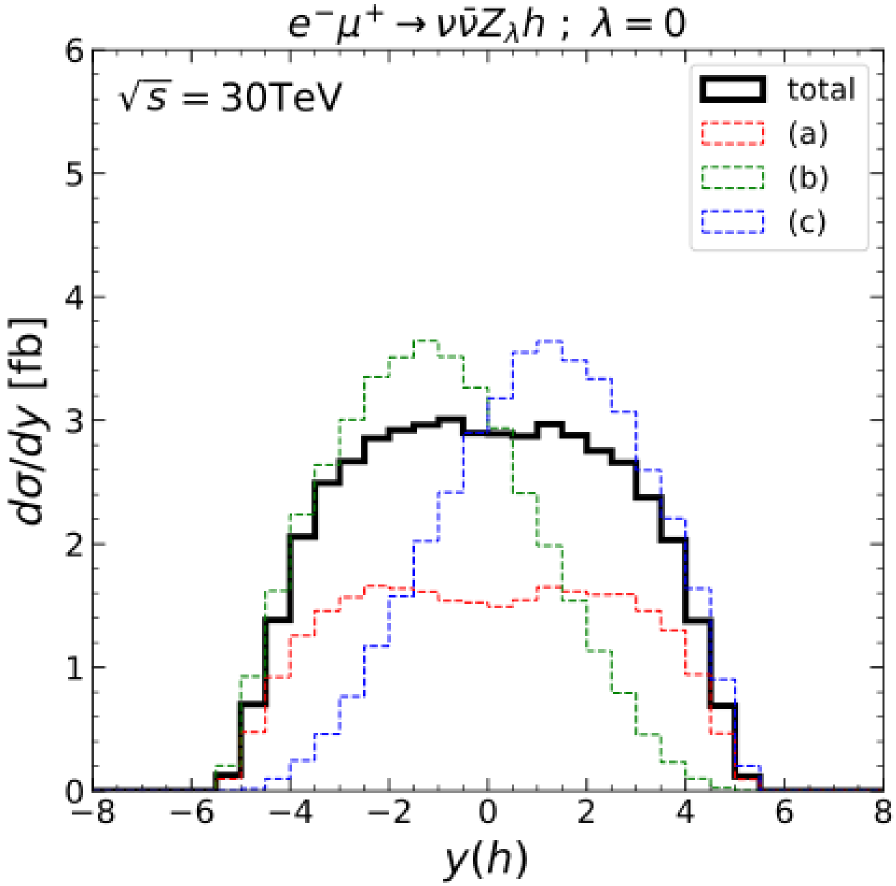
(c) : Backward scattering.



In the FD gauge,
The behavior of each group shows good agreement with our expectation.

3. $e^- \mu^+ \to \nu_e \bar{\nu}_\mu Z h$

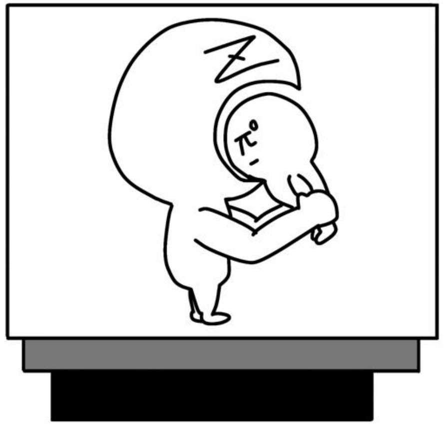




In $y(h)$,

- The $y(h)$ distributed in central region.
  →It does not receive collinear enhancement and thus tends to be emitted more isotropically, the Higgs is often produced as part of WW fusion or Zh system.
- (b) and (c) are swapped.
  → The boost effect.

Contents



4. Summary

We studied $e^-\mu^+ \to \nu\bar{\nu}Zh$.

We found

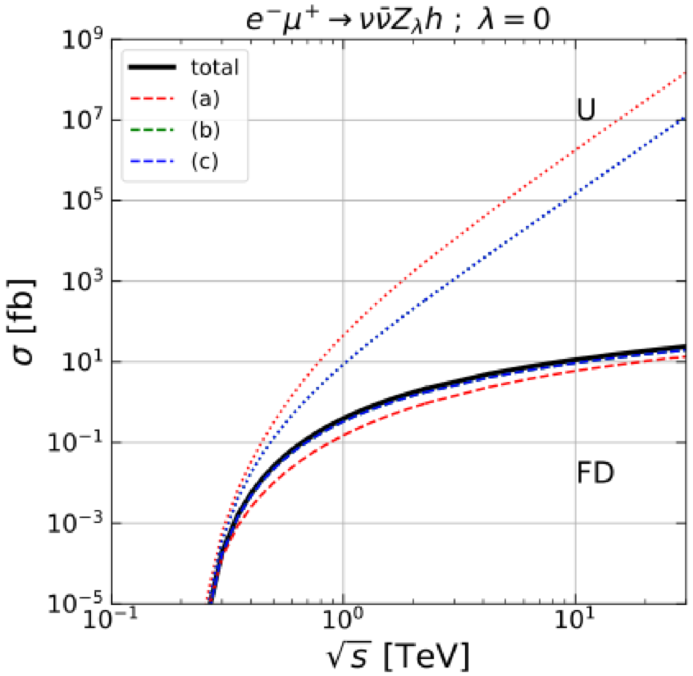
U gauge:

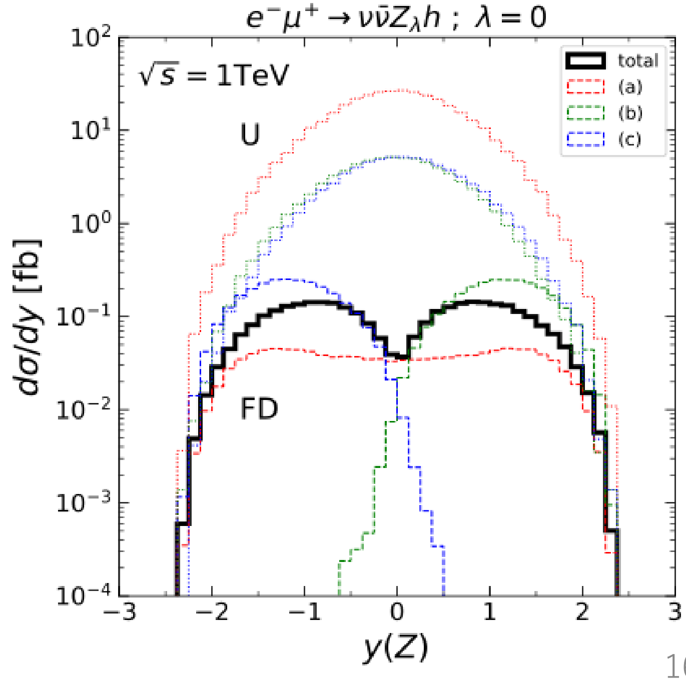
- Each category of the amplitudes has energy growth.
  → Large cancellation among the three groups ((a),(b),(c)) at high energies.
- No clue for the physical distribution from each contribution.

FD gauge :

- No such energy growth of each category at all.
- Each contribution is clearly separated, and the interference patterns among the diagrams can be clearly seen.
- Naive expectations for the contribution of each group agree. In particular, it becomes clear which diagrams dominate in specific regions of phase space.

Thanks to FD gauge!
For the search for new physics!?



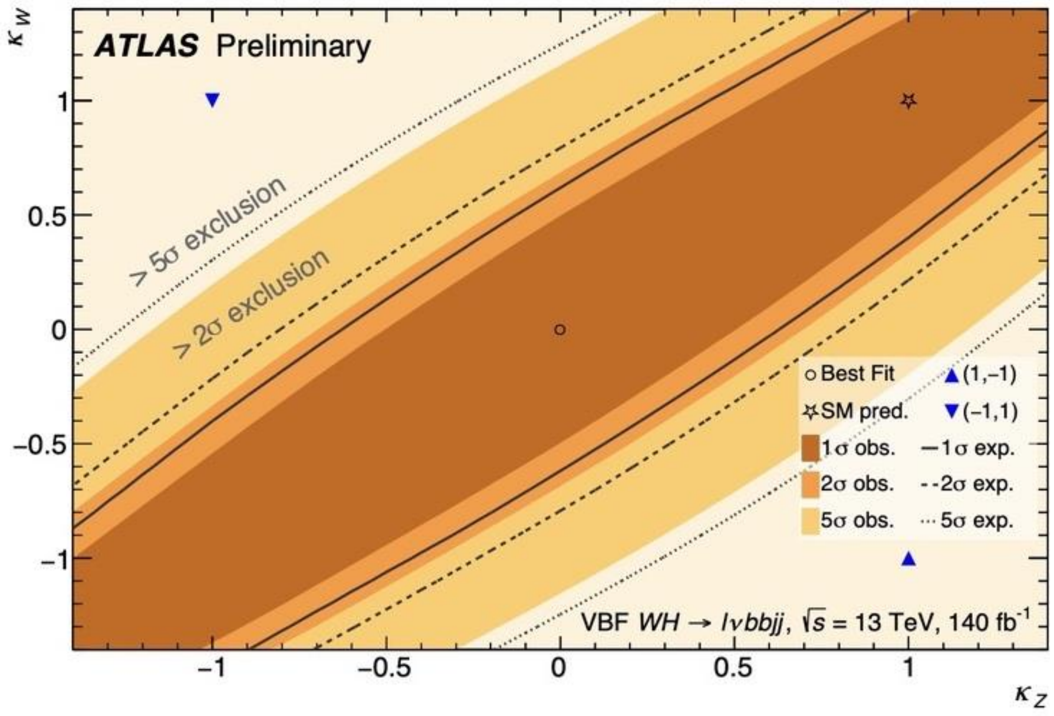


Back up

1. Motivation

・Determining the relative sign of the Higgs boson couplings to $\boldsymbol{W}$ and $\boldsymbol{Z}$ bosons using VBF $\boldsymbol{WH}$ production at ATLAS[1] and CMS[2]

[1] The ATLAS collaboration. "Determining the relative sign of the Higgs boson coupling to W and Z boson using VBF WH production with the ATLAS detector", Phys. Rev. Lett 133,141801, 2024.
[2] "Hayrapetyan, Aram and others", The CMS collaboration, "Study of WH production through vector boson scattering and extraction of the relative sign of the W and Z couplings to the Higgs boson in proton-proton collisions at $\sqrt{s} = 13$TeV", Phys. Lett. B (2025), 860, 139202.

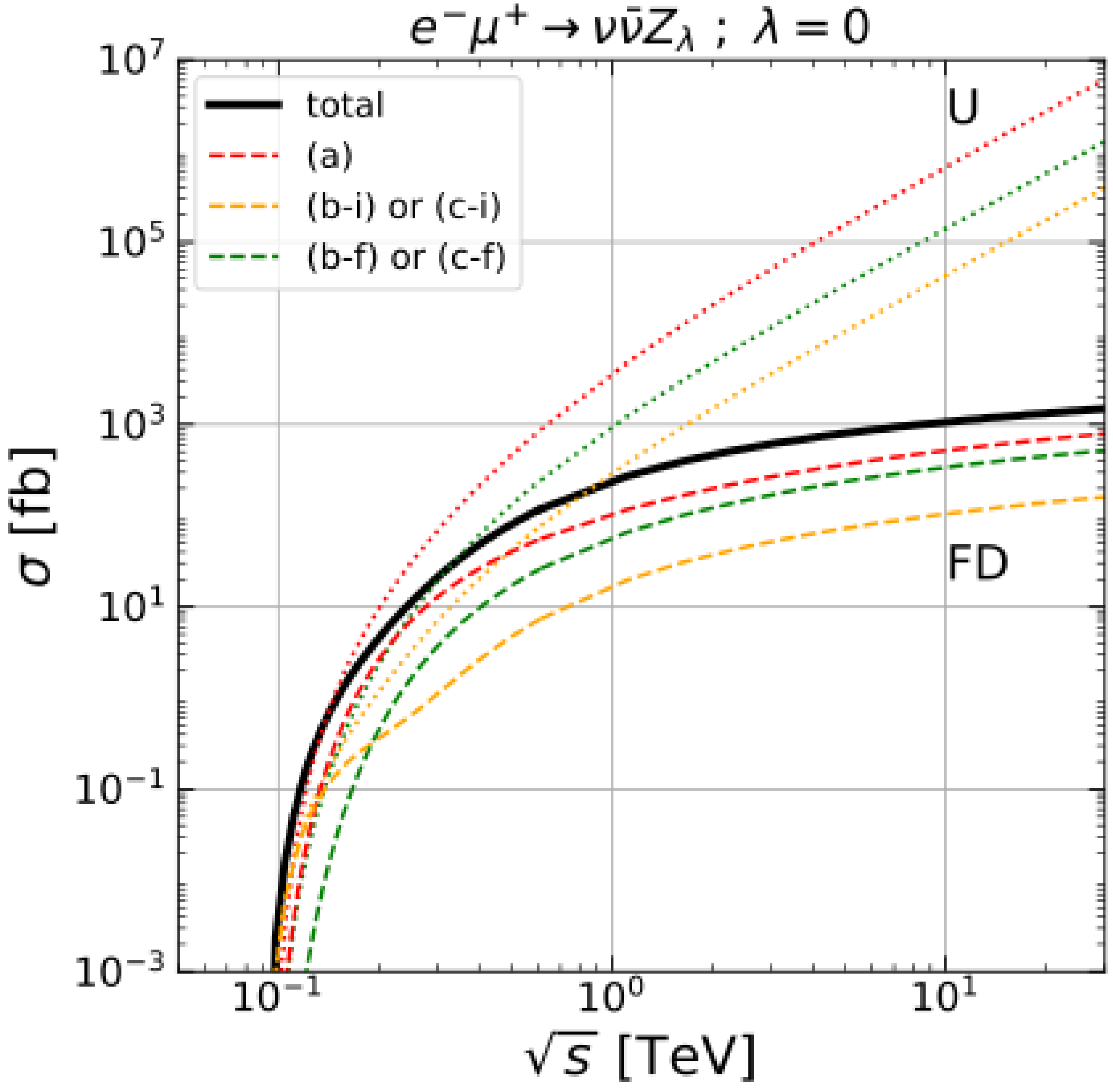


The scenario with opposite-sign couplings of the W and Z to the Higgs
→This hypothesis is excluded with significance greater than 8σ.

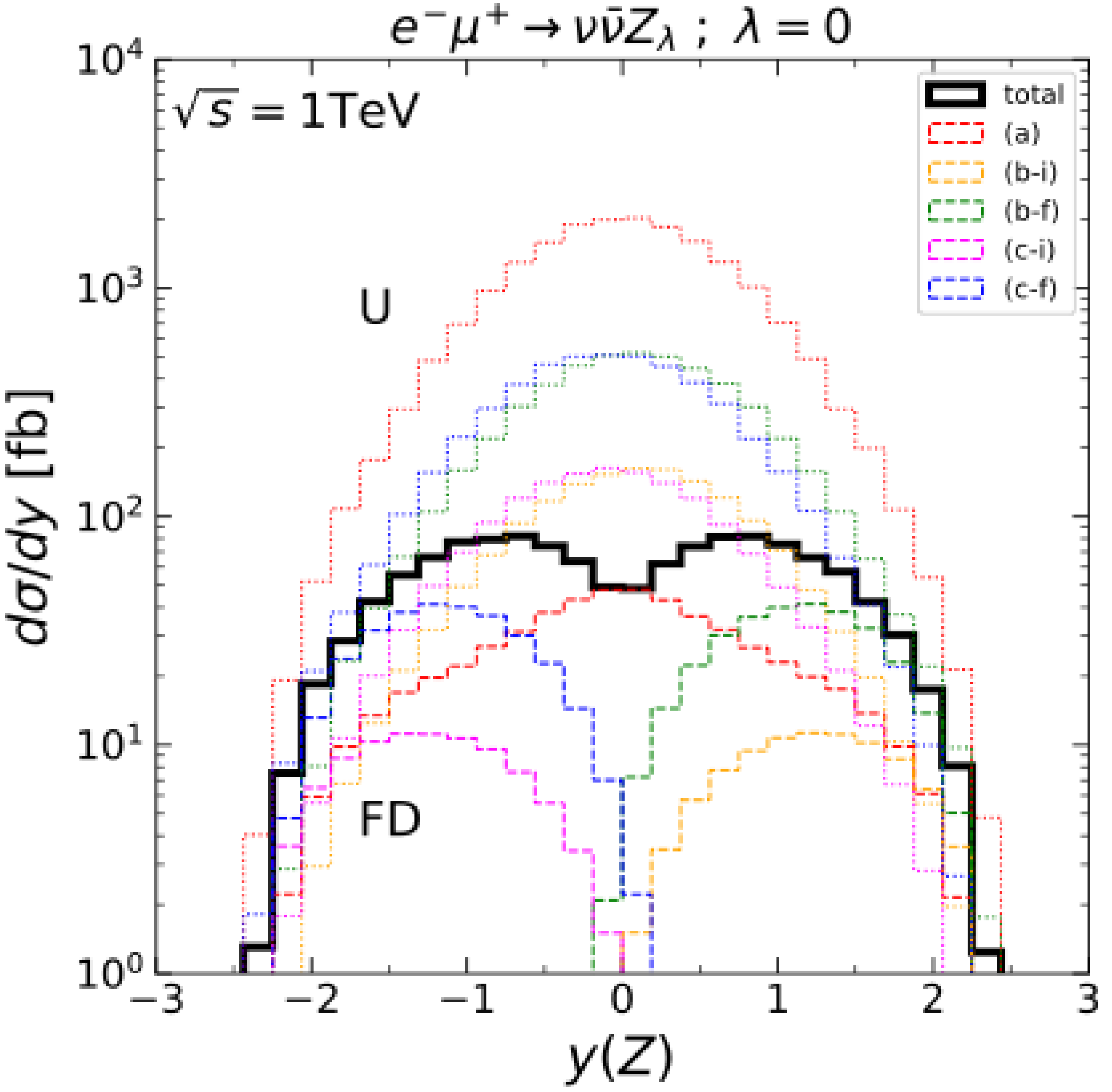
The cross section is highly dependent on the relative sign of the couplings.

This process is sensitive to new physics contributions involving W, Z and Higgs.

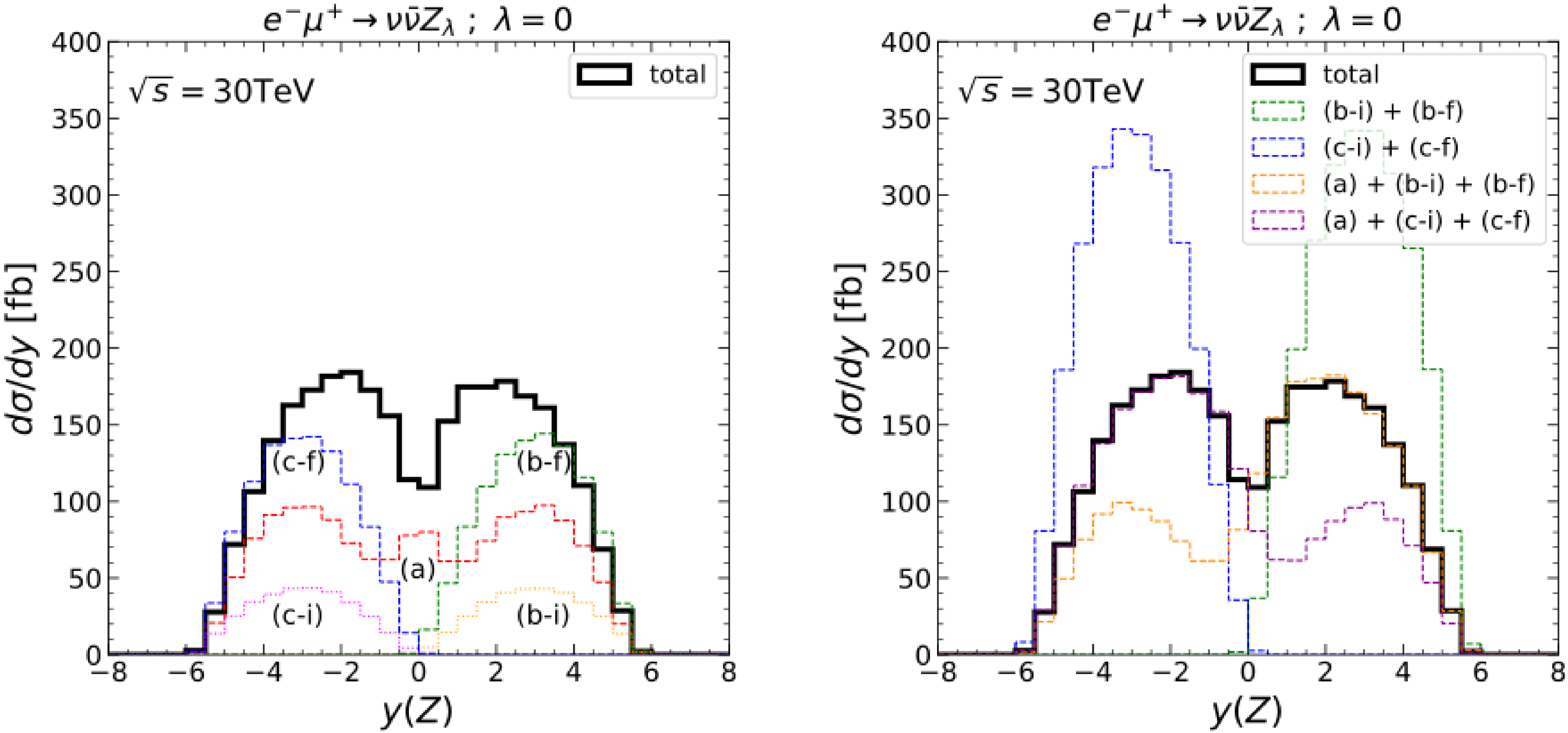
$$e^{-}\mu^{+} \rightarrow \nu_e \bar{\nu}_\mu Z$$



$e^- \mu^+ \to \nu\bar{\nu} Z_\lambda$; $\lambda = 0$
total
(a)
(b-i) or (c-i)
(b-f) or (c-f)
U
FD
σ [fb]
$\sqrt{s}$ [TeV]



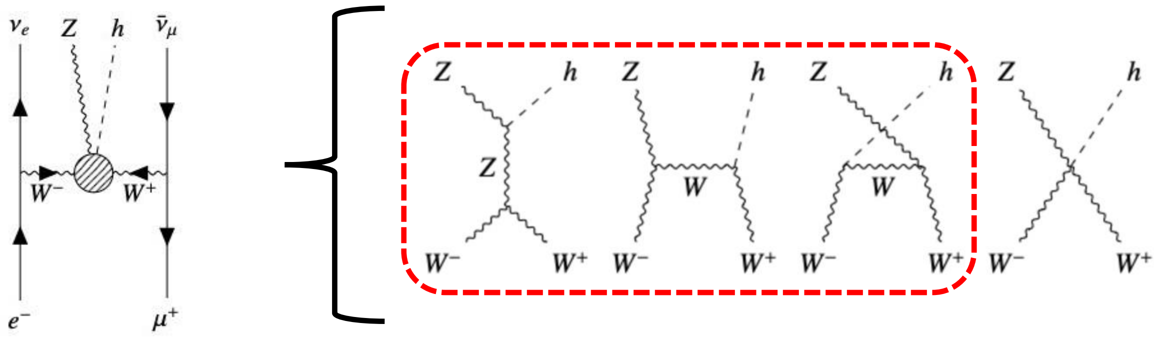
$e^- \mu^+ \to \nu\bar{\nu} Z_\lambda$; $\lambda = 0$
$\sqrt{s} = 1$TeV
total
(a)
(b-i)
(b-f)
(c-i)
(c-f)
U
FD
$d\sigma/dy$ [fb]
$y(Z)$



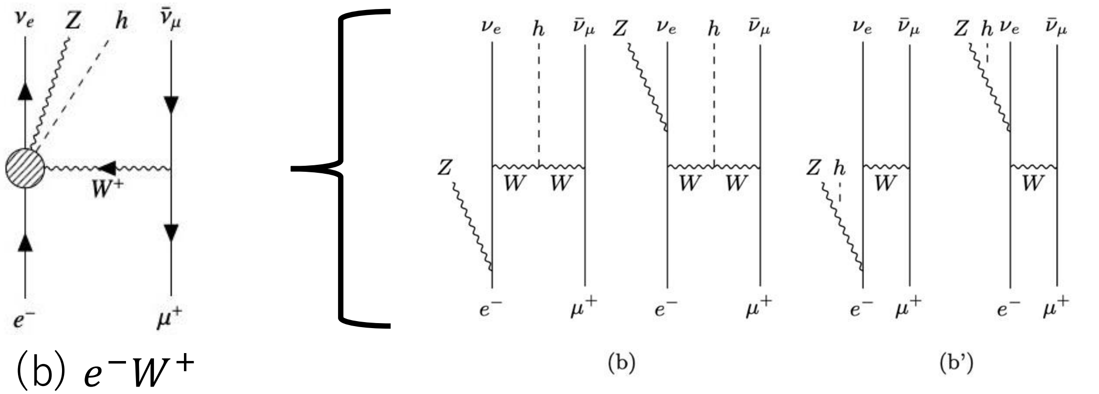
$e^-\mu^+ \to \nu\bar{\nu}Z_\lambda$; $\lambda = 0$
$\sqrt{s}$ = 30TeV
total
(c-f)
(b-f)
(a)
(c-i)
(b-i)
$d\sigma/dy$ [fb]
$y(Z)$
$e^-\mu^+ \to \nu\bar{\nu}Z_\lambda$; $\lambda = 0$
$\sqrt{s}$ = 30TeV
total
(b-i) + (b-f)
(c-i) + (c-f)
(a) + (b-i) + (b-f)
(a) + (c-i) + (c-f)
$d\sigma/dy$ [fb]
$y(Z)$

Show the contribution of all groups

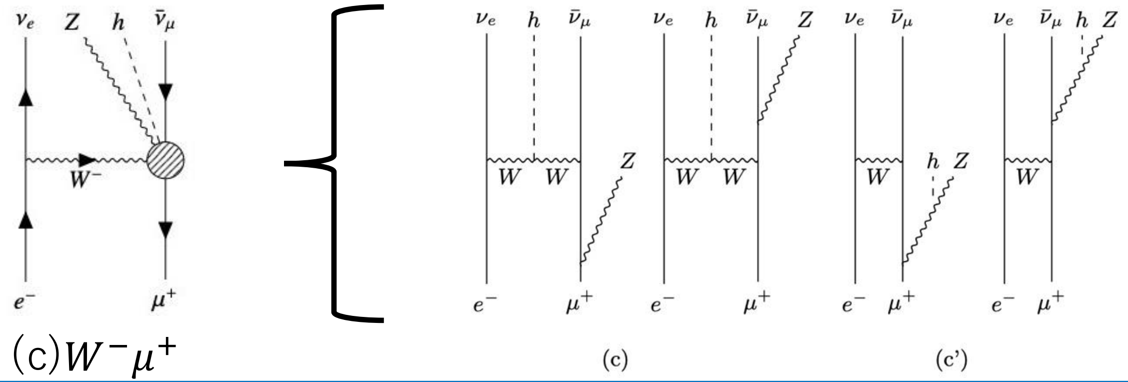
3. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$



(a) VBS



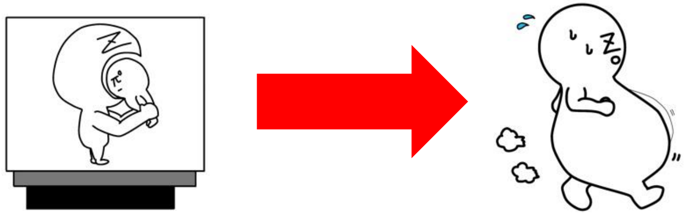
(b) e^-W^+



(c)$W^-\mu^+$

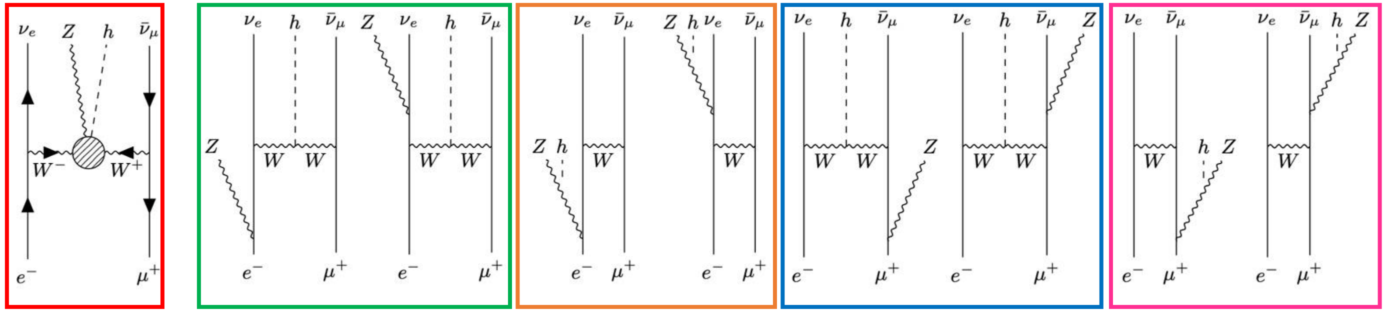
In the Feynman-diagram (FD) gauge, 4-point interactions such as $\boldsymbol{W^-W^+ \to Zh}$, since the Nambu–Goldstone bosons are explicitly treated.

In the unitary (U) gauge, they are absorbed into the gauge bosons.



3. $e^-\mu^+ \to \nu_e \bar{\nu}_\mu Zh$

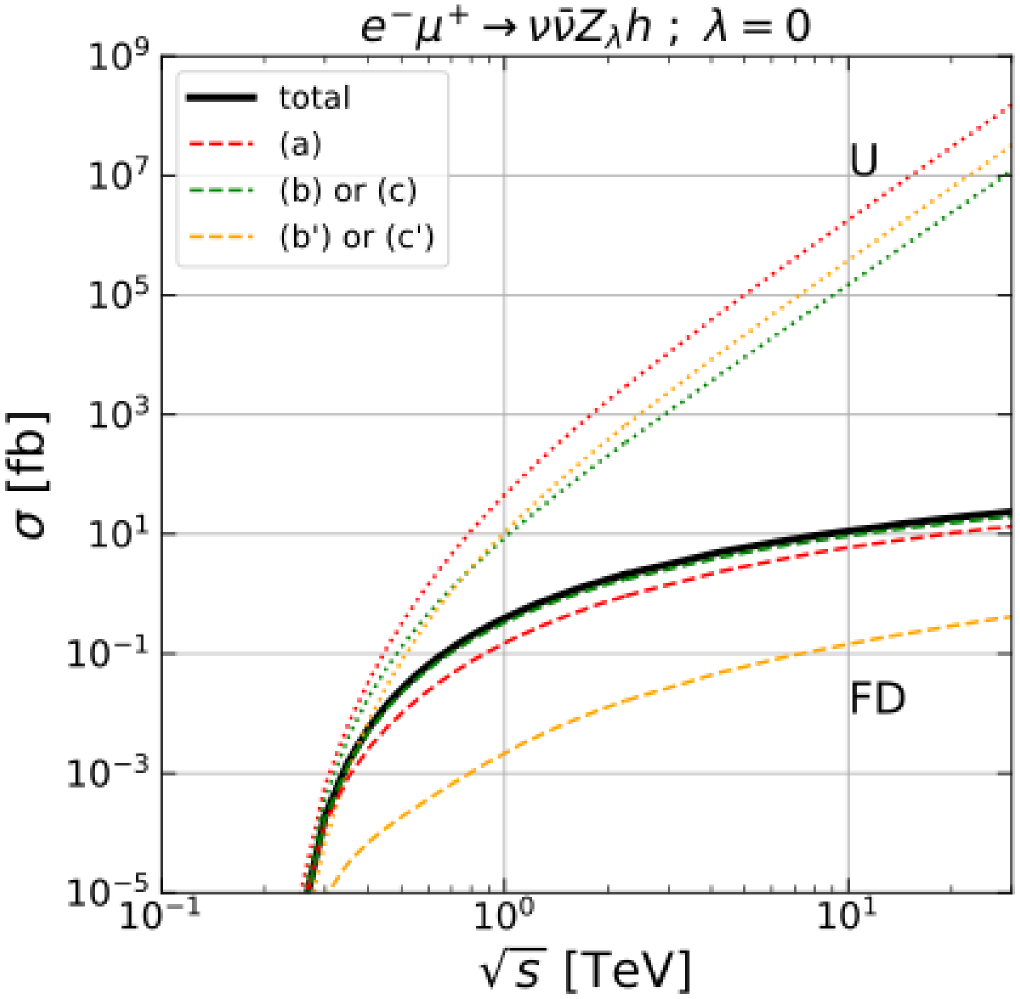
- **The solid black line** is the physical observable.
  total$\propto |(a) + (b) + (b') + (c) + (c')|^2$



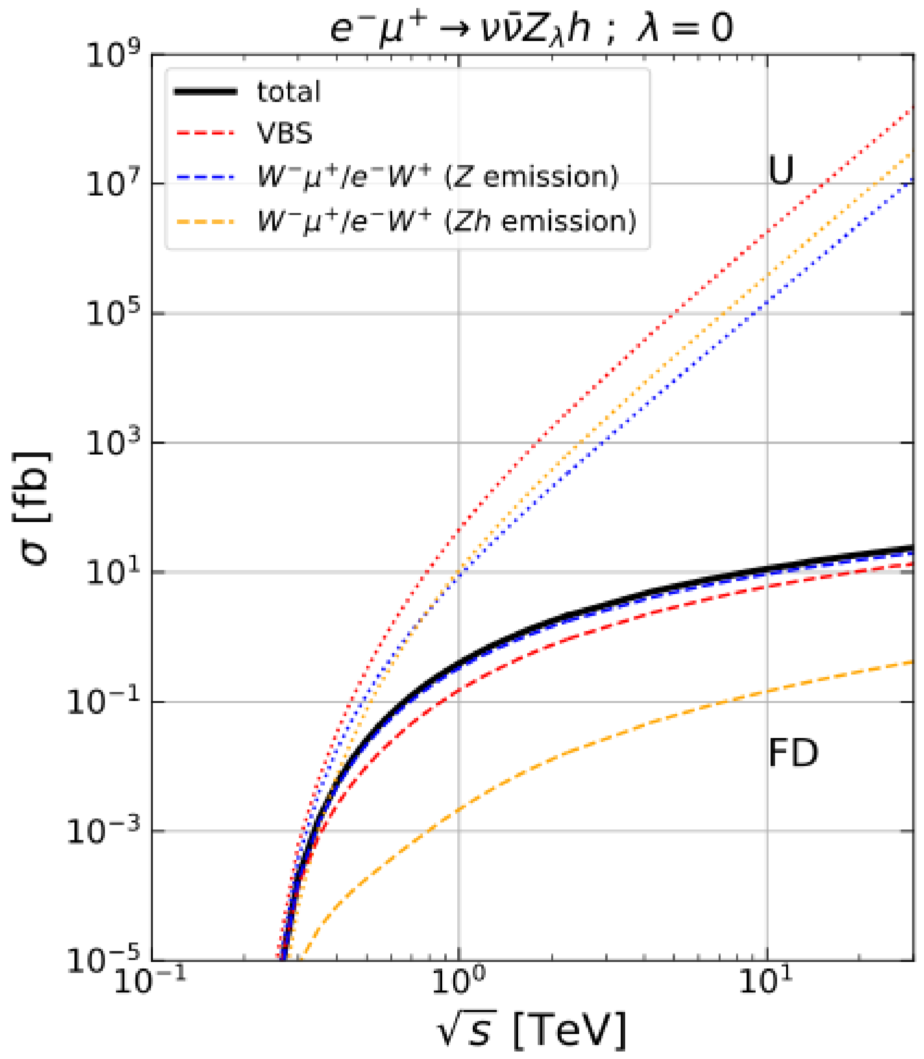
- U gauge ⋯⋯⋯ , FD gauge – – – –
  unphysical quantities.

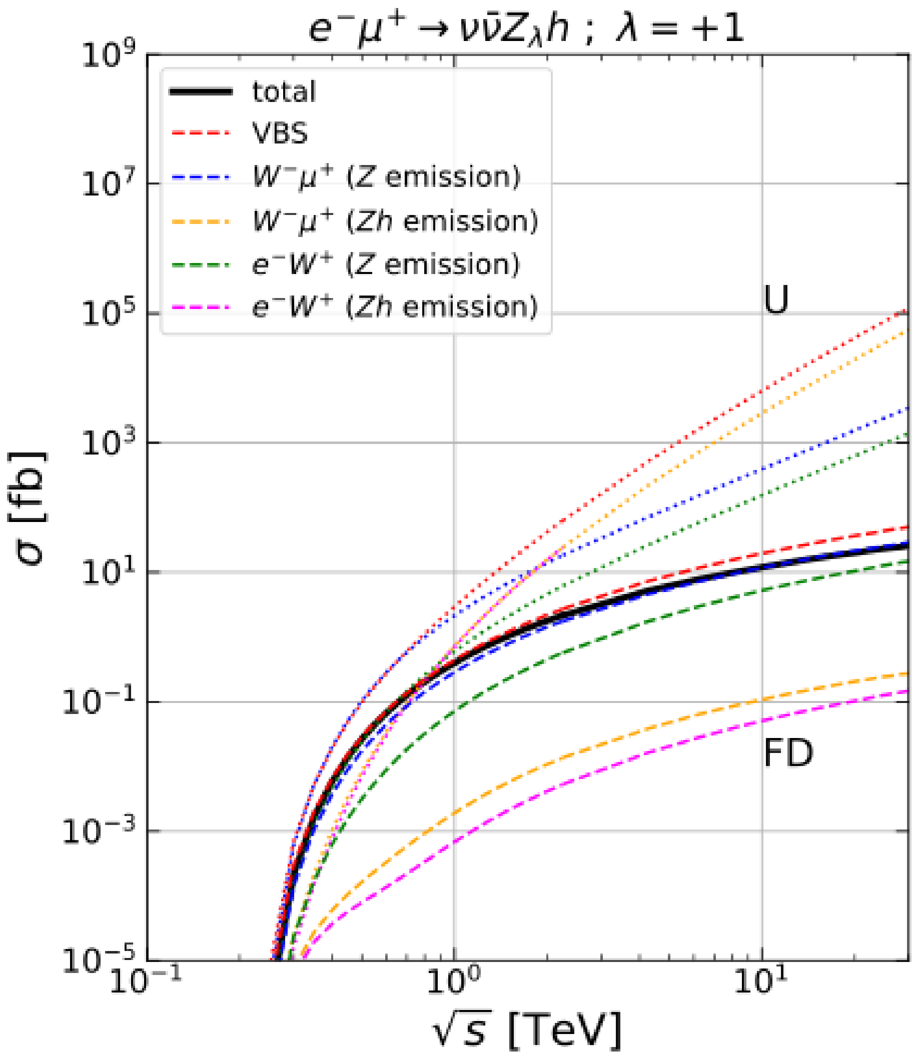
U gauge:
- Each category of the amplitudes has energy growth.
- Large cancellation among ((a),(b), (b'),(c), (c')) at high energies.

FD gauge :
- No such energy growth of each category at all.
- the suppression of Zh emission is clearly observed.



Total cross section





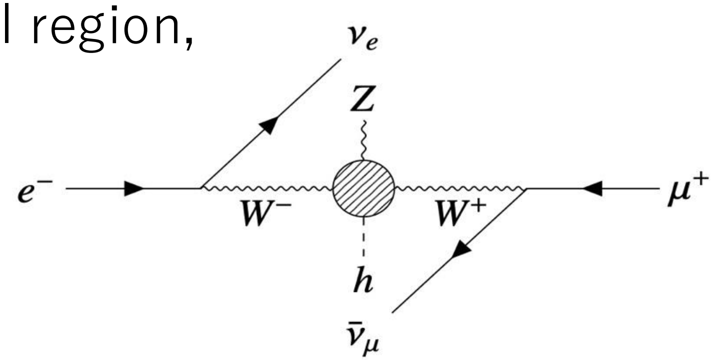
3. $e^-\mu^+ \to \nu_e \bar{\nu}_\mu Zh$

U gauge:

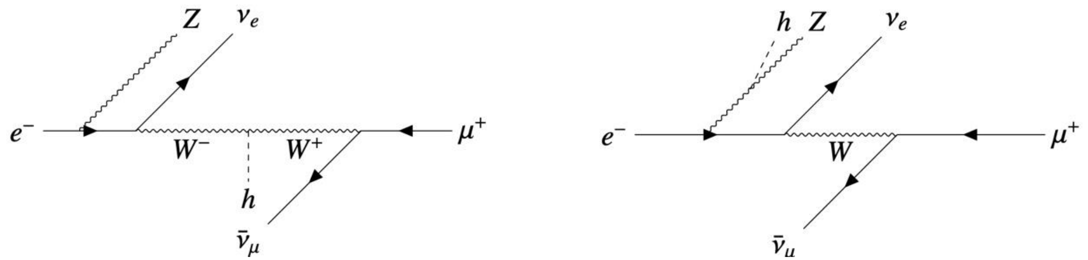
- No clue for the physical distribution from each contribution.

FD gauge :

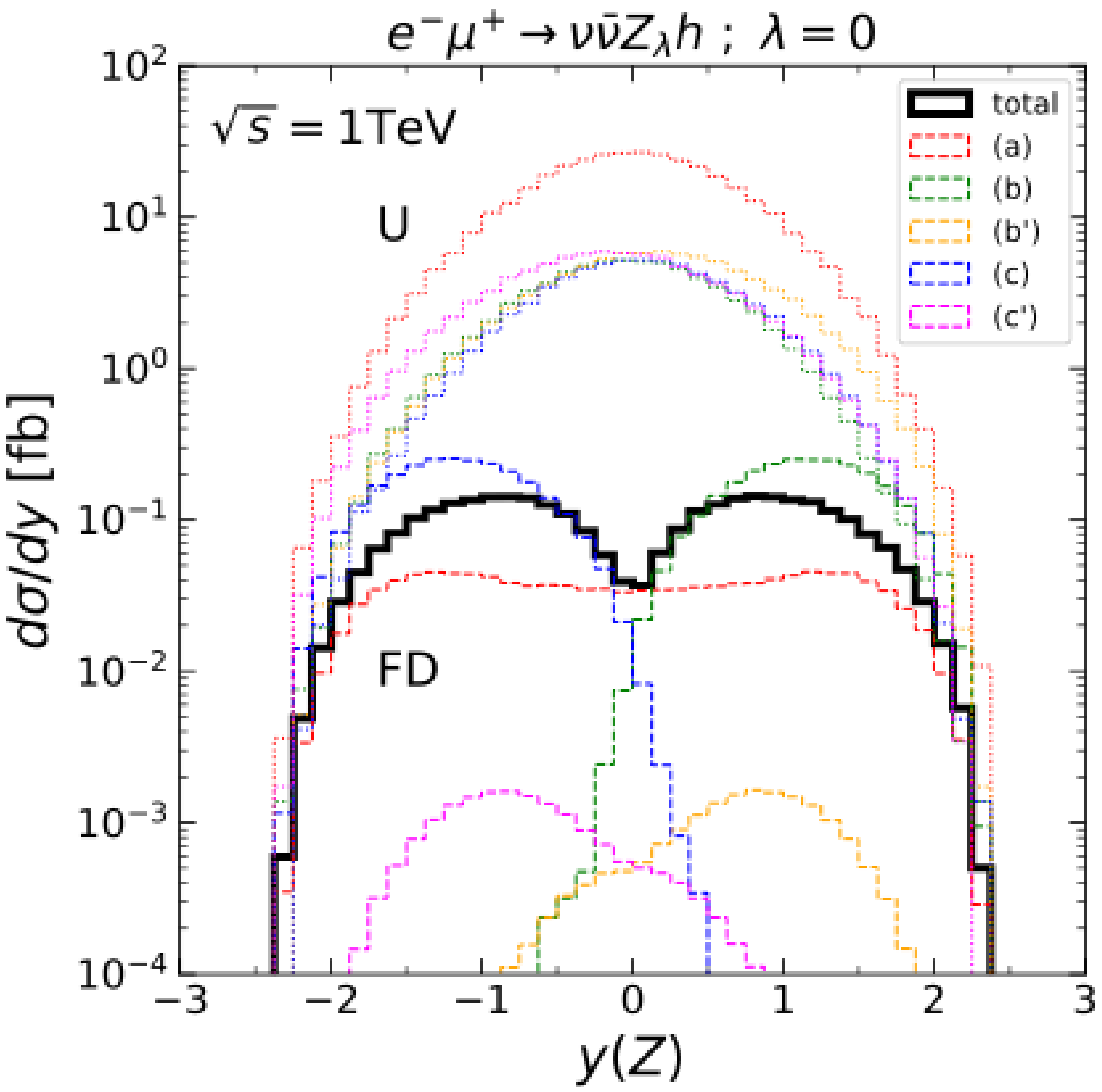
- (a) : Central region,



(b) : Forward scattering,

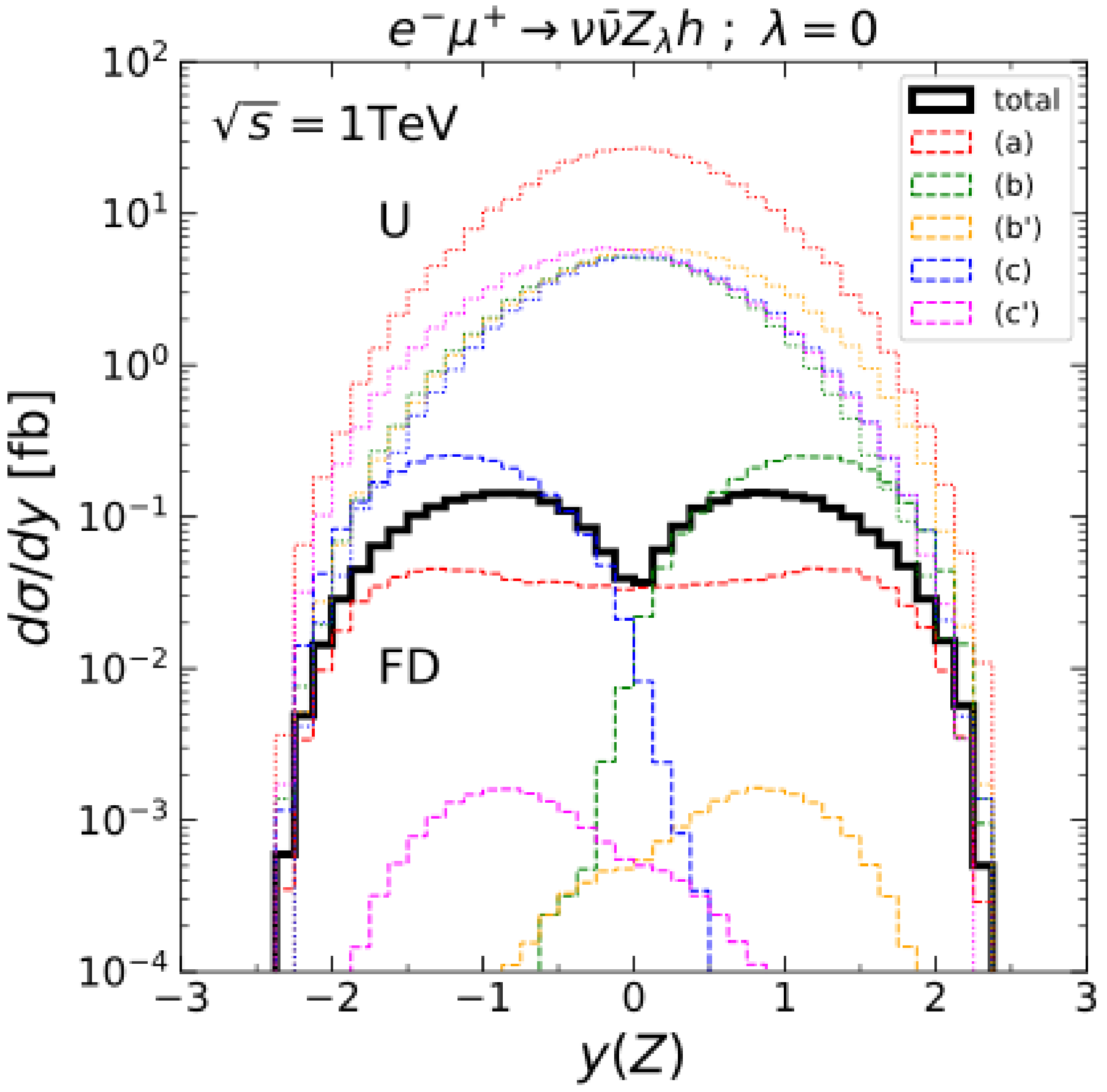


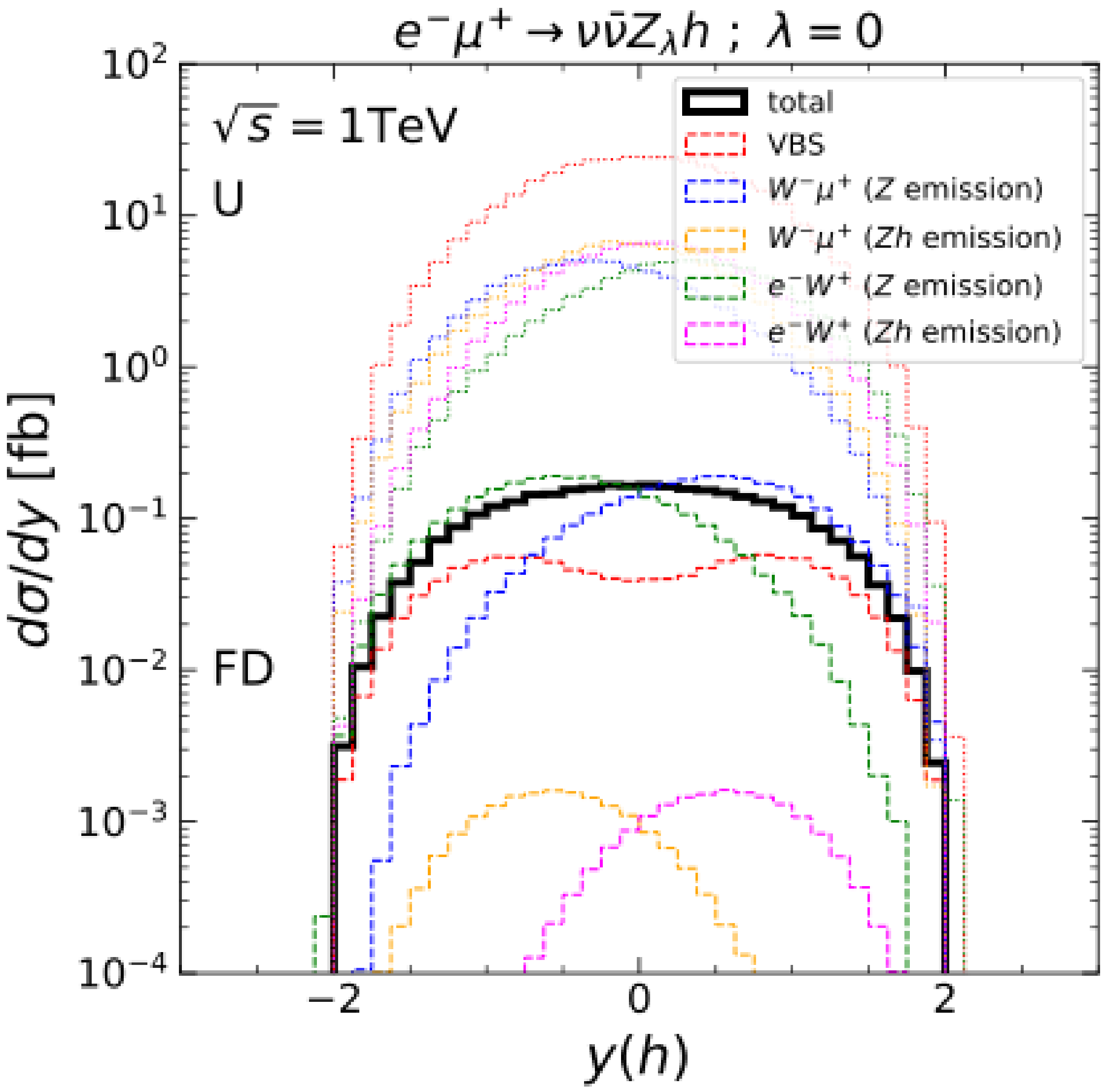
(c) : Backward scattering.



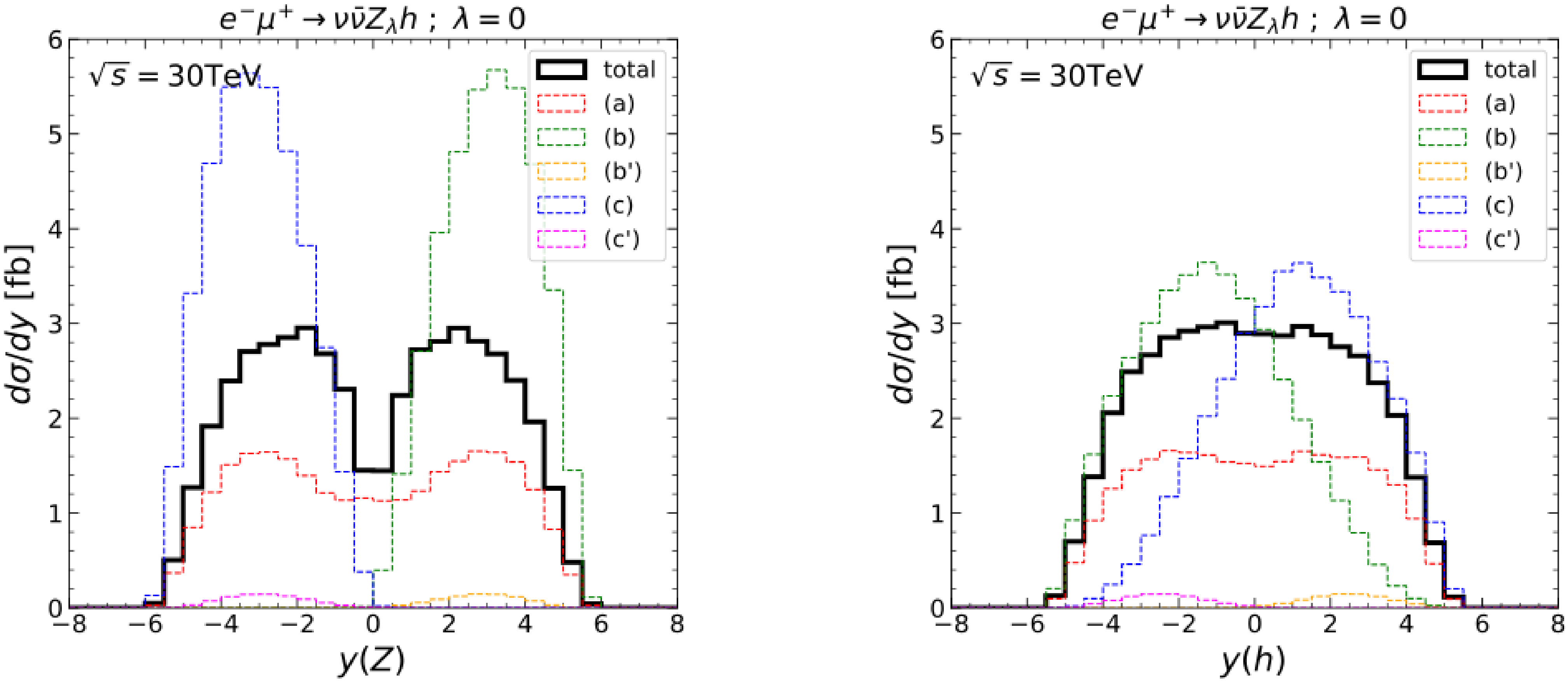
In the FD gauge,
The behavior of each group shows good agreement with our expectation.

3. $e^-\mu^+ \to \nu_e \bar{\nu}_\mu Z h$





3. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$

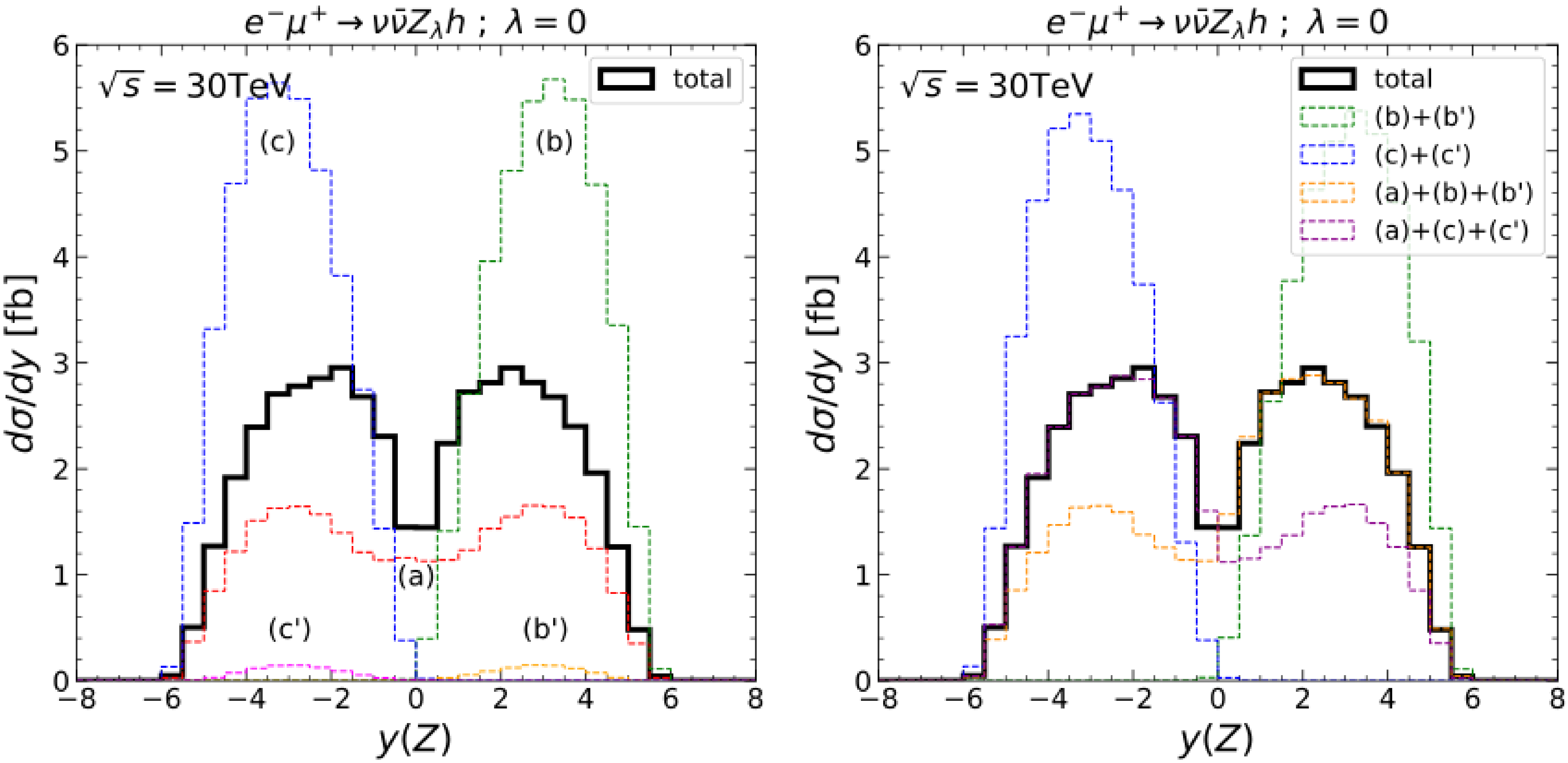


In $y(h)$,

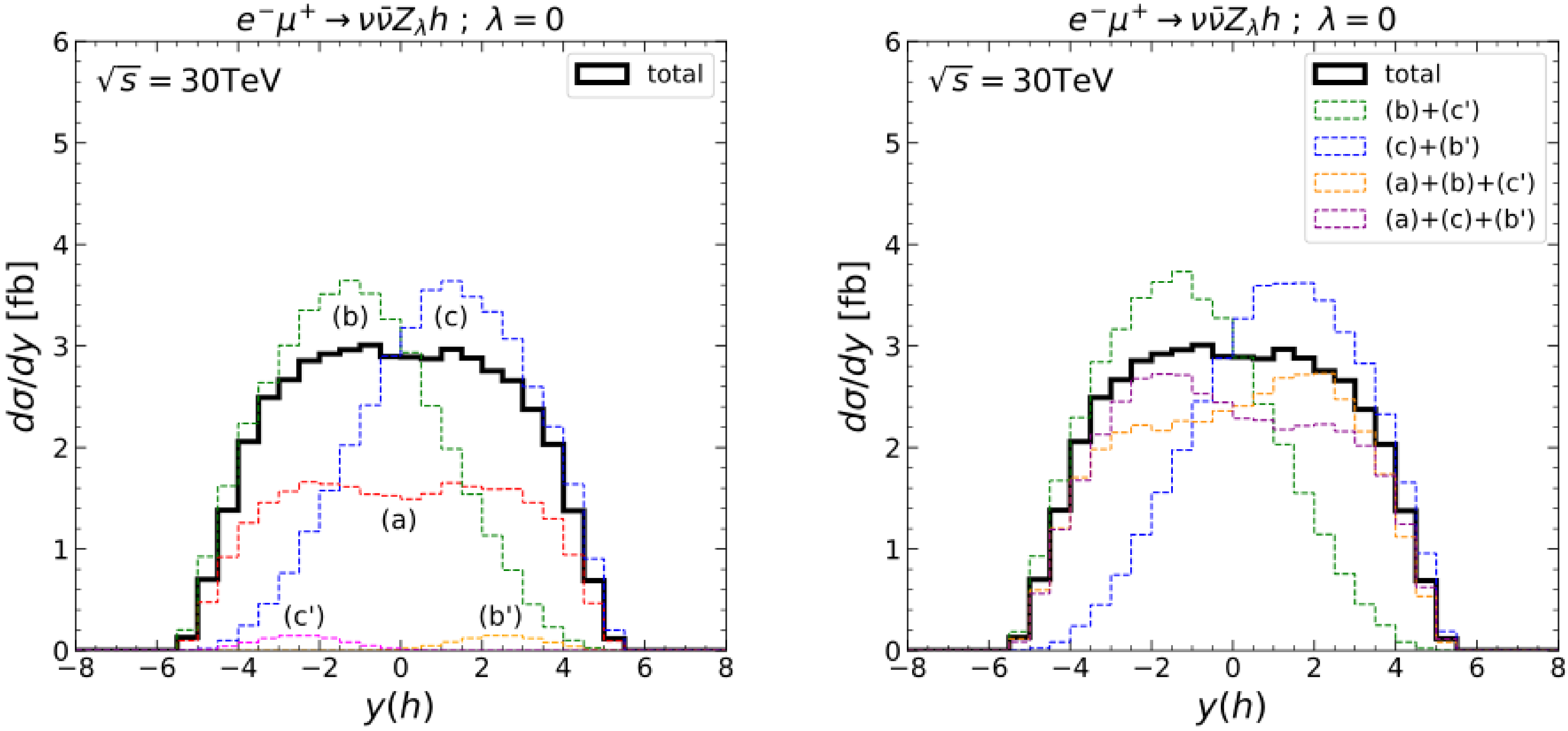
- The $y(h)$ distributed in central region.
  →It does not receive collinear enhancement and thus tends to be emitted more isotropically, the Higgs is often produced as part of WW fusion or Zh system.
- (b) and (c) are swapped.
  → The boost effect.

Interferences

3. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$

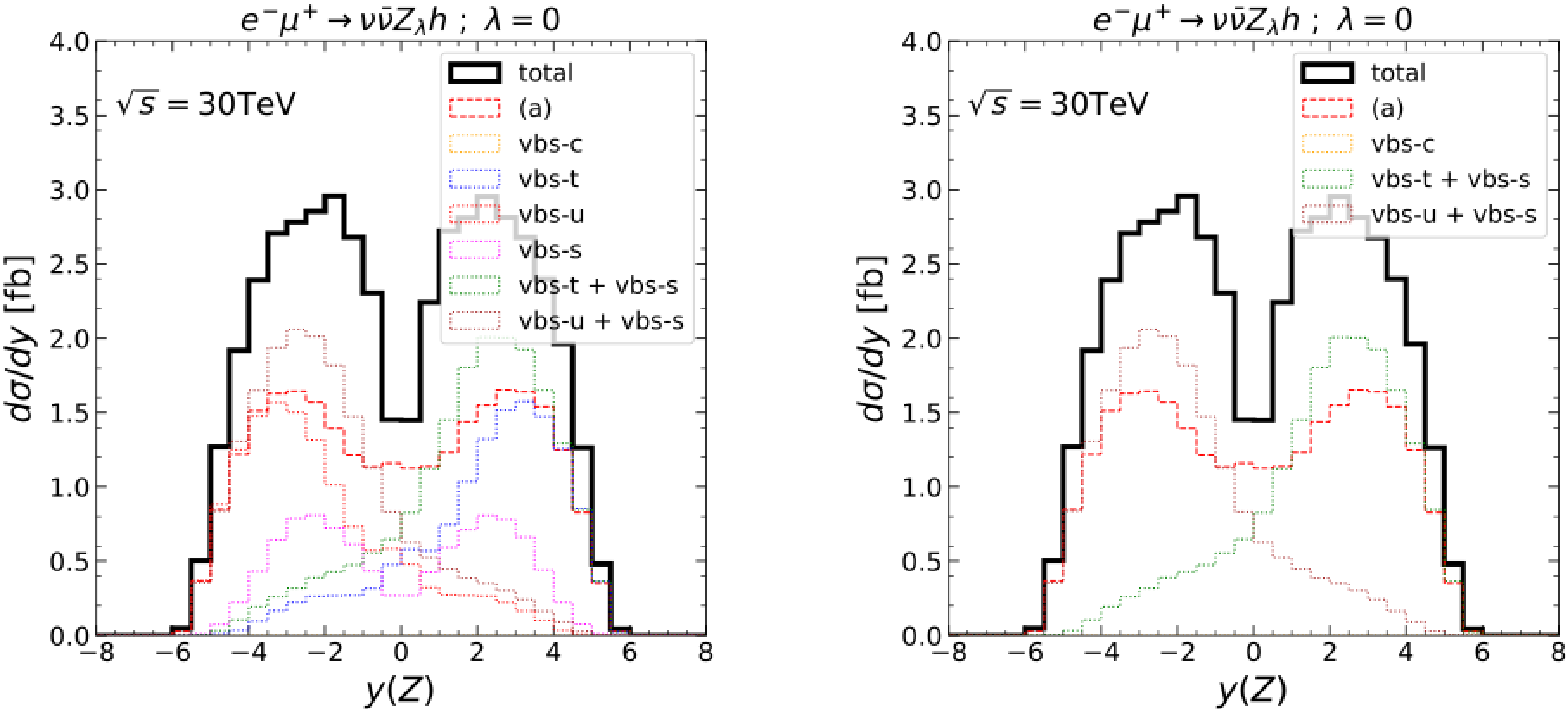


3. $e^-\mu^+ \to \nu_e \bar{\nu}_\mu Z h$

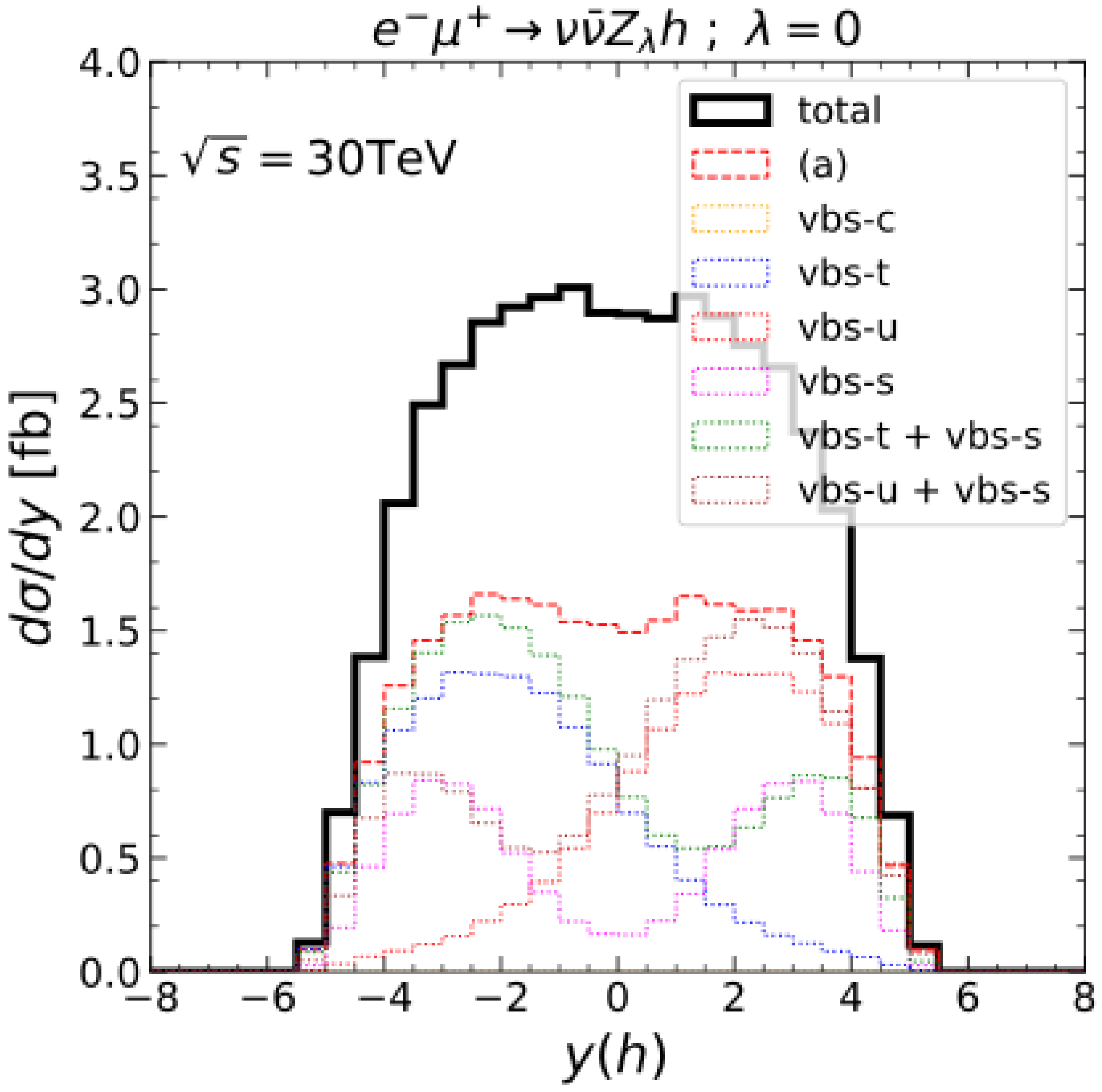


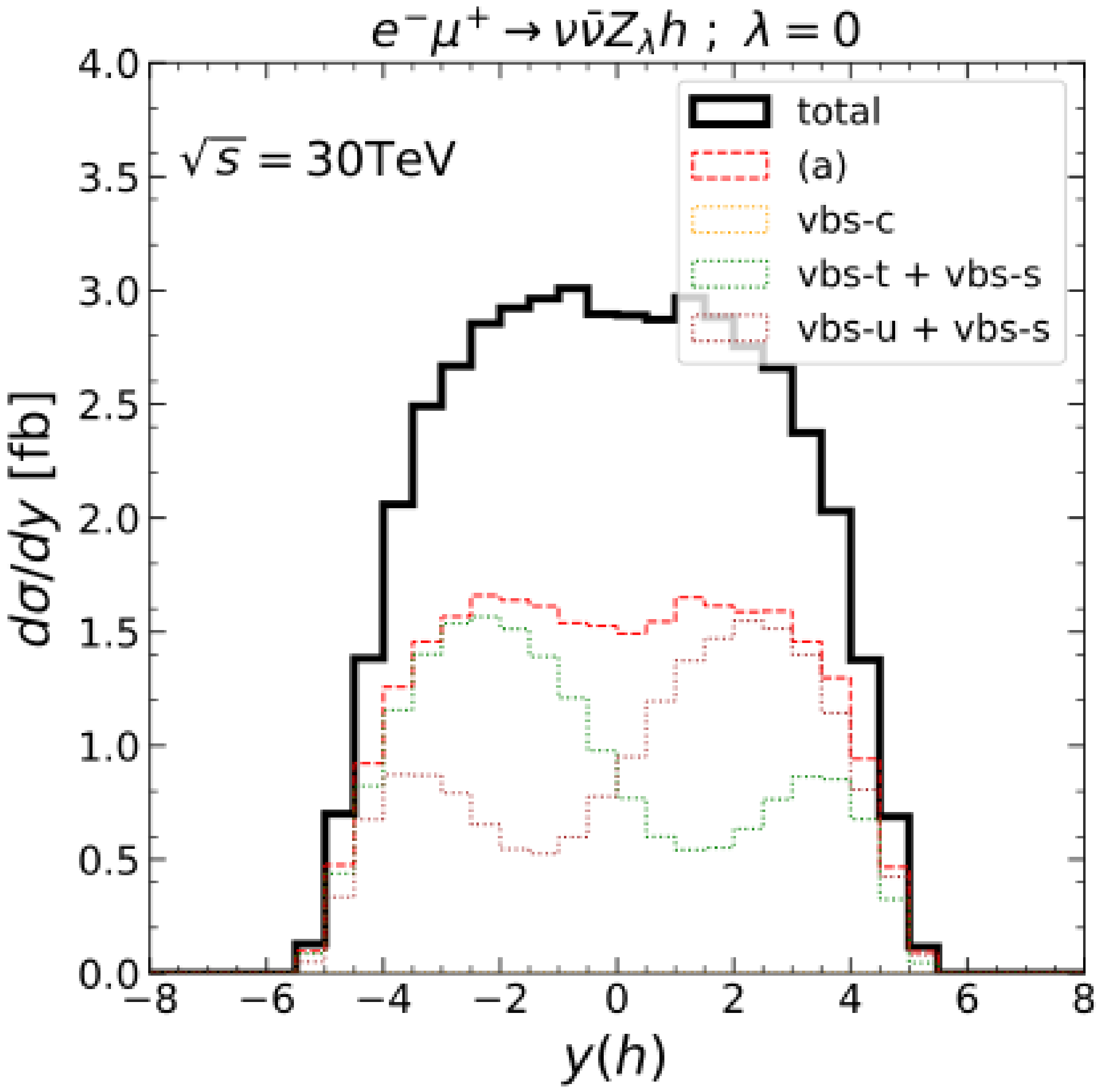
VBS

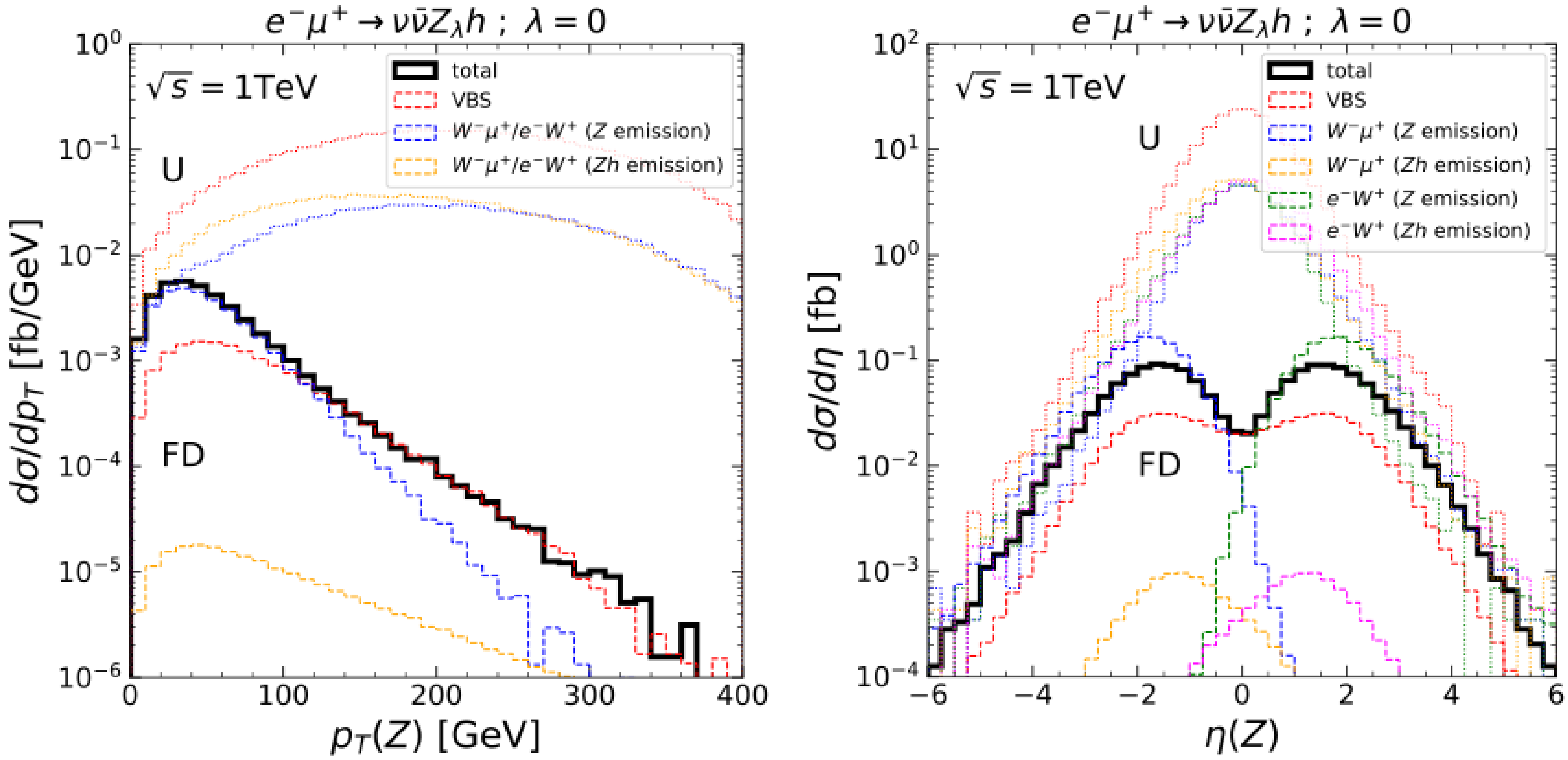
3. $e^- \mu^+ \to \nu_e \bar{\nu}_\mu Z h$



3. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$





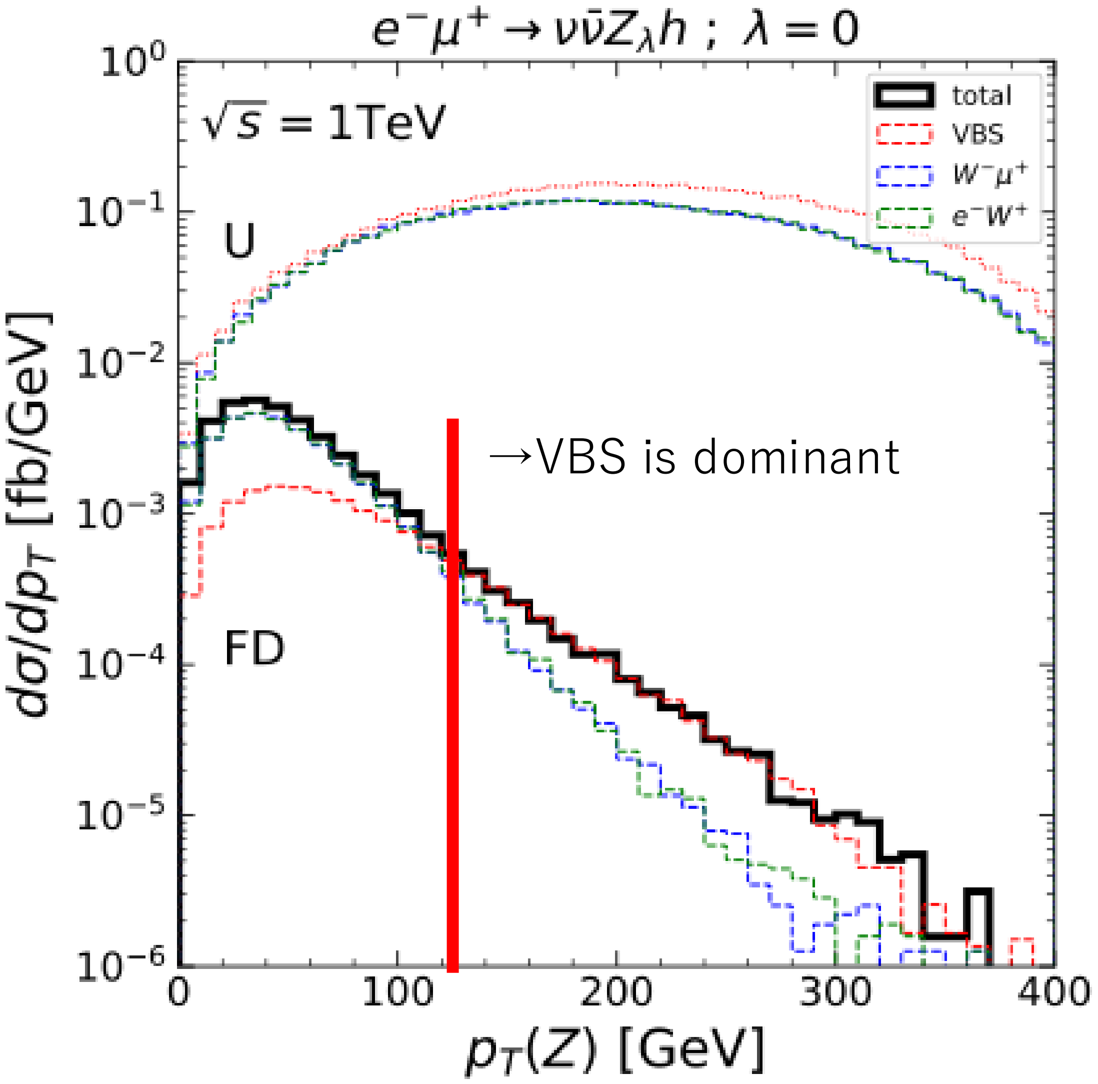


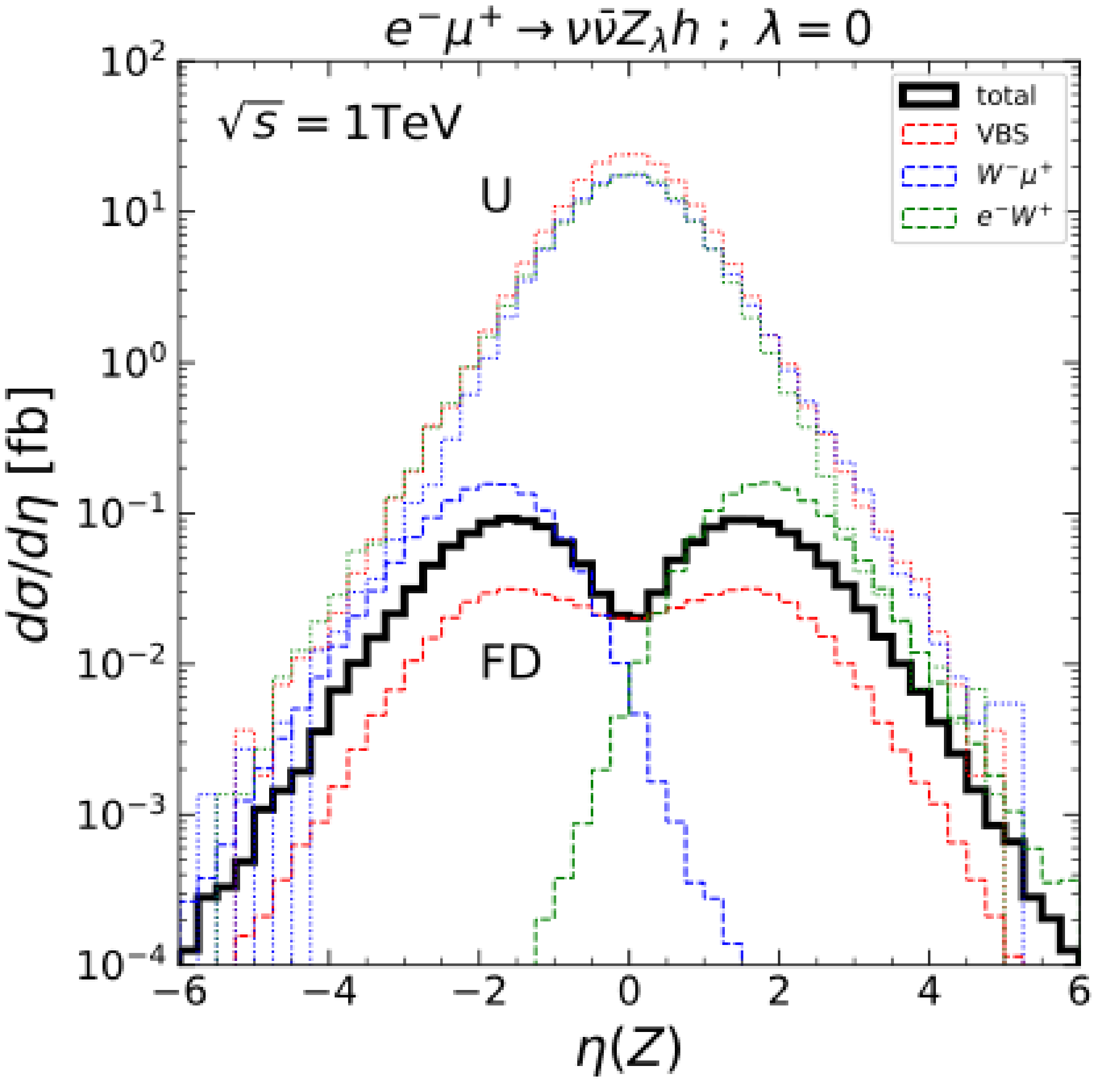
$e^-\mu^+ \to \nu\bar{\nu}Z_\lambda h\ ;\ \lambda = 0$
$\sqrt{s} = 1\text{TeV}$
total
VBS
$W^-\mu^+/e^-W^+$ (Z emission)
$W^-\mu^+/e^-W^+$ (Zh emission)
U
FD
$d\sigma/dp_T$ [fb/GeV]
$p_T(Z)$ [GeV]
$e^-\mu^+ \to \nu\bar{\nu}Z_\lambda h\ ;\ \lambda = 0$
$\sqrt{s} = 1\text{TeV}$
total
VBS
$W^-\mu^+$ (Z emission)
$W^-\mu^+$ (Zh emission)
e^-W^+ (Z emission)
e^-W^+ (Zh emission)
U
FD
$d\sigma/d\eta$ [fb]
$\eta(Z)$

e- mu+ -> ve vm~ Z_{0} h

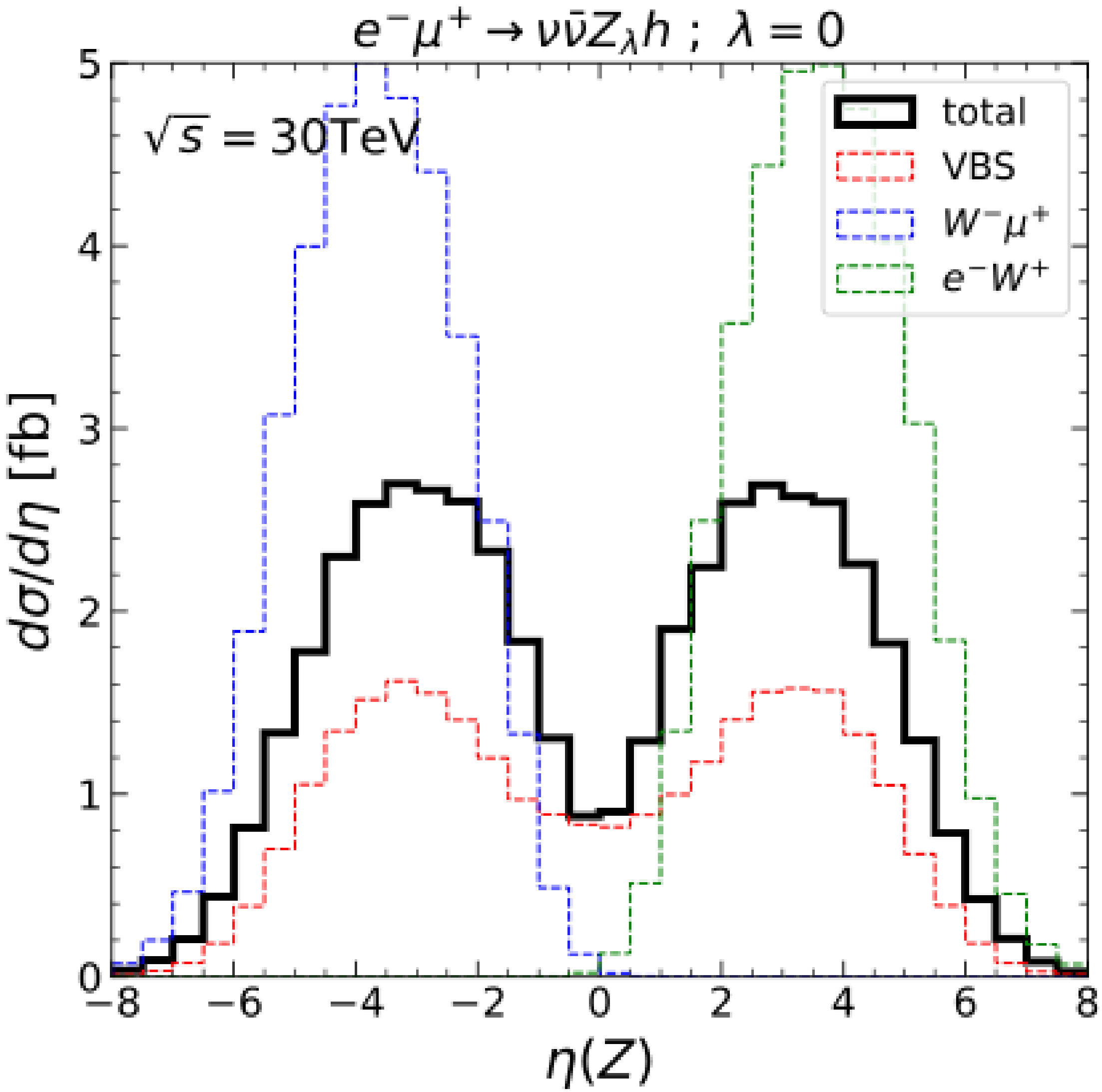
Kinematical cut pT(Z,h)>150GeV

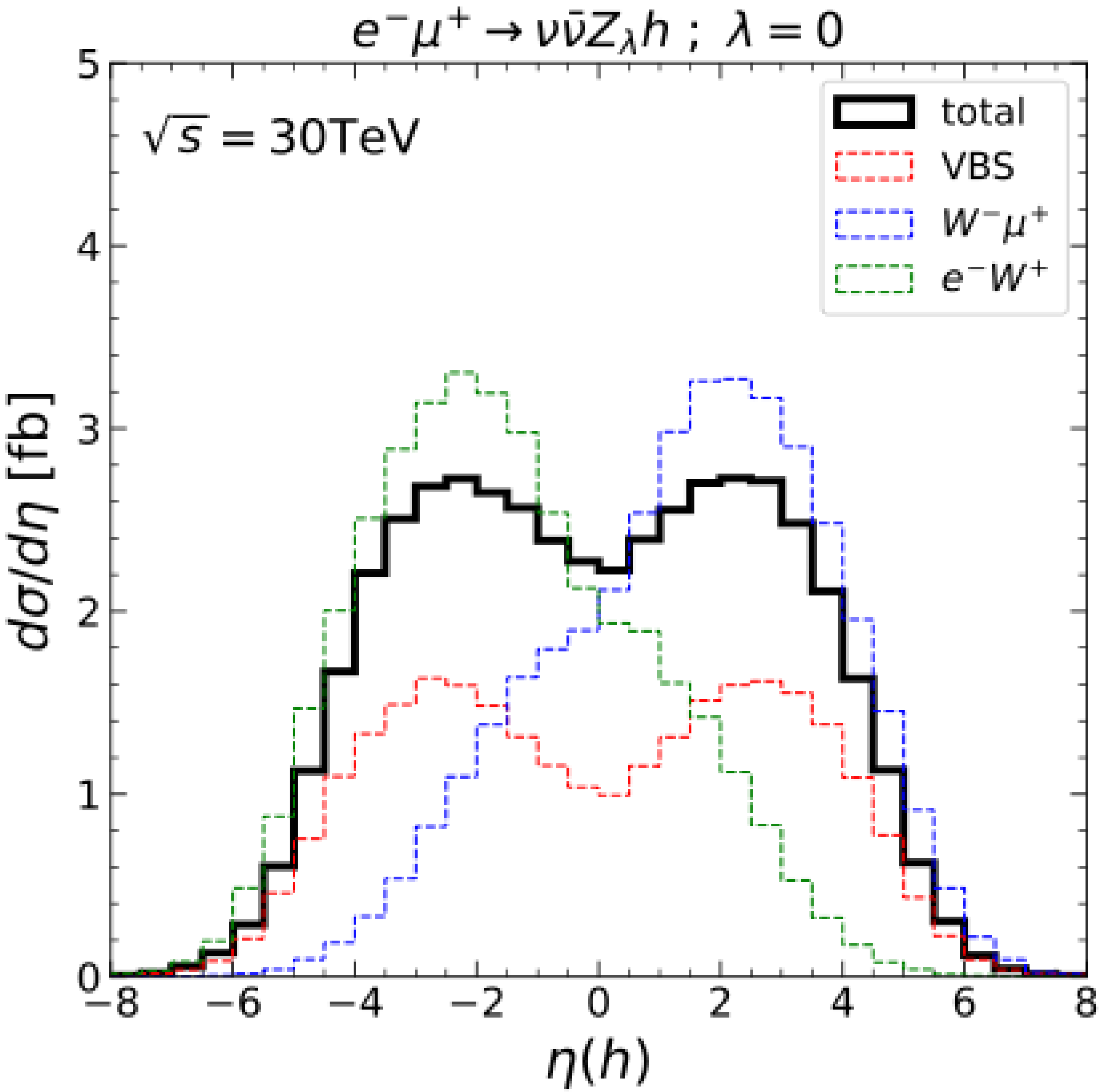
$p_T(Z)$ distribution



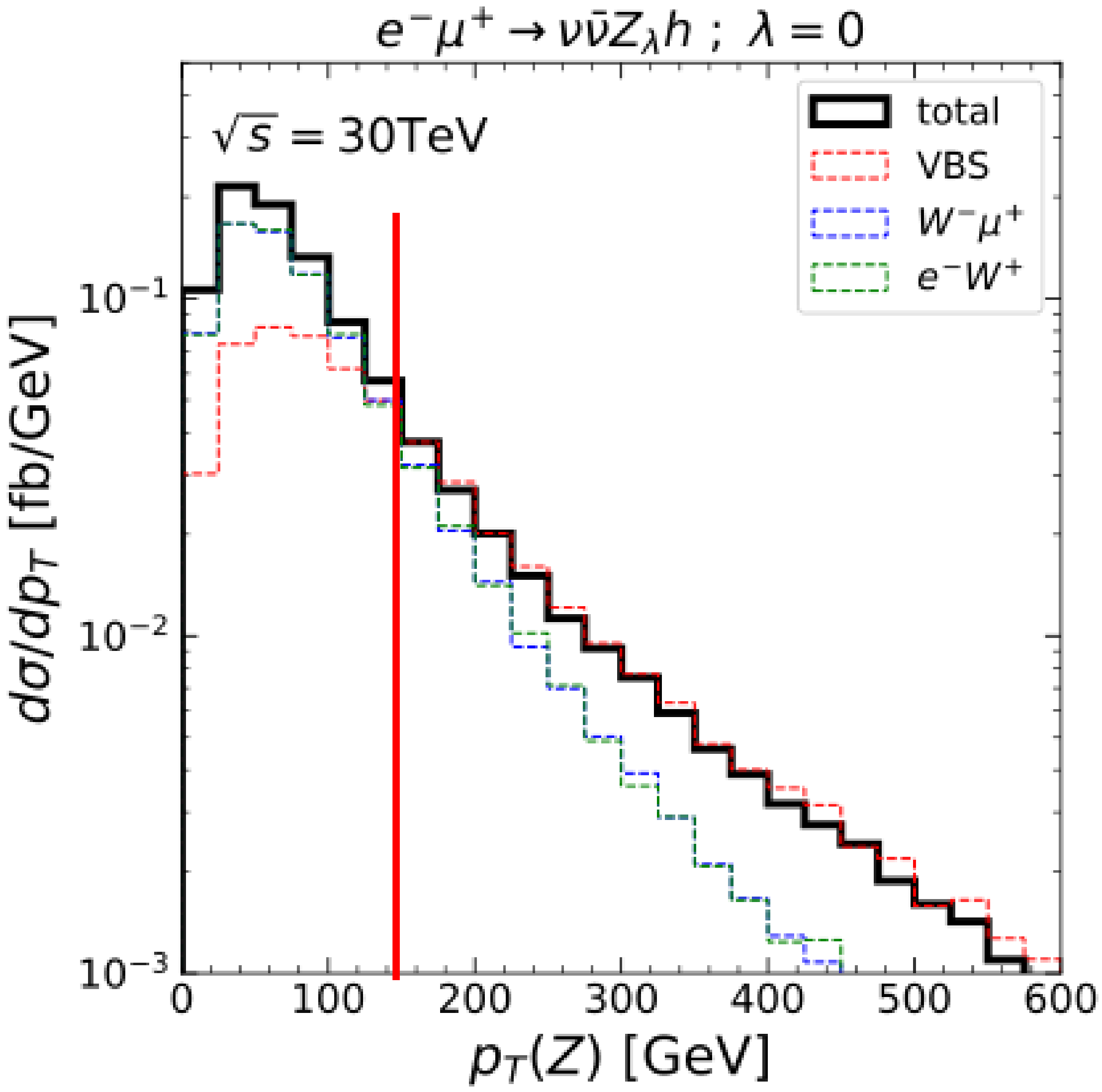


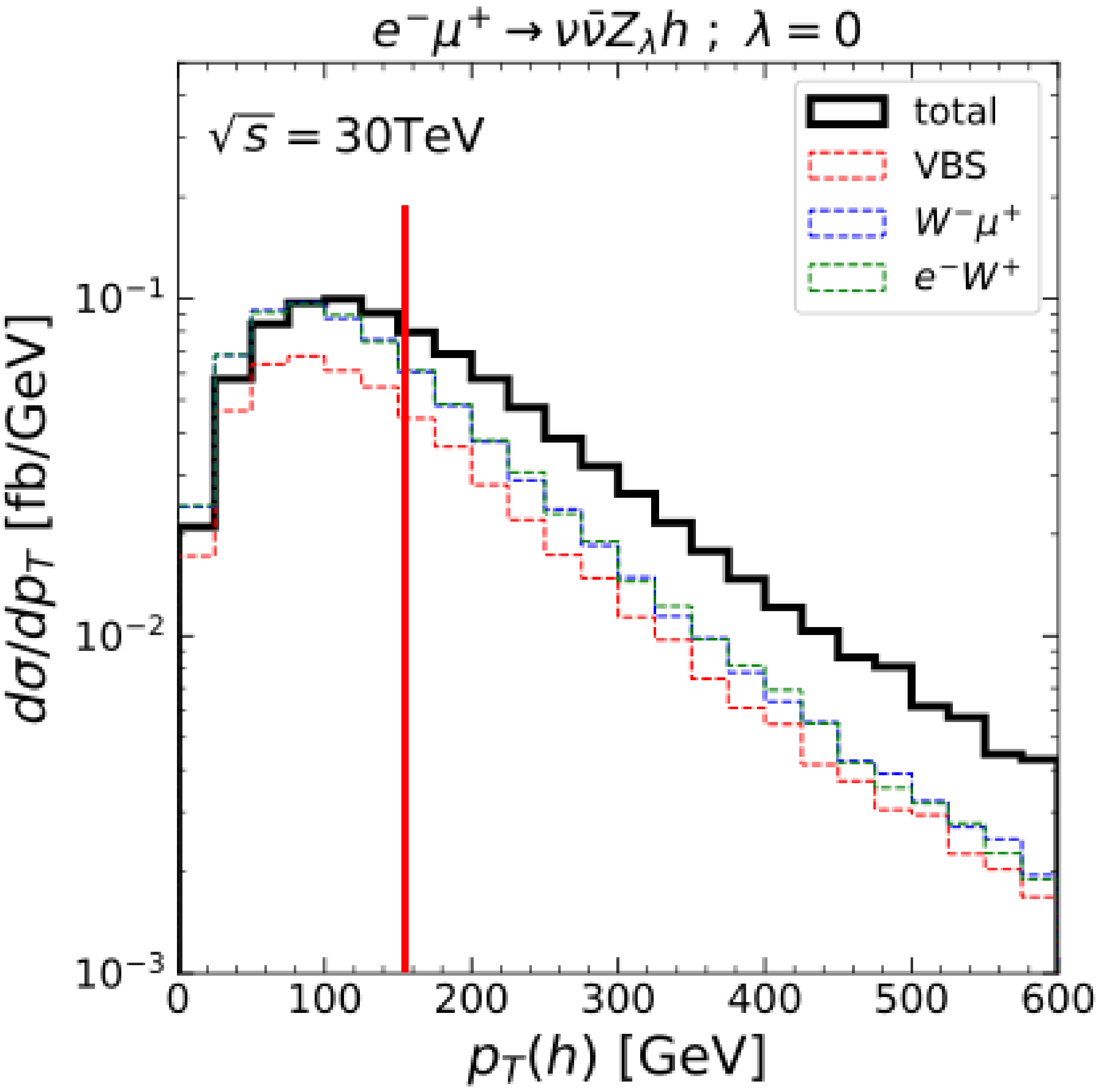
Enhancement of the VBS



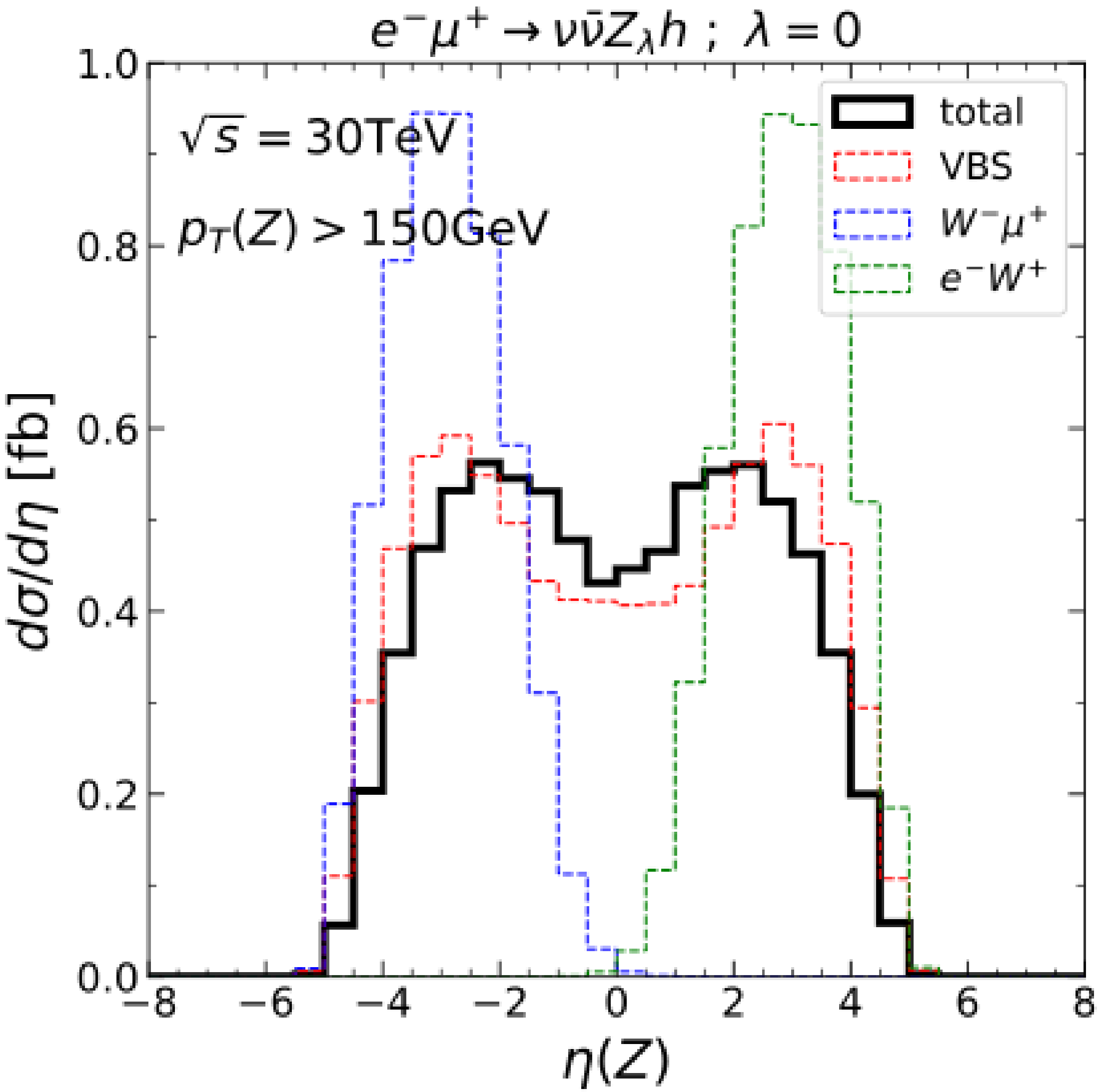


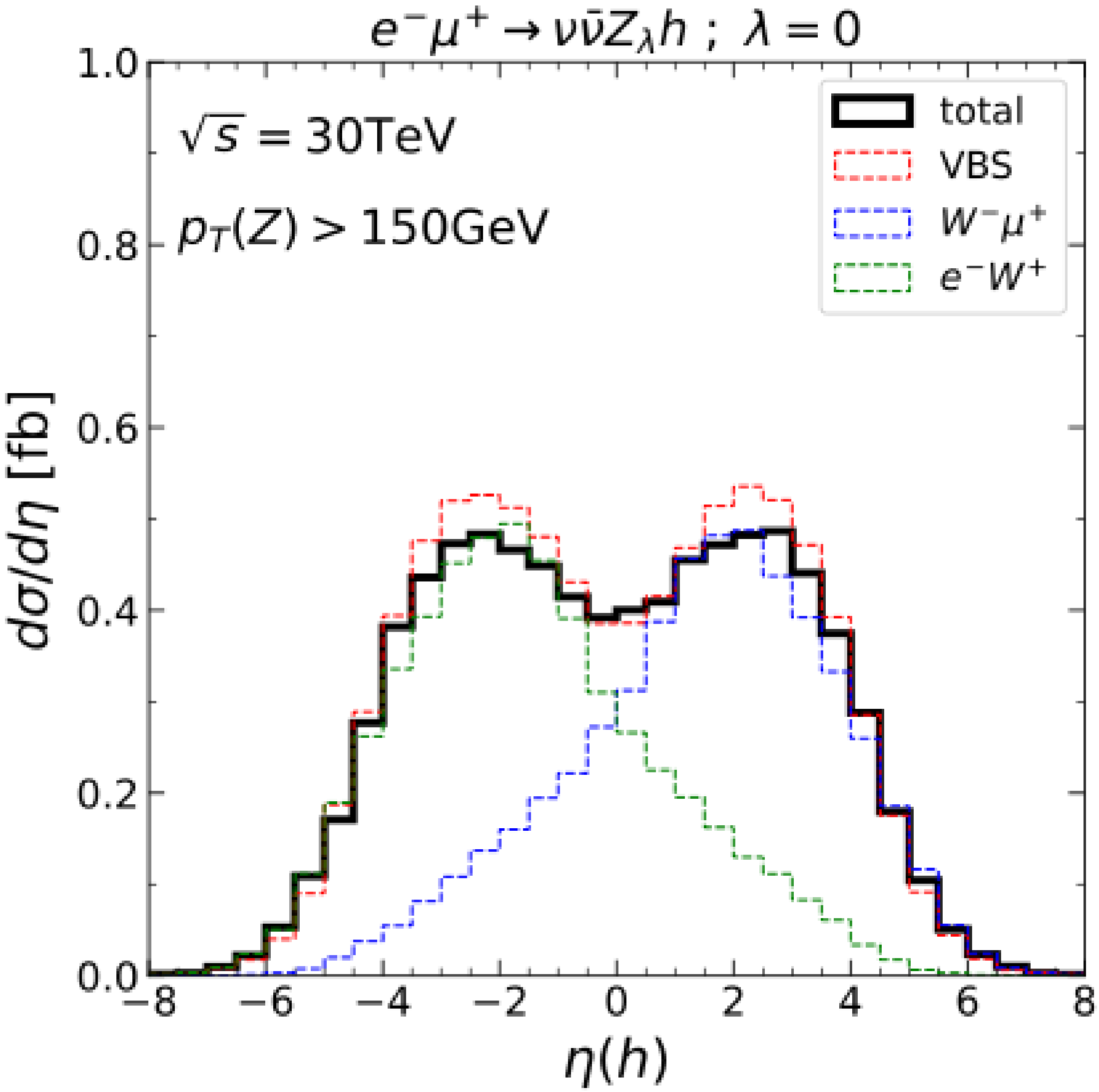
Enhancement of the VBS



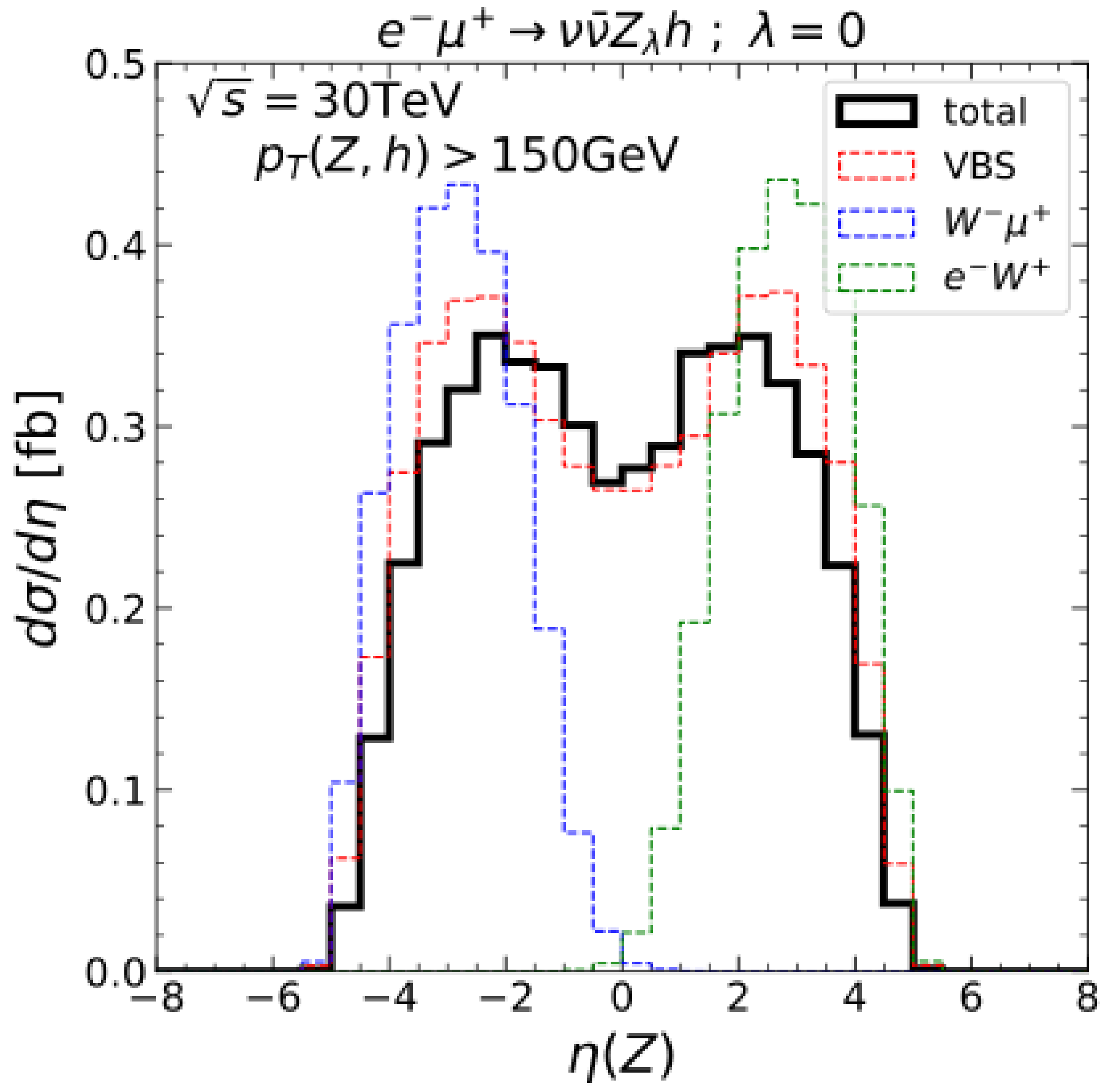


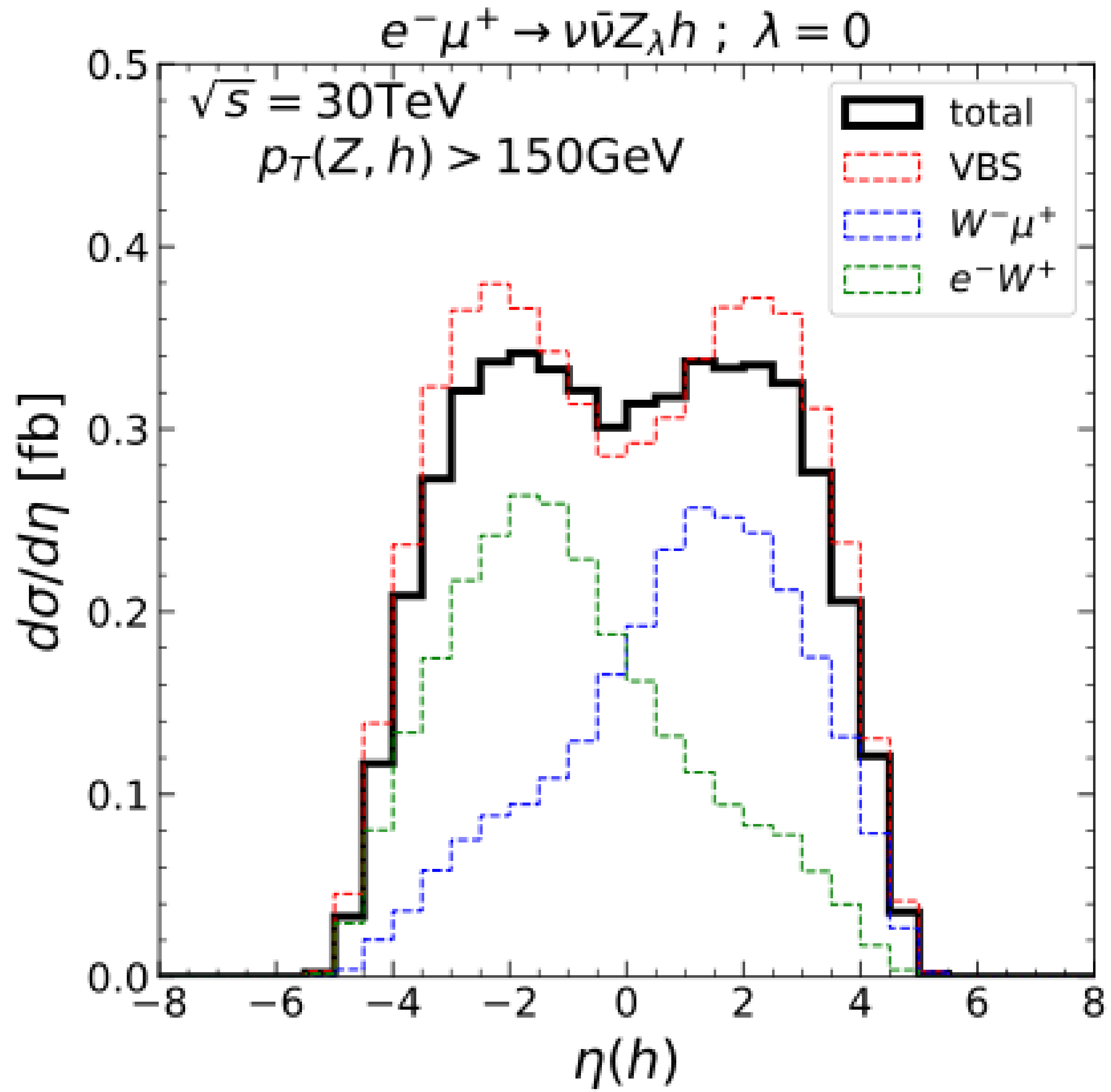
Enhancement of the VBS





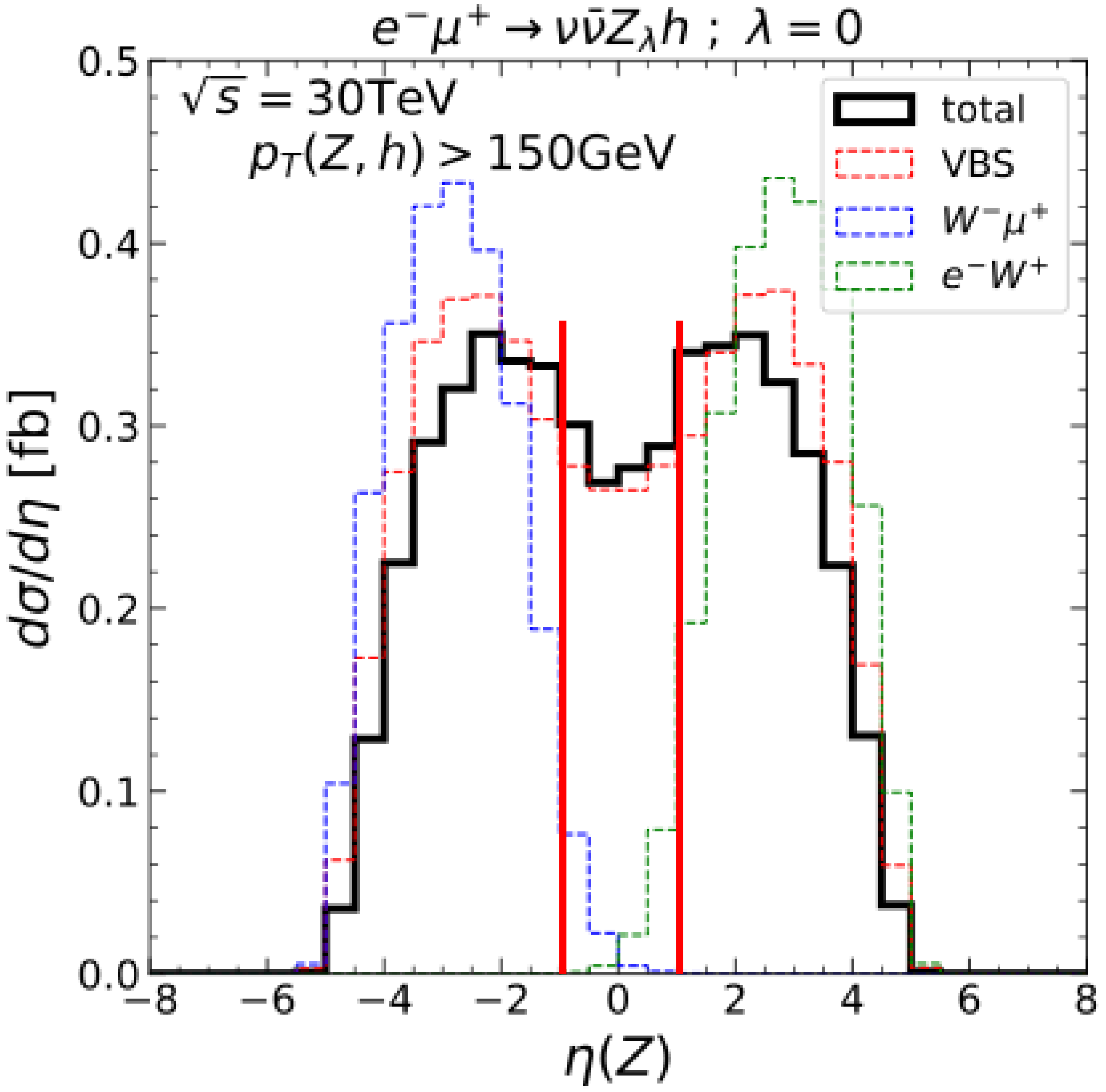
Enhancement of the VBS





Enhancement of the VBS

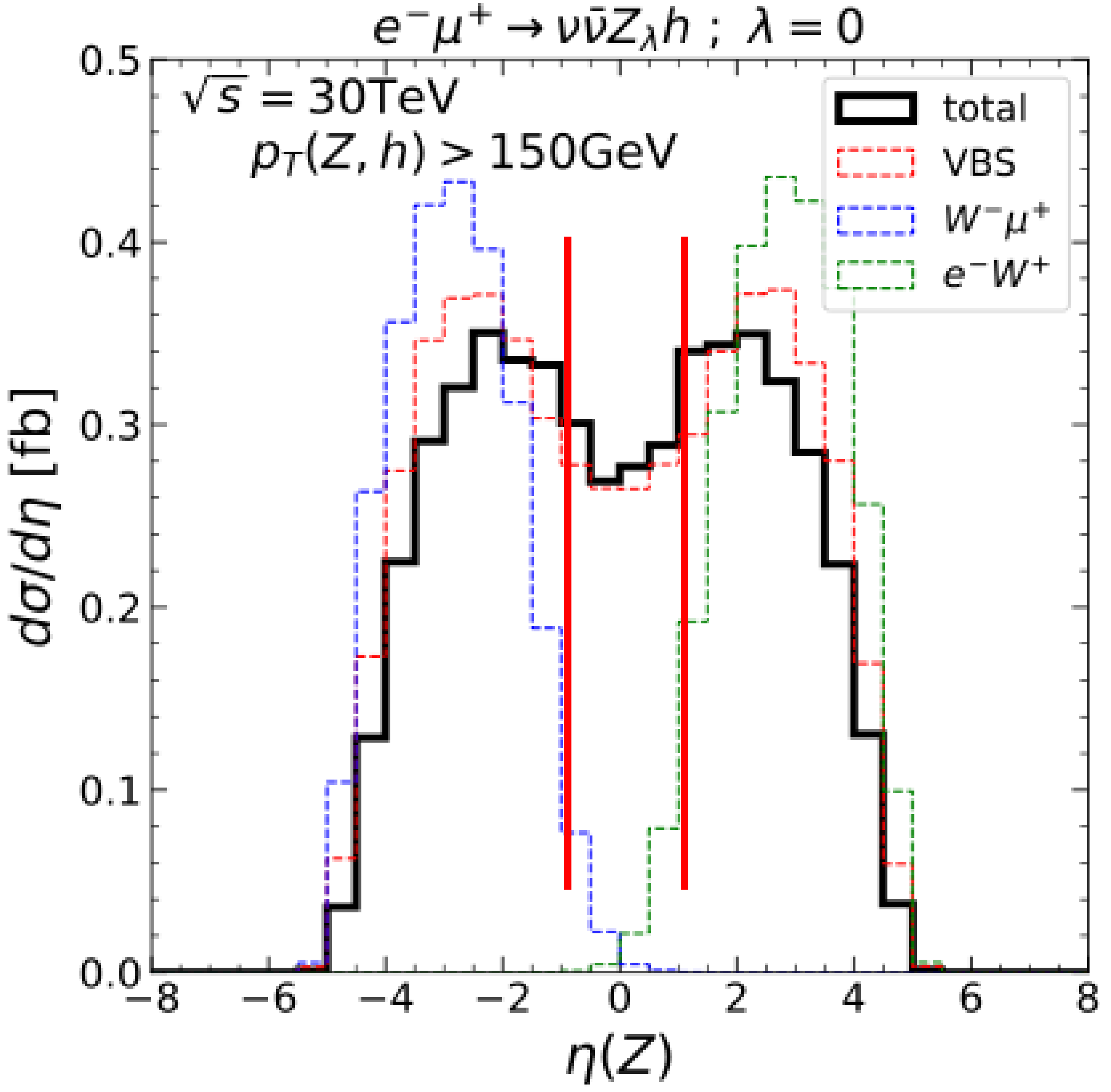
The VBS contribution can be enhanced by some kinematical cuts suggested by the distributions of the subgroups in the FD gauge.
At the same time, we also find that the dominance of the VBS contribution depends on the observables, and it is hard to isolate it from the W $-\mu+$ and e$-$W $+$ contribution, especially in the observables related to the Higgs boson in our naive study.



Enhancement of the VBS

The Weak boson PDFs provide a very good approximation under conditions where VBS is dominant. However, in this process the $W-\mu+$ and $e-W+$ contributions are dominant, so the conditions under which VBS dominates are very limited, making the use of PDFs for the analysis rather inefficient.

Moreover, in the unitary gauge, the amplitude groups exhibit energy growth at high-energy region, which makes it extremely difficult to study the VBS contribution alone.



e- e+ -> W- W+

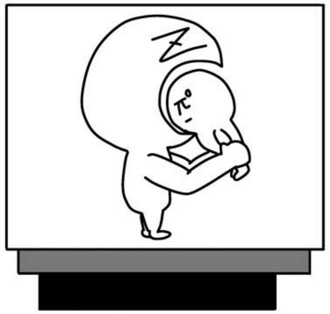
2. Gauge choices

Electroweak symmetry breaking

NG boson

In the early universe, gauge bosons run at the speed of light (mass 0).

Unitary (U) gauge



Gauge bosons eat NG bosons and become fat (run slower = get the mass).

Feynman-Diagram (FD) gauge

Gauge bosons and NG bosons run together (run slower = get the mass).

3. Helicity amplitudes

$$\beta = \frac{q}{E} = \sqrt{1 - \frac{m^2}{E^2}},$$
$$\gamma = \frac{E}{m} = \frac{1}{\sqrt{1-\beta^2}}.$$

U

$\Delta\lambda$	$(\lambda, \bar{\lambda})$	$\tilde{\mathcal{M}}_\gamma{}^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_Z{}^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_\nu{}^{\lambda\bar{\lambda}}(\theta)$
0	$(0,0)$	$-2\gamma^2\beta + \beta$	$2\gamma^2\beta - \beta$	$2\gamma^2(\beta - \cos\theta) - \beta$

$$\tilde{\mathcal{M}} \propto E^2$$

FD

$\Delta\lambda$	$(\lambda, \bar{\lambda})$	$\tilde{\mathcal{M}}_\gamma{}^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_Z{}^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_\nu{}^{\lambda\bar{\lambda}}(\theta)$
0	$(0,0)$	$\frac{1}{\gamma^2}\frac{3+\beta}{(1+\beta)^2} + 1$	$-\frac{1}{\gamma^2}\frac{3+\beta}{(1+\beta)^2} - \frac{s_W^2}{c_W^2}\left(\frac{\beta}{2s_W^2} - 1\right)$	$-\frac{1}{\gamma^2}\frac{2}{(1+\beta)^2}(1+\cos\theta)$

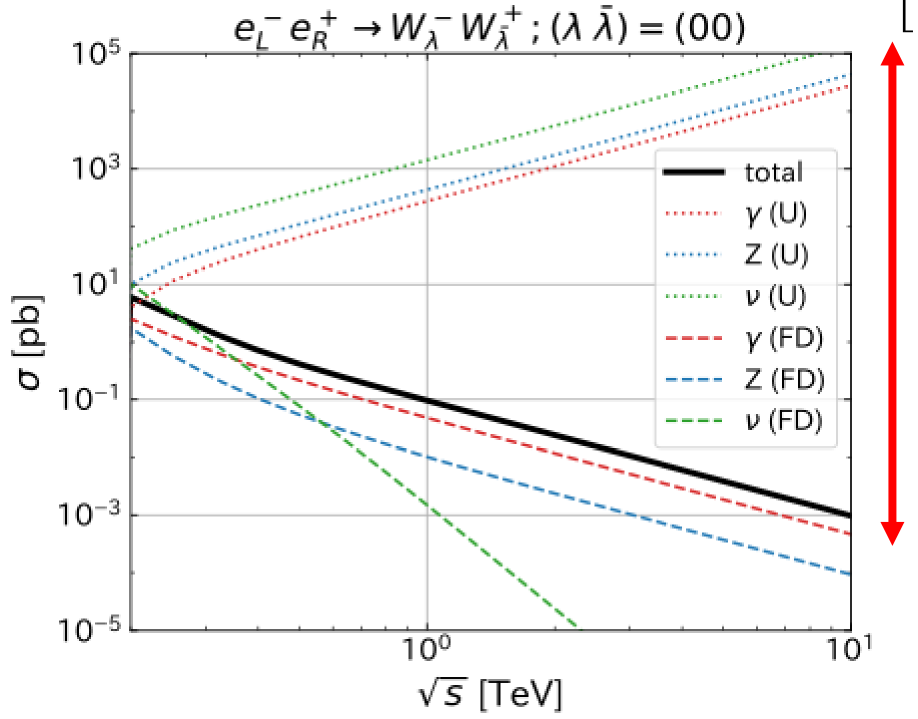
$$\tilde{\mathcal{M}} \not\propto E^2$$

2. FD gauge

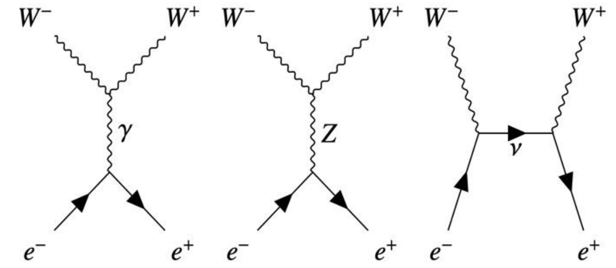
[3]

In the longitudinal massive gauge boson scattering, Gauge cancellation (energy growth)
→loss of significant digits

This is a major obstacle for future high-energy experiments.



A new gauge fixing was proposed and it indicates gauge cancellation among the interfering amplitudes can be avoided!!



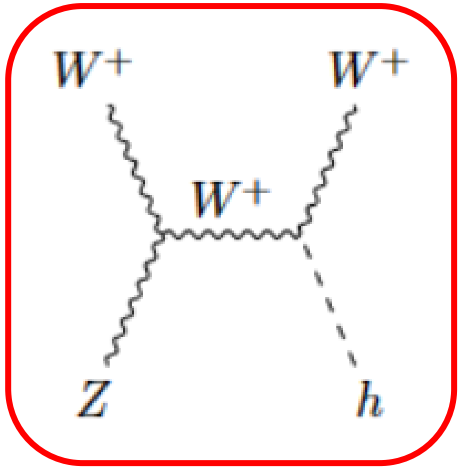
Feynman-Diagram (FD) gauge [4],[5],[6]

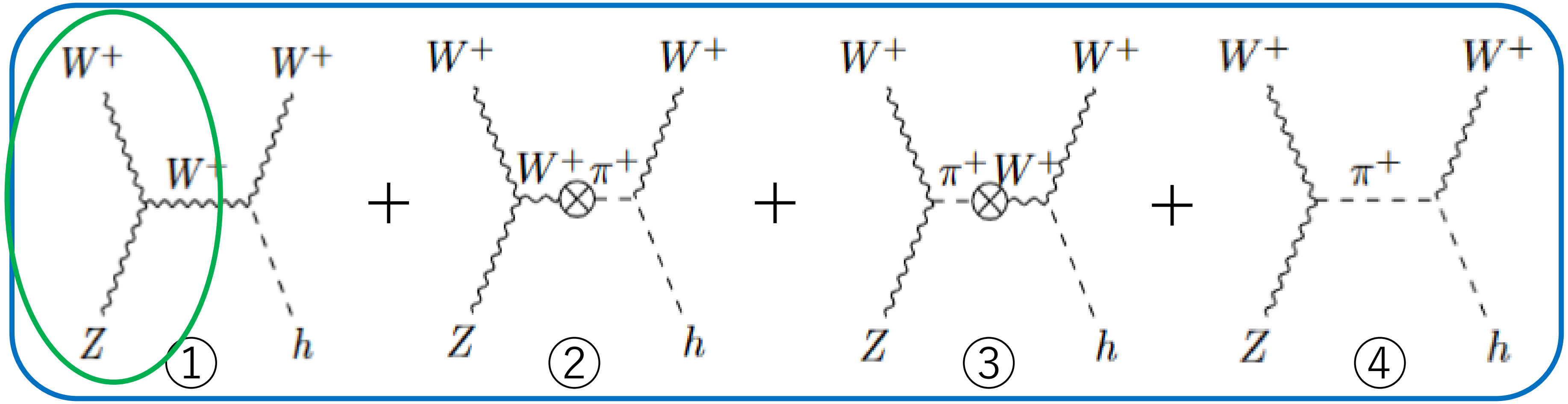
total$\propto |M_\gamma + M_Z + M_\nu|^2$,
$\gamma \propto |M_\gamma|^2, Z \propto |M_Z|^2, \nu \propto |M_\nu|^2$

[3] H.Furusato, K.Mawatari, Y.Suzuki, Y.Zheng, “W-boson pair production at lepton colliders in the Feynman-Diagram gauge”, *Phys.Rev.D 110 (2024) 5, 053005*
[4] J. Chen, K. Hagiwara, J. Kanzaki and K. Mawatari, “Helicity amplitudes without gauge cancellation for electroweak processes”, Eur.Phys.J.C 83 (2023) 10, 922.
[2] J. Chen, K. Hagiwara, J. Kanzaki, K. Mawatari and Y. Zheng , “Helicity amplitudes in light-cone and Feynman-diagram gauges”, Eur.Phys.J.Plus 139 332 (2024)
[3] K. Hagiwara, J. Kanzaki, O. Mattelaer, K. Mawatari and Y. Zheng, “Automatic generation of helicity amplitudes in Feynman-Diagram gauge”, Phys. ReV. D 110, 056024 (2024)

Subprocess VhVBS
W+ Z -> W+ h

Ex. the s-channel amplitude for $\lambda_1 = \lambda_2 = \lambda_3 = +1$ in the U and FD gauges

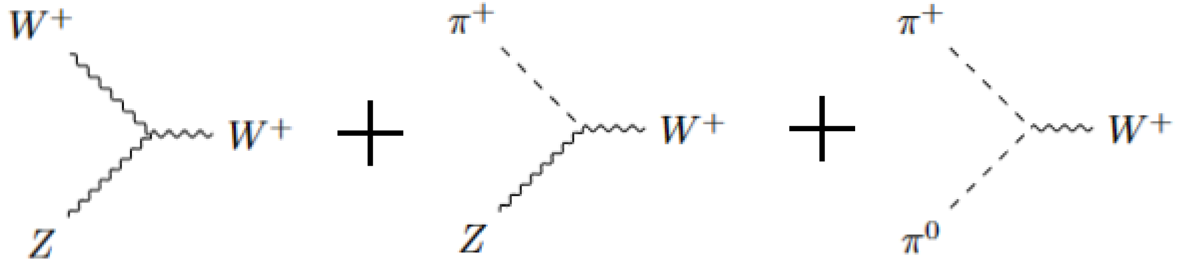




$$\mathcal{M}_s^{\mathrm{U}}(+,+,+) = J^{\mu}_{s\,++} P^{\mathrm{U}}_{s\,\mu\nu} J^{\nu}_{s\,+}$$

$$= J^{\mu}_{s\,++} \frac{i}{q^2 - m^2 + i\epsilon} \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2} \right) J^{\nu}_{s\,+}$$

for $\lambda_1 = \lambda_2 = \lambda_3 = 0$



$$\mathcal{M}_s^{\mathrm{FD}}(+,+,+) = J^{M}_{s\,++} P^{\mathrm{FD}}_{s\,MN} J^{N}_{s\,+}$$

$$= J^{\mu}_{s\,++} \frac{i}{q^2 - m^2 + i\epsilon} \left(-g_{\mu\nu} + \frac{n_\mu q_\nu + q_\mu n_\nu}{n \cdot q} \right) J^{\nu}_{s\,+} \quad \cdots ①$$

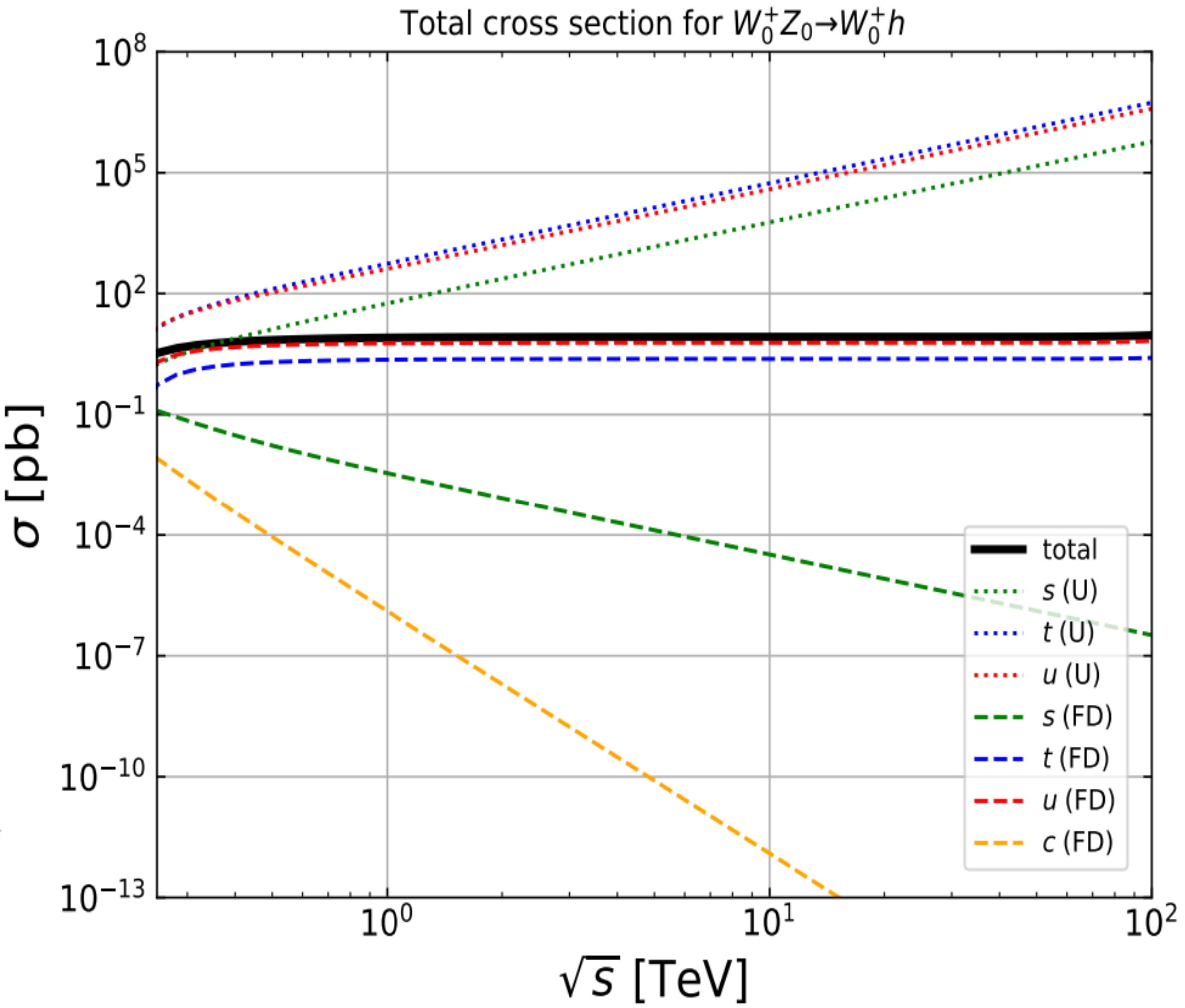
$$+ J^{\mu}_{s\,++} \frac{i}{q^2 - m^2 + i\epsilon} \left(im \frac{n_\mu}{n \cdot q} \right) J^{4}_{s\,+} \quad \cdots ②$$

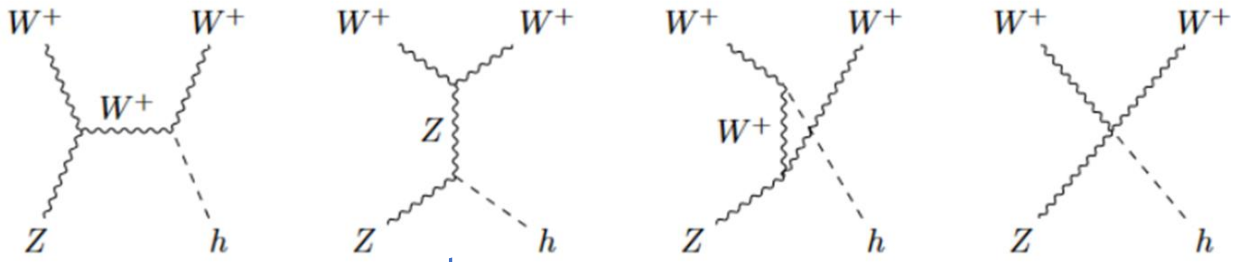
$$+ J^{4}_{s\,++} \frac{i}{q^2 - m^2 + i\epsilon} \left(-im \frac{n_\nu}{n \cdot q} \right) J^{\nu}_{s\,+} \quad \cdots ③$$

$$+ J^{4}_{s\,++} \frac{i}{q^2 - m^2 + i\epsilon} J^{4}_{s\,+} \quad \cdots ④$$

Comparison of cross sections in the U gauge and FD gauge

- **The solid black line** is the physical observable (obtained by summing the amplitudes of all channels (s, t, u, c) together and squaring them.)
- Dotted lines (U gauge) and dashed lines (FD gauge) are non-physical quantities (obtained by squaring the individual amplitudes s, t, u, c).
- Whichever one is chosen, the physical cross section (**solid black**) remains the same as expected.
  →That is, Gauge invariant.
- However, the contribution from each amplitude is quite different between the gauges!



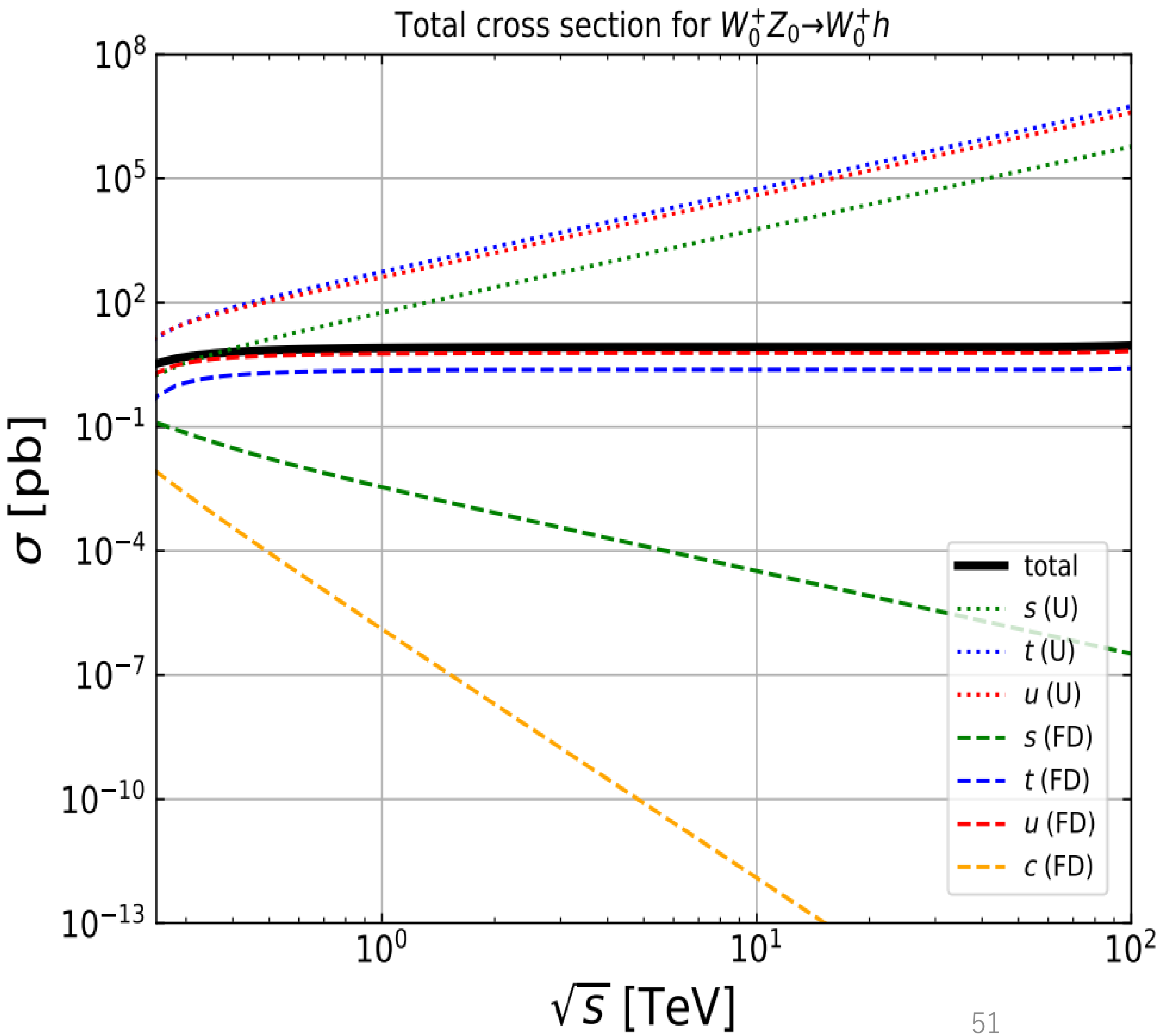


s t u c

Total cross section for $\lambda_1 = \lambda_2 = \lambda_3 = 0$

- U gauge :
  - Each amplitude has energy growth.
  - Large cancellation between the three amplitudes (s, t, u) at high energies.

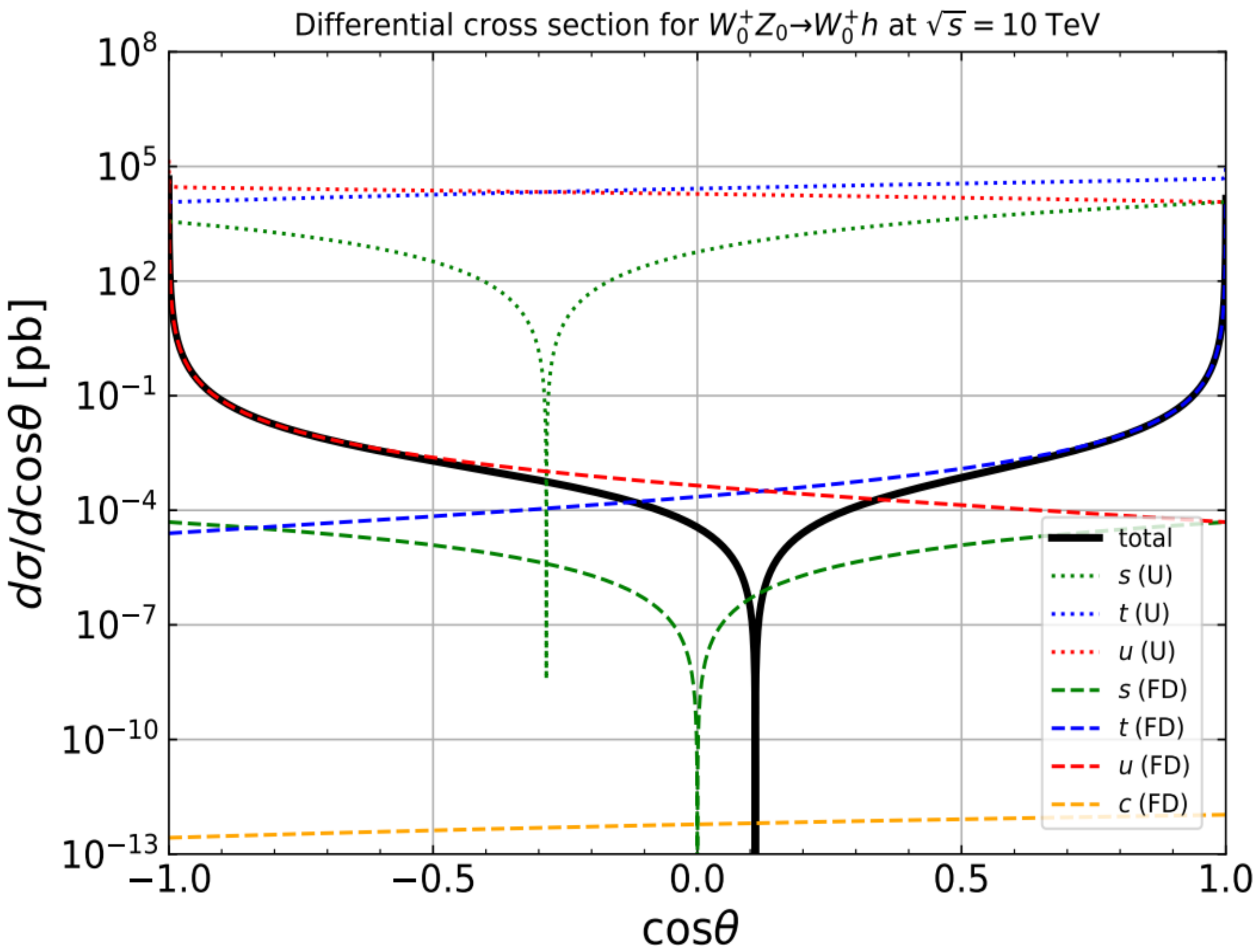
- FD gauge :
  - No such energy growth of each amplitude at all, i.e. no such gauge cancellation.
  - u and t are the main contributors.
  - At higher energies, s and c become smaller.



Angular Distribution for $\lambda_1 = \lambda_2 = \lambda_3 = 0$

- U gauge :
  - No clue for the physical distribution from each contribution.

- FD gauge :
  - Clear indication to the physical distribution from each contribution.
  - Forward scattering ($\cos\theta = 1$) : t is dominant.
  - Backward scattering ($\cos\theta = -1$) : u is dominant.



The FD gauge allows us to interpret the property of each Feynman amplitude and the interference among them.

We studied $pp \to Whjj$, especially the $W^+Z \to W^+h$ subprocess for longitudinally polarized gauge bosons.
We found

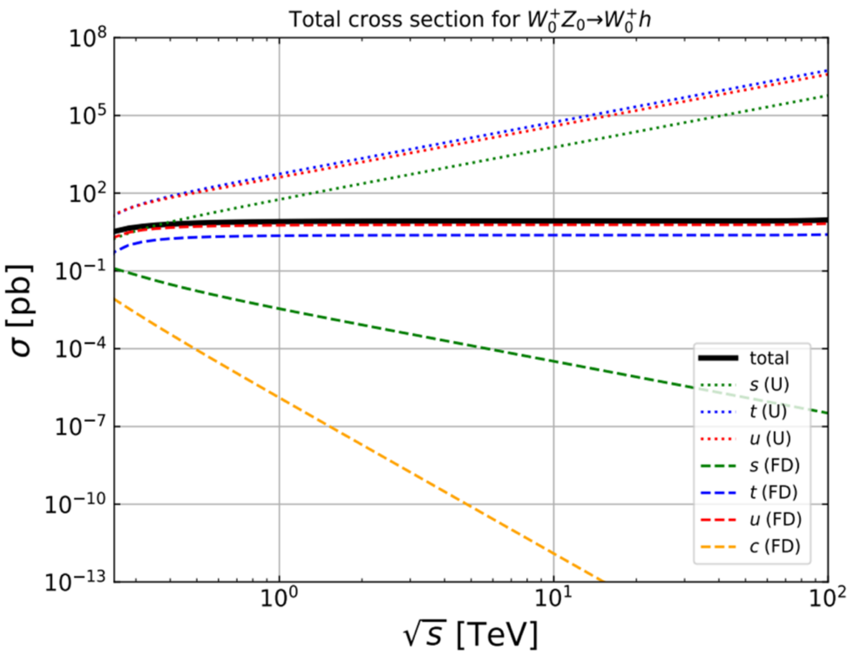
in the U gauge

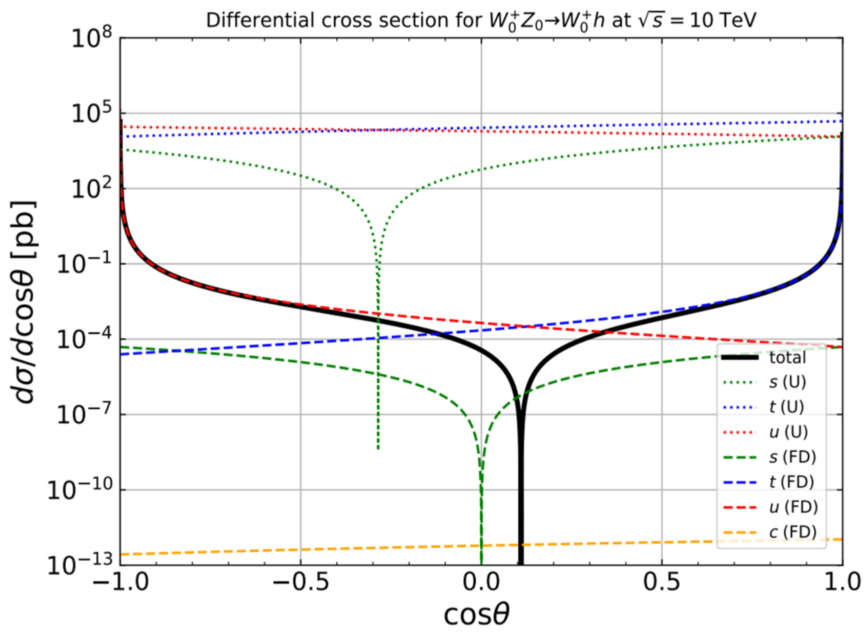
・huge gauge cancellation is needed at high energies.

in the FD

・no such cancellation.

・it is possible to see which are the dominant contributions and to interpret the property of the individual amplitudes.





B Goldstone boson couplings in the SM

In this appendix, we present all the Goldstone-boson couplings of the SM explicitly, because not only the magnitude but also the relative signs of all the gauge-boson and the corresponding Goldstone-boson couplings should be kept exact in order to keep the BRST invariance of the amplitudes. The Goldstone bosons appear in the four sector of the SM Lagrangian, the Higgs potential $\mathcal{V}_H$, the Higgs gauge interactions $\mathcal{K}_H$, the gauge-fixing term $\mathcal{L}_{GF}$, and the Yukawa term $\mathcal{L}_Y$. We show all of them explicitly below.

The minimal SM has just one $\mathrm{SU}(2)_L$ doublet Higgs field $\phi(x)$. The Lagrangian of the Higgs field is given by

$$\mathcal{L}_H = \mathcal{K}_H - \mathcal{V}_H \tag{B.1}$$

with

$$\mathcal{K}_H = (D^\mu\phi)^\dagger(D_\mu\phi), \tag{B.2}$$

$$\mathcal{V}_H = \frac{\lambda}{4}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2, \tag{B.3}$$

$$\mathcal{V}_H = \frac{m_H^2}{2}\left[H + \frac{H^2 + (\pi^1)^2 + (\pi^2)^2 + (\pi^3)^2}{2v}\right]^2, \tag{B.12}$$

$$\mathcal{K}_H = \frac{1}{2}(\partial H)^2 + (\partial\pi^+)(\partial\pi^-) + \frac{1}{2}(\partial\pi^0)^2 \tag{B.15a}$$

$$+ \left[\frac{g^2}{4}W^+W^- + \frac{g_Z^2}{4}\frac{Z^2}{2}\right]\left[(v+H)^2 + (\pi^0)^2\right] \tag{B.15b}$$

$$+ \frac{g_Z}{2}Z\left[(\partial\pi^0)(v+H) - (\partial H)\pi^0\right] \tag{B.15c}$$

$$+ \frac{g}{2}\left[W^+(\partial\pi^-) + W^-(\partial\pi^+)\right](v+H) \tag{B.15d}$$

$$- i\frac{g}{2}\left[W^+(\partial\pi^-) - W^-(\partial\pi^+)\right]\pi^0 \tag{B.15e}$$

$$+ i\frac{g}{2}(s_W^2 g_Z Z - eA)(W^+\pi^- - W^-\pi^+)(v+H) \tag{B.15f}$$

$$+ \frac{g}{2}(s_W^2 g_Z Z - eA)(W^+\pi^- + W^-\pi^+)\pi^0 \tag{B.15g}$$

$$- \frac{g}{2}\left[(W^+\pi^- + W^-\pi^+)(\partial H) + i(W^+\pi^- - W^-\pi^+)(\partial\pi^0)\right] \tag{B.15h}$$

$$- i\left[\left(\frac{1}{2} - s_W^2\right)g_Z Z + eA\right]\left[(\partial\pi^+)\pi^- - (\partial\pi^-)\pi^+\right] \tag{B.15i}$$

$$+ \left[\frac{g^2}{2}W^+W^- + \left(\left(\frac{1}{2} - s_W^2\right)g_Z Z + eA\right)^2\right]\pi^+\pi^-, \tag{B.15j}$$

J. Chen, K. Hagiwara, J. Kanzaki and K. Mawatari,
"Helicity amplitudes without gauge cancellation for electroweak processes", Eur.Phys.J.C 83 (2023) 10, 922.