# Impact of quark flavor violating SUSY on h(125) decays at future lepton colliders

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# 1. Introduction

- **What is the SM-like Higgs boson discovered at LHC?**

- **This is one of the most important issues in the present particle physics field!**

- **Here we study a possibility that it is the lightest Higgs boson $h^0$ of the Minimal Supersymmetric Standard Model (MSSM), focusing on the decays $h^0(125) \rightarrow c\,\bar{c}\,, b\,\bar{b}\,, b\,\bar{s}\,, \gamma\,\gamma,\ g\,g\,, Z\gamma$ with special emphasis on Supersymmetric Quark Flavor Violation (QFV).**

# 2. *MSSM with general QFV*

**Key parameters** *in the MSSM with general QFV* **are:**

 * *QFV parameters*: $\tilde{c}_{L/R} - \tilde{t}_{L/R}$ **&** $\tilde{s}_{L/R} - \tilde{b}_{L/R}$ *mixing parameters*

 * *QFC parameter*: $\tilde{t}_L - \tilde{t}_R$ **&** $\tilde{b}_L - \tilde{b}_R$ *mixing parameters*

*In the MSSM with MFV (minimal flavor violation),*

*these QFV parameters are absent!*

# 3. *Parameter scan*

- *In the MSSM with general QFV, we compute*

  *widths of decays $h^0(125) \to c\,\bar{c}$ , $b\,\bar{b}$ , $b\,\bar{s}$  at full one-loop level.*

  $h^0(125) \to \gamma\,\gamma$, $g\,g$ *at NLO QCD level.*

  $h^0(125) \to Z\gamma$ *at LO level.*

- *We perform a systematic MSSM parameter scan respecting all the*

  *relevant experimental and theoretical constraints on the MSSM*

  *with general QFV.*

- *We also take into account the expected SUSY particle mass*

  *limits from the future HL-LHC experiment in our analysis.*

# 3.1 *Constraints on the MSSM*

*We respect the following experimental and theoretical constraints:*

**(1)  The LHC limits on the SUSY particle masses.**

**(2)  The constraint on ($m_{A/H^+}$, tan$\beta$) from MSSM Higgs boson search at LHC.**

**(3)  The constraints on the QFV parameters from the B & K meson data.**

$$ B(b \rightarrow s\,\gamma) \quad \Delta M_{Bs} \quad B(B_s \rightarrow \mu^+ \mu^-) \quad B(B_u^+ \rightarrow \tau^+ \nu) \quad \textit{etc.} $$

**(4)  The constraints from the observed Higgs boson mass and couplings at LHC ; e.g.**
   *121.6 GeV < m_h⁰ < 128.6 GeV (allowing for theoretical uncertainty) ,*
   *0.68 < $\kappa_b$ < 1.11 (ATLAS),    0.68 < $\kappa_b$ < 1.32 (CMS)*

**(5) The experimental limit on SUSY contributions to the electroweak ρ parameter**
   *$\Delta\rho$ (SUSY) < 0.0012.*

**(6)  Theoretical constraints from the vacuum stability conditions for the**
   *trilinear couplings $T_{U\alpha\beta}$ and $T_{D\alpha\beta}$.*

# *3.2. Parameter scan*

- *We compute the $h^0(125)$ decay widths in the* *MSSM with general QFV.*

- *We take parameter scan ranges as follows:*

$$1 \; TeV < \; M_{SUSY} \; < 5 \; TeV$$

*10 < $\tan\beta$ < 80*
*2500 < $M_3$ < 5000 GeV*
*100   < $M_2$ < 2500 GeV*
*100   < $M_1$ < 2500 GeV*
*100   <  $\mu$  < 2500 GeV*
*1350 < $m_A$(pole) < 6000 GeV*
*etc. etc.*

- *In the parameter scan, all of the relevant experimental and theoretical constraints are imposed.*

- *377180 parameter points are generated and 3208 points survive the constraints.*

# 4. $h^0 \rightarrow c\,\bar{c}\,,\,b\,\bar{b}\,,\,b\,\bar{s}$ in the MSSM

- *Main 1-loop correction to $h^0 \rightarrow c\,\bar{c}$ :*

  *gluino - su loops [ su = ($\tilde{t}$ - $\tilde{c}$ mixture)]*

  *can be enhanced by large trilinear couplings $T_{U23}$ , $T_{U32}$ , $T_{U33}$*

- *Main 1-loop corrections to $h^0 \rightarrow b\,\bar{b}$ & $b\,\bar{s}$ :*

  *gluino – sd loops [ sd = ($\tilde{b}$ - $\tilde{s}$ mixture)]*

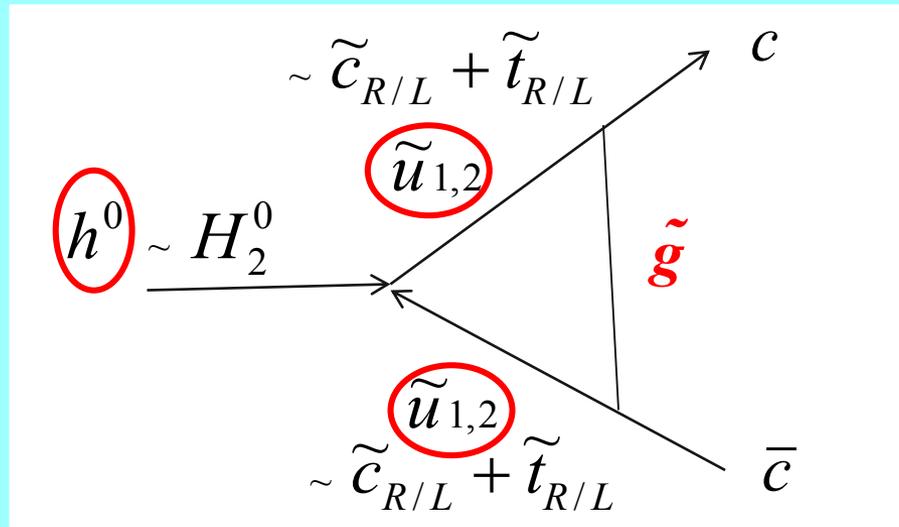  *can be enhanced by large trilinear couplings $T_{D23}$ , $T_{D32}$ , $T_{D33}$*

  *chargino - su loops [ su = ($\tilde{t}$ - $\tilde{c}$ mixture)]*

  *can be enhanced by large trilinear couplings $T_{U23}$ , $T_{U32}$ , $T_{U33}$*

**In large $\widetilde{c}_{R/L} - \widetilde{t}_{R/L}$ & $\widetilde{t}_L - \widetilde{t}_R$ mixing scenario;**

$h^0 \sim H_2^0$

$\widetilde{u}_{1,2} \sim \widetilde{c}_{R/L} + \widetilde{t}_{R/L}$



**In our scenario, "trilinear couplings" ($\widetilde{c}_R - \widetilde{t}_L - H_2^0$, $\widetilde{c}_L - \widetilde{t}_R - H_2^0$, $\widetilde{t}_L - \widetilde{t}_R - H_2^0$ couplings) = ($T_{U23}\, T_{U32}$, $T_{U33}$) are large!**

$\widetilde{u}_{1,2} - \widetilde{u}_{1,2} - h^0$ **couplings are large!**
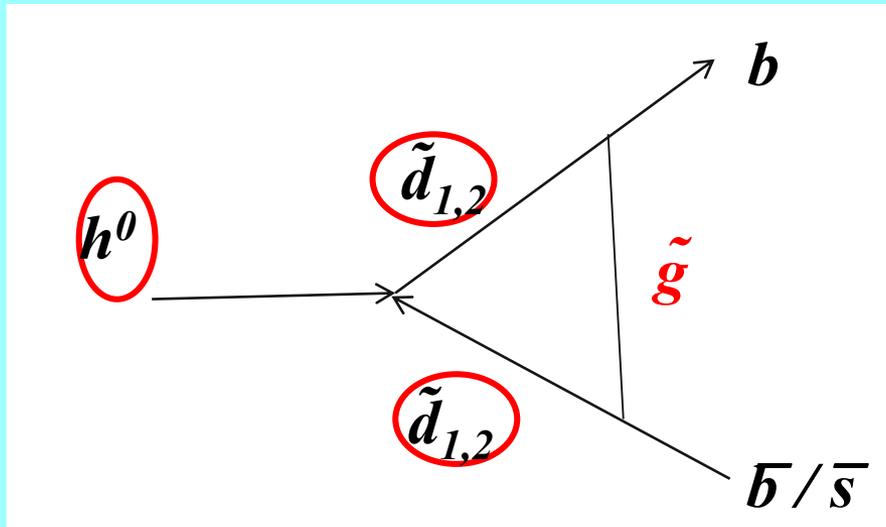
**Gluino loop contributions can be large!**

**Deviation of $\Gamma\,(h^0 \to c\,\overline{c})$ from SM width can be large!**

In large $\tilde{s}_{R/L}$ - $\tilde{b}_{R/L}$ & $\tilde{b}_L$ - $\tilde{b}_R$ mixing scenario;

$$h^0 \sim -s\alpha H_1^0 + c\alpha H_2^0$$

$$\tilde{d}_{1,2} \sim \tilde{s}_{R/L} + \tilde{b}_{R/L}$$



In our scenario, "trilinear couplings"($T_{D23}$ $T_{D32}$ , $T_{D33}$) =
($\tilde{s}_R$ - $\tilde{b}_L$ - $H_1^0$ , $\tilde{s}_L$ - $\tilde{b}_R$ - $H_1^0$ , $\tilde{b}_L$ - $\tilde{b}_R$ - $H_1^0$ couplings) are large!

$\tilde{d}_{1,2}$ - $\tilde{d}_{1,2}$ - $h^0$ couplings are large!
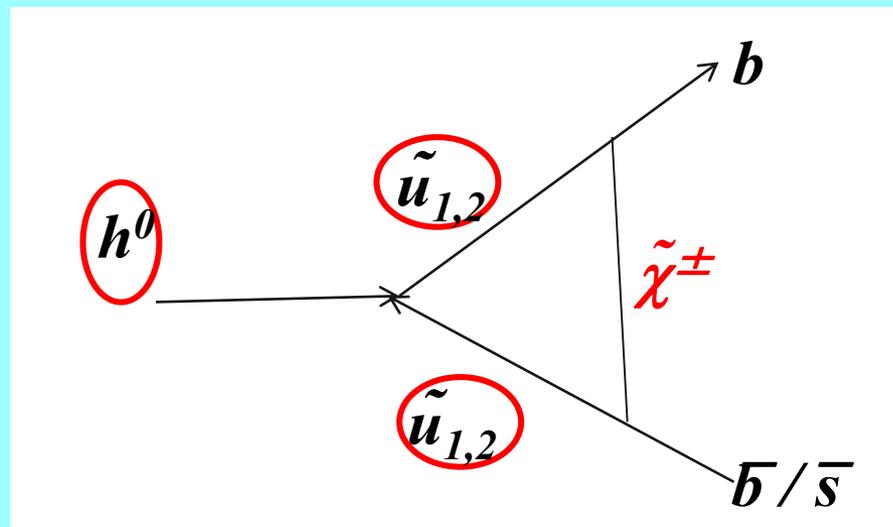
Gluino loop contributions can be large!

Deviation of $\Gamma(h^0 \rightarrow b\,\bar{b}/\bar{s})$ from SM width can be large!

*In large $\tilde{c}_{R/L}$ - $\tilde{t}_{R/L}$ & $\tilde{t}_L$ - $\tilde{t}_R$ mixing scenario;*

$h^0 \sim H_2^0$

$\tilde{u}_{1,2} \sim \tilde{c}_{R/L} + \tilde{t}_{R/L}$

$\tilde{\chi}^\pm \sim \tilde{W}^\pm + \tilde{H}^\pm$



*In our scenario, "trilinear couplings" ($\widetilde{c}_R - \widetilde{t}_L - H_2^0$, $\widetilde{c}_L - \widetilde{t}_R - H_2^0$, $\widetilde{t}_L - \widetilde{t}_R - H_2^0$ couplings) = ($T_{U23}$ $T_{U32}$, $T_{U33}$) are large!*

$\widetilde{u}_{1,2} - \widetilde{u}_{1,2} - h^0$ *couplings are large!*

*Chargino loop contributions can be large!*

*Deviation of $\Gamma(h^0 \to b\,\bar{b}/\bar{s})$ from SM width can be large!*

# 4.1 Deviation of the width from the SM prediction

- *Deviation of the width from the SM prediction:*

$$DEV(h^0 \to X\bar{X}) = \Gamma(h^0 \to X\bar{X})_{MSSM} / \Gamma(h^0 \to X\bar{X})_{SM} - 1$$

$$X = c, b$$

# *4.1 Deviation of the width from the SM prediction*

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$$DEV(h^0 \rightarrow X\bar{X}) = \Gamma(h^0 \rightarrow X\bar{X})_{MSSM} \,/\, \Gamma(h^0 \rightarrow X\bar{X})_{SM} \,-\, 1$$
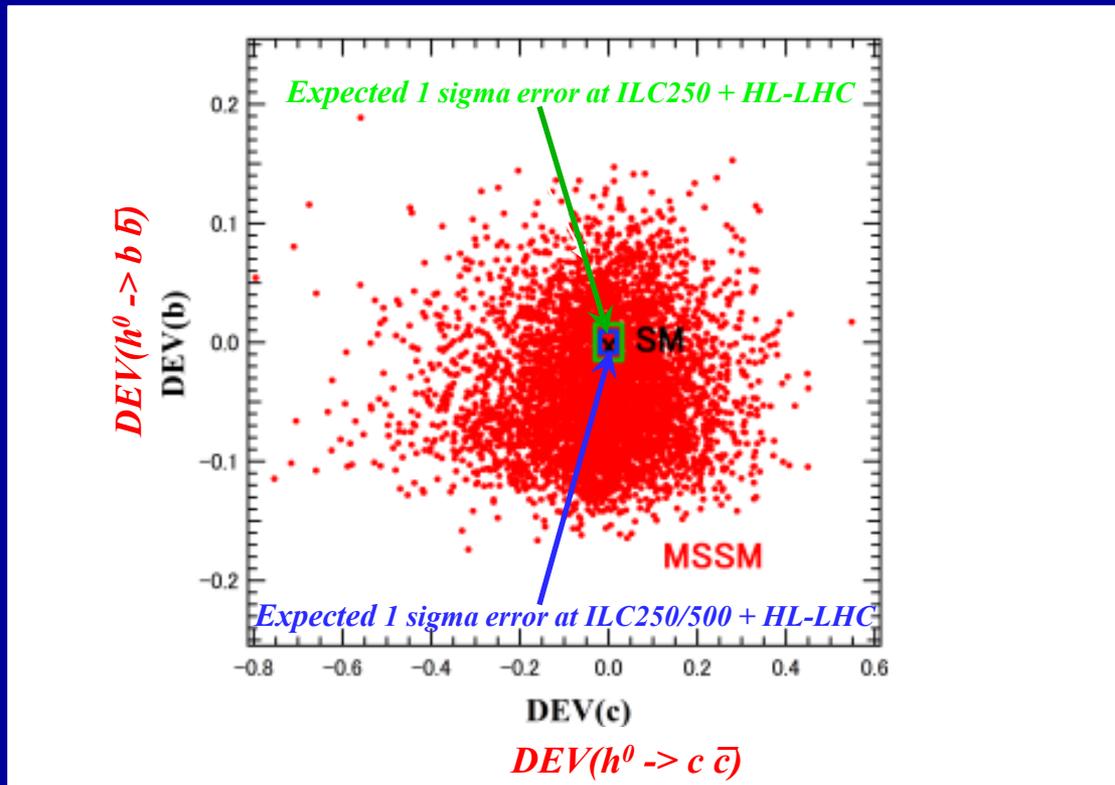
$$X = c, b$$

- *Coupling modifier:*

$$\kappa_X = g(h^0 X X) \,/\, g(h^0 X X)_{SM}$$

- *DEV -* $\kappa_X$ *relation:*

$$DEV(h^0 \rightarrow X X) = \kappa_X^2 - 1$$

**Scatter plot in $DEV(h^0 \to c\bar{c})$ - $DEV(h^0 \to b\bar{b})$ plane**

Expected 1 sigma error at ILC250 + HL-LHC

SM

MSSM

$DEV(h^0 \to b\bar{b})$
DEV(b)

Expected 1 sigma error at ILC250/500 + HL-LHC

DEV(c)

$DEV(h^0 \to c\bar{c})$

- $DEV(h^0 \to c\bar{c})$ and $DEV(h^0 \to b\bar{b})$ can be very large simultaneously!:
  $DEV(h^0 \to b\bar{b})$ can be as large as ~ **±20%**
  $DEV(h^0 \to c\bar{c})$ can be as large as ~ **±60%**

  $DEV(h^0 \to c\bar{c})$ is very small < **O(1)%** in MSSM with **MFV!**

- *ILC can observe such large deviations from SM at high significance (arXiv:2206.08326)!:*
  *$\Delta DEV(h^0 \to c\bar{c})$ = (3.6%, 2.4%, 1.8%) at (ILC250, ILC250/500, ILC250/500/1000)*
  *$\Delta DEV(h^0 \to b\bar{b})$ = (1.7%, 1.1%, 0.9%) at (ILC250, ILC250/500, ILC250/500/1000)*

# *Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders*

Table 6: The expected *absolute* $1\sigma$ error of the deviations DEV(X) and DEV(X/Y) (denoted by $\Delta DEV(X)$ and $\Delta DEV(X/Y)$) at future lepton colliders: ILC-I = ILC250 + Giga-Z, ILC-II = ILC250+500 + Giga-Z, ILC-III = ILC250+500+1000 + Giga-Z; CLIC-I = CLIC380, CLIC-II = CLIC380+1500, CLIC-III = CLIC380+1500+3000; FCC-ee I = FCC-ee240 + Z/WW, FCC-ee II = FCC-ee240+365 + Z/WW; CEPC-I = CEPC240 + Z/WW, CEPC-II = CEPC240+360 + Z/WW; MuC-I = MuC3TeV, MuC-II = MuC10TeV, MuC-III = MuC10TeV+125GeV. As for ILC the results without Giga-Z run are almost identical to those with Giga-Z one. The Z/WW denote Z-pole and WW threshold runs. All results except for MuC-I and MuC-II are those from the free-$\Gamma_H$ fit and the results for MuC-I and MuC-II are those from the constrained-$\Gamma_H$ fit, where $\Gamma_H$ is the total width of the Higgs boson $h^0$. The details of run scenarios of the lepton colliders are explained in Ref. [14]. The HL-LHC and LEP/SLD measurements are combined with all lepton collider run scenarios.

*Jorge de Blas et al., Snowmass2021 Report: arXiv:2206.08326 [hep-ph];*
*Private communication with Jorge de Blas.*

# Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders

|  | ILC-I | ILC-II | ILC-III | CLIC-I | CLIC-II | CLIC-III |
|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.7% | 1.1% | 0.9% | 2.2% | 1.2% | 1.1% |
| $\Delta DEV(c)$ | 3.6% | 2.4% | 1.8% | 8.6% | 3.8% | 3.0% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 2.0% | 2.6% | 2.4% | 2.2% |
| $\Delta DEV(g)$ | 1.8% | 1.4% | 1.1% | 2.2% | 1.6% | 1.4% |
| $\Delta DEV(b/c)$ | 3.12% | 2.07% | 1.3% | 8.16% | 3.5% | 2.54% |
| $\Delta DEV(\gamma/g)$ | 3.29% | 2.82% | 2.3% | 3.41% | 3.05% | 2.61% |
| $\Delta DEV(g/b)$ | 1.51% | 1.14% | 0.85% | 1.62% | 1.3% | 1.1% |
| $\Delta DEV(g/c)$ | 3.4% | 2.34% | 1.52% | 8.3% | 3.72% | 2.75% |

|  | FCC-ee I | FCC-ee II | CEPC-I | CEPC-II | MuC-I | MuC-II | MuC-III |
|---|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.3% | 1.2% | 0.86% | 0.84% | 1.8% | 0.92% | 1.1% |
| $\Delta DEV(c)$ | 2.8% | 2.6% | 2.4% | 2.2% | 12.4% | 3.8% | 3.6% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 1.8% | 1.8% | 2.4% | 1.4% | 1.5% |
| $\Delta DEV(g)$ | 1.5% | 1.4% | 0.9% | 0.88% | 1.7% | 0.92% | 1.0% |
| $\Delta DEV(b/c)$ | 2.22% | 2.06% | 2.0% | 1.94% | 11.92% | 3.53% | 3.37% |
| $\Delta DEV(\gamma/g)$ | 3.03% | 2.86% | 2.07% | 2.03% | 3.18% | 1.55% | 1.55% |
| $\Delta DEV(g/b)$ | 1.29% | 1.16% | 0.73% | 0.7% | 1.69% | 0.73% | 0.72% |
| $\Delta DEV(g/c)$ | 2.54% | 2.33% | 2.13% | 2.06% | 12.0% | 3.59% | 3.43% |

# 4.2. $\underline{BR(h^0 \to b\,\bar{s}\,/\,s\,\bar{b})}$

$BR(h^0 \to b\,\bar{s}\,/\,s\,\bar{b}) \cong 0\;(\,<10^{-7}\,)\;\;(SM)$

$BR(h^0 \to b\,\bar{s}\,/\,s\,\bar{b}) \cong 0\;\;(MSSM\;with\;MFV)$

$BR(h^0 \to b\,\bar{s}\,/\,s\,\bar{b})$ can be as large as ~ 0.15% (MSSM with QFV)!

*(See also Gomez-Heinemeyer-Rehman, PR D93 (2016) 095021 [arXiv:1511.04342]. )*

**ILC(250+500+1000) sensitivity could be ~ 0.1% (at 4 $\sigma$ significance)!**

*Private communication with Junping Tian;*
*See also Barducci et al., JHEP 12 (2017) 105 [arXiv:1710.06657].*

*Expected FCC-ee bound is $BR(h^0 \to b\,\bar{s}\,/\,s\,\bar{b}) < 4.5 \cdot 10^{-4}$ (at 95% CL)*

*See M. Selvaggi, talk at FCC Physics Performance Meeting, 18 Mar 2024.*

*Expected CEPC bound is $BR(h^0 \to b\,\bar{s}\,/\,s\,\bar{b}) < 2.2 \cdot 10^{-4}$ (at 95% CL)*

*See H. Liang et al., arXiv:2310.03440.*

## 4.3 *Deviation of width ratio from the SM prediction*

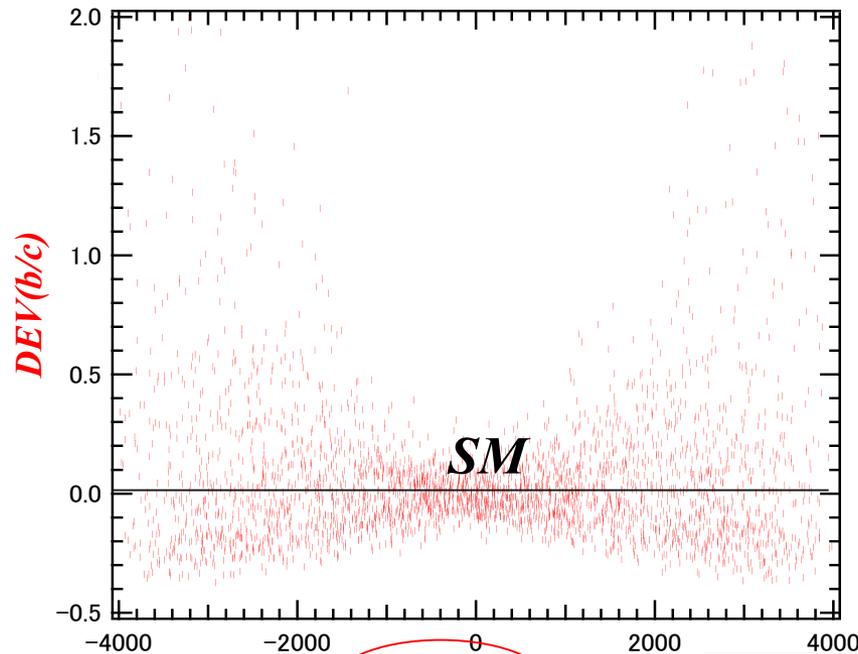- *The deviation of the width ratio from the SM prediction:*

$$DEV(b/c) = [\Gamma(b) / \Gamma(c)]_{MSSM} \ / \ [\Gamma(b) / \Gamma(c)]_{SM} \ - \ 1$$

$$\Gamma(X) = \Gamma(h^0 -> X \bar{X})$$

- *We find;*
  *the experimental measurement errors as*
  *well as the MSSM prediction errors*
  *tend to cancel out significantly in the width ratios.*

# Scatter plot in $T_{U32} - DEV(b/c)$ plane



There is a strong correlation between $T_{U32} - DEV(b/c)$!

DEV(b/c) can exceed ~ +100% for large $T_{U32}$ !

Expected absolute 1 σ error of DEV(b/c) at ILC:

$\Delta DEV(b/c) = (3.1\%, 2.1\%, 1.3\%)$ at (ILC250, ILC250+500, ILC250+500+1000).

[*Jorge de Blas et al., Snowmass2021 Report: arXiv:2206.08326;
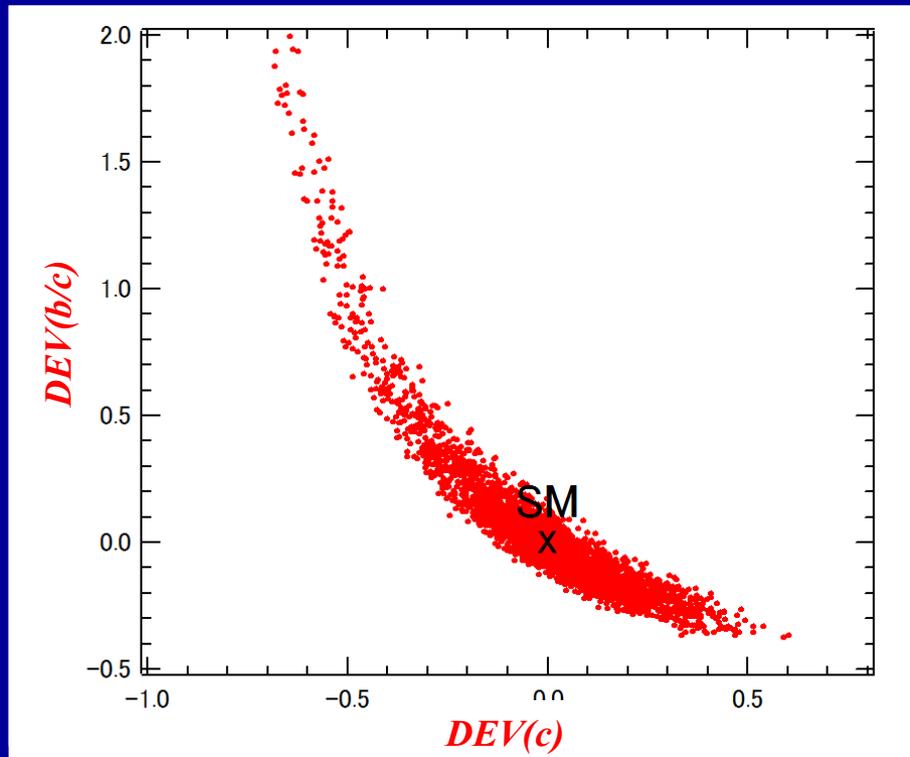Private communication with Jorge de Blas.*]

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# Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders

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| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 2.0% | 2.6% | 2.4% | 2.2% |
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# Scatter plot in DEV(b/c) − DEV(c) plane



- *There is a very strong correlation between DEV(b/c) − DEV(c)!*
- *DEV(b/c) and DEV(c) can be very large simultaneously!*
- *Expected absolute 1 σ error of DEV(c) and DEV(b/c) at ILC:*
   *Δ DEV(c) = (3.6%, 2.4%, 1.8%) at (ILC250, ILC250+500, ILC250+500+1000)*
   *ΔDEV(b/c) = (3.1%, 2.1%, 1.3%) at (ILC250, ILC250+500, ILC250+500+1000).*

*[Jorge de Blas et al., Snowmass2021 Report: arXiv:2206.08326; Private communication with Jorge de Blas.]*

- **ILC can observe such large deviation from SM at very high significance!**

# *Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders*

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# 5. $h^0 \rightarrow \gamma\gamma, g\,g, Z\gamma$ in the MSSM

- *Main 1-loop contributions to $h^0 \rightarrow \gamma\gamma, Z\gamma$ :*

  *[$W^+$/ top-quark / su] - loops  [ su = ($\tilde{t}$ - $\tilde{c}$ mixture)]*

  *The su-loops can be enhanced by large trilinear couplings $T_{U23}$ , $T_{U32}$ , $T_{U33}$ ,*

  *resulting in sizable deviation of $\Gamma$($h^0 \rightarrow \gamma\gamma, Z\gamma$) from the SM width!*


- *Main 1-loop contributions to $h^0 \rightarrow g\,g$:*

  *[top-quark / su] - loops  [ su = ($\tilde{t}$ - $\tilde{c}$ mixture)]*
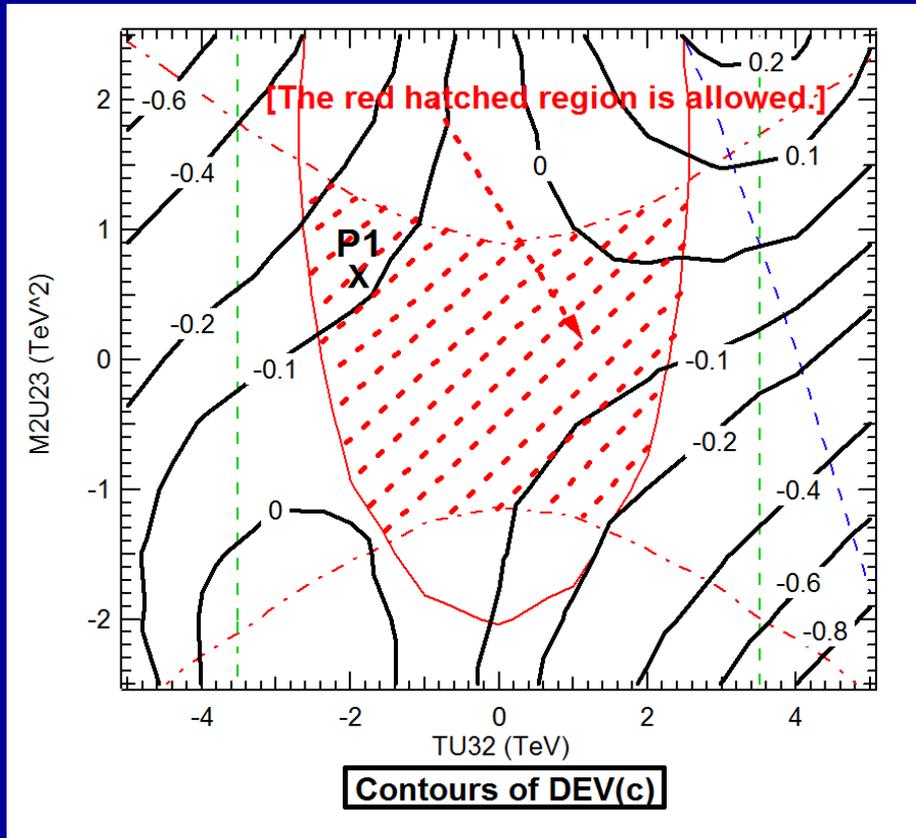
  *The su-loops can be enhanced by large trilinear couplings $T_{U23}$ , $T_{U32}$ , $T_{U33}$ ,*

  *resulting in sizable deviation of $\Gamma$($h^0 \rightarrow g\,g$) from the SM width!*

# Scatter plot in DEV($h^0 \to \gamma\gamma$) - DEV($h^0 \to gg$) plane
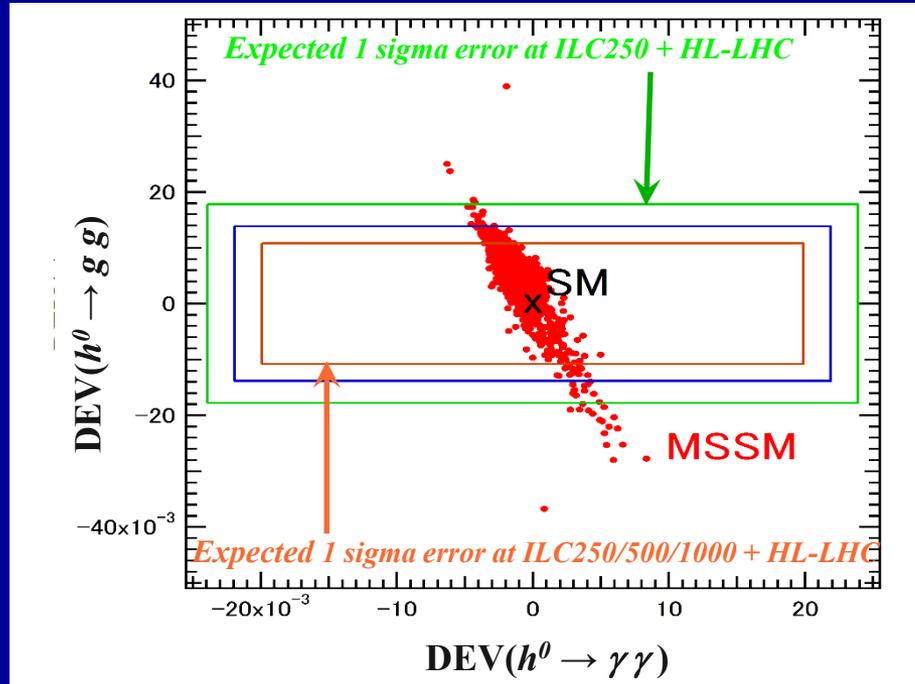


- DEV($h^0 \to \gamma\gamma$) and DEV($h^0 \to gg$) can be sizable simultaneously!:

  DEV($h^0 \to \gamma\gamma$) can be as large as $\pm 1\%$

  DEV($h^0 \to gg$) can be as large as $\pm 4\%$.

  DEV($h^0 \to gg$) can be as large as $+4\%$ and $-7\%$ from a contour plot analysis.

- Expected absolute 1 $\sigma$ errors at (ILC250, ILC250+500, ILC250+500+1000):

  $\Delta$ DEV($h^0 \to \gamma\gamma$) = (2.4%, 2.2%, 2.0%)

  $\Delta$ DEV($h^0 \to gg$) = (1.8%, 1.4%, 1.1%)

  [*Jorge de Blas et al., Snowmass2021 Report: arXiv:2206.08326*]

- ILC can observe such sizable deviation DEV($h^0 \to gg$) at high significance!

# 6. *Benchmark scenario*

*From the allowed MSSM parameter points in the scan, we have selected
a benchmark point P1 shown in Table 2 .*

Table 2: The MSSM parameters for the reference point P1 (in units of GeV or GeV$^2$ expect for $\tan\beta$). All parameters are defined at scale $Q = 1$ TeV, except $m_A(pole)$. The parameters that are not shown here are taken to be zero.

| $\tan\beta$ | $M_1$ | $M_2$ | $M_3$ | $mu$ | $m_A(pole)$ |
|---|---|---|---|---|---|
| 33 | 1660 | 765 | 4615 | 870 | 5325 |

| $M_{Q22}^2$ | $M_{Q33}^2$ | $M_{Q23}^2$ | $M_{U22}^2$ | $M_{U33}^2$ | $M_{U23}^2$ |
|---|---|---|---|---|---|
| $3975^2$ | $3160^2$ | $920^2$ | $3465^2$ | $1300^2$ | $795^2$ |

| $M_{D22}^2$ | $M_{D33}^2$ | $M_{D23}^2$ | $T_{U23}$ | $T_{U32}$ | $T_{U33}$ |
|---|---|---|---|---|---|
| $2620^2$ | $2425^2$ | $-1625^2$ | -2040 | -1880 | -4945 |

| $T_{D23}$ | $T_{D32}$ | $T_{D33}$ | $T_{E33}$ |
|---|---|---|---|
| -2360 | 1670 | -2395 | -300 |

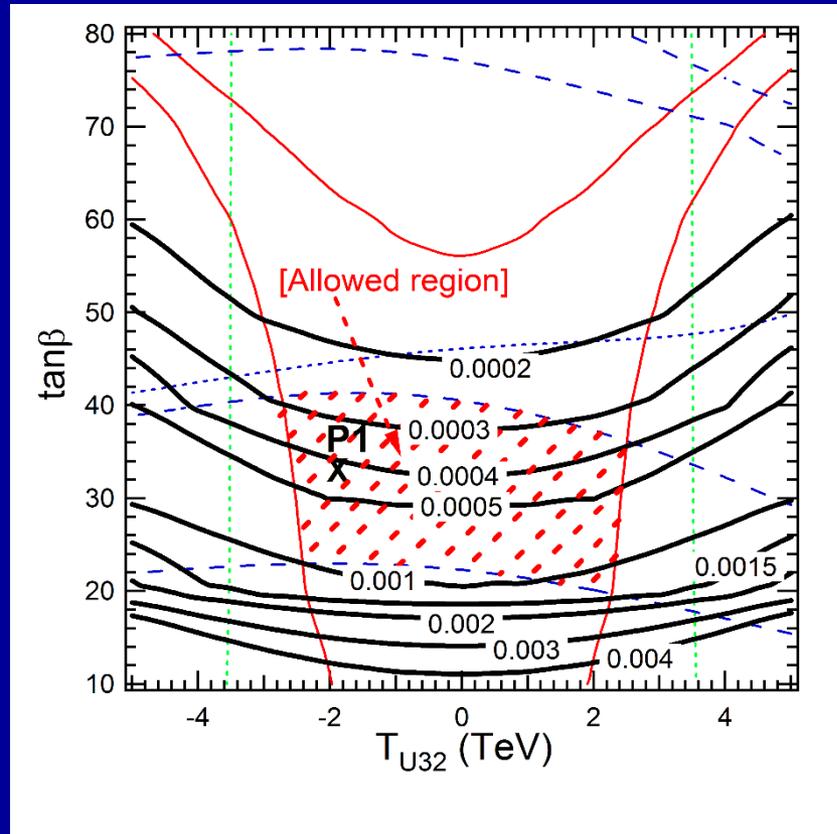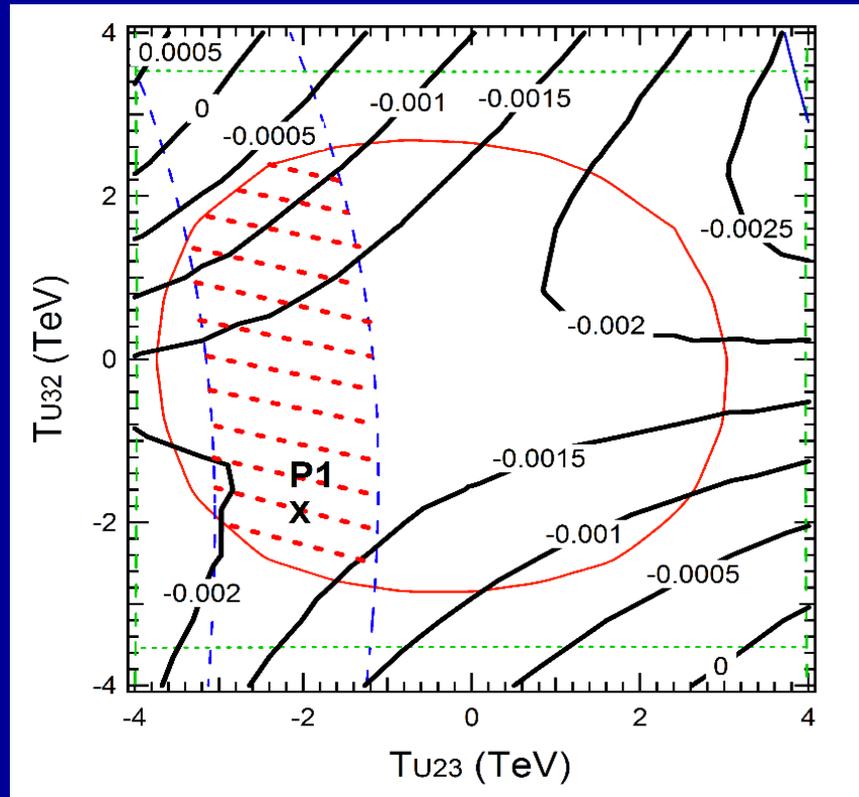| $M_{Q11}^2$ | $M_{U11}^2$ | $M_{D11}^2$ | $M_{L11}^2$ | $M_{L22}^2$ | $M_{L33}^2$ | $M_{E11}^2$ | $M_{E22}^2$ | $M_{E33}^2$ |
|---|---|---|---|---|---|---|---|---|
| $4500^2$ | $4500^2$ | $4500^2$ | $1500^2$ | $1500^2$ | $1500^2$ | $1500^2$ | $1500^2$ | $1500^2$ |

Contours of DEV(c)

*We find that DEV(c) can be very large (about -30% to 10%) in a sizable region allowed by all the constraints including the expected sparticle mass limits from the future HL-LHC experiment!*

# *Contours of DEV($h^0$ -> b $\bar{b}$) in $T_{U32}$ - $M^2_{U23}$ plane around P1*



Contours of DEV(b)

- *We find that **DEV(b) can be very large (about -10% to -18%)** in a sizable region allowed by all the constraints **including the expected sparticle mass limits from the future HL-LHC experiment!***

# Contours of $B(h^0 \to b\,\bar{s}\,/\,s\,\bar{b})$ in $T_{U32} - \tan\beta$ plane around P1



- $B(h^0 \to b\,\bar{s}\,/\,s\,\bar{b})$ **can be as large as ~ 0.15%** in a sizable region allowed by all the constraints **including the expected sparticle mass limits from the future HL-LHC!**

- **ILC(250 + 500 + 1000) sensitivity could be ~ 0.1% at $4\sigma$ significance!**
  *Private communication with Junping Tian;*
  *See also Barducci et al., JHEP 12 (2017) 105 [arXiv:1710.06657].*

*DEV($Z\gamma$) is very small in the $T_{U23}$ - $T_{U32}$ plane around P1.*

# 8. *Conclusion*

- *We have studied the decays*

  *$h^0$ (125GeV) → $c\,\bar{c}$, $b\,\bar{b}$, $b\,\bar{s}$, $\gamma\,\gamma$, $g\,g$, $Z\gamma$ in the MSSM with general QFV.*


- *For the first time, we have performed a systematic MSSM parameter scan*
  *respecting all of the relevant theoretical and experimental constraints.*


- *In strong contrast to the usual studies in the MSSM with minimal flavor violation (MFV),*
  *we have found that the deviations of these MSSM decay widths and width ratios*
  *from the SM values can be quite sizable in the MSSM with general QFV.*


- *All of these large deviations in the h (125) decays are due to*
  *large $\tilde{c}$ - $\tilde{t}$ mixing & large $\tilde{c}$ / $\tilde{t}$ involved trilinear couplings $T_{U23}$, $T_{U32}$, $T_{U33}$ and*
  *large $\tilde{s}$ - $\tilde{b}$ mixing & large $\tilde{s}$ / $\tilde{b}$ involved trilinear couplings $T_{D23}$, $T_{D32}$, $T_{D33}$ .*


- *Future lepton colliders such as ILC, CLIC, CEPC, FCC-ee and MuC can*
  *observe such sizable deviations from the SM at high signal significance*
  *even after the failure of SUSY particle discovery at the HL-LHC.*


- *In case the deviation pattern shown here is really observed at the lepton colliders,*
  *then it would strongly suggest the discovery of QFV SUSY (the MSSM with general QFV).*

- *Our analysis suggests the following:*

   *PETRA/TRISTAN e- e+ collider discovered virtual $Z^0$ effect for the first time.*
   *Later, CERN p $\bar{p}$ collider discovered the $Z^0$ boson.*

   *Similarly, lepton colliders, such as ILC, could discover virtual Sparticle effects for the first time in $h^0(125)$ decays!*
   *Later, FCC-hh p p collider could discover the Sparticles!*

- *In summary:*

*The effect of quark flavor violating (QFV) MSSM on h(125) decays to cc, bb, bs, γγ, gg can be quite significant, making* <span style="color:yellow">*measurement of these decay widths*</span> *at future lepton colliders* <span style="color:yellow">*a very sensitive probe of QFV SUSY.*</span>

# END

# *Thank you!*

# Backup

# *Impact of quark flavor violating SUSY on h(125) decays at future lepton colliders*

*K. Hidaka*
*Tokyo Gakugei University*

*Collaboration with*
*H. Eberl, E. Ginina (HEPHY Vienna)*

# 1. Introduction

- *What is the SM-like Higgs boson discovered at LHC?*

- *It can be the SM Higgs boson.*

- *It can be a Higgs boson of New Physics.*

- *This is one of the most important issues in the present particle physics field!*

- *Here we study a possibility that it is the lightest Higgs boson $h^0$ of the Minimal Supersymmetric Standard Model (MSSM), focusing on the decays $h^0(125) \rightarrow c\,\bar{c}\,, b\,\bar{b}\,, b\,\bar{s}\,, \gamma\,\gamma,\ g\,g$ with special emphasis on Supersymmetric Quark Flavor Violation (QFV).*

# 2. *MSSM with general QFV*

*Key parameters in this study are:*

\* *QFV parameters*: $\tilde{c}_{L/R} - \tilde{t}_{L/R}$ & $\tilde{s}_{L/R} - \tilde{b}_{L/R}$ *mixing parameters*

\* *QFC parameter*: $\tilde{t}_L - \tilde{t}_R$ & $\tilde{b}_L - \tilde{b}_R$ *mixing parameters*

$M^2_{Q23} = (\tilde{c}_L - \tilde{t}_L$ *mixing parameter*$) = (\tilde{s}_L - \tilde{b}_L$ *mixing parameter*$)$

$M^2_{U23} = (\tilde{c}_R - \tilde{t}_R$ *mixing parameter*$)$

$M^2_{D23} = (\tilde{s}_R - \tilde{b}_R$ *mixing parameter*$)$

$T_{U23} = (\tilde{c}_R - \tilde{t}_L$ *mixing parameter*$)$

$T_{U32} = (\tilde{c}_L - \tilde{t}_R$ *mixing parameter*$)$

$T_{U33} = (\tilde{t}_L - \tilde{t}_R$ *mixing parameter*$)$

$T_{D23} = (\tilde{s}_R - \tilde{b}_L$ *mixing parameter*$)$

$T_{D32} = (\tilde{s}_L - \tilde{b}_R$ *mixing parameter*$)$

$T_{D33} = (\tilde{b}_L - \tilde{b}_R$ *mixing parameter*$)$

# 2. *MSSM with general QFV*

*Key parameters in this study are:*

* *QFV parameters*: $\tilde{c}_{L/R} - \tilde{t}_{L/R}$ & $\tilde{s}_{L/R} - \tilde{b}_{L/R}$ *mixing parameters*

* *QFC parameter*: $\tilde{t}_L - \tilde{t}_R$ & $\tilde{b}_L - \tilde{b}_R$ *mixing parameters*

*These QFV parameters are absent in the MSSM with minimal flavor violation (MFV)!*

# 3. *Constraints on the MSSM*

*We respect the following experimental and theoretical constraints:*

**(1)  The LHC limits on the SUSY particle masses.**

**(2)  The constraint on $(m_{A/H^+}, \tan\beta)$ from MSSM Higgs boson search at LHC.**

**(3)  The constraints on the QFV parameters from the B & K meson data.**

$$B(b \to s\,\gamma) \quad \Delta M_{Bs} \quad B(B_s \to \mu^+\mu^-) \quad B(B_u^+ \to \tau^+\nu) \quad etc.$$

**(4)  The constraints from the observed Higgs boson mass and couplings at LHC ; e.g.**
   *121.6 GeV < m_h⁰ < 128.6 GeV (allowing for theoretical uncertainty) ,*
   *0.68 < $\kappa_b$ < 1.11 (ATLAS),    0.68 < $\kappa_b$ < 1.32 (CMS)*

**(5) The experimental limit on SUSY contributions to the electroweak $\rho$ parameter**
   *$\Delta\rho$ (SUSY) < 0.0012.*

**(6)  Theoretical constraints from the vacuum stability conditions for the**
   *trilinear couplings $T_{U\alpha\beta}$ and $T_{D\alpha\beta}$.*

***\* We also take into account the expected SUSY particle mass limits from the future HL-LHC experiment in our analysis.***

[1] X. Cid Vidal et al., "Report from Working Group 3: Beyond the Standard Model physics at the HL-LHC and HE-LHC", CERN Yellow Rep: Monogr. 7 (2019) 585-865 [arXiv:1812.07831 [hep-ph]].

[2] T. Bose et al., "Report of the Topical Group on Physics Beyond the Standard Model at Energy Frontier for Snowmass 2021", arXiv:2209.13128 [hep-ph].

[3] ATLAS Collaboration, ATLAS Notes ATL-PHYS-PUB-2012-001, ATL-PHYS-PUB2013-002, ATL-PHYS-PUB-2014-010, and ATL-PHYS-PUB-2018-048.

[4] C. M. Berggren, talk at The European Physical Society Conference on High Energy Physics (EPS-HEP2023), 21-25 August 2023, Hamburg, Germany.

[5] H. Gilmer, talk at the 40th International Conference on High Energy Physics, virtual conference (ICHEP2020), 28 Jul - 6 Aug 2020, Prague, Czech Republic.

[6] M. Narain et al., "The Future of US Particle Physics - The Snowmass 2021 Energy Frontier Report", arXiv:2211.11084 [hep-ex].

[7] H. Bahl, P. Bechtle, S. Heinemeyer, S. Liebler, T. Stefaniak and G. Weiglein, Eur. Phys. J. C 80 (2020) 916 [arXiv:2005.14536 [hep-ph]].

*** Constraints from W boson mass data:**

*The recent $m_W$ data from CDF II [1] disagrees significantly with the previous world average. (-> See backup slides.)*
*[1] CDF Collaboration, Science 376, 170–176 (2022)*

*This issue of the $m_W$ data is not yet settled.*

*Hence, we do not take into accont this $m_W$ constraint on the MSSM parameters in our analysis.*

# * Constraint from muon g-2:

- Tension btw the new Fermilab muon g-2 data [1] and the SM prediction is ~ 5 sigma..

- In our scenario with heavy sleptons/sneutrinos with masses of about 1.5 TeV the MSSM loop contributions to muon g-2 are too small to explain this tension.

## However;

- The tension btw the muon g-2 data and the SM prediction with lattice QCD [2] is only ~ 1.6 sigma!

- Moreover, the SM prediction in the data-driven approach using the recent CDM-3 data [3] supports the SM prediction with the lattice QCD [2]!

[1]  Muon g-2 Collaboration, arXiv:2308.06230 [hep-ex].

[2] BMW Collaboration, Nature 593 (2021) 51 [arXiv:2002.12347[hep-lat]].

[3] CMD-3 Collaboration, arXiv:2302.08834 [hep-ex].

*** Constraint on the MSSM QFV parameters from B(Z - > b s):**

*The current best experimental upper limit on B(Z - > b s) can not give any significant constraint on the sstrange-sbottom and scharm -stop mixings.
(see D. Atwood et al., Phys. Rev. D66(2002) 093005 [arXiv:hep-ph/0203200])*

*Note that no experimental upper limit on B(Z -> b s) is listed in PDG2022 !*

# 4. Parameter scan

- *We compute the $h^0$(125) decay widths in the MSSM with general QFV.*

- *We take parameter scan ranges as follows:*

$$1\ TeV < M_{SUSY} < 5\ TeV$$

$10 < tan\beta < 80$
$2500 < M_3 < 5000\ GeV$
$100\ \ < M_2 < 2500\ GeV$
$100\ \ < M_1 < 2500\ GeV$
$100\ \ <\ \mu\ < 2500\ GeV$
$1350 < m_A(pole) < 6000\ GeV$
etc. etc.

- *In the parameter scan, all of the relevant experimental and theoretical constraints are imposed.*

- *377180 parameter points are generated and 3208 points survive the constraints.*

# 5. $h^0 \rightarrow c\,\bar{c}\,,\,b\,\bar{b}\,,\,b\,\bar{s}$ in the MSSM

- *We compute the decay widths $\Gamma\,(h^0 \rightarrow c\,\bar{c}),\, \Gamma\,(h^0 \rightarrow b\,\bar{b}),$*
  *and $\Gamma\,(h^0 \rightarrow b\,\bar{s})$ at full 1-loop level in the DRbar renormalization*
  *scheme in the MSSM with general QFV.*

- *Main 1-loop correction to $h^0 \rightarrow c\,\bar{c}$ :*

  *gluino - su loops [ su = ($\tilde{t}$ - $\tilde{c}$ mixture)]*
  *can be enhanced by large trilinear couplings $T_{U23}\,,\,T_{U32}\,,\,T_{U33}$*

- *Main 1-loop corrections to $h^0 \rightarrow b\,\bar{b}$ & $b\,\bar{s}$ :*

  *gluino – sd loops [ sd = ($\tilde{b}$ - $\tilde{s}$ mixture)]*
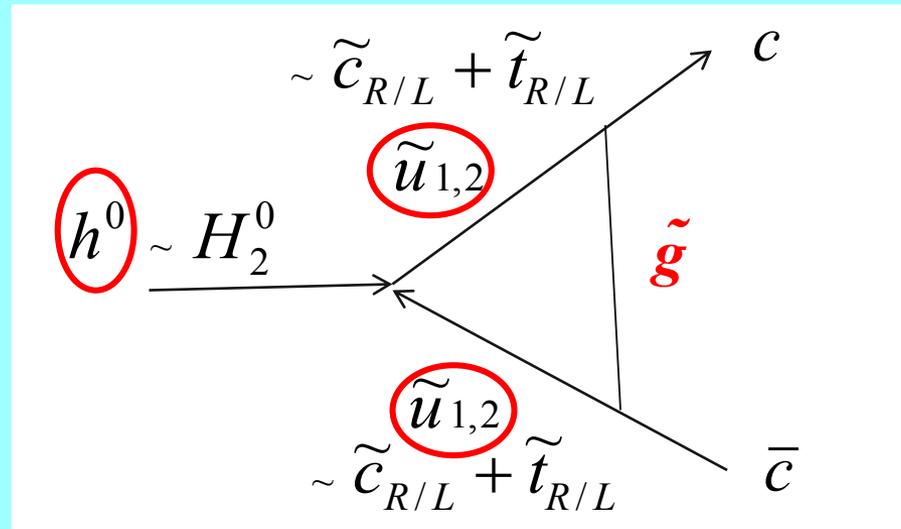  *can be enhanced by large trilinear couplings $T_{D23}\,,\,T_{D32}\,,\,T_{D33}$*

  *chargino - su loops [ su = ($\tilde{t}$ - $\tilde{c}$ mixture)]*
  *can be enhanced by large trilinear couplings $T_{U23}\,,\,T_{U32}\,,\,T_{U33}$*

*In large $\widetilde{c}_{R/L} - \widetilde{t}_{R/L}$ & $\widetilde{t}_L - \widetilde{t}_R$ mixing scenario;*

$h^0 \sim H_2^0$

$\widetilde{u}_{1,2} \sim \widetilde{c}_{R/L} + \widetilde{t}_{R/L}$



*In our scenario, "trilinear couplings" ($\widetilde{c}_R - \widetilde{t}_L - H_2^0$, $\widetilde{c}_L - \widetilde{t}_R - H_2^0$, $\widetilde{t}_L - \widetilde{t}_R - H_2^0$ couplings) = ($T_{U23}\, T_{U32}$, $T_{U33}$) are large!*

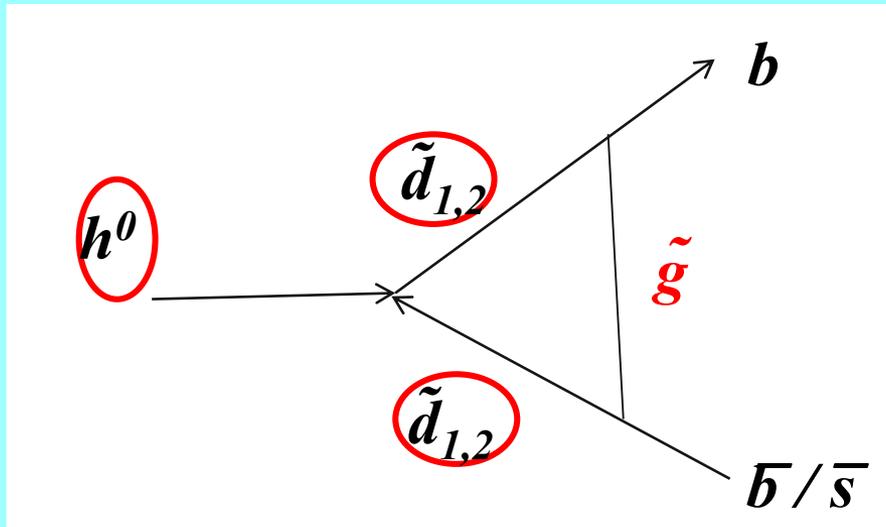$\widetilde{u}_{1,2} - \widetilde{u}_{1,2} - h^0$ *couplings are large!*

*Gluino loop contributions can be large!*

*Deviation of $\Gamma(h^0 \to c\,\overline{c})$ from SM width can be large!*

In large $\tilde{s}_{R/L} - \tilde{b}_{R/L}$ & $\tilde{b}_L - \tilde{b}_R$ mixing scenario;

$$h^0 \sim -s\alpha\, H_1^0 + c\alpha\, H_2^0$$

$$\tilde{d}_{1,2} \sim \tilde{s}_{R/L} + \tilde{b}_{R/L}$$



In our scenario, "trilinear couplings"$(T_{D23}\, T_{D32},\, T_{D33}) =$
$(\tilde{s}_R - \tilde{b}_L - H_1^0,\, \tilde{s}_L - \tilde{b}_R - H_1^0,\, \tilde{b}_L - \tilde{b}_R - H_1^0$ couplings$)$ are large!

$\tilde{d}_{1,2} - \tilde{d}_{1,2} - h^0$ couplings are large!

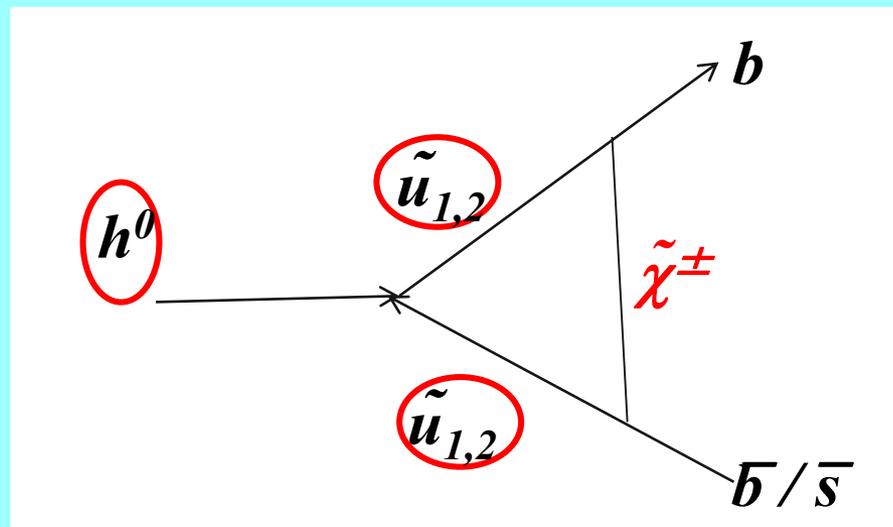Gluino loop contributions can be large!

Deviation of $\Gamma(h^0 \rightarrow b\, \bar{b}/\bar{s})$ from SM width can be large!

*In large $\tilde{c}_{R/L} - \tilde{t}_{R/L}$ & $\tilde{t}_L - \tilde{t}_R$ mixing scenario;*

$h^0 \sim H_2^0$

$\tilde{u}_{1,2} \sim \tilde{c}_{R/L} + \tilde{t}_{R/L}$

$\tilde{\chi}^{\pm} \sim \tilde{W}^{\pm} + \tilde{H}^{\pm}$



*In our scenario, "trilinear couplings" ($\tilde{c}_R - \tilde{t}_L - H_2^0$, $\tilde{c}_L - \tilde{t}_R - H_2^0$, $\tilde{t}_L - \tilde{t}_R - H_2^0$ couplings) = ($T_{U23}\, T_{U32}$, $T_{U33}$) are large!*

$\widetilde{u}_{1,2} - \widetilde{u}_{1,2} - h^0$ *couplings are large!*

*Chargino loop contributions can be large!*

*Deviation of $\Gamma(h^0 \to b\,\bar{b}/\bar{s})$ from SM width can be large!*

# 5.1 *Deviation of the width from the SM prediction*

- *Deviation of the width from the SM prediction:*

$$DEV(h^0 \to X\bar{X}) = \Gamma(h^0 \to X\bar{X})_{MSSM} / \Gamma(h^0 \to X\bar{X})_{SM} - 1$$
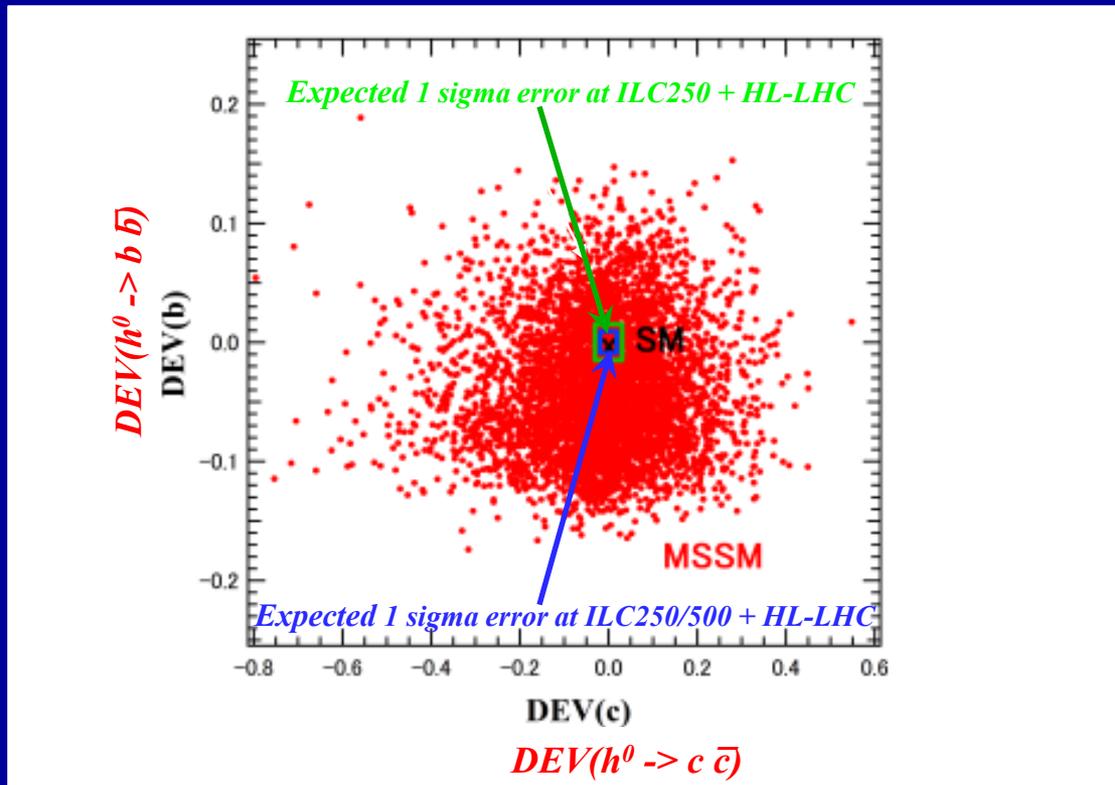
$$X = c, b$$

- *Coupling modifier:*

$$\kappa_X = g(h^0 X X) / g(h^0 X X)_{SM}$$

- *DEV - $\kappa_X$ relation:*

$$DEV(h^0 \to X X) = \kappa_X^2 - 1$$

# Scatter plot in $DEV(h^0 \to c\bar{c})$ - $DEV(h^0 \to b\bar{b})$ plane



*Expected 1 sigma error at ILC250 + HL-LHC*

*Expected 1 sigma error at ILC250/500 + HL-LHC*

$DEV(h^0 \to b\bar{b})$

DEV(b)

SM

MSSM

DEV(c)

$DEV(h^0 \to c\bar{c})$

- $DEV(h^0 \to c\bar{c})$ and $DEV(h^0 \to b\bar{b})$ can be very large simultaneously!:
  $DEV(h^0 \to c\bar{c})$ can be as large as ~ **±60%**.
  $DEV(h^0 \to b\bar{b})$ can be as large as ~ **±20%**.

- ILC can observe such large deviations from SM at high significance *(arXiv:2206.08326)!:*
  $\Delta DEV(h^0 \to c\bar{c})$ = (3.6%, 2.4%, 1.8%) at (ILC250, ILC250/500, ILC250/500/1000)
  $\Delta DEV(h^0 \to b\bar{b})$ = (1.7%, 1.1%, 0.9%) at (ILC250, ILC250/500, ILC250/500/1000)

# *Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders*

Table 6: The expected *absolute* $1\sigma$ error of the deviations DEV(X) and DEV(X/Y) (denoted by $\Delta DEV(X)$ and $\Delta DEV(X/Y)$) at future lepton colliders: ILC-I = ILC250 + Giga-Z, ILC-II = ILC250+500 + Giga-Z, ILC-III = ILC250+500+1000 + Giga-Z; CLIC-I = CLIC380, CLIC-II = CLIC380+1500, CLIC-III = CLIC380+1500+3000; FCC-ee I = FCC-ee240 + Z/WW, FCC-ee II = FCC-ee240+365 + Z/WW; CEPC-I = CEPC240 + Z/WW, CEPC-II = CEPC240+360 + Z/WW; MuC-I = MuC3TeV, MuC-II = MuC10TeV, MuC-III = MuC10TeV+125GeV. As for ILC the results without Giga-Z run are almost identical to those with Giga-Z one. The Z/WW denote Z-pole and WW threshold runs. All results except for MuC-I and MuC-II are those from the free-$\Gamma_H$ fit and the results for MuC-I and MuC-II are those from the constrained-$\Gamma_H$ fit, where $\Gamma_H$ is the total width of the Higgs boson $h^0$. The details of run scenarios of the lepton colliders are explained in Ref. [14]. The HL-LHC and LEP/SLD measurements are combined with all lepton collider run scenarios.
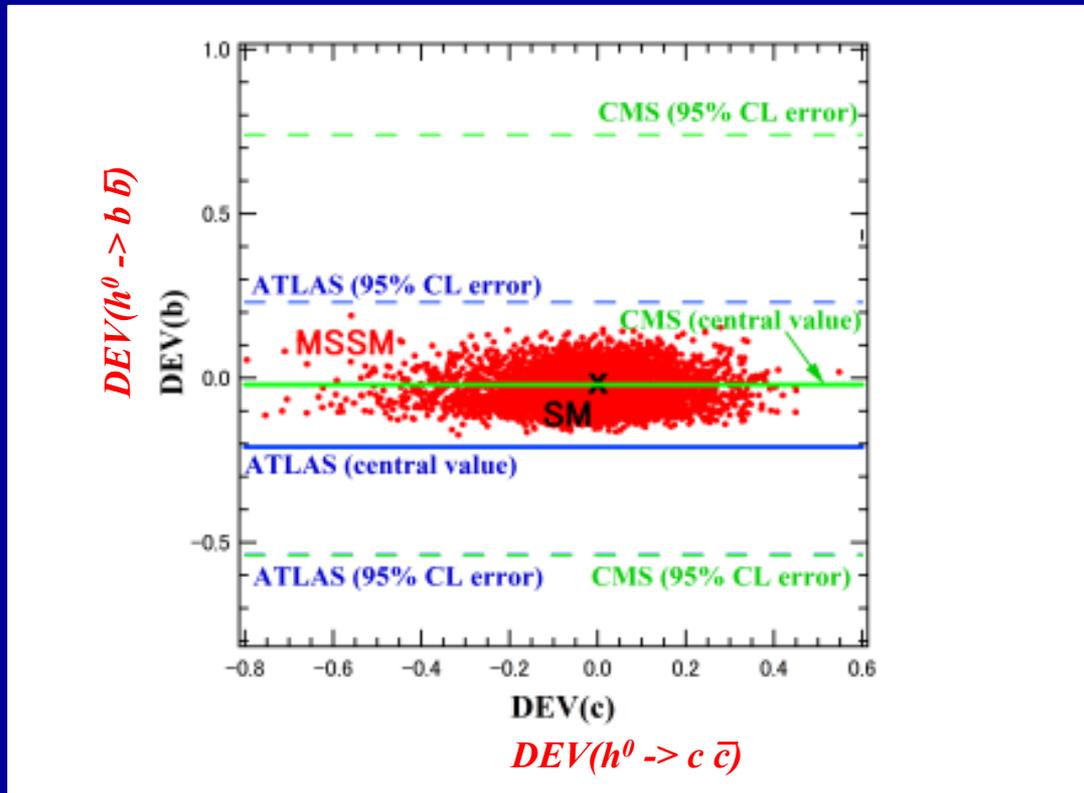
*Jorge de Blas et al., Snowmass2021 Report: arXiv:2206.08326 [hep-ph];*
*Private communication with Jorge de Blas.*

# *Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders*

|  | ILC-I | ILC-II | ILC-III | CLIC-I | CLIC-II | CLIC-III |
|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.7% | 1.1% | 0.9% | 2.2% | 1.2% | 1.1% |
| $\Delta DEV(c)$ | 3.6% | 2.4% | 1.8% | 8.6% | 3.8% | 3.0% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 2.0% | 2.6% | 2.4% | 2.2% |
| $\Delta DEV(g)$ | 1.8% | 1.4% | 1.1% | 2.2% | 1.6% | 1.4% |
| $\Delta DEV(b/c)$ | 3.12% | 2.07% | 1.3% | 8.16% | 3.5% | 2.54% |
| $\Delta DEV(\gamma/g)$ | 3.29% | 2.82% | 2.3% | 3.41% | 3.05% | 2.61% |
| $\Delta DEV(g/b)$ | 1.51% | 1.14% | 0.85% | 1.62% | 1.3% | 1.1% |
| $\Delta DEV(g/c)$ | 3.4% | 2.34% | 1.52% | 8.3% | 3.72% | 2.75% |

|  | FCC-ee I | FCC-ee II | CEPC-I | CEPC-II | MuC-I | MuC-II | MuC-III |
|---|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.3% | 1.2% | 0.86% | 0.84% | 1.8% | 0.92% | 1.1% |
| $\Delta DEV(c)$ | 2.8% | 2.6% | 2.4% | 2.2% | 12.4% | 3.8% | 3.6% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 1.8% | 1.8% | 2.4% | 1.4% | 1.5% |
| $\Delta DEV(g)$ | 1.5% | 1.4% | 0.9% | 0.88% | 1.7% | 0.92% | 1.0% |
| $\Delta DEV(b/c)$ | 2.22% | 2.06% | 2.0% | 1.94% | 11.92% | 3.53% | 3.37% |
| $\Delta DEV(\gamma/g)$ | 3.03% | 2.86% | 2.07% | 2.03% | 3.18% | 1.55% | 1.55% |
| $\Delta DEV(g/b)$ | 1.29% | 1.16% | 0.73% | 0.7% | 1.69% | 0.73% | 0.72% |
| $\Delta DEV(g/c)$ | 2.54% | 2.33% | 2.13% | 2.06% | 12.0% | 3.59% | 3.43% |

$$DEV(h^0 \to X\,X) = \kappa_X^2 - 1$$

- *Recent LHC data:*

  $DEV(h^0 \to b\, \bar{b}) = -0.21 +0.44/-0.33$ (95% CL) (ATLAS)    (arXiv:2207.00092)
  $DEV(h^0 \to b\, \bar{b}) = -0.02 +0.76/-0.52$ (95% CL) (CMS)      (arXiv:2207.00043)

- *Both SM and MSSM are consistent with the recent ATLAS/CMS data!*
  *The errors of the recent ATLAS/CMS data are too large!*

# 5.2 $\underline{BR(h^0 \rightarrow b\,\overline{s}\,/\,s\,\overline{b})}$

$BR(h^0 \rightarrow b\,\overline{s}\,/\,s\,\overline{b}) \cong 0\ (\,<10^{-7}\,)$ **(SM)**

$BR(h^0 \rightarrow b\,\overline{s}\,/\,s\,\overline{b}) \cong 0$ **(MSSM with MFV)**

$BR(h^0 \rightarrow b\,\overline{s}\,/\,s\,\overline{b})$ *can be as large as ~ 0.15% (MSSM with QFV)!*

*(See also Gomez-Heinemeyer-Rehman, PR D93 (2016) 095021 [arXiv:1511.04342]. )*

*ILC(250+500+1000) sensitivity could be ~ 0.1% (at 4 $\sigma$ significance)!*

*Private communication with Junping Tian;*
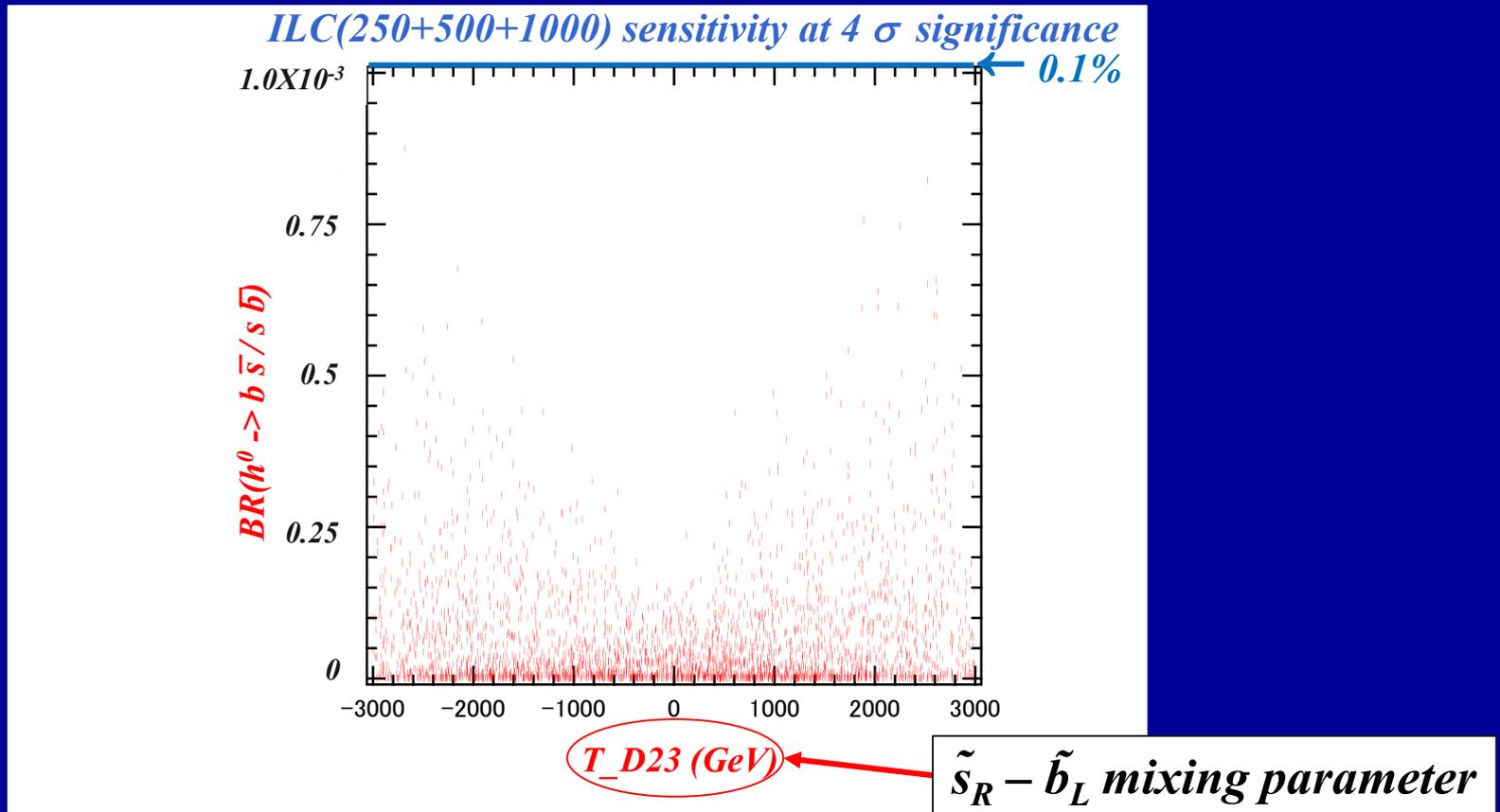*See also Barducci et al., JHEP 12 (2017) 105 [arXiv:1710.06657].*

*Expected FCC-ee bound is $BR(h^0 \rightarrow b\,\overline{s}\,/\,s\,\overline{b}) < 4.5 \cdot 10^{-4}$ (at 95% CL)*

*See M. Selvaggi, talk at FCC Physics Performance Meeting, 18 Mar 2024.*

*Expected CEPC bound is $BR(h^0 \rightarrow b\,\overline{s}\,/\,s\,\overline{b}) < 2.2 \cdot 10^{-4}$ (at 95% CL)*

*See H. Liang et al., arXiv:2310.03440.*

# Scatter plot in $T_{D23}$ - $BR(h^0 \rightarrow b\,\bar{s} / s\,\bar{b})$ plane



*ILC(250+500+1000) sensitivity at 4 σ significance*

$1.0 \times 10^{-3}$ — *0.1%*

$BR(h^0 \rightarrow b\,\bar{s} / s\,\bar{b})$

$T\_D23$ (GeV)

$\tilde{s}_R - \tilde{b}_L$ *mixing parameter*

- *There is a strong correlation between $T_{D23}$ - $BR(h^0 \rightarrow b\,\bar{s} / s\,\bar{b})$!*

- *$BR(h^0 \rightarrow b\,\bar{s} / s\,\bar{b})$ can be as large as **0.1%** for large $T_{D23}$ !*

- *ILC(250 + 500 + 1000) sensitivity could be ~ 0.1% at 4 sigma significance!*
    *Private communication with Junping Tian;*
    *See also Barducci et al., JHEP 12 (2017) 105 [arXiv:1710.06657].*

- *LHC & HL-LHC sensitivity should not be so good due to huge QCD BG!*

## 5.3 *Deviation of <u>width ratio</u> from the SM prediction*

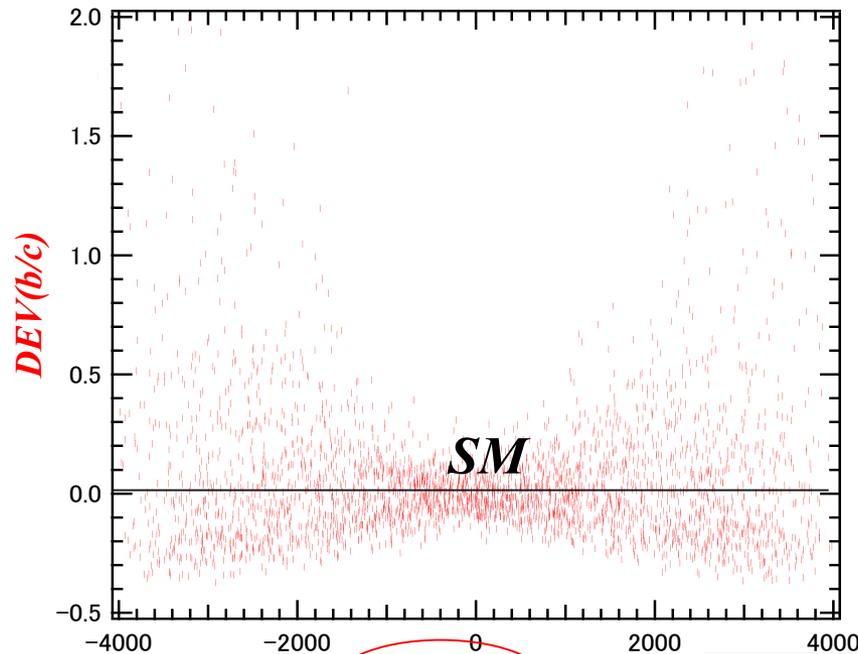*- The deviation of the width ratio from the SM prediction:*

$$DEV(b/c) = [\Gamma(b) / \Gamma(c)]_{MSSM} \; / \; [\Gamma(b) / \Gamma(c)]_{SM} \; - \; 1$$

$$\Gamma(X) = \Gamma(h^0 -> X \, \overline{X})$$

- *We find;*
  *the experimental measurement errors as*
  *well as the MSSM prediction errors*
  *tend to cancel out significantly in the width ratios.*

# Scatter plot in $T_{U32} - DEV(b/c)$ plane



- *There is a strong correlation between $T_{U32} - DEV(b/c)$!*
- *DEV(b/c) can exceed ~ +100% for large $T_{U32}$ !*
- *Expected absolute 1 σ error of DEV(b/c) at ILC:*
  *ΔDEV(b/c) = (3.1%, 2.1%, 1.3%) at (ILC250, ILC250+500, ILC250+500+1000).*

  [*Jorge de Blas et al., Snowmass2021 Report: arXiv:2206.08326;
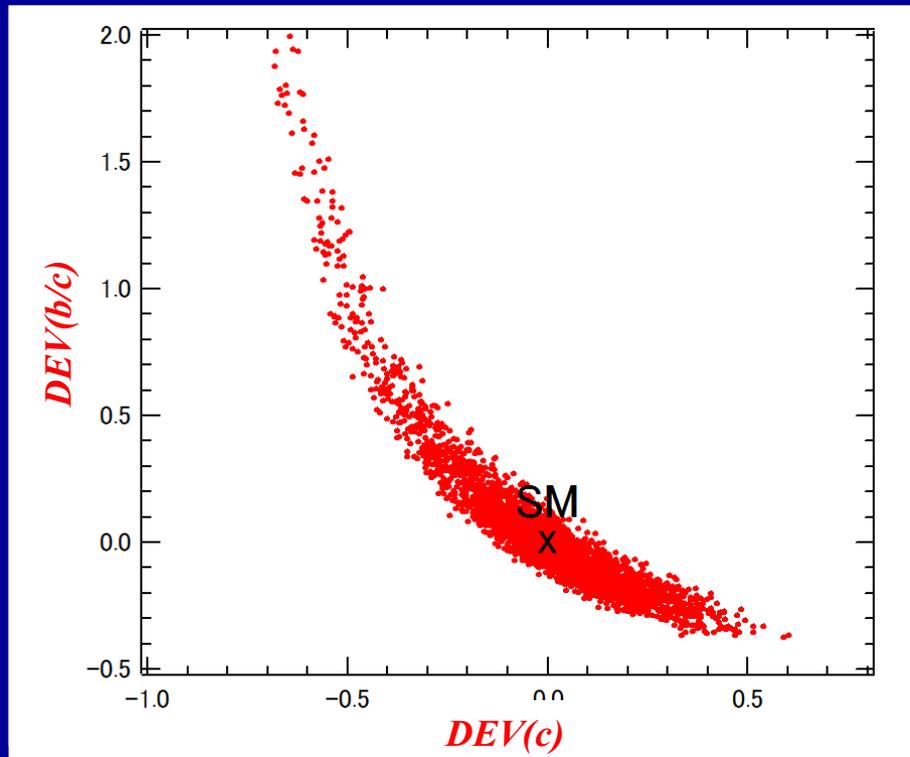  Private communication with Jorge de Blas.*]

- **ILC can observe such large deviation from SM at very high significance!**

# Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders

|  | ILC-I | ILC-II | ILC-III | CLIC-I | CLIC-II | CLIC-III |
|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.7% | 1.1% | 0.9% | 2.2% | 1.2% | 1.1% |
| $\Delta DEV(c)$ | 3.6% | 2.4% | 1.8% | 8.6% | 3.8% | 3.0% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 2.0% | 2.6% | 2.4% | 2.2% |
| $\Delta DEV(g)$ | 1.8% | 1.4% | 1.1% | 2.2% | 1.6% | 1.4% |
| $\Delta DEV(b/c)$ | 3.12% | 2.07% | 1.3% | 8.16% | 3.5% | 2.54% |
| $\Delta DEV(\gamma/g)$ | 3.29% | 2.82% | 2.3% | 3.41% | 3.05% | 2.61% |
| $\Delta DEV(g/b)$ | 1.51% | 1.14% | 0.85% | 1.62% | 1.3% | 1.1% |
| $\Delta DEV(g/c)$ | 3.4% | 2.34% | 1.52% | 8.3% | 3.72% | 2.75% |

|  | FCC-ee I | FCC-ee II | CEPC-I | CEPC-II | MuC-I | MuC-II | MuC-III |
|---|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.3% | 1.2% | 0.86% | 0.84% | 1.8% | 0.92% | 1.1% |
| $\Delta DEV(c)$ | 2.8% | 2.6% | 2.4% | 2.2% | 12.4% | 3.8% | 3.6% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 1.8% | 1.8% | 2.4% | 1.4% | 1.5% |
| $\Delta DEV(g)$ | 1.5% | 1.4% | 0.9% | 0.88% | 1.7% | 0.92% | 1.0% |
| $\Delta DEV(b/c)$ | 2.22% | 2.06% | 2.0% | 1.94% | 11.92% | 3.53% | 3.37% |
| $\Delta DEV(\gamma/g)$ | 3.03% | 2.86% | 2.07% | 2.03% | 3.18% | 1.55% | 1.55% |
| $\Delta DEV(g/b)$ | 1.29% | 1.16% | 0.73% | 0.7% | 1.69% | 0.73% | 0.72% |
| $\Delta DEV(g/c)$ | 2.54% | 2.33% | 2.13% | 2.06% | 12.0% | 3.59% | 3.43% |

# Scatter plot in DEV(b/c) − DEV(c) plane



- *There is a very strong correlation between DEV(b/c) − DEV(c)!*
- *DEV(b/c) and DEV(c) can be very large simultaneously!*
- *Expected absolute 1 σ error of DEV(c) and DEV(b/c) at ILC:*
   *Δ DEV(c) = (3.6%, 2.4%, 1.8%) at (ILC250, ILC250+500, ILC250+500+1000)*
   *ΔDEV(b/c) = (3.1%, 2.1%, 1.3%) at (ILC250, ILC250+500, ILC250+500+1000).*

   *[Jorge de Blas et al., Snowmass2021 Report: arXiv:2206.08326;*
   *Private communication with Jorge de Blas.]*

- **ILC can observe such large deviation from SM at very high significance!**

# Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders

|  | ILC-I | ILC-II | ILC-III | CLIC-I | CLIC-II | CLIC-III |
|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.7% | 1.1% | 0.9% | 2.2% | 1.2% | 1.1% |
| $\Delta DEV(c)$ | 3.6% | 2.4% | 1.8% | 8.6% | 3.8% | 3.0% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 2.0% | 2.6% | 2.4% | 2.2% |
| $\Delta DEV(g)$ | 1.8% | 1.4% | 1.1% | 2.2% | 1.6% | 1.4% |
| $\Delta DEV(b/c)$ | 3.12% | 2.07% | 1.3% | 8.16% | 3.5% | 2.54% |
| $\Delta DEV(\gamma/g)$ | 3.29% | 2.82% | 2.3% | 3.41% | 3.05% | 2.61% |
| $\Delta DEV(g/b)$ | 1.51% | 1.14% | 0.85% | 1.62% | 1.3% | 1.1% |
| $\Delta DEV(g/c)$ | 3.4% | 2.34% | 1.52% | 8.3% | 3.72% | 2.75% |

|  | FCC-ee I | FCC-ee II | CEPC-I | CEPC-II | MuC-I | MuC-II | MuC-III |
|---|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.3% | 1.2% | 0.86% | 0.84% | 1.8% | 0.92% | 1.1% |
| $\Delta DEV(c)$ | 2.8% | 2.6% | 2.4% | 2.2% | 12.4% | 3.8% | 3.6% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 1.8% | 1.8% | 2.4% | 1.4% | 1.5% |
| $\Delta DEV(g)$ | 1.5% | 1.4% | 0.9% | 0.88% | 1.7% | 0.92% | 1.0% |
| $\Delta DEV(b/c)$ | 2.22% | 2.06% | 2.0% | 1.94% | 11.92% | 3.53% | 3.37% |
| $\Delta DEV(\gamma/g)$ | 3.03% | 2.86% | 2.07% | 2.03% | 3.18% | 1.55% | 1.55% |
| $\Delta DEV(g/b)$ | 1.29% | 1.16% | 0.73% | 0.7% | 1.69% | 0.73% | 0.72% |
| $\Delta DEV(g/c)$ | 2.54% | 2.33% | 2.13% | 2.06% | 12.0% | 3.59% | 3.43% |

# 6. $h^0 \to \gamma\,\gamma, g\,g$ in the MSSM

- *As the $h^0$ decays to photon photon and gluon gluon are loop-induced decays, these decays are very sensitive to New Physics!*

- *We compute the widths $\Gamma(h^0 \to \gamma\,\gamma)$ and $\Gamma(h^0 \to g\,g)$ at NLO QCD level in the MSSM with general QFV .*

- *Main 1-loop contributions to $h^0 \to \gamma\,\gamma$ :*

   *$[W^+/\,top\text{-}quark\,/\,su]$ - loops $[\,su = (\tilde{t} - \tilde{c}\ mixture)]$*

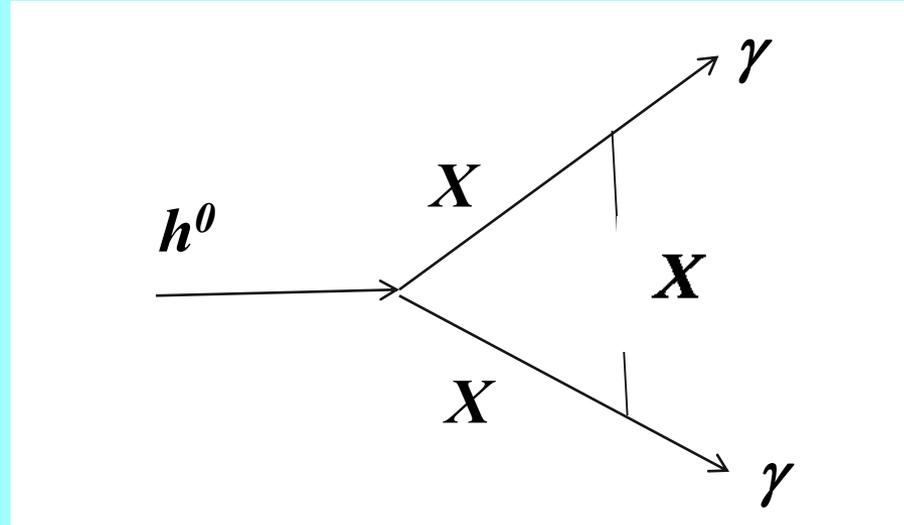   *The su-loops can be enhanced by large trilinear couplings $T_{U23}$ , $T_{U32}$ , $T_{U33}$ , resulting in sizable deviation of $\Gamma(h^0 \to \gamma\,\gamma)$ from the SM width!*

- *Main 1-loop contributions to $h^0 \to g\,g$:*

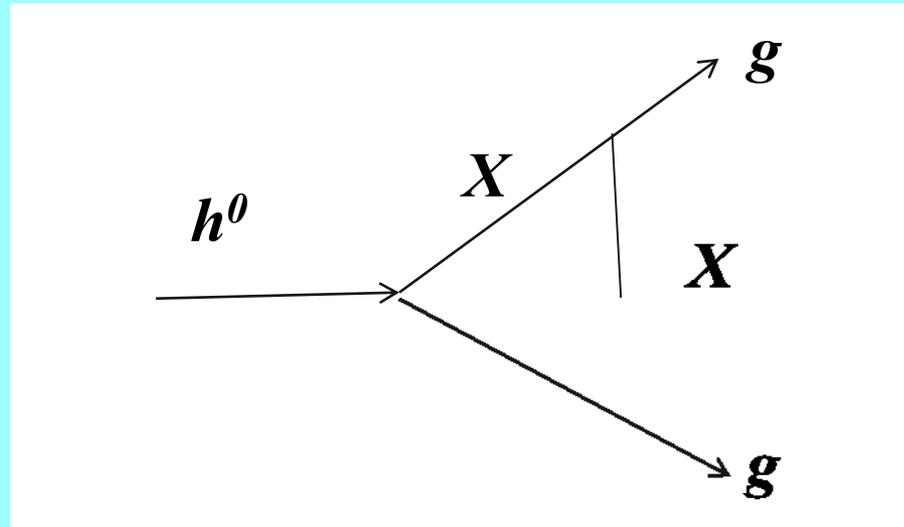   *$[top\text{-}quark\,/\,su]$ - loops $[\,su = (\tilde{t} - \tilde{c}\ mixture)]$*

   *The su-loops can be enhanced by large trilinear couplings $T_{U23}$ , $T_{U32}$ , $T_{U33}$ , resulting in sizable deviation of $\Gamma(h^0 \to g\,g)$ from the SM width!*
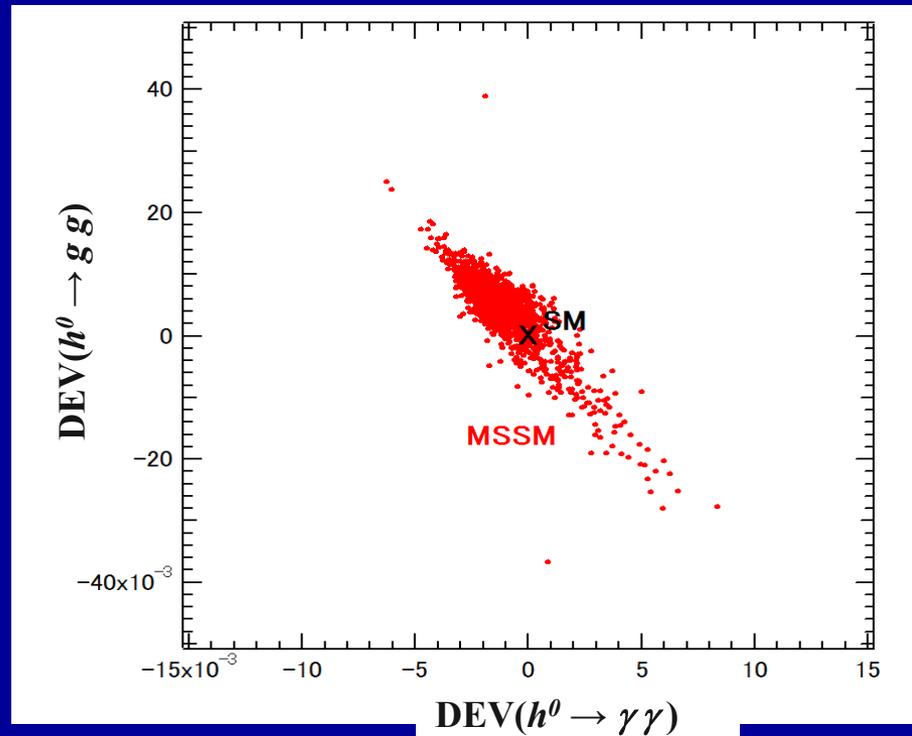
$$X = W^+ / t / \tilde{u}_{1,2}$$

$$X = t / \tilde{u}_{1,2}$$

# Scatter plot in DEV($h^0 \to \gamma\gamma$) - DEV($h^0 \to g\,g$) plane



- *There is a strong correlation between* DEV($h^0 \to \gamma\gamma$) *and* DEV($h^0 \to g\,g$)!
- DEV($h^0 \to \gamma\gamma$) *and* DEV($h^0 \to g\,g$) *can be sizable simultaneously!:*
  DEV($h^0 \to \gamma\gamma$) *can be as large as ~ ±1%*
  DEV($h^0 \to g\,g$) *can be as large as ~ ±4%.*
- *Expected absolute 1 σ errors at (ILC250, ILC250+500, ILC250+500+1000):*
  *Δ DEV($h^0 \to \gamma\gamma$) = (2.4%, 2.2%, 2.0%)*
  *Δ DEV($h^0 \to g\,g$) = (1.8%, 1.4%, 1.1%)*
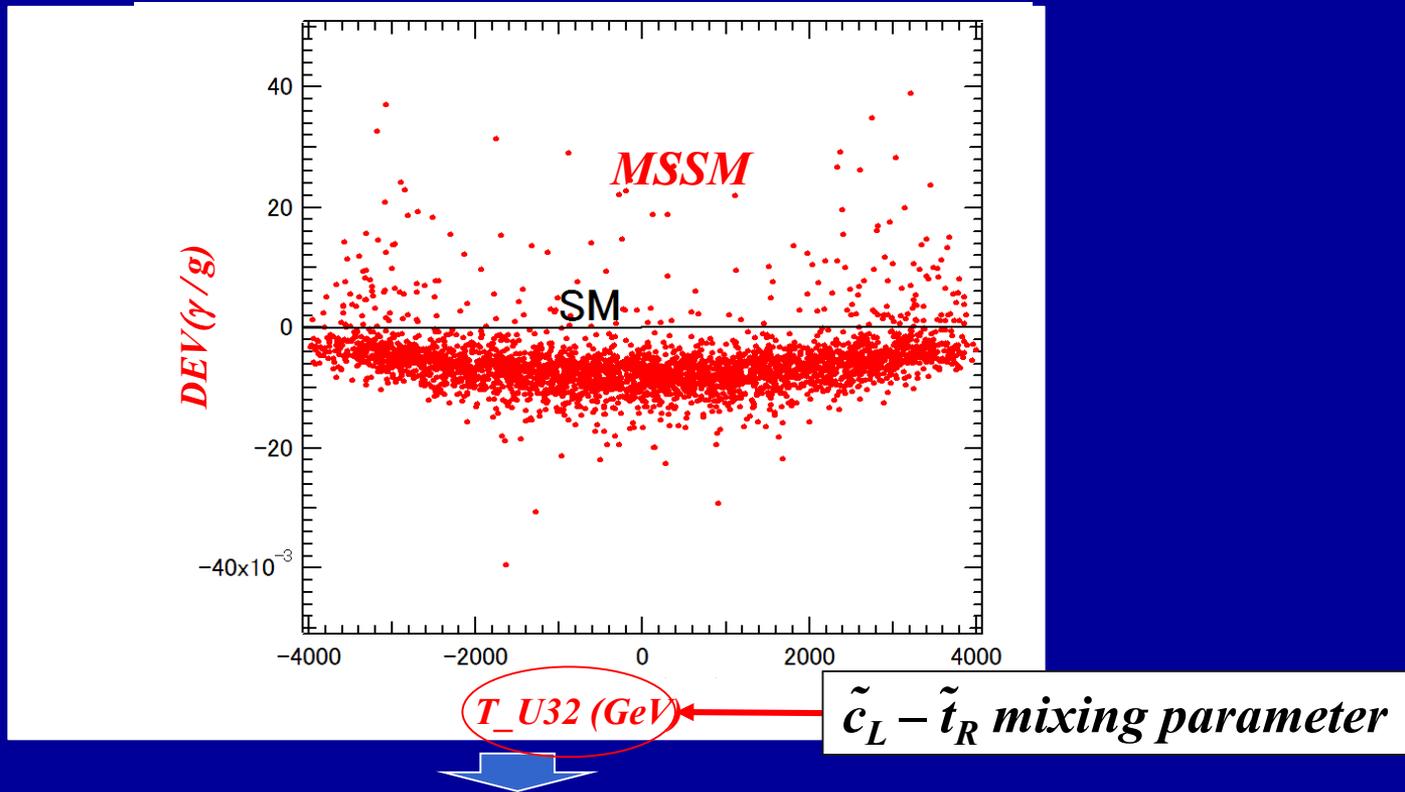    [*Jorge de Blas et al., Snowmass2021 Report: arXiv:2206.08326;*
    *Private communication with Jorge de Blas.*] *(See Backup slides)*
- **ILC can observe such sizable deviation DEV($h^0 \to g\,g$) at high significance!**

# Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders

|  | ILC-I | ILC-II | ILC-III | CLIC-I | CLIC-II | CLIC-III |
|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.7% | 1.1% | 0.9% | 2.2% | 1.2% | 1.1% |
| $\Delta DEV(c)$ | 3.6% | 2.4% | 1.8% | 8.6% | 3.8% | 3.0% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 2.0% | 2.6% | 2.4% | 2.2% |
| $\Delta DEV(g)$ | 1.8% | 1.4% | 1.1% | 2.2% | 1.6% | 1.4% |
| $\Delta DEV(b/c)$ | 3.12% | 2.07% | 1.3% | 8.16% | 3.5% | 2.54% |
| $\Delta DEV(\gamma/g)$ | 3.29% | 2.82% | 2.3% | 3.41% | 3.05% | 2.61% |
| $\Delta DEV(g/b)$ | 1.51% | 1.14% | 0.85% | 1.62% | 1.3% | 1.1% |
| $\Delta DEV(g/c)$ | 3.4% | 2.34% | 1.52% | 8.3% | 3.72% | 2.75% |

|  | FCC-ee I | FCC-ee II | CEPC-I | CEPC-II | MuC-I | MuC-II | MuC-III |
|---|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.3% | 1.2% | 0.86% | 0.84% | 1.8% | 0.92% | 1.1% |
| $\Delta DEV(c)$ | 2.8% | 2.6% | 2.4% | 2.2% | 12.4% | 3.8% | 3.6% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 1.8% | 1.8% | 2.4% | 1.4% | 1.5% |
| $\Delta DEV(g)$ | 1.5% | 1.4% | 0.9% | 0.88% | 1.7% | 0.92% | 1.0% |
| $\Delta DEV(b/c)$ | 2.22% | 2.06% | 2.0% | 1.94% | 11.92% | 3.53% | 3.37% |
| $\Delta DEV(\gamma/g)$ | 3.03% | 2.86% | 2.07% | 2.03% | 3.18% | 1.55% | 1.55% |
| $\Delta DEV(g/b)$ | 1.29% | 1.16% | 0.73% | 0.7% | 1.69% | 0.73% | 0.72% |
| $\Delta DEV(g/c)$ | 2.54% | 2.33% | 2.13% | 2.06% | 12.0% | 3.59% | 3.43% |

## Scatter plot in $T_{U32} - DEV(\gamma/g)$ plane

MSSM

SM

$DEV(\gamma/g)$

$T\_U32$ (GeV)

$\tilde{c}_L - \tilde{t}_R$ mixing parameter

- **There is a strong correlation between $T_{U32} - DEV(\gamma/g)$ !**

- **$DEV(\gamma/g)$ can be as large as $\sim +4\%$ for large $T_{U32}$! (from scatter plot analysis)**
  **$\sim +8\%$ for large $T_{U32}$! (from contour plot analysis)**

- **Expected absolute 1 sigma error of $DEV(\gamma/g)$ at ILC:**

  **$\Delta DEV(\gamma/g) = (3.3\%, 2.8\%, 2.3\%)$ at (ILC250, ILC250+500, ILC250+500+1000).**

  [*Jorge de Blas et al., Snowmass2021 Report: arXiv:2206.08326;*
  *Private communication with Jorge de Blas.*] *(See Backup slides)*

- **ILC can observe such sizable deviation from SM at high significance.**

# Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders

|  | ILC-I | ILC-II | ILC-III | CLIC-I | CLIC-II | CLIC-III |
|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.7% | 1.1% | 0.9% | 2.2% | 1.2% | 1.1% |
| $\Delta DEV(c)$ | 3.6% | 2.4% | 1.8% | 8.6% | 3.8% | 3.0% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 2.0% | 2.6% | 2.4% | 2.2% |
| $\Delta DEV(g)$ | 1.8% | 1.4% | 1.1% | 2.2% | 1.6% | 1.4% |
| $\Delta DEV(b/c)$ | 3.12% | 2.07% | 1.3% | 8.16% | 3.5% | 2.54% |
| $\Delta DEV(\gamma/g)$ | 3.29% | 2.82% | 2.3% | 3.41% | 3.05% | 2.61% |
| $\Delta DEV(g/b)$ | 1.51% | 1.14% | 0.85% | 1.62% | 1.3% | 1.1% |
| $\Delta DEV(g/c)$ | 3.4% | 2.34% | 1.52% | 8.3% | 3.72% | 2.75% |

|  | FCC-ee I | FCC-ee II | CEPC-I | CEPC-II | MuC-I | MuC-II | MuC-III |
|---|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.3% | 1.2% | 0.86% | 0.84% | 1.8% | 0.92% | 1.1% |
| $\Delta DEV(c)$ | 2.8% | 2.6% | 2.4% | 2.2% | 12.4% | 3.8% | 3.6% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 1.8% | 1.8% | 2.4% | 1.4% | 1.5% |
| $\Delta DEV(g)$ | 1.5% | 1.4% | 0.9% | 0.88% | 1.7% | 0.92% | 1.0% |
| $\Delta DEV(b/c)$ | 2.22% | 2.06% | 2.0% | 1.94% | 11.92% | 3.53% | 3.37% |
| $\Delta DEV(\gamma/g)$ | 3.03% | 2.86% | 2.07% | 2.03% | 3.18% | 1.55% | 1.55% |
| $\Delta DEV(g/b)$ | 1.29% | 1.16% | 0.73% | 0.7% | 1.69% | 0.73% | 0.72% |
| $\Delta DEV(g/c)$ | 2.54% | 2.33% | 2.13% | 2.06% | 12.0% | 3.59% | 3.43% |

# 7. *Benchmark scenario*

- *In our analysis we also take into account the expected sparticle mass limits from the future HL-LHC experiment .*


- *From the allowed MSSM parameter points in the scan, we have selected a benchmark point P1 shown in Table 2 .*


- *This benchmark scenario P1  satisfies also all the expected sparticle mass limits [including ($m_{A/H^+}$ , tan$\beta$) limits] from future HL-LHC experiment .*
  *[ see CERN Yellow Rep., arXiv:1812.07831;*
  *Snowmass Rep. of EF, arXiv:2209.13128; ... ]*


- *The resulting physical masses of the particles are shown in Table 3.*

# *Benchmark scenario P1*

Table 2:    The MSSM parameters for the reference point P1 (in units of GeV or GeV$^2$ expect for $\tan\beta$). All parameters are defined at scale Q = 1 TeV, except $m_A(pole)$. The parameters that are not shown here are taken to be zero.

| $\tan\beta$ | $M_1$ | $M_2$ | $M_3$ | $mu$ | $m_A(pole)$ |
|---|---|---|---|---|---|
| 33 | 1660 | 765 | 4615 | 870 | 5325 |

| $M^2_{Q22}$ | $M^2_{Q33}$ | $M^2_{Q23}$ | $M^2_{U22}$ | $M^2_{U33}$ | $M^2_{U23}$ |
|---|---|---|---|---|---|
| $3975^2$ | $3160^2$ | $920^2$ | $3465^2$ | $1300^2$ | $795^2$ |

| $M^2_{D22}$ | $M^2_{D33}$ | $M^2_{D23}$ | $T_{U23}$ | $T_{U32}$ | $T_{U33}$ |
|---|---|---|---|---|---|
| $2620^2$ | $2425^2$ | $-1625^2$ | $-2040$ | $-1880$ | $-4945$ |

| $T_{D23}$ | $T_{D32}$ | $T_{D33}$ | $T_{E33}$ |
|---|---|---|---|
| $-2360$ | $1670$ | $-2395$ | $-300$ |

| $M^2_{Q11}$ | $M^2_{U11}$ | $M^2_{D11}$ | $M^2_{L11}$ | $M^2_{L22}$ | $M^2_{L33}$ | $M^2_{E11}$ | $M^2_{E22}$ | $M^2_{E33}$ |
|---|---|---|---|---|---|---|---|---|
| $4500^2$ | $4500^2$ | $4500^2$ | $1500^2$ | $1500^2$ | $1500^2$ | $1500^2$ | $1500^2$ | $1500^2$ |

# *Physical masses for Benchmark scenario P1*

Table 3: Physical masses in GeV of the particles for the scenario of Table 2.

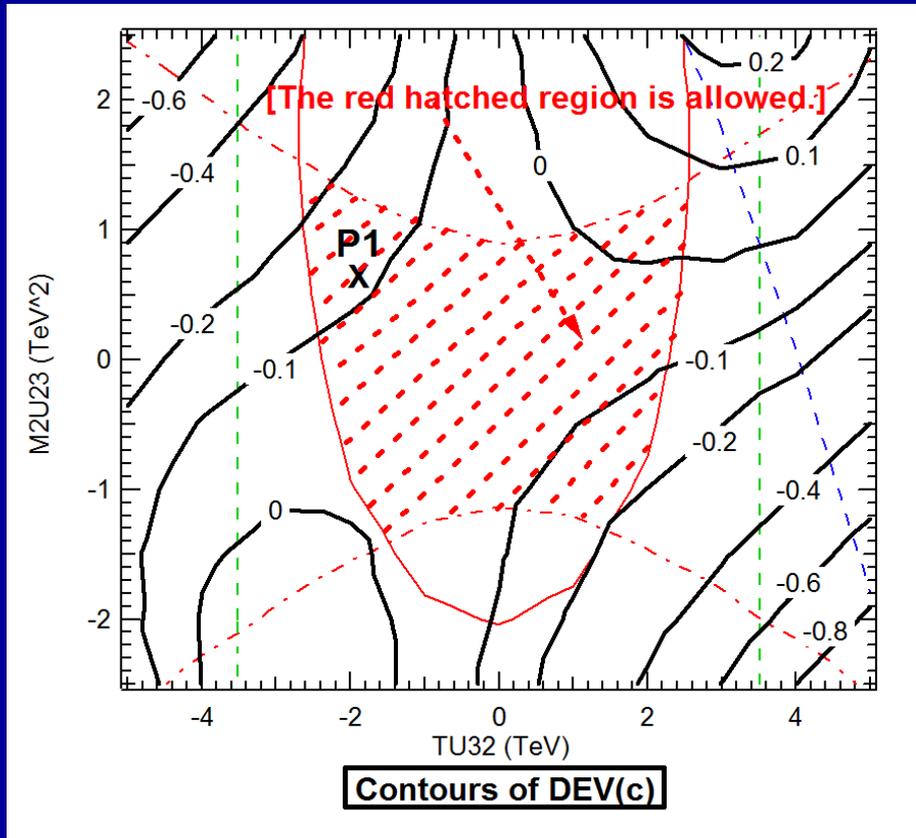| $m_{\tilde{\chi}_1^0}$ | $m_{\tilde{\chi}_2^0}$ | $m_{\tilde{\chi}_3^0}$ | $m_{\tilde{\chi}_4^0}$ | $m_{\tilde{\chi}_1^+}$ | $m_{\tilde{\chi}_2^+}$ |
|---|---|---|---|---|---|
| 781 | 882 | 911 | 1669 | 782 | 914 |

| $m_{h^0}$ | $m_{H^0}$ | $m_{A^0}$ | $m_{H^+}$ |
|---|---|---|---|
| 124 | 5325 | 5325 | 5359 |

| $m_{\tilde{g}}$ | $m_{\tilde{u}_1}$ | $m_{\tilde{u}_2}$ | $m_{\tilde{u}_3}$ | $m_{\tilde{u}_4}$ | $m_{\tilde{u}_5}$ | $m_{\tilde{u}_6}$ |
|---|---|---|---|---|---|---|
| 4424 | 868 | 3011 | 3331 | 3877 | 4402 | 4402 |

| $m_{\tilde{d}_1}$ | $m_{\tilde{d}_2}$ | $m_{\tilde{d}_3}$ | $m_{\tilde{d}_4}$ | $m_{\tilde{d}_5}$ | $m_{\tilde{d}_6}$ |
|---|---|---|---|---|---|
| 1705 | 2833 | 3010 | 3877 | 4397 | 4403 |

| $m_{\tilde{\nu}_1}$ | $m_{\tilde{\nu}_2}$ | $m_{\tilde{\nu}_3}$ | $m_{\tilde{l}_1}$ | $m_{\tilde{l}_2}$ | $m_{\tilde{l}_3}$ | $m_{\tilde{l}_4}$ | $m_{\tilde{l}_5}$ | $m_{\tilde{l}_6}$ |
|---|---|---|---|---|---|---|---|---|
| 1509 | 1509 | 1528 | 1489 | 1489 | 1509 | 1512 | 1512 | 1545 |

Contours of DEV(c)

*We find that DEV(c) can be very large (about -30% to 10%) in a sizable region allowed by all the constraints including the expected sparticle mass limits from the future HL-LHC experiment!*

# Contours of DEV($h^0$ -> b $\bar{b}$) in $T_{U32}$ - $M^2_{U23}$ plane around P1



Contours of DEV(b)
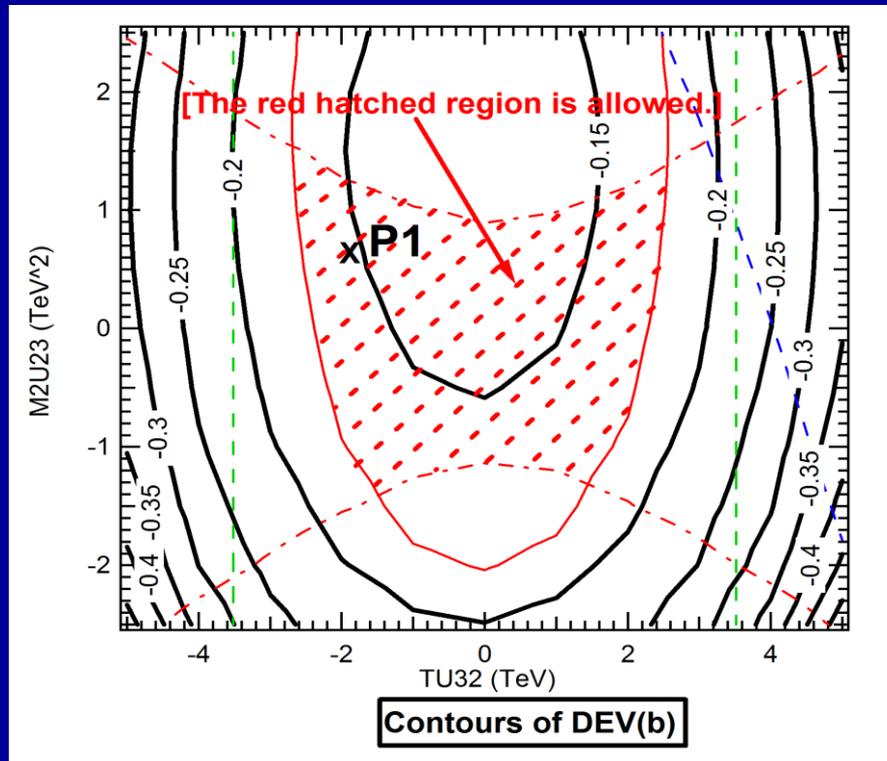
- *We find that DEV(b) can be very large (about -10% to -18%) in a sizable region allowed by all the constraints including the expected sparticle mass limits from the future HL-LHC experiment!*

- *For DEV(g), DEV($\gamma$), DEV($\gamma$/g) and DEV(b/c) we have obtained similar results to those for DEV(c) and DEV(b).*

# *Contours of $B(h^0 \to b\bar{s}/s\bar{b})$ in $T_{U32} - \tan\beta$ plane around P1*
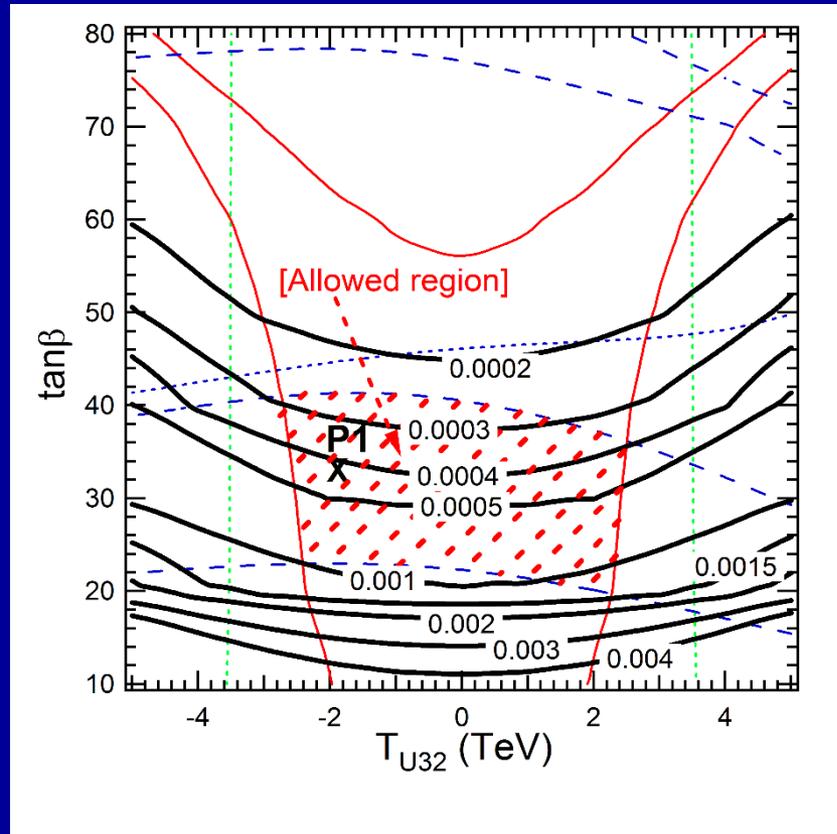


- *$B(h^0 \to b\bar{s}/s\bar{b})$ can be as large as ~ 0.15%* *in a sizable region allowed by all the constraints including the expected sparticle mass limits from the future HL-LHC!*

- *ILC(250 + 500 + 1000) sensitivity could be ~ 0.1% at $4\sigma$ significance!*
  *Private communication with Junping Tian;*
  *See also Barducci et al., JHEP 12 (2017) 105 [arXiv:1710.06657].*

# 8. *Conclusion*

- *We have studied the decays*

  $h^0$ *(125GeV)* $\rightarrow$ $c\,\bar{c}$ , $b\,\bar{b}$ , $b\,\bar{s}$ , $\gamma\,\gamma$, $g\,g$ *in the MSSM with general QFV.*

- *For the first time, we have performed a systematic MSSM parameter scan
  respecting all of the relevant theoretical and experimental constraints.*

- *In strong contrast to the usual studies in the MSSM with minimal flavor violation (MFV),
  we have found that the deviations of these MSSM decay widths and width ratios
  from the SM values can be quite sizable in the MSSM with general QFV.*

- *All of these large deviations in the h (125) decays are due to
  large $\tilde{c}$ - $\tilde{t}$ mixing & large $\tilde{c}\,/\,\tilde{t}$ involved trilinear couplings $T_{U23}$, $T_{U32}$, $T_{U33}$ and
  large $\tilde{s}$ - $\tilde{b}$ mixing & large $\tilde{s}\,/\,\tilde{b}$ involved trilinear couplings $T_{D23}$, $T_{D32}$, $T_{D33}$ .*

- *Future lepton colliders such as ILC, CLIC, CEPC, FCC-ee and MuC can
  observe such sizable deviations from the SM at high signal significance
  even after the failure of SUSY particle discovery at the HL-LHC.*

- *In case the deviation pattern shown here is really observed at the lepton colliders,
  then it would strongly suggest the discovery of QFV SUSY (the MSSM with general QFV).*

- *Our analysis suggests the following:*

*PETRA/TRISTAN e- e+ collider discovered virtual $Z^0$ effect for the first time.*

*Later, CERN p p̄ collider discovered the $Z^0$ boson.*

*Similarly, lepton colliders, such as ILC, could discover virtual Sparticle effects for the first time in $h^0(125)$ decays!*

*Later, FCC-hh p p collider could discover the Sparticles!*

# END

# *Thank you!*

# *Backup Slides*

# Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders

Table 6: The expected *absolute* $1\sigma$ error of the deviations DEV(X) and DEV(X/Y) (denoted by $\Delta DEV(X)$ and $\Delta DEV(X/Y)$) at future lepton colliders: ILC-I = ILC250 + Giga-Z, ILC-II = ILC250+500 + Giga-Z, ILC-III = ILC250+500+1000 + Giga-Z; CLIC-I = CLIC380, CLIC-II = CLIC380+1500, CLIC-III = CLIC380+1500+3000; FCC-ee I = FCC-ee240 + Z/WW, FCC-ee II = FCC-ee240+365 + Z/WW; CEPC-I = CEPC240 + Z/WW, CEPC-II = CEPC240+360 + Z/WW; MuC-I = MuC3TeV, MuC-II = MuC10TeV, MuC-III = MuC10TeV+125GeV. As for ILC the results without Giga-Z run are almost identical to those with Giga-Z one. The Z/WW denote Z-pole and WW threshold runs. All results except for MuC-I and MuC-II are those from the free-$\Gamma_H$ fit and the results for MuC-I and MuC-II are those from the constrained-$\Gamma_H$ fit, where $\Gamma_H$ is the total width of the Higgs boson $h^0$. The details of run scenarios of the lepton colliders are explained in Ref. [14]. The HL-LHC and LEP/SLD measurements are combined with all lepton collider run scenarios.

*Jorge de Blas et al., Snowmass2021 Report: arXiv:2206.08326 [hep-ph];*
*Private communication with Jorge de Blas.*

# Expected absolute 1 sigma errors of the deviations DEVs at future lepton colliders

|  | ILC-I | ILC-II | ILC-III | CLIC-I | CLIC-II | CLIC-III |
|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.7% | 1.1% | 0.9% | 2.2% | 1.2% | 1.1% |
| $\Delta DEV(c)$ | 3.6% | 2.4% | 1.8% | 8.6% | 3.8% | 3.0% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 2.0% | 2.6% | 2.4% | 2.2% |
| $\Delta DEV(g)$ | 1.8% | 1.4% | 1.1% | 2.2% | 1.6% | 1.4% |
| $\Delta DEV(b/c)$ | 3.12% | 2.07% | 1.3% | 8.16% | 3.5% | 2.54% |
| $\Delta DEV(\gamma/g)$ | 3.29% | 2.82% | 2.3% | 3.41% | 3.05% | 2.61% |
| $\Delta DEV(g/b)$ | 1.51% | 1.14% | 0.85% | 1.62% | 1.3% | 1.1% |
| $\Delta DEV(g/c)$ | 3.4% | 2.34% | 1.52% | 8.3% | 3.72% | 2.75% |

|  | FCC-ee I | FCC-ee II | CEPC-I | CEPC-II | MuC-I | MuC-II | MuC-III |
|---|---|---|---|---|---|---|---|
| $\Delta DEV(b)$ | 1.3% | 1.2% | 0.86% | 0.84% | 1.8% | 0.92% | 1.1% |
| $\Delta DEV(c)$ | 2.8% | 2.6% | 2.4% | 2.2% | 12.4% | 3.8% | 3.6% |
| $\Delta DEV(\gamma)$ | 2.4% | 2.2% | 1.8% | 1.8% | 2.4% | 1.4% | 1.5% |
| $\Delta DEV(g)$ | 1.5% | 1.4% | 0.9% | 0.88% | 1.7% | 0.92% | 1.0% |
| $\Delta DEV(b/c)$ | 2.22% | 2.06% | 2.0% | 1.94% | 11.92% | 3.53% | 3.37% |
| $\Delta DEV(\gamma/g)$ | 3.03% | 2.86% | 2.07% | 2.03% | 3.18% | 1.55% | 1.55% |
| $\Delta DEV(g/b)$ | 1.29% | 1.16% | 0.73% | 0.7% | 1.69% | 0.73% | 0.72% |
| $\Delta DEV(g/c)$ | 2.54% | 2.33% | 2.13% | 2.06% | 12.0% | 3.59% | 3.43% |

# *Higgs couplings at future colliders*

*Snowmass2021 Report: arXiv:2206.08326*
*(See Fig. 3 in page 37; Table 29 in page 40)*

**Higgs coupling precision at future colliders**



precision reach on effective couplings from SMEFT global fit
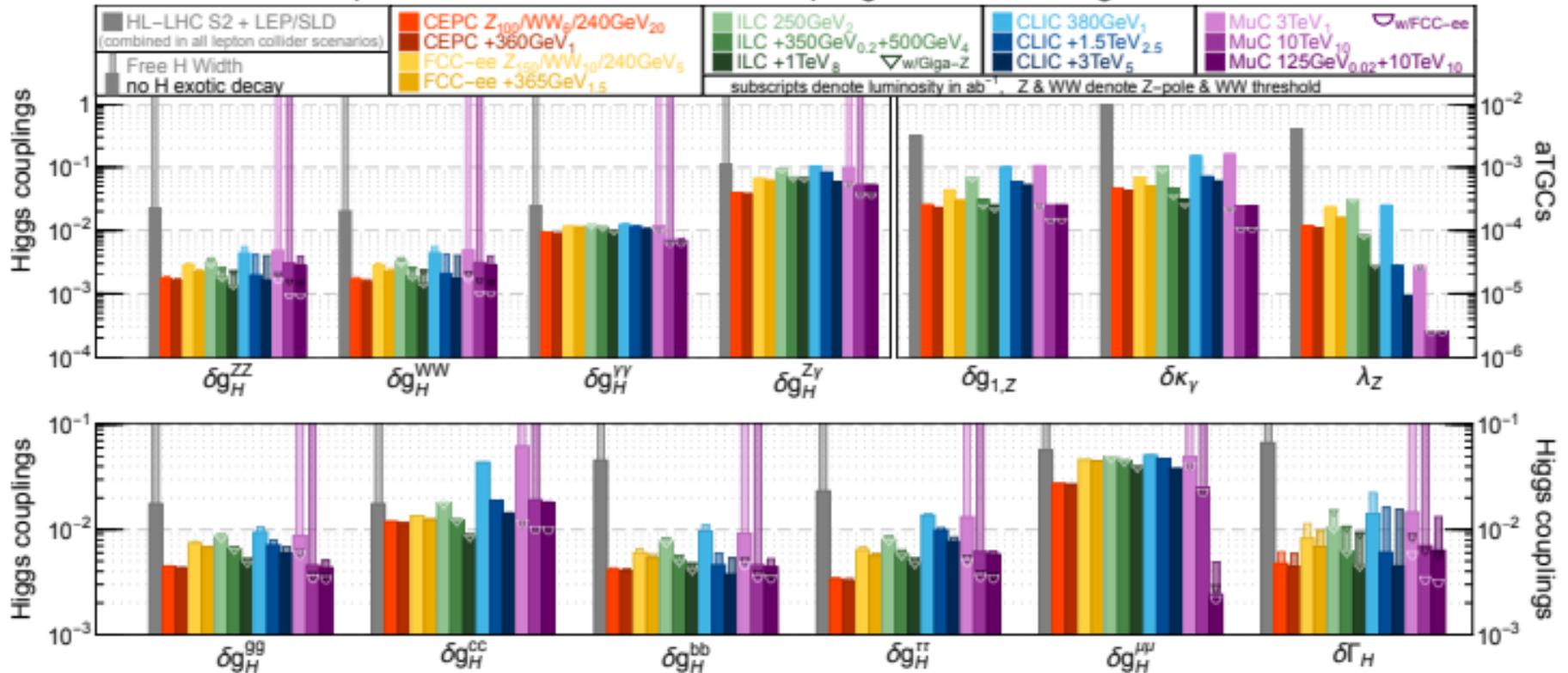
## *ESU2020 Report: arXiv:1905.03764*

**Higgs coupling precision in % at future colliders**

arXiv:1910.11775, arXiv:1905.03764
CERN-LPCC-2018-04

- Future colliders under consideration will improve with respect to the HL-LHC the understanding of the Higgs boson couplings - 1-5%

  - **Coupling to charm** quark could be measured with an accuracy of ~1% in future e+e- machines
  - **Couplings to μ/γ/Zγ** benefit the most from the large dataset available at HL-LHC
  - At low energy top-Higgs coupling is not accessible at future lepton colliders
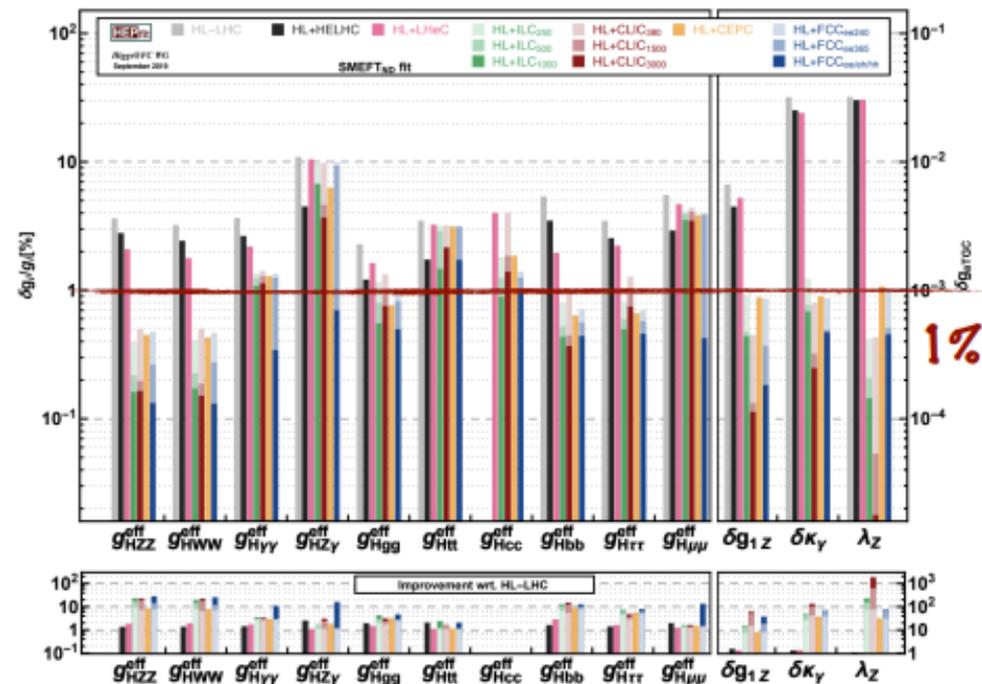
# *Higgs couplings at future colliders*

*ILC White Paper (Snowmass2021): arXiv:2203.07622*
*(See Fig. 12.1 in page 255; Table 12.2 in page 256)*
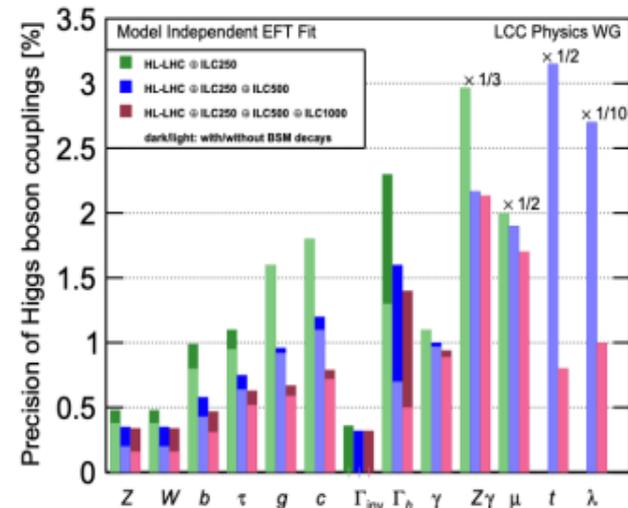
**Higgs coupling precision in % for ILC**

SLAC

| coupling | ILC250 full | ILC250 no BSM | ILC500 full | ILC500 no BSM | ILC1000 full | ILC1000 no BSM |
|---|---|---|---|---|---|---|
| $hZZ$ | 0.49 | 0.38 | 0.35 | 0.20 | 0.34 | 0.16 |
| $hWW$ | 0.48 | 0.38 | 0.35 | 0.20 | 0.34 | 0.16 |
| $hbb$ | 0.99 | 0.80 | 0.58 | 0.43 | 0.47 | 0.31 |
| $h\tau\tau$ | 1.1 | 0.95 | 0.75 | 0.63 | 0.63 | 0.52 |
| $hgg$ | 1.6 | 1.6 | 0.96 | 0.91 | 0.67 | 0.59 |
| $hcc$ | 1.8 | 1.7 | 1.2 | 1.1 | 0.79 | 0.72 |
| $h\gamma\gamma$ | 1.1 | 1.0 | 1.0 | 0.96 | 0.94 | 0.89 |
| $h\gamma Z$ | 8.9 | 8.9 | 6.5 | 6.5 | 6.4 | 6.4 |
| $h\mu\mu$ | 4.0 | 4.0 | 3.8 | 3.7 | 3.4 | 3.4 |
| $htt$ | — | — | 6.3 | 6.3 | 1.0 | 1.0 |
| $hhh$ | — | — | 20 | 20 | 10 | 10 |
| $\Gamma_{tot}$ | 2.3 | 1.3 | 1.6 | 0.70 | 1.4 | 0.50 |
| $\Gamma_{inv}$ | 0.36 | — | 0.32 | — | 0.32 | — |

*Note C³ would run at 550 GeV, a factor 2 improvement to the top-Yukawa coupling (\*)*
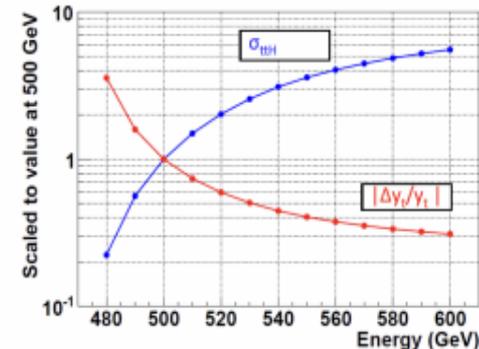
# *Higgs coupling precision at future colliders*

## *Physics Briefing Book: arXiv:2511.03883*
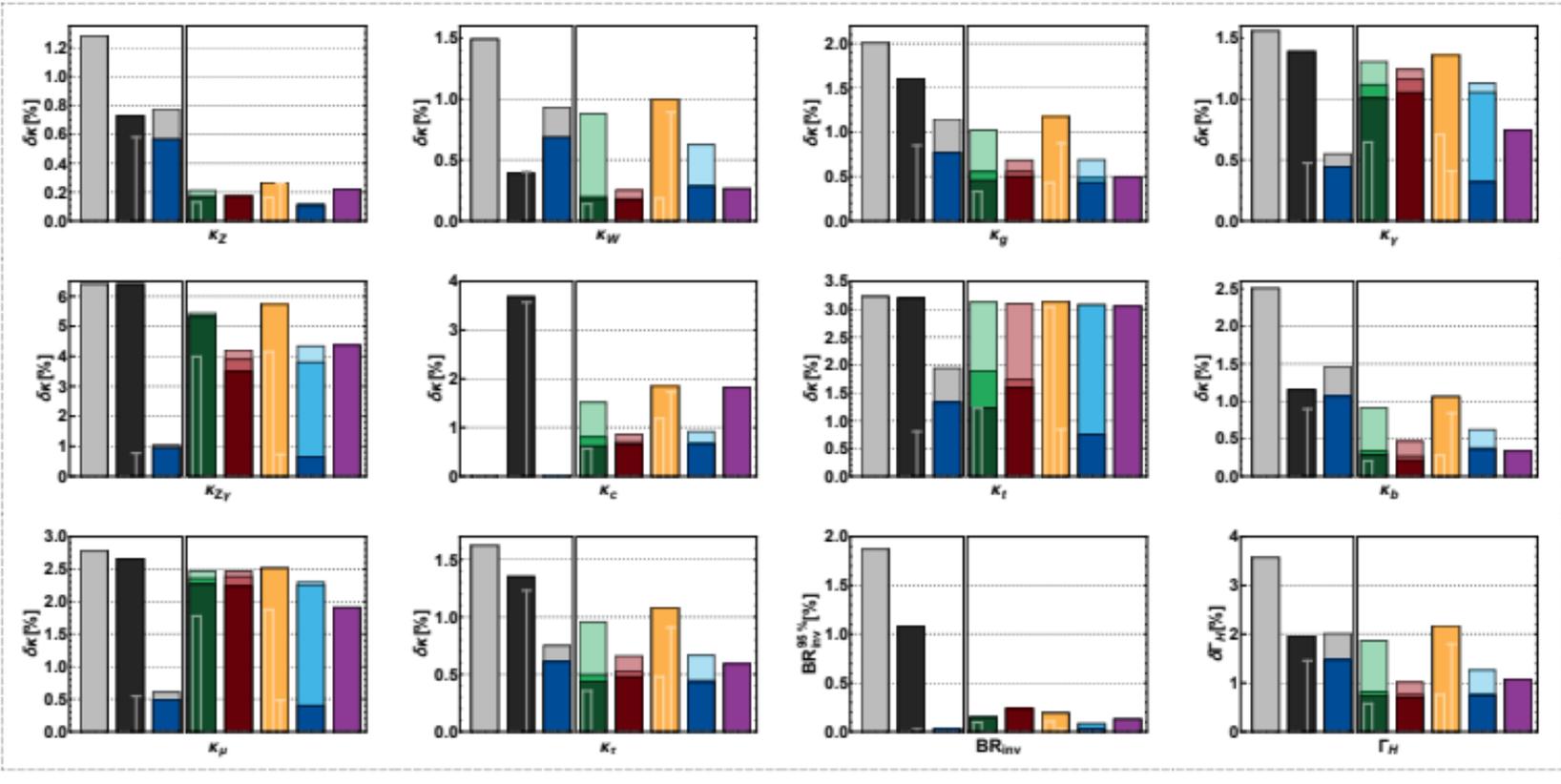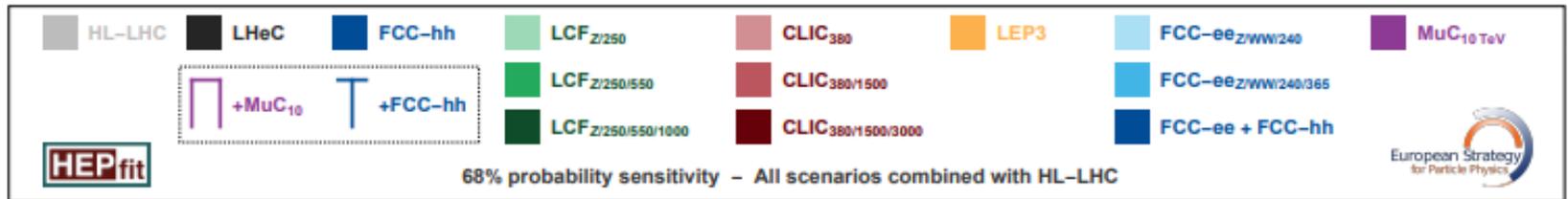
# DEV error - coupling error relation

$\Delta DEV(h \rightarrow X\,X) = 2\ \delta g(hXX)$

$\delta g(hXX) = $ *[Expected relative error of coupling g(hXX)]*

$\Delta DEV(h \rightarrow X\,X) = $ *[Expected absolute error of deviation*

*DEV(h → X X)]*

# 2. *MSSM with QFV*

## *The basic parameters of the MSSM with QFV :*

$$\{tan\beta, m_A, M_1, M_2, M_3, \mu, M^2_{Q,\alpha\beta}, M^2_{U,\alpha\beta}, M^2_{D,\alpha\beta}, T_{U\alpha\beta}, T_{D\alpha\beta}\}$$

(*at Q = 1 TeV scale* )     (*α,β = 1,2,3 = u, c, t  or  d, s, b*)

*tanβ :*      ratio of VEV of the two Higgs doublets $<H^0_2>/<H^0_1>$

$m_A :$      *CP odd Higgs boson mass (pole mass)*

$M_1, M_2, M_3 :$      *U(1), SU(2),SU(3)  gaugino masses*

$\mu :$          *higgsino mass parameter*

$M^2_{Q,\alpha\beta} :$  *left squark  soft mass matrix*

$M^2_{U\alpha\beta} :$   *right up-type squark  soft mass matrix*

$M^2_{D\alpha\beta} :$   *right down-type squark  soft mass matrix*

$T_{U\alpha\beta} :$      *trilinear coupling matrix of up-type squark and  Higgs boson*

$T_{D\alpha\beta} :$      *trilinear coupling matrix of down-type squark and  Higgs boson*

# 4. _Parameter scan for $h^0$ decay in the MSSM_

Table 1: Scanned ranges and fixed values of the MSSM parameters (in units of GeV or GeV$^2$, except for $\tan\beta$). The parameters that are not shown explicitly are taken to be zero. $M_{1,2,3}$ are the U(1), SU(2), SU(3) gaugino mass parameters.

| $\tan\beta$ | $M_1$ | $M_2$ | $M_3$ | $\mu$ | $m_A(pole)$ |
|---|---|---|---|---|---|
| $10 \div 80$ | $100 \div 2500$ | $100 \div 2500$ | $2500 \div 5000$ | $100 \div 2500$ | $1350 \div 6000$ |

| $M^2_{Q22}$ | $M^2_{Q33}$ | $|M^2_{Q23}|$ | $M^2_{U22}$ | $M^2_{U33}$ | $|M^2_{U23}|$ |
|---|---|---|---|---|---|
| $2500^2 \div 4000^2$ | $2500^2 \div 4000^2$ | $< 1000^2$ | $1000^2 \div 4000^2$ | $600^2 \div 3000^2$ | $< 2000^2$ |

| $M^2_{D22}$ | $M^2_{D33}$ | $|M^2_{D23}|$ | $|T_{U23}|$ | $|T_{U32}|$ | $|T_{U33}|$ |
|---|---|---|---|---|---|
| $2500^2 \div 4000^2$ | $1000^2 \div 3000^2$ | $< 2000^2$ | $< 4000$ | $< 4000$ | $< 5000$ |

| $|T_{D23}|$ | $|T_{D32}|$ | $|T_{D33}|$ | $|T_{E33}|$ |
|---|---|---|---|
| $< 3000$ | $< 3000$ | $< 4000$ | $< 500$ |

| $M^2_{Q11}$ | $M^2_{U11}$ | $M^2_{D11}$ | $M^2_{L11}$ | $M^2_{L22}$ | $M^2_{L33}$ | $M^2_{E11}$ | $M^2_{E22}$ | $M^2_{E33}$ |
|---|---|---|---|---|---|---|---|---|
| $4500^2$ | $4500^2$ | $4500^2$ | $1500^2$ | $1500^2$ | $1500^2$ | $1500^2$ | $1500^2$ | $1500^2$ |

# Constraints on the MSSM parameters from K & B meson and $h^0$ data:

Table 5: Constraints on the MSSM parameters from the $K$- and $B$-meson data relevant mainly for the mixing between the second and the third generations of squarks and from the data on the $h^0$ mass and couplings $\kappa_b$, $\kappa_g$, $\kappa_\gamma$. The fourth column shows constraints at 95% CL obtained by combining the experimental error quadratically with the theoretical uncertainty, except for $B(K_L^0 \to \pi^0 \nu\bar{\nu})$, $m_{h^0}$ and $\kappa_{b,g,\gamma}$.

| Observable | Exp. data | Theor. uncertainty | Constr. (95%CL) |
|---|---|---|---|
| $10^3 \times |\epsilon_K|$ | $2.228 \pm 0.011$ (68% CL) [20] | $\pm 0.28$ (68% CL) [28] | $2.228 \pm 0.549$ |
| $10^{15} \times \Delta M_K$ [GeV] | $3.484 \pm 0.006$ (68% CL) [20] | $\pm 1.2$ (68% CL) [28] | $3.484 \pm 2.352$ |
| $10^9 \times B(K_L^0 \to \pi^0 \nu\bar{\nu})$ | $< 3.0$ (90% CL) [20] | $\pm 0.002$ (68% CL) [20] | $< 3.0$ (90% CL) |
| $10^{10} \times B(K^+ \to \pi^+ \nu\bar{\nu})$ | $1.7 \pm 1.1$ (68% CL) [20] | $\pm 0.04$ (68% CL) [20] | $1.7^{+2.16}_{-1.70}$ |
| $\Delta M_{B_s}$ [ps$^{-1}$] | $17.757 \pm 0.021$ (68% CL) [20, 29] | $\pm 2.7$ (68% CL) [30] | $17.757 \pm 5.29$ |
| $10^4 \times B(b \to s\gamma)$ | $3.32 \pm 0.15$ (68% CL) [20, 29] | $\pm 0.23$ (68% CL) [17] | $3.32 \pm 0.54$ |
| $10^6 \times B(b \to s\, l^+ l^-)$ $(l = e$ or $\mu)$ | $1.60^{+0.48}_{-0.45}$ (68% CL) [31] | $\pm 0.11$ (68% CL) [32] | $1.60^{+0.97}_{-0.91}$ |
| $10^9 \times B(B_s \to \mu^+\mu^-)$ | $2.69^{+0.37}_{-0.35}$ (68%CL) [33] | $\pm 0.23$ (68% CL) [34] | $2.69^{+0.85}_{-0.82}$ |
| $10^4 \times B(B^+ \to \tau^+\nu)$ | $1.06 \pm 0.19$ (68%CL) [29] | $\pm 0.29$ (68% CL) [35] | $1.06 \pm 0.69$ |
| $m_{h^0}$ [GeV] | $125.09 \pm 0.24$ (68% CL) [36] | $\pm 3$ [37] | $125.09 \pm 3.48$ |
| $\kappa_b$ | $0.89^{+0.22}_{-0.21}$ (95% CL) [38] | | $0.89^{+0.22}_{-0.21}$ (ATLAS) |
| | $0.99^{+0.33}_{-0.31}$ (95% CL) [39] | | $0.99^{+0.33}_{-0.31}$ (CMS) |
| $\kappa_g$ | $0.95^{+0.14}_{-0.13}$ (95% CL) [38] | | $0.95^{+0.14}_{-0.13}$ (ATLAS) |
| | $0.92 \pm 0.16$ (95% CL) [39] | | $0.92 \pm 0.16$ (CMS) |
| $\kappa_\gamma$ | $1.01 \pm 0.12$ (95% CL) [38] | | $1.01 \pm 0.12$ (ATLAS) |
| | $1.10 \pm 0.16$ (95% CL) [39] | | $1.10 \pm 0.16$ (CMS) |

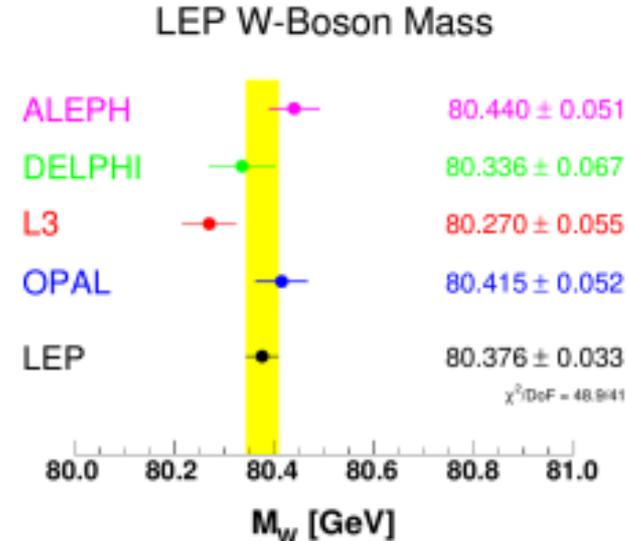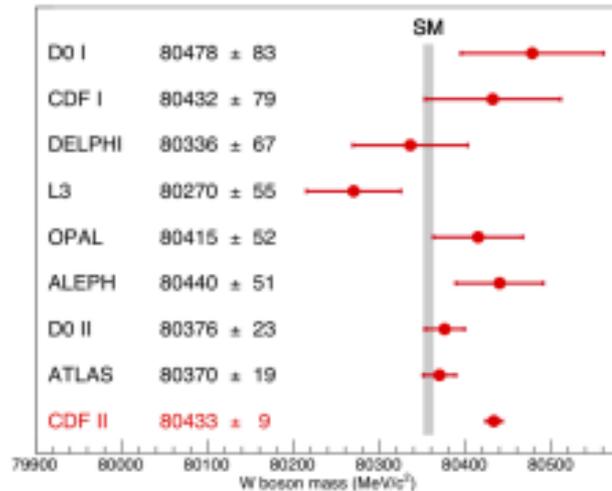# * _Constraints from W boson mass data_:

_The recent $m_W$ data from CDF II [1] disagrees significantly with the previous world average. (-> See backup slides.)_
_[1] CDF Collaboration, Science 376, 170–176 (2022)_

_This issue of the $m_W$ data is not yet settled._

_Hence, we do not take into accont this $m_W$ constraint on the MSSM parameters in our analysis._

## What to think of $m_W$ measurements?



- The LEP results are based on 42 separate measurements with a **healthy** $\chi^2$.
- The LEP-combined (33 MeV), LHCb (32 MeV), D0 Run II (23 MeV), ATLAS (19 MeV) and CDF Run II (9.4 MeV) measurements have a $\chi^2/DoF = 17.1/4$, with p-value of **0.2%** for compatibility (neglecting correlations).
- So reasonably strong evidence that the ensemble of experimental results are **inconsistent with each other** independent of any SM prediction.
- The standard PDG procedure is to add a scale factor "democratically" to all measurements to parametrize our ignorance.

# PDG scale factors

(What can happen with supposed high precision measurements)
The new world average $m_W$ uncertainty should be scaled up by about 2.1 leading to an uncertainty of 15 MeV in PDG-2022 compared with 12 MeV in PDG-2020.



The charged kaon mass has been in this scale-factored state for 30 years!
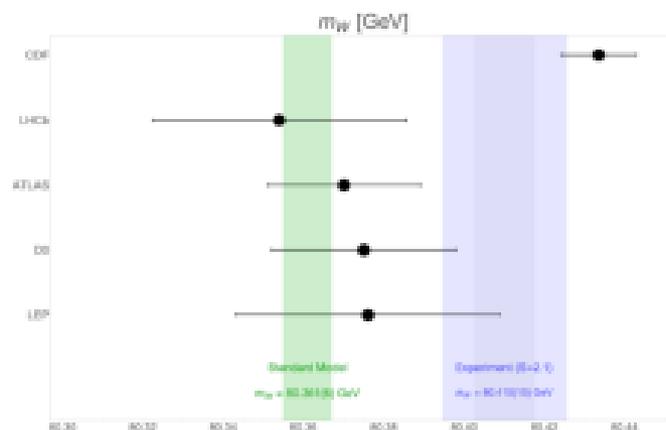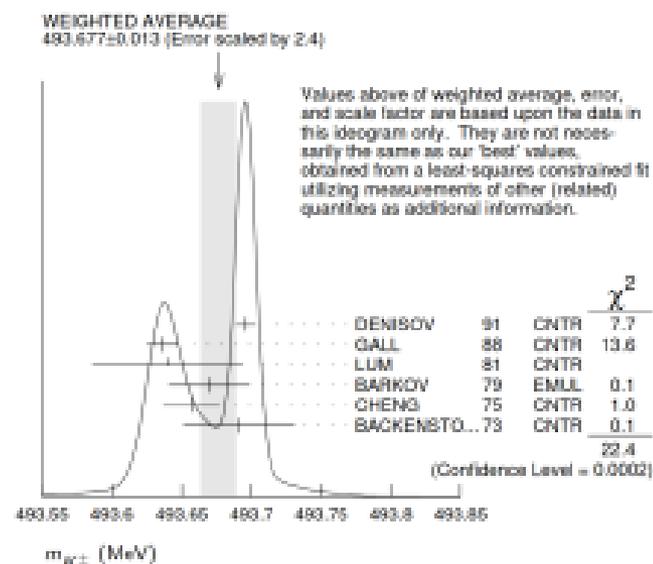
Plot from Resonaances blog (Adam Falkowski). Independently I had also done this and concluded that the scale-factored world-average is $+3.2\sigma$ off the SM value used by CDF

Perhaps one or more experiments has underestimated uncertainties. Also may be difficult to measure the same thing in $p\bar{p}$, $pp$, and $e^+e^-$ collisions.
Strong motivation to measure $m_W$ well in complementary ways in $e^+e^-$ collisions!

**Main SUSY one-loop contributions to $h^0 \to c\bar{c}$**
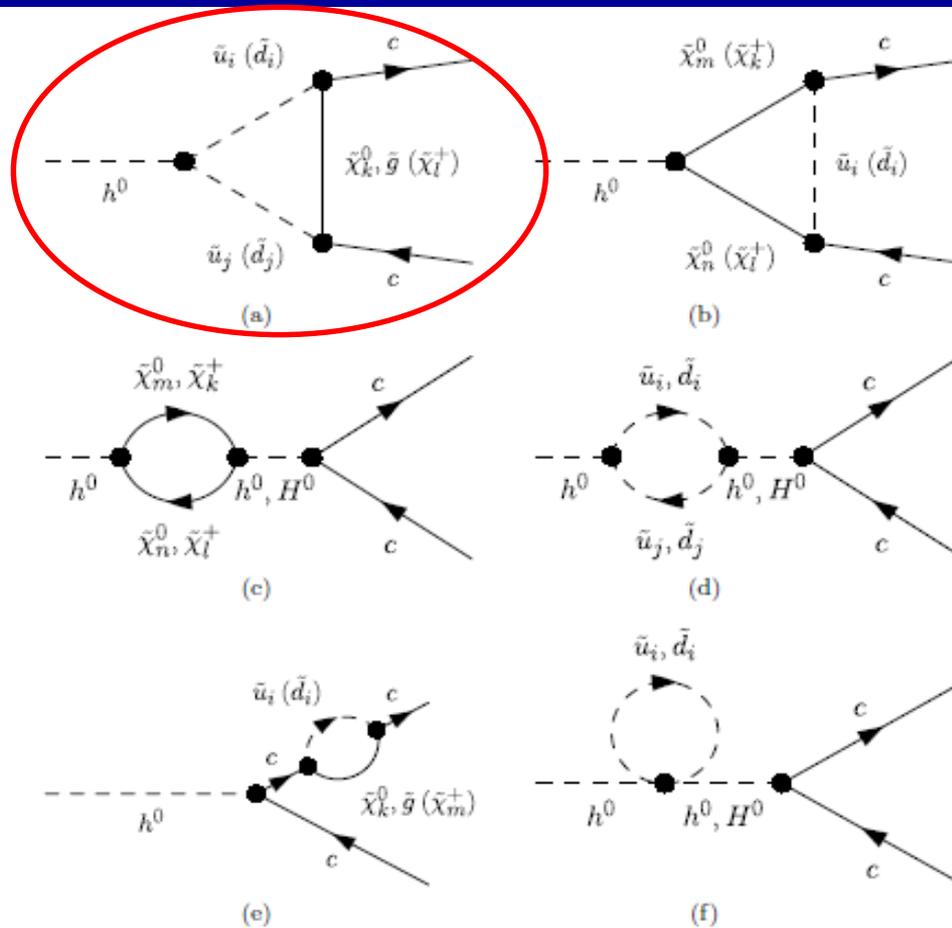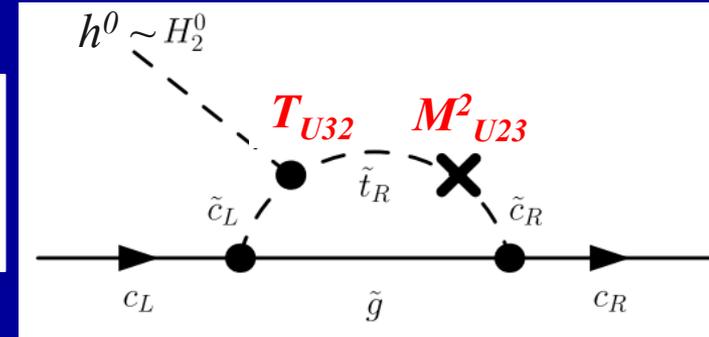
Figure 2: The main one-loop contributions with SUSY particles in $h^0 \to c\bar{c}$. The corresponding diagram to (e) with the self-energy contribution to the other charm quark is not shown explicitly.

Caveat for very large *DEV(h⁰ -> c c̄)*
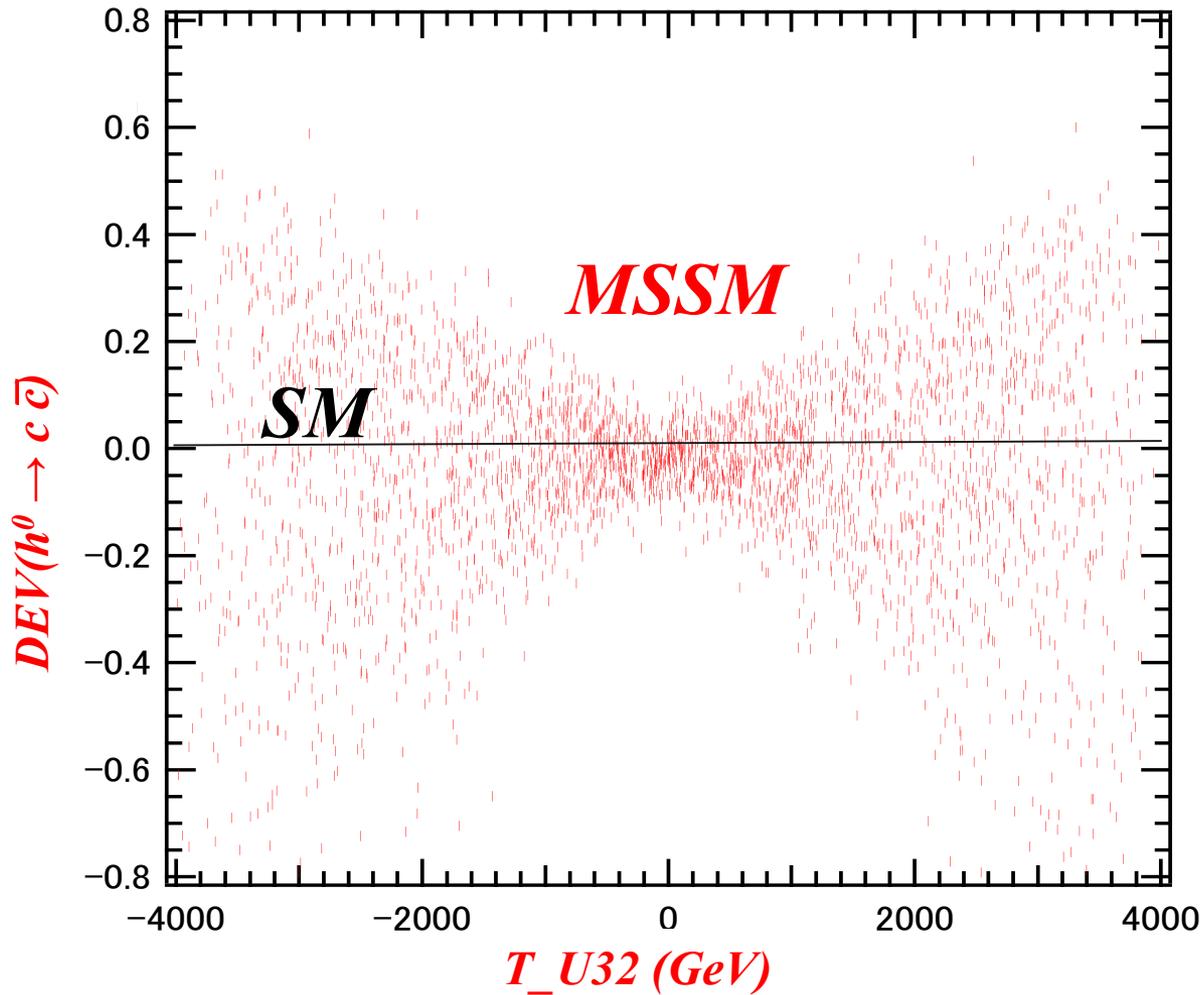
# *Caveat for very large DEV($h^0 \to c\bar{c}$)*

**Gluino loop** *contribution to* $h^0 \to c\bar{c}$ *can be **very large** (positive and negative) **for large** $T_{U32}*M^2{}_{U23}$!*



*The **interference term** between the tree diagram and the gluino one-loop diagram can be **very large** (positive and negative) **for large** $T_{U32}*M^2{}_{U23}$, which can lead to even **NEGATIVE width** $\Gamma(h^0 \to c\bar{c})$ at one-loop level !*

*In this case **perturbation theory breaks down!***

*A large deviation of $\Gamma(h^0 \to c\bar{c})$ from the SM value is in principle possible due to large values of the product $T_{U32}*M^2{}_{U23}$ .*

*Since **there is no significant physical constraint on this product**, the deviation DEV($h^0 \to c\bar{c}$) can be unnaturally large. So, we show only the results with a deviation from the SM up to ~ +/-60%.*

Contours of $DEV(h^0 \to c\bar{c})$ in $T_{U32}$ - $M^2_{U23}$ plane

[The red hatched region is allowed.]

Contours of DEV(c)
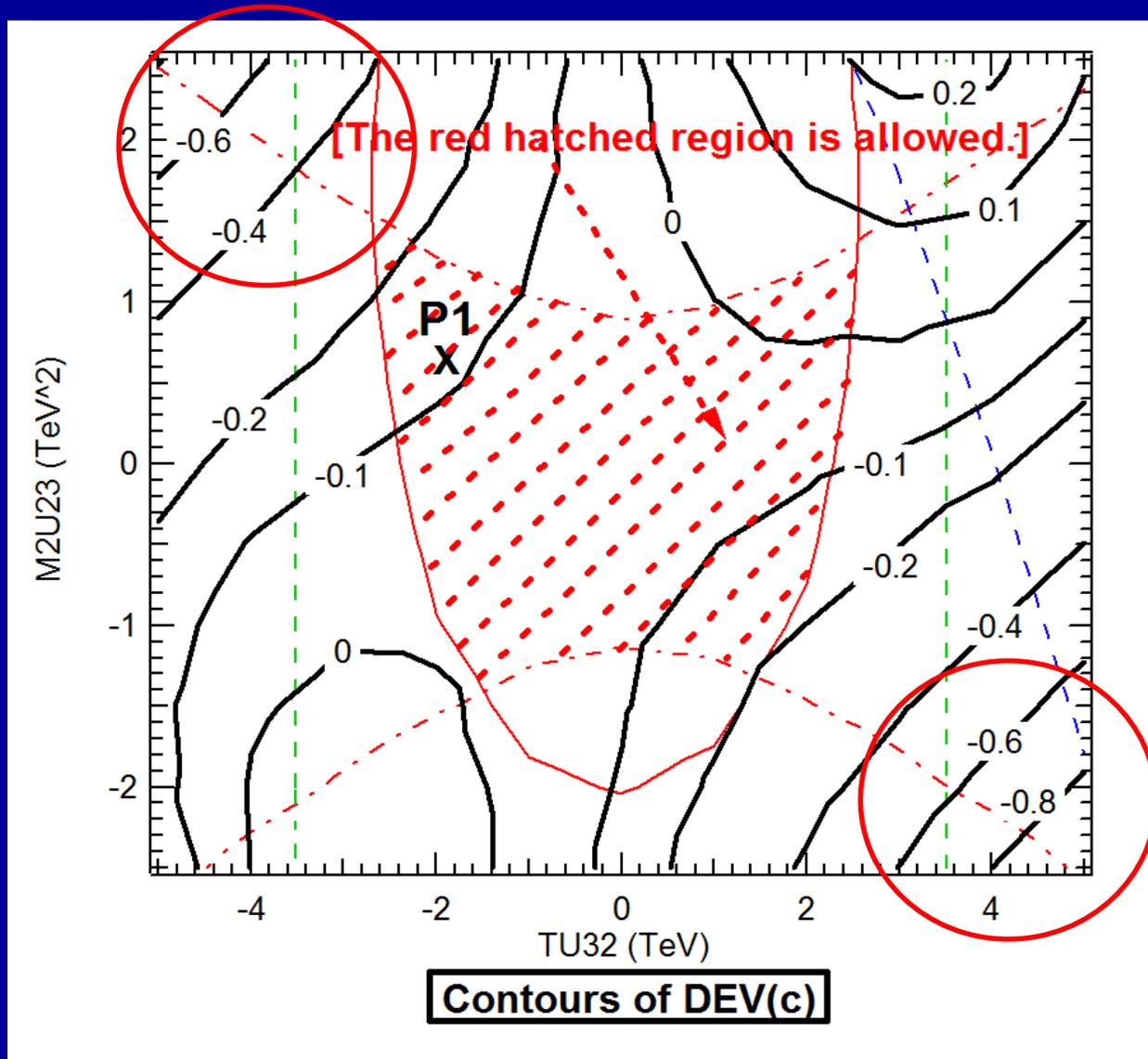
## Effect of Resummation of the bottom Yukawa coupling at large $\tan\beta$

*As for  $\Gamma(h^0 \rightarrow b\,\bar{b})$ & $\Gamma(h^0 \rightarrow b\,\bar{s}\,/\,s\,\bar{b})$,  we have considered the large $\tan\beta$ enhancement and the resummation of the bottom Yukawa coupling [1].*
*It turns out, however, that in our case with large $m_A$ close to the decoupling Higgs limit, the resummation effect ($\Delta_b$ effect) is very small (< 0.1%) [2].*

[1] *M. Carena et al., Nucl. Phys. B 577 (2000) 88 [hep-ph/9912516].*
[2] *H. Eberl, E. Ginina, A. Bartl, K. Hidaka and W. Majerotto, JHEP 06 (2016) 143 [arXiv:1604.02366 [hep-ph]];*
   *E. Ginina, A. Bartl, H. Eberl, K. Hidaka and W. Majerotto, PoS(EPS-HEP2015)146 [arXiv:1510.03714 [hepph]].*

Scatter plot in $T_{U33} - DEV(\gamma/g)$ plane



MSSM

SM
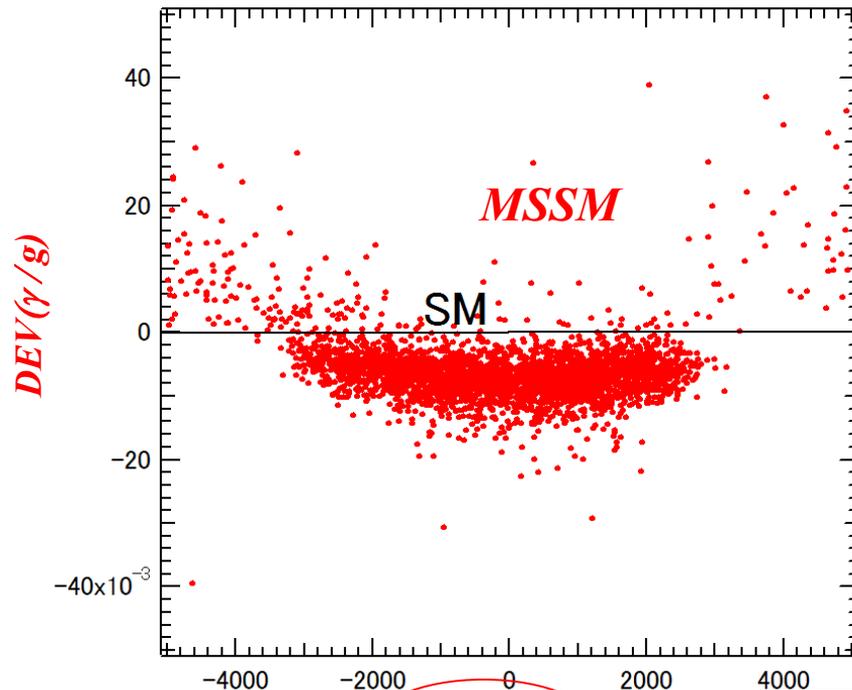
$DEV(\gamma/g)$

T_U33 (GeV)

$\tilde{t}_L - \tilde{t}_R$ mixing parameter

-There is a strong correlation between $T_{U33} - DEV(\gamma/g)$ !

- $DEV(\gamma/g)$ can be as large as ~ +4% for large $T_{U33}$ !

# 8. *Conclusion*

- *We have studied the decays*

  *$h^0$ (125GeV) → c c̄ , b b̄ , b s̄ , γ γ, g g in the MSSM with general QFV.*

- *Performing a systematic MSSM parameter scan respecting all of the relevant theoretical and experimental constraints , we have found the followings:*

  *\* DEV($h^0$ -> c c̄) and DEV($h^0$ -> b b̄) can be very large simultaneously! :*
  *DEV($h^0$ -> c c̄) can be as large as ~ ± 60%,*
  *DEV($h^0$ -> b b̄) can be as large as ~ ± 20%.*

  *\* The deviation of the width ratio Γ ($h^0$ -> b b̄) / Γ ($h^0$ -> c c̄) from the SM value can exceed ~ +100%.*

  *\* BR($h^0$ -> b s̄ / s b̄) can be as large as ~ 0.15%!*
  *ILC(250 + 500 + 1000) sensitivity could be ~ 0.1% at 4 sigma signal significance!*

**\* DEV($h^0 \to \gamma\gamma$) and DEV($h^0 \to gg$) can be sizable simultaneously! :**

DEV($h^0 \to \gamma\gamma$) can be as large as ~ ±1%,

DEV($h^0 \to gg$) can be as large as ~ ±4%.

**\* The deviation of the width ratio $\Gamma(h^0 \to \gamma\gamma)/\Gamma(h^0 \to gg)$ from the SM value can be as large as ~ -4% to +8% .**

**\* There is a very strong correlation between DEV($h^0 \to \gamma\gamma$) and DEV($h^0 \to gg$). This correlation is due to the fact that the stop-loop (stop-scharm mixture loop) contributions dominate the two DEVs.**

- **All of these large deviations in the $h^0$ (125) decays are due to large $\tilde{c}$ - $\tilde{t}$ mixing & large $\tilde{c}/\tilde{t}$ involved trilinear couplings $T_{U23}$, $T_{U32}$, $T_{U33}$ and large $\tilde{s}$ - $\tilde{b}$ mixing & large $\tilde{s}/\tilde{b}$ involved trilinear couplings $T_{D23}$, $T_{D32}$, $T_{D33}$.**

- **Future lepton colliders such as ILC, CLIC, CEPC, FCC-ee can observe such large deviations from SM at high significance!**

- **In case the deviation pattern shown here is really observed at the future lepton colliders, then it would strongly suggest the discovery of QFV SUSY (MSSM with general QFV)!**