Analysis of Monopoles in Multi-Step Spontaneous Gauge Symmetry Breaking

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Ongoing Research

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Phenomenological context

Various Beyond Standard Model (BSM) scenarios include monopoles:

- Candidates for dark matter

't Hooft–Polyakov monopoles originating from the dark sector contribute to the relic dark matter abundance

V.V. Khoze & G. Ro (2014).
Yang Bai et al. (2014).
S. Baek et al. (2020)…

- Physics involving axions

Contributions to axion mass via the Witten effect

M. Kawasaki et al. (2016).
A. Banerjee & M.A. Buen-Abad (2025)…

- Possible origin of ultra-high-energy cosmic rays (UHECRs)

=Cosmic rays $(> 10^{18}\,\mathrm{eV})$

Monopole annihilation and radiation from accelerated monopoles

E. Huguet & P. Peter (2000)
Ł. Bratek & J. Jałocha (2025)…

Can we consider magnetic monopoles within the SM?

Talk Overview

Indeed, The Cho–Maison monopole exists within the SM

However, the energy diverges at the spatial origin

$$E_0 \propto \int_L^\infty \frac{\mathrm{d}r}{r^2} \qquad \frac{1}{L} \text{ : cut off energy}$$

The SM description break down for $E > \dfrac{1}{L}$

➡ What is the ultraviolet completion of the Cho–Maison monopole?

Our research

’t Hooft–Polyakov monopole serves as a UV completion of the Cho–Maison monopole.

[1] Y. M. Cho & D. Maison (1996).

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't Hooft–Polyakov monopole

$$\mathrm{SU}(2) \to \mathrm{U}(1)$$

[1] G. 't Hooft (1974). [2] A. M. Polyakov (1974).

$$\mathcal{L} = -\frac{1}{4} F^{a\mu\nu} F^{a}_{\mu\nu} + \frac{1}{2} (D^{\mu}\Phi)^{a} (D_{\mu}\Phi)^{a} - V(\Phi) \quad V(\Phi) = \frac{\lambda}{8} (\Phi^{a}\Phi^{a} - v^{2})^{2}$$

Adjoint Representation $\Phi = \Phi^{a} \frac{\sigma^{a}}{2} = \frac{1}{2} \begin{pmatrix} \Phi^{3} & \Phi^{1} - \mathrm{i}\Phi^{2} \\ \Phi^{1} + \mathrm{i}\Phi^{2} & -\Phi^{3} \end{pmatrix}$

Hedgehog Ansatz (Solution of EoM)

Boundary Condition: $|\Phi| \to \langle \Phi \rangle \ (r \to \infty)$

$$\Phi^{a}(x) = v_{\Phi} \chi(r) \frac{x^{a}}{r}, \quad A^{a}_{0} = 0, \quad A^{a}_{i} = \frac{1}{e} (1 - f(r)) \varepsilon_{aij} \frac{x^{j}}{r^{2}}$$

The spatial rotation is compensated by a rotation in field space.

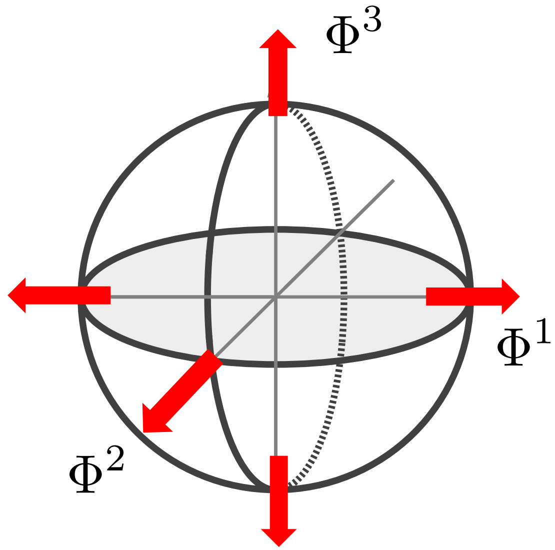
$$\vec{K} = \vec{J} + \vec{I} = \vec{x} \times \frac{1}{\mathrm{i}} \nabla + \frac{\vec{\sigma}}{2} \qquad [\vec{K}, \Phi(x)] = 0$$

$$[L_i, A^a_j] = i\,\varepsilon_{imn} x_m \partial_n A^a_j,$$
$$[S_i, A^a_j] = -i\,\varepsilon_{ijk} A^a_k,$$
$$[I_i, A^a_j] = -i\,\varepsilon_{iab} A^b_j,$$

$$K_i = L_i + S_i + I_i$$

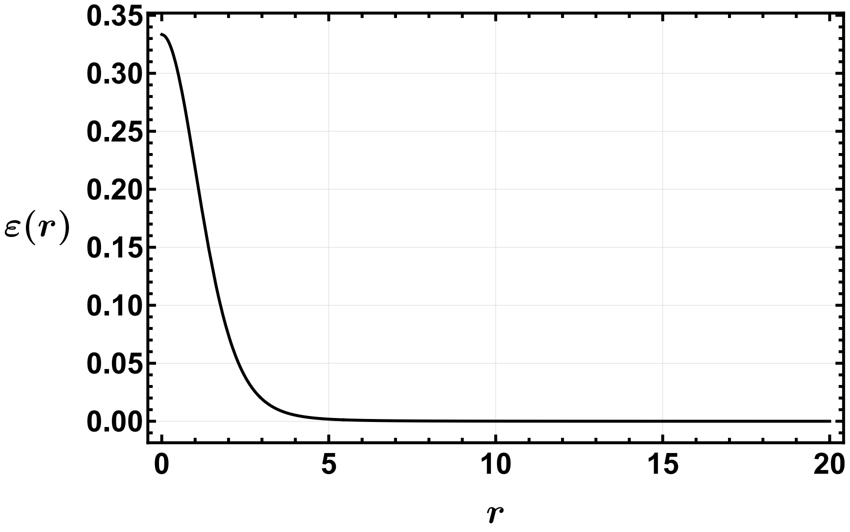
$$[K_i, A^a_j] = 0$$

A nonsingular, spherically symmetric configuration

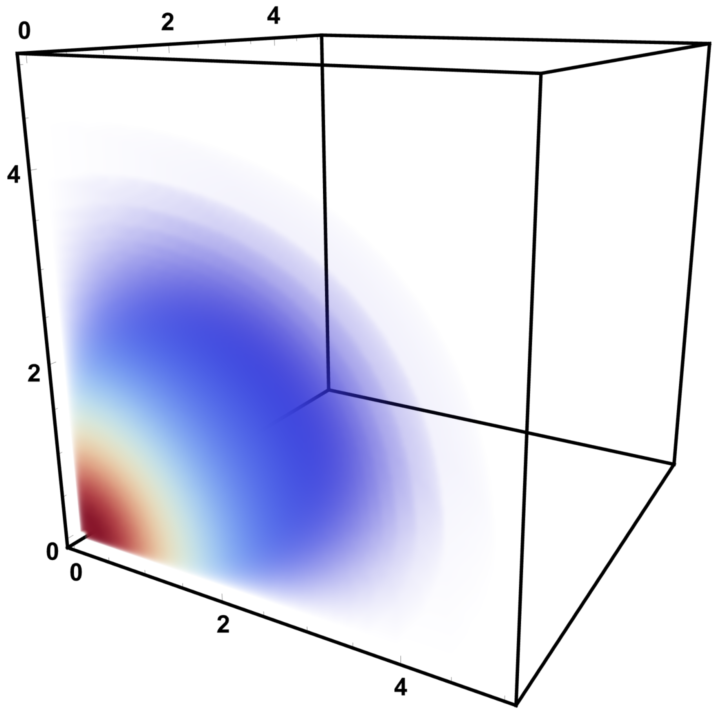


Finite Energy of the ’t Hooft–Polyakov Monopole

$$E = \int \mathrm{d}^3x \left[\frac{1}{4} F^a_{ij} F^a_{ij} + \frac{1}{2} (D_i \Phi^a)(D_i \Phi^a) + V(\Phi) \right]$$



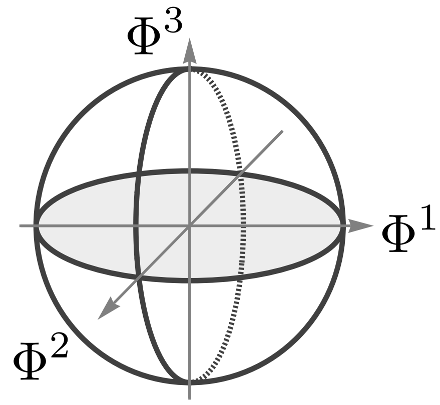
➡ Finite Energy



3D Plot of the Energy Density

Structure of the Vacuum

$$\frac{\partial V}{\partial \Phi} = 0 \Rightarrow (\Phi^1)^2 + (\Phi^2)^2 + (\Phi^3)^2 = \frac{v^2}{2}$$



$$\langle \Phi \rangle : S^2_\infty \to S^2_{\mathrm{vacuum}}$$
$$\pi_2(S^2) = \mathbb{Z}$$

Nontrivial Homotopy ensures stability

$\pi_2(\mathcal{M}_{\mathrm{vac}}) \neq \{e\} \implies$ The 't Hooft–Polyakov monopole exists

$$\pi_2\left(\mathcal{M}^{\mathrm{SM}}_{\mathrm{vac}}\right) = \pi_2(S^3) = \{e\}$$

$\implies$ The 't Hooft–Polyakov monopole is absent in the Standard Model.

Does the Standard Model admit any other type of monopole?

Cho–Maison monopole $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}} \longrightarrow \mathrm{U}(1)_{\mathrm{EM}}$

Hedgehog configuration

$$H = \frac{1}{\sqrt{2}}\rho(r)\xi, \quad \xi = \mathrm{i}\begin{pmatrix} \sin(\theta/2)\,\mathrm{e}^{-\mathrm{i}\varphi} \\ -\cos(\theta/2) \end{pmatrix}$$

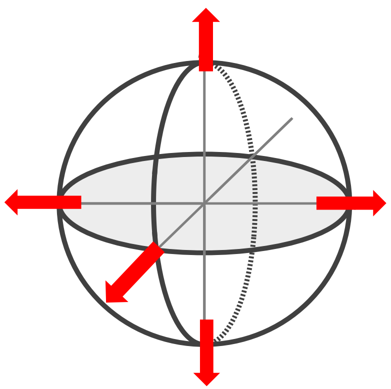
This configuration is undefined at $\theta = \pi$

$$W_i^a = \frac{1}{g}\left[1 - f(r)\right]\varepsilon_{aij}\frac{x^j}{r^2}$$

Same configuration as the 't Hooft -Polyakov monopole

$$B_i = -\frac{1}{g'}\frac{1-\cos\theta}{r\sin\theta}\left(-\sin\varphi,\ \cos\varphi,\ 0\right)$$

Same configuration as the Dirac monopole



$$E = E_0 + E_1$$

$$E_0 \propto \int_0^\infty \frac{\mathrm{d}r}{r^2}$$

Energy diverges at the spatial origin

→ SM description break down at the high energy region

Existence of solutions: [Y. Yang (2001)]
Stability of solutions: [R. Gervalle & M.S. Volkov (2022)]

- 't Hooft–Polyakov monopole

$$\mathrm{SU}(2) \to \mathrm{U}(1)$$

- Is a nonsingular configuration
- Has a finite energy solution

Stability is guaranteed

$$\langle \Phi \rangle : S^2_\infty \to S^2_{\mathrm{vacuum}}$$

$$\Pi_2(\mathcal{M}_{\mathrm{vac}}) \neq \{e\}$$

- Cho–Maison monopole

$$\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}} \longrightarrow \mathrm{U}(1)_{\mathrm{EM}}$$

- Is a singular configuration
- Energy diverges at the spatial origin

In the SM, this mathematical requirement is not satisfied; consequently, the 't HP monopole is absent.

$$\Pi_2\left(\mathcal{M}^{\mathrm{SM}}_{\mathrm{vac}}\right) = \{e\}$$

High energy : 't Hooft–Polyakov monopole

Low energy : Cho–Maison monopole

➡ we study a concrete model that reproduces this scenario.

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- Background and Open Problems in Magnetic Monopole Research
- Theoretical Review of Magnetic Monopoles
- Overview of Our Research
  - The UV completion of the Cho-Maison monopole

UV Completion of the Cho–Maison Monopole

- Pati-Salam Model

J. C. Pati & A. Salam (1974).

$$\mathrm{SU}(4)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$$
$$\downarrow \ \langle \Phi \rangle$$
$$\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}$$
$$\downarrow \ \langle H \rangle$$
$$\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{U}(1)_{\mathrm{EM}}$$

Conditions for the Existence of the ’t HP Monopole

$$\Pi_2(G/H) = \mathbb{Z}$$

In this effective theory, the monopole appears as a Cho–Maison monopole.

Hierarchy of Vacuum Expectation Values

$$\langle \Phi \rangle \gg \langle H \rangle$$

Goal

Confirm that the scalar field $H(x)$ behaves as a ’t HP Monopole in the high-energy regime.

Next Steps

- Assume a hedgehog configuration and solve the equations of motion
- Investigate the behavior of the scalar field $H(x)$ in high- and low-energy regions

Pati-Salam Model

$$V(H,\Phi) = -\mu_\Phi^2\,\mathrm{Tr}\,\Phi^\dagger\Phi - \mu_H^2\,\mathrm{Tr}\,H^\dagger H + \lambda_1\left(\mathrm{Tr}\,\Phi^\dagger\Phi\right)^2 + \lambda_2\,\mathrm{Tr}\,\Phi^\dagger\Phi\,\Phi^\dagger\Phi + \lambda_3\left(\mathrm{Tr}\,H^\dagger H\right)^2 + \lambda_4\,\mathrm{Tr}\,H^\dagger H\,H^\dagger H$$

Hedgehog configuration $\hat{r}^a = \frac{x^a}{r} = \left(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta\right)$

Scalar field

$\Phi(4,1,2)\quad \Phi \to \Phi' = U_4\,\Phi\,U_R^\dagger \quad \langle\Phi\rangle = v_\Phi \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$

$$\Phi(x) = v_\Phi\chi(r)\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \cos\theta & e^{-\mathrm{i}\varphi}\sin\theta \\ e^{\mathrm{i}\varphi}\sin\theta & -\cos\theta \end{pmatrix}$$

$H(1,2,2)\quad H \to H' = U_\mathrm{L} H U_\mathrm{R}^\dagger \quad \langle H\rangle = v_H \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$

$$H(x) = v_H h(r)\left(1_2 + \frac{x^a\sigma^a}{r}\right) = v_H h(r)\begin{pmatrix} 1+\cos\theta & e^{-\mathrm{i}\varphi}\sin\theta \\ e^{\mathrm{i}\varphi}\sin\theta & 1-\cos\theta \end{pmatrix}$$

Gauge field

$$G_{4i}^a = \frac{1}{g_4}\left(-f_4(r)+1\right)\varepsilon_{aij}\frac{x^j}{r^2}$$

$$W_{\mathrm{R}i}^a = \frac{1}{g_\mathrm{R}}\left(-f_\mathrm{R}(r)+1\right)\varepsilon_{aij}\frac{x^j}{r^2}$$

$$W_{\mathrm{L}i}^a = \frac{1}{g_\mathrm{L}}\left(-f_\mathrm{L}(r)+1\right)\varepsilon_{aij}\frac{x^j}{r^2}$$

$$G_{40}^a = W_{\mathrm{L}0}^a = W_{\mathrm{R}0}^a = 0$$

$$\mathrm{SU}(4)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}$$
$$\downarrow\ \langle\Phi\rangle$$
$$\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$$
$$\downarrow\ \langle H\rangle$$
$$\mathrm{SU}(3)_\mathrm{C}\times\mathrm{U}(1)_\mathrm{EM}$$

$$\langle\Phi\rangle \gg \langle H\rangle$$

Assume this spherically symmetric field behavior and solve the EoM

● Pati-Salam Model

$$\Phi(x) = v_\Phi \chi(r) \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \cos\theta & e^{-i\varphi}\sin\theta \\ e^{i\varphi}\sin\theta & -\cos\theta \end{pmatrix}$$

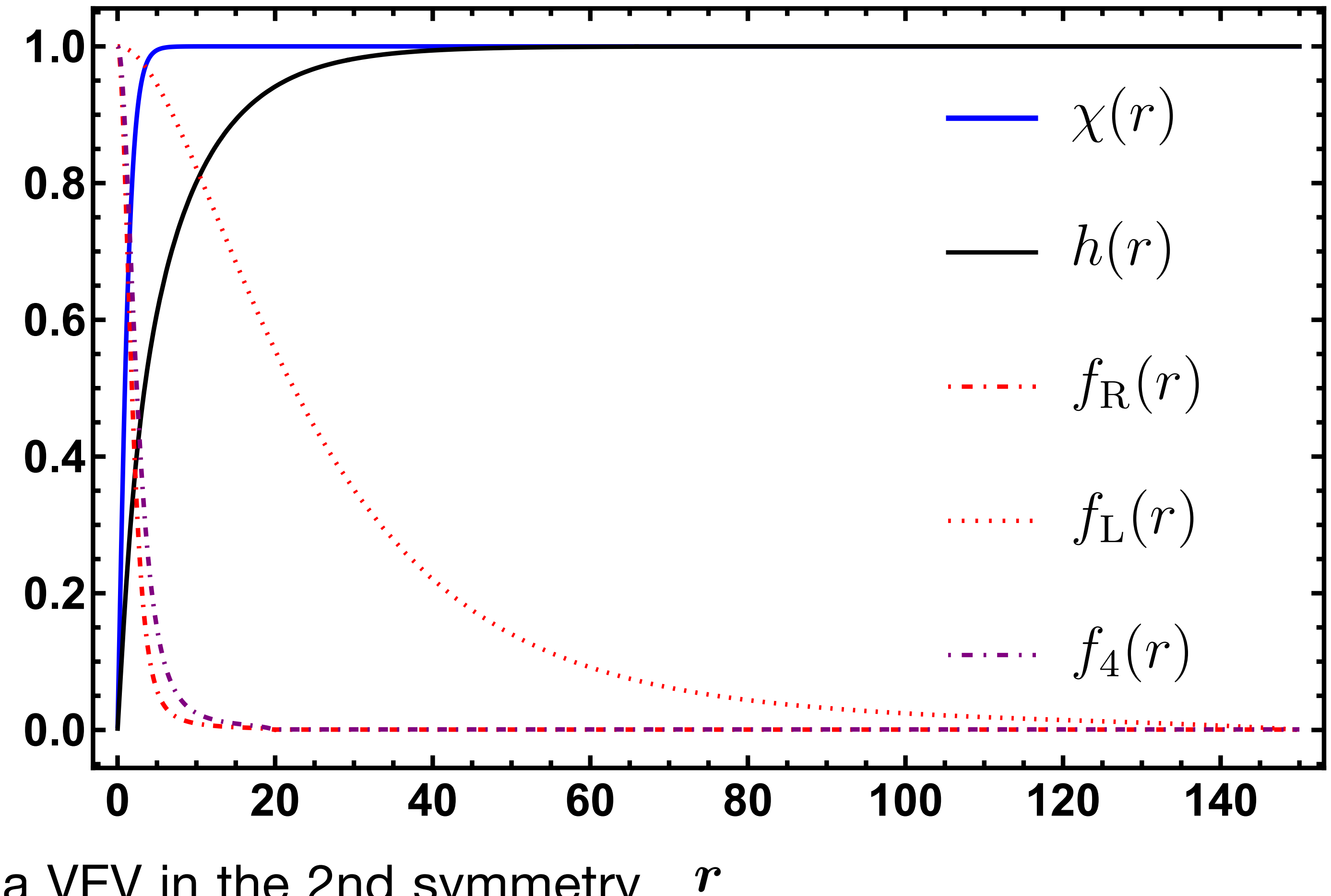
$$H(x) = v_H h(r) \begin{pmatrix} 1+\cos\theta & e^{-i\varphi}\sin\theta \\ e^{i\varphi}\sin\theta & 1-\cos\theta \end{pmatrix}$$

$$G^a_{4i} = \frac{1}{g_4}\left(-f_4(r)+1\right)\varepsilon_{aij}\frac{x^j}{r^2}$$

$$W^a_{\mathrm{R}i} = \frac{1}{g_\mathrm{R}}\left(-f_\mathrm{R}(r)+1\right)\varepsilon_{aij}\frac{x^j}{r^2}$$

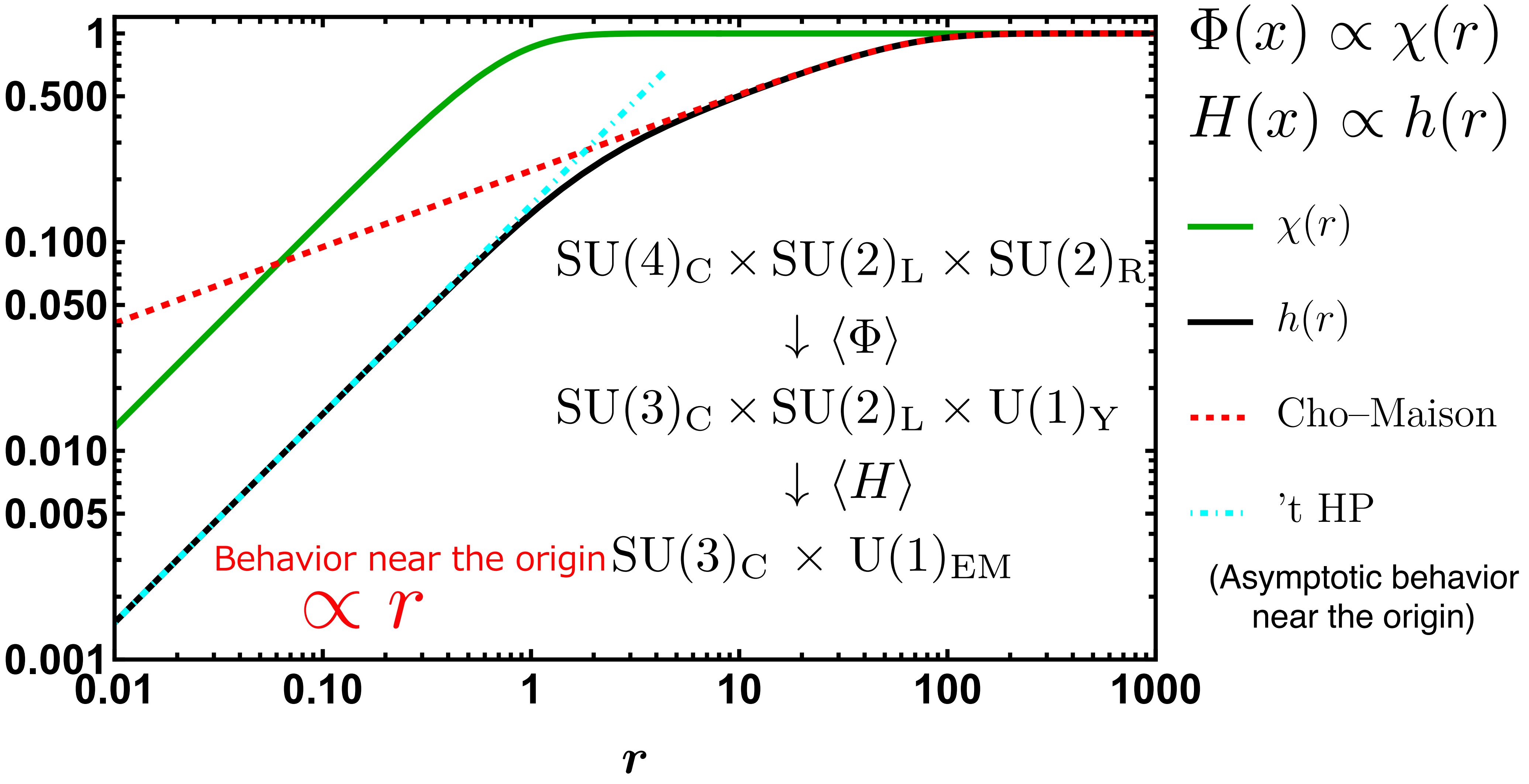
$$W^a_{\mathrm{L}i} = \frac{1}{g_\mathrm{L}}\left(-f_\mathrm{L}(r)+1\right)\varepsilon_{aij}\frac{x^j}{r^2}$$

Solutions to the EoM

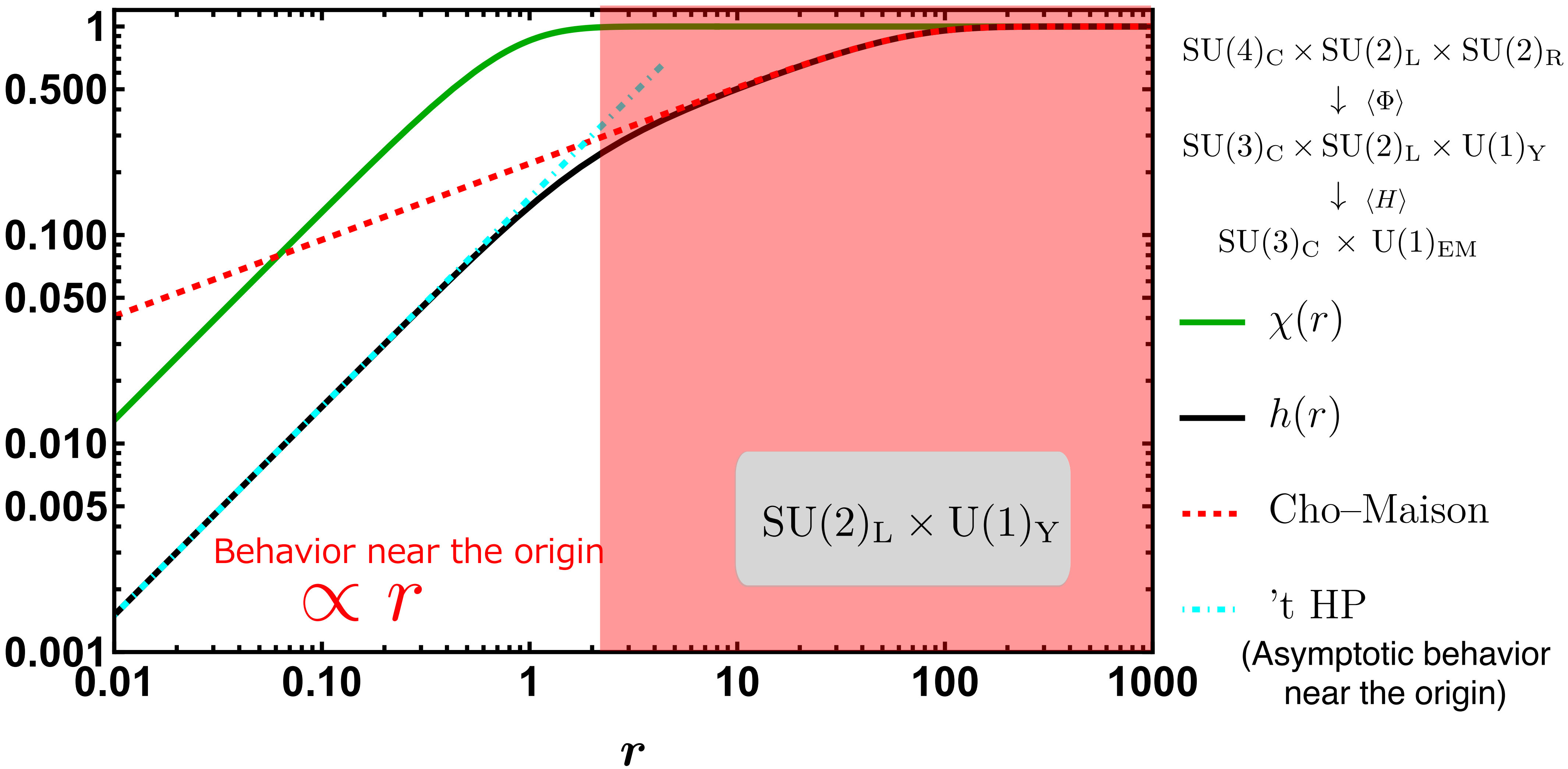


How does $h(r)$, acquiring a VEV in the 2nd symmetry breaking, behave at high energies?

● Pati-Salam Model



● Pati-Salam Model



● Summary

– Cho–Maison monopole exists within the Standard Model, but its high-energy behavior is not described by the SM

– We studied the Pati–Salam model realizing the scenario where the ’t Hooft–Polyakov monopole provides a UV completion of the Cho–Maison monopole.

● Future Directions

– Classification of gauge symmetry breaking patterns and scalar representations reproducing the Cho–Maison monopole

– Systematic organization of the relation between gauge groups and magnetic charge

– Phenomenological applications

- SU(5) Model

$$U(\theta,\varphi)\,\sigma^3\,U^\dagger(\theta,\varphi)=\hat{r}^a\cdot\sigma^a$$

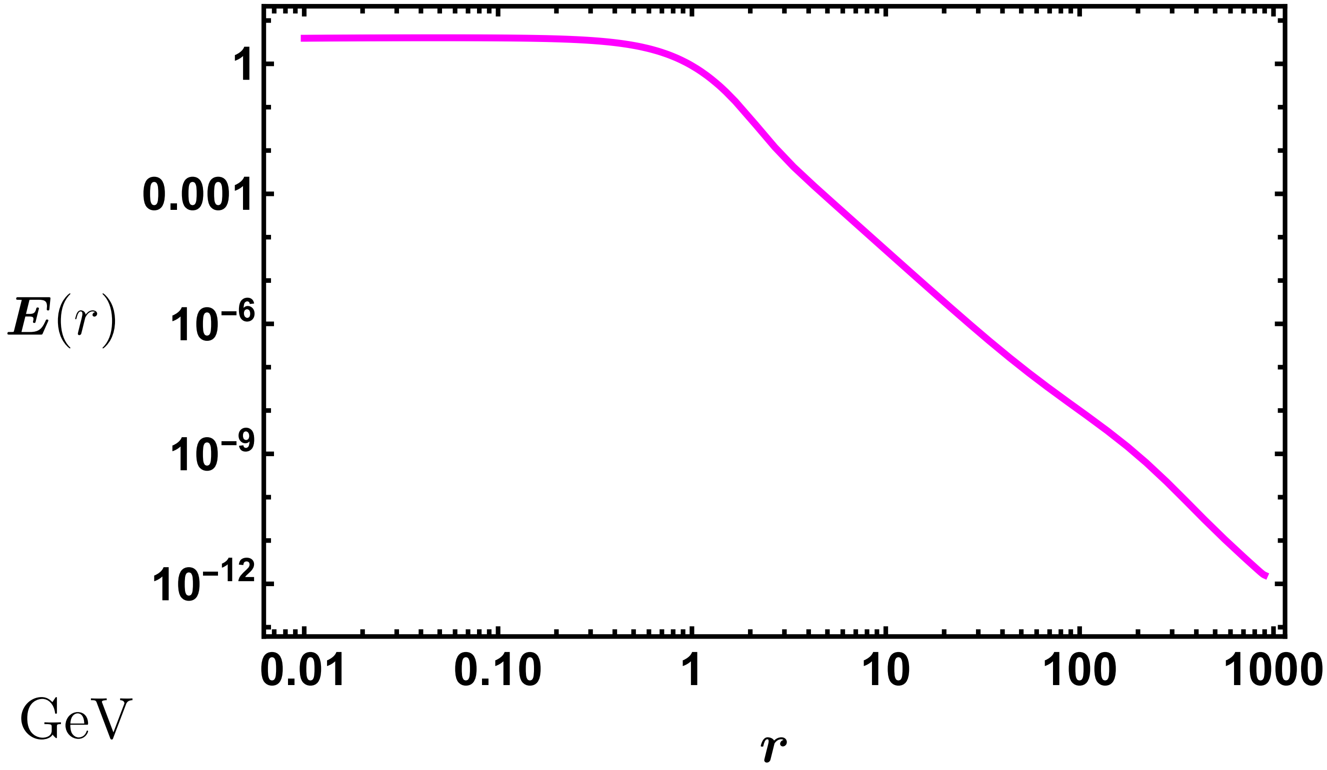
$$U(\theta,\varphi)=\exp\Big(-\tfrac{\mathrm{i}\varphi}{2}\sigma^3\Big)\exp\big(-\tfrac{\mathrm{i}\theta}{2}\sigma^2\big)\exp\Big(\tfrac{\mathrm{i}\varphi}{2}\sigma^3\Big)$$

$$=\begin{pmatrix}\cos(\theta/2) & -e^{-\mathrm{i}\varphi}\sin(\theta/2)\\ e^{\mathrm{i}\varphi}\sin(\theta/2) & \cos(\theta/2)\end{pmatrix}.$$

$$\langle\Phi\rangle=v_\Phi\begin{pmatrix}2&0&0&0&0\\0&2&0&0&0\\0&0&2&0&0\\0&0&0&-3&0\\0&0&0&0&-3\end{pmatrix}\qquad\langle H\rangle=\frac{v_H}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$$

$$U\,\langle\Phi\rangle\,U^\dagger\qquad\qquad U\,\langle H\rangle$$

- Finite Energy



$v_\Phi \sim 10^{16}$ GeV

Monopole energy $E_\mathrm{M} = \int \mathrm{d}\Omega\, E(r) \sim 10^{17}$ GeV

Monopole size $L_\mathrm{M} \sim 10^{-31}\mathrm{cm}$

‘t Hooft-Polyakov monopole

$$\frac{\mathrm{d}^2\chi}{\mathrm{d}r^2}+\frac{2}{r}\frac{\mathrm{d}\chi}{\mathrm{d}r}=2\frac{f^2\chi}{r^2}+\lambda\left(\chi^2-1\right)\chi$$

$$\frac{\mathrm{d}^2 f}{\mathrm{d}r^2}=\frac{f(f^2-1)}{r^2}+\chi^2 f$$

Near the origin

$$\chi(r)\sim a_1 r+\cdots$$

$\mathrm{U}(1)_{\mathrm{Y}}$ contribution

Cho-Maison monpole

$$\frac{\mathrm{d}^2\rho}{\mathrm{d}r^2}+\frac{2}{r}\frac{\mathrm{d}\rho}{\mathrm{d}r}=\frac{1}{2}\frac{f^2}{r^2}\rho+\lambda\left(\rho^2-1\right)\rho$$

$$\frac{\mathrm{d}^2 f}{\mathrm{d}r^2}=\frac{f(f^2-1)}{r^2}+\rho^2 f$$

Near infinity

$$\rho(r)\sim 1-\frac{a_2}{r}\exp(-\kappa r)+\cdots$$

Near the origin

$$\rho(r)\sim a_1 r^{\delta}+\cdots\qquad \delta=\frac{\sqrt{3}-1}{2}$$

- # Pati-Salam Model

$$V(H,\Phi) = -\mu_\Phi^2 \,\mathrm{Tr}\,\Phi^\dagger\Phi - \mu_H^2\,\mathrm{Tr}\,H^\dagger H + \lambda_1\,\left(\mathrm{Tr}\,\Phi^\dagger\Phi\right)^2 + \lambda_2\,\mathrm{Tr}\,\Phi^\dagger\Phi\,\Phi^\dagger\Phi + \lambda_3\,\left(\mathrm{Tr}\,H^\dagger H\right)^2 + \lambda_4\,\mathrm{Tr}\,H^\dagger H\,H^\dagger H$$

Boundary condition

$$\begin{aligned}
\chi(0) &= 0, \quad \chi(\infty) = 1, \\
h(0) &= 0, \quad h(\infty) = 1, \\
f_4(0) &= 1, \quad f_4(\infty) = 0, \\
f_{\mathrm{L}}(0) &= 1, \quad f_{\mathrm{L}}(\infty) = 0, \\
f_{\mathrm{R}}(0) &= 1, \quad f_{\mathrm{R}}(\infty) = 0.
\end{aligned}$$

$$v_\Phi^2 = \frac{\mu_\Phi^2}{2\,(\lambda_1+\lambda_2)}$$

$$v_H^2 = \frac{\mu_H^2}{2\,(\lambda_3+\lambda_4)}$$

EoM

$$\chi''(r) = -\frac{2}{r}\,\chi'(r) + \frac{\left(f_4(r)+f_R(r)\right)^2}{2r^2}\,\chi(r) + \mu_\Phi^2(\chi(r)^2-1)\chi(r)$$

$$h''(r) = -\frac{2}{r}\,h'(r) + \frac{\left(f_{\mathrm{L}}(r)+f_{\mathrm{R}}(r)\right)^2}{2\,r^2}\,h(r) + \mu_H^2(h(r)^2-1)h(r)$$

$$f_4''(r) = \frac{f_4(r)^2-1}{r^2} f_4(r) + \tfrac{1}{2} g_4^2\,v_\Phi^2\,f_4(r)\,\chi(r)^2$$

$$f_R''(r) = \frac{f_R(r)^2-1}{r^2} f_R(r) + \tfrac{1}{2} g_R^2\,f_R(r)\,\left(v_H^2 h(r)^2 + v_\Phi^2\chi(r)^2\right)$$

$$f_L''(r) = \frac{f_L(r)^2-1}{r^2} f_L(r) + \tfrac{1}{2} g_L^2\,v_H^2\,f_L(r)\,h(r)^2$$

● Pati-Salam Model

$$V(H,\Phi) = -\mu_\Phi^2 \,\mathrm{Tr}\,\Phi^\dagger\Phi - \mu_H^2 \,\mathrm{Tr}\,H^\dagger H + \lambda_1 \left(\mathrm{Tr}\,\Phi^\dagger\Phi\right)^2 + \lambda_2 \,\mathrm{Tr}\,\Phi^\dagger\Phi\,\Phi^\dagger\Phi + \lambda_3 \left(\mathrm{Tr}\,H^\dagger H\right)^2 + \lambda_4 \,\mathrm{Tr}\,H^\dagger H\, H^\dagger H$$

$$\chi''(r) = -\frac{2}{r}\,\chi'(r) + \frac{\left(f_4(r)+f_R(r)\right)^2}{2r^2}\,\chi(r) + \mu_\Phi^2(\chi(r)^2-1)\chi(r)$$

EoM

$$h''(r) = -\frac{2}{r}\,h'(r) + \frac{\left(f_{\mathrm{L}}(r)+f_{\mathrm{R}}(r)\right)^2}{2\,r^2}\,h(r) + \mu_H^2(h(r)^2-1)h(r)$$

$$f_L''(r) = \frac{f_L(r)^2-1}{r^2}\,f_L(r) + \tfrac{1}{2}g_L^2\,v_H^2\,f_L(r)\,h(r)^2$$

$$v_\Phi^2 = \frac{\mu_\Phi^2}{2\,(\lambda_1+\lambda_2)}$$

$$v_H^2 = \frac{\mu_H^2}{2\,(\lambda_3+\lambda_4)}$$

$$f_R''(r) = \frac{f_R(r)^2-1}{r^2}\,f_R(r) + \tfrac{1}{2}g_R^2\,f_R(r)\left(v_H^2 h(r)^2 + v_\Phi^2\chi(r)^2\right)$$

$$f_4''(r) = \frac{f_4(r)^2-1}{r^2}\,f_4(r) + \tfrac{1}{2}g_4^2\,v_\Phi^2\,f_4(r)\,\chi(r)^2$$

$$\Phi(x) \to \langle\Phi\rangle$$

EoM after the 1st step SSB

$$h''(r) = -\frac{2}{r}\,h'(r) + \frac{\left(f_{\mathrm{L}}(r)\right)^2}{2\,r^2}\,h(r) + \mu_H^2(h(r)^2-1)h(r)$$

$$f_{\mathrm{L}}''(r) = \frac{f_{\mathrm{L}}(r)}{r^2}\left(-1+f_{\mathrm{L}}(r)^2\right) + g_{\mathrm{L}}^2 v_{\mathrm{H}}^2\,f_{\mathrm{L}}(r)\,h(r)^2$$

EoM for the Cho-Maison

$$\rho'' = -\frac{2}{r}\rho' + \frac{f^2}{2r^2}\rho + \lambda\left(\rho^2-1\right)\rho$$

$$f'' = \frac{f}{r^2}(-1+f^2) + \frac{g^2}{4}f\rho^2$$