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Screening Effects of Finite-Mass Fermions in Monopole Baryogenesis Scenario

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Background and Motivation: Baryon Number Asymmetry and Monopoles

[1] : Sakharov, 1967
[2] : 't Hooft, 1974
[3] : Polyakov, 1974
[4] : Witten, 1979

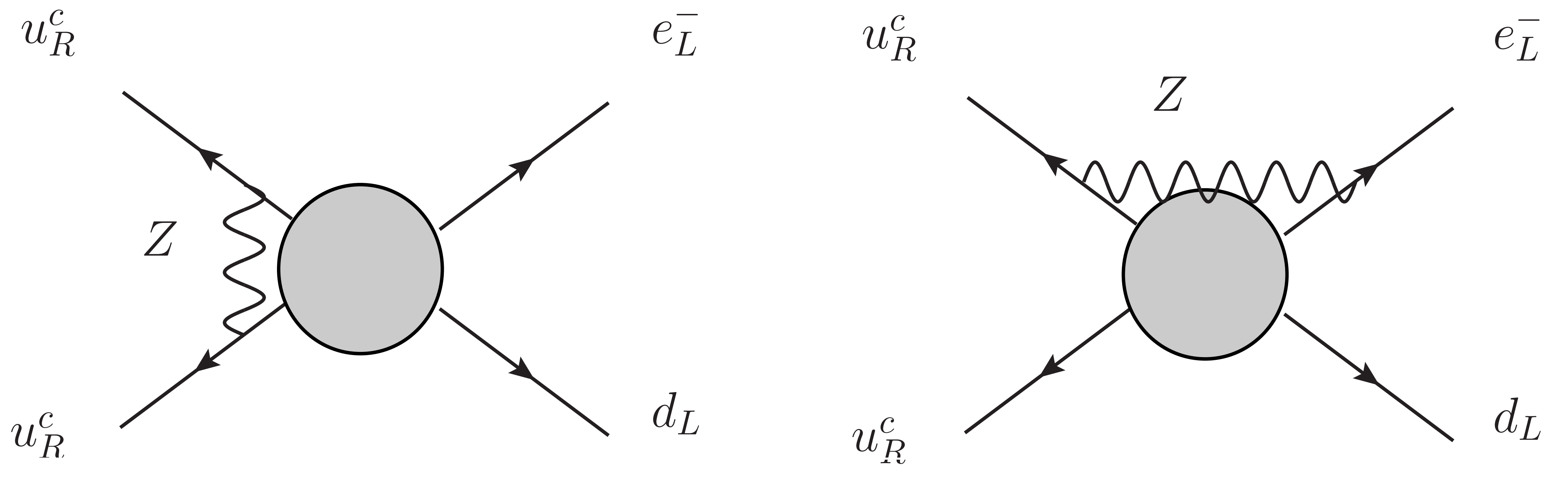
- Sakharov's 3 Conditions [1]:
  Requirements for Matter-Antimatter Asymmetry:
  - **Baryon Number (B) Violation**: Quark: B = 1/3, Anti-quark: B = -1/3
  - **C and CP Symmetry Violation:** Asymmetry in the reaction probabilities
  - **Deviation from Thermal Equilibrium**: (Equilibrium → Asymmetry Vanishes)
- GUT (Grand Unified Theory) Monopoles [2, 3]:
  - In SU(5) GUT, **'t Hooft-Polyakov monopoles** (magnetic monopoles) exist as solutions to the background gauge field.
- Witten Effect [4]:

  CP-violating θ-term in Lagrangian $\Longrightarrow$ Induced Electric Charge: $q_e = -\frac{\theta}{2\pi} q_m$

Background and Motivation: Baryogenesis and Loop Corrections

[5] : Callan, 1974
[6] : Brennan, 2024

- Baryon Number Violating Reactions Inside Monopoles (CR Effect) [5, 6]: Reactions not inhibited by potential barriers
  - Reaction with Baryon Number Change $\Delta B = +1$ (Process A):
    $e_R^+ + \bar{d}_{3R} + M \to u_{1L} + u_{2L} + M$、 $\bar{u}_{1R} + \bar{u}_{2R} + \bar{M} \to e_L^- + d_{3L} + \bar{M}$
  - Reaction with Baryon Number Change $\Delta B = -1$ (Process B):
    $u_{1R} + u_{2R} + M \to e_L^+ + \bar{d}_{3L} + M$、 $e_R^- + d_{3R} + \bar{M} \to \bar{u}_{1L} + \bar{u}_{2L} + \bar{M}$
- Rate Asymmetry from Z-Boson Loop:
  - Particles in Process A couple to SU(2) bosons; Process B particles do not.
  - **Z-loop corrections**
    → Rate asymmetry
    → Net Baryon Number



Caption: Quantum corrections occur only in Process A

Problem Setting: Purpose of This Research

[7] : Fujikawa, 1979

- Chiral Rotation [7] and Charge Screening Effects:
  - Massless limit ($m = 0$): Chiral rotation $\psi \rightarrow e^{i\alpha(x)\gamma_5}\psi$ cancels θ-term.
  - Finite Mass ($m \neq 0$): Rotation invalid. Must match $m \rightarrow 0$ limit.
  - Key Question: Can screening effects mask monopole charge?
  - **This effect is not considered in current monopole baryogenesis scenarios.**
- Purpose of This Research:
  - **Evaluate finite-mass screening on baryogenesis.**

[8] : Abel, 2020
[9] : Takenaka, 2020
[10] : Hook, 2025

Research Method: Setup

- Target Scale: EW Phase Transition ($T \sim 100$ GeV)
- Adopted Model:
  - Setup: Model with 2 fermion flavors ($N_f = 2$) with mass $M = 1$ MeV.
  - **Generalization**: Assign arbitrary charges q_1 and q_2.
  - In this study, we adopt the observational upper limit $\theta \sim 10^{-10}$ [8].
  - Monopole mass is $M_{GUT} \sim 10^{16}$ GeV (Experimental lower limit is $\gtrsim 5 \times 10^{15}$ GeV [9]).
- Bosonization [10]:
  - Bosonization: (1+1)D fermion $\chi(r, t) \rightarrow$ **boson** $\phi(r, t)$.

    Boundary condition for field values: $\phi_1(r_c) = \phi_2(r_c)$
    Boundary condition for field derivatives: $\partial_r \phi_1(r_c) = -\partial_r \phi_2(r_c)$

There are 2 fermions ($N_f = 2$);
the corresponding boson solutions are ϕ_1, ϕ_2.

Research Method: Boundary Conditions and Charge

- Equation of Motion for Boson Fields:

$$\partial_r^2 \phi_i = M^2 \sin\phi_i + \frac{\lambda q_i}{r^2}(q_1\phi_1 + q_2\phi_2 - 2\theta) \quad \left(M^2 \equiv \left(\frac{\pi\mu}{2}\right)^2, \quad \lambda \equiv \frac{\alpha}{4\pi}\right)$$

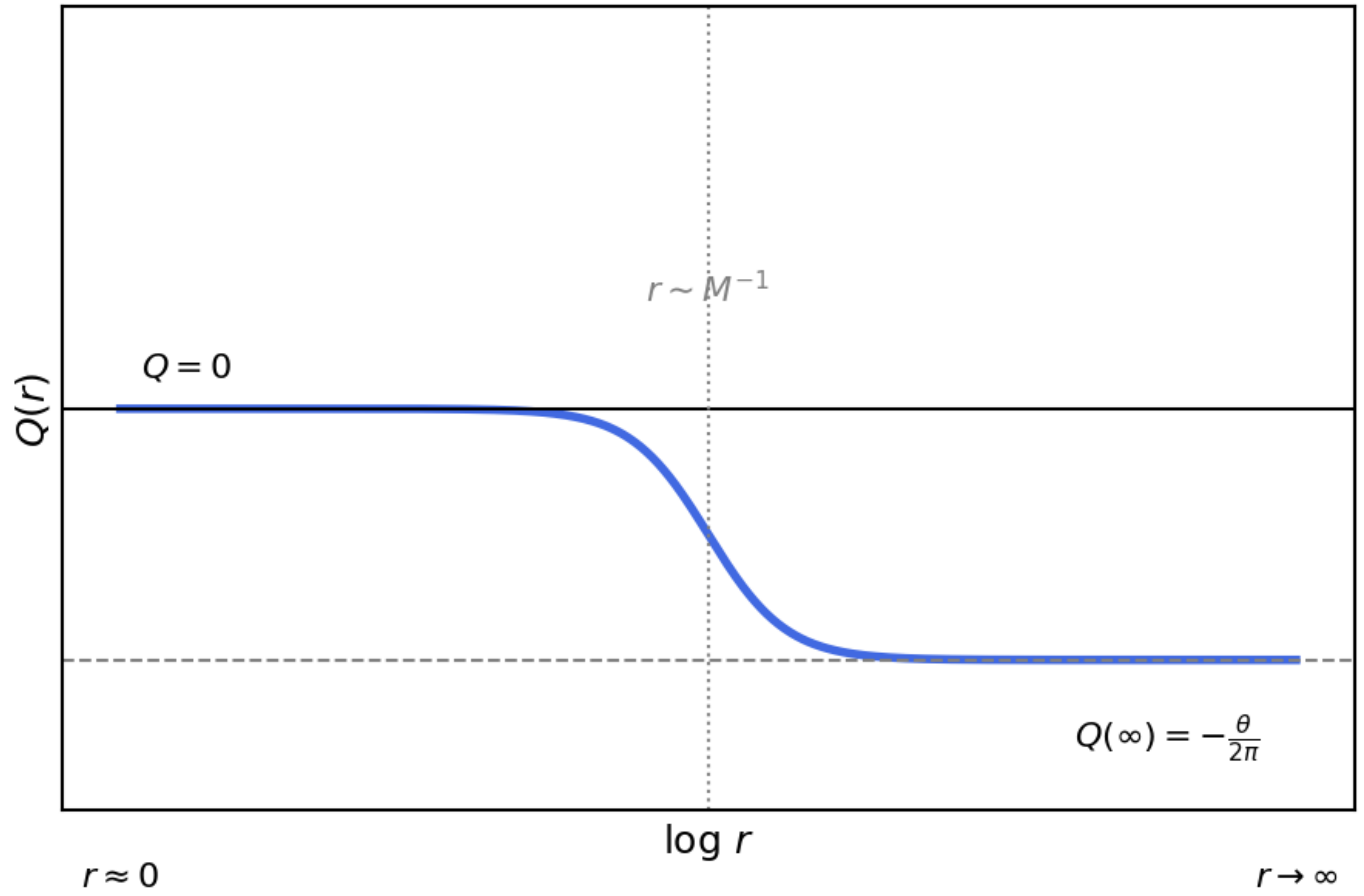
- Boundary Conditions at $r \to r_c$:
  - Limit $r \to r_c$ (Potential dominates): Finite Energy $\Longrightarrow q_1\phi_1(r_c) + q_2\phi_2(r_c) = 2\theta$.
  - With **BC** $\phi_1(r_c) = \phi_2(r_c)$: $\phi_1(r_c) = \phi_2(r_c) = \dfrac{2\theta}{q_1 + q_2}$
- Boundary Conditions at $r \to \infty$:
  - Limit $r \to \infty$: the first term dominates.
    For the energy to be minimized, we require $\phi_i(\infty) = 2\pi q$ (q: integer).
  - The monopole charge is minimized when $q = 0$.

Research Method: Monopole Charge Distribution and Effective Potential

[9] : Hook, 2025

- Monopole Charge Distribution [9]:
  - Witten effect charge is **not core-localized** but **radially distributed**.
  - Effective monopole charge at distance r (charge within a sphere of radius r):

$$Q(r) = \sum_i \int_0^r dr' j^0(r') = \frac{1}{2\pi} \sum_i \frac{q_i}{2} \left(\phi_i(r) - \phi_i(0)\right)$$



- Effective Potential Definition:
  Potential barrier assumed in literature (Brennan et al.):

$$V(r) = \frac{\alpha\theta}{2\pi r}$$

Effective potential definition considering screening effects:

$$V_{\text{eff}}(r) = \left| \int_r^\infty dr' \frac{\alpha Q(r')}{r'^2} \right| = \frac{\alpha\theta}{2\pi} \int_r^\infty \frac{dr'}{r'^2} \frac{Q(r')}{Q(\infty)} < V(r)$$

- Impact on Baryogenesis:
  Screening lowers the potential barrier, potentially **suppressing** the CR effect **asymmetry**.

Analysis Method: Mode Decomposition

- Equation of Motion for Boson Fields:

$$\partial_r^2 \phi_i = M^2 \sin \phi_i + \frac{\lambda q_i}{r^2}(q_1\phi_1 + q_2\phi_2 - 2\theta) \quad (i = 1,\, 2) \qquad (\lambda \equiv \frac{\alpha}{4\pi})$$

- Our Approach:
  Series solution based on mode decomposition using diagonalization
  - Linearization ($\phi \sim \theta \simeq 10^{-10} \ll 1$): Approx. $\sin \phi \sim \phi$.
  - Mode Decomposition (Diagonalization)
  - Rewrite the boundary conditions for ϕ into boundary conditions between modes.
  - Find the particular solution for the screening mode as a series solution.
  - Select the physical solution based on boundary conditions.

Analysis Method: Mode Decomposition (Details)

- Linearization:

  Since $\phi \sim \theta \simeq 10^{-10} \ll 1$, we linearize the equation using $\sin\phi \sim \phi$.

  $$\left[\partial_r^2 - M^2 \cdot \mathbf{1} - \frac{\lambda}{r^2}\mathbf{Q}\right]\begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix} = -\frac{2\lambda\theta}{r^2}\begin{pmatrix}q_1 \\ q_2\end{pmatrix} \quad \left(\mathbf{Q} = \begin{pmatrix}q_1^2 & q_1 q_2 \\ q_1 q_2 & q_2^2\end{pmatrix}\right)$$

- Mode Decomposition:

  Diagonalize matrix $\mathbf{Q}$ and decompose modes: $\begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix} = u_S(r)\vec{v}_S + u_F(r)\vec{v}_F$

  Screening Mode (Eigenvalue: $q_1^2 + q_2^2 \equiv Q_{\text{eff}}^2$): $\vec{v}_S = \begin{pmatrix}q_1 \\ q_2\end{pmatrix}$

  Free Mode (Eigenvalue: 0): $\vec{v}_F = \begin{pmatrix}q_2 \\ -q_1\end{pmatrix}$

- Equation of Motion for Each Mode:

  Screening Mode: $\left[\partial_r^2 - M^2 - \frac{\lambda Q_{\text{eff}}^2}{r^2}\right] u_S(r) = -\frac{2\lambda\theta}{r^2}$

  Free Mode: $\left[\partial_r^2 - M^2\right] u_F = 0$

Analysis Method: Mode Decomposition (Boundary Conditions)

- Boundary Conditions Between Modes:
  Rewrite boundary conditions using the relation $\phi_1, \phi_2 \leftrightarrow u_S(r), u_F(r)$.

  - BC for field values: $\phi_1(r_c) = \phi_2(r_c) \implies (q_1 - q_2)u_S(r_c) + (q_1 + q_2)u_F(r_c) = 0$

  - BC for field derivatives: $\partial_r\phi_1(r_c) = -\partial_r\phi_2(r_c) \implies (q_1 + q_2)u_S'(r_c) + (q_2 - q_1)u_F'(r_c) = 0$

- Screening Mode and Charge:

$$\sum_i q_i\phi_i = q_1\phi_1 + q_2\phi_2 = q_1(q_1u_S + q_2u_F) + q_2(q_2u_S - q_1u_F) = Q^2_{\text{eff}}u_S$$

Therefore, $Q(r)_{EM} = \frac{1}{2\pi}\sum_i \frac{q_i}{2}\left(\phi_i(r) - \phi_i(0)\right) = \frac{1}{2\pi}\left(\frac{Q^2_{\text{eff}}}{2}u_S(r) - \theta\right)$

Only the screening mode contributes to the effective charge.

Analysis Method: Screening Mode

- Equation of Motion: $\left[\partial_r^2 - M^2 - \frac{\lambda Q_{\text{eff}}^2}{r^2}\right] u_S(r) = -\frac{2\lambda\theta}{r^2}$

  Construct the solution as the sum of the homogeneous equation solution $u_S^{\text{hom}}(r)$

  and the particular solution $u_S^{\text{part}}(r)$: $u_S(r) = u_S^{\text{hom}}(r) + u_S^{\text{part}}(r)$

- Homogeneous Equation: $\left[\partial_r^2 - \left(M^2 + \frac{\lambda Q_{\text{eff}}^2}{r^2}\right)\right] u_S^{\text{hom}}(r) = 0$

  Using modified Bessel functions $I_\nu(z), K_\nu(z)$: $u_S^{\text{hom}}(r) = \sqrt{Mr}\left[AI_\nu(Mr) + BK_\nu(Mr)\right]$

- Particular Solution: $u_S^{\text{part}}(r) = c_0\left[1 + \frac{M^2}{2-\Lambda}r^2 + \frac{M^4}{(12-\Lambda)(2-\Lambda)}r^4 + \dots\right]$

- Determination of Coefficients: $u_S^{\text{part}}(r)$ and $I_\nu(Mr)$ increase with r and diverge at $r \to \infty$.

  Coefficient A is tuned numerically to ensure regularity at infinity.

  From the connection condition at $r = r_c$: $u_S(r_c) = c_0 = \frac{2\theta}{Q_{\text{eff}}^2}$, we determine coefficient B: $B = \frac{2\theta}{Q_{\text{eff}}^2}\frac{1}{\sqrt{Mr_c}K_\nu(Mr_c)}$

Results: Charge Behavior

- Analysis Content:

  Charge ratio $\frac{Q(r)_{EM}}{Q(\infty)_{EM}}$ for different fermion masses M

- Parameter Settings:

  - Charges:
    $q_1 = -2$ (corresponding to electron),
    $q_2 = -2/3$ (corresponding to down quark)

  - Screening Evaluation Point:
    $r_{EM} = (100\ \text{GeV})^{-1}$ (Electroweak phase transition scale)

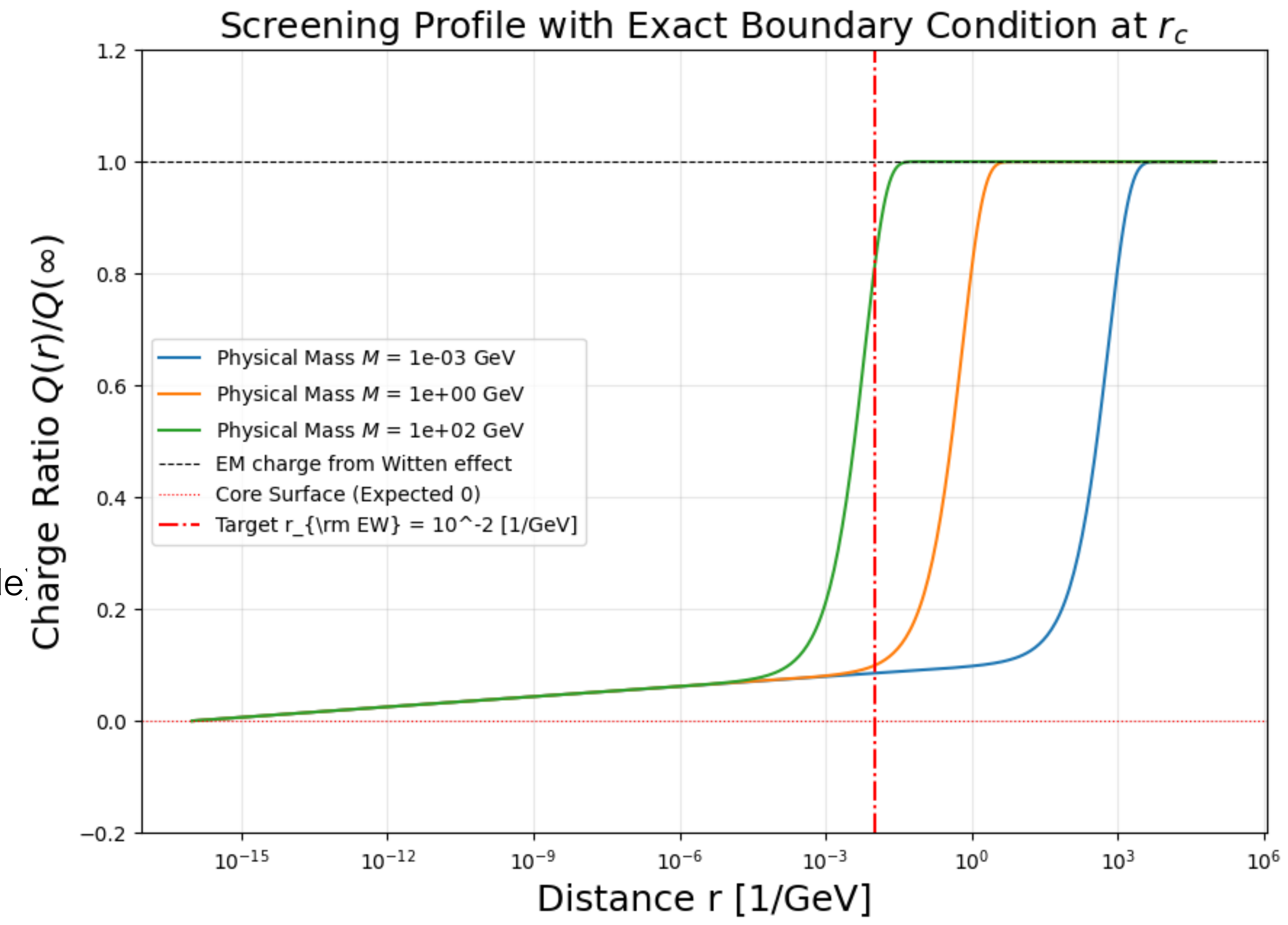
  - GUT Scale:
    $M_{\text{GUT}} = 10^{16}$ GeV

- Numerical Results:

  - $M = 1$ MeV: Ratio ≈ 0.085

  - $M = 1$ GeV: Ratio ≈ 0.099

  - $M = 100$ GeV: Ratio ≈ 0.809



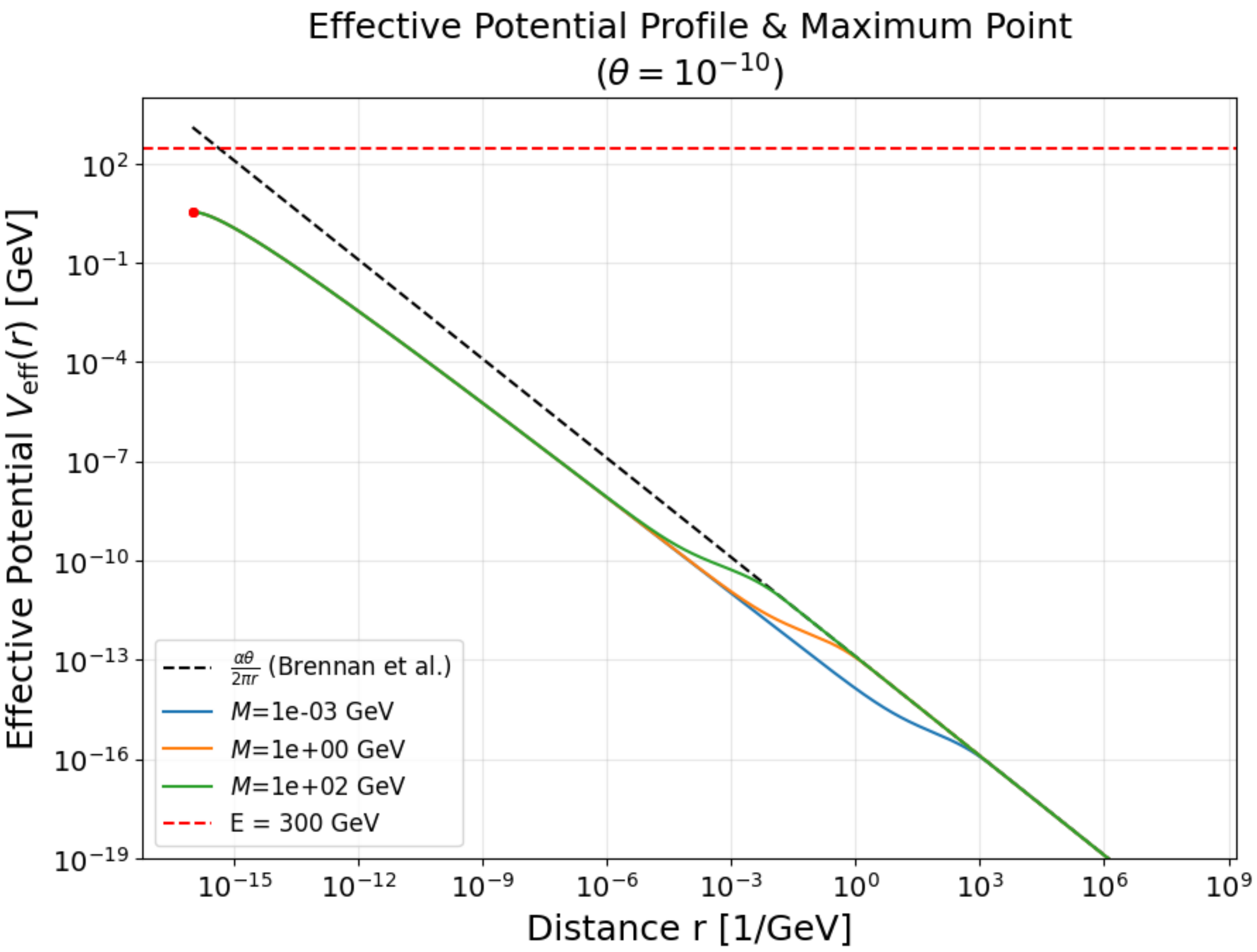
Mass dependence of screening confirmed.

Results: Effective Potential Behavior

- Analysis Content:
  Maximum value of the effective potential
  $V_{\text{eff}}(r) = \frac{\alpha\theta}{2\pi}\int_r^\infty \frac{dr'}{r'^2}\frac{Q(r')}{Q(\infty)}$ for different fermion masses M

- Parameter Settings:
  - Charges:
    $q_1 = -2$ (corresponding to electron),
    $q_2 = -2/3$ (corresponding to down quark)
  - GUT Scale:
    $M_{\text{GUT}} = 10^{16}$ GeV

- Numerical Results:
  - $M = 1$ MeV: Max value ≈ 3.41 GeV
  - $M = 1$ GeV: Max value ≈ 3.41 GeV
  - $M = 100$ GeV: Max value ≈ 3.41 GeV



Result: Regardless of Mass M, $V_{\text{max}} \approx 3.41$ GeV $\ll T \sim 100$ GeV

Conclusion and Outlook

- Comparison with Potential Barrier:
  Value of effective potential $V_{\text{eff}} \sim 3$ GeV   <<   Energy scale $E \sim 300$ GeV

- Conclusion:
  With realistic masses, repulsion suppression is **ineffective**.
  (Mechanism by Brennan et al. fails in this setup)

- Outlook:
  - Analytical proof of cancellation mechanism.
  - Detailed BC investigation.
  - Non-degenerate fermion masses
  - Extension to Multi-flavor ($N_f \geq 4$).

Research Method: Partial Wave Expansion and Bosonization

[12] : Callan, 1982
[10] : Hook, 2025

- Partial Wave Expansion [12]:
  - Angular momentum operator in the presence of a background monopole field: $J = L + S + T$ (L : Orbital, S : Spin, T : Isospin)
  - Modes with $J \neq 0$ experience repulsion due to centrifugal force. The $J = 0$ mode does not experience centrifugal repulsion and reaches the monopole core.
  - The $J = 0$ mode has no angular dependence (spherically symmetric) and can be effectively represented by a (1+1) dimensional spinor $\chi(r, t)$.
  - Boundary conditions are required at the monopole surface ($r = r_c$).
- Bosonization [10]:
  - There exists a boson field theory $\phi(r, t)$ equivalent to the (1+1) dimensional fermion field theory $\chi(r, t)$.

    Boundary condition for field values: $\phi_1(r_c) = \phi_2(r_c)$
    Boundary condition for field derivatives: $\partial_r\phi_1(r_c) = -\partial_r\phi_2(r_c)$

    There are 2 fermions ($N_f = 2$);
    the corresponding boson solutions are ϕ_1, ϕ_2.

Analysis Method: Screening Mode (Particular Solution)

- Inhomogeneous Equation:

$$\left[\partial_r^2 - M^2 - \frac{\lambda Q_{\text{eff}}^2}{r^2}\right] u_S(r) = -\frac{2\lambda\theta}{r^2}$$

- Particular Solution:

Assume a series solution: $u_S^{\text{part}}(r) = r^s \sum_{n=0}^{\infty} c_n r^n = \sum_{n=0}^{\infty} c_n r^{n+s} \quad (c_0 \neq 0)$ and substitute.

LHS $= \sum_{n=0}^{\infty} c_n \left[(n+s)(n+s-1) - \Lambda\right] r^{n+s-2} - \sum_{n=0}^{\infty} c_n M^2 r^{n+s}$

Comparing with LHS: $s = 0, \quad c_0 = \frac{2\theta}{Q_{\text{eff}}^2}, \quad c_{2k-1} = 0, \quad c_{2k} = \frac{M^2}{(2k)(2k-1) - \Lambda} c_{2k-2}$

$$u_S^{\text{part}}(r) = c_0 \left[1 + \frac{M^2}{2-\Lambda} r^2 + \frac{M^4}{(12-\Lambda)(2-\Lambda)} r^4 + \dots\right]$$

Analysis Method: Screening Mode (Selection of Physical Solution)

- Charge Ratio and Behavior at Infinity:
  - Ratio of effective charge $Q(r)_{EM}$ at distance r to the charge at infinity $Q(\infty)_{EM}$: $\dfrac{Q(r)_{EM}}{Q(\infty)_{EM}} = 1 - \dfrac{Q_{\text{eff}}^2}{2\theta} u_S(r)$

    At distance $r \to \infty$, the screening effect should vanish,
    and the bare monopole charge should be visible; thus, the ratio should approach 1.
  - The particular solution part $u_S^{\text{part}}(r)$ and the homogeneous component $I_\nu(Mr)$ increase with r and diverge at $r \to \infty$.
  - The coefficient A is determined numerically to cancel these out,
    leaving only the $K_\nu(Mr)$ component as dominant.
- Determination of Coefficients:
  Due to cancellation with the singular term of the equation near the origin,
  the screening mode has the constant value c_0.

  From the connection condition at $r = r_c$: $u_S(r_c) = c_0 = \dfrac{2\theta}{Q_{\text{eff}}^2}$, we determine coefficient B: $B = \dfrac{2\theta}{Q_{\text{eff}}^2} \dfrac{1}{\sqrt{Mr_c} K_\nu(Mr_c)}$

Discussion: Energy vs Potential Barrier

[11] : Anchordoqui, 2011

- Incident Fermion Energy:
  Using the fact that the average energy of relativistic fermions is about 3 times the temperature [11]: $E \sim 3T \approx 300$ GeV.

- Comparison with Potential Barrier:
  Value of effective potential $V_{\text{eff}} \sim 3$ GeV << Energy scale $E \sim 300$ GeV

- Conclusion:
  When realistic fermion masses are considered, the "Monopole Baryogenesis using repulsion suppression" proposed by Brennan et al. does not work, at least under this simple setup.