Forbidden Dark Matter with Sommerfeld Enhancement

Takumu Yamanaka The University of Osaka

Collaboration with Tomohiro Abe (Tokyo Univ. of Sci.) and Ryosuke Sato (UOsaka)

Feb. 17th , 2026

[Paper In Preparation]

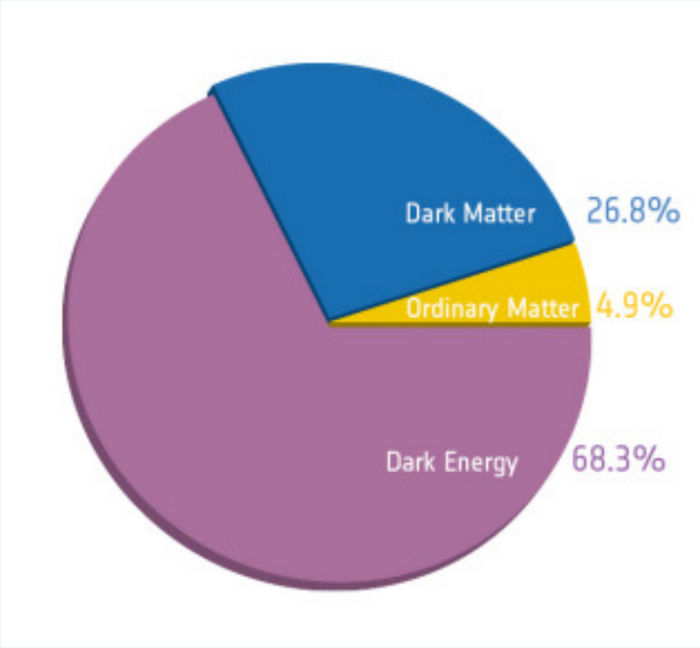
KEK Theory Meeting on Particle Physics Phenomenology (KEK-PH 2026Winter)

Introduction

Dark Matter (DM)

energy density: $\Omega h^2 \simeq 0.12$

[Planck Collaboration (2018)]



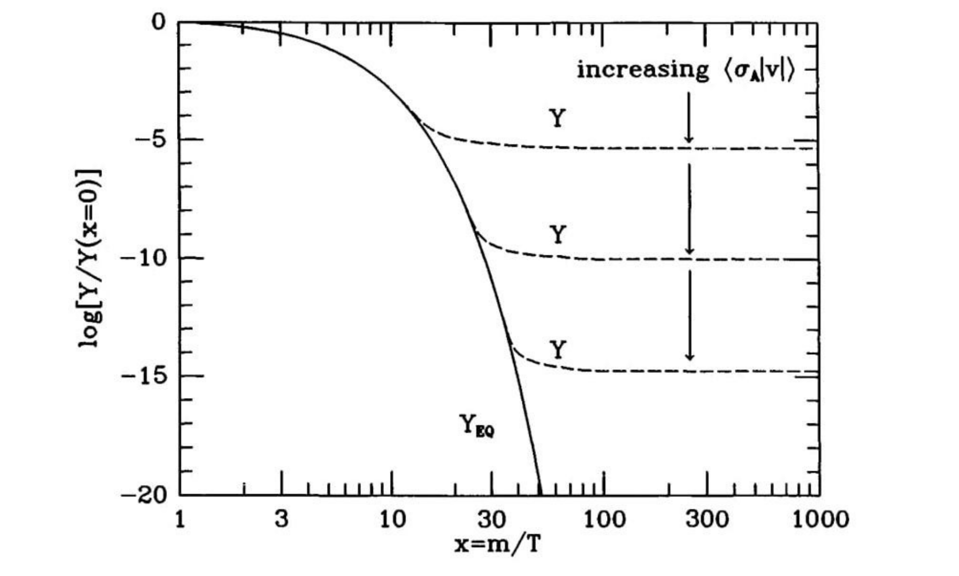
https://sci.esa.int/web/planck/-/51557-planck-new-cosmic-recipe

Freeze-out mechanism

DM decouples from thermal bath in the early universe

$$\Omega h^2 \sim 0.12 \frac{2\times 10^{-26}\ \mathrm{cm}^3/\mathrm{s}}{\langle \sigma v \rangle}$$

$\langle \sigma v \rangle$: thermally averaged annihilation cross section of DM



Kolb, Turner *"The Early Universe"*

Sommerfeld Enhancement (SE)

[Hisano, Matsumoto, Nojiri, Saito (2005)]

[Arkani-Hamed, Finkbeiner, Slatyer, Weiner (2009)]

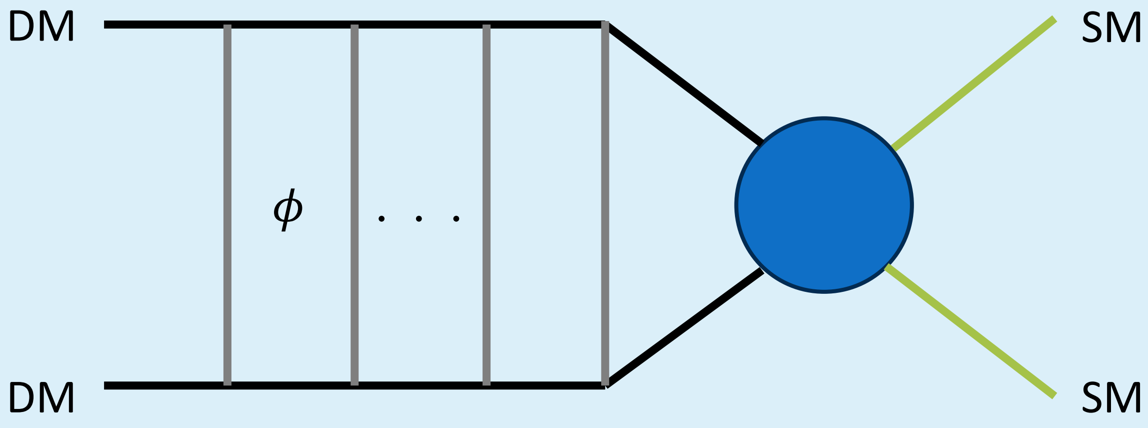
Sommerfeld Enhancement

σv_{rel} is enhanced if

- two-body states feel attractive long-range potential

&

- two-body states moves w/ low velocity $v \ll 1$

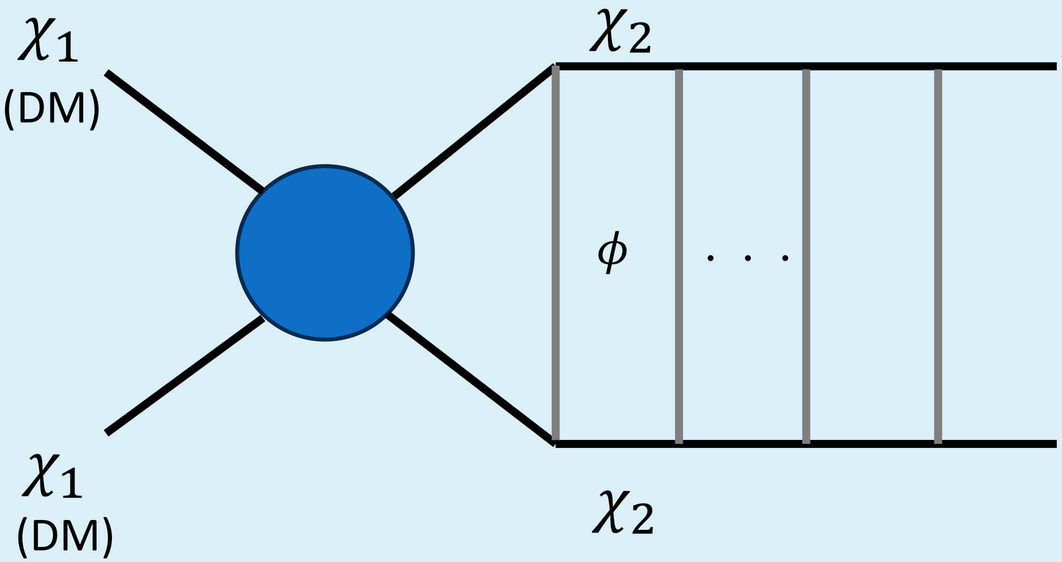


ϕ: light mediator ($m_\phi \ll m_{\mathrm{DM}}$)

Enhanced cross section: $\sigma_{\mathrm{SE}} v = S(v) \times \sigma_{\mathrm{LO}} v$

$S(v)$ obtained by solving Schrödinger eq. of two-body states.

Final-state SE [Cui, Luo (2020)]



Forbidden channels

[Griest, Seckel (1991), D’ Agnolo, Ruderman (2016)]

χ_1: DM w/ mass m_1

χ_2: Annihilation product w/ mass $m_2 > m_1$

The velocity of χ_2 is smaller than χ_1 velocity

→ SE from final state particles can be more significant

Instability of Annihilation Products

χ_2 **must decay → Final-state SE depends on lifetime of** χ_2

Previous work [Cui, Luo (2020)] cutoff velocity is introduced (cutoff method)

$$S_f^{(\mathrm{cut})}(v_2) \sim \begin{cases} S_f(v_2) & (v_{cut} < v_2) \\ 1 & (0 < v_2 < v_{cut}) \end{cases}$$

$$v_{cut} = \sqrt{\frac{\Gamma}{m_2}}$$

Γ: decay width of χ_2

$t\bar{t}$ **production** [Fadlin, Khoze (1987), Strassler, Peskin (1991), Hagiwara, Sumino (2008)]

Schrödinger eq. which replace $E \to E + i\Gamma$ is solved.

$t\bar{t}$ bound states enhance the production cross section

Bound states of annihilation products → impact on Ωh^2 **prediction**

Formulation of Final-state SE

Schrödinger eqs. of wave functions of $\chi_1\overline{\chi_1}$ ($\psi_1(\boldsymbol{r})$) and $\chi_2\overline{\chi_2}$ ($\psi_2(\boldsymbol{r})$) (we focus on the s-wave processes.)

$$\begin{cases} \left[-\frac{1}{2\mu_1}\nabla^2 - E_1\right]\psi_1(\boldsymbol{r}) = u\delta^3(\boldsymbol{r})\psi_2(\boldsymbol{0}) \\ \left[-\frac{1}{2\mu_2}\nabla^2 + V(r) - (E_2 + i\Gamma)\right]\psi_2(\boldsymbol{r}) = u\delta^3(\boldsymbol{r})\psi_1(\boldsymbol{0}) \end{cases}$$

boundary conditions at $r \to \infty$

$$\begin{pmatrix} \psi_1(\boldsymbol{r}) \\ \psi_2(\boldsymbol{r}) \end{pmatrix} \to \begin{pmatrix} e^{ip_1 z} + f_1(\theta)\frac{e^{ip_1 r}}{r} \\ f_2(\theta)\frac{e^{ip_2 r}}{r} \end{pmatrix}$$

$$p_1 \equiv \sqrt{2\mu_1 E_1}, \quad p_2 \equiv \sqrt{2\mu_2(E_2 + i\Gamma)}$$

$$E_2 = E_1 - 2(m_2 - m_1)$$

annihilation cross section (Born approximation)

$$\sigma v = -\int d^3r\, \nabla\cdot\boldsymbol{j_1}(\boldsymbol{r}) \simeq 4\mu_2 u^2 \mathrm{Im}\, G_2(\boldsymbol{0}; E_2 + i\Gamma)$$

$$\begin{cases} \left[-\frac{1}{2\mu_2}\nabla^2 + V(r) - (E_2 + i\Gamma)\right] G_2(\boldsymbol{r}; E_2 + i\Gamma) = \frac{1}{2\mu_2}\delta^3(\boldsymbol{r}) \\ G_2(\boldsymbol{r}; E_2 + i\Gamma) \to \frac{d_2}{4\pi r}e^{ip_2 r} \quad (r \to \infty) \end{cases}$$

→ Final state SE factor $S_f(E_2, \Gamma) = \frac{\mathrm{Im}G_2(\boldsymbol{0}; E_2 + i\Gamma)}{\mathrm{Im}G_2^{\mathrm{free}}(\boldsymbol{0}; E_2 + i\Gamma)}$

Cross Section

e.g.) Attractive Coulomb potential

$$V(r) = -\frac{\alpha}{r}$$

$\Gamma \neq 0$ allows off-shell final state particles

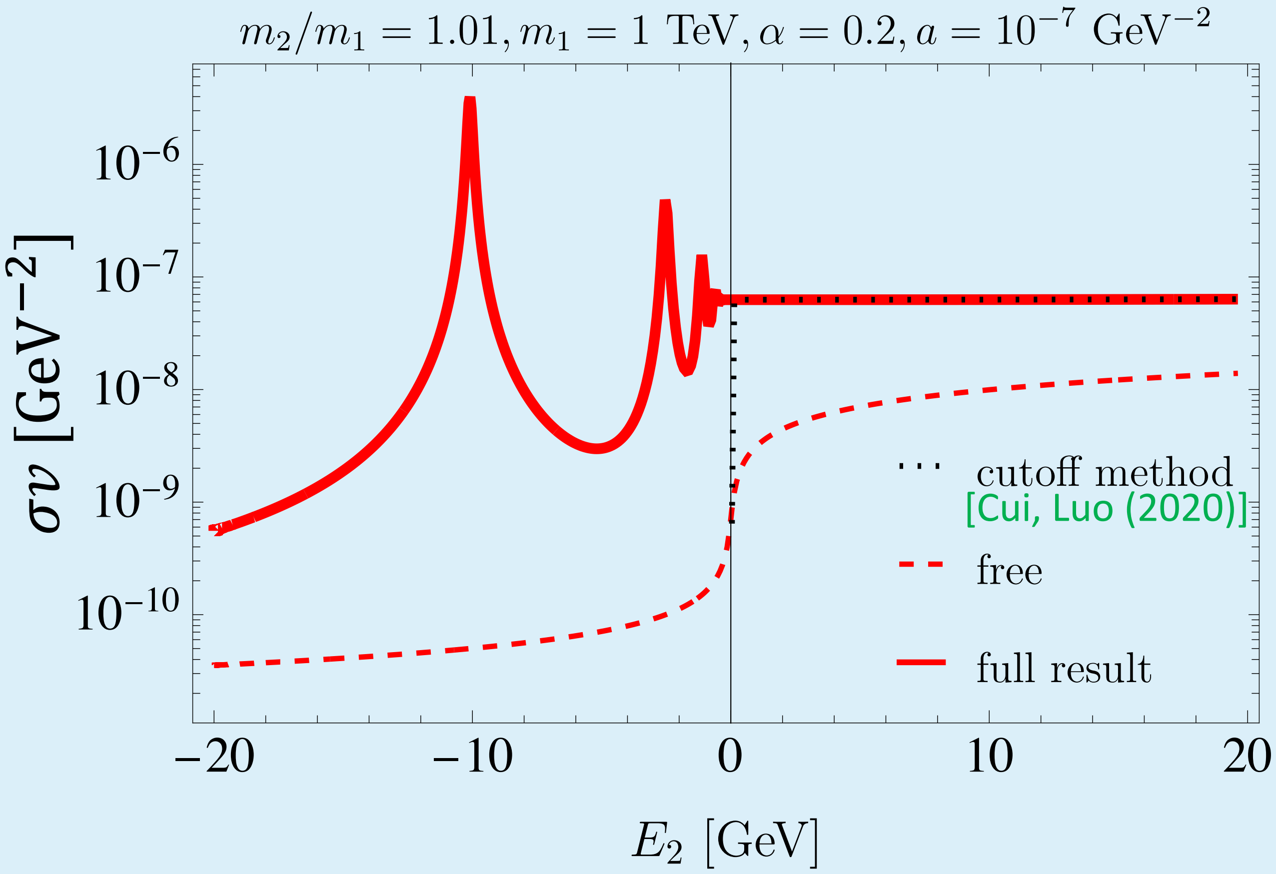
Resonances of bound states

$$\mathcal{E}_n = -\frac{\mu_2 \alpha^2}{2n^2}, \qquad (n = 1,2,\cdots)$$

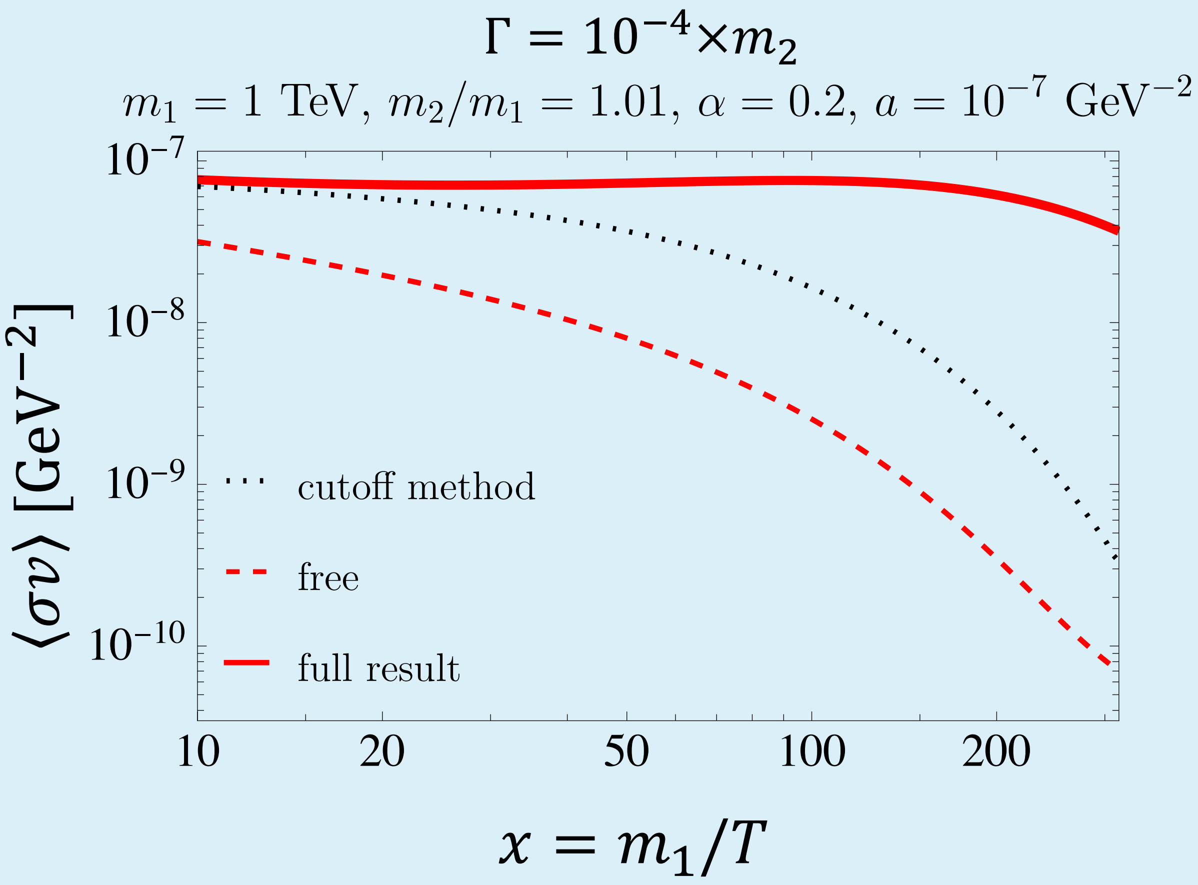
Around $E_2 = \mathcal{E}_n$,

$$S_f(E_2, \Gamma) \propto \frac{\Gamma}{(E_2 - \mathcal{E}_n)^2 + \Gamma^2}$$

$\Gamma/m_2 = 10^{-4}$



Temperature Dependence



Forbidden channels w/o SE

$$\langle \sigma v \rangle \propto \exp\left(-2\left(\frac{m_2}{m_1} - 1\right)x\right)$$

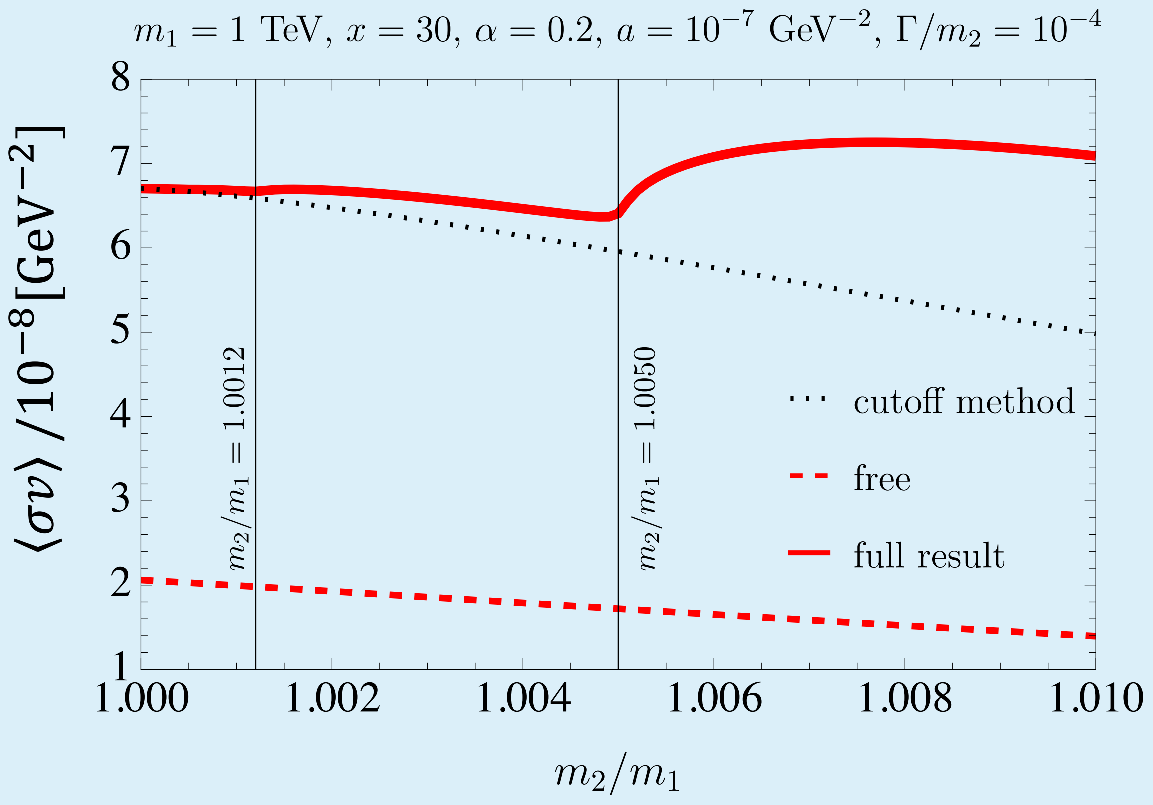
[Griest, Seckel (1991), D' Agnolo, Ruderman (2016)]

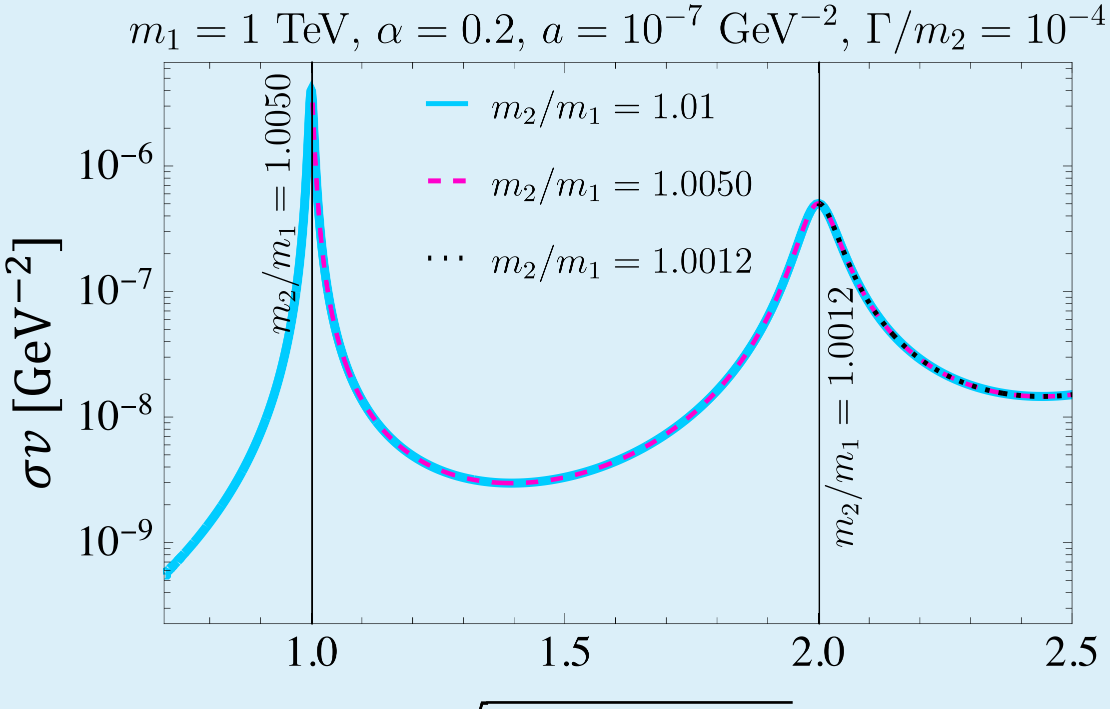
Our full result deviates from the cutoff method result.

Effect of Resonances

binding energy

$$\mathcal{E}_n = -\frac{\mu_2 \alpha^2}{2n^2}, \qquad (n = 1,2,\cdots)$$





with $E_2 < 0$

The minimum of E_2: $E_2^{\min} = -2(m_2 - m_1) = -2m_1 \left(\frac{m_2}{m_1} - 1\right)$

Resonances are kinematically allowed if $2m_2 + \mathcal{E}_n \geq 2m_1$

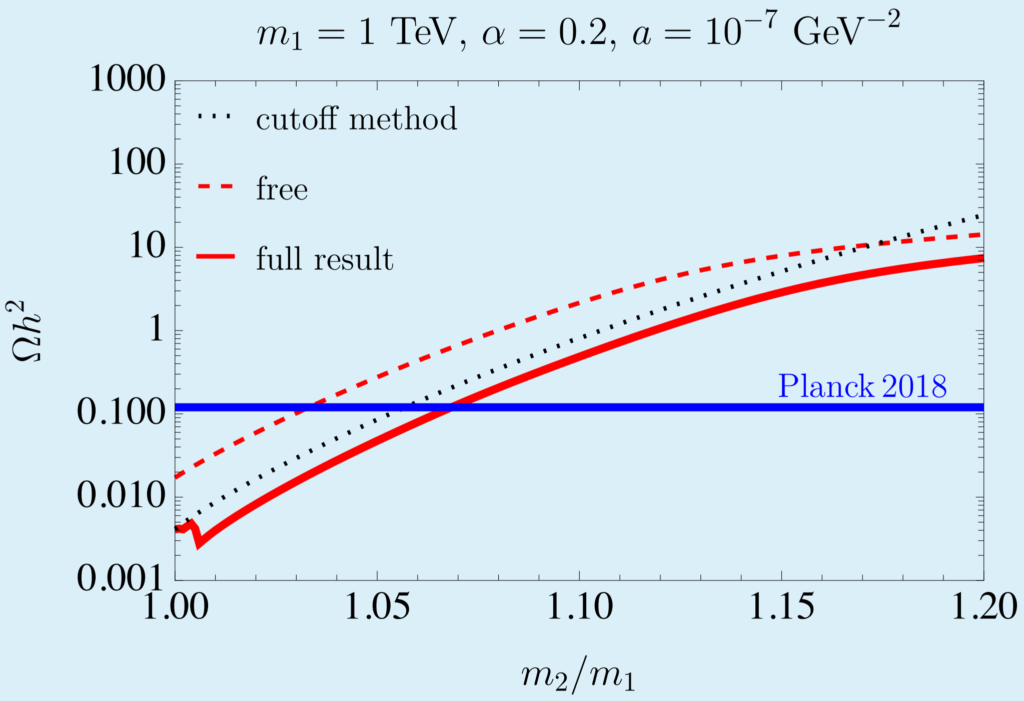
n-th bound state resonance contributes if $E_2^{\min} \leq \mathcal{E}_n$

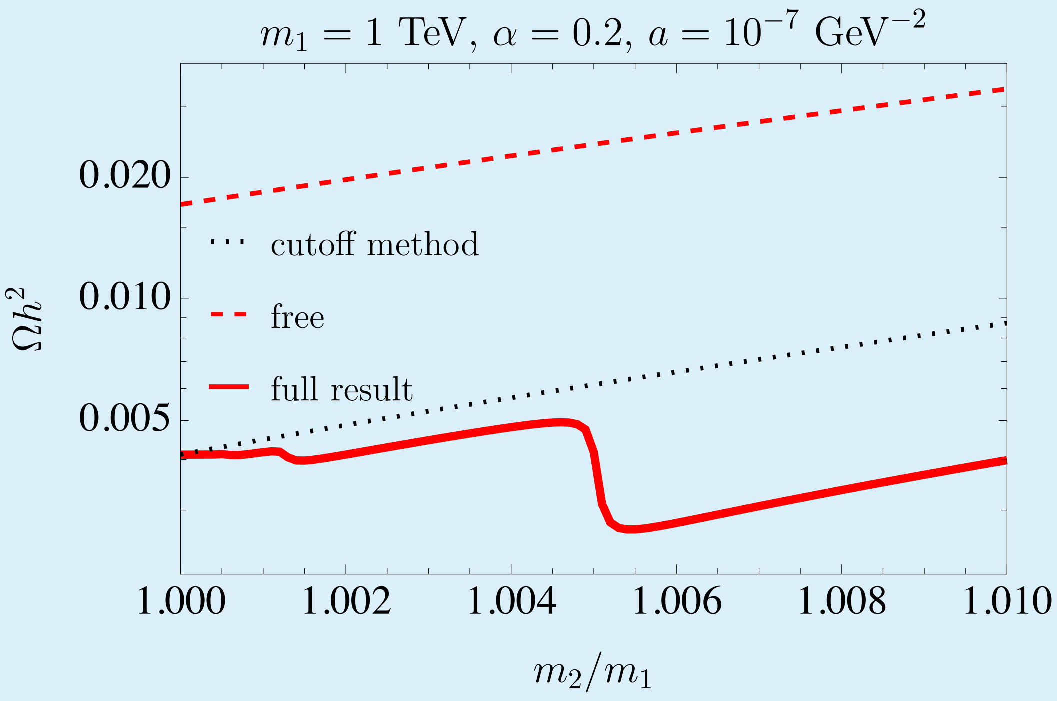
DM Relic Abundance

Boltzmann eq.

$$\frac{dn_1}{dt} + 3Hn_1 = -\langle\sigma v\rangle(n_1^2 - n_{1,eq}^2)$$

$$\Gamma = 10^{-4} \times m_2$$





The predicted mass ratio is modified by resonances

Summary

- In forbidden channel, the final-state SE is more significant than the initial-state SE.
- Instability of annihilation products needs to be considered when calculating the final-state SE.
- We formulate the final-state SE by solving the Schrödinger eq. with the decay width.
- Resonances of bound states enhances $\langle \sigma v \rangle$ → decreases Ωh^2

Future prospect

Application to specific models including forbidden channels

Back up

Bound States in Finite Temperature

Mediator gains thermal mass $m_{\text{th}}^2 \sim \alpha^{\frac{1}{2}} T^2 \rightarrow$ Long-range int. is screened.

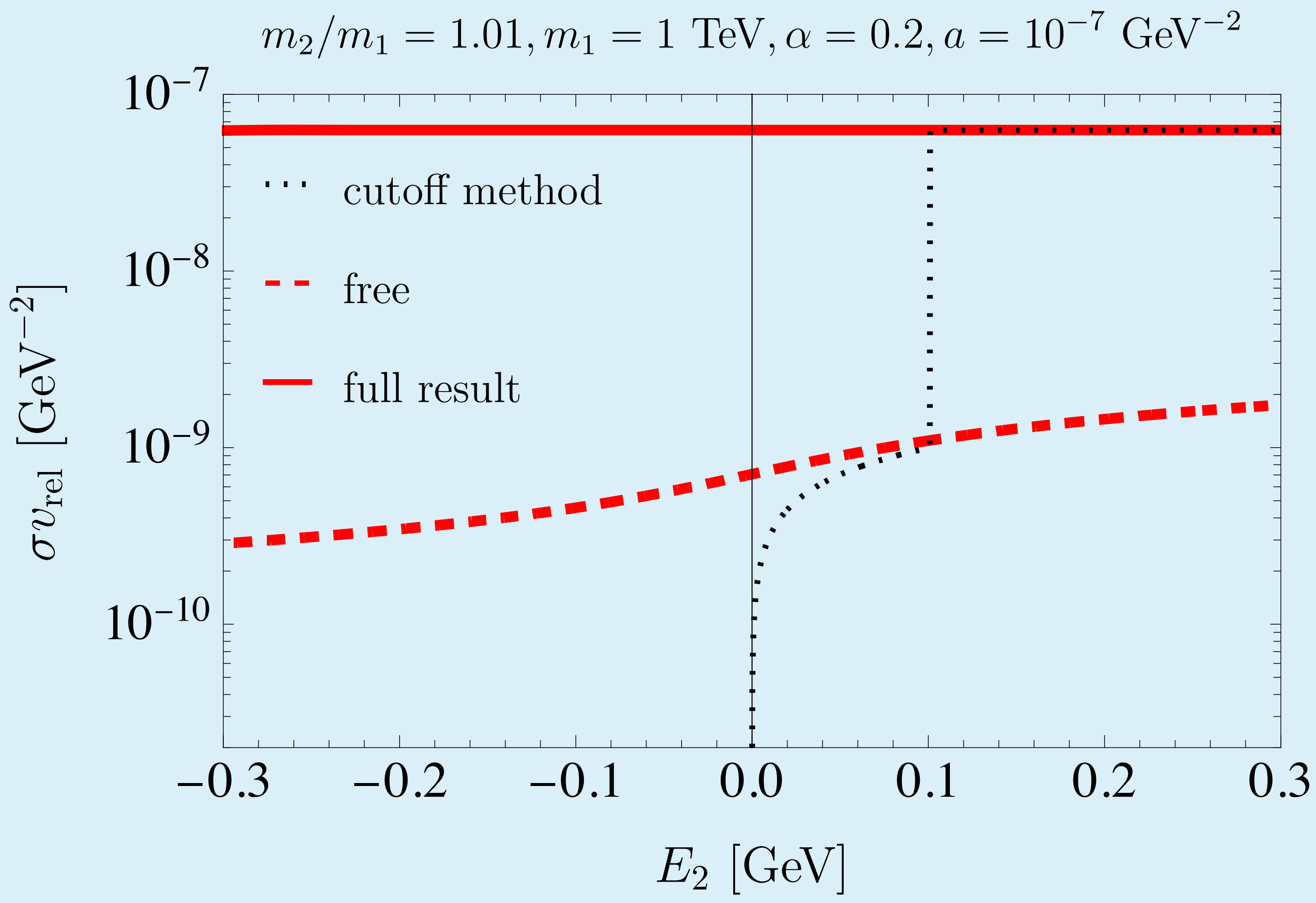
Criteria for the existence of bound states [Kim, Laine (2017)]

1. (screening length) > (Bohr radius) : $\frac{1}{m_{\text{th}}} \gtrsim \frac{1}{\mu_2 \alpha} \Leftrightarrow T \lesssim \alpha^{\frac{1}{2}} m_2$
2. (interaction rate) <(binding energy) : $\frac{T^3}{m_2^2} \lesssim m_2 \alpha^2 \Leftrightarrow T \lesssim \alpha^{\frac{2}{3}} m_2$

the simple power counting: $T \lesssim \alpha m_2$

If DM freezes out at $T_f = \frac{m_1}{30}$, $\alpha \gtrsim \frac{1}{30} \times \frac{m_1}{m_2}$

The Cutoff method



SE in the cutoff method

$$S_f^{(\mathrm{cut})}(E_2) = \begin{cases} S_f(E_2, 0) & (v_{cut} < v_2) \\ 1 & (0 < v_2 < v_{cut}) \end{cases}$$

In this plot,

$$v_{cut} = \sqrt{\frac{\Gamma}{m_2}} \simeq 10^{-2}$$

Thermal Average

$$(\sigma v_{\rm rel})_{\rm free} = 4\mu_2 u^2 \times \frac{1}{4\pi}\ {\rm Re}p_2 \equiv a\tilde{v}_2$$

$$\begin{cases} a = \frac{m_2^2 u^2}{2\pi} \\ \tilde{v}_2 = {\rm Re}\frac{p_2}{m_2} = {\rm Re}\sqrt{\frac{E_1 - 2(m_2 - m_1) + i\Gamma}{m_2}} \end{cases}$$

Averaged cross section

$$\langle \sigma v_{\rm rel} \rangle = a\int_0^\infty dv_{\rm rel}\ v_{\rm rel}^2 S_f(E_2, \Gamma)\tilde{v}_2 \exp\left(-\frac{v_{\rm rel}^2 x}{4}\right)$$

$$x = m_1/T$$

cutoff method [Cui, Luo (2020)]

$$\langle \sigma v_{\rm rel} \rangle = a\int_{v_{\rm th}}^\infty dv_{\rm rel}\ v_{\rm rel}^2 S_f^{\rm (cut)}(E_2) v_2 \exp\left(-\frac{v_{\rm rel}^2 x}{4}\right)$$

$$\begin{cases} S_f^{\rm (cut)}(E_2) = \begin{cases} S_f(E_2, 0) & (\sqrt{\Gamma/m_2} < v_2) \\ 1 & (0 < v_2 < \sqrt{\Gamma/m_2}) \end{cases} \\ v_2 = \sqrt{\frac{E_1 - 2(m_2 - m_1)}{m_2}} \end{cases}$$

$v_{\rm th}$: threshold velocity of χ_1

Formulation of Final State SE

Schrödinger eqs. of wave functions of $\chi_1\overline{\chi_1}$ ($\psi_1(\boldsymbol{r})$) and $\chi_2\overline{\chi_2}$ ($\psi_2(\boldsymbol{r})$) (we focus on the s-wave processes.)

$$\begin{cases} \left[-\frac{1}{2\mu_1}\nabla^2 - E_1\right]\psi_1(\boldsymbol{r}) = u\delta^3(\boldsymbol{r})\psi_2(\boldsymbol{0}) \\ \left[-\frac{1}{2\mu_2}\nabla^2 + V(r) - (E_2 + i\Gamma)\right]\psi_2(\boldsymbol{r}) = u\delta^3(\boldsymbol{r})\psi_1(\boldsymbol{0}) \end{cases}$$

boundary conditions at $r \to \infty$

$$\begin{pmatrix} \psi_1(\boldsymbol{r}) \\ \psi_2(\boldsymbol{r}) \end{pmatrix} \to \begin{pmatrix} e^{ip_1 z} + f_1(\theta)\frac{e^{ip_1 r}}{r} \\ f_2(\theta)\frac{e^{ip_2 r}}{r} \end{pmatrix}$$

$$p_1 \equiv \sqrt{2\mu_1 E_1}, \quad p_2 \equiv \sqrt{2\mu_2(E_2 + i\Gamma)}$$

$$E_2 = E_1 - 2(m_2 - m_1)$$

Probability current

$$\boldsymbol{j_1}(\boldsymbol{r}) = \frac{1}{\mu_2}\mathrm{Im}\left(\psi_1^*(\boldsymbol{r})\nabla\psi_1(\boldsymbol{r})\right) \Rightarrow \nabla\cdot\boldsymbol{j_1}(\boldsymbol{r}) = -2u\delta^3(\boldsymbol{r})\mathrm{Im}\left(\psi_1^*(\boldsymbol{0})\psi_2(\boldsymbol{0})\right)$$

annihilation cross section

$$\sigma v = -\int d^3r\, \nabla\cdot\boldsymbol{j_1}(\boldsymbol{r}) = 2u\mathrm{Im}\left(\psi_1^*(\boldsymbol{0})\psi_2(\boldsymbol{0})\right)$$

Formulation of Final State SE

Let us solve $$\left[-\frac{1}{2\mu_2}\nabla^2 + V(r) - (E_2 + i\Gamma)\right]\psi_2(\boldsymbol{r}) = u\delta^3(\boldsymbol{r})\psi_1(\boldsymbol{0})$$

Green's function

$$\left[-\frac{1}{2\mu_2}\nabla^2 + V(r) - (E_2 + i\Gamma)\right]G_2(\boldsymbol{r}; E_2 + i\Gamma) = \frac{1}{2\mu_2}\delta^3(\boldsymbol{r})$$

boundary condition at $r \to \infty$

$$G_2(\boldsymbol{r}; E_2 + i\Gamma) \to \frac{d_2}{4\pi r}e^{ip_2 r}$$

$$\Rightarrow \quad \psi_2(\boldsymbol{r}) = 2\mu_2 u G_2(\boldsymbol{r}; E_2 + i\Gamma)\psi_1(\boldsymbol{0})$$

Using Born approximation,

$$\sigma v = 2u\,\mathrm{Im}\big(\psi_1^*(\boldsymbol{0})\psi_2(\boldsymbol{0})\big) \simeq 4\mu_2 u^2\,\mathrm{Im}G_2(\boldsymbol{0};\ E_2 + i\Gamma)$$

→ SE factor $S_f(E_2, \Gamma) = \dfrac{\mathrm{Im}G_2(\boldsymbol{0}; E_2 + i\Gamma)}{\mathrm{Im}G_2^{\mathrm{free}}(\boldsymbol{0}; E_2 + i\Gamma)}$

Source of the Resonance

SE under Coulomb potential

$$S_f(E_2,\Gamma) = \frac{\mu_2\alpha}{\mathrm{Re}p_2}\left[\pi - 2\arg p_2 - 2\mathrm{Im}\,\psi\left(-\frac{i\mu_2\alpha}{p_2}\right) - \mathrm{Re}\left(\frac{p_2}{\mu_2\alpha}\right)\right]$$

Around $E_2 = \mathcal{E}_n$, and $\frac{\Gamma}{\mu_2\alpha} \ll 1$,

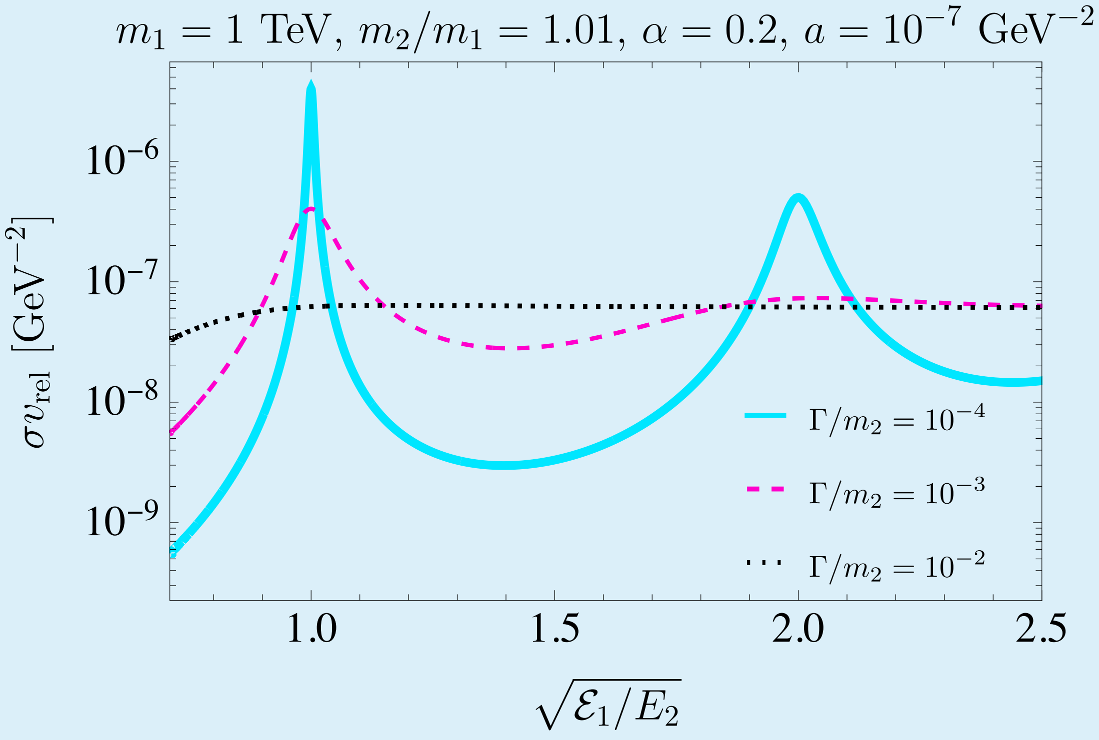
$$-\frac{i\mu_2\alpha}{p_2} \simeq -n - \frac{n^3}{\mu_2\alpha^2}[(E_2 - \mathcal{E}_n) + i\Gamma]$$

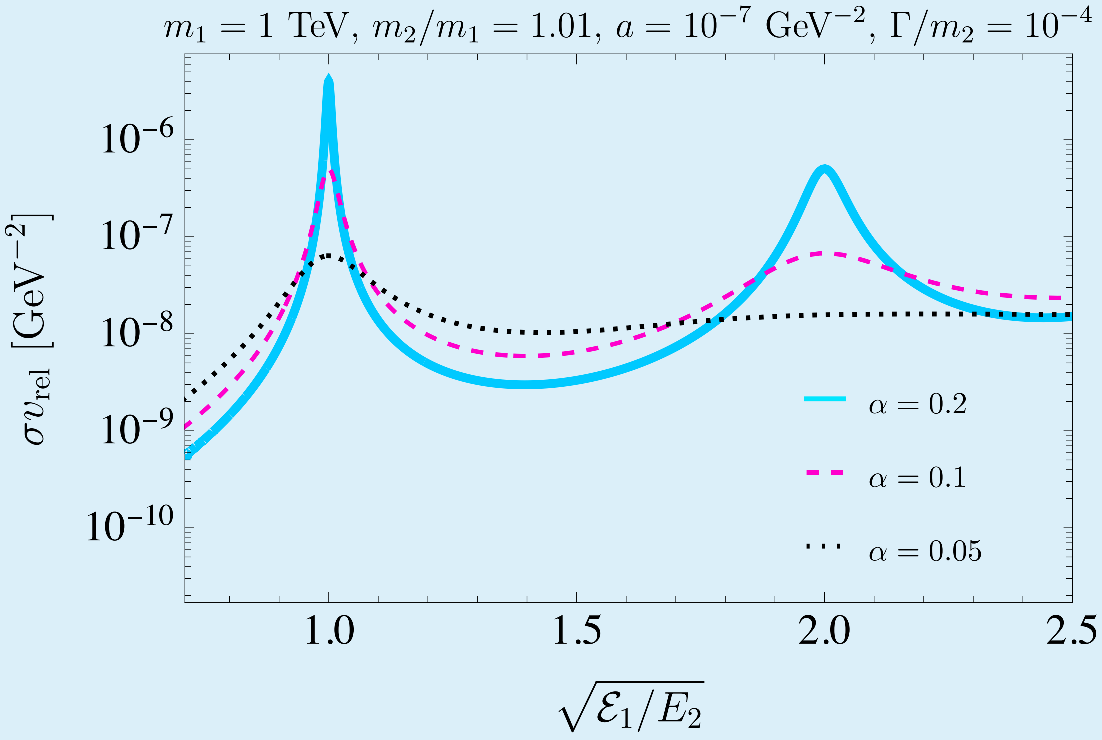
$z = -n$ is a pole of $\psi(z)$ $\quad \psi(z) \simeq -\frac{1}{z+n}$ $\Rightarrow$ $\psi\left(-\frac{i\mu_2\alpha}{p_2}\right) \simeq -\frac{\mu_2\alpha^2}{n^3}\frac{1}{(E_2 - \mathcal{E}_n) + i\Gamma}$

$\Rightarrow$ $S_f(E_2,\Gamma) \simeq \frac{\mu_2^2\alpha^3}{n^3}\frac{\Gamma}{(E_2 - \mathcal{E}_n)^2 + \Gamma^2}$ around $E_2 = \mathcal{E}_n$

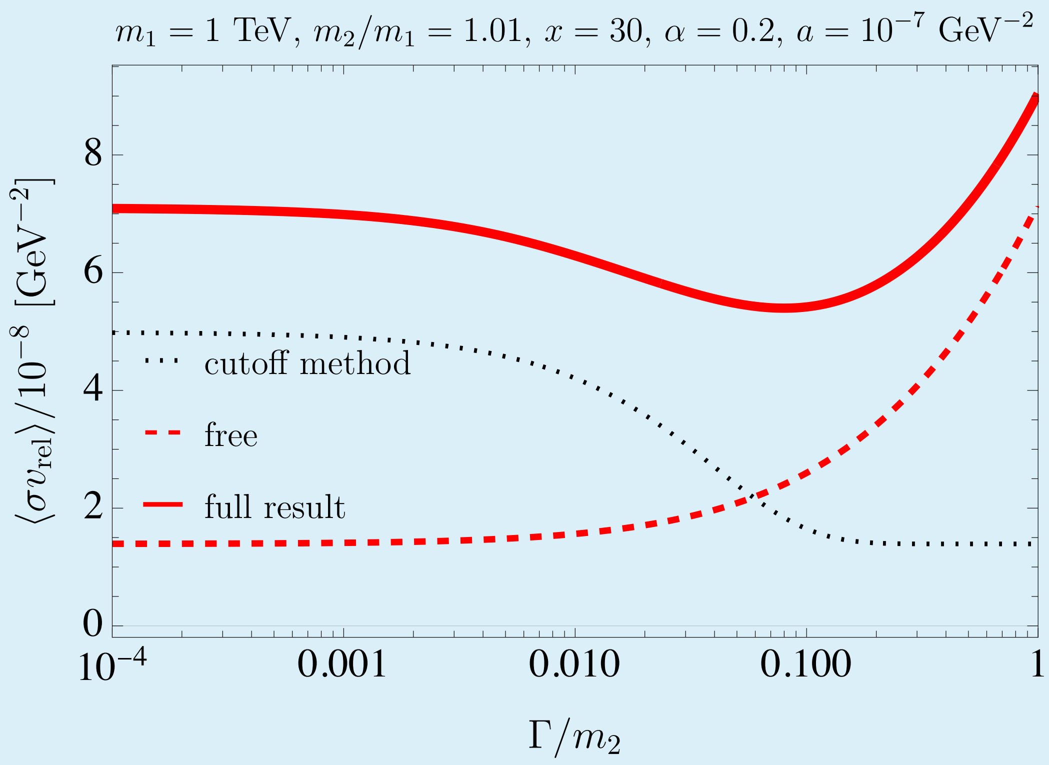
Size of the Peak

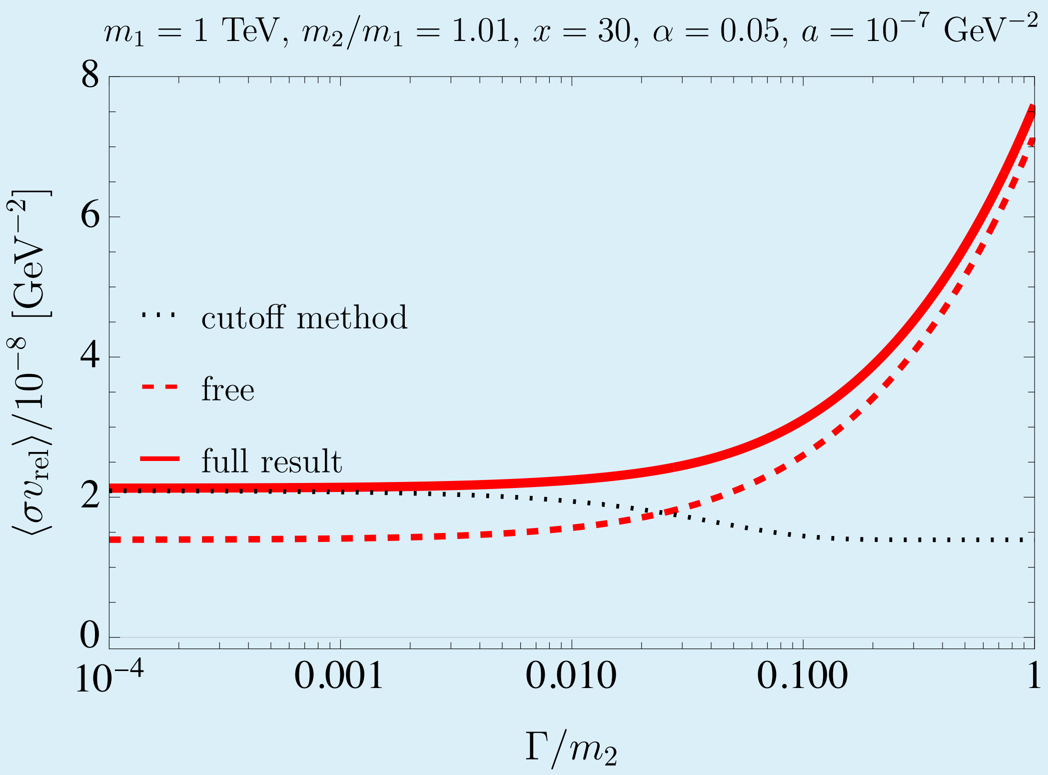
$$S_f(E_2,\Gamma) \simeq \frac{\mu_2^2\alpha^3}{n^3}\frac{\Gamma}{(E_2-\mathcal{E}_n)^2+\Gamma^2} \quad \text{if } \Gamma \ll \mu_2\alpha^2$$





Dependence on Decay Width





narrow width : bound state resonances

wide width: off-shell final states

$$S_f(E_2, \Gamma) \simeq 1 + \frac{\pi}{2}\alpha\sqrt{\frac{\mu_2}{\Gamma}} + O(\alpha^2)$$

Attractive Hulthén Potential ①

$$V_H(r) = -\frac{\alpha m_* e^{-m_* r}}{1 - e^{-m_* r}}$$

(Approximation of the Yukawa potential)

$$V_Y(r) = -\frac{\alpha}{r} e^{-m_\phi r}, \qquad m_\phi = \frac{\pi^2}{6} m_*$$

$$g_2(r; E_2 + i\Gamma) = \frac{\Gamma(1-\alpha_+)\Gamma(1-\alpha_-)}{\Gamma(1-\alpha_+-\alpha_-)}\, e^{ip_2 r}\, {}_2F_1(-\alpha_-, -\alpha_+; 1-\alpha_+-\alpha_-; e^{-m_* r})$$

where $G_2(\boldsymbol{r}; E_2 + i\Gamma) = \frac{g_2(r; E_2 + i\Gamma)}{4\pi r}$

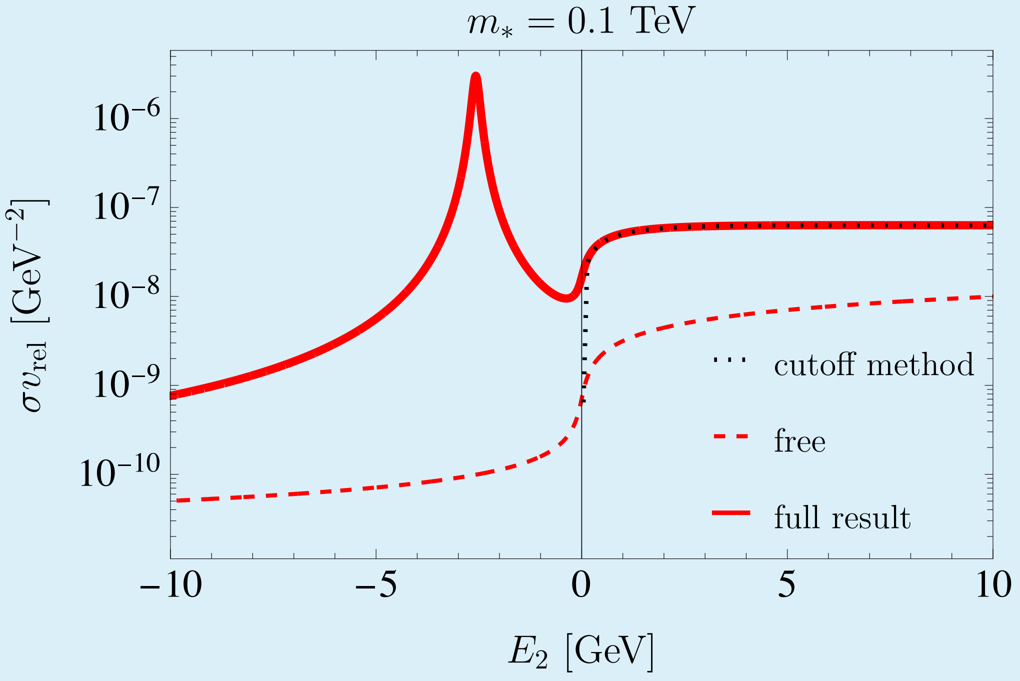
Binding energy

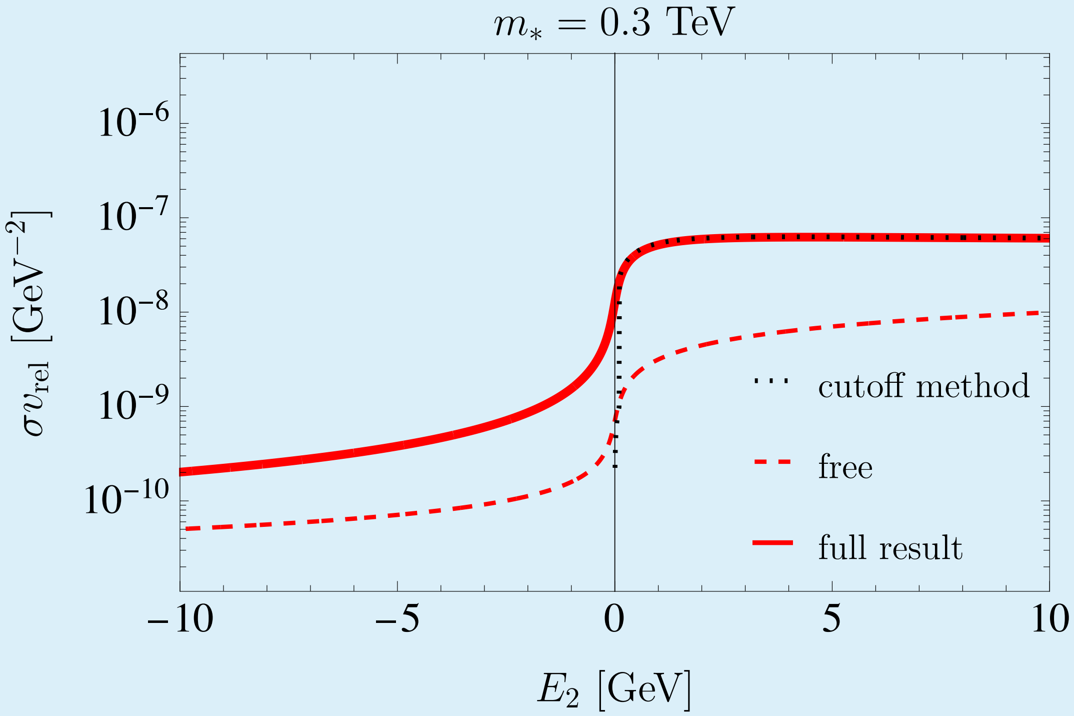
$$\mathcal{E}_n = -\frac{\mu_2 \alpha^2}{2n^2}\left(1 - \frac{n^2 m_*}{2\mu_2 \alpha}\right)^2$$

Bound states exists if $m_* \leq 2\mu_2 \alpha$

Attractive Hulthén Potential ②

$$\frac{m_2}{m_1} = 1.01, \quad \alpha = 0.2, \quad m_1 = 1\ \text{TeV}, \quad \frac{\Gamma}{m_2} = 10^{-4}, \quad a = 10^{-7}\ \text{GeV}^{-2}$$

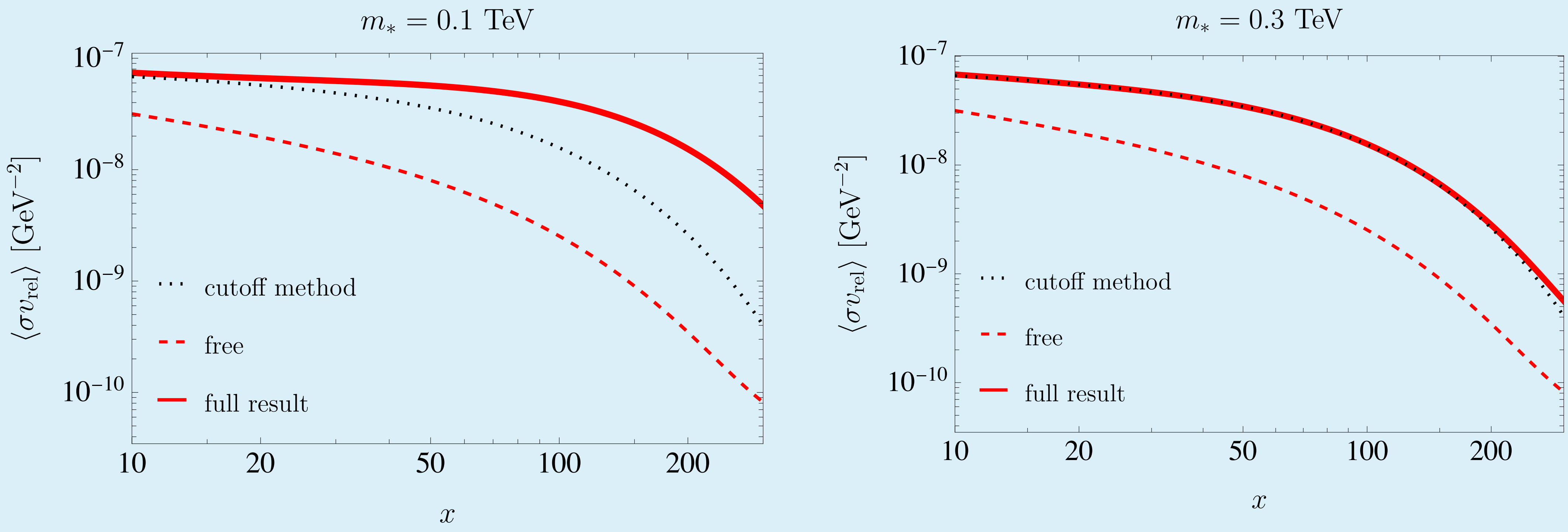




Bound state exists if $m_* \leq 2\mu_2\alpha \simeq 0.2$ TeV

Attractive Hulthén Potential ③

$$\frac{m_2}{m_1} = 1.01, \qquad \alpha = 0.2, \qquad m_1 = 1\ \text{TeV}, \qquad \frac{\Gamma}{m_2} = 10^{-4}$$



Attractive Hulthén Potential ④