# One-Loop QED Corrections to Leptonic $D_s^+$ Decays and CKM Unitarity Test


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# CKM matrix and New Physics

- Cabibbo-Kobayashi-Maskawa (CKM) matrix is unitary in the SM

- Each component of the CKM matrix is measured without assuming unitarity

Kaon decays

B meson decays

$\beta$ decays

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

D meson decays

K and B mixing

SM : $V_{CKM}^{\dagger} V_{CKM} = V_{CKM} V_{CKM}^{\dagger} = \mathbf{1}$

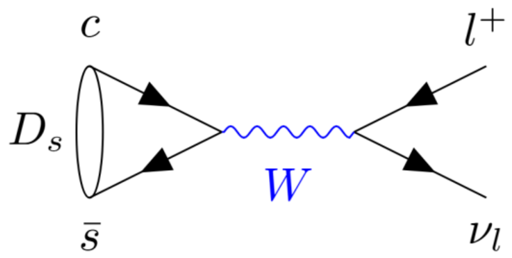
VS

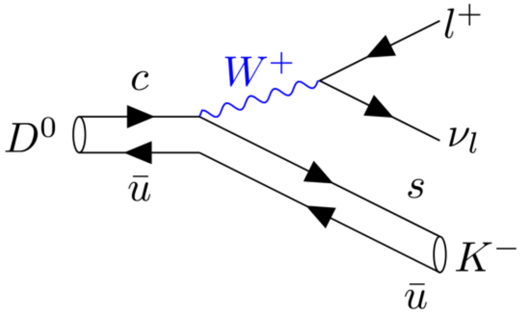
NP : $V_{CKM}^{\dagger} V_{CKM} \neq \mathbf{1},\ V_{CKM} V_{CKM}^{\dagger} \neq \mathbf{1}$

- Testing the CKM unitarity → New Physics (NP) exploring

# $|V_{cs}|$ and Radiative Corrections

- $|V_{cs}|$ is determined by $D_s \to \ell\nu, D \to \bar{K}\ell\nu$ (PDG)




- Radiative correction for D meson decays : not fully calculated

- Recently uncertainties from lattice form factors become small

  → Theoretical precise calculation is getting more important

- meson decays : short-distance (SD) and long-distance (LD) corrections

# Short-Distance corrections to $c \to s\ell\nu$

- $\mathcal{O}(\alpha)$ short-distance correction is common in (semi) leptonic decays

$$\frac{\mathcal{M}}{\mathcal{M}_0} = 1 + \frac{\alpha}{\pi} \ln \frac{m_Z}{\mu}$$

(Sirlin,et al. 1982), (Sirlin,et al. 1993)

$$C(\mu) = 1 + \frac{\alpha}{2\pi}\left[\ln\left(\frac{M_Z^2}{\mu^2}\right) - \frac{11}{6}\right]$$

Brod, Gorbahn, 0805.4119 etc.

- 2nd column Unitarity →

$$\Delta_{\rm CKM} \equiv |V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 - 1$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

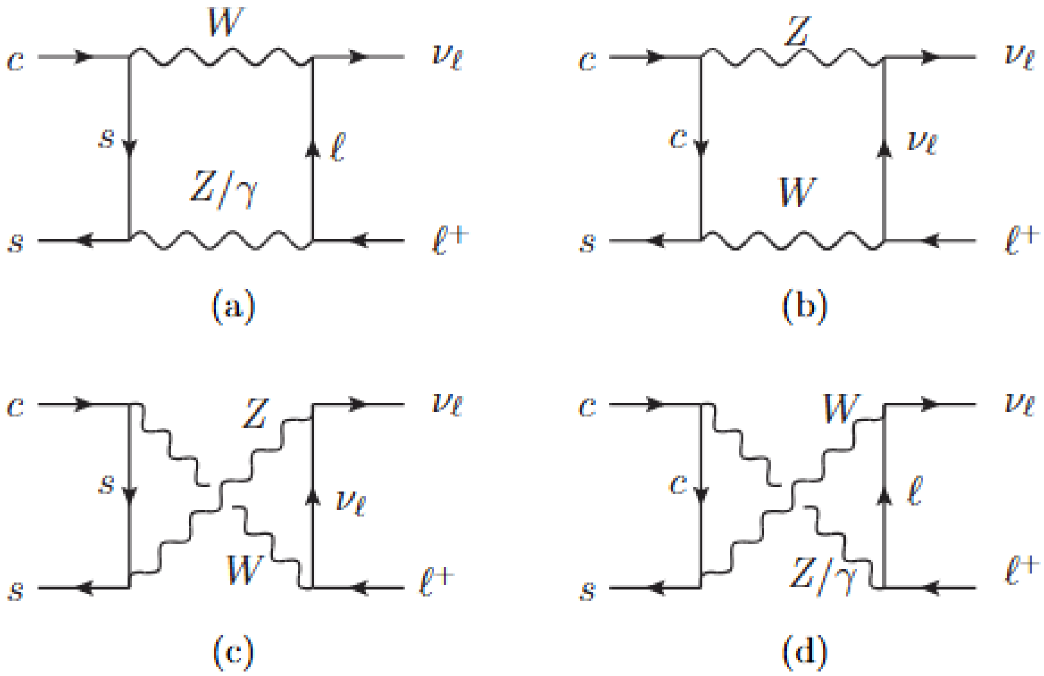
$$= \begin{cases} 0.032 \pm 0.006 \quad [5.2\ \sigma]\ (\text{nominal}) \\ 0.021 \pm 0.001 \quad [2.0\ \sigma]\ (\text{scale factor}) \end{cases}$$

(van Dyk, et al. arXiv 2407.06145)

# Short-Distance corrections to $c \to s\ell\nu$

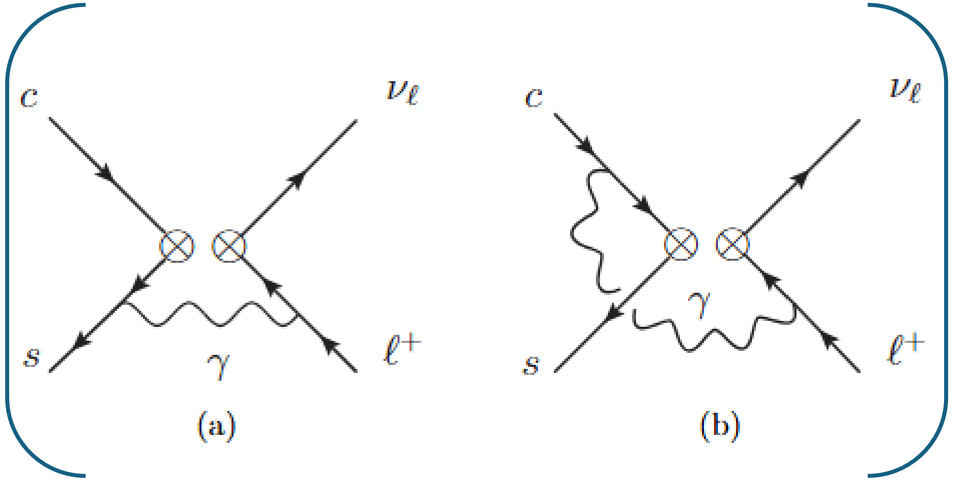
- SD correction is given by Wilson coefficient (high energy)

- Matching full theory and 4-Fermi EFT




$$= C(\mu)$$

High energy




Low energy

$$
\begin{aligned}
i\mathcal{M}^{\gamma W}_{(\mathrm{a})} &= i\mathcal{M}_0 Q_s Q_\ell \\
&\times \frac{\alpha}{\pi}\Bigg\{ \underbrace{\frac{3}{8} + \frac{1}{4}\ln\left[\frac{2M_W^2}{s(1-\cos\theta)}\right]}_{\ell^2} \underbrace{- \frac{\pi^2}{6} - \frac{1}{4}\ln^2\left[\frac{2m_\gamma^2}{s(1-\cos\theta)}\right] - \ln\left[\frac{2m_\gamma^2}{s(1-\cos\theta)}\right] - \frac{5}{4}}_{\ell^0} \Bigg\},
\end{aligned}
$$

photon box

$$
\begin{aligned}
i\mathcal{M}^{\gamma}_{(\mathrm{a})} &= i\mathcal{M}_0 Q_s Q_\ell \\
&\times \frac{\alpha}{\pi}\Bigg\{ \underbrace{\frac{1}{4\bar\epsilon} + \frac{1}{4} + \frac{1}{4}\ln\left[\frac{2\mu^2}{s(1-\cos\theta)}\right]}_{\ell^2} \underbrace{- \frac{\pi^2}{6} - \frac{1}{4}\ln^2\left[\frac{2m_\gamma^2}{s(1-\cos\theta)}\right] - \ln\left[\frac{2m_\gamma^2}{s(1-\cos\theta)}\right] - \frac{5}{4}}_{\ell^0} \Bigg\},
\end{aligned}
\tag{A.4}
$$

4-Fermi

photon box

$$
\begin{aligned}
i\mathcal{M}^{\gamma W}_{(\mathrm{d})} &= i\mathcal{M}_0 Q_c Q_\ell \\
&\times \frac{\alpha}{\pi}\Bigg\{ \underbrace{-\frac{3}{2} - \ln\left[\frac{2M_W^2}{s(1+\cos\theta)}\right]}_{\ell^2} + \underbrace{\frac{\pi^2}{6} + \frac{1}{4}\ln^2\left[\frac{2m_\gamma^2}{s(1+\cos\theta)}\right] + \ln\left[\frac{2m_\gamma^2}{s(1+\cos\theta)}\right] + \frac{3}{2}}_{\ell^0} \Bigg\},
\end{aligned}
$$

4-Fermi

$$
\begin{aligned}
i\mathcal{M}^{\gamma}_{(\mathrm{b})} &= i\mathcal{M}_0 Q_c Q_\ell \\
&\times \frac{\alpha}{\pi}\Bigg\{ \underbrace{-\frac{1}{\bar\epsilon} - \frac{11}{4} - \ln\left[\frac{2\mu^2}{s(1+\cos\theta)}\right]}_{\ell^2} + \underbrace{\frac{\pi^2}{6} + \frac{1}{4}\ln^2\left[\frac{2m_\gamma^2}{s(1+\cos\theta)}\right] + \ln\left[\frac{2m_\gamma^2}{s(1+\cos\theta)}\right] + \frac{3}{2}}_{\ell^0} \Bigg\}.
\end{aligned}
$$

UV div → $\overline{MS}$ renormalization

IR div, scattering angle, invariant mass dependences all cancel !

# Short-Distance corrections to $c \to s\ell\nu$

- Fermi coupling constant is derived from muon lifetime, includes only LD

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(\rho)\left[1 + \frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]$$

Kinoshita and Sirlin, 1959

- Similarly to $c \to s\ell\nu$, SD correction to $\mu \to e\bar{\nu}\nu$ is needed

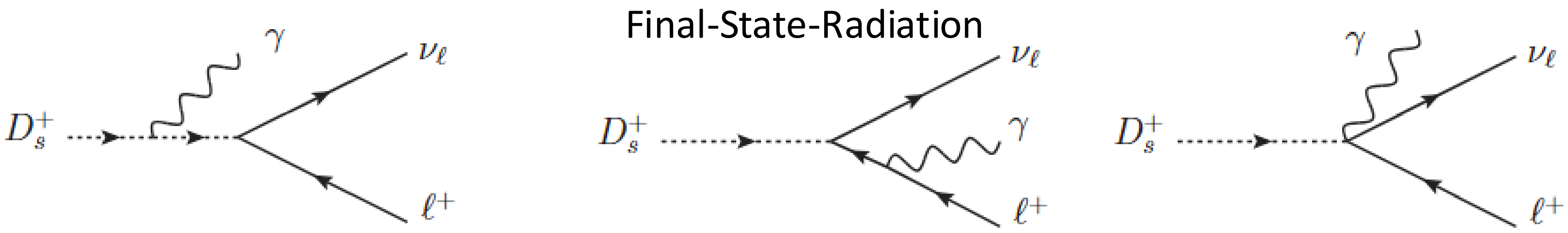
- Result :

$$\eta_{\mathrm{EW}} \equiv \frac{\mathcal{M}}{\mathcal{M}_0} = \begin{cases} 1 + \frac{\alpha(\mu)}{2\pi}\left[\ln\left(\frac{M_Z^2}{\mu^2}\right) + \frac{1}{2}A_{g_s} - \frac{11}{6}\right] & \text{(NDR)}, \\ 1 + \frac{\alpha(\mu)}{2\pi}\left[\ln\left(\frac{M_Z^2}{\mu^2}\right) + \frac{1}{2}A_{g_s} - \frac{7}{6}\right] & \text{(HV)}, \end{cases}$$

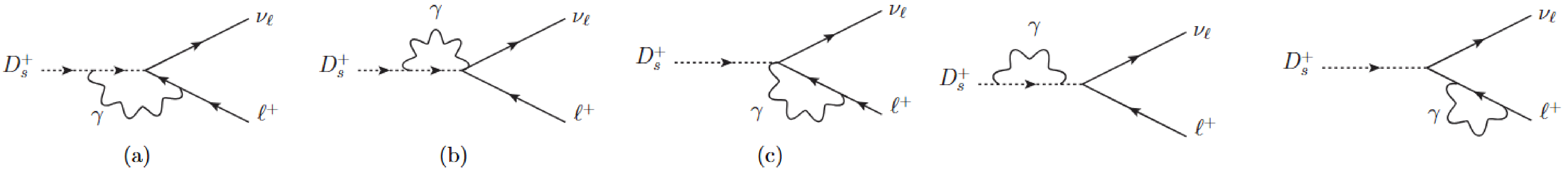
# Long-Distance corrections to $D_s \to \ell\nu$

- long-distance correction → Scalar QED (low energy)

- soft photon emission diagrams : exclusive (depend on photon energy $E_{max}$)




- Contributions of virtual corrections



# Long-Distance corrections to $D_s \to \ell\nu$

$$\frac{\Delta\Gamma^{(a)}_{(\mathrm{vertex})}}{\Gamma_0} = -Q_\ell Q_{D_s}\frac{\alpha_0}{2\pi}\left[\frac{5}{2}\left(\frac{1}{\bar\epsilon}+\ln\frac{\mu^2}{m_{D_s}^2}\right)+\frac{11}{2}+\boxed{\kappa_c}\right.$$
$$\left. -2\left(\frac{1+x_\ell}{1-x_\ell}\right)\ln x_\ell \ln\frac{m_\gamma}{\sqrt{m_{D_s}m_\ell}}-\frac{2}{1-x_\ell}\ln x_\ell\right],$$

$$\frac{\Delta\Gamma^{(b)}_{(\mathrm{vertex})}}{\Gamma_0} = Q_{D_s}^2\frac{\alpha_0}{2\pi}\left[-\frac{3}{2}\left(\frac{1}{\bar\epsilon}+\ln\frac{\mu^2}{m_{D_s^2}}\right)-\frac{7}{2}\right],$$

$$\frac{\Delta\Gamma^{(c)}_{(\mathrm{vertex})}}{\Gamma_0} = -Q_\ell Q_{D_s}\frac{\alpha_0}{2\pi}\left[-3\left(\frac{1}{\bar\epsilon}+\ln\frac{\mu^2}{m_\ell^2}\right)-4+\boxed{\kappa_d}\right],$$

$$\frac{\Delta\Gamma^{(a)}_{(\mathrm{self})}}{\Gamma_0} = 2\left(\frac{1}{2}\left.\frac{d\Sigma}{dp^2}\right|_{p^2=m_{D_s}^2}\right)$$
$$= Q_{D_s}^2\frac{\alpha_0}{2\pi}\left(\frac{1}{\bar\epsilon}+\ln\frac{\mu^2}{m_{D_s}^2}-2\ln\frac{m_\gamma}{\sqrt{m_{D_s}m_\ell}}-\frac{1}{2}\ln x_\ell\right),$$

$$\frac{\Delta\Gamma^{(b)}_{(\mathrm{self})}}{\Gamma_0} = 2\left(\frac{1}{2}\left.\frac{d\Sigma}{d\not p}\right|_{\not p=m_\ell}\right)$$
$$= Q_\ell^2\frac{\alpha_0}{2\pi}\left[-\frac{1}{2}\left(\frac{1}{\bar\epsilon}+\ln\frac{\mu^2}{m_\ell^2}\right)-2-2\ln\frac{m_\gamma}{\sqrt{m_{D_s}m_\ell}}+\frac{1}{2}\ln x_\ell\right]$$

There are renormalization scheme dependence $\kappa_i$, this dependence cancel accidentally in total !

# Long-Distance corrections to $D_s \to \ell\nu$

- Formula of $|V_{cs}|$

Tree level

Short-Distance (Wilson Coefficient)

$$|V_{cs}|^2 = \frac{\mathcal{B}(D_s^+ \to \ell^+\nu_\ell)}{\tau_{D_s}} \left( \frac{G_F^2}{8\pi} m_{D_s} m_\ell^2 f_{D_s}^2 \left(1 - x_\ell\right)^2 \left\{ 1 + \frac{\alpha(m_{D_s})}{2\pi} \left[ \ln\left(\frac{M_Z^2}{m_{D_s}^2}\right) + \frac{1}{2}\mathcal{A}_{g_s} - \frac{11}{6} \right] \right\}^2 \right.$$

$$\left. \times \Omega_B \left(\Omega_B^{\mathrm{PHOTOS}}\right)^{-1} \left\{ 1 + \frac{\alpha_0}{2\pi} \left[ -2 - \frac{1+x_\ell}{1-x_\ell} \left( \frac{1}{2}\ln^2 x_\ell + 2\,\mathrm{Li}_2\left(1-x_\ell\right) \right) - \frac{3}{2}\ln\frac{\mu^2}{m_{D_s} m_\ell} - \frac{1+15x_\ell}{4(1-x_\ell)}\ln x_\ell \right] + F^{\mathrm{hard}} + \frac{\alpha_0}{2\pi}\frac{1+x_\ell}{1-x_\ell}\ln x_\ell \right\} \right)^{-1},$$

Long-Distance

$E_{max}$ dependence

$$\Omega_B(E_{\max}) \equiv \left( \frac{2E_{\max}}{\sqrt{m_{D_s} m_\ell}} \right)^{-\frac{2\alpha_0}{\pi}\left(1 + \frac{1}{2}\frac{1+x_\ell}{1-x_\ell}\ln x_\ell\right)}$$

Multi-photon resummation

$$E_\gamma < E_{\max} \simeq 54\ \mathrm{MeV} \quad (\text{for } \mu \text{ mode}) \qquad \text{tau mode : inclusive } (m_{D_s} \sim m_\tau)$$

# Determination of $E_{max}$

**BESIII** $E_{\mathrm{max}}$

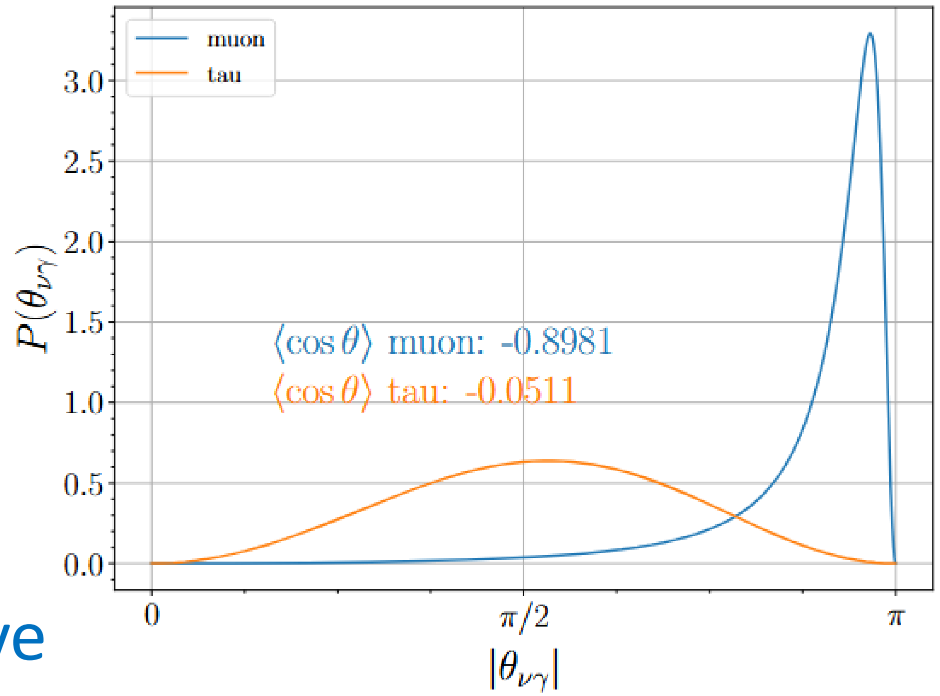
Energy cut

$$M^2_{\mathrm{miss}} = (p_\nu + \underbrace{p_\gamma}_{\text{invisible}})^2 = 2E_\nu E_\gamma(1-\cos\theta_{\nu\gamma}) = m_{D_s}E_\gamma(1-\cos\theta_{\nu\gamma}) \le 0.2[\mathrm{GeV}^2]$$

$$E_\gamma < E_{\mathrm{max}} \simeq 54\ \mathrm{MeV} \qquad (\text{for } \mu \text{ mode})\,.$$

→ exclusive

$$P(\theta_{\nu\gamma}) = \frac{2}{1+\beta_\ell\cos\theta_{\nu\gamma}} - 1 - \frac{1-\beta_\ell^2}{(1+\beta_\ell\cos\theta_{\nu\gamma})^2}\,.$$



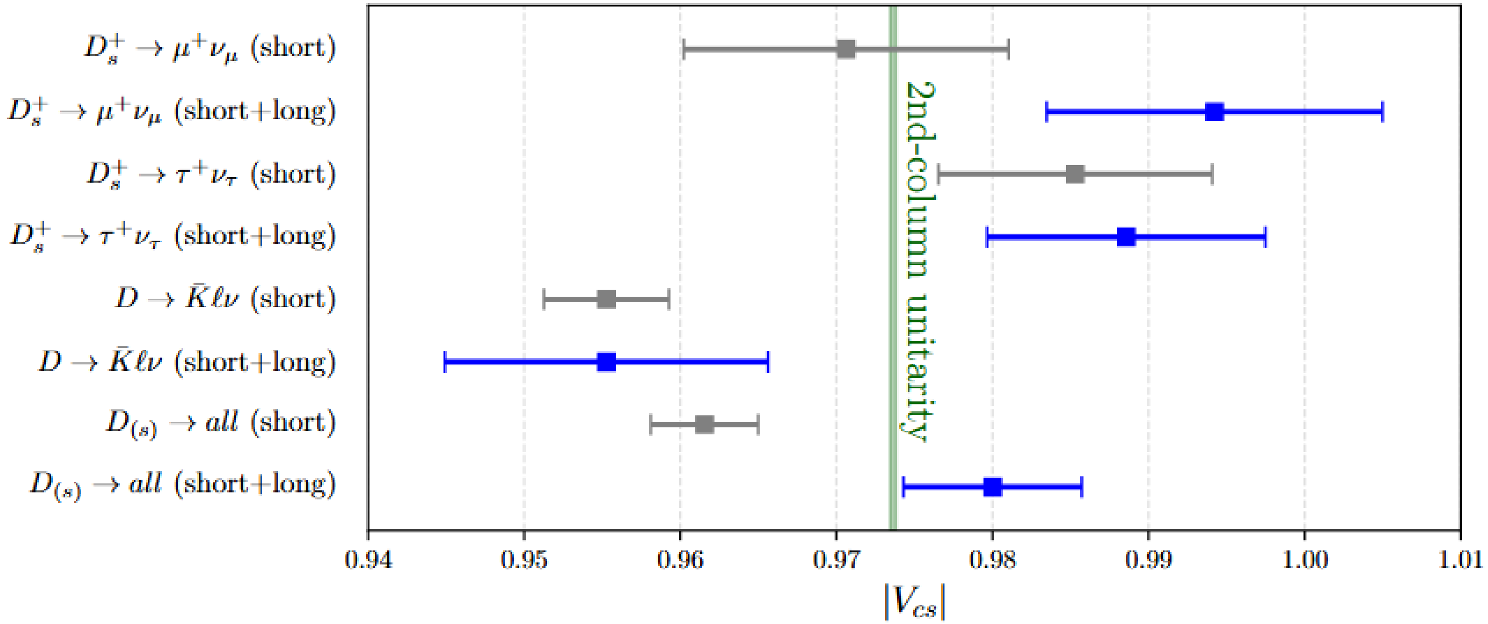

$$M^2_{\mathrm{miss}} = (p_{\nu_\tau} + p_{\nu_{\bar\tau}} + p_\gamma)^2 \simeq 2E_{\nu_{\bar\tau}}(E_\gamma + E_{\nu_\tau}) < 0.6\,\mathrm{GeV}^2$$

$$E_\gamma + E_{\nu_\tau} \lesssim 340\,\mathrm{MeV}\,. \qquad \text{for } \tau \text{ mode}\,.$$

more than maximum photon energy → inclusive

# Determination of $|V_{cs}|$

$|V_{cs}|$ values of each decay mode and average




For $D \to \bar{K}\ell\nu$,

- LD is hard to perform

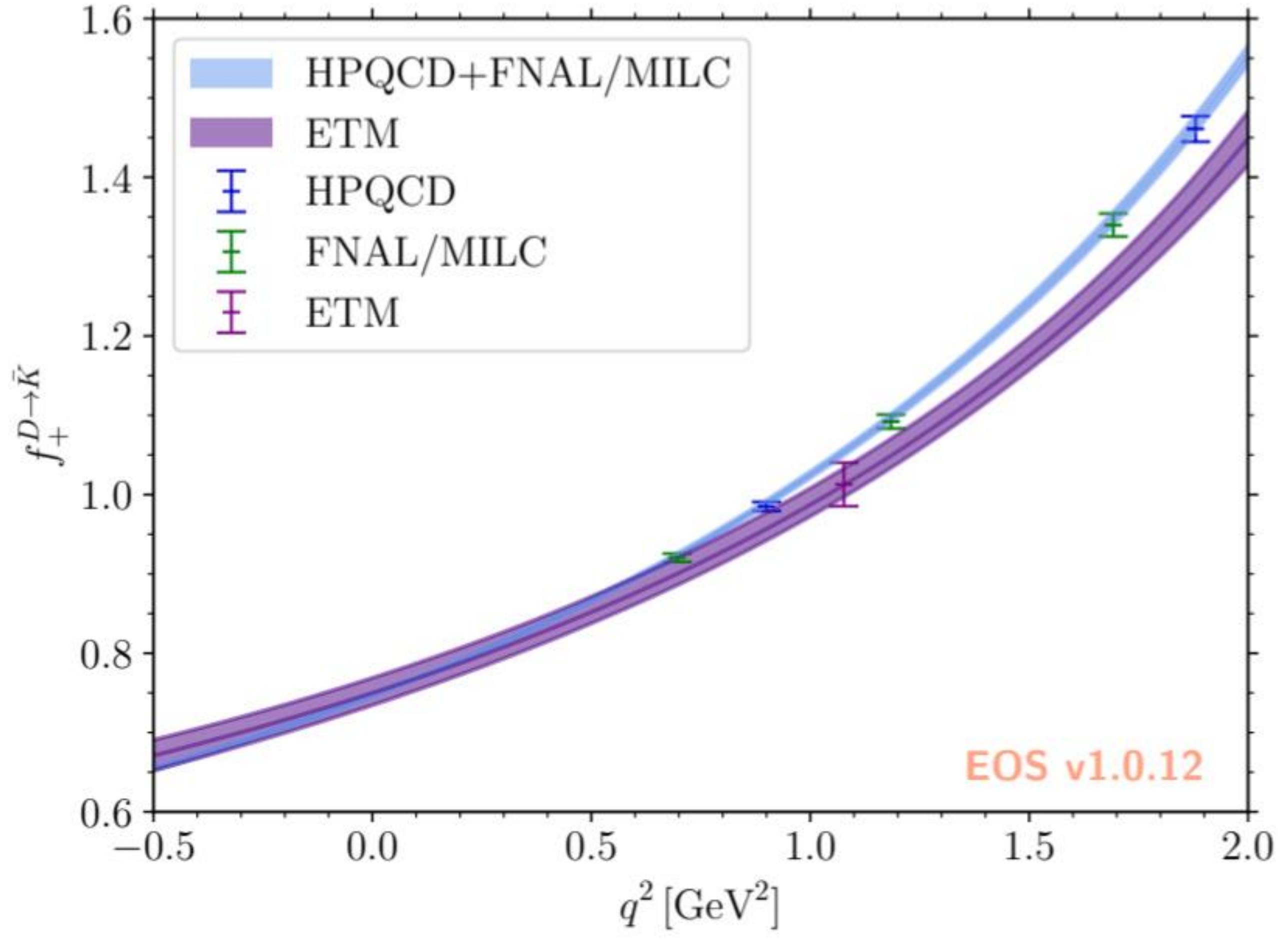
→ We use estimation of lattice, as 1% uncertainty

FLAG2024, 2411.04268,
Bozavov et al, 2212.12648

Global Fit : $$|V_{cs}| = \begin{cases} 0.980 \pm 0.006 & \text{(nominal)}\,, \\ 0.983 \pm 0.006 & \text{(scale factor)}\,, \end{cases}$$

→ Unitarity Test

# Form Factors of Lattice Groups




- Form factor tension between lattice groups
- Nominal : Assume that ETM is outlier
- Scale factor : study the impact of removing the ETM result

(van Dyk, et al. arXiv 2407.06145)

# CKM Unitarity Test

- CKM Unitarity → $\left(V^\dagger V\right)_{22} = |V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1\,.$

$$\Delta_{\rm CKM} \equiv |V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 - 1$$

only SD $= \begin{cases} 0.032 \pm 0.006 & [5.2\ \sigma]\ \text{(nominal)} \\ 0.021 \pm 0.010 & [2.0\ \sigma]\ \text{(scale factor)} \end{cases}$

(van Dyk, et al. arXiv 2407.06145)

↓ + LD corrections etc.

$$= \begin{cases} 0.012 \pm 0.011 & [1.1\ \sigma]\ \text{(nominal)}, \\ 0.018 \pm 0.012 & [1.6\ \sigma]\ \text{(scale factor)}, \end{cases}$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM Unitarity recovers → No NP or Constrain on NP ?

# Summary

- Unitarity recovers by consider long-distance corrections for $D_s \to \ell\nu$ (and $D \to K\ell\nu$ as a 1% uncertainty from the lattice estimation)

| scenario | data set | $\lvert V_{cs}\rvert$ | $\Delta_{\text{CKM}}$ | result |
|---|---|---|---|---|
| nominal | $D \to \overline{K}\ell\nu$<br>all average | $0.955 \pm 0.004 \pm 0.010$<br>$0.980 \pm 0.006$ | $0.012 \pm 0.011$ | $1.1\ \sigma$ |
| scale factor | $D \to \overline{K}\ell\nu$<br>all average | $0.959 \pm 0.007 \pm 0.010$<br>$0.983 \pm 0.006$ | $0.018 \pm 0.012$ | $1.6\ \sigma$ |

- By incorporating the structure-dependent QED and $D \to \overline{K}\ell\nu$ long-distance correction, one can perform more accurate CKM unitarity test (future work) (Rowe and Zwicky, 2404.07648)

# Back up

# Renormalization Scheme

When we use dimensional regularization : $d = 4 - 2\epsilon$ ⇒ $\gamma_\mu = \tilde{\gamma}_\mu + \hat{\gamma}_\mu$

- Naive dimensional regularization (NDR) : anti-commutates with $\gamma_5$

⇒ $$\gamma_5 \gamma_\mu = -\gamma_\mu \gamma_5$$

- ‘t Hooft-Veltman scheme (HV) : commutates with $\gamma_5$

⇒ $$\gamma_5 \gamma_\mu = \gamma_5(\tilde{\gamma}_\mu + \hat{\gamma}_\mu) = -(\tilde{\gamma}_\mu - \hat{\gamma}_\mu)\gamma_5$$

$$\gamma_\mu \gamma_\rho \Gamma \gamma^\rho \gamma^\mu \otimes \Gamma = A_1(\epsilon) \Gamma \otimes \Gamma + E_1 \,,$$

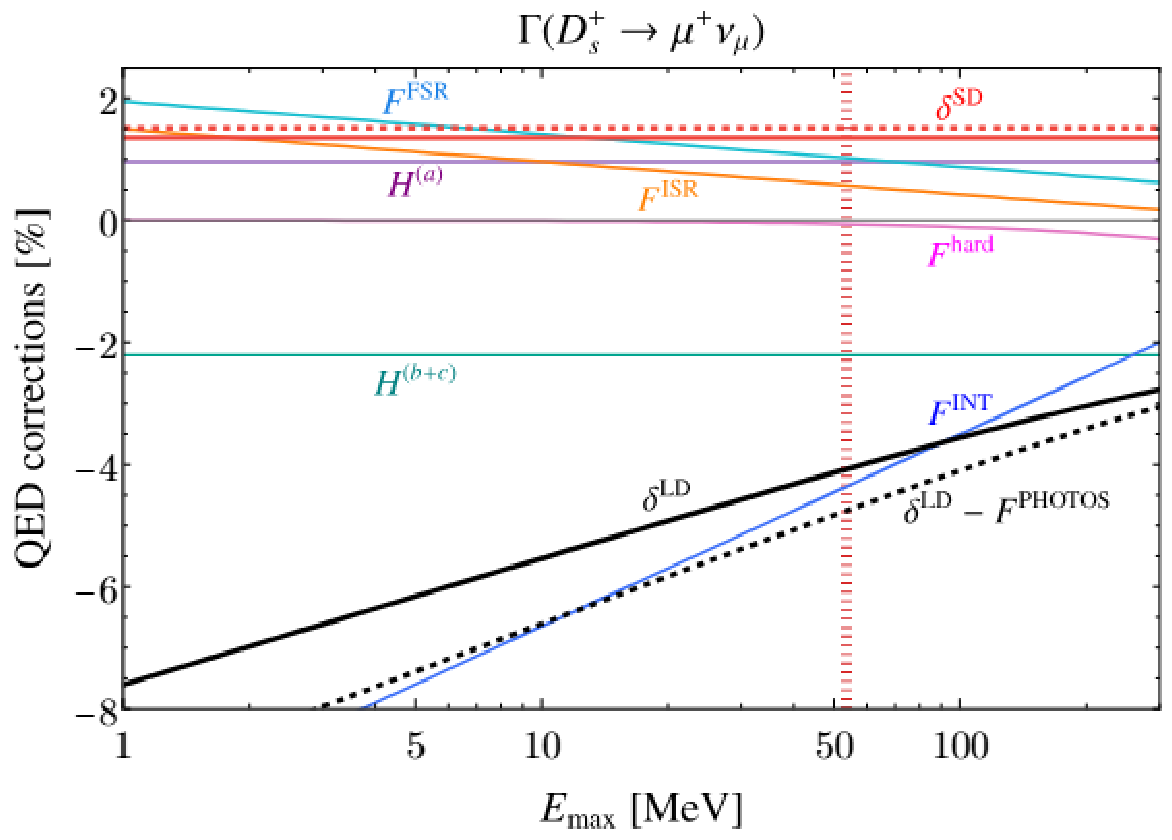
$$\Gamma \gamma_\mu \gamma_\rho \otimes \Gamma \gamma^\mu \gamma^\rho = A_2(\epsilon) \Gamma \otimes \Gamma + E_2 \,,$$

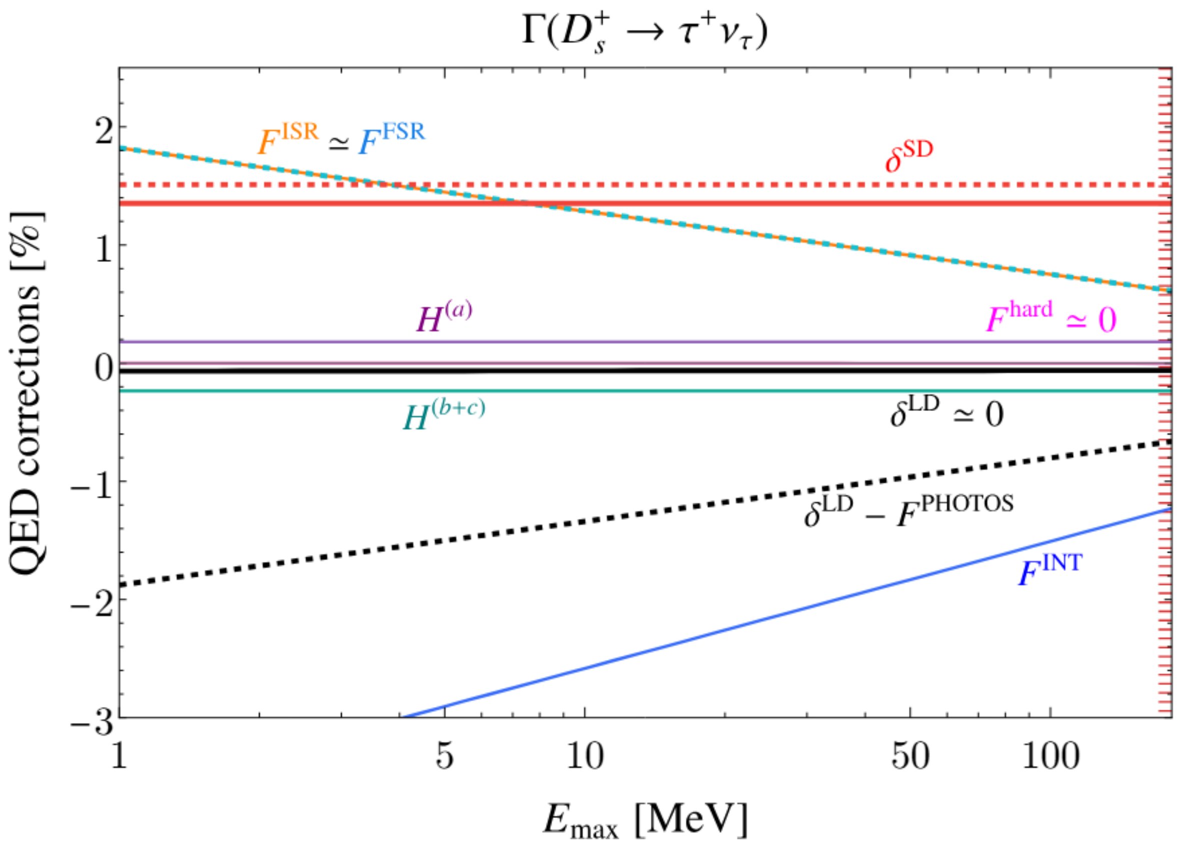
$$\Gamma \gamma_\mu \gamma_\rho \otimes \gamma^\rho \gamma^\mu \Gamma = A_3(\epsilon) \Gamma \otimes \Gamma + E_3 \,,$$

$$A_1(\epsilon) = A_3(\epsilon) = \begin{cases} 4(1-\epsilon)^2 & (\mathrm{NDR}) \,, \\ 4(1+\epsilon^2) & (\mathrm{HV}) \,, \end{cases}$$

$$A_2(\epsilon) = 4(4 - \epsilon - \epsilon^2) \quad (\mathrm{NDR}, \mathrm{HV}) \,.$$

# Determination of $E_{max}$

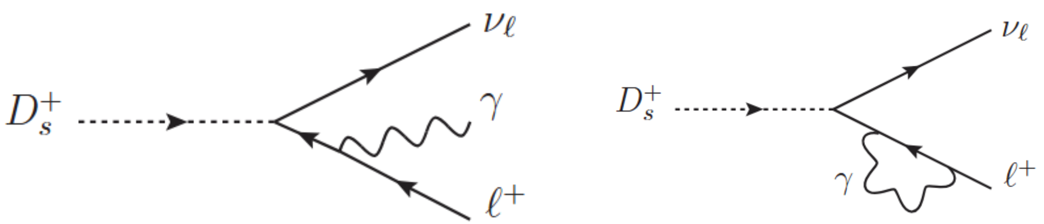


# PHOTOS

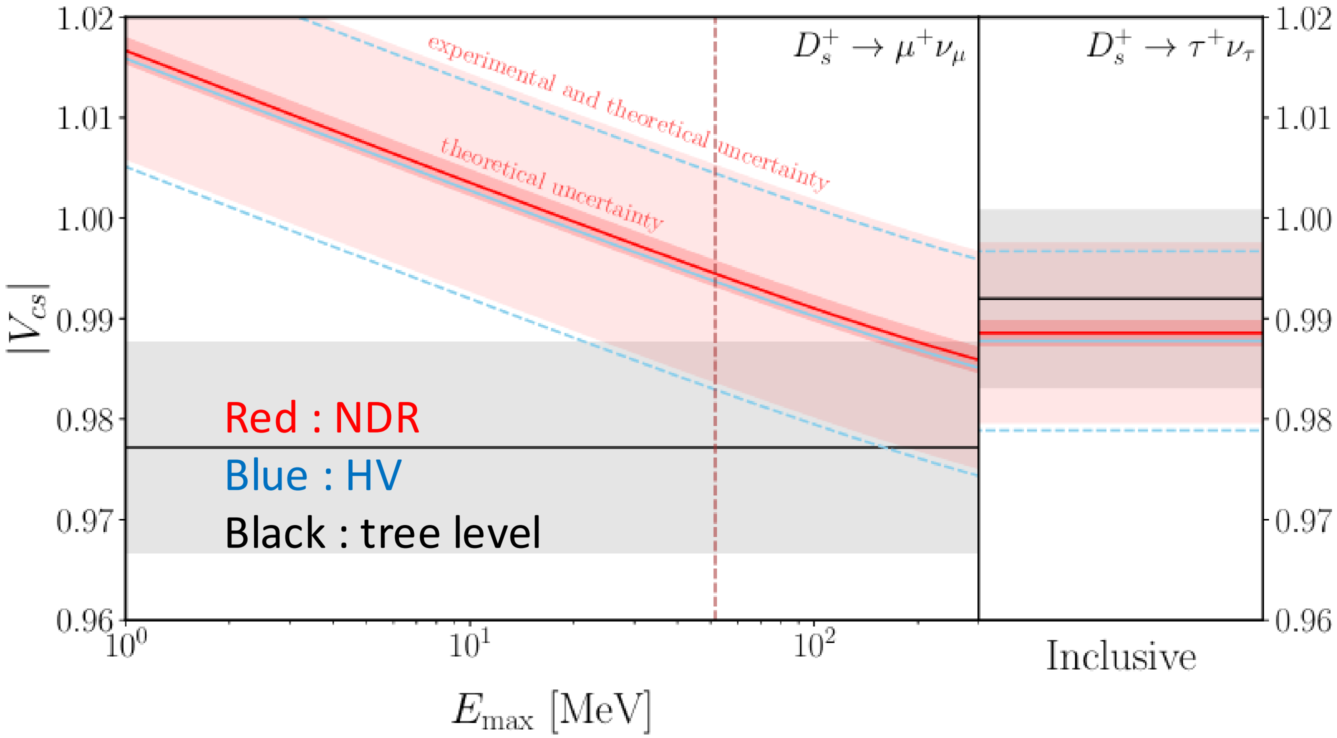
- Correspondence with PHOTOS, which is experimental analysis tool
  → subtract final-state-radiation from theoretical calculation

Final-State-Radiation

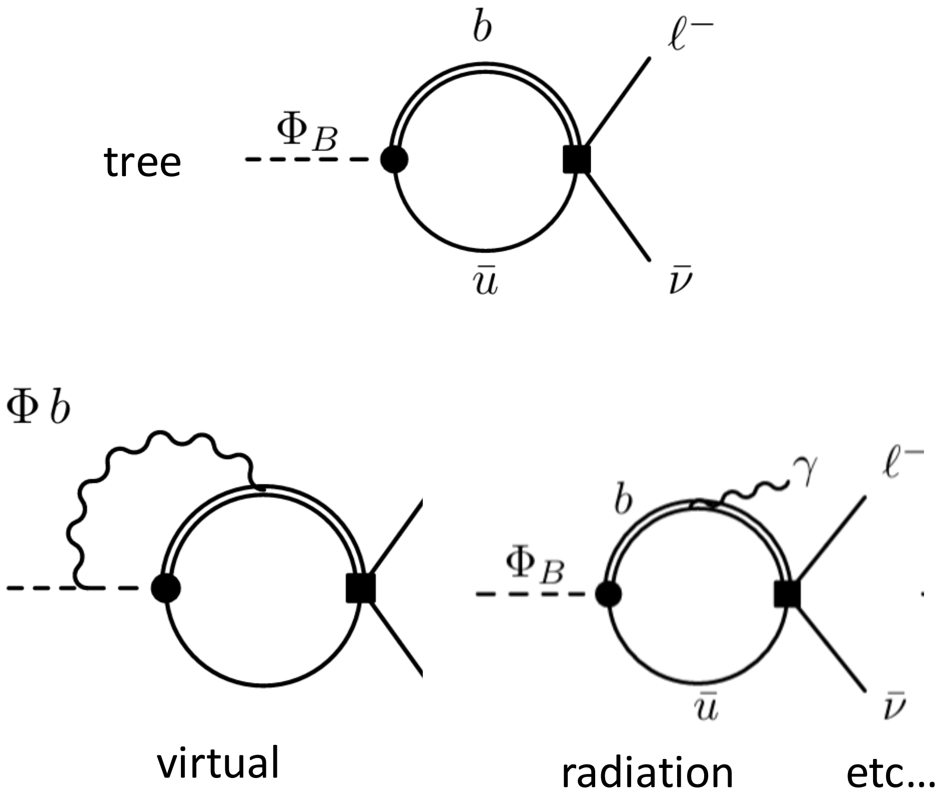



(b)

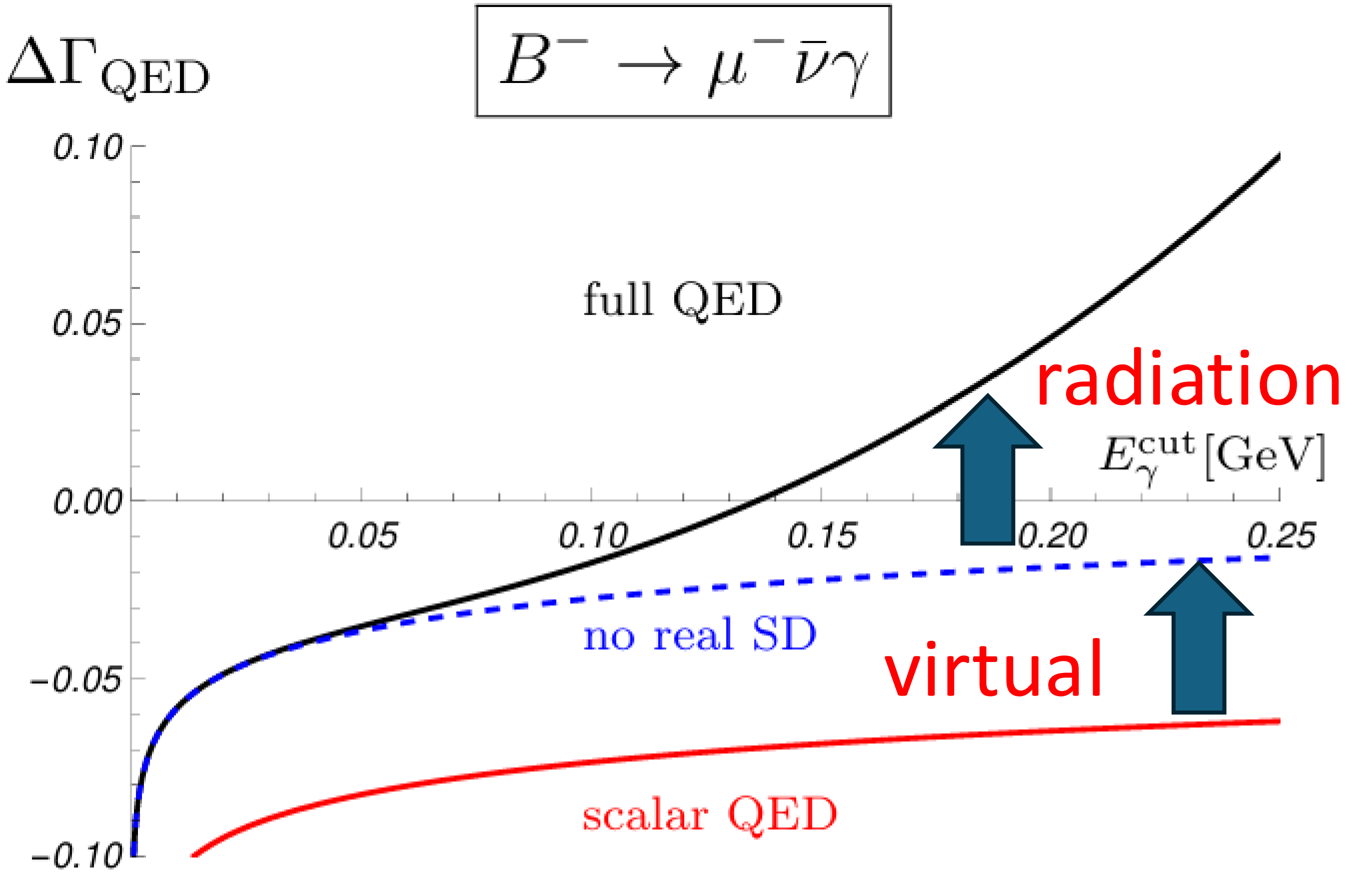
$$F^{\mathrm{PHOTOS}} = \frac{\alpha_0}{2\pi}\left(-2\ln\frac{2E_{\mathrm{max}}}{m_\ell} - \frac{1+x_\ell}{1-x_\ell}\ln x_\ell\right)$$



# Structure-Dependent QED




New contribution




(Rowe and Zwicky, 2404.07648)