Cutting rule for cosmological correlators and its applications

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Based on 2409.07521, 2506.15780, 260x.xxxxx
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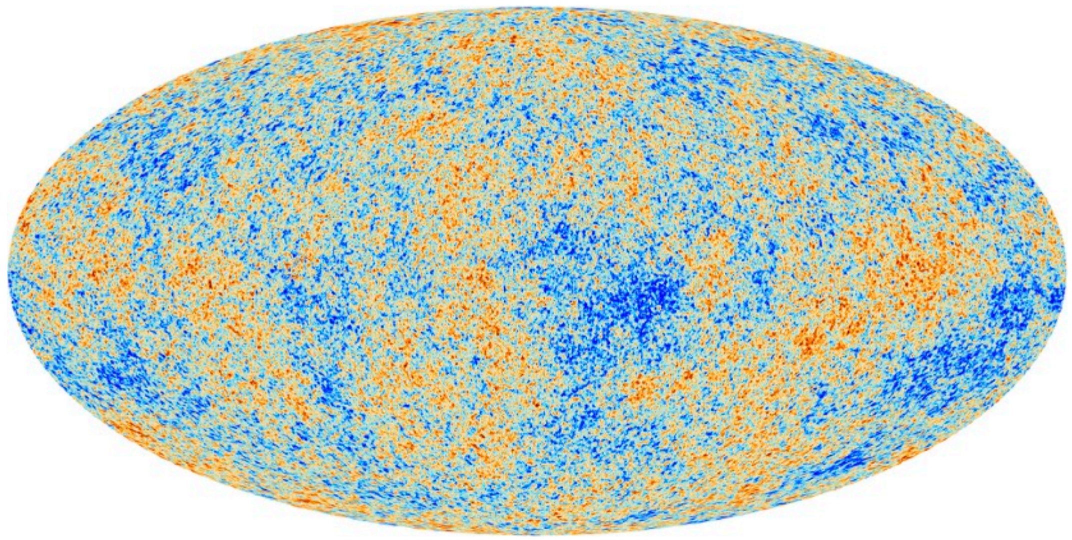
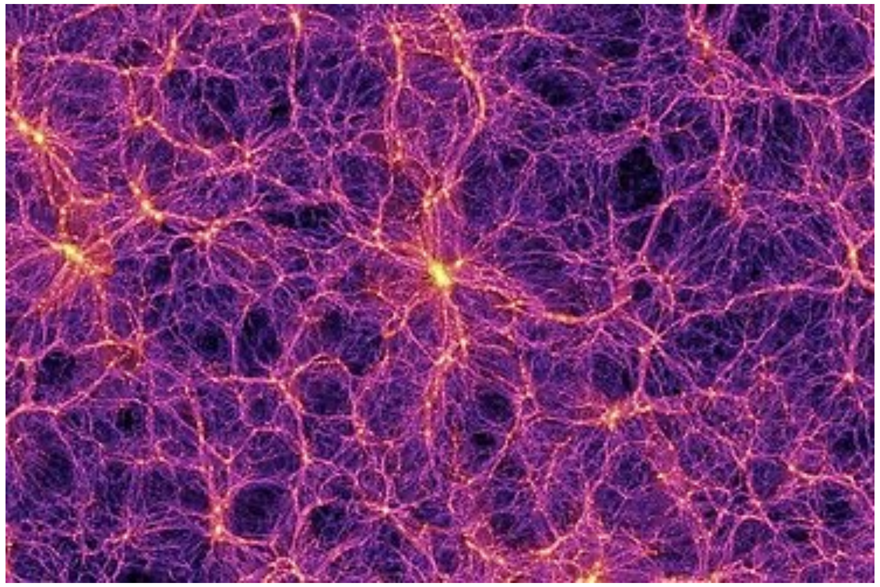
Cosmological correlators

- Structures of the universe originate from cosmic inflation.



[Planck collaboration]



[https://wwwmpa.mpa-garching.mpg.de/galform/data_vis/]

- Initial conditions provided by "cosmological correlators"

$$\langle \phi_{\vec{k}_1}\phi_{\vec{k}_2}\cdots\phi_{\vec{k}_n}\rangle$$: generated during inflation.

➡ Correlators encode: inflaton potential, couplings to other particles, etc.

- No time translation invariance → calculations quite involved in general.

➡ QFT technique to simplify/classify calculations?

Outline

1. In-in formalism and cutting rule
2. Application 1: non-local CC signals
3. Application 2: one-loop correction to soft mode
4. Summary

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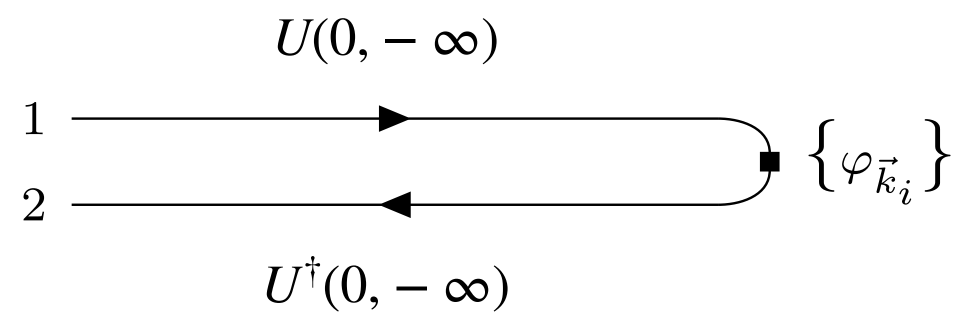
In-in formalism

- Cosmological correlators: evaluated at end of inflation with initial condition fixed.

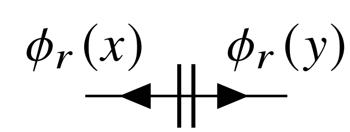
"in-in" formalism: $\langle \varphi_{\vec{k}_1} \cdots \varphi_{\vec{k}_n} \rangle = \langle 0 | U^\dagger(0, -\infty) \varphi_{\vec{k}_1} \cdots \varphi_{\vec{k}_n} U(0, -\infty) | 0 \rangle \, .$



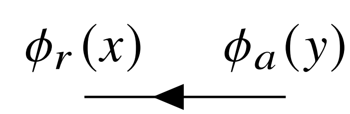
- Two types of fields: $\phi_1(x)$ from $U(0, -\infty)$ and $\phi_2(x)$ from $U^\dagger(0, -\infty)$.

- Keldysh basis : $\phi_r = \frac{1}{2}\left(\phi_1 + \phi_2\right), \quad \phi_a = \phi_1 - \phi_2.$ [Keldysh 64]

$$G_{rr}(x,y) = \frac{1}{2}\left[G_>(x,y) + G_<(x,y)\right] =$$



$$G_{ra}(x,y) = \theta(x_0 - y_0)\left[G_>(x,y) - G_<(x,y)\right] =$$

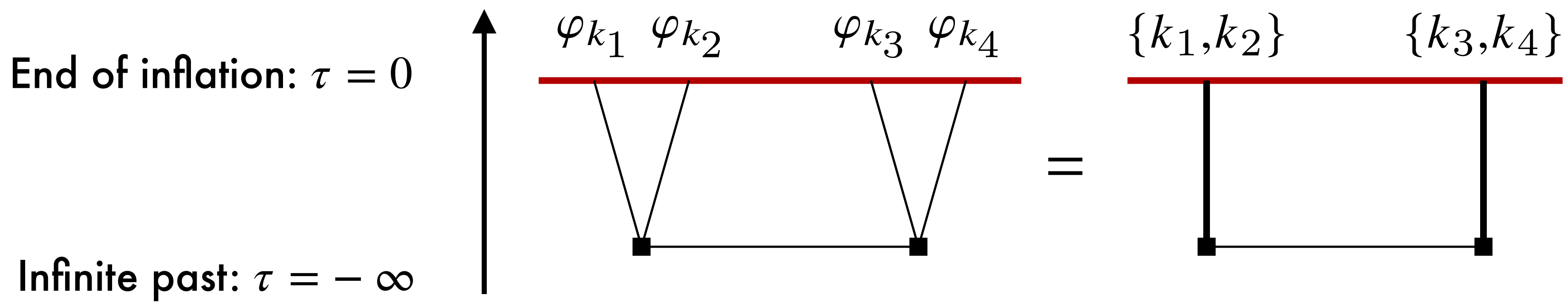


Arrow: causal flow $a \to r$.

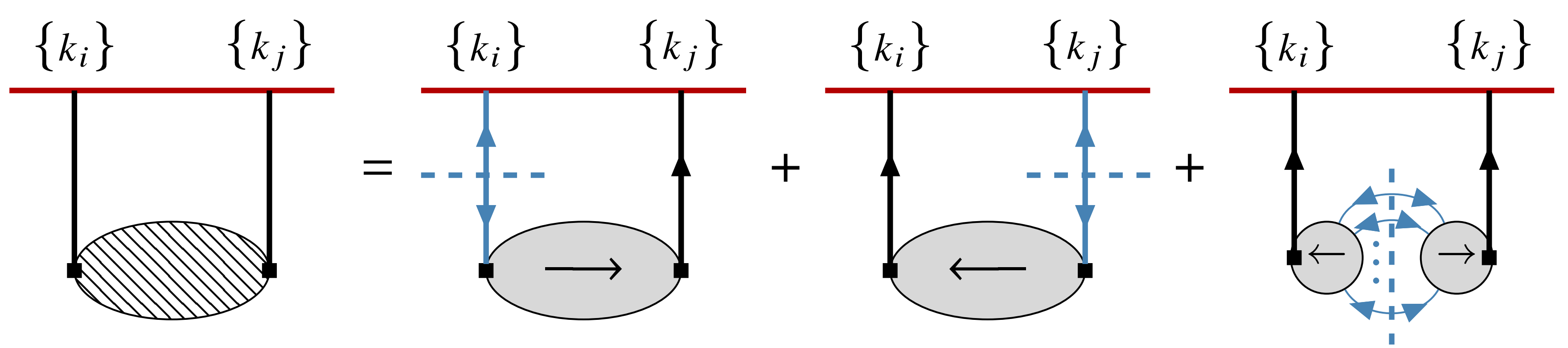
$$G_{aa}(x,y) = 0$$

Cutting rule for in-in correlators

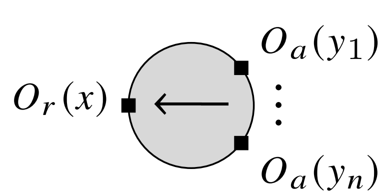
- In-in correlators: expressed by "in-in Feynman" diagrams.



- A general cutting rule exists for in-in correlators: [YE, Mukaida 24]



where blue cut : average of $G_>(x,y)$ and $G_<(x,y)$,

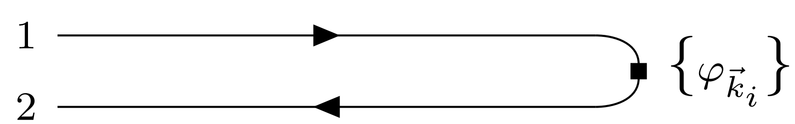


$= \langle O_r(x) O_a(y_1) \cdots O_a(y_n) \rangle$: fully retarded function

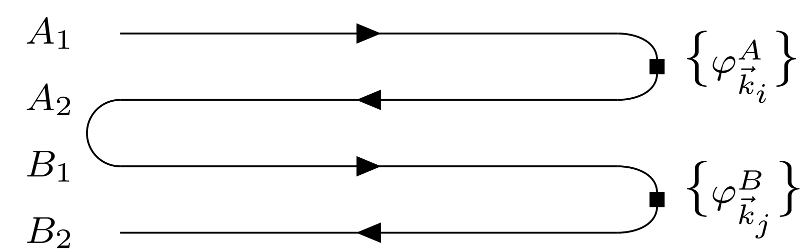
with a straightforward generalization to n-pt bulk correlators.

Sketch of proof

1. Duplicate the in-in contour (See also [Caron-Huot 07]):

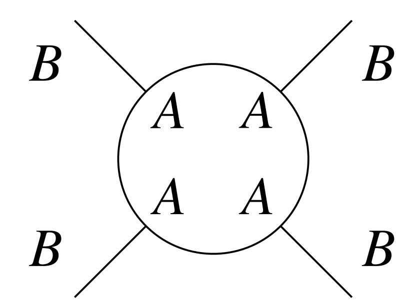


$$\langle 0|\underbrace{U^\dagger(0,-\infty)}_{\text{“2”}}\varphi_{\vec{k}_1}\cdots\varphi_{\vec{k}_n}\underbrace{U(0,-\infty)}_{\text{“1”}}|0\rangle$$



$$\langle 0|\underbrace{U^\dagger}_{\text{“}B_2\text{”}}\varphi_{\vec{k}_1}\cdots\varphi_{\vec{k}_i}\underbrace{UU^\dagger}_{\text{“}B_1+A_2\text{”}}\varphi_{\vec{k}_j}\cdots\varphi_{\vec{k}_n}\underbrace{U}_{\text{“}A_1\text{”}}|0\rangle$$

2. Show that ''isolated islands'' of A/B vertices vanish:

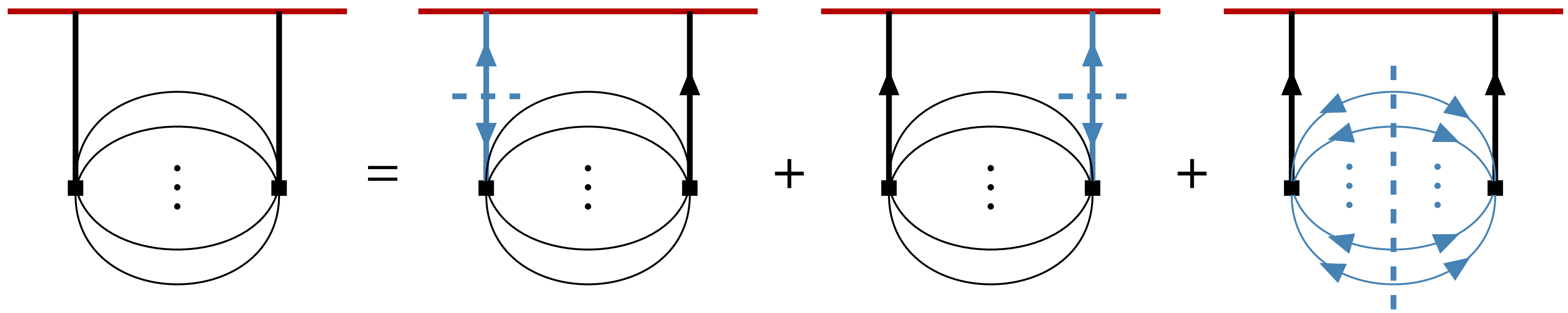


$= 0.$

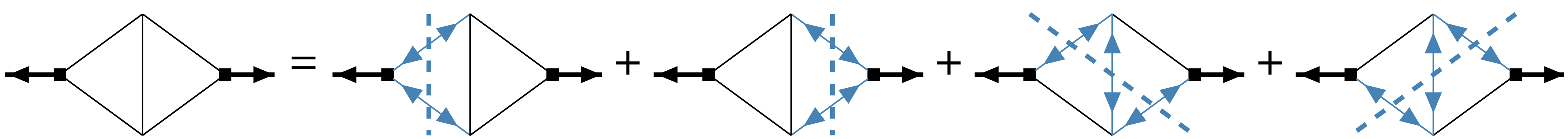
Each diagram has a boundary of A/B regions connected to $\{\varphi^{A/B}_{\vec{k}_i}\}$ = cut.

Cutting rule: examples

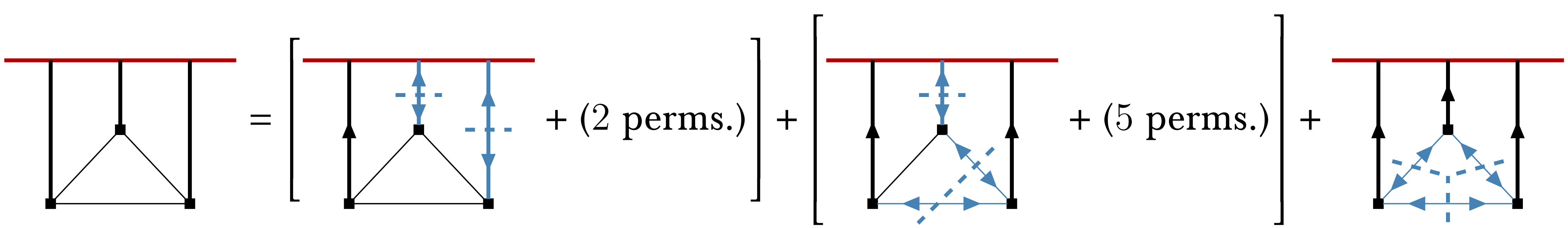
- "Melon" diagrams:



- A two-loop diagram:



- One-loop 3pt diagram:



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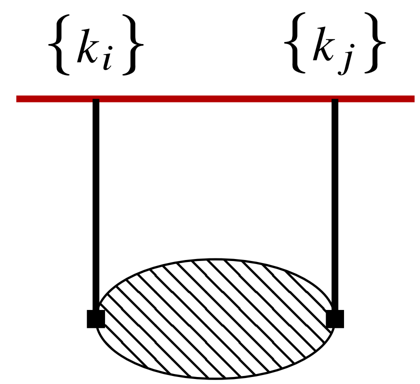
Cosmological collider

- Cosmological collider (CC) = "particle production" signals

$$\langle \phi_{\vec{k}_1}\phi_{\vec{k}_2}\cdots\phi_{\vec{k}_n}\rangle =$$



with $\sum_i k_i = k_L,\ \sum_j k_j = k_R,\ \left|\sum \vec{k}_i\right| = k_s$

CC signals: $\langle \phi_{\vec{k}_1}\phi_{\vec{k}_2}\cdots\phi_{\vec{k}_n}\rangle \propto \left[\frac{k_s}{k_{L/R}}\right]^{\pm\nu},\ \left[\frac{k_L}{k_R}\right]^{\pm\nu}$, non-analytic in momentum $\left(\nu \sim \frac{im}{H}\right)$.

"non-local" "local"

[Chen, Wang 09;...; Arkani-hamed, Maldacena 15; ...]

Non-analytic = particle production

- Higher dimensional operators: analytic in momentum.

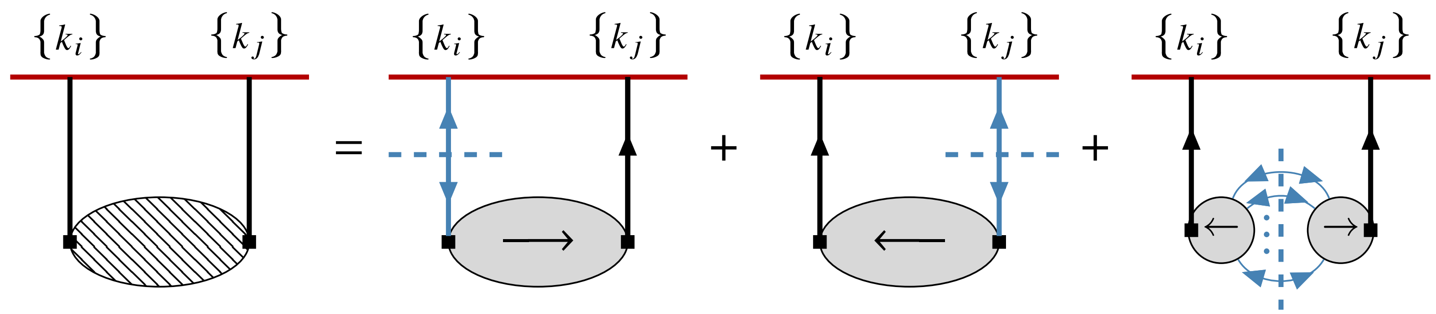
$\mathscr{L} \sim (\partial\phi)^{2n} \sim k^{2n}\phi^{2n}$ with n : integer for higher dimensional operators.

- No time translation invariance → calculations quite involved in general.

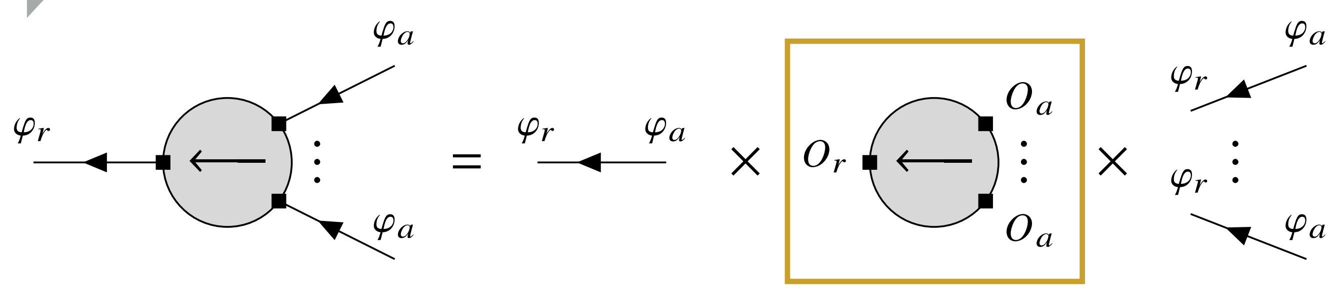
Cutting rules for cosmological correlators simplify?

Fully retarded function

- Cutting rule:



"Fully retarded function" is fundamental



$$= \mathscr{V}_{ra\cdots a}(x, y_1, \cdots, y_n) = \langle \mathcal{O}_r(x)\mathcal{O}_a(y_1)\cdots\mathcal{O}_a(y_n)\rangle \,.$$

- Microcausality: fully retarded functions satisfy

$$\mathscr{V}_{ra\cdots a}(x, y_1, \cdots, y_n) = 0 \text{ if } y_i \text{ is outside past light-cone of } x.$$

<u>E.g. 2pt:</u> $\mathscr{V}_{ra}(x, y) = \theta(x^0 - y^0)\langle[\mathcal{O}(x), \mathcal{O}(y)]\rangle$ ➡ microcausality directly applicable.

Generalizable to multi-pt functions by mathematical induction.

Cutting rule: non-local CC

- Fully retarded function has a finite support in its spatial arguments:

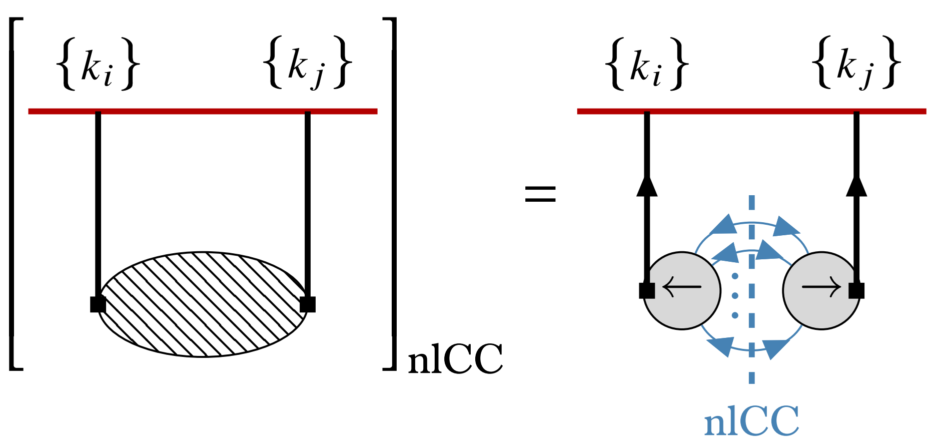
$$\mathscr{V}_{ra\cdots a}(x, y_1, \cdots, y_n) = 0 \ \text{ if } |\vec{x} - \vec{y}_i|^2 > (x^0 - y_i^0)^2$$: FT analytic in spatial momentum $\vec{k}_i$.

"Paley-Wiener theorem"

- Non-local CC: non-analytic dependence on spatial momentum.

Non-local CC arises solely from cut-propagators

[YE, Mukaida 24]



Time integrals factorized, simplify calculations.

(See also [Tong+ 21] for tree-level, [Qin, Xianyu 23,24] for loops but with explicit forms of propagators)

Non-local CC: example

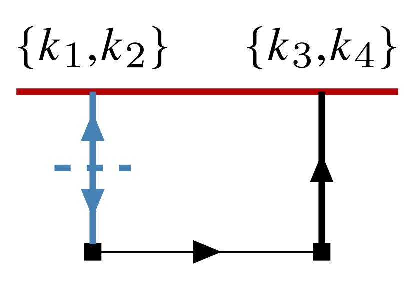
- Consider $\mathscr{L} = \lambda_3 a^2 \varphi'^2 \sigma$ with massless φ and massive σ :

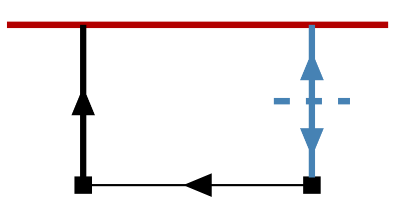
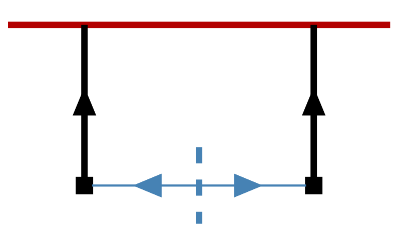
$$\begin{cases} \varphi : \Delta_>(k;\tau_1,\tau_2) = \dfrac{H^2}{2k^3}(1+ik\tau_1)(1-ik\tau_2)e^{-ik(\tau_1-\tau_2)}, \\ \sigma : G_>(k;\tau_1,\tau_2) = \dfrac{\pi}{4Ha_1^{3/2}a_2^{3/2}} H_\nu^{(1)}(-k\tau_1)H_\nu^{(2)}(-k\tau_2), \quad \nu = \sqrt{\dfrac{9}{4}-\dfrac{m^2}{H^2}}\,. \end{cases}$$

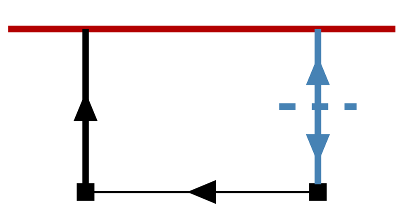
$$\Rightarrow \quad G_{ra}(k;\tau_1,\tau_2) = \frac{i\pi\theta(\tau_1-\tau_2)}{2Ha_1^{3/2}a_2^{3/2}}\left[\frac{J_\nu(-k\tau_1)J_{-\nu}(-k\tau_2)}{\sin(\pi\nu)} + (\nu \to -\nu)\right]$$: analytic in k.

$$J_\nu(e^{2i\pi}z) = e^{2i\pi\nu}J_\nu(z)$$

- Tree-level "s-channel" process:

$$\langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\varphi_{\vec{k}_4}\rangle\Big|_s =$$ 

$+$  $+$ 



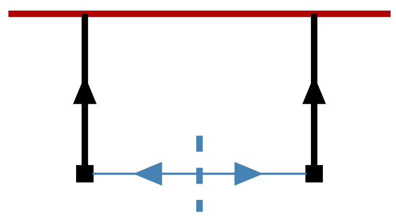
$$= -\frac{\lambda_3^2 H^4}{2k_1k_2k_3k_4}\int_{-\infty}^{0} d\tau_L \int_{-\infty}^{\tau_L} d\tau_R \sin(k_{12}\tau_L)\cos(k_{34}\tau_R)G_{ra}(k_s;\tau_L,\tau_R),$$

nested but no non-local CC

where $k_{ij} = k_i + k_j$, and $k_s = |\vec{k}_1 + \vec{k}_2| = |\vec{k}_3 + \vec{k}_4|$.

Non-local CC: example

- “Bulk-cut” diagram has factorized time integrals.

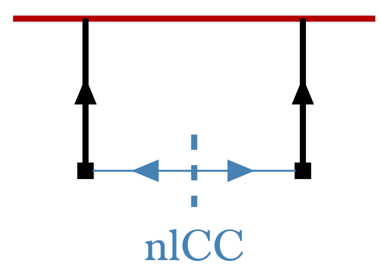


$$= \frac{\lambda_3^2 H^4}{2k_1k_2k_3k_4}\int_{-\infty}^{0} d\tau_L \int_{-\infty}^{0} d\tau_R \sin(k_{12}\tau_L)\sin(k_{34}\tau_R)G_{rr}(k_s;\tau_L,\tau_R),$$

where $G_{rr} \sim J_{\pm\nu}(-k_s\tau_L)J_{\pm\nu}(-k_s\tau_R)$.

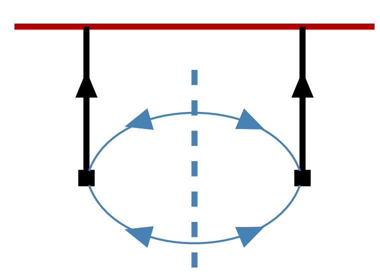
- Factorized time integrals simplify the calculation, in particular

$$\langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\varphi_{\vec{k}_4}\rangle\Big|_{s,\text{nlCC}} =$$



$$= \frac{\lambda_3^2 H^6[1+\sin(\pi\nu)]\Gamma^2[-\nu]\Gamma^2\left[\frac{5}{2}+\nu\right]}{8\pi k_1k_2k_3k_4k_{12}^{5/2}k_{34}^{5/2}}\underbrace{\left[\frac{k_s^2}{4k_{12}k_{34}}\right]^{\nu}}_{\text{non-local CC}} {}_2F_1\left[\begin{matrix}\frac{5+2\nu}{4},\frac{7+2\nu}{4}\\ 1+\nu\end{matrix};\frac{k_s^2}{k_{12}^2}\right] {}_2F_1\left[\begin{matrix}\frac{5+2\nu}{4},\frac{7+2\nu}{4}\\ 1+\nu\end{matrix};\frac{k_s^2}{k_{34}^2}\right] + (\nu \to -\nu)\,.$$

- Can be extended to loops:



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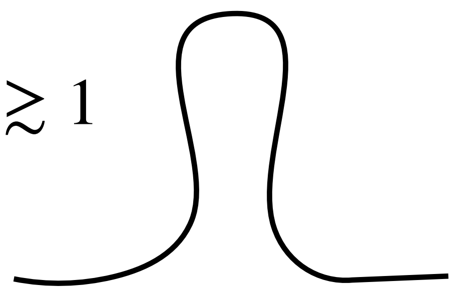
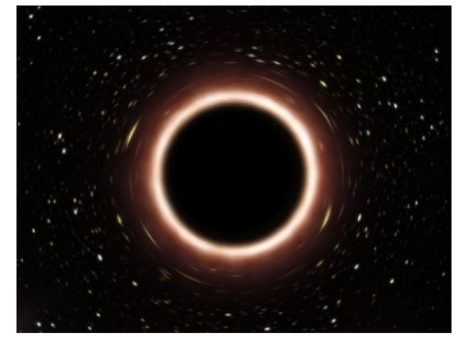
Motivation

- Enhanced scaler perturbations to produce primordial black holes.

$$\frac{\delta\rho}{\rho} \gtrsim 1$$





Claims on *scale-invariant* corrections to large scale perturbations.

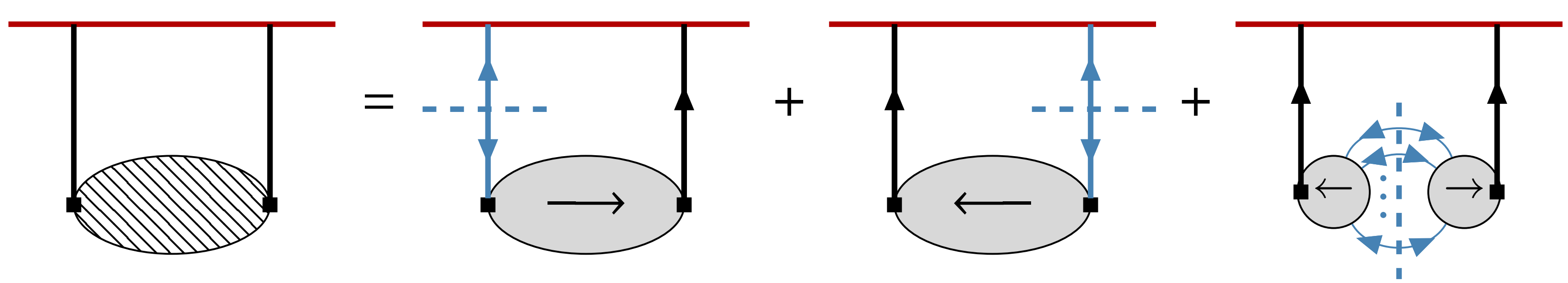
Scalar: [Kristiano, Yokoyama 22,23], tensor: [Ota, Sasaki, Wang 22a, 22b]

- Inconsistent with expectation from gauge invariance.

<u>e.g.</u> corrections to soft tensor mode → no derivative = graviton mass: $h_{\mu\nu}h^{\mu\nu}$.

How to make symmetry structures manifest in actual calculations?

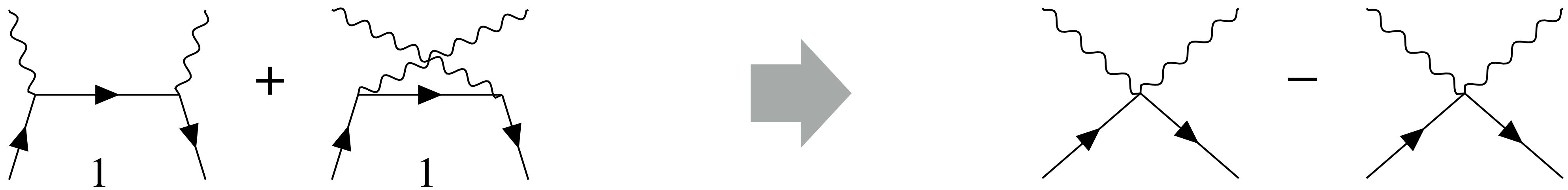
- Cutting-rule classification:



Causal production of GW $\sim k^3$

"Shrinking" structure

- QED example:

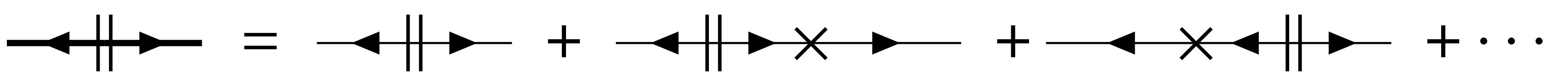


$$\frac{1}{t - m_e^2} \qquad \frac{1}{u - m_e^2} \qquad q^\mu \Gamma_\mu(q) = S^{-1}(p+q) - S^{-1}(p)$$

- Similar "shrinking" structure for cosmological correlators: [YE, Hong, Jinno, Mukaida 25]

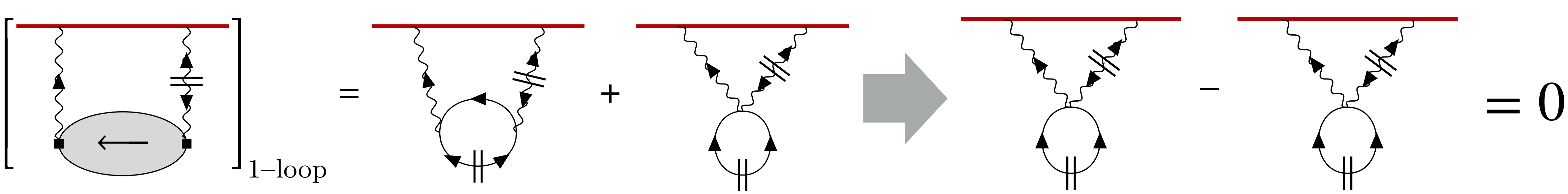
$$S = \frac{1}{2}\int d^{d-1}x d\tau f^2(\tau)\left[\left(\frac{d\chi}{d\tau}\right)^2 - c_s^2(\tau)(1+\epsilon)^2(\partial_i\chi)^2 - m_\chi^2(\tau)\chi^2\right]$$

Include ϵ to full order v.s. treat ϵ perturbatively



$$\frac{\partial}{\partial \log l} G_{rr} \sim G_{ra} \times G_{rr}$$: reduce number of propagators = "shrinking"

- As a result, one-loop correction to soft tensor mode cancels out:



[Ota, Sasaki, Wang 22a, 22b] didn't solve the mode equation with a given background consistently.

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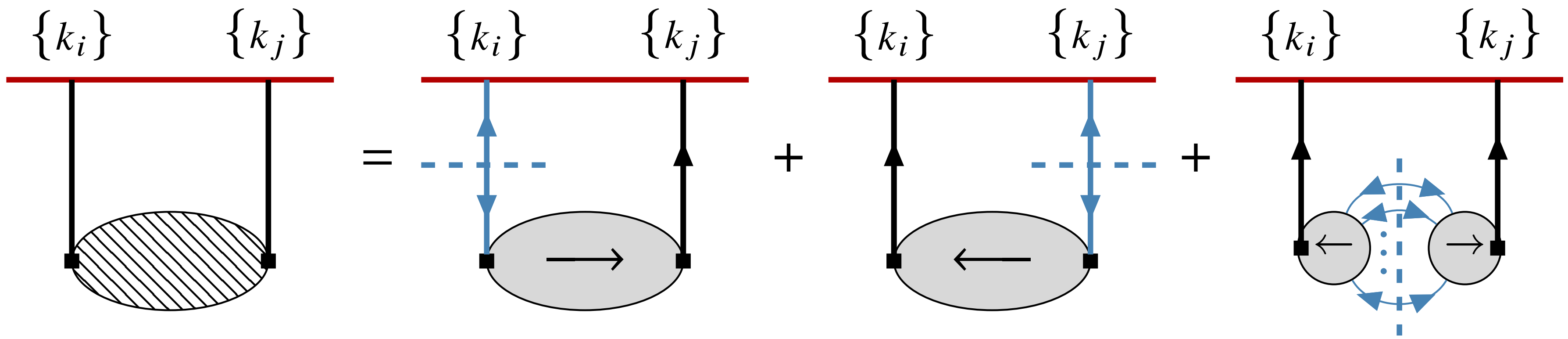
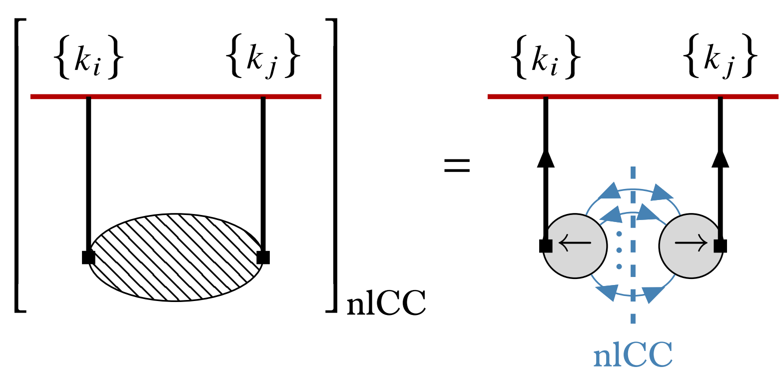
Summary

- A general cutting rule exists for in-in correlators:

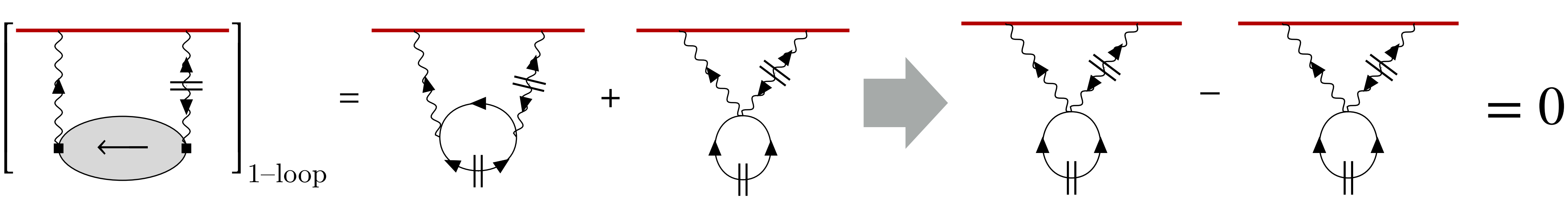


- Several applications to classify/simplify calculations:

1. Factorization of non-local cosmo-collider signal:

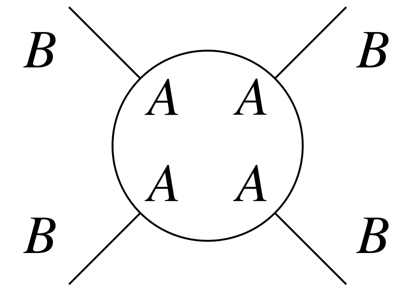


2. Cancellation of one-loop correction to large scale perturbations:

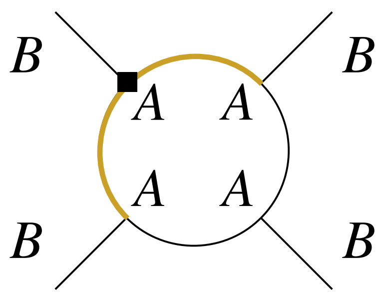


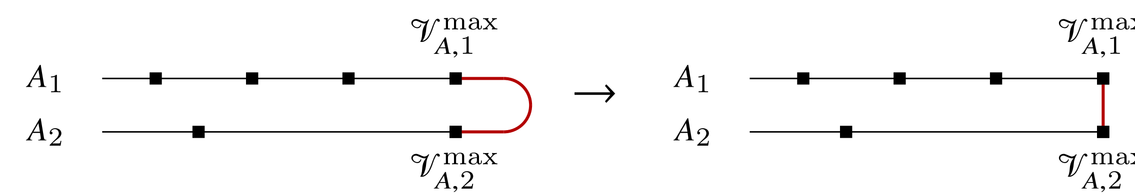
Back up

Proof of cutting rule: 2nd step

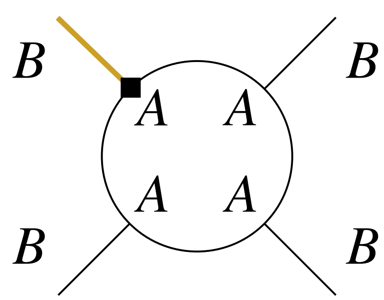
- Show that ''isolated islands'' of A/B vertices vanish:  $= 0.$

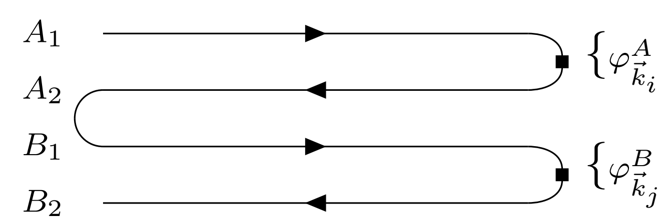
1. Propagators connected to $\mathscr{V}_A^{\max}$ with the largest time argument independent of $1/2$.



Ordering independent of $1/2$: 

2. Propagators connecting $\mathscr{V}_A^{\max}$ to external B vertices independent of $1/2$.



"B" always comes later than "A": 

3. Vertices have "±" due to $S[\phi_1] - S[\phi_2]$ → cancellation after summing $1,\ 2$.

A/B regions should be connected to boundary fields $\{\varphi_{\vec{k}_i}^{A/B}\}$.

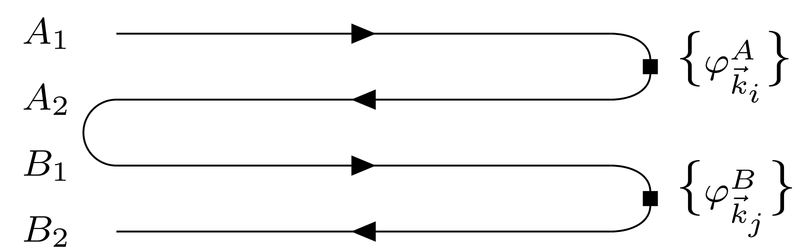
(Similar argument holds for Cutkosky rule in Veltman's largest time equation.)

Proof of cutting rule: cont'd

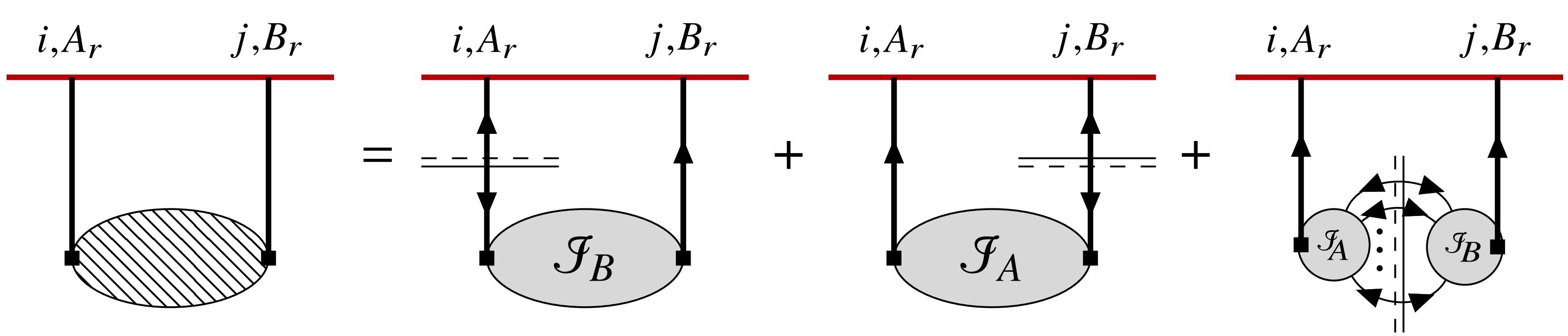
- Step 1+2: internal A/B region should be connected to external A/B fields.

  ➡ Each diagram has a boundary of A/B vertices = cut.

- Ordering of A/B fields independent of $1/2$.



➡ - Propagator is Wightman function $G_{>/<}(x, y)$, without time ordering.
  - Fields consumed are $1 + 2 = "r" \rightarrow$ time arrow flowing into vertex.

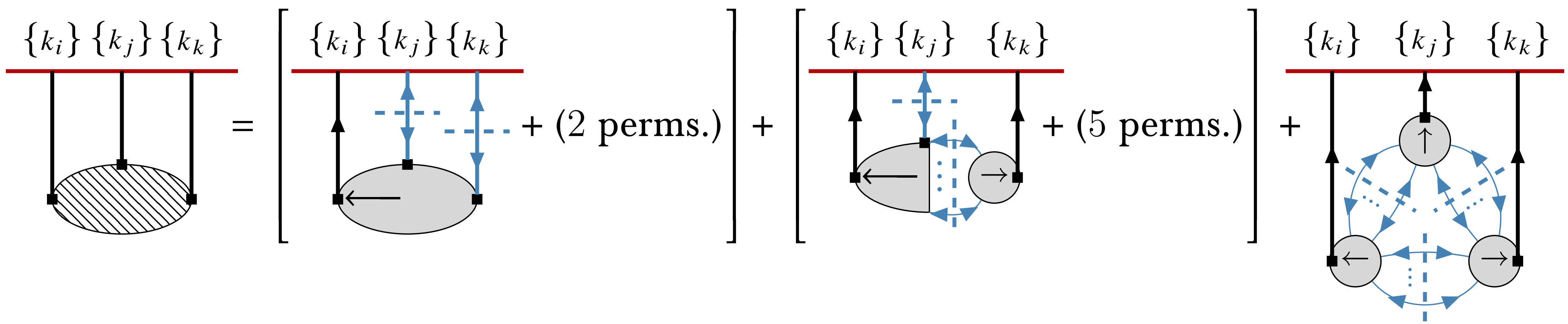


- Symmetrizing A/B assignments gives the desired result.

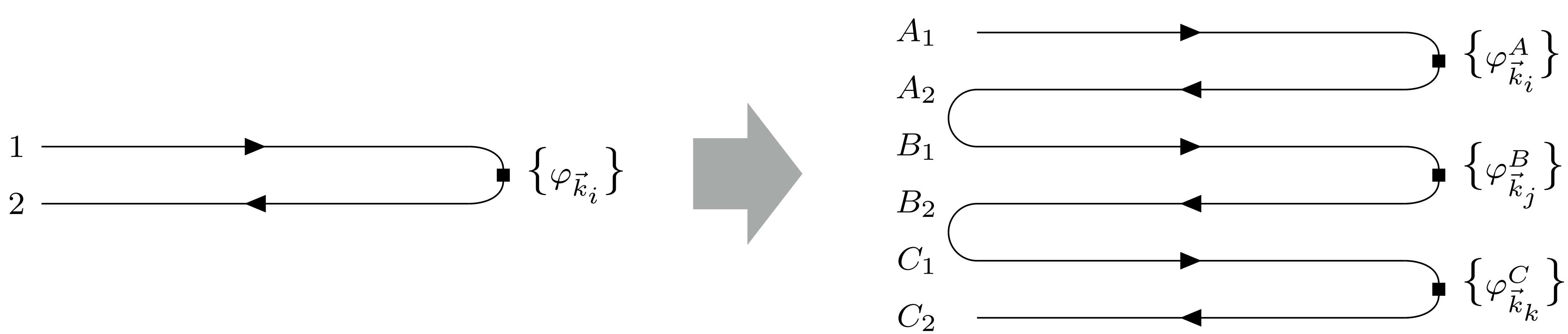
Cutting rule for higher-pt correlator

- Cutting rule can be generalized to higher-pt bulk correlators.

<u>ex. 3pt bulk correlator</u>



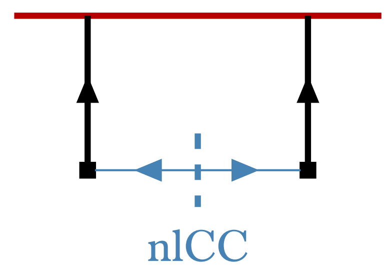
- Proof: further duplicate the contours.



Non-local CC: example

$$\mathscr{L} = \lambda_3 a^2 \varphi'^2 \sigma + \frac{\lambda_4}{4} a^2 \varphi'^2 \sigma^2$$

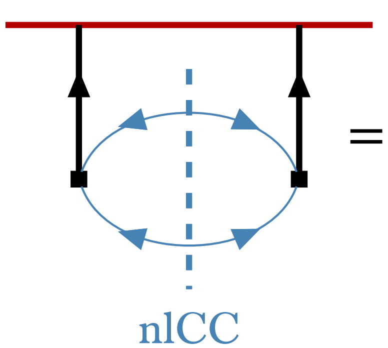
- **Tree-level:**



$$= \frac{\lambda_3^2 H^6 [1+\sin(\pi\nu)]\Gamma^2[-\nu]\Gamma^2\left[\frac{5}{2}+\nu\right]}{8\pi k_1 k_2 k_3 k_4 k_{12}^{5/2} k_{34}^{5/2}} \underbrace{\left[\frac{k_s^2}{4k_{12}k_{34}}\right]^{\nu}}_{\text{non-local CC}} {}_2F_1\left[\begin{matrix}\frac{5+2\nu}{4}, \frac{7+2\nu}{4} \\ 1+\nu\end{matrix}; \frac{k_s^2}{k_{12}^2}\right] {}_2F_1\left[\begin{matrix}\frac{5+2\nu}{4}, \frac{7+2\nu}{4} \\ 1+\nu\end{matrix}; \frac{k_s^2}{k_{34}^2}\right] + (\nu \to -\nu),$$

where $\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$, $k_{ij} = k_i + k_j$, and $k_s = |\vec{k}_1 + \vec{k}_2| = |\vec{k}_3 + \vec{k}_4|$.

- **One-loop:**

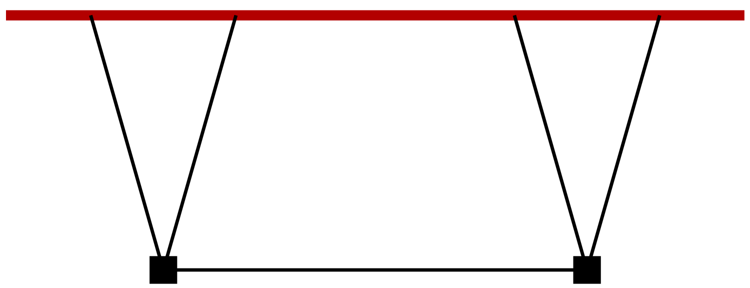


$$= \frac{\lambda_4^2 H^8 (2\nu+3)\sin^2(\pi\nu)\Gamma^2[-\nu]\Gamma[2\nu+4]\Gamma^2[\nu+3/2]\Gamma[-2\nu-3/2]}{128\pi^{7/2} k_1 k_2 k_3 k_4 k_{12}^{5/2} k_{34}^{5/2}} \underbrace{\left[\frac{k_s^2}{4k_{12}k_{34}}\right]^{3/2+2\nu}}_{\text{non-local CC}} + \cdots.$$

Correlators and amplitude

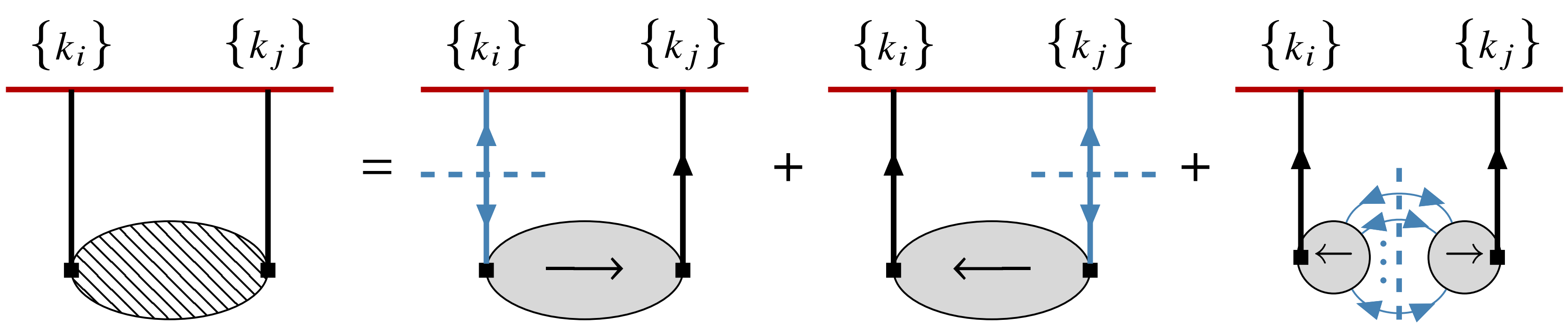
- Indication that correlators reduce to amplitudes at "energy poles"

[Maldacena, Pimentel 11; Raju 12; ...]



$$\sim \frac{A_4}{k_1 + k_2 + k_3 + k_4} + \cdots .$$

- Cutting rule decomposes diagrams into fully retarded functions

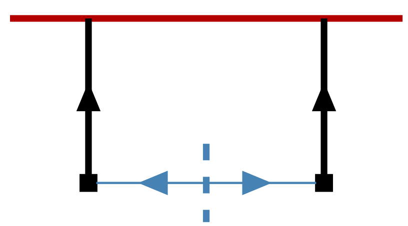


Fully retarded functions encode amplitudes??

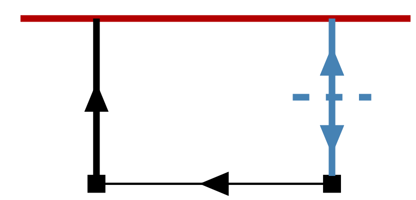
- Established at tree-level → how about loops??

Future directions

- Understanding "local CC signals" and "background"?

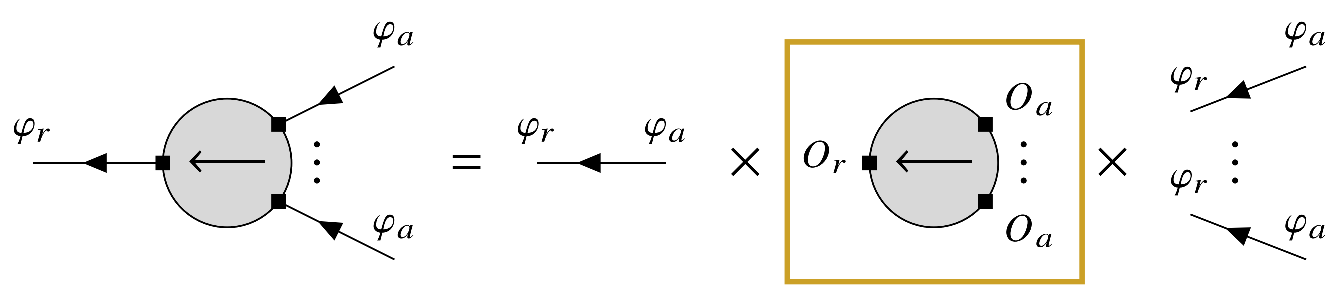


$\sim$ (nlCC) + (lCC),

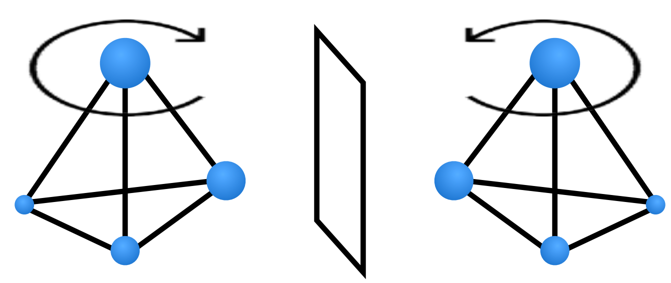


$\sim$ (lCC) + (BG) .

- Analytic property of fully retarded functions: "energy singularities"



- Factorization of parity-odd 4pt signals?



Parity-odd signal: $\mathrm{Im}\langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\varphi_{\vec{k}_4}\rangle$, "Im" $\rightarrow$ cutting rule?

- Relation to cutting rule on wavefunction of the universe?