Quantum sensing of high-frequency gravitational waves with ion crystals

Ryoto Takai (SOKENDAI)

Based on arXiv:**2512.19053**

with A. Ito, R. Kitano and W. Nakano

High-frequency GWs

- GWs from early universe physics [Caprini+, 2014], e.g., phase transition, cosmic strings, preheating after inflation

  → stochastic … strong bound from BBN [Yeh+, 2022]

- Exotic sources, e.g., merger of light PBHs [Franciolini+, 2022], BH superradiance [Brito+, 2015], exotic compact objects [Giudice+, 2016]

  → non-stochastic

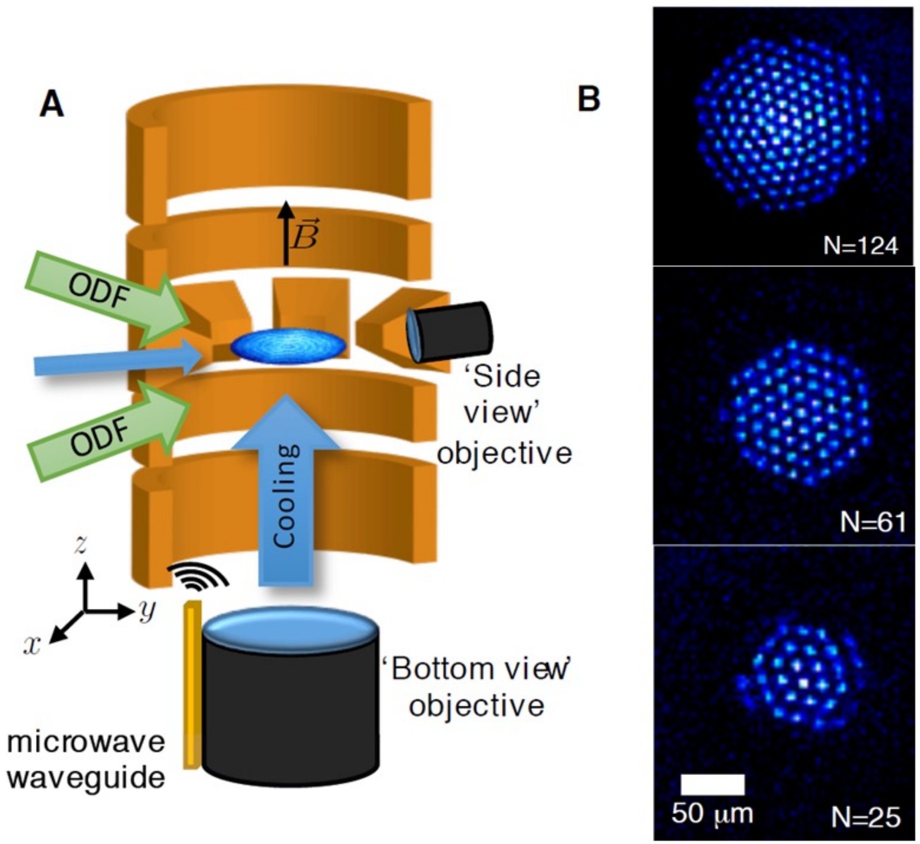
High-frequency GWs

- We focus on MHz range
- GW wavelength ≫ typical detector size
- Interaction Hamiltonian [Ito, 2021]

$$H_{\mathrm{GW}} = \frac{1}{2} m_{\mathrm{ion}} R_{0k0l}\, x^k x^l \sim h_0 m_{\mathrm{ion}} \omega^2 xz$$

- Larger wavefunction, stronger coupling

Ion crystal in Penning trap



- Ions are trapped in a vacuum chamber with static voltages and a magnetic field
- Ions lie at the 2D triangular lattice points
- Each ion has its spin

[NIST webpage]

Oscillation states

- Ions are cooled by lasers so that quantum d.o.f. of collective oscillation appears

- Focus on z-direction (along the magnetic field)

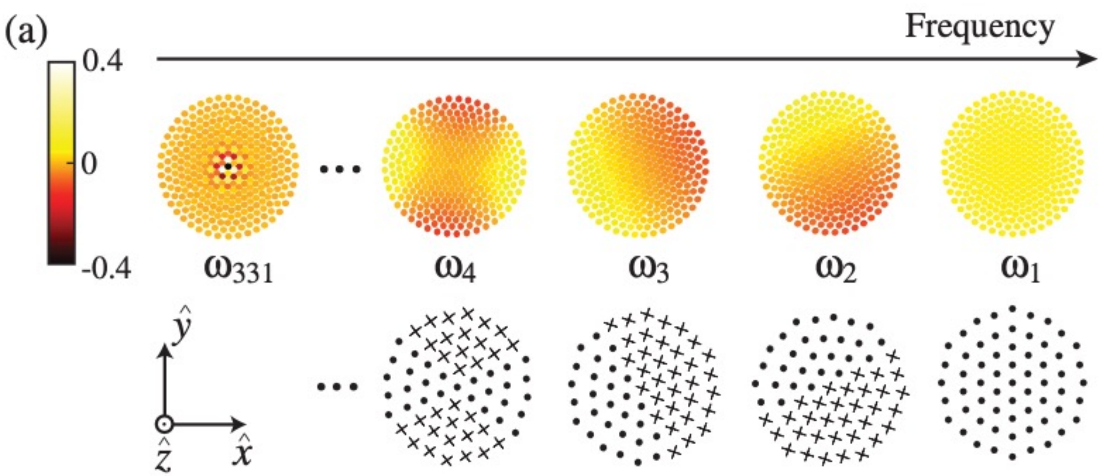
$$V_z = \frac{1}{2} m_{\rm ion} \omega_z^2 \sum_{i=1}^{N} z_i^2 + \frac{1}{2} \sum_{i \neq j} \frac{\alpha_{\rm EM}}{\sqrt{|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|^2 + (z_i - z_j)^2}}$$

$$= \frac{1}{2} m_{\rm ion} \omega_z^2 \sum_{i} z_i^2 - \frac{\alpha_{\rm EM}}{4} \sum_{i \neq j} \frac{(z_i - z_j)^2}{|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|^3} + \cdots ,$$

Oscillation states

- Ions are cooled by lasers so that quantum d.o.f. of collective oscillation appears

- Focus on z-direction (along the magnetic field)



[Sawyer+, 2012]

Entanglement b/w spin & motion

- Two lasers with frequency difference ω_{ODF} are employed by using AC Stark effect to make entanglement between spin and oscillation states (optical dipole force)

$$H_{\mathrm{ODF}} = \sum_i F_0 \cos(\omega_{\mathrm{ODF}}\, t)\, z_i\, \sigma_i^z \simeq \frac{g}{\sqrt{N}} \left(\hat{a}_1 + \hat{a}_1^\dagger\right) \hat{J}_z$$

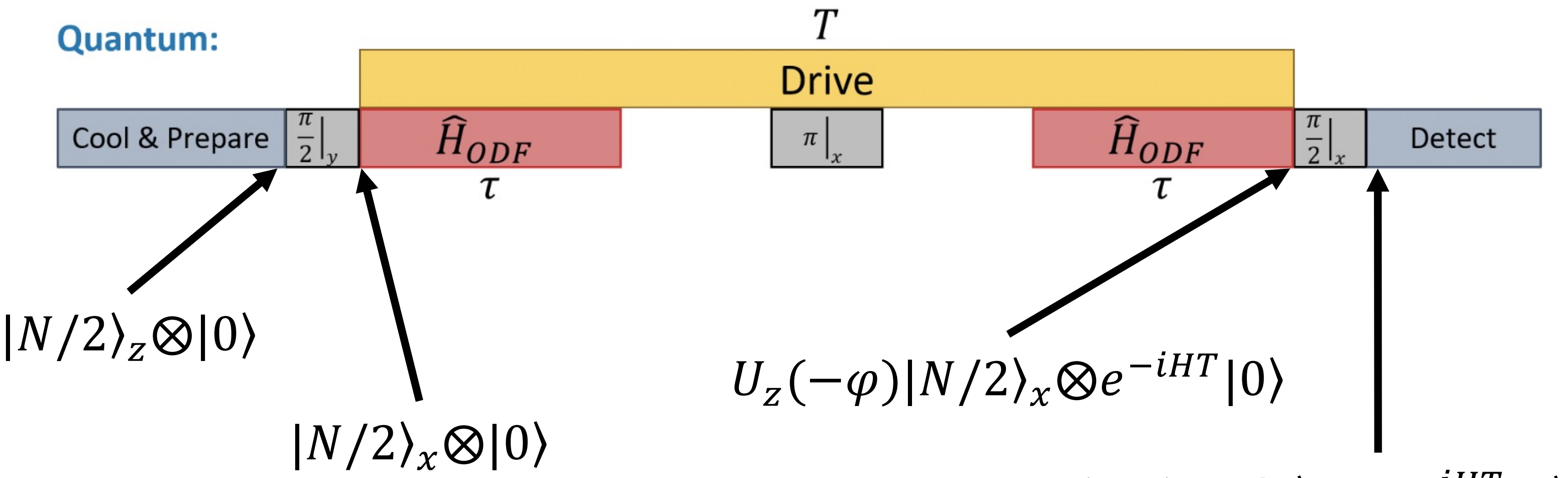
when $\omega_{\mathrm{ODF}} = \omega_1$

- Extend to other mode by deformation mirror

Center-of-mass mode: ω_1

- COM mode is sensitive to oscillating electric fields $H = eEz \simeq \alpha a_1 + \alpha^* a_1^\dagger$ by resonant excitation of COM mode

[Gilmore+ 2021]

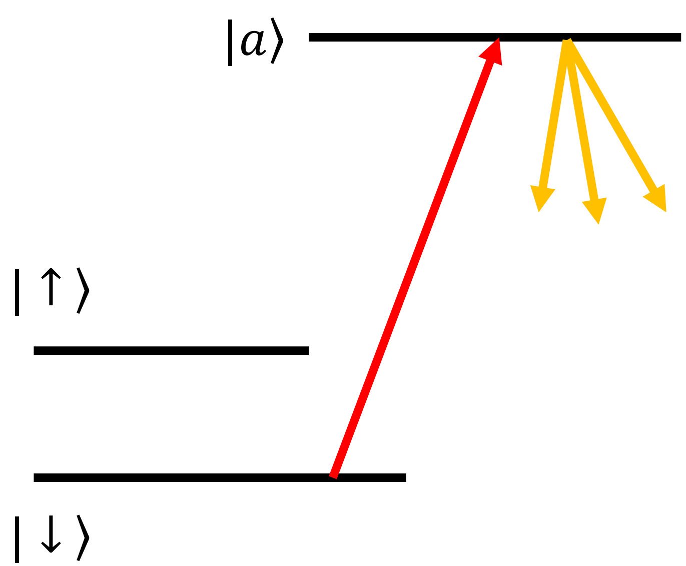


$$\varphi = \frac{2g\tau(T-\tau)}{\sqrt{N}}(\mathrm{Im}\,\alpha)$$

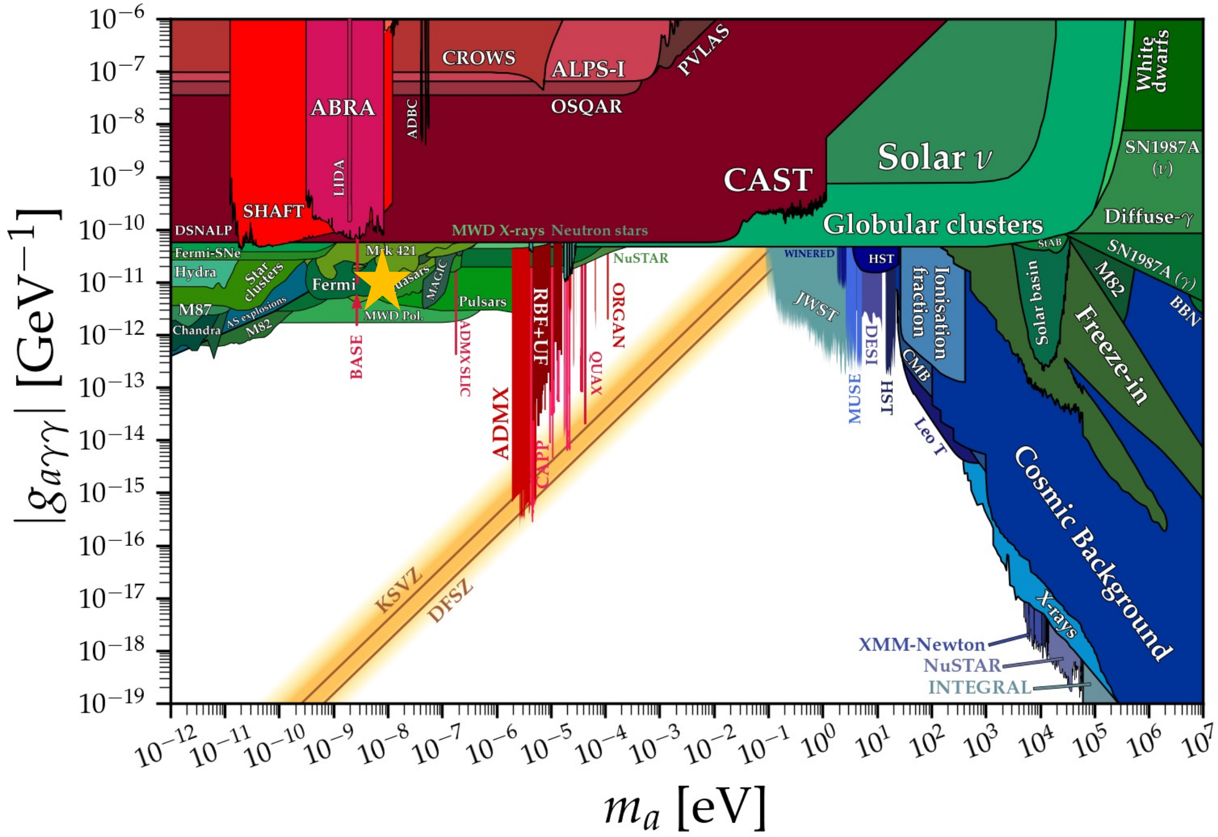
Detection

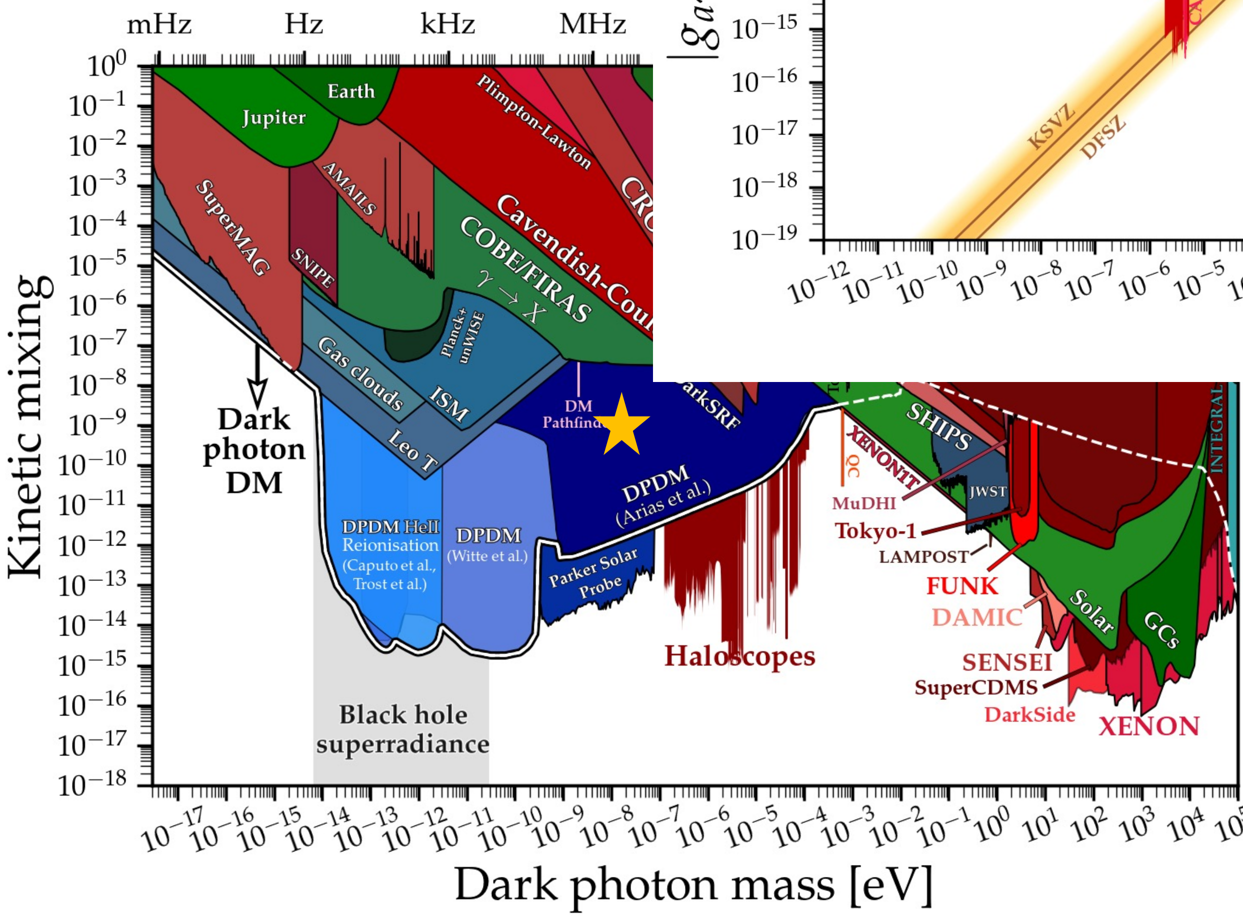
- Measure J_z by the spin-dependent global resonance fluorescence method [Norcia+, 2018]

- Strength of fluorescence is proportional to the number of ions in ground spin-state



[Axion Limits]



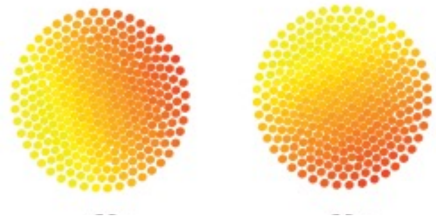


$N \sim 150$

$B = 4.5$ T

$R = 6.5$ cm

Parity-odd mode: $\omega_2 = \omega_3$



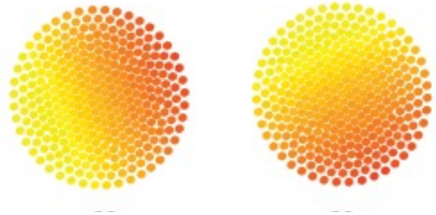
- Interaction Hamiltonian

$$\hat{H}_{\mathrm{GW}} \simeq \sum_i \frac{m_{\mathrm{ion}}\omega^2 e^{i\omega t}}{8\sqrt{2}} \Big[h^{(+)}\, e^{i\phi^{(+)}} \left(x_i^2 \cos^2\theta + z_i^2 \sin^2\theta - y_i^2 \underline{- 2x_i z_i \sin\theta\cos\theta} \right) + 2h^{(\times)}\, e^{i\phi^{(\times)}} \left(x_i y_i \cos\theta \underline{- y_i z_i \sin\theta} \right) \Big] + \mathrm{h.c.},$$

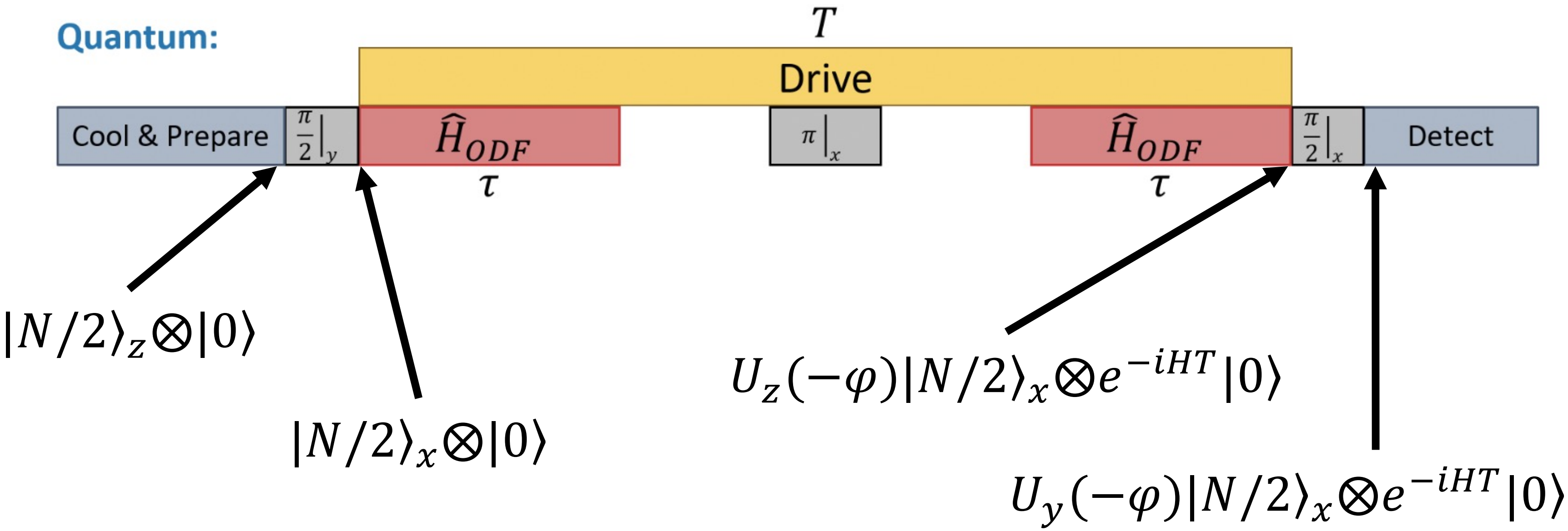
- GWs excite the parity-odd modes!

$$H = \alpha a + \alpha^* a^\dagger,\ a = \frac{1}{\sqrt{2}}(a_2 + i a_3)$$

Parity-odd mode: $\omega_2 = \omega_3$



[Gilmore+ 2021]



$$\varphi = \frac{2g\tau(T-\tau)}{\sqrt{N}}(\mathrm{Im}\,\alpha) \qquad \alpha = -\frac{\sqrt{N m_{\mathrm{ion}}}(\omega_2-\omega_r)^2 R}{8\zeta_{11}\sqrt{(\zeta_{11}^2-1)\,\omega_2}}\, h_0 \sin\theta\big(\cos\theta - i\big)e^{i\phi}$$

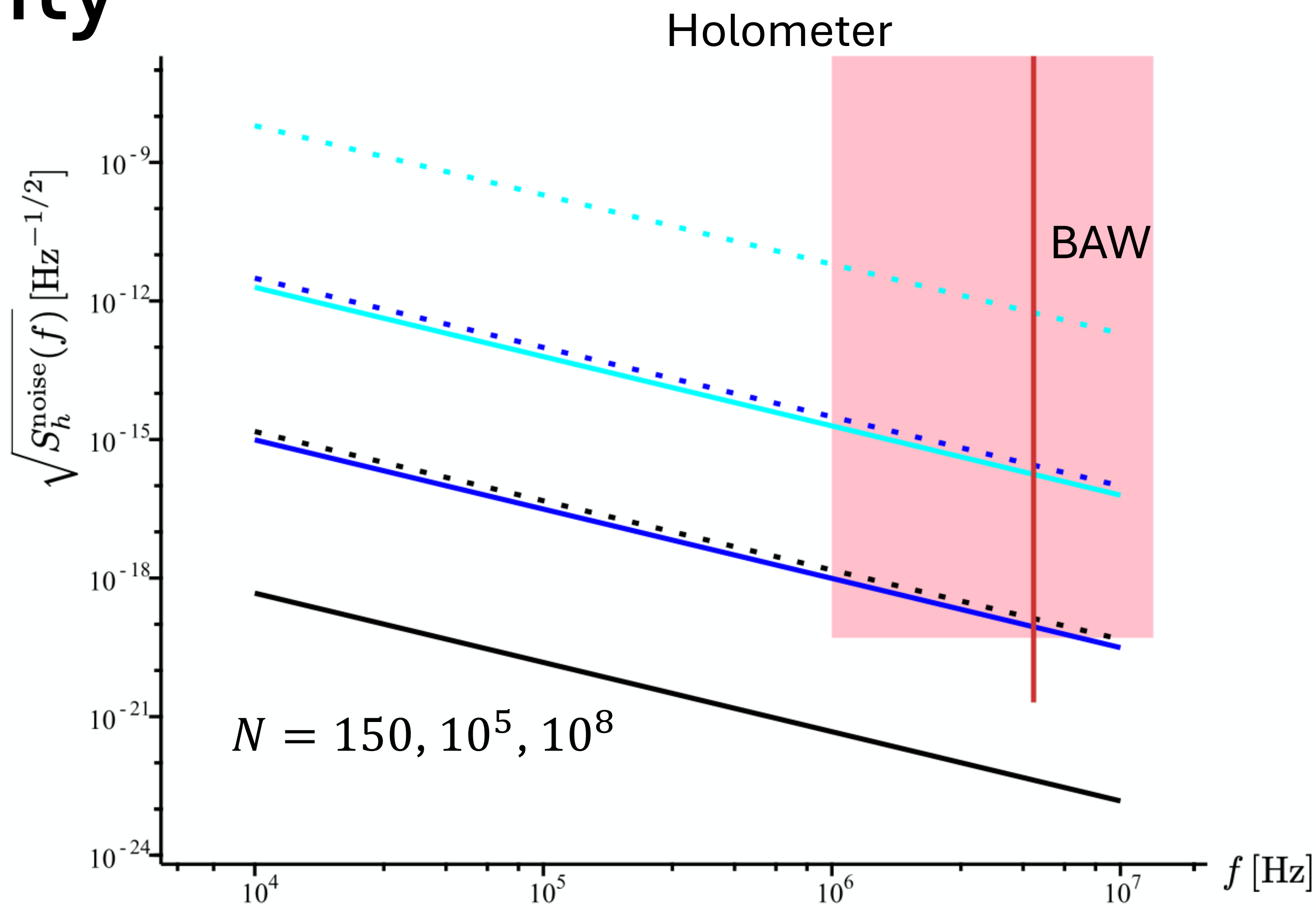
Sensitivity

- For single measurement, the sensitivity to $\eta = |\mathrm{Im}\,\alpha|$ is given by

$$(\delta\eta)^2 \simeq \frac{e^{2\Gamma\tau}}{4g^2\tau^2(T-\tau)^2} + \frac{e^{2\Gamma\tau}\sigma^2(2T^2-\tau T+\tau^2)}{8g^2\tau^2(T-\tau)^2} + \frac{\sigma^2(2\bar{n}+1)}{4} + \frac{g^2\tau^2\sigma^2(T-\frac{4}{3}\tau)^2}{4(T-\tau)^2}$$

- Γ is decoherence rate of spins, σ is frequency detuning of ODF, $\bar{n}$ is initial occupation number of the phonon mode

Sensitivity



$$\sqrt{S_h^{\text{noise}}(f)} = 2.3\times10^{-22}\,\text{Hz}^{-1/2}\times\left(\frac{N}{10^8}\right)^{-1/2}\left(\frac{m_{\text{ion}}}{8.3\text{ GeV}}\right)^{-1/2}\left(\frac{R}{80\text{ mm}}\right)^{-1}\left(\frac{f}{1.6\text{ MHz}}\right)^{-3/2}$$

$$= \sqrt{T h_0^2}$$

Summary

- Ion crystals are useful to search MHz gravitational wave (and wave-like dark matter)

- GWs excite the parity-odd mode, unique for GWs

- Good scaling with respect to the measurement time T and ion number N beyond the standard quantum limit