The Electric Dipole Moment of electron induced by Electroweak Multiplets at full Three-loop

arXiv: 2602.11888

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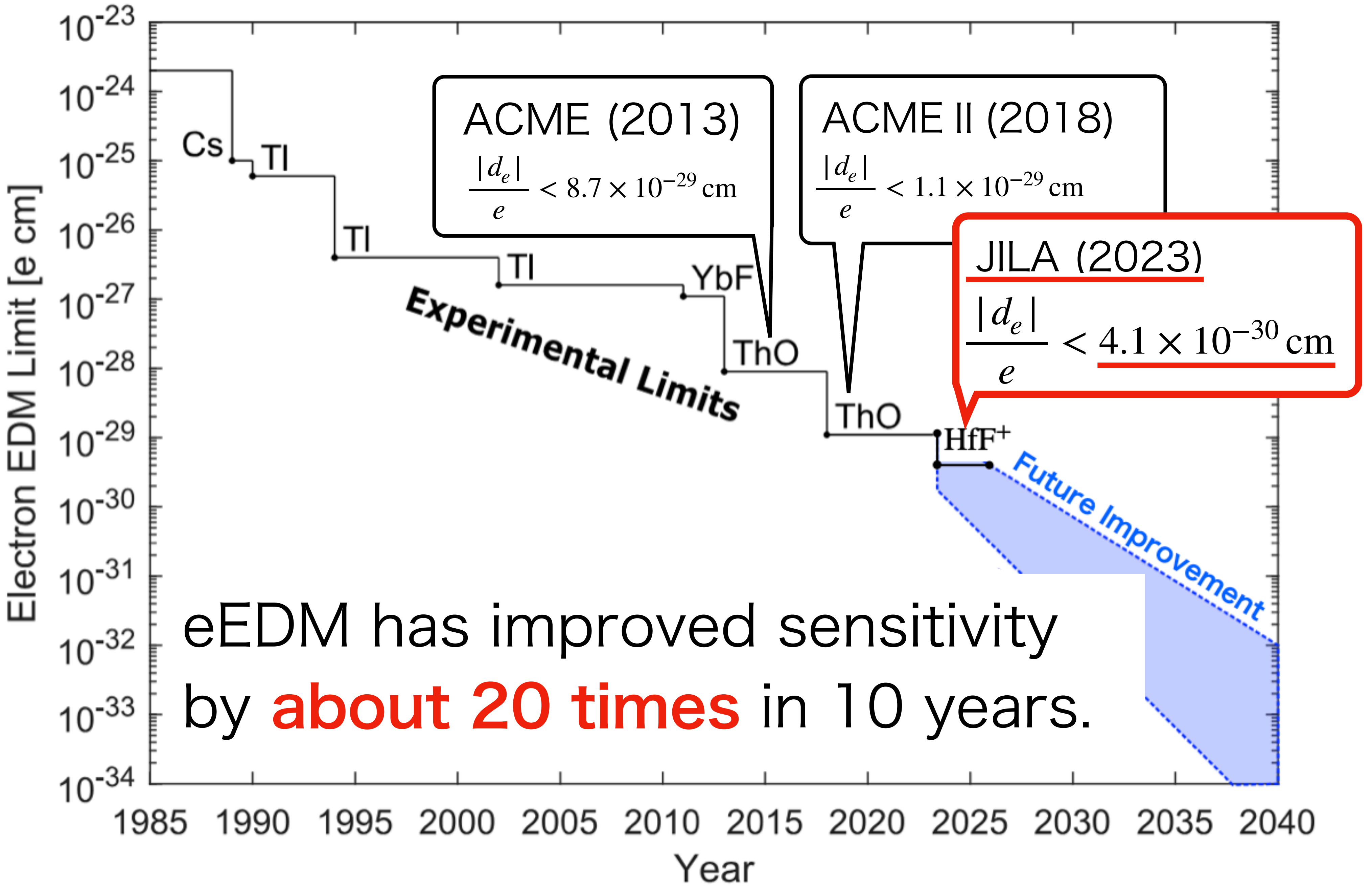
NAGOYA UNIVERSITY

Electron EDM (eEDM)

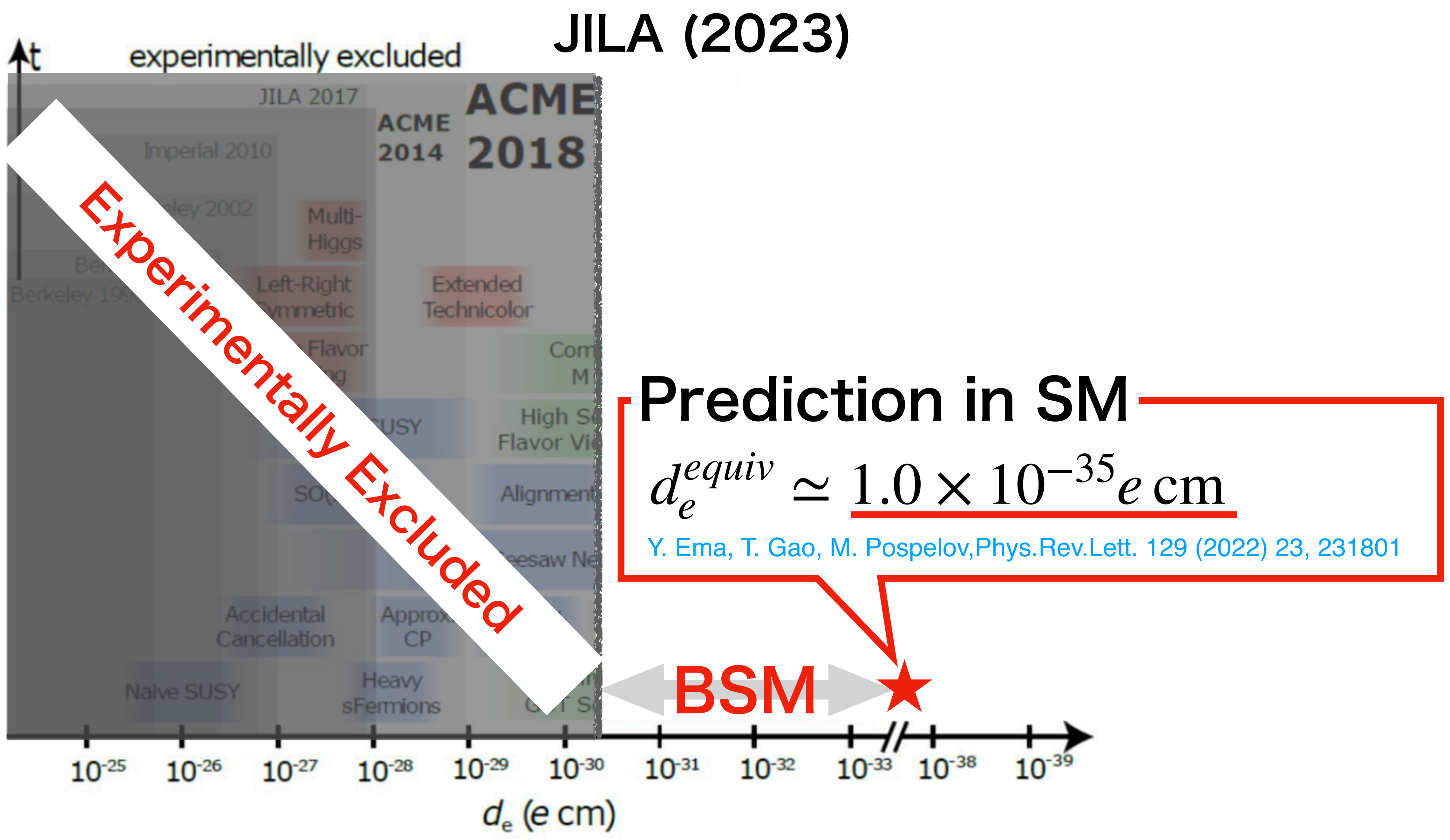
- **Electric Dipole Moment (EDM)**

$$\mathcal{L} = -d\frac{i}{2}\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\gamma_5\psi$$ Parity(P) & Time reversal(T = CP) symmetries are violated

- **Progress of electron EDM (eEDM)**



R. Alarcon et al. “Electric dipole moments and the search for new physics” in Snowmass,(2021),(2022)
T. S. Roussy et al. Science 381 (2023)



https://cfp.physics.northwestern.edu/gabrielse-group/acme-electron-edm.html
M. Pospelov, A. Ritz, Phys.Rev.D 89 (2014) 5, 056006

eEDM is attractive observable to probe BSM!

Electron EDM (eEDM)

◆ order estimate on the eEDM (n-loop)

$$\frac{d_e}{e} \approx \left(\frac{\lambda^2}{16\pi^2}\right)^n \frac{m_e}{\Lambda^2} \sin\phi_{CP}$$

(n : loop order, Λ : BSM scale, λ : coupling constants, $\sin\phi_{CP}$: CP phase)

◆ constraints to BSM scale

Current

$$\frac{|d_e|}{e} < 4.1 \times 10^{-30}\,\mathrm{cm}$$

($\lambda \simeq g$ (SU(2)$_L$ gauge coupling), $\sin\phi_{CP} = O(1)$)

$\Lambda \gtrsim 80$ TeV (n = 1)
$\Lambda \gtrsim 4$ TeV (n = 2)

Future Experiments

$\frac{|d_e|}{e} < O(10^{-31})\,\mathrm{cm}$ ▶ $\Lambda \gtrsim 1$ TeV (n = 3)

$\frac{|d_e|}{e} < O(10^{-32})\,\mathrm{cm}$ ▶ $\Lambda \gtrsim 3$ TeV (n = 3)

sensitive to TeV scale at 3-loop level

In the future eEDM experiments,

TeV-scale BSM that induces the eEDM at 3-loop can be probed!

Set Up

BSM(TeV)

- $SU(2)_L$ multiplets : ψ_A, ψ_B (fermion), S (scalar)

CP violating Yukawa interaction $\mathcal{L} \supset -\bar{\psi}_B g_{\bar{B}AS} \psi_A S - \bar{\psi}_A g_{\bar{A}B\bar{S}} \psi_B S^*$

$$g_{\bar{B}AS} = X_{\bar{B}AS}(s + \gamma_5 a)$$
$$g_{\bar{A}B\bar{S}} = X_{\bar{A}B\bar{S}}(s^* - \gamma_5 a^*)$$

~~CP~~

(s, a : complex number)

($X_{\bar{B}AS}, X_{\bar{A}B\bar{S}}$: coefficients that depend on representation)

※assumption: $SU(2)_L$ multiplets couple to electron only through W

SM

- **The motivation of the $SU(2)_L$ multiplets**

▶ **Minimal Dark Matter model** M. Cirelli and A. Strumia, New J. Phys. 11(2009) 105005

  - The neutral component is DM candidate.
  - mass $\sim O(\mathrm{TeV})$ (Relic abundance)
  - quintuplet fermion is favor (DM stability)

Set Up

BSM(TeV)

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$g_{\bar{B}AS} = X_{\bar{B}AS}(s + \gamma_5 a)$ (s, a : complex number)

$g_{\bar{A}B\bar{S}} = X_{\bar{A}B\bar{S}}(s^* - \gamma_5 a^*)$ ($X_{\bar{B}AS}, X_{\bar{A}B\bar{S}}$: coefficients that depend representation)

~~CP~~

※assumption: $SU(2)_L$ multiplets couple to electron only through W

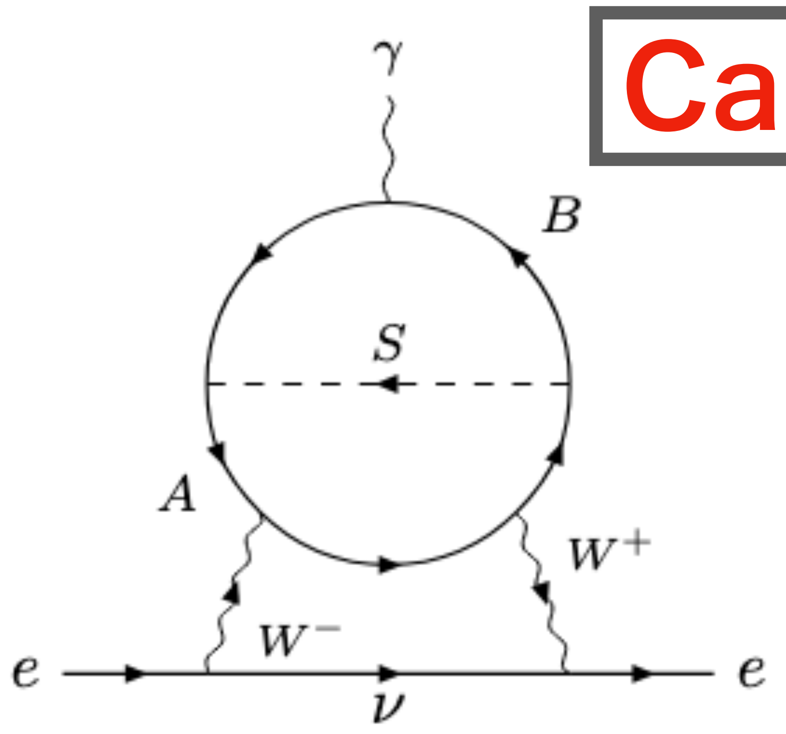
SM

Our Work : Evaluate the eEDM in this model

The 3-loop contribution is Leading Order(LO)!

Can this model de probed in the future experiment?

d_e



Set Up

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Integrating out $\psi_{A/B}, S$ **2-loop**

3-loop

SM

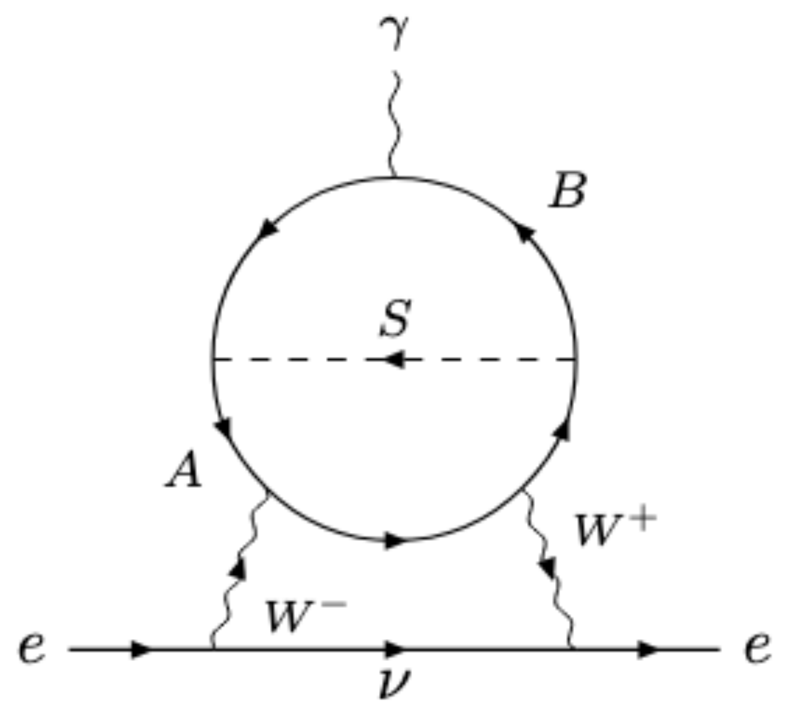
Electroweak-Weinberg Operator

$$\mathcal{L}_{\mathrm{SMEFT}} = -\frac{g^3}{3} C_W \epsilon^{abc} W^a_{\mu\nu} W^{b\nu}{}_{\rho} \widetilde{W}^{c\rho\mu} - d_e \frac{i}{2} \bar{e} \sigma_{\mu\nu} F^{\mu\nu} \gamma_5 e$$

threshold-correction in SMEFT

Integrating out m_W **1-loop**

$$d_e$$



Previous Study

BSM(TeV)

- $SU(2)_L$ multiplets : ψ_A, ψ_B (fermion), S (scalar)

CP violating Yukawa interaction $\mathcal{L} \supset -\bar{\psi}_B g_{\bar{B}AS} \psi_A S - \bar{\psi}_A g_{\bar{A}B\bar{S}} \psi_B S^*$

Integrating out $\psi_{A/B}, S$ 2-loop

SM

Electroweak-Weinberg Operator

$$\mathcal{L}_{\text{SMEFT}} = -\frac{g^3}{3} C_W \epsilon^{abc} W^a_{\mu\nu} W^{b\nu}{}_\rho \widetilde{W}^{c\rho\mu}$$

Integrating out m_W 1-loop

d_e

Matching condition to eEDM

$$\frac{d_e^{C_W}}{e} = \frac{\alpha_2^2}{6} m_e C_W + \mathcal{O}\left(\frac{m_e^2}{m_W^2}\right)$$

K. Ogawa et al., JHEP 02 (2025) 082

- No Log-enhancement because anomalous-dim = 0

This result is composed of the threshold-correction in LEFT!

W. Dekens and P. Stoffer, JHEP 10 (2019) 197

This Study

BSM(TeV)

- $SU(2)_L$ multiplets : ψ_A, ψ_B (fermion), S (scalar)

CP violating Yukawa interaction $\mathcal{L} \supset -\bar{\psi}_B g_{\bar{B}AS} \psi_A S - \bar{\psi}_A g_{\bar{A}B\bar{S}} \psi_B S^*$

Integrating out $\psi_{A/B}, S$ **2-loop**

3-loop

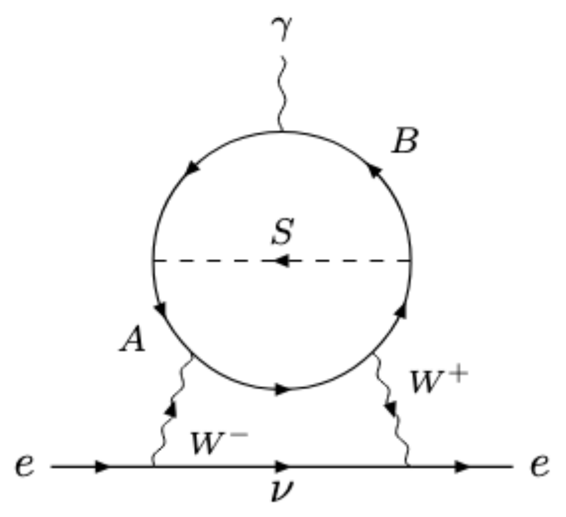
SM

Electroweak-Weinberg Operator

$$\mathcal{L}_{\text{SMEFT}} = -\frac{g^3}{3} C_W \epsilon^{abc} W^a_{\mu\nu} W^{b\nu}{}_{\rho} \widetilde{W}^{c\rho\mu} - d_e \frac{i}{2} \bar{e} \sigma_{\mu\nu} F^{\mu\nu} \gamma_5 e$$

threshold-correction in SMEFT

Integrating out m_W **1-loop**



d_e

Threshold correction in SMEFT
would contribute at the same order

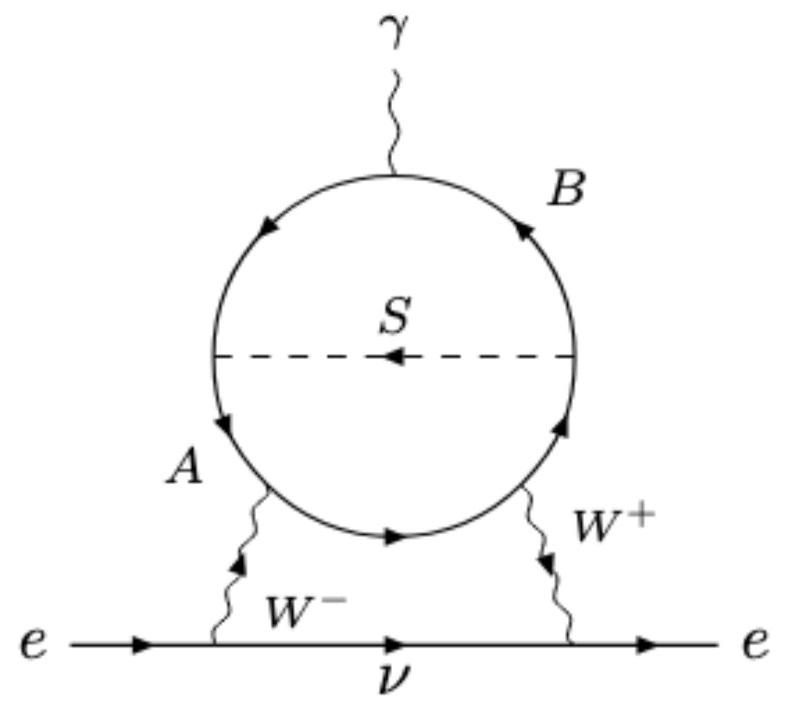
This Study

BSM(TeV)

- $SU(2)_L$ multiplets : ψ_A, ψ_B (fermion), S (scalar)

CP violating Yukawa interaction $\mathcal{L} \supset -\bar{\psi}_B g_{\bar{B}AS} \psi_A S - \bar{\psi}_A g_{\bar{A}B\bar{S}} \psi_B S^*$

SM



3-loop

d_e

This Study

▶ We evaluate Full 3-loop eEDM directly

Have already evaluated

$$d_e^{\mathrm{Full}} = d_e^{CW} + d_e^{\mathrm{th}}$$

Threshold-correction in SMEFT

Full Calculation

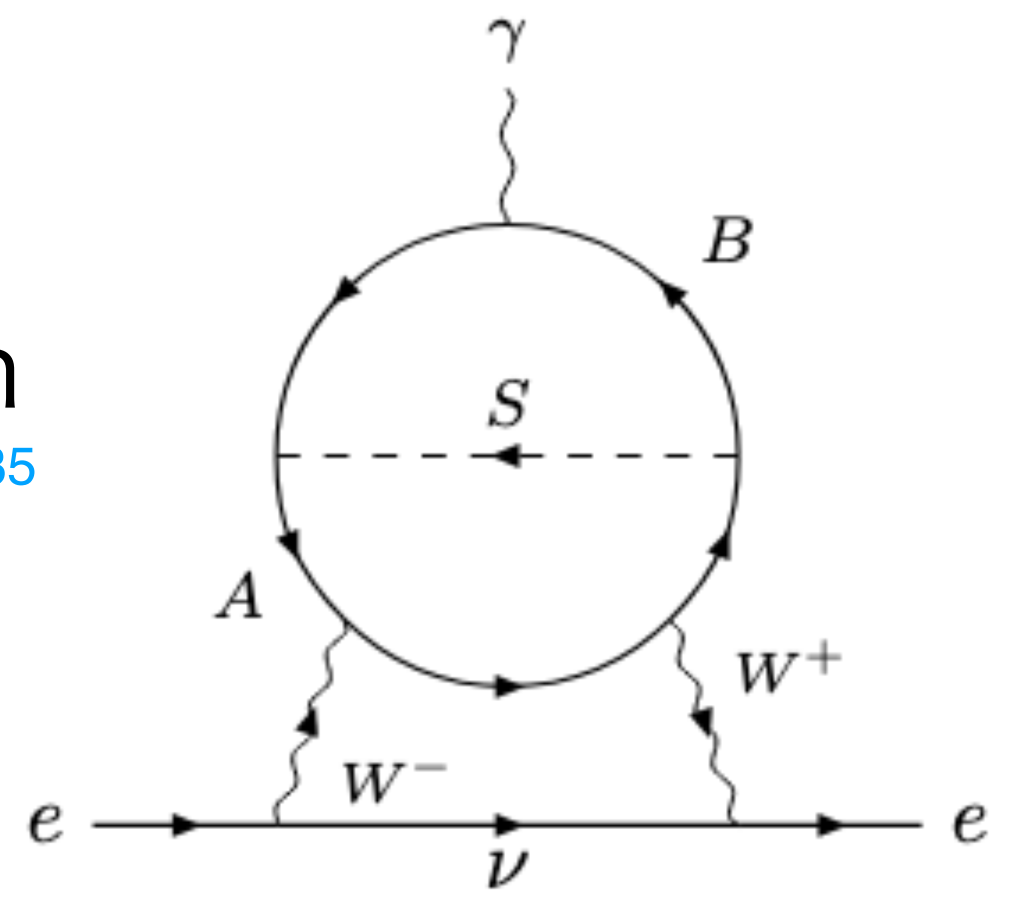
Result of calculation

$(A, B, S) = (r, r, 1)$

Combination of 3-loop vacuum integrals

$$\frac{d_e^{\mathrm{Full}}}{e} = \frac{\alpha_2^2}{(16\pi^2)^2}\frac{m_e}{2}\frac{r(r^2-1)}{12}\mathrm{Im}(sa^*)m_A m_B B(m_A, m_B, m_S, m_W)$$

Rephasing-invariant factor



- $\mathrm{Im}(sa^*)m_A m_B$ obtains rephasing invariance under the field redefinition

  T. Banno et al., JHEP 02 (2024) 195, JHEP 09 (2025) 135

  A γ_5 - part also composes of $\mathrm{Re}(sa^*)$ ▶ Loop-integral = 0 !

  m_A, m_B can be picked up by Chirality-flips in each fermions

- $B(m_A, m_B, m_S, m_W)$ can be evaluated analytically in the case of $m_A = m_B$

Result

◆ **Comparison of $d_e^{\rm Full}$ and $d_e^{C_W}$** $(m_A = m_B = m_S)$

electron EDM in the full theory

$$\frac{d_e^{\rm Full}}{e} = \frac{\alpha_2^2 m_e}{(16\pi^2)^2}\frac{r(r^2-1)}{12}\,{\rm Im}(sa^*)\left(\frac{0.41}{m_A^2} + \frac{0.22 m_W^2}{m_A^4}\right) + \mathcal{O}\left(\frac{m_W^4}{m_A^4}\right)$$

electron EDM through the electroweak-Weinberg operator

$$\frac{d_e^{C_W}}{e} = \frac{\alpha_2^2 m_e}{(16\pi^2)^2}\frac{r(r^2-1)}{12}\,{\rm Im}(sa^*)\frac{0.14}{m_A^2}$$

Result

◆ **Comparison of d_e^{Full} and $d_e^{C_W}$** $(m_A = m_B = m_S)$

electron EDM in the full theory

$$\frac{d_e^{\mathrm{Full}}}{e} = \frac{\alpha_2^2 m_e}{(16\pi^2)^2} \frac{r(r^2-1)}{12} \mathrm{Im}(sa^*) \left(\frac{0.41}{m_A^2} + \frac{0.22 m_W^2}{m_A^4} \right) + \mathcal{O}\left(\frac{m_W^4}{m_A^4} \right)$$

electron EDM through the electroweak-Weinberg operator

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$$d_e^{\mathrm{Full}} \simeq 3 d_e^{C_W}$$

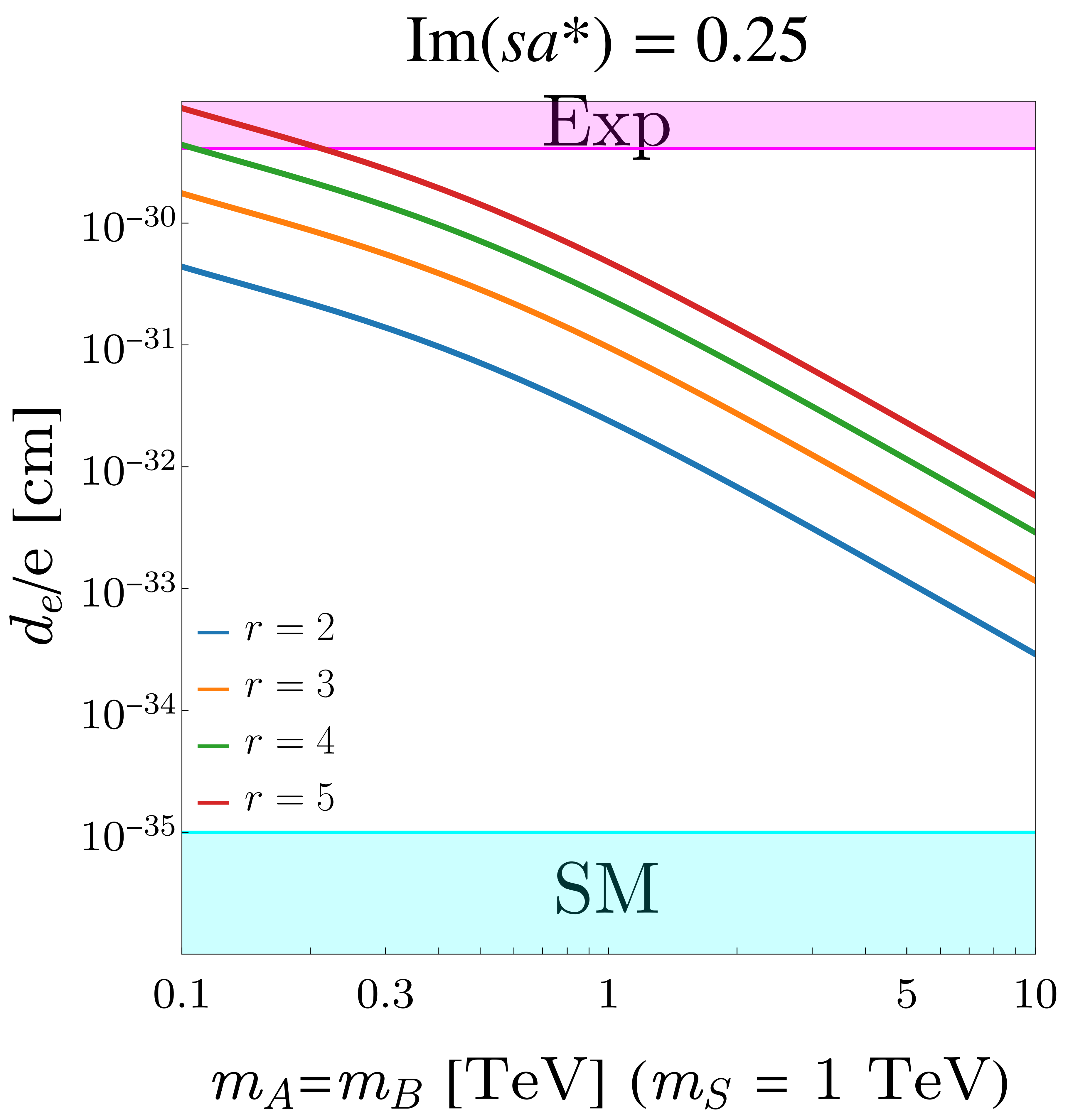
Result

◆ Prospects for the future experiments $(m_A = m_B \neq m_S)$

- This scenario can be tested in the future eEDM!

- In the case of large r, eEDM is enhanced

$$d_e^{\rm Full} \propto r(r^2 - 1)$$



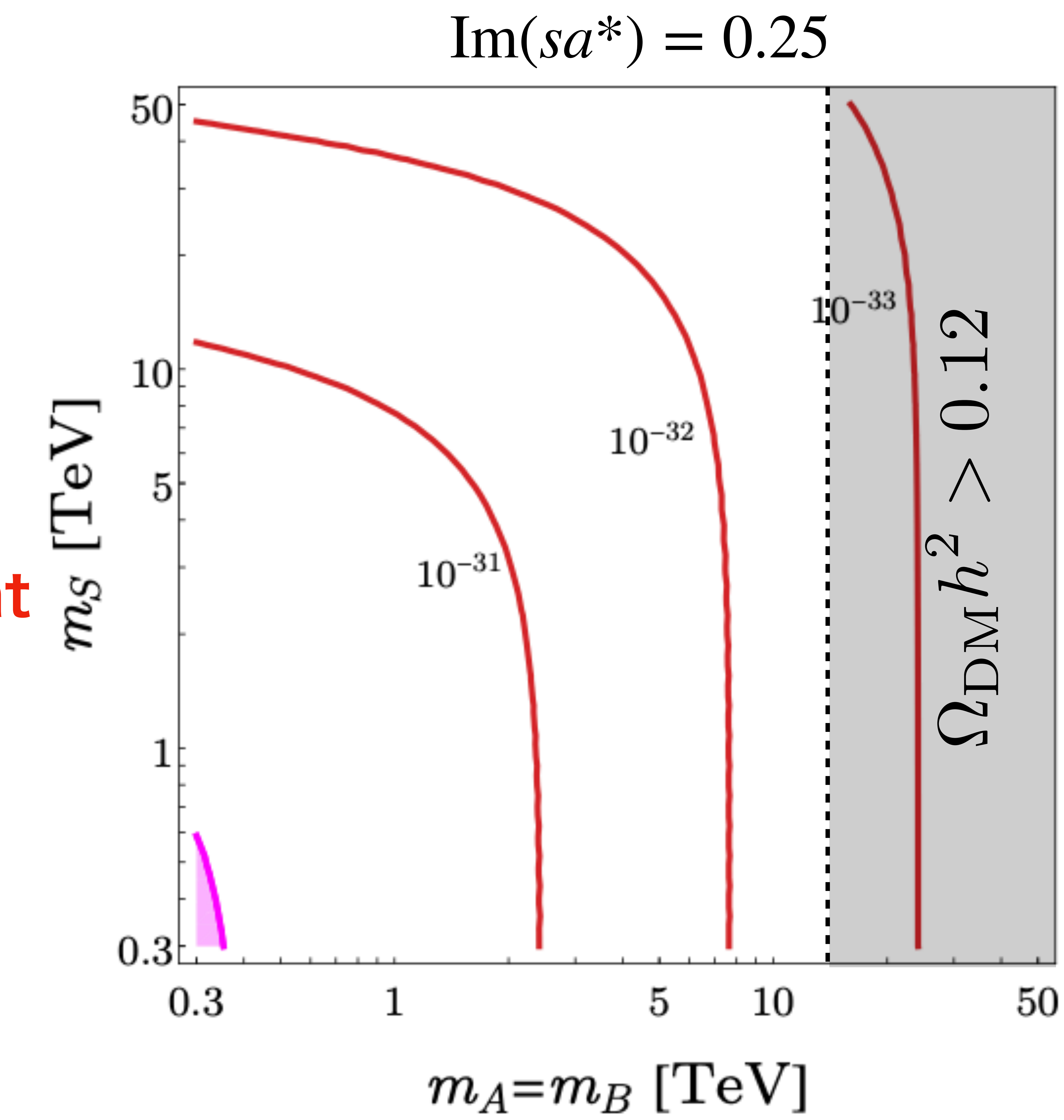
Result

◆ Prospects for the future experiments $(m_A = m_B \neq m_S)$

- 5-plet fermion case is favored as Minimal DM

$$\Omega_{\rm DM}h^2 \leq 0.12 \quad \blacktriangleright \quad m_A \leq 14\text{TeV}$$

- DM candidate would be probed by improvement of two order of magnitudes



Conclusion

- TeV-scale BSM that induces the eEDM at 3-loop can be probed!

- We evaluate the contribution of the eEDM in the full 3-loop diagram

Result1

Threshold-correction in SMEFT

$$d_e^{\mathrm{Full}} = d_e^{CW} + d_e^{\mathrm{th}} \simeq 3 d_e^{CW}$$

Give large contribution

Result2

This scenario would be tested in the future eEDM!

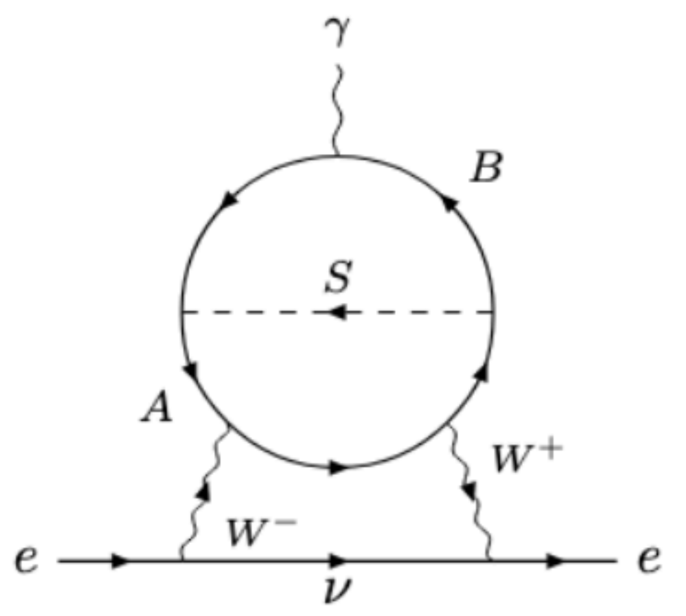
BSM(TeV)

- $SU(2)_L$ multiplets : ψ_A, ψ_B (fermion), S (scalar)

CP violating Yukawa interaction

$$\mathcal{L} \supset -\bar{\psi}_B g_{\bar{B}AS} \psi_A S - \bar{\psi}_A g_{\bar{A}B\bar{S}} \psi_B S^*$$

SM



3-loop

d_e

Back Up

Electric Dipole Moment(EDM)

- EDM has many observables (nucleon, pala-magnetic atom, etc)

- The most recently results

Neutron EDM (2018)

$|d_n| < 1.8 \times 10^{-26} e\text{cm}$ C. Abel, *et al.*, Phys. Rev. Lett. 124 (2020) 081803

Electron EDM (2023)

$|d_e| < 4.1 \times 10^{-30} e\text{cm}$ T. S. Roussy et al. Science 381 (2023)

Mercury EDM (2023)

$|d_{\text{Hg}}| < 7.4 \times 10^{-30} e\text{cm}$ B. Graner et al., Phys. Rev. Lett. **116** (2016) 161601

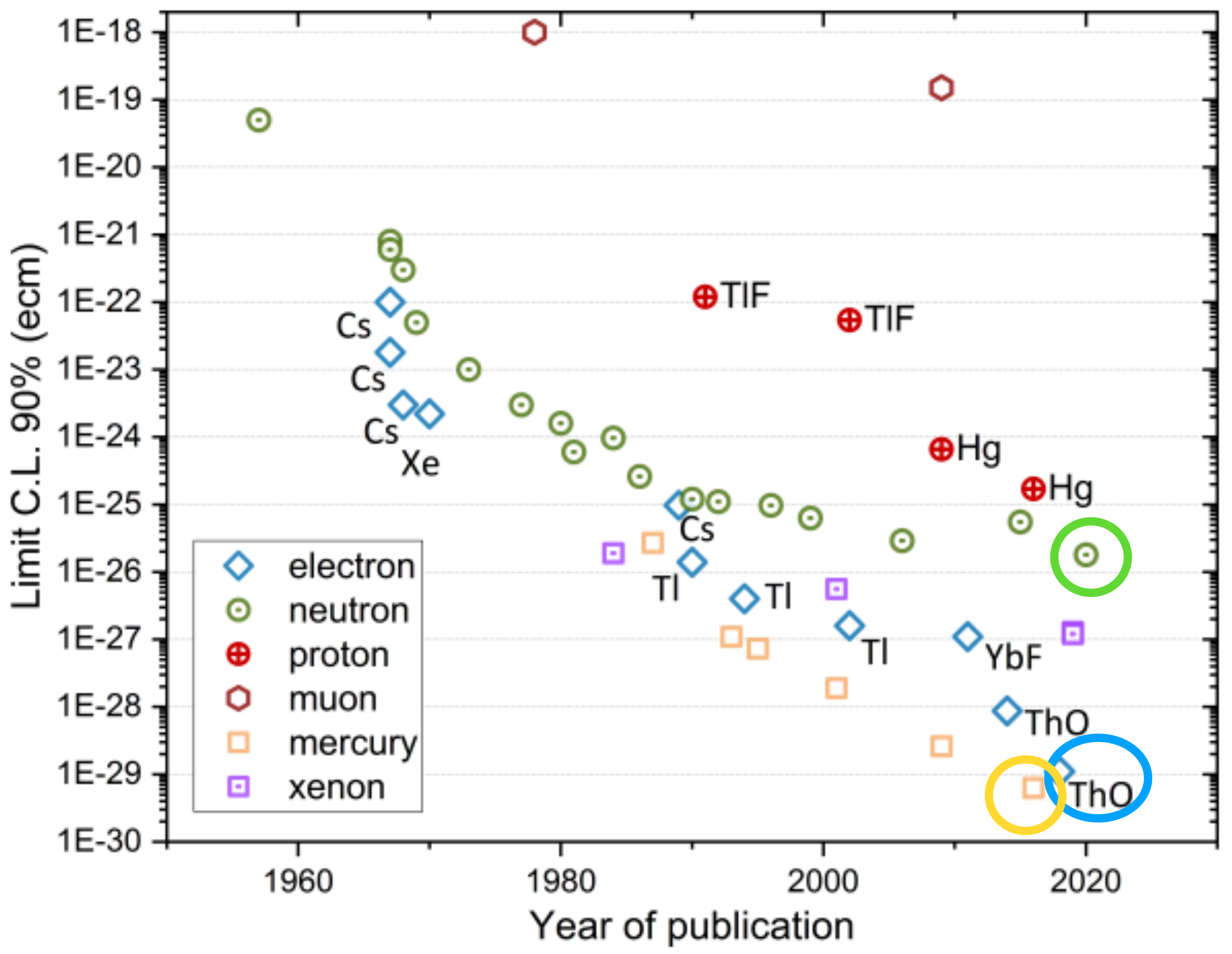


Figure 2. Plot of the history of upper EDM limits (CL 90%) as function of the year of publication.

Future Experiment in eEDM

- ACME III

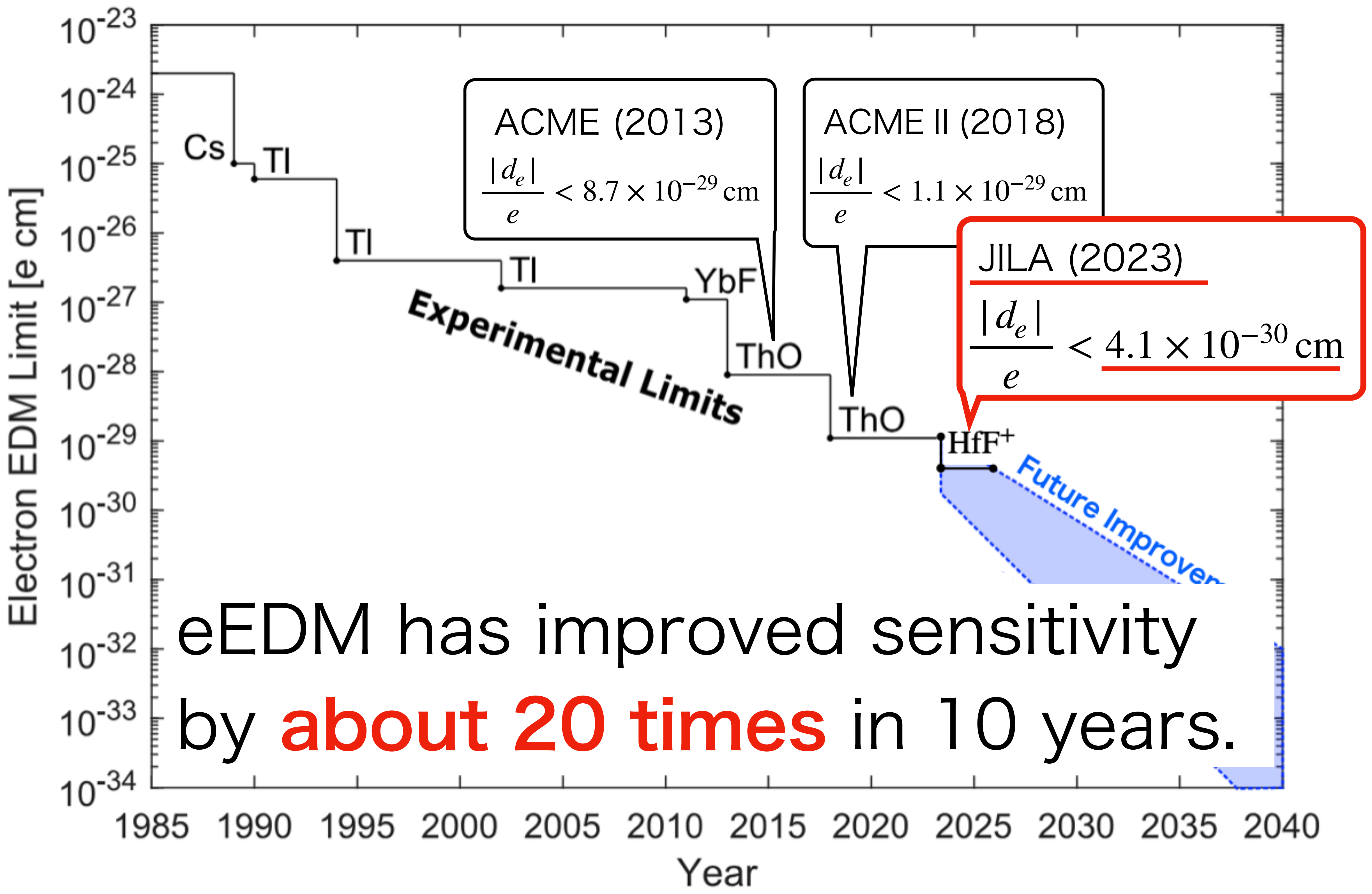
  The aim is to improve sensitivity by a factor of 30 over previous(ACME II) results.

A.Hiramoto et al., Nucl. Instrum. Meth. A 1045 (2023) 167513

- JILA

  $HfF^+ \rightarrow ThF^+$

  Aim for sensitivity improvements of more than one order of magnitude.



R. Alarcon et al. "Electric dipole moments and the search for new physics" in Snowmass,(2021),(2022)

T. S. Roussy et al. Science 381 (2023)

Equivalent EDM

- In theoretically, We have to discuss eEDM including **nuclei effect**
- **Equivalent EDM** is defined as the eEDM including nuclei effect
  - Effective Lagrangian

$$\mathscr{L} = -i\frac{d_e}{2}\bar{e}\sigma_{\mu\nu}\gamma_5 eF^{\mu\nu} + \frac{G_F}{\sqrt{2}}C_S\bar{e}i\gamma_5 e\bar{N}N \qquad N = \binom{p}{n}$$

M. Pospelov and A. Ritz, Annals Phys. 318 (2005) 119 [hep-ph/0504231]

↓

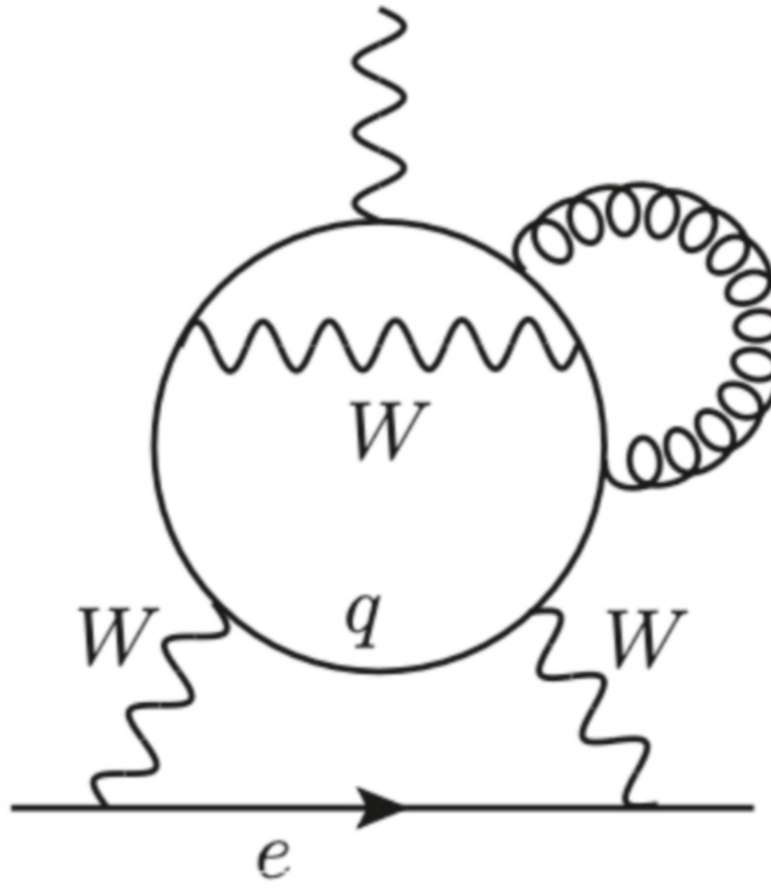
$$d_e^{\text{equiv}} = d_e + C_S \times X(\text{molcule depend})\ \text{e cm}$$

e.g.) $X = 0.9 \times 10^{-20}$ (HfF^+)

$X = 1.5 \times 10^{-20}$ (ThO)

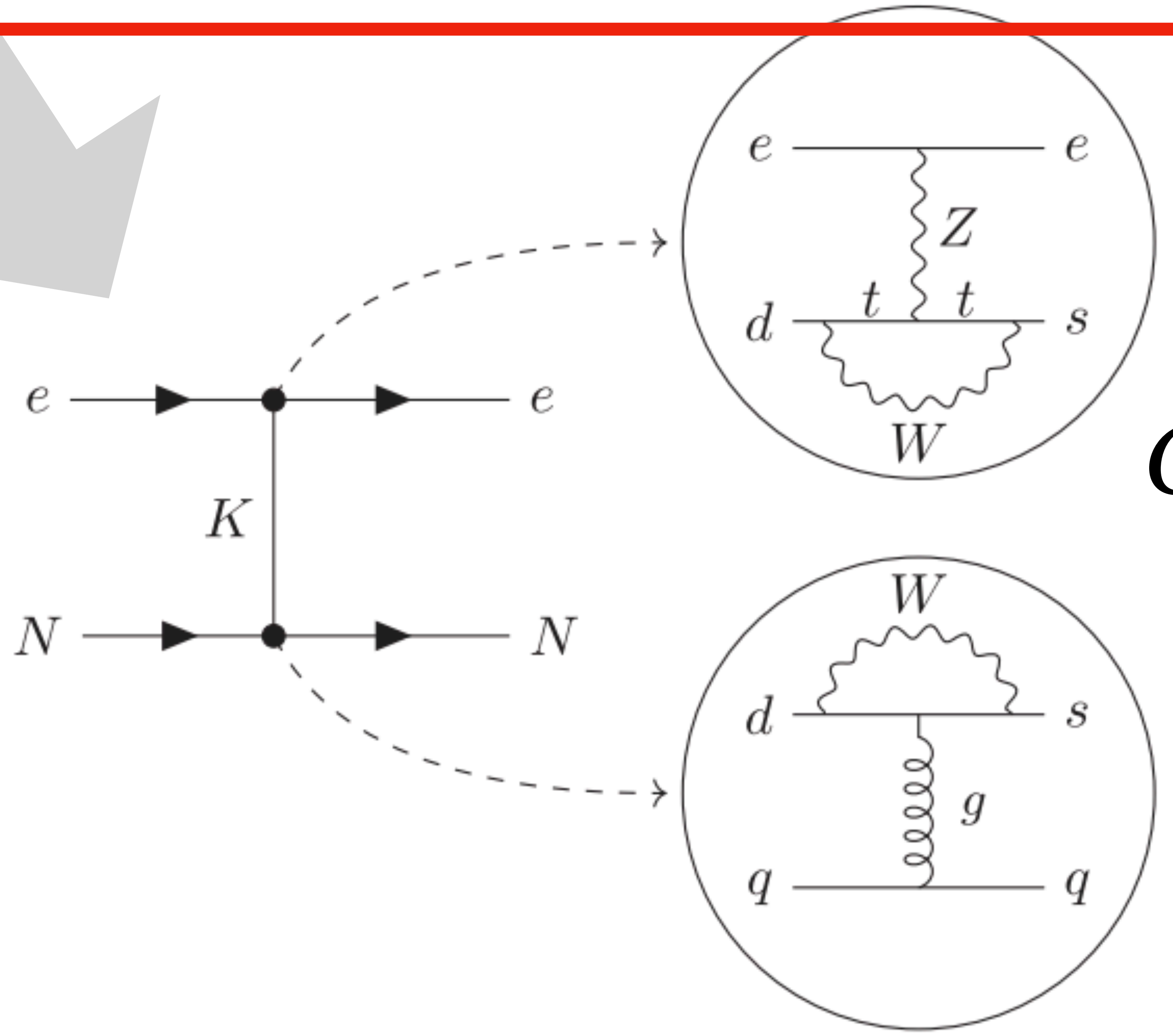
Equivalent EDM induced by SM

- Equivalent EDM predicted by Standard Model
  - In ThO case $d_e^{\text{equiv}} = d_e + C_S \times 1.5 \times 10^{-20} e\ \text{cm} \simeq 1.0 \times 10^{-35} e\text{cm}$



$d_e \sim \mathcal{O}(10^{-44}) e\text{cm}$

M. Pospelov, A. Ritz, Phys.Rev.D 89 (2014) 5, 056006



$C_S \simeq 6.9 \times 10^{-16}$

Y. Ema, T. Gao, M. Pospelov,Phys.Rev.Lett. 129 (2022) 23, 231801

$$d_e^{equiv} \simeq 1.0 \times 10^{-35} e\text{cm}$$

In the SM, C_s contribution is dominant!

Minimal Dark Matter Model

M. Cirelli, A. Strumia, and M. Tamburini, Nucl. Phys. B 787 (2007) 152–175.

- **Representation**

Quantum numbers $SU(2)_L$	$U(1)_Y$	Spin	DM can decay into	DD bound?	Stable?
2	1/2	S	EL	×	×
2	1/2	F	EH	×	×
3	0	S	HH^*	√	×
3	0	F	LH	√	×
3	1	S	HH, LL	×	×
3	1	F	LH	×	×
4	1/2	S	HHH^*	×	×
4	1/2	F	(LHH^*)	×	×
4	3/2	S	HHH	×	×
4	3/2	F	(LHH)	×	×
5	0	S	(HHH^*H^*)	√	×
5	0	F	–	√	√
5	1	S	$(HH^*H^*H^*)$	×	×
5	1	F	–	×	√
5	2	S	$(H^*H^*H^*H^*)$	×	×
5	2	F	–	×	√
6	1/2, 3/2, 5/2	S	–	×	√
7	0	S	–	√	√
8	1/2, 3/2 ...	S	–	×	√

- **Collider search**

Mass splitting

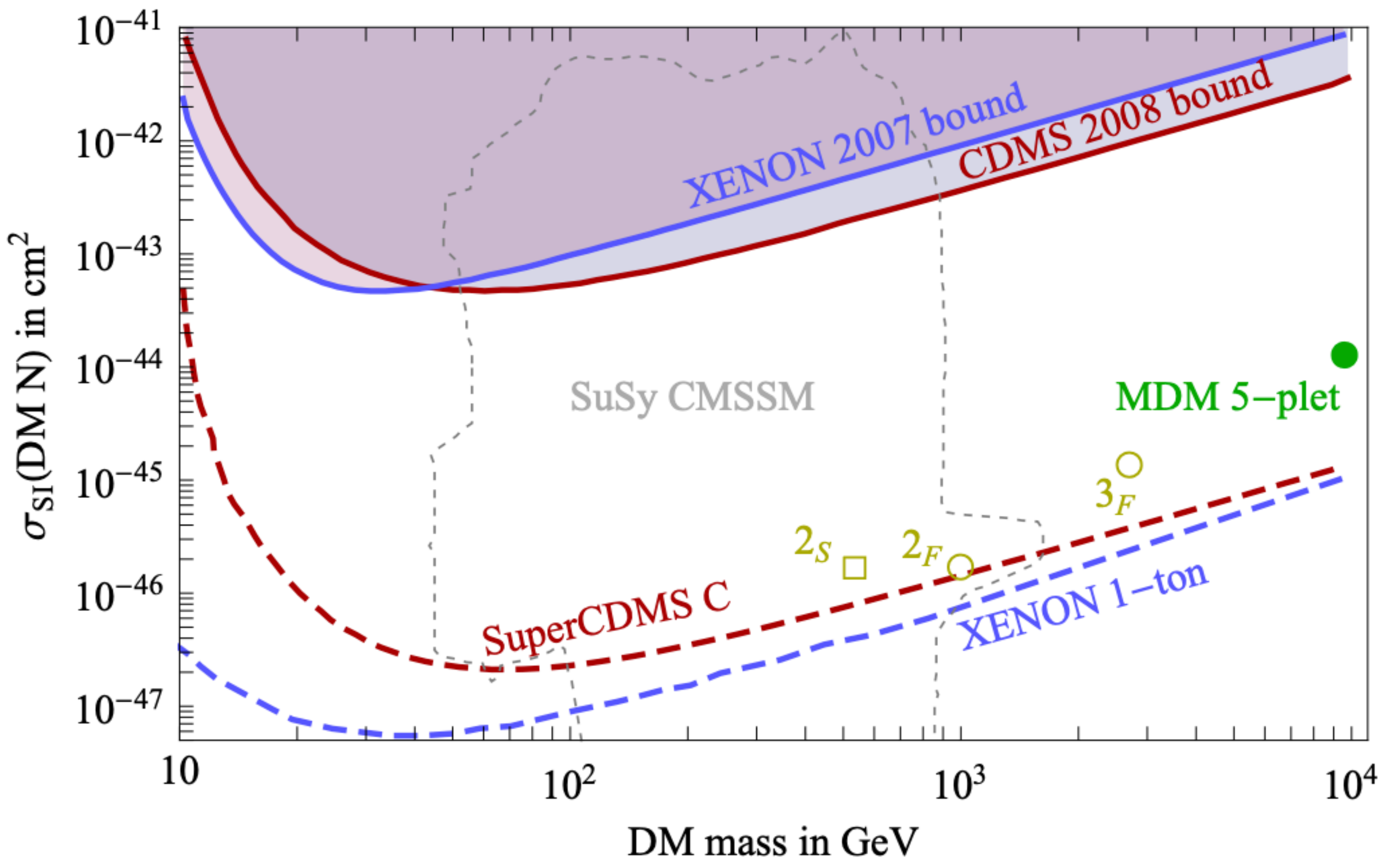
$$M_{\chi^\pm} - M_\chi = \Delta M = O(100)\ \mathrm{MeV}$$

The constraints to $SU(2)_L$ triplets by LHC

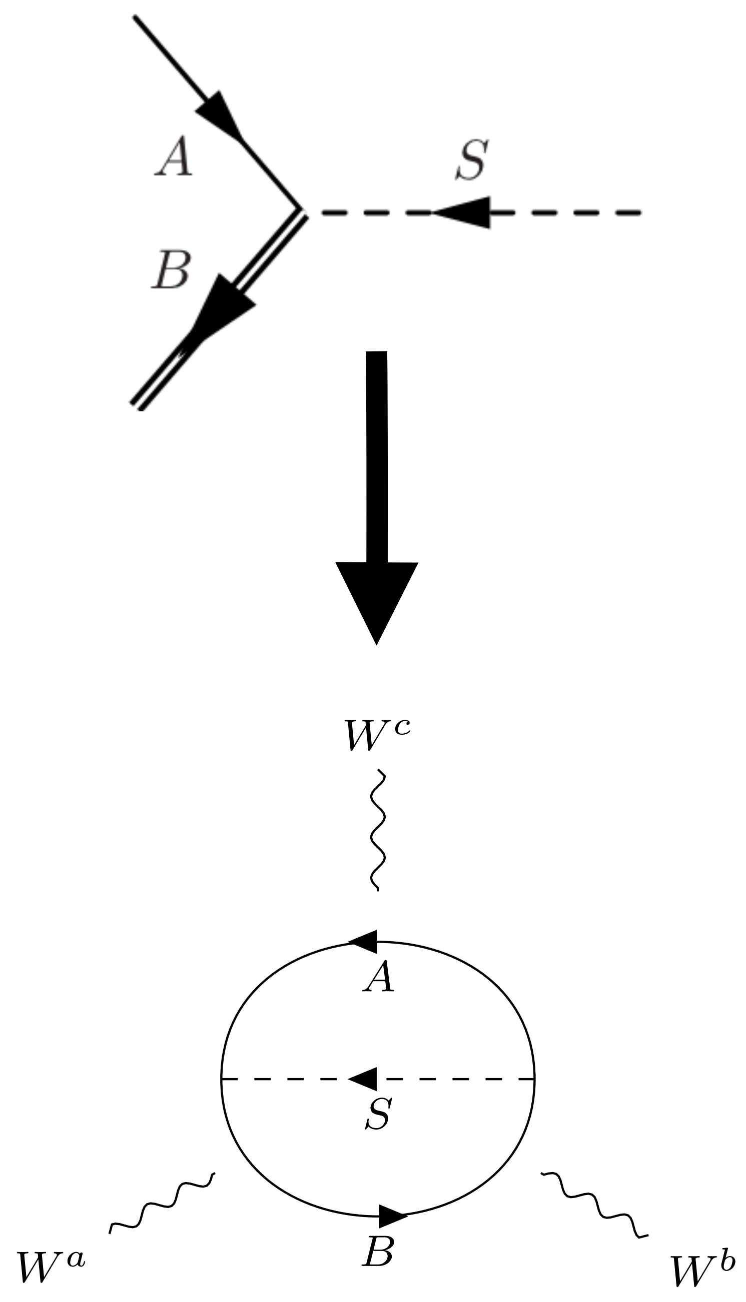
$$m_{r=3} \geq 660\mathrm{GeV}$$

ATLAS,Aad et al, 2201.02472,CERN-EP-2021-209,Eur.Phys.J.C,(2022)

- **Direct Detection**



Yukawa interaction → EW-Weinberg Operator



✓ Yukawa interaction→ QCD Weinberg Operator

T. Abe, J. Hisano, and R. Nagai, JHEP 03 (2018) 175

- We derive the EW-Weinberg Operator **with 2 changes.**

JHEP 03 (2018) 175		Our Work
① SU(3)	→	SU(N) → N=2
② Numerical	→	Analytic

S.P.Martin,Phys.Rev.D.65 (2002) 116003

(A,B,S) Group factors

(A,B,S)	$\psi_A\psi_B S$	$X_{\bar{B}AS}$	$XT_AT_AX^\dagger$	$XT_AX^\dagger T_B$	$XX^\dagger T_BT_B$
$(2,2,1)$	$(\psi_A)_a(\psi_B)_bS$	δ^{ab}	$1/2$	$1/2$	$1/2$
$(2,1,2)$	$(\psi_A)_a\psi_B S_i$	δ^{ai}	$1/2$	0	0
$(3,3,1)$	$(\psi_A)_\alpha(\psi_B)_\beta S$	$\delta^{\alpha\beta}$	2	2	2
$(3,1,3)$	$(\psi_A)_\alpha\psi_B S_\gamma$	$\delta^{\alpha\gamma}$	2	0	0
$(3,2,2)$	$(\psi_A)_\alpha(\psi_B)_bS_i$	$(T^\alpha)^{bi}$	1	$1/2$	$3/8$
$(2,2,3)$	$(\psi_A)_a(\psi_B)_bS_\gamma$	$(T^\gamma)^{ba}$	$3/8$	$-1/8$	$3/8$
$(3,3,3)$	$(\psi_A)_\alpha(\psi_B)_\beta S_\gamma$	$\epsilon^{\beta\alpha\gamma}$	4	2	4
$(r,r,1)$	$(\psi_A)_{a_r}(\psi_B)_{b_r}S$	$\delta^{a_rb_r}$	$r(r^2-1)/12$	$r(r^2-1)/12$	$r(r^2-1)/12$
$(r,1,r)$	$(\psi_A)_{a_r}\psi_B S_{i_r}$	$\delta^{a_ri_r}$	$r(r^2-1)/12$	0	0

Table 2: Group factors in the $SU(2)_L$ representations. The last two rows show the simple cases of the r-dimensional representations.

How to derive EW-Weinberg Operator

- The Wilson coefficient for EW-Weinberg operator

$$C_W = \frac{6}{(4\pi)^4}\,\mathrm{Im}(sa^*) m_A m_B$$
$$\times \Big\{ \boxed{\big(XT_AT_AX^\dagger\big)} g_1(m_A^2, m_B^2, m_S^2) + \boxed{\big(XX^\dagger T_BT_B\big)} g_1(m_B^2, m_A^2, m_S^2)$$
$$+ \boxed{\big(XT_AX^\dagger T_B\big)} \big[g_2(m_A^2, m_B^2, m_S^2) + g_2(m_B^2, m_A^2, m_S^2)\big] \Big\}.$$

$\boxed{}$: group factor

loop functions

$$g_1(x_1, x_2, x_3) = \left(2\bar{I}_{(4;1)} + 4x_1\bar{I}_{(5;1)}\right)(x_1; x_2; x_3),$$
$$g_2(x_1, x_2, x_3) = \left(\bar{I}_{(3;2)} + x_1\bar{I}_{(4;2)}\right)(x_1; x_2; x_3),$$

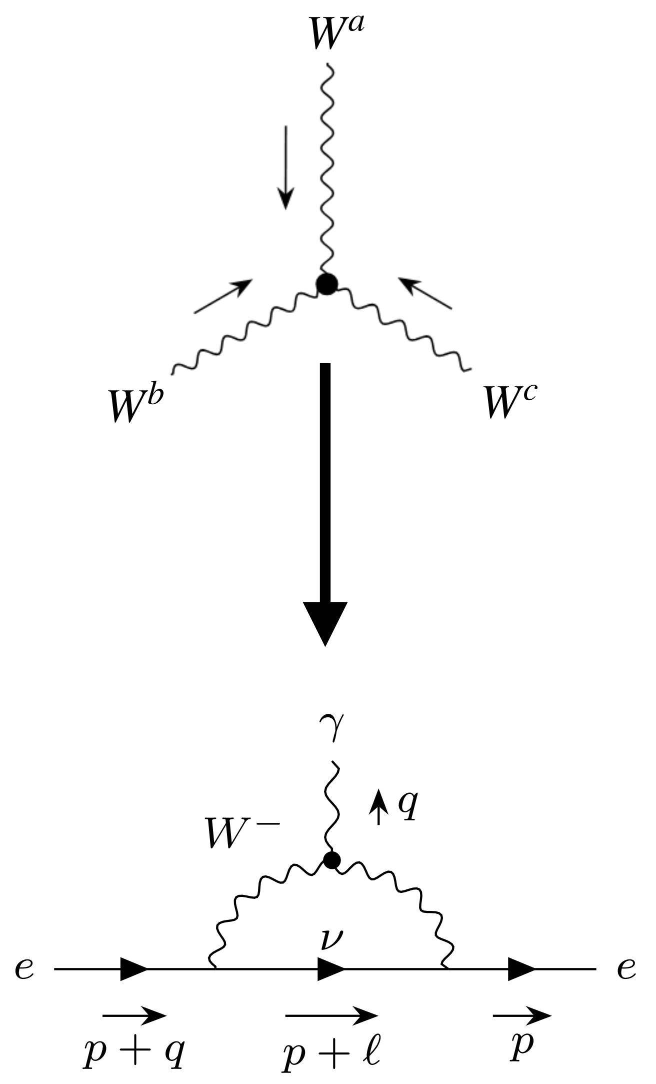
$$\bar{I}_{(n;m)}(x_1; x_2; x_3) = \frac{1}{(n-1)!(m-1)!}\frac{d^{n-1}}{dx_1^{n-1}}\frac{d^{m-1}}{dx_2^{m-1}}\bar{I}(x_1; x_2; x_3),$$

Master Integral

$$I(x_1; x_2; x_3) = \int d^dk \int d^dq \frac{1}{(k^2 - x_1)(q^2 - x_2)[(k-q)^2 - x_3]}$$
$$= \bar{I}(x_1; x_2; x_3) + I_{div}(x_1; x_2; x_3)$$

S.P.Martin, Phys.Rev.D.65 (2002) 116003

EW-Weinberg Operator → eEDM



- **Evaluate 1-loop diagram in DR**

▶ **Lagrangian 4-dim → d-dim** $\mathcal{L}^{(d)} = \mathcal{L}^{(4)} + \mathcal{L}^{(d-4)}$

Evanescent operator is added

▶ Treatment for γ_5 & Levi-Chivita symbol

Bollini-**M**éndez-**H**oleman-**V**eltman(**BMHV**) Scheme

G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B 44 (1972) 189–213.

- **γ_5 & Levi-Chivita symbol are defined as 4-dimensional objects**

$$\gamma_5 = -\frac{i}{4!}\bar{\epsilon}^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma \qquad \hat{\epsilon}^{\mu\nu\rho\sigma} = 0$$

- **give up anticommutation property**

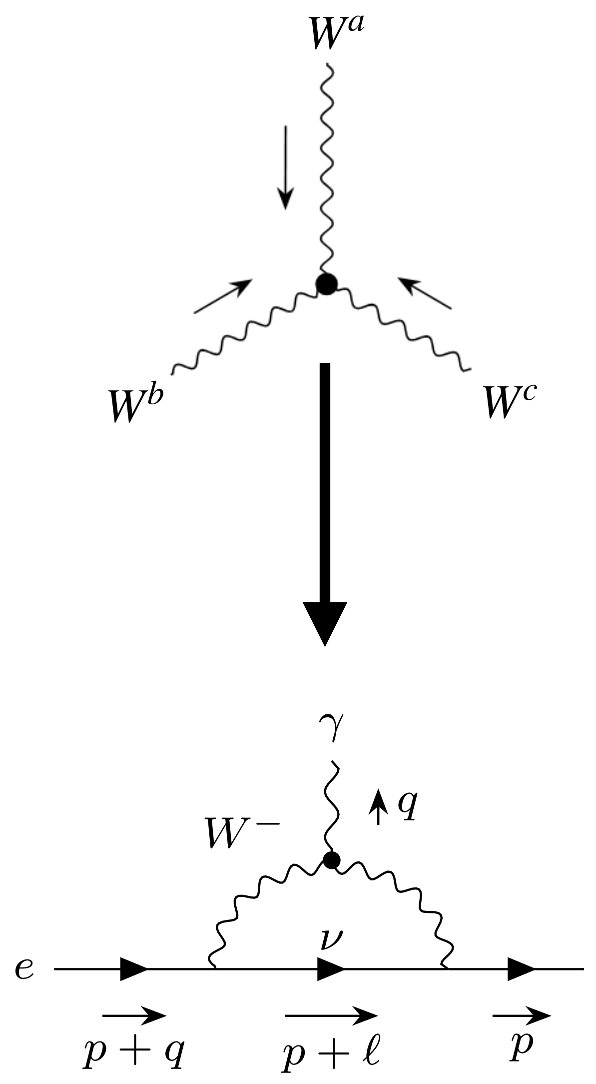
$$\{\bar{\gamma}^\mu, \gamma_5\} = 0, \; [\hat{\gamma}^\mu, \gamma_5] = 0$$

※ bars and hats conventionally represent 4 and (d – 4)-dim.

EW-Weinberg Operator in d-dim

$$\mathcal{L}_W^d = -\frac{g^3}{3}C_W^{(4)} f^{abc}\bar{W}^a_{\mu\nu}\bar{W}^{b\nu}{}_\rho\tilde{\bar{W}}^{c\rho\mu} - \frac{g^3}{3}C_W^{(d-4)} f^{abc}W^a_{\bar{\mu}\hat{\nu}}W^{b\hat{\nu}}{}_{\bar{\rho}}\tilde{\bar{W}}^{c\rho\mu}$$

EW-Weinberg Operator → eEDM



- Evaluate 1-loop diagram in DR

We separately evaluate the 4- & (d-4)-dim contributions.

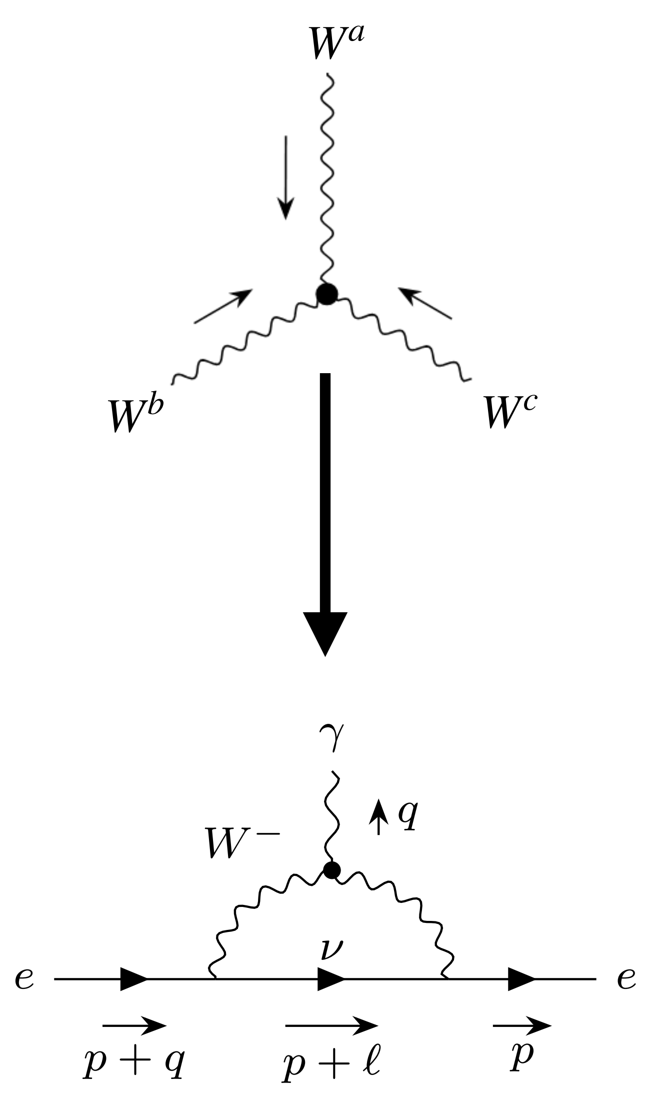
	4-dim.	$(d-4)$-dim.
$-ieg^2\frac{2}{3}W^-W^+\widetilde{F}$	$\bar{O}_1$: $(1/18)C_W$	$\hat{O}_1$: $(1/9)C_{W^{(d-4)}}$
$-ieg^2\frac{2}{3}F(W^-\widetilde{W}^+ - W^+\widetilde{W}^-)$	$\bar{O}_2$: $(1/18+1/18)C_W$	$\hat{O}_2$: 0
total	$(1/6)C_W$	$(1/9)C_{W^{(d-4)}}$

$$\hat{O}_2 \neq 2\hat{O}_1$$

$$F_{\bar{\mu}\hat{\nu}}\left(W^{-\hat{\nu}}_{\ \bar{\lambda}}\widetilde{W}^{+\lambda\mu} - W^{+\hat{\nu}}_{\ \bar{\lambda}}\widetilde{W}^{-\lambda\mu}\right)$$

$\hat{O}_2$ do not contribute because $F_{\bar{\mu}\hat{\nu}}$ has (d-4)-dimensional momentum or polarization.

EW-Weinberg Operator → eEDM



- **Evaluate 1-loop diagram in DR**

EW-Weinberg Operator in d-dim

$$\mathcal{L}_W^d = -\frac{2ieg^2}{3} C_W^{(4)} \left[\bar{W}^-_{\mu\nu} \bar{W}^{+\nu}{}_{\lambda} \tilde{\bar{F}}^{\lambda\mu} + \bar{F}_{\mu\nu} \left(\bar{W}^{-\nu}{}_{\lambda} \tilde{\bar{W}}^{+\lambda\mu} - \bar{W}^{+\nu}{}_{\lambda} \tilde{\bar{W}}^{-\lambda\mu} \right) \right] = 3\bar{W}^-_{\mu\nu} \bar{W}^{+\nu}{}_{\lambda} \tilde{\bar{F}}^{\lambda\mu}$$

$$- \frac{2ieg^2}{3} C_W^{(d-4)} \left[W^-_{\bar{\mu}\hat{\nu}} W^{+\hat{\nu}}{}_{\bar{\lambda}} \tilde{\bar{F}}^{\lambda\mu} + F_{\bar{\mu}\hat{\nu}} \left(W^{-\hat{\nu}}{}_{\bar{\lambda}} \tilde{\bar{W}}^{+\lambda\mu} - W^{+\hat{\nu}}{}_{\bar{\lambda}} \tilde{\bar{W}}^{-\lambda\mu} \right) \right]$$

We separately evaluate the 4- & (d-4)-dim contributions.

$$\frac{d_e}{e} = \frac{\alpha_2^2}{6} m_e C_W^{(4)} + \frac{\alpha_2^2}{9} m_e C_W^{(d-4)}$$

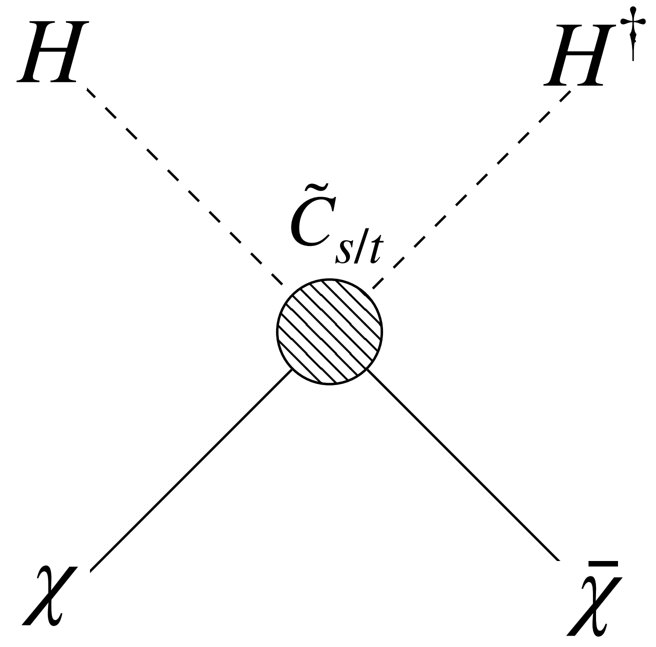
In $C_W^{(4)} = C_W^{(d-4)}$ case, this result is consistent to the Naive Dimensional Regularization.

W. Dekens and P. Stoffer, JHEP 10 (2019) 197

In the case of S = SM Higgs

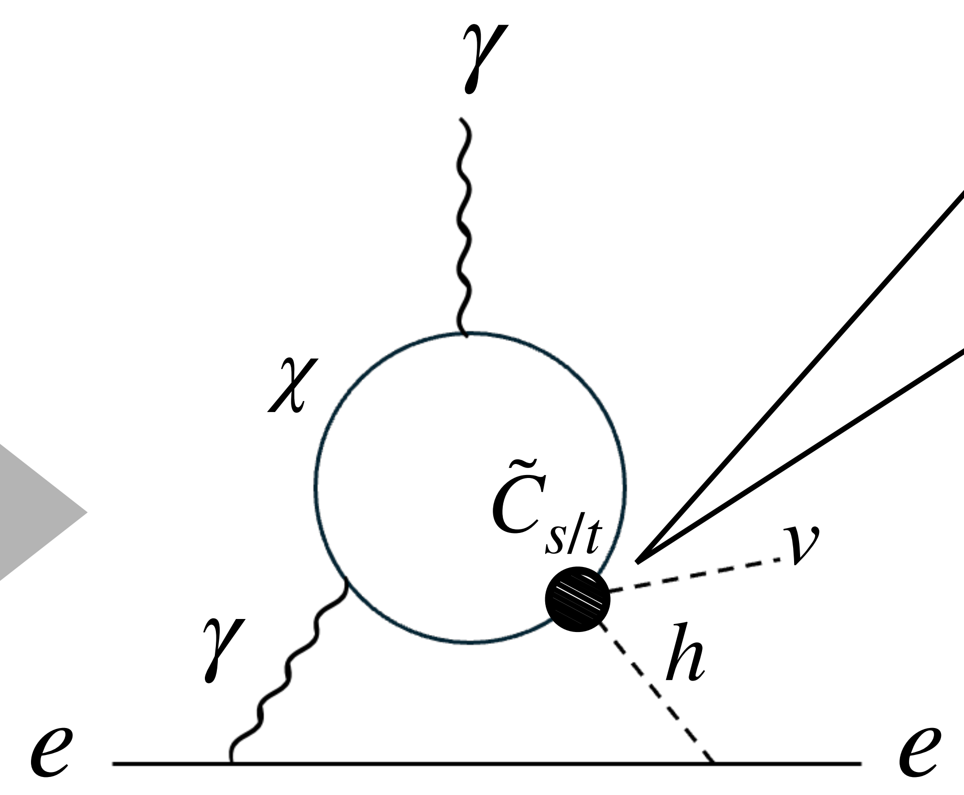
$(A, B, S) = (3,2,2^H)$

CP violating operator



$\chi = \psi_{A/B}$

2-loop level



Barr-Zee diagram(LO)

N. Nagata and S. Shirai, JHEP 01 (2015)

3-loop level

RG effect for CP violating operator at 1-loop level

$$\bar{\gamma}_{ss}^{(r,Y_r)} = -\left[6\alpha_2\left(C_2(r)+\frac{3}{4}\right)+6\alpha_Y\left(Y_r^2+\frac{1}{4}\right)-3\lambda'-6\alpha_t\right]$$

$$\bar{\gamma}_{tt}^{(r,Y_r)} = -\left[6\alpha_2\left(C_2(r)-\frac{1}{4}\right)+6\alpha_Y\left(Y_r^2+\frac{1}{4}\right)-\lambda'-6\alpha_t\right]$$

$\alpha_2/\alpha_Y/\alpha_t/\lambda'$: SM coupling $C_2(r)$: casimir operator

Barr-Zee diagram(NLO)

W. Kuramoto, T. Kuwahara, and R. Nagai, Phys. Rev. D99 (2019) 095024

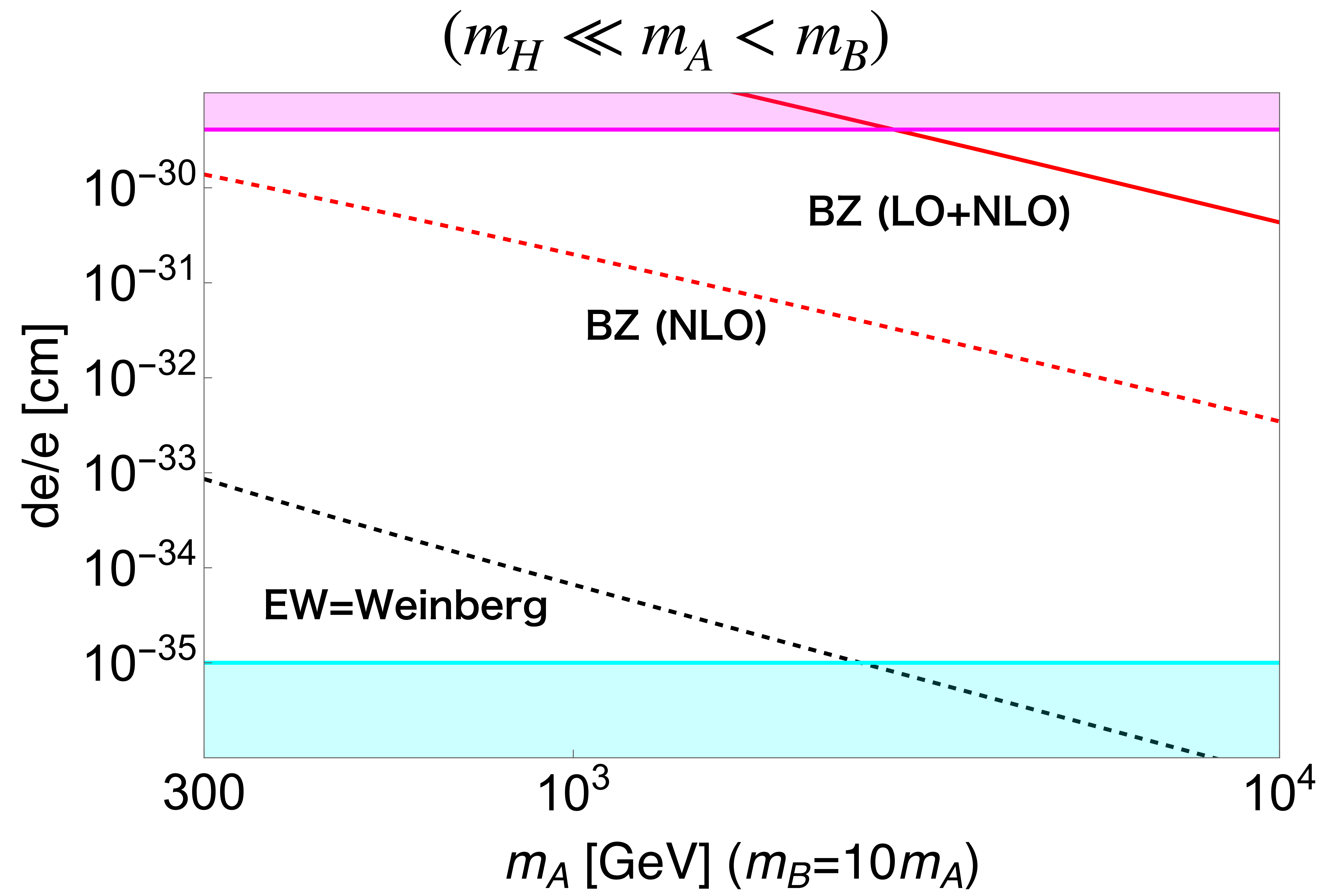
vs

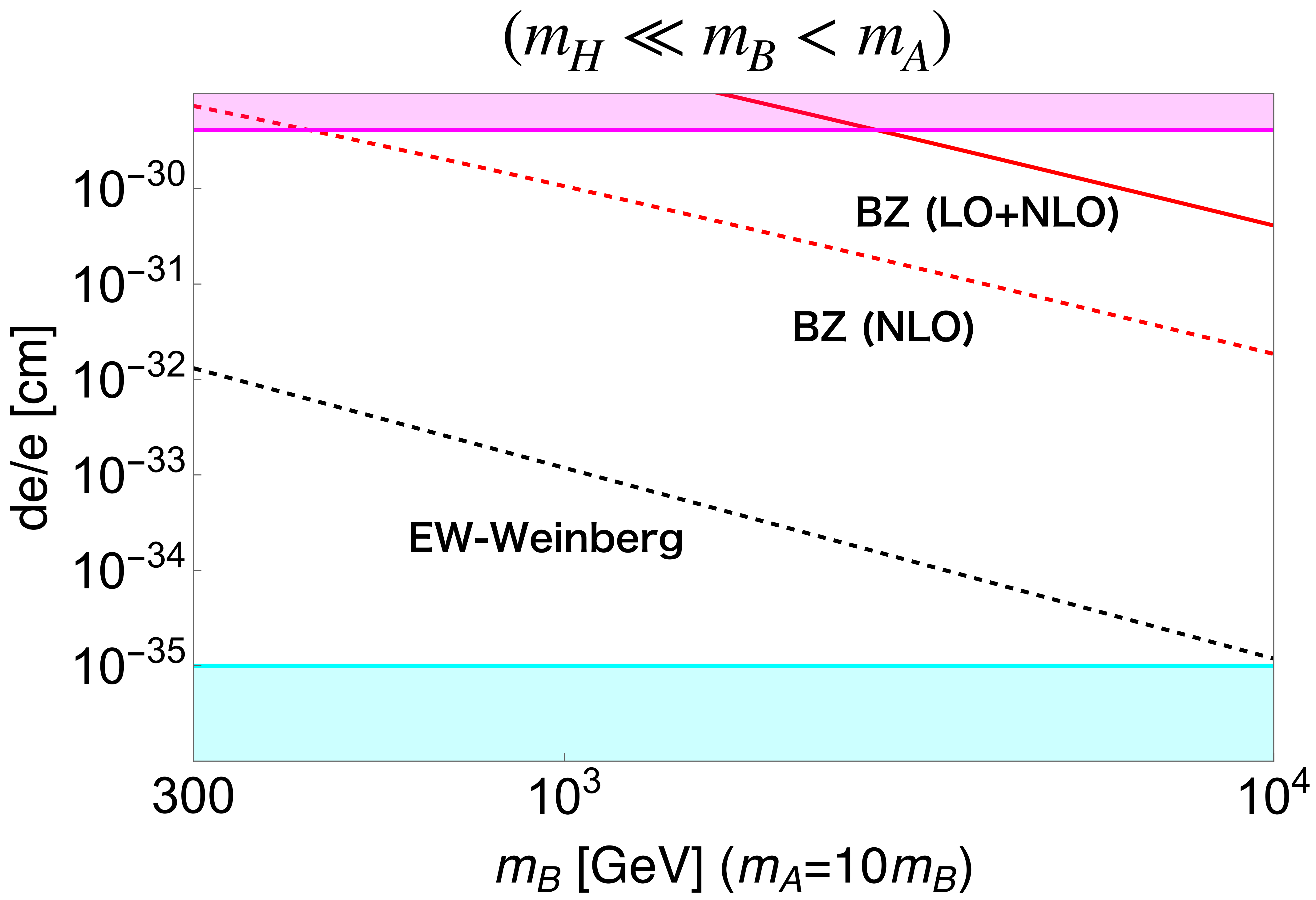
EW-Weinberg

In the case of S = SM Higgs

Barr-Zee diagram(NLO) vs EW-Weinberg

$(A, B, S) = (3,2,2^H)$



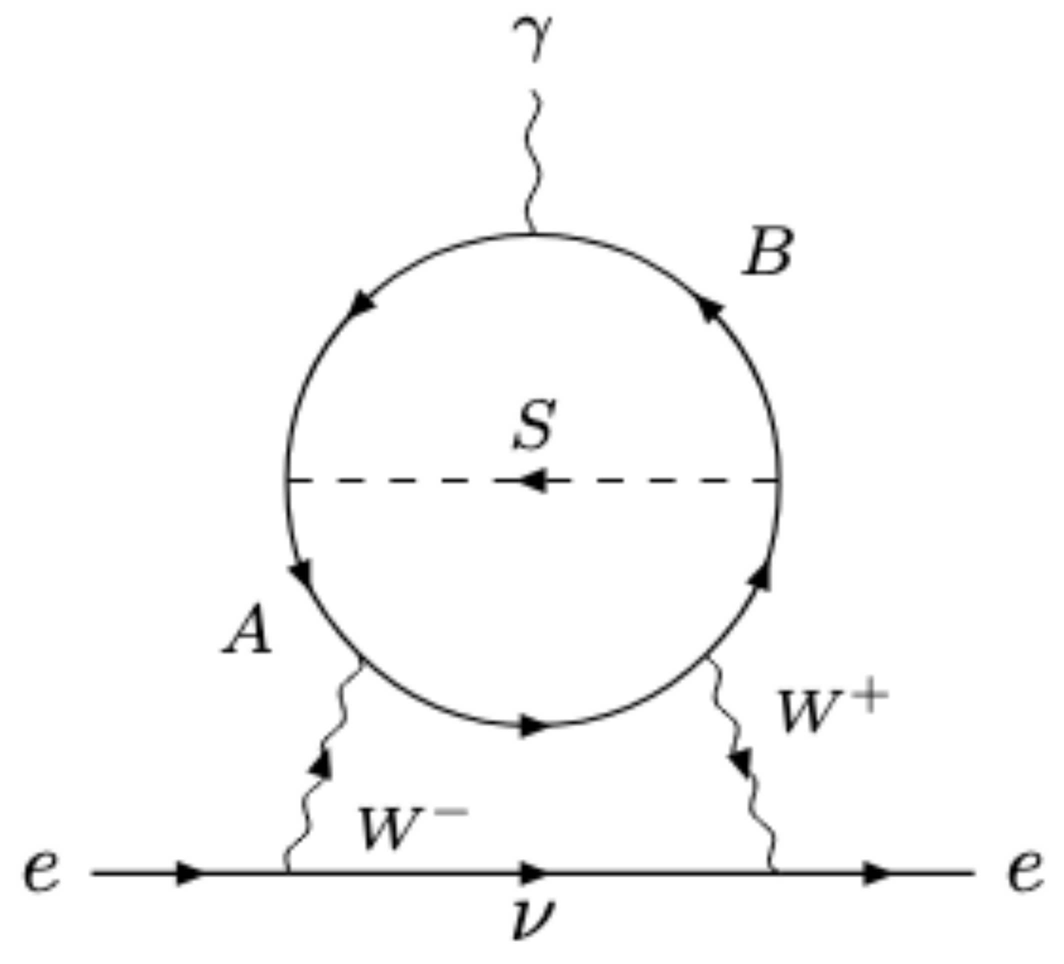


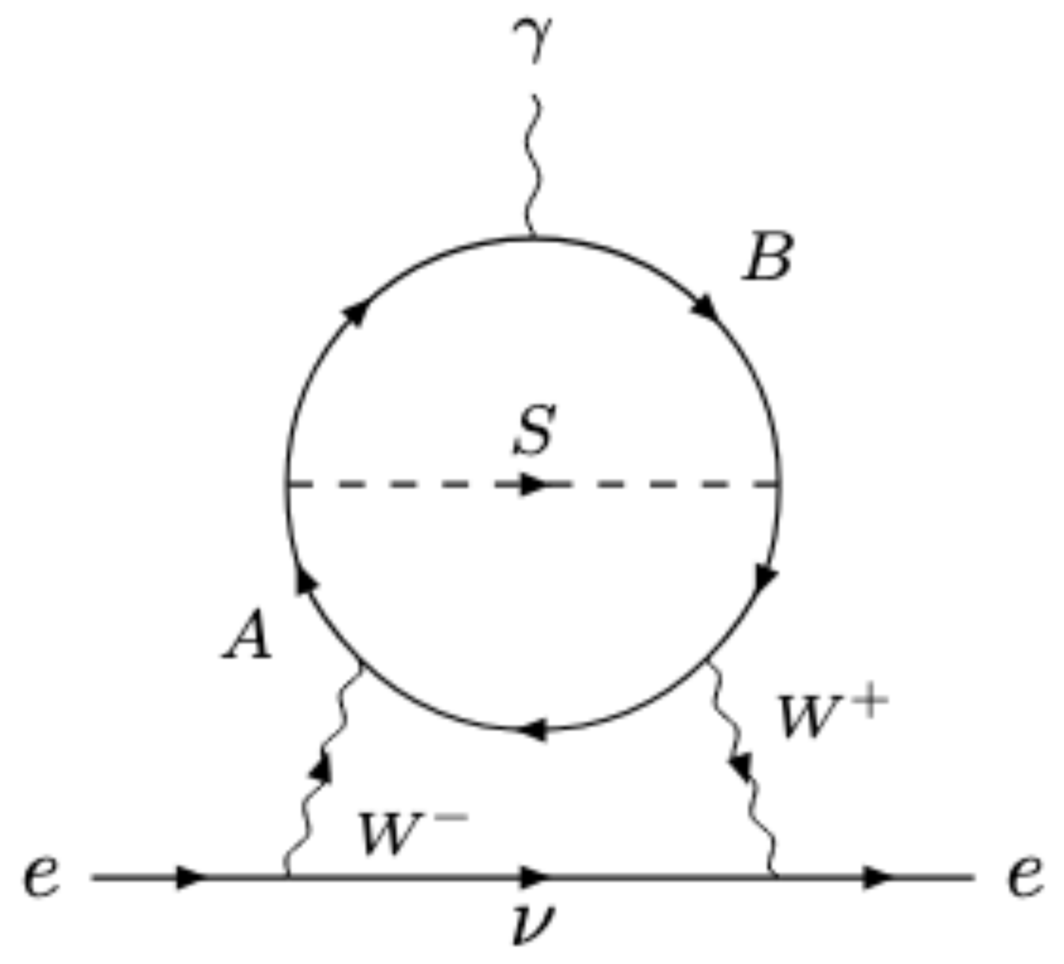
EW-Weinberg $\ll$ Barr-Zee diagram(NLO)

In the Yukawa interaction with Higgs, EW-Weinberg contribution can be neglected!

Cancelation of $U(1)_Y$ dependence

- **We can write clockwise flow diagrams and counterclockwise flow diagrams**





Fermion trace is same up to sign

$$\mathrm{Tr}\left[Q[T^+, T^-]\right] = \frac{r(r^2-1)}{12} . \quad T^{\pm} = T^1 \pm iT^2, \; Q = T^3 + Y.$$

This result is independent of $\mathrm{U}(1)_Y$ because of $\mathrm{Tr}[T^a] = 0$.

How to calculate three-loop integrals

- Expand for W-boson

$$B(m_A, m_B, m_S, m_W) \simeq B_0(m_A, m_B, m_S) + m_W^2 B_1(m_A, m_B, m_S) + \mathcal{O}\left(\frac{m_W^4}{\Lambda^4}\right)$$

where Λ stands for heavy particle scale ($\psi_{A/B}$ or S). $B_0(m_A, m_B, m_S)$ and $B_1(m_A, m_B, m_S)$ are sum of the three-loop integrals defined as

$$J[n_1, n_2, n_3, n_4, n_5, n_6] = \left(\frac{i}{16\pi^2}\right)^{-3} \int \frac{d^d l_1}{(2\pi)^d} \frac{d^d l_2}{(2\pi)^d} \frac{d^d l_3}{(2\pi)^d}$$

$$\frac{1}{(l_1^2 - m_A^2)^{n_1} (l_2^2 - m_B^2)^{n_2} (l_3^2)^{n_3} [(l_1 - l_2)^2 - m_S^2]^{n_4} [(l_2 - l_3)^2 - m_B^2]^{n_5} [(l_3 - l_1)^2 - m_A^2]^{n_6}} .$$

How to calculate three-loop integrals

- **We use the Integration-by-Parts (IBP) method**

Complex Integral → (Reduce) → The combination of Simplified(Master) Integral

We reduce the complex integral by public code "Kira" P. Maierhöfe et al., Comput. Phys. Commun. 230 (2018) 99–112

$m_A = m_B$ case,

$B_0(m_A, m_B, m_S)$

$B_1(m_A, m_B, m_S)$

→ (Reduce)

1loop-integral × 3 $J[1,0,0,1,1,0]$ and $J[1,1,0,0,1,0]$

1loop-integral × 2-loop integral $J[1,1,0,1,1,0]$

3-loop integral $J[0,1,1,2,0,1]$, $J[0,2,1,1,0,1]$, $J[2,1,0,0,1,1]$, $J[1,1,0,1,1,1]$

These integrals have already known analytical solutions

S. P. Martin, Phys. Rev. D 65 (2002) 116003

S. P. Martin and D. G. Robertson, Phys. Rev. D 95 (2017) 016008

Result

- Comparison of $d_e^{\rm Full}$ and $d_e^{C_W}$ $(m_A = m_B \neq m_S)$

electron EDM in the full theory

$$\frac{d_e^{\rm Full}}{e} \simeq \begin{cases} -\dfrac{\alpha_2^2 m_e}{(16\pi^2)^2}\dfrac{r(r^2-1)}{12}{\rm Im}(sa^*) \times \left(\dfrac{3+2\log\frac{m_A^2}{m_S^2}}{m_S^2} + m_W^2 \dfrac{7+6\log\frac{m_A^2}{m_S^2}}{9m_A^2 m_S^2}\right) & (m_A = m_B \ll m_S)\,, \\ \dfrac{\alpha_2^2 m_e}{(16\pi^2)^2}\dfrac{r(r^2-1)}{12}{\rm Im}(sa^*) \times \left(\dfrac{1}{2m_A^2} + m_W^2 \dfrac{8}{27m_A^4}\right) & (m_S \ll m_A = m_B)\,. \end{cases}$$

electron EDM through the electroweak-Weinberg operator

$$\frac{d_e^{C_W}}{e} \simeq \begin{cases} -\dfrac{\alpha_2^2 m_e}{(16\pi^2)^2}\dfrac{r(r^2-1)}{12}{\rm Im}(sa^*) \times \left(\dfrac{3+2\log\frac{m_A^2}{m_S^2}}{3m_S^2}\right) & (m_A = m_B \ll m_S)\,, \\ \dfrac{\alpha_2^2 m_e}{(16\pi^2)^2}\dfrac{r(r^2-1)}{12}{\rm Im}(sa^*) \times \left(\dfrac{1}{6m_A^2}\right) & (m_S \ll m_A = m_B)\,. \end{cases}$$

$$d_e^{\rm Full} \simeq 3 d_e^{C_W}$$