

Sweeping the pion chimney for axion-like particles with KOTO

2507.01947 & 2508.08402

with Reuven Balkin, Stefania Gori & Dean Robinson

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BERKELEY LAB

Motivation

axion-like particles (ALPs) are well motivated and naturally arise in many extension of the SM:

- QCD axion/strong CP problem
- dark matter
- portal to dark sector
- 'axiverse'
- composite models
- supersymmetry

Motivation

Flavor Probes of ALPs

e.g. Bauer et al. 2110.10698
Camalich et al. 2002.04623

- flavor and CP violation are suppressed in the SM
- flavor violating & CP violating observables are very sensitive to small BSM contributions
- charged kaon decays place strong bounds on axions

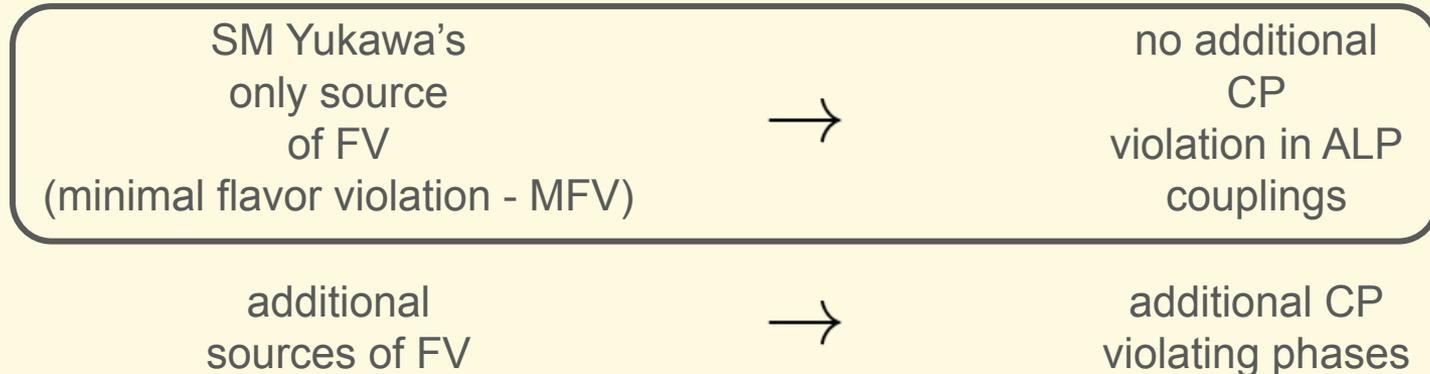
$$K^+ \rightarrow \pi^+ a(inv) \rightarrow f_a/c_{sd} > 10^9 \text{ GeV}$$

NA62 2103.15389

Motivation

Flavor & CP Probes of ALPs

- **neutral kaon decays** are unique probes of CP-violation (CPV) in BSM sector
- for ALP couplings, CPV and FV are **related**



Motivation

Flavor & CP Probes of ALPs

- **goal:** study 3-body decays of neutral kaons:

$$K_L \rightarrow \pi^0 \pi^0 a \qquad K_L \rightarrow \pi^+ \pi^- a$$

$$\frac{\Gamma(K_L \rightarrow \pi^0 \pi^0 a)}{\Gamma(K_L \rightarrow \pi^0 a)} = ?$$

SM:

$$\frac{\Gamma(K_L \rightarrow \pi^0 \pi^0 \pi^0)}{\Gamma(K_L \rightarrow \pi^0 \pi^0)} = 200$$

Plan

$$K_L \rightarrow \pi^0 \pi^0 a$$

- I. Theory: focus on FV and EW contributions

(R. Balkin, S. Gori, CS: 2507.01947)

- II. Application using KOTO calibration data

(R. Balkin, S. Gori, D. Robinson, CS: 2508.08402)

Flavor violating ALP couplings - from UV to IR

$$\begin{aligned}\mathcal{L}_a(\Lambda) = & \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{1}{2}m_a^2 a^2 + \frac{\partial_\mu a}{f} \sum_F \bar{F} c_F \gamma_\mu F \\ & + c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f} W_{\mu\nu}^i \tilde{W}^{i,\mu\nu} + c_{BB} \frac{\alpha_1}{4\pi} \frac{a}{f} B_{\mu\nu} \tilde{B}^{\mu\nu}\end{aligned}$$

Flavor violating ALP couplings - from UV to IR

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$$+ c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f} W_{\mu\nu}^i \tilde{W}^{i,\mu\nu} + c_{BB} \frac{\alpha_1}{4\pi} \frac{a}{f} B_{\mu\nu} \tilde{B}^{\mu\nu}$$

in MFV:

$$\kappa_U = c_0^Q \mathbb{1} + \epsilon c_1^Q (\mathbf{Y}_t)^2 + \mathcal{O}(\epsilon^2)$$

$$\kappa_D = c_0^Q \mathbb{1} + \epsilon c_1^Q \mathbf{V}^\dagger (\mathbf{Y}_t)^2 \mathbf{V} + \mathcal{O}(\epsilon^2)$$

real coefficients!

$$\kappa_u = c_0^u \mathbb{1} + \epsilon c_1^u (\mathbf{Y}_t)^2 + \mathcal{O}(\epsilon^2)$$

$$\kappa_d = c_0^d \mathbb{1}$$

Flavor violating ALP couplings - from UV to IR

above the QCD scale:

$$\mathcal{L}_a(\mu) \supset \frac{\partial_\mu a}{f} (\kappa_R \bar{d}_R \gamma_\mu s_R + \kappa_L \bar{d}_L \gamma_\mu s_L + \text{h.c.})$$

with

$$\kappa_R = [\mathbf{c}_{d_R}(\Lambda)]_{12}$$

$$\kappa_L = [\hat{\mathbf{c}}_{Q_L}(\Lambda)]_{12} + \Delta\kappa_L$$

mass basis

matching
contribution

Flavor violating ALP couplings - from UV to IR

above the QCD scale:

$$\mathcal{L}_a(\mu) \supset \frac{\partial_\mu a}{f} (\kappa_R \bar{d}_R \gamma_\mu s_R + \kappa_L \bar{d}_L \gamma_\mu s_L + \text{h.c.})$$

with

$$\kappa_R = [\mathbf{c}_{d_R}(\Lambda)]_{12}$$

$$\kappa_L = \underbrace{[\hat{\mathbf{c}}_{Q_L}(\Lambda)]_{12}}_{\text{mass basis}} + \underbrace{\Delta\kappa_L}_{\text{matching contribution}}$$

$$\begin{aligned} [\hat{\mathbf{c}}_{Q_L}(\Lambda)]_{12} &= [\mathbf{V}^\dagger \mathbf{c}_{Q_L} \mathbf{V}] \\ &= [\mathbf{c}_{Q_L}]_{12} + V_{cd}^* V_{cs} ([\mathbf{c}_{Q_L}]_{22} - [\mathbf{c}_{Q_L}]_{11}) \\ &\quad + V_{td}^* V_{ts} ([\mathbf{c}_{Q_L}]_{33} - [\mathbf{c}_{Q_L}]_{11}) + \dots \end{aligned}$$

in MFV: $\mathbf{c}_{Q_L} \approx \text{diag}(0, 0, y_t^2)$

Flavor violating ALP couplings - from UV to IR

above the QCD scale:

$$\mathcal{L}_a(\mu) \supset \frac{\partial_\mu a}{f} (\kappa_R \bar{d}_R \gamma_\mu s_R + \kappa_L \bar{d}_L \gamma_\mu s_L + \text{h.c.})$$

RGE evolution

with

$$\kappa_R = [\mathbf{c}_{d_R}(\Lambda)]_{12}$$

$$\kappa_L = [\hat{\mathbf{c}}_{Q_L}(\Lambda)]_{12} + \boxed{\Delta\kappa_L}$$

mass basis matching contribution

$$\Delta\kappa_L = \boxed{n_t} c_{tt}(\Lambda) + \boxed{n_G} \tilde{c}_{GG}(\Lambda) + \boxed{n_W} \tilde{c}_{WW}(\Lambda) + \boxed{n_B} \tilde{c}_{BB}(\Lambda)$$

$$c_{tt} = [\mathbf{c}_{u_r}]_{33} - [\mathbf{c}_{Q_L}]_{33}$$

$$\tilde{c}_{GG} = c_{GG} + \frac{1}{2} \text{Tr} [\mathbf{c}_{u_R} + \mathbf{c}_{d_R} - 2\mathbf{c}_{Q_L}]$$

$$\tilde{c}_{WW} = c_{WW} - \frac{1}{2} \text{Tr} [3\mathbf{c}_{Q_L} + \mathbf{c}_{L_L}]$$

$$\tilde{c}_{BB} = c_{BB} + \text{Tr} \left[\frac{4}{3} \mathbf{c}_{u_R} + \frac{1}{3} \mathbf{c}_{d_R} - \frac{1}{6} \mathbf{c}_{Q_L} - \frac{1}{2} \mathbf{c}_{L_L} + \mathbf{c}_{e_R} \right]$$

Flavor violating ALP couplings - from UV to IR

below confinement scale: chiral Lagrangian

$$\mathcal{L}_{kin} = \frac{1}{2}(\partial_\mu a)^2 + \frac{f_\pi^2}{8} \text{Tr} [D^\mu \Sigma (D_\mu \Sigma^\dagger)]$$

$$\mathcal{L}_m = \frac{f_\pi^2}{4} B_0 \text{Tr} \left[\hat{\mathbf{m}}_q \Sigma^\dagger + h.c. \right] - \frac{1}{2} m_{a,0}^2 a^2$$

$$\mathcal{L}_{weak} = \frac{4N_8}{f_\pi^2} [L_\mu L^\mu]^{32} + h.c.$$

$$\Sigma = \exp [i\Pi/f_\pi]$$

$$D_\mu \Sigma = \partial_\mu \Sigma - i \frac{\partial_\mu a}{f} \left(\hat{\mathbf{k}}_Q \Sigma - \Sigma \hat{\mathbf{k}}_q \right)$$

$$\hat{\mathbf{m}}_q = \exp \left[-2i\kappa_q c_{GG} \frac{a(x)}{f} \right] \mathbf{m}_q$$

Flavor violating ALP couplings - from UV to IR

below confinement scale: chiral Lagrangian

$$\mathcal{L}_{kin} = \frac{1}{2}(\partial_\mu a)^2 + \frac{f_\pi^2}{8} \text{Tr} [D^\mu \Sigma (D_\mu \Sigma^\dagger)]$$
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Kinetic term:

- FV interactions proportional to off-diagonal ALP couplings
- kinetic mixing

Flavor violating ALP couplings - from UV to IR

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Kinetic term:

- FV interactions proportional to off-diagonal ALP couplings
- kinetic mixing

Mass term:

- mass mixing
- flavor preserving interactions

Flavor violating ALP couplings - from UV to IR

below confinement scale: chiral Lagrangian

$$\mathcal{L}_{kin} = \frac{1}{2}(\partial_\mu a)^2 + \frac{f_\pi^2}{8} \text{Tr} [D^\mu \Sigma (D_\mu \Sigma^\dagger)]$$

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$$\mathcal{L}_{weak} = \frac{4N_8}{f_\pi^2} [L_\mu L^\mu]^{32} + h.c.$$

$$\Sigma = \exp [i\Pi/f_\pi]$$

$$D_\mu \Sigma = \partial_\mu \Sigma - i \frac{\partial_\mu a}{f} \left(\hat{\mathbf{k}}_Q \Sigma - \Sigma \hat{\mathbf{k}}_q \right)$$

$$\hat{\mathbf{m}}_q = \exp \left[-2i\kappa_q c_{GG} \frac{a(x)}{f} \right] \mathbf{m}_q \quad L_\mu^{ji} = -\frac{if_\pi^2}{4} e^{i(\phi_{q_i}^- - \phi_{q_j}^-)a(x)/f} [\Sigma (D_\mu \Sigma)^\dagger]^{ji}$$

Kinetic term:

- FV interactions proportional to off-diagonal ALP couplings
- kinetic mixing

Mass term:

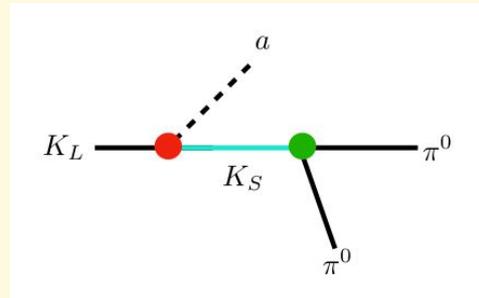
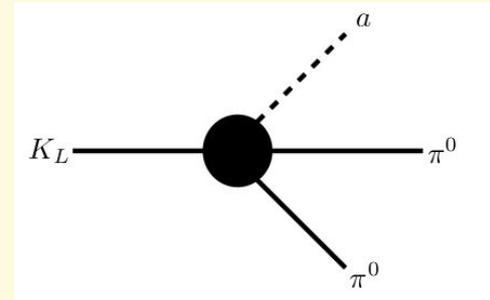
- mass mixing
- flavor preserving interactions

Weak-mediated term:

- flavor violating interactions proportional to diagonal ALP couplings
- kinetic mixing (neutral and charged meson, neutral and ALP mixing)

Kaon decay amplitudes

- 1) move to physical basis (remove all mixings)
- 2) sum up all relevant 4-point contact terms
- 3) sum up all relevant factorizable contributions



FV and CPV - some numbers

$$\varepsilon = 2.228(11) \times 10^{-3} e^{i\theta_\varepsilon}$$

$$\theta_\varepsilon \approx 0.76$$

$$|N_8| \approx 1.53 \times 10^{-7}$$

$$\delta_8 \approx \text{Im} \left[-\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \right] \approx -6 \times 10^{-4}$$

contributes to ε'

1107.6001

hep-ph/0309172

hep-ph/0005189

$$V_{td}^* V_{ts}$$

$$= (0.33 + 0.13i) \times 10^{-3}$$

Kaon decay amplitudes

$$\mathcal{M}(K_L \rightarrow a \pi^0 \pi^0) = \frac{m_K^2}{2\sqrt{2}f_\pi f} \left(\underbrace{\text{Re } \mathcal{M}_0}_{\text{axial}} + i\varepsilon \underbrace{\text{Im } \mathcal{M}_0} \right) \quad \text{CP-preserving}$$

$$\mathcal{M}(K_S \rightarrow a \pi^0 \pi^0) = \frac{m_K^2}{2\sqrt{2}f_\pi f} \left(\underbrace{\text{Im } \mathcal{M}_0}_{\text{axial}} - i\varepsilon \underbrace{\text{Re } \mathcal{M}_0} \right) \quad \text{CP-violating}$$

$$\mathcal{M}(K_L \rightarrow \pi^0 a) = \frac{(m_K^2 - m_\pi^2)}{2f} \left(\underbrace{\text{Im } \widetilde{\mathcal{M}}}_{\text{vector}} - i\varepsilon \underbrace{\text{Re } \widetilde{\mathcal{M}}} \right)$$

Ratio of rates

$$R_0 = \frac{\text{Br}(K_L \rightarrow \pi^0 \pi^0 a)}{\text{Br}(K_L \rightarrow \pi^0 a)}$$

$$R_{\pm} = \frac{\text{Br}(K_L \rightarrow \pi^+ \pi^- a)}{\text{Br}(K_L \rightarrow \pi^0 a)}$$

$$R_0, R_{\pm} \sim \left| \frac{\text{largest CP-conserving coupling}}{\text{largest CP-violating coupling}} \right|^2 \cdot I$$

phase space ratio $I \sim \mathcal{O}(10^{-3} - 10^{-2})$

Scenarios

1. maximal flavor violation $\kappa_L \gg V_{td}^* V_{ts}$

2. minimal flavor violation $\kappa_L = c_{MFV} V_{td}^* V_{ts}$

“directly mediated” $N_8 \ll c_{MFV} V_{td}^* V_{ts}$ $R_0 \sim \mathcal{O}(10^{-2})$

“mix” $\epsilon N_8 \ll c_{MFV} V_{td}^* V_{ts} \ll N_8$ $R_0 \sim \mathcal{O}(10^{-1} - 10)$

“indirectly mediated” $c_{MFV} V_{td}^* V_{ts} \ll \epsilon N_8$ $R_0 \sim \mathcal{O}(10^2 - 10^3)$

Minimal flavor violation - neutral

$$c_{MFV} V_{td}^* V_{ts} \ll \epsilon N_8$$

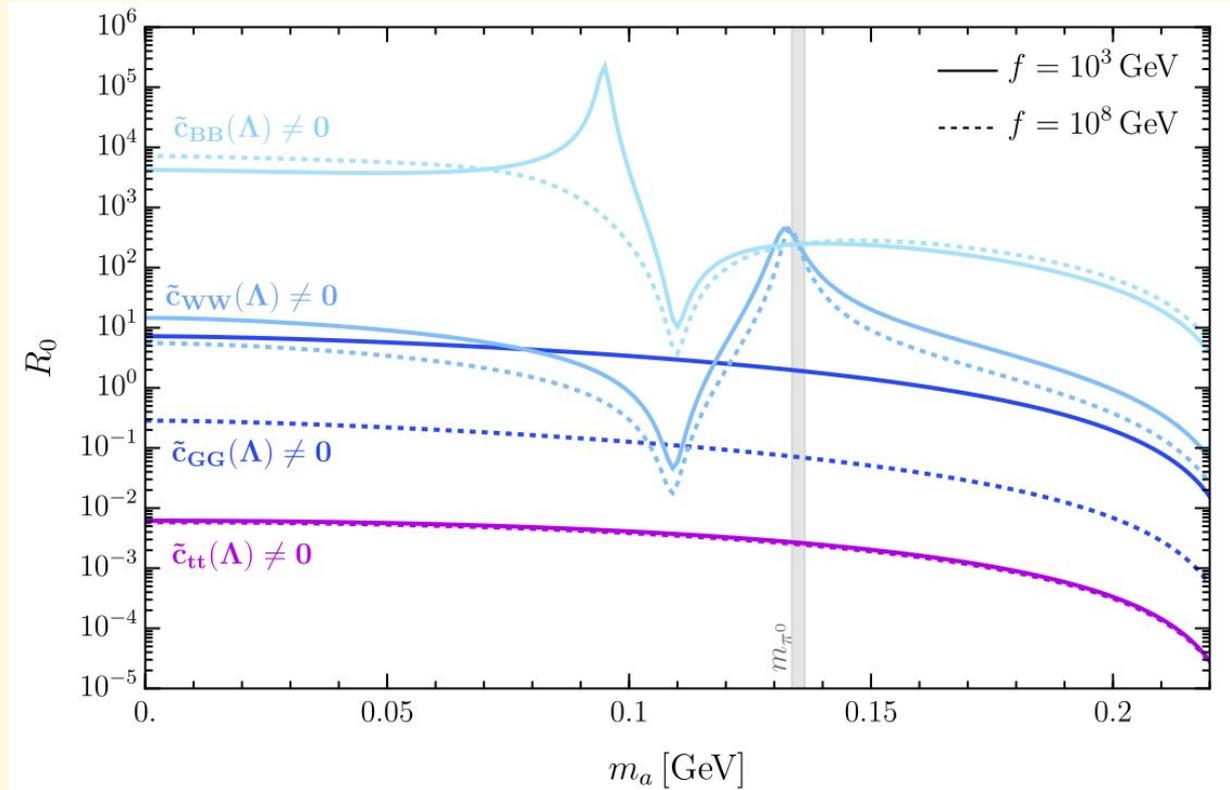
“directly mediated”

$$\epsilon N_8 \ll c_{MFV} V_{td}^* V_{ts} \ll N_8$$

“mix”

$$N_8 \ll c_{MFV} V_{td}^* V_{ts}$$

“indirectly mediated”



Minimal flavor violation - neutral

$$N_8 \ll c_{MFV} V_{td}^* V_{ts}$$

“directly mediated”

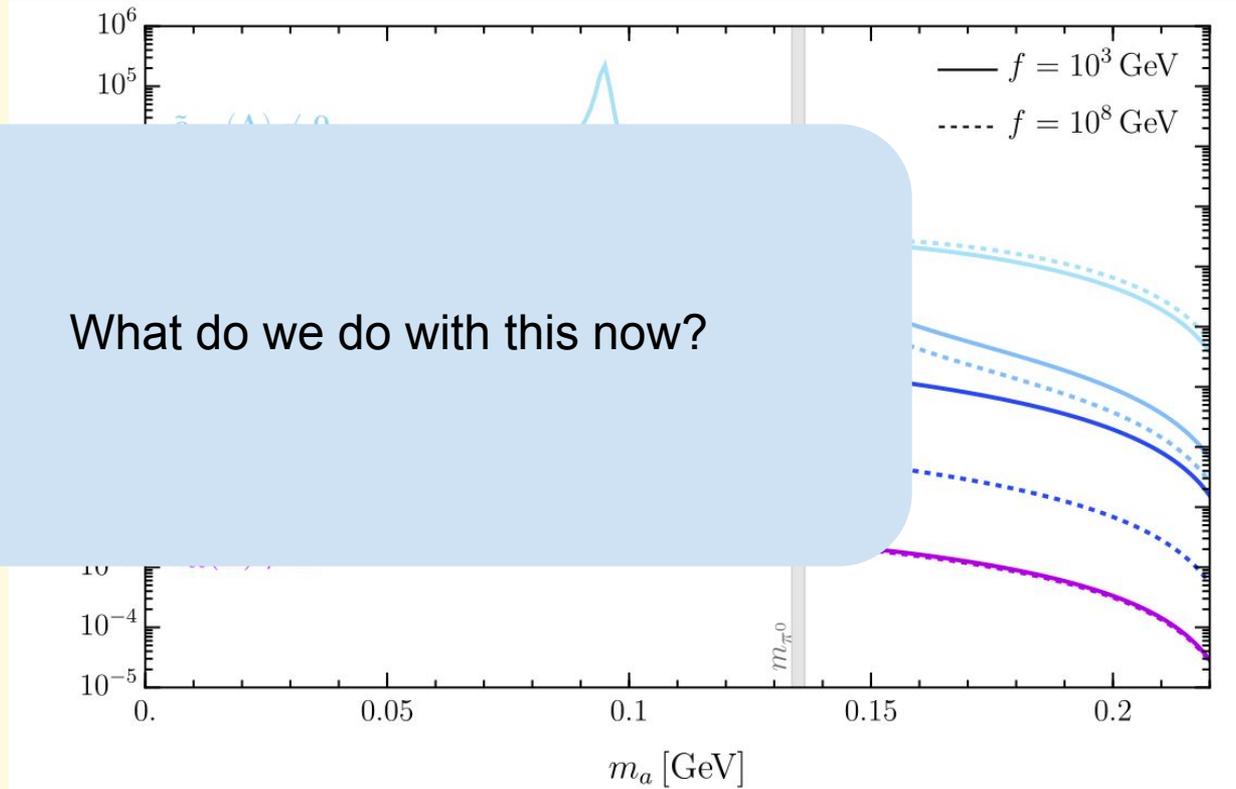
$$\epsilon N_8 \ll c_{MFV} V_{td}^* V_{ts} <$$

“mix”

$$N_8 \ll c_{MFV} V_{td}^* V_{ts}$$

“indirectly mediated”

What do we do with this now?



Minimal flavor violation - neutral

$$N_8 \ll c_{MFV} V_{td}^* V_{ts}$$

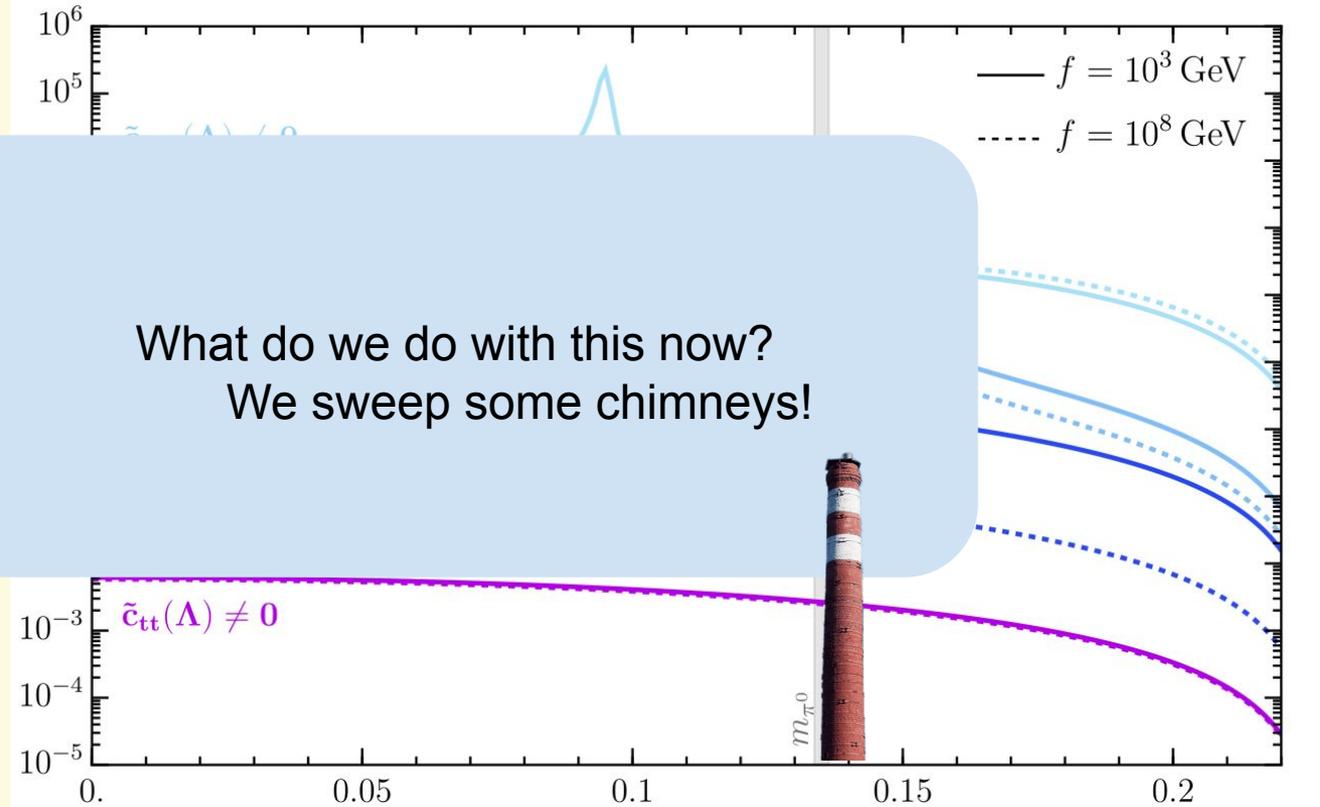
“directly mediated”

$$\epsilon N_8 \ll c_{MFV} V_{td}^* V_{ts}$$

“mix”

$$N_8 \ll c_{MFV} V_{td}^* V_{ts}$$

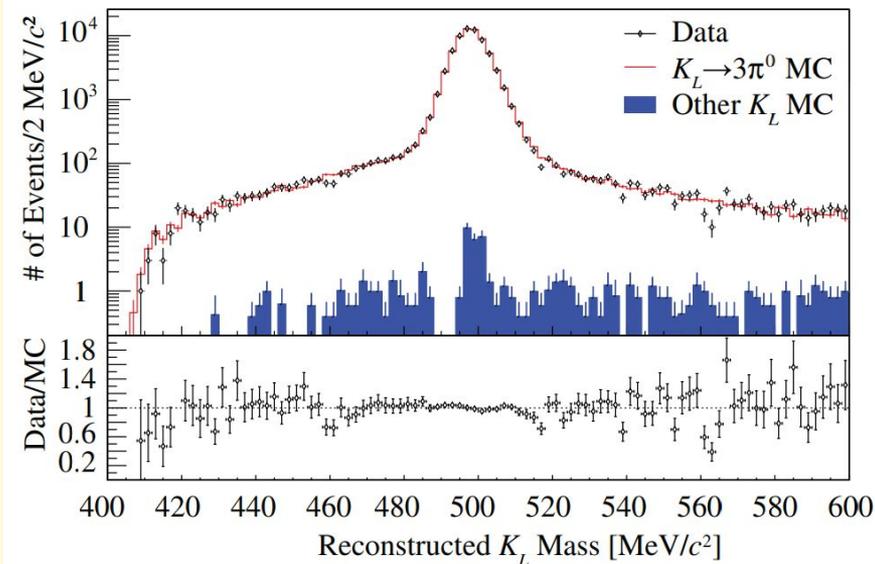
“indirectly mediated”



Look at some data!

Long-lived neutral-kaon flux measurement for the KOTO experiment

T. Masuda^{1,16,*}, J. K. Ahn², S. Banno^{3,17}, M. Campbell⁴, J. Comfort⁵, Y. T. Duh⁶, T. Hineno^{1,18}, Y. B. Hsiung⁶, T. Inagaki⁷, E. Iwai^{3,19}, N. Kawasaki¹, E. J. Kim⁸, Y. J. Kim⁹, J. W. Ko⁹, T. K. Komatsubara⁷, A. S. Kurilin^{10,†}, G. H. Lee⁸, J. W. Lee^{3,20}, S. K. Lee⁸, G. Y. Lim⁷, J. Ma^{11,21}, D. MacFarland⁵, Y. Maeda^{1,22}, T. Matsumura¹², R. Murayama³, D. Naito¹, Y. Nakaya^{3,‡}, H. Nanjo¹, T. Nomura⁷, Y. Odani^{13,23}, H. Okuno⁷, Y. D. Ri^{3,24}, N. Sasao¹⁴, K. Sato^{3,25}, T. Sato⁷, S. Seki¹, T. Shimogawa^{13,19}, T. Shinkawa¹², K. Shiomi^{3,19}, J. S. Son⁸, Y. Sugiyama³, S. Suzuki¹³, Y. Tajima¹⁵, G. Takahashi¹, Y. Takashima^{3,26}, M. Tecchio⁴, M. Togawa³, T. Toyoda^{3,27}, Y. C. Tung^{6,28}, Y. W. Wah¹¹, H. Watanabe⁷, J. K. Woo⁹, J. Xu^{4,31}, T. Yamanaka³, Y. Yanagida^{3,29}, H. Y. Yoshida¹⁵, and H. Yoshimoto^{3,30}



KOTO

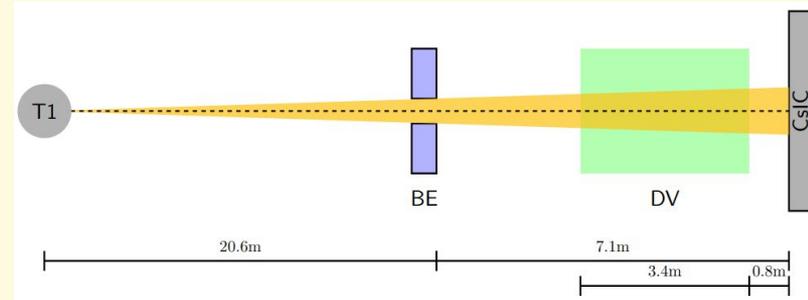
KOTO measures photon energies and positions

to reconstruct kaon mass:

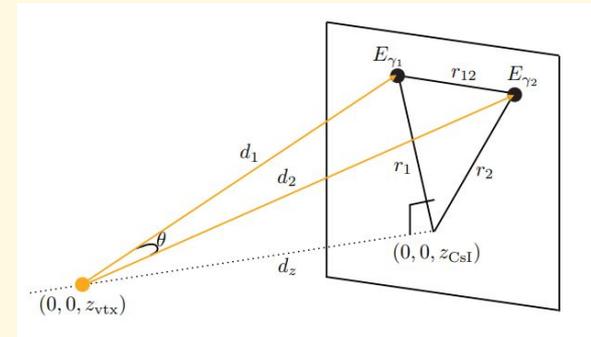
1. use opening angle between any pair of photons to find potential kaon decay vertex
2. choose photon pairing combination that minimizes

$$\chi_Z^2 = \sum_{i=1}^{15} \frac{(Z_i - \bar{Z})^2}{\sigma_{zi}^2} \quad \text{with} \quad \bar{Z} = \frac{\sum_{n=1}^{15} Z_i / \sigma_{zi}^2}{\sum_{n=1}^{15} 1 / \sigma_{zi}^2}$$

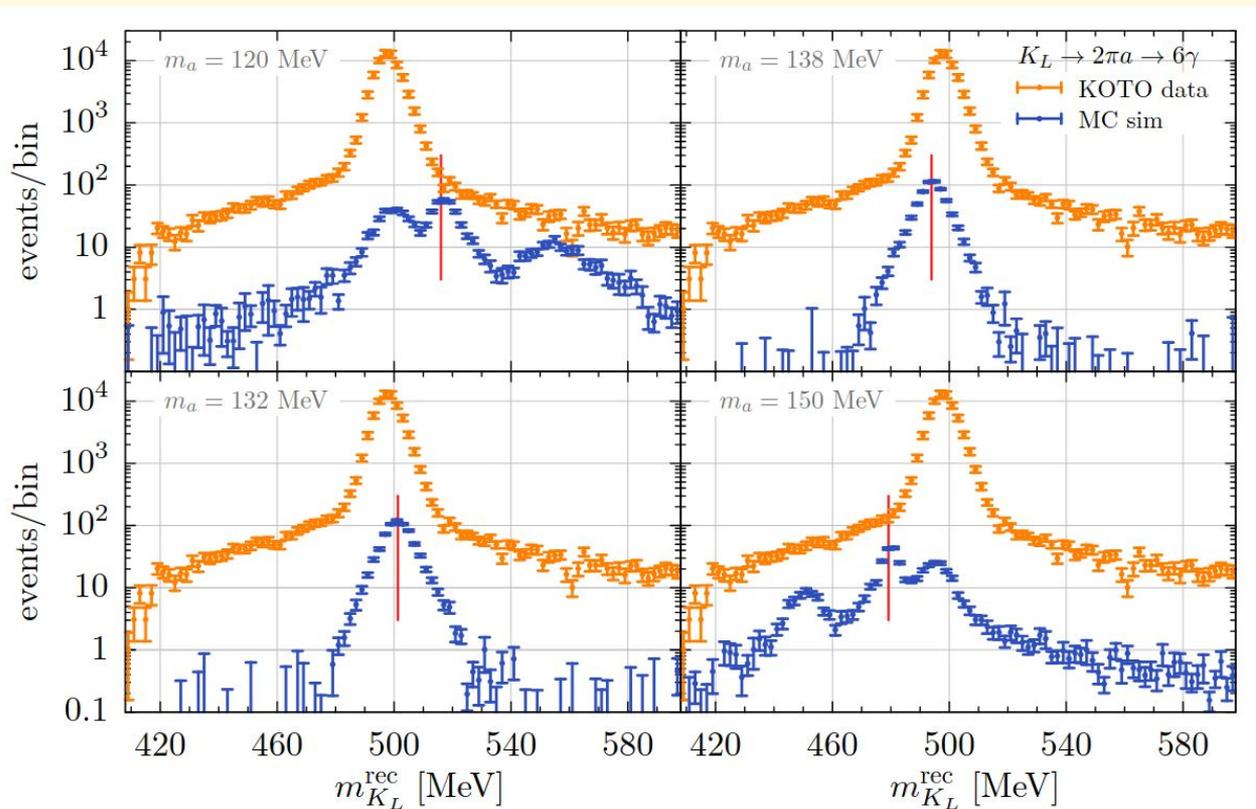
3. reconstruct photon momenta and pion and kaon mass
4. choose events with $\Delta m_{\pi^0} \leq 10 \text{ MeV}$



$$m_{\pi^0}^2 = 2E_1E_2(1 - \cos \theta)$$



How would that look for ALPs?



one photon pair originates from an ALP

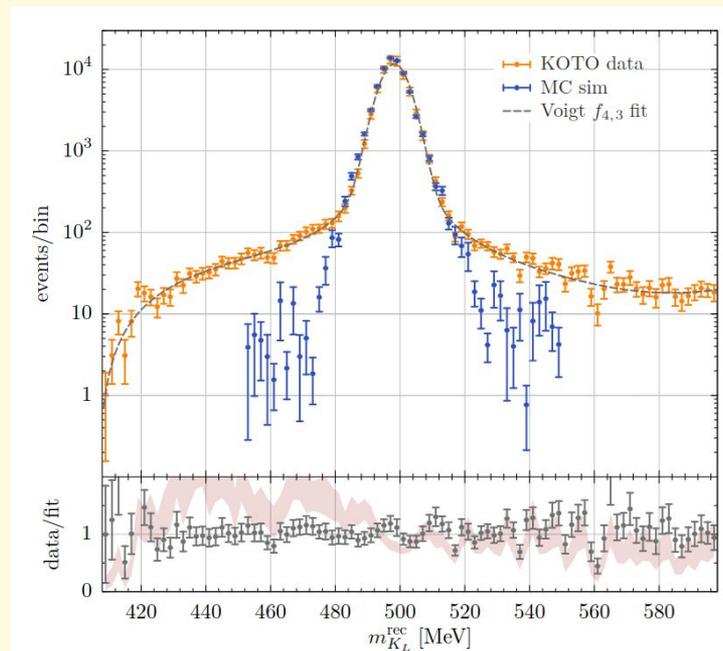
ALP mass must be close to pion mass to be selected



$$\delta m_{K_L}^{\text{rec}} \simeq -[m_{K_L}/3m_{\pi^0}] \delta m \simeq -1.2 \delta m$$

Fits and limits

KOTO signal contains detector backgrounds that we can't simulate!



Fits and limits

KOTO signal contains detector backgrounds that we can't simulate!

use lineshape fit instead

$$f_{2n_a, n_b}(\mu) = \sqrt{\mu^2 - 9m_{\pi^0}^2} \left\{ \sum_{k=1}^{n_a} a_{2k} \frac{[\mu^2 - 9m_{\pi^0}^2]^{2k-2}}{\mu^{2k}} + \left[\sum_{k=0}^{n_b} b_k \frac{[\mu^2 - 9m_{\pi^0}^2]^k}{\mu^k} \right] V_{\delta, \sigma}(\mu - m_{K_L} + \delta m) \right\}$$

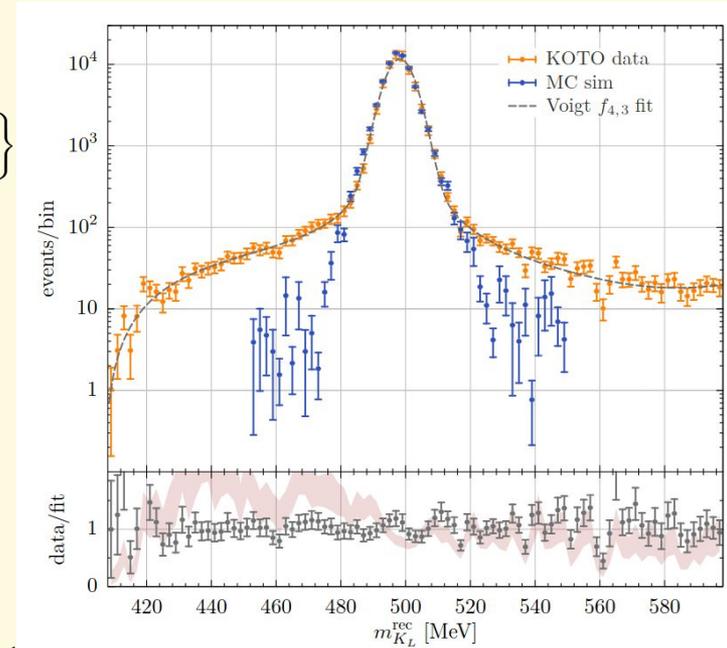
construct likelihood ratio

$$\delta\chi^2(m_a, \rho) = \chi_{\text{sub}}^2(m_a, \rho) - \chi_0^2$$

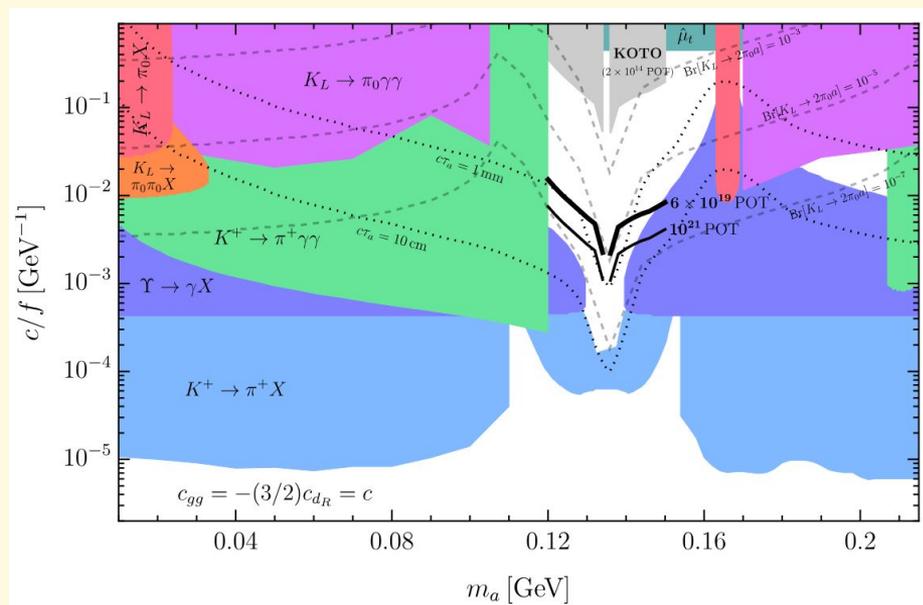
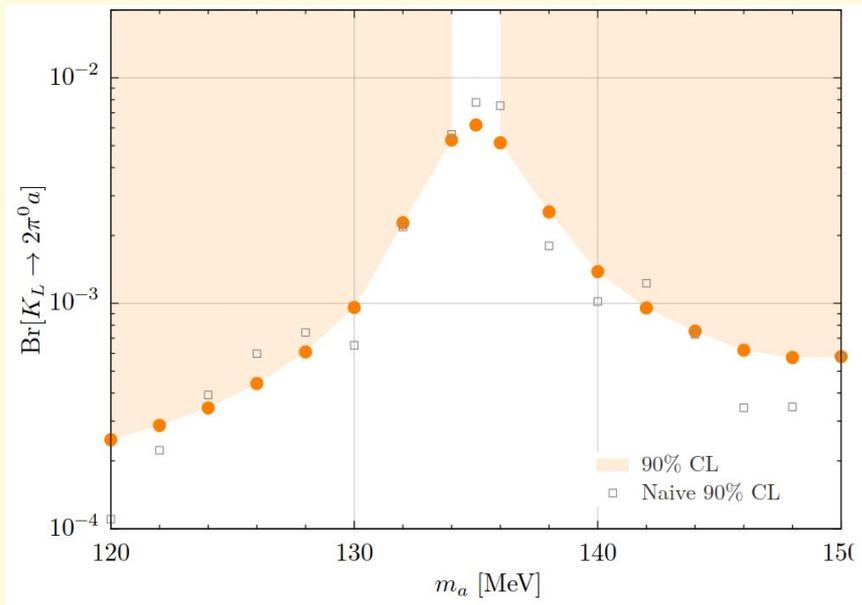
fit residuals of
re-fitted lineshape -
(KOTO data + ρ ALP
contribution)

fit residuals of
lineshape - KOTO data

$$\rho = \text{Br}[K_L \rightarrow 2\pi^0 a] / \text{Br}[K_L \rightarrow 3\pi^0] \geq 0$$



Results



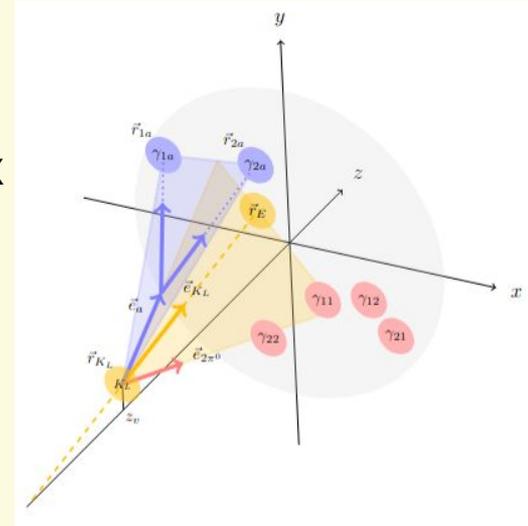
Beyond the chimney

for small ALP masses, ALPs become long-lived

alternative selection:

- only require two photon pairs to reconstruct kaon vertex (now 45 possible combinations of photon pairs)
- ALP momentum coplanar to kaon-two-pion momentum plane and plane of kaon vertex and last two photons
- introduce displacement parameter in ALP momentum, so that

$$\sqrt{[p_{2\pi^0} + k_{1a}(\lambda_{K_L}) + k_{2a}(\lambda_{K_L})]^2} = m_{K_L}$$



Summary

- first calculation of ALPs in 3-body neutral kaon decays, probing the CP properties of ALP couplings
- in some scenarios, 3-body decays are the **dominant** channel for neutral kaon decays
- can potentially provide the strongest probe in some part of the parameter space, e.g. in the pion chimney using

KOTO $K_L \rightarrow 3\pi^0 \rightarrow 6\gamma$ data



Backup

Starting point : $N_f = 3$ QCD

$$\mathcal{L}_a(\mu_{\text{QCD}}) \supset c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G\tilde{G} + \frac{\partial^\mu a}{f} (\bar{q}_L \mathbf{k}_Q \gamma_\mu q_L + \bar{q}_R \mathbf{k}_q \gamma_\mu q_R),$$

Field redefinition:

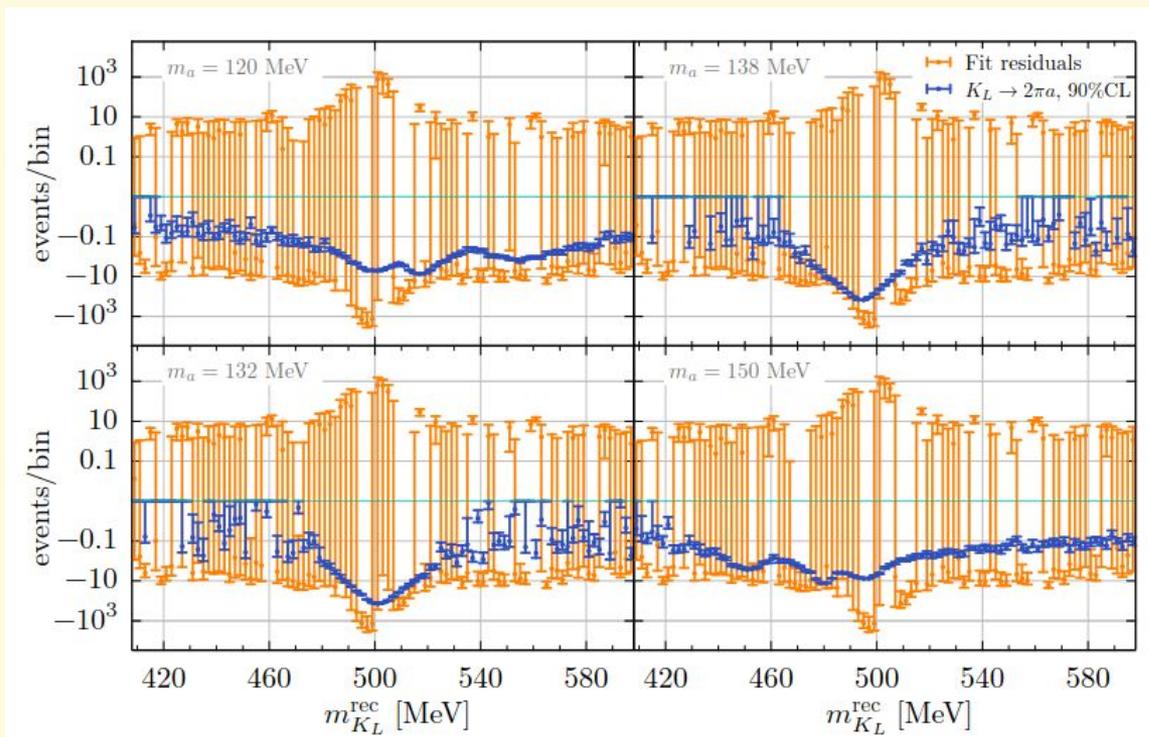
$$q(x) \rightarrow \exp \left[-i(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5) c_{GG} \frac{a(x)}{f} \right] q(x)$$

- 1 $\mathbf{k}_Q \rightarrow \hat{\mathbf{k}}_Q(a) = U_- (\mathbf{k}_Q + \phi_q^-) U_-^\dagger,$
- 2 $\mathbf{m}_q \rightarrow \hat{\mathbf{m}}_q = \exp \left[-2i \boldsymbol{\kappa}_q c_{GG} \frac{a(x)}{f} \right] \mathbf{m}_q$
- 3 $\text{Tr}[\boldsymbol{\kappa}_q] = 1 \rightarrow$ removes $G\tilde{G}$ term

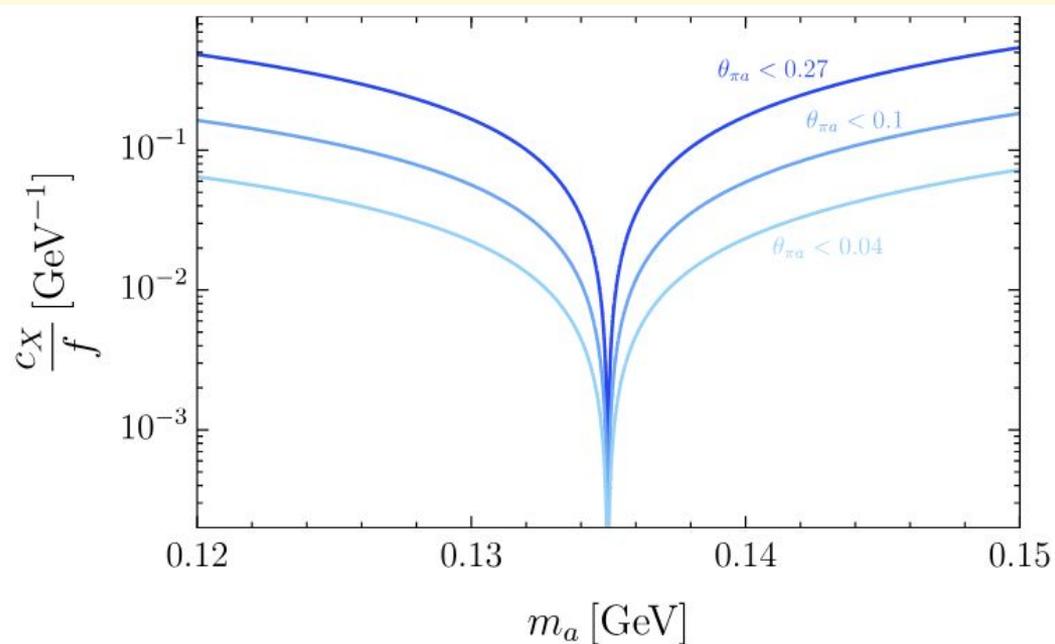
Classification of decay amplitudes in terms of CP and P

	$P = -1$ (κ_V, N_8)	$P = +1$ (κ_A, N_8)
$CP = +1$ ($\text{Re}[c], \varepsilon \cdot \text{Im}[c]$)	$K_S \rightarrow \pi^0 a$	$K_L \rightarrow \pi^0 \pi^0 a$ or $\pi^+ \pi^- a$
$CP = -1$ ($\text{Im}[c], \varepsilon \cdot \text{Re}[c]$)	$K_L \rightarrow \pi^0 a$	$K_S \rightarrow \pi^0 \pi^0 a$ or $\pi^+ \pi^- a$

Residuals



Validity of ChPT



$$\theta_{\pi a} \sim f_{\pi}/f \times c_X/|\delta m_a| \ll 1$$

Modified reconstruction

