Thermal Lepton Oscillations in Leptogenesis

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2026.02.19

@ KEK-PH2026

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Lisp & A. Pilaftsis, arXiv: 2601.15921; 2603.xxxxx

Outline

- The Behavior of Oscillation
- Thermal Lepton Oscillations in Flavor-Covariant Kadanoff-Baym Equation
- Applications in Leptogenesis
- Summary

Oscillation Pattens

- Active & steile neutrino oscillations—density-matrix formalism

G. Sigl & G. Raffelt, NPB 1993

Production term

Depletion/destruction term

$$i\frac{d\rho}{dt} = [\hat{H},\rho] - \frac{i}{2}\{\Gamma,\rho\} + \frac{i}{2}\{\Gamma^p, 1-\rho\}$$

Time evolution of density matrix

Diagonal entries: distribution/occupation number

Off-diagonal entries: flavor correlation/violation

Commutation with Hamiltonian (free+background/medium corrections)

Relativistic Oscillations

$$i\frac{d\rho}{dt} = [\hat{H}, \rho] - \frac{i}{2}\{\Gamma, \rho\} + \frac{i}{2}\{\Gamma^p, 1-\rho\}$$

Feeble interactions or quasi-thermal equilibrium

$\hat{H} = H_0 = P + \frac{M^2}{2P}$, with diagonal, real mass M

$$\longrightarrow \quad \rho_{ij} = \mathcal{N}\exp\left[-i\frac{M_i^2 - M_j^2}{2P}t\right]$$

- ★ Oscillation occurs at off-diagonal entries; triggered by vacuum mass difference—off-diagonal entries are NOT mass eigenstates
- ★ No oscillations in diagonal entries: one mass eigenstate CANNOT oscillate into another mass eigenstate—energy-momentum conservation
- ★ Oscillation without CP violation

$\hat{H} = H_0 = P + \frac{M^2}{2P}$, with non-diagonal, complex mass M

$$\longrightarrow \quad \frac{d\rho_{ij}}{dt} = -i\frac{M_i^2 - M_j^2}{2P}\rho_{ij} - i[\delta M, \rho]_{ij}$$

- ★ If δM is complex, oscillations lead to CP violation
- ★ For active & sterile neutrinos, complex δM is caused by complex Yukawa matrix

$$\begin{cases} \frac{d\rho_{ii}}{dt} = -i[\delta M, \rho]_{ii}\,, i = j, \\ \frac{d\rho_{ij}}{dt} \approx -i\frac{M_i^2 - M_j^2}{2P}\rho_{ij}\,, i \neq j \end{cases} \quad \xrightarrow{\delta M = \delta M^T} \quad \frac{d\rho_{ii}}{dt} = 2\delta M_{ik}\sin\left(\frac{M_k^2 - M_i^2}{2P}t\right), \frac{d\bar{\rho}_{ii}}{dt} = 2\delta M_{ik}^*\sin\left(\frac{M_k^2 - M_i^2}{2P}t\right)$$

How oscillations appear from massless particles

H. Jukkala, arXiv:2211.11785, a detailed record of closed-time-path formalism

- Flavor-covariant Kadanoff-Baym formalism—the starting point

$$i\partial_t[\gamma^0 iS_{<,>}]_{\alpha\beta} - \left[\mathrm{Re}\Sigma_R\gamma^0, \gamma^0 iS_{<,>}\right]_{\alpha\beta} = -\frac{i}{2}\left(\mathcal{C}_{\alpha\beta} + \mathcal{C}^\dagger_{\alpha\beta}\right)$$

Time evolution of Wightman propagator matrix

$$\gamma^0[iS_<]_{\alpha\beta} = -\langle\gamma^0\bar{\psi}_\beta\psi_\alpha\rangle$$

$$\gamma^0[iS_>]_{\alpha\beta} = \langle\gamma^0\psi_\alpha\bar{\psi}_\beta\rangle$$

Commutation with the background plasma

$\mathrm{Re}\Sigma_R$: real part of the retarded self-energy correction

$$\mathcal{C}_{\alpha\beta} \equiv [i\Sigma_>]_{\alpha\gamma}[iS_<]_{\gamma\beta} - [i\Sigma_<]_{\alpha\gamma}[iS_>]_{\gamma\beta}$$

Collision terms: depletion and production rates

$\Sigma_{<,>}$: N-loop self-energy corrections

- Comparison with density-matrix formalism

$$i\frac{d\rho}{dt} = [\hat{H}, \rho] - \frac{i}{2}\{\Gamma, \rho\} + \frac{i}{2}\{\Gamma^p, 1-\rho\}$$

How oscillations appear from massless particles

- Flavor-covariant Kadanoff-Baym formalism—the starting point

$$i\partial_t[\gamma^0 iS_{<,>}]_{\alpha\beta} - \left[\mathrm{Re}\Sigma_R\gamma^0, \gamma^0 iS_{<,>}\right]_{\alpha\beta} = -\frac{i}{2}\left(\mathcal{C}_{\alpha\beta} + \mathcal{C}^\dagger_{\alpha\beta}\right)$$

- Particle & anti-particle number density

$$n_{\alpha\beta}(t) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} f_{\alpha\beta}(E_\mathbf{k}, t) = -\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \int_0^\infty \frac{\mathrm{d}k_0}{2\pi} \mathrm{Tr}[\gamma^0 iS_<]_{\alpha\beta}$$

$$\bar{n}_{\alpha\beta}(t) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \bar{f}_{\alpha\beta}(E_\mathbf{k}, t) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \int_{-\infty}^0 \frac{\mathrm{d}k_0}{2\pi} \mathrm{Tr}[\gamma^0 iS_>]_{\alpha\beta}$$

Correspondence to a generalized Noether charge:
$j^0_{\alpha\beta} = \bar{\psi}_\beta\gamma^0\psi_\alpha$ integrated over the momentum space

$$-\mathrm{Tr}[\gamma^0\bar{\psi}_\beta\psi_\alpha] = \mathrm{Tr}[\gamma^0\psi_\alpha\bar{\psi}_\beta] = \mathrm{Tr}j^0_{\alpha\beta}$$

How oscillations appear from massless particles

- Kinetic equations for $n_{\alpha\beta}\,,\bar{n}_{\alpha\beta}$

$$\frac{\mathrm{d}n_{\alpha\beta}}{\mathrm{d}t} - i[b,n]_{\alpha\beta} = \frac{1}{2}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\int_0^\infty\frac{\mathrm{d}k_0}{2\pi}\mathrm{Tr}\left(\mathcal{C}_{\alpha\beta}+\mathcal{C}^\dagger_{\alpha\beta}\right)$$

$$\frac{\mathrm{d}\bar{n}_{\alpha\beta}}{\mathrm{d}t} + i[b,\bar{n}]_{\alpha\beta} = -\frac{1}{2}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\int_{-\infty}^0\frac{\mathrm{d}k_0}{2\pi}\mathrm{Tr}\left(\mathcal{C}_{\alpha\beta}+\mathcal{C}^\dagger_{\alpha\beta}\right)$$

The commutation also appears
—the source for oscillations

- Kinetic equations for $\Delta n_{\alpha\beta} \equiv n_{\alpha\beta} - \bar{n}_{\alpha\beta}\,, \sum n_{\alpha\beta} \equiv n_{\alpha\beta} + \bar{n}_{\alpha\beta}$

$$\frac{\mathrm{d}\Delta n_{\alpha\beta}}{\mathrm{d}t} - i(b_\alpha - b_\beta)\Sigma n_{\alpha\beta} = \frac{1}{2}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\int_{-\infty}^\infty\frac{\mathrm{d}k_0}{2\pi}\mathrm{Tr}\left(\mathcal{C}_{\alpha\beta}+\mathcal{C}^\dagger_{\alpha\beta}\right)$$

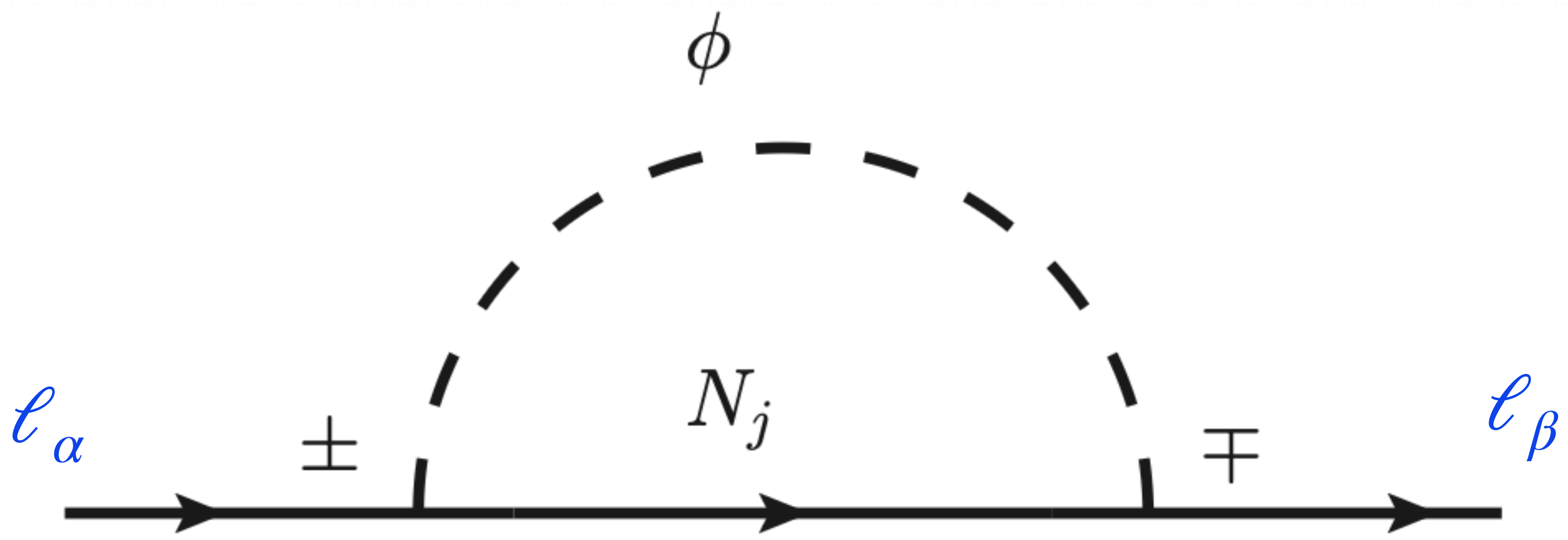
What is the oscillation phase b ?

$$\frac{\mathrm{d}\Sigma n_{\alpha\beta}}{\mathrm{d}t} - i(b_\alpha - b_\beta)\Delta n_{\alpha\beta} = \frac{1}{2}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\int_{-\infty}^\infty\frac{\mathrm{d}k_0}{2\pi}\mathrm{sign}(k_0)\mathrm{Tr}\left(\mathcal{C}_{\alpha\beta}+\mathcal{C}^\dagger_{\alpha\beta}\right)$$

Oscillation Phase via Thermal Masses

- What is b?

H. A. Weldon, PRD 26 (1982) 2789; PRD 26 (1982) 1394



$$[\mathbf{Re}\Sigma_R(k)]_{\alpha\beta} \equiv -a_{\alpha\beta} P_R \not{k} P_L - b_{\alpha\beta} P_R \not{u} P_L$$

$\mathrm{Re}\Sigma_R$: real part of the retarded self-energy correction

A Lorentz-covariant formalism introduces the four-velocity of the background plasma.

In the rest plasma frame: $u = (1,0,0,0)$

- The coefficient b includes all the interactions with the plasma

Lepton doublets in the Standard Model before the electroweak gauge symmetry breaking

$$a_\alpha(k_0, \mathbf{k}) = \frac{\tilde{m}_\alpha^2}{|\mathbf{k}|^2}\left[1 + \frac{k_0}{2|\mathbf{k}|}\ln\left(\frac{k_0 - |\mathbf{k}|}{p_0 + |\mathbf{k}|}\right)\right]$$

$$b_\alpha(k_0, \mathbf{k}) = -\frac{\tilde{m}_\alpha^2}{|\mathbf{k}|}\left[\frac{k_0}{|\mathbf{k}|} - \frac{1}{2}\left(1 - \frac{k_0^2}{|\mathbf{k}|^2}\right)\ln\left(\frac{k_0 - |\mathbf{k}|}{k_0 + |\mathbf{k}|}\right)\right]$$

Thermal masses of lepton doublets

$$\tilde{m}_\alpha^2 = \left(\frac{3}{32}g_2^2 + \frac{1}{32}g_1^2 + \frac{1}{16}y_\alpha^2\right)T^2$$

Oscillation Phase via Thermal Masses

- Quasi-thermal leptons, $n_{\alpha\beta} + \bar{n}_{\alpha\beta} = 2n^{\text{eq}}\delta_{\alpha\beta} + \mathcal{O}(\mu^2)$

Vanish up to $\mathcal{O}(\mu)$ for $\alpha = \beta$, & $\alpha \neq \beta$

$$\frac{\mathrm{d}\Delta n_{\alpha\beta}}{\mathrm{d}t} - i(b_\alpha - b_\beta)\Sigma n_{\alpha\beta} = \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_0}{2\pi}\mathrm{Tr}\left(\mathcal{C}_{\alpha\beta} + \mathcal{C}^\dagger_{\alpha\beta}\right)$$

$d\Delta n_{\alpha\beta}/dt$, $\alpha \neq \beta$ & $d\Delta n_{\alpha\alpha}/dt$, $\alpha = \beta$, grow via non-thermal collision rate

$$\frac{\mathrm{d}\Sigma n_{\alpha\beta}}{\mathrm{d}t} - i(b_\alpha - b_\beta)\Delta n_{\alpha\beta} = \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_0}{2\pi}\mathrm{sign}(k_0)\mathrm{Tr}\left(\mathcal{C}_{\alpha\beta} + \mathcal{C}^\dagger_{\alpha\beta}\right)$$

★ $\Delta n_{\alpha\beta}$, $\alpha \neq \beta$, appears in collision rates as the *CP-violating source*

★ $\Delta n_{\alpha\alpha}$ appears in the collision rates as the *washout effect*

$$\Delta n_{\alpha\beta} = \frac{i}{2(b_\alpha - b_\beta)}\int \frac{\mathrm{d}^3\mathbf{k}_\ell}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_{\ell 0}}{2\pi}\mathrm{sign}(k_{\ell 0})\mathrm{Tr}\left(\mathcal{C}_{\alpha\beta} + \mathcal{C}^\dagger_{\alpha\beta}\right)$$

If vacuum contributions included, MSW-like resonance may occur

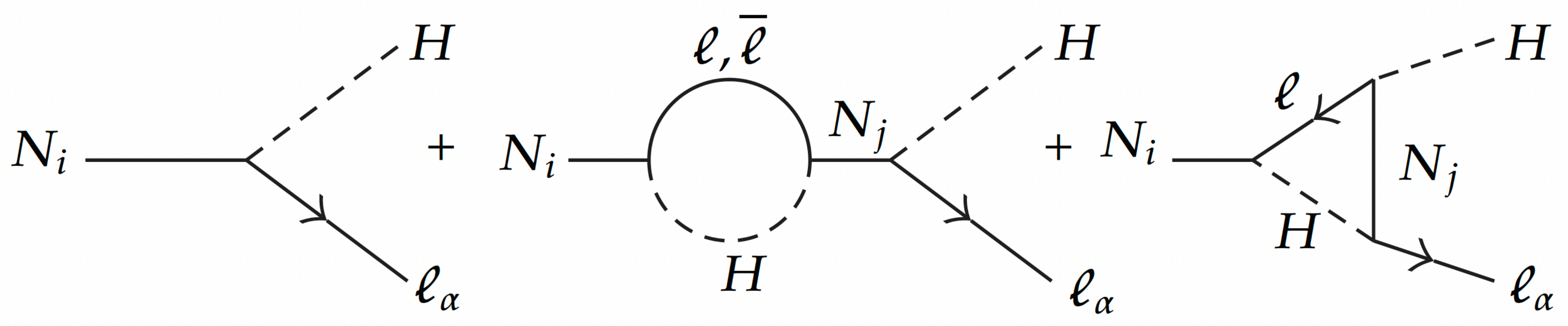
$$b_\alpha(E_\mathbf{k}) - b_\beta(E_\mathbf{k}) = \frac{\tilde{m}_\beta^2 - \tilde{m}_\alpha^2}{|\mathbf{k}|} = \frac{T^2}{16|\mathbf{k}|}\left(y_\beta^2 - y_\alpha^2\right)$$

Applications in Leptogenesis

Traditional leptogenesis

- High-scale leptogenesis from heavy neutrino decay

$$\mathcal{L}_I = \mathcal{L}_{\rm SM} + i\overline{N_{R_i}}\partial\!\!\!/ N_{R_i} - \left(\frac{1}{2}M_i\overline{N^c_{R_i}}N_{R_i} + \epsilon_{ab}Y_{\alpha i}\overline{N_{R_i}}\ell^a_\alpha H^b + \text{h.c.}\right)$$



Heavy neutrino mass above 10^9 GeV

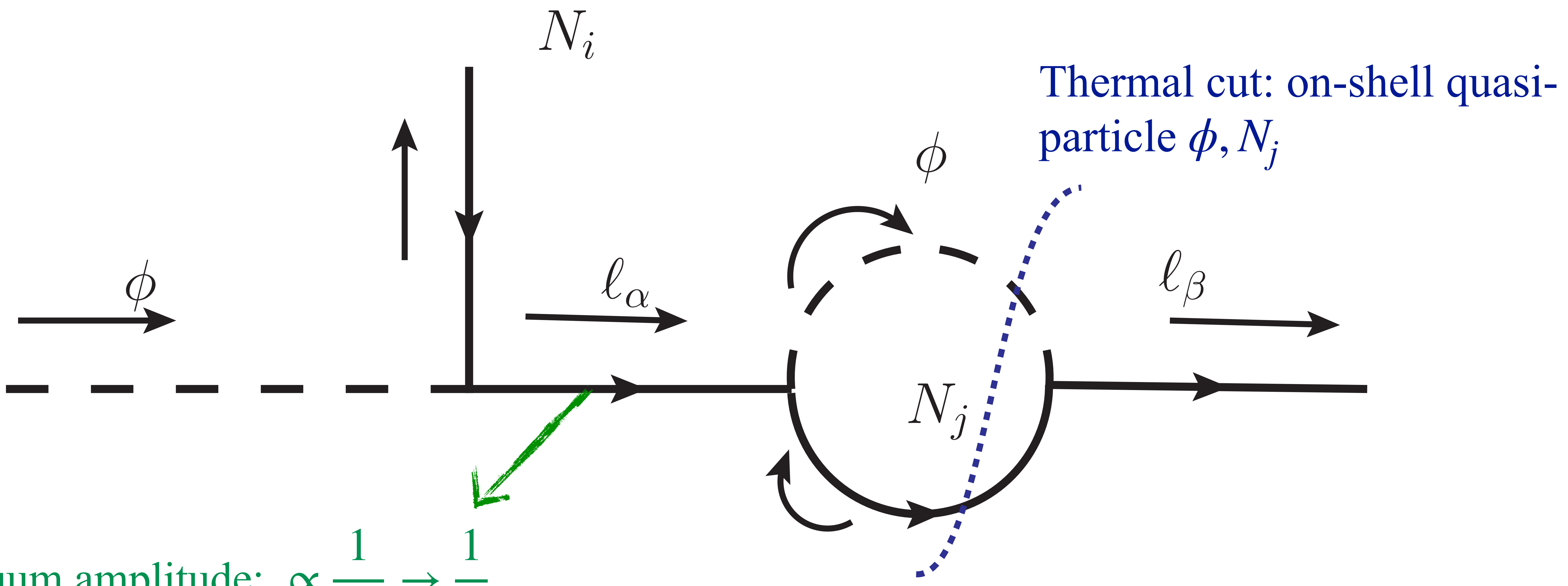
- Leading low-scale leptogenesis mechanism: resonant leptogenesis (Pilaftsis 1997); leptogenesis via sterile neutrino oscillations (Akhmedov-Rubakov-Smirnov, ARS mechanism, 1998);

Heavy neutrino mass below 1 TeV, but quasi degeneracy of neutrino mass required

A New Channel of Leptogenesis in Type-I Seesaw

Lisp & A. Pilaftsis, arXiv: 2601.15921

$$\mathcal{L} = -y_i \bar{\ell}_i \phi P_R e_i - y'_{ij} \bar{\ell}_i \tilde{\phi} P_R N_j - \frac{1}{2} M_i \overline{N_i^c} P_R N_i + \text{h.c.}$$



Vacuum amplitude: $\propto \frac{1}{p_\beta^2} \to \frac{1}{0}$

Thermal amplitude: $\propto \frac{1}{p_\beta^2 - \tilde{m}_\alpha^2} \to \frac{1}{b_\beta^2 - b_\alpha^2}$ → *Resonance and flavor oscillation may be the same thing*

Leptogenesis conditions

- Out-of-equilibrium SM Higgs decay to relativistic sterile neutrinos
- CP violation from neutrino Yukawa couplings y'
- Electroweak sphaleron processes on the asymmetry of Y_ℓ but not on Y_N

Pre-condition: lepton thermal mass is SM dominated ↔ oscillation rate not much smaller than damping date

$$\Gamma_{\text{osc}} \sim b_\alpha - b_\beta \sim 2 \times 10^{-8}\, T(\text{min}) \gtrsim \Gamma_{\text{col}} \sim y'^2 T \to y' \lesssim 10^{-4}$$

Thermal Resonant Leptogenesis

- An easy-to-remember result

$$\mathcal{F} \equiv [f_\phi^{\rm eq}(E_\ell + E_N) + f_{N_i}(E_N)][f_\ell^{\rm eq}(E_\ell) + f_\phi^{\rm eq}(E_\ell + E'_N)]\delta f_{N_j}(E'_N)$$

Leptogenesis @ $\mathcal{O}(y'^4)$

$$\frac{\mathrm{d}\Delta n_\ell}{\mathrm{d}t} = \sum_{\substack{\alpha,\gamma=1,\\ \alpha\neq\gamma}}^{3} \sum_{i,j=1}^{2} \frac{\Delta m^4 \mathrm{Im}(y'_{\gamma i} y'^*_{\alpha i} y'_{\alpha j} y'^*_{\gamma j})}{256\pi^4(\tilde{m}_\gamma^2 - \tilde{m}_\alpha^2)} \int \frac{dE_\ell}{E_\ell} \int_{E_{\rm low}}^{\infty} dE_N \int_{E_{\rm low}}^{\infty} dE'_N \mathcal{F}$$

Thermal resonance:

Phase-space integration

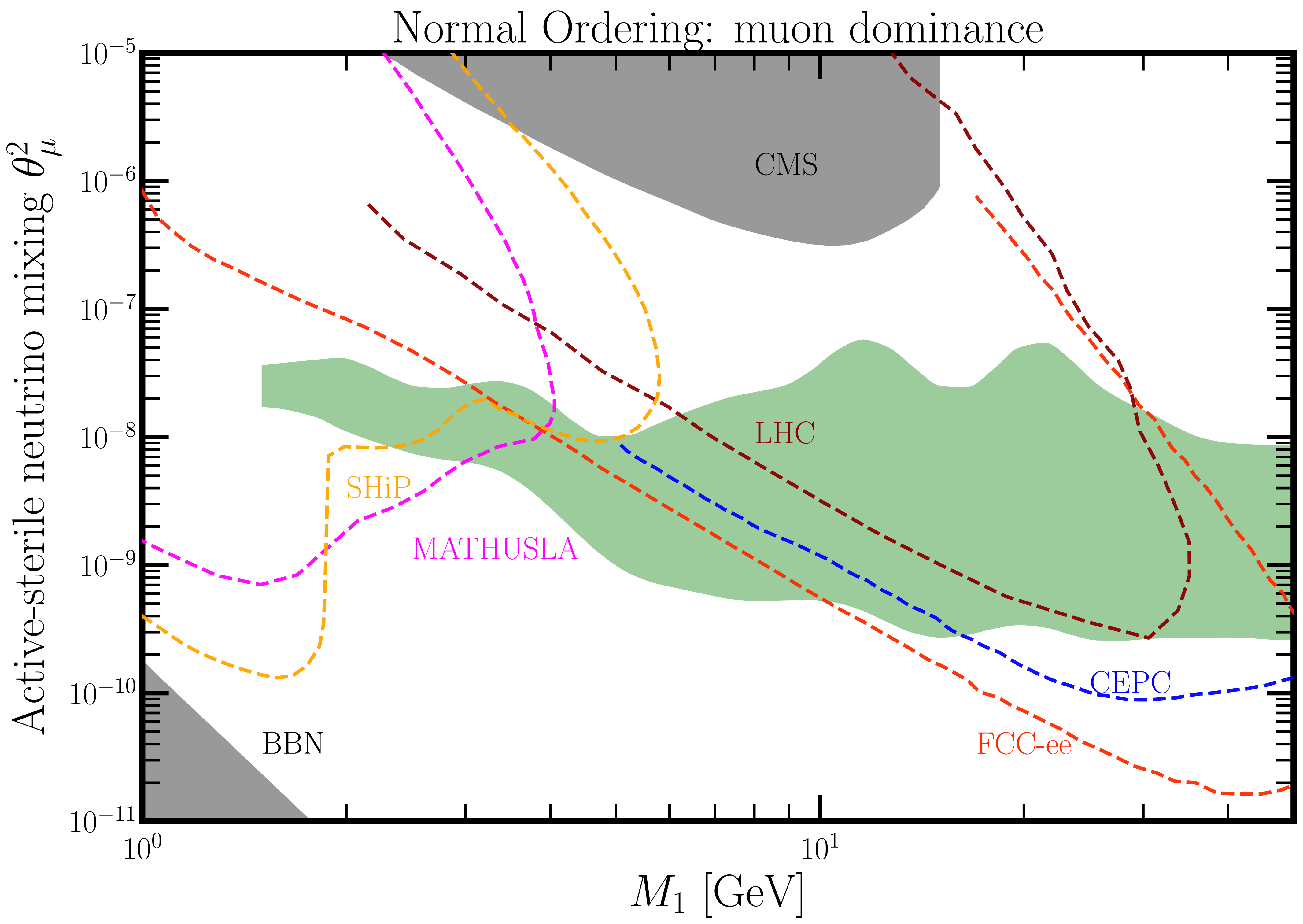
$$m_{\ell_i}^2 = \left(\frac{3g_2^2}{32} + \frac{g_1^2}{32} + \frac{y_{\ell_i}^2}{16}\right) T^2$$

$$\frac{1}{y_\mu^2 - y_e^2} \simeq 10^7$$

- Quasi degeneracy of neutrino masses not required

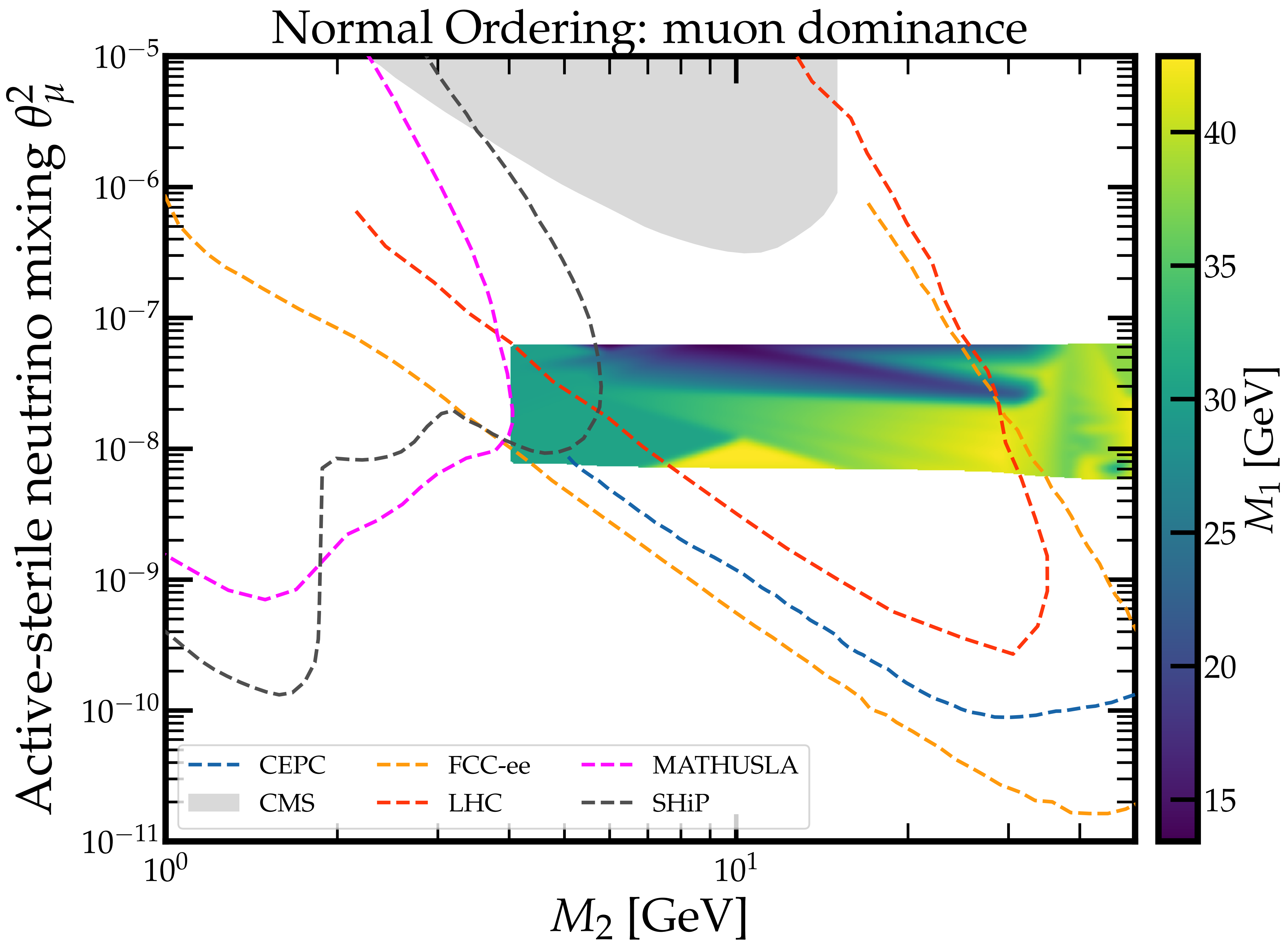
Experimental probes

- Markov Chain Monte Carlo



Produce $Y_B = [10^{-11}, 10^{-10}]$, where $Y_B^{obs} \approx 8.75 \times 10^{-11}$

- Deep Neural Network (machine learning)



Produce $Y_B = [Y_B^{obs}/2, 2Y_B^{obs}]$

Summary

- Oscillations from massless thermal (quasi)particles can occur in a thermal plasma—A proper technique: flavor-covariant non-thermal quantum field theory
- Thermal lepton oscillations can boost leptogenesis to explain the baryon asymmetry of the universe—open a new channel in the well-motivated type-I seesaw framework
- The *maximal* effect of SM lepton oscillations in leptogenesis at finite temperatures leads to a new mechanism—*quadratic leptogenesis*—that depends only on small Yukawa couplings at quadratic order

Thank you!