Model-independent methods in Boundary & Interface

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[JHEP 01 (2023) 108]

[JHEP 11 (2023) 126]

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[Phys.Rev.Lett. 135 (2025) 6, 061603]

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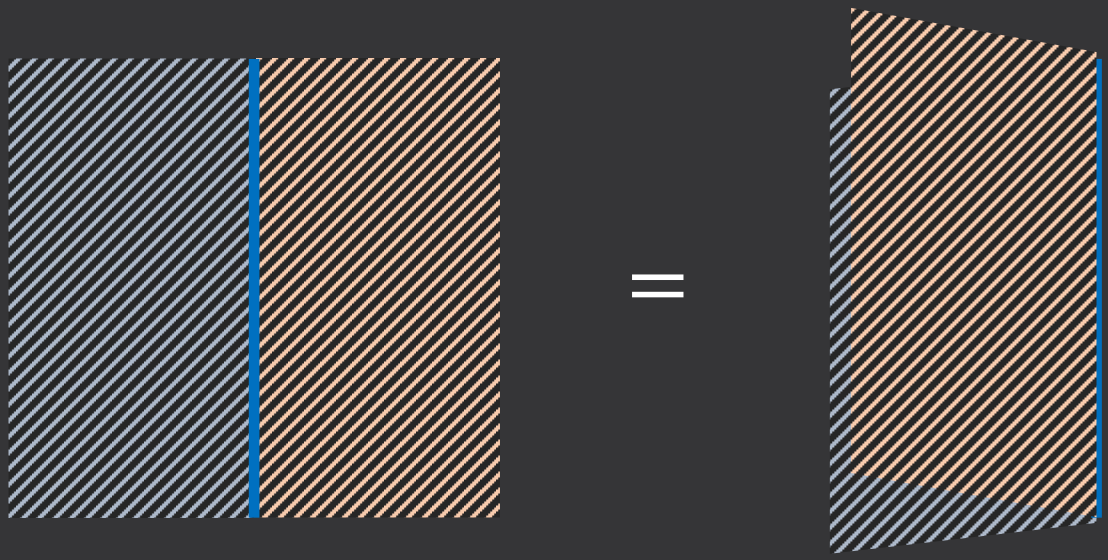
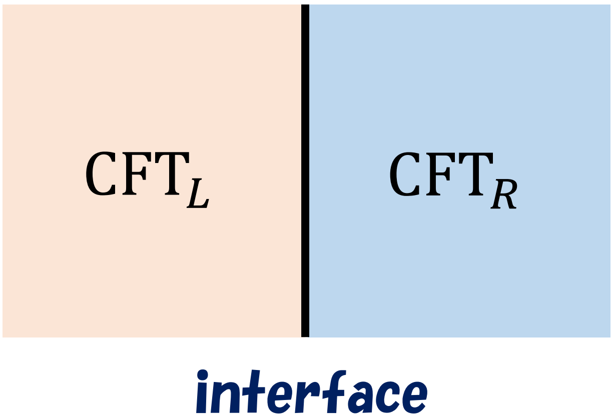
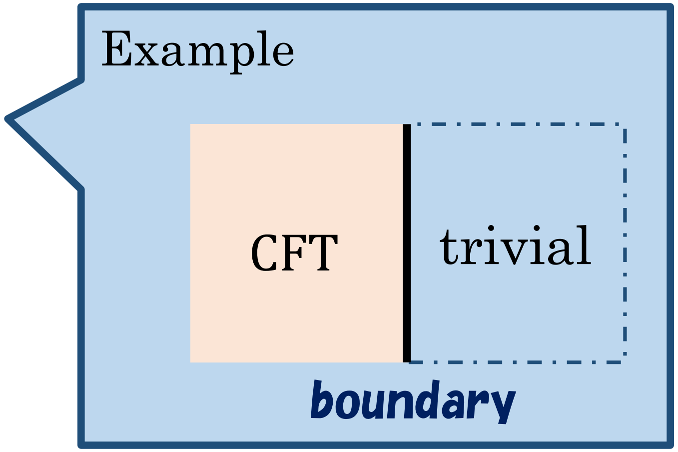


Introduction

Why Interface?



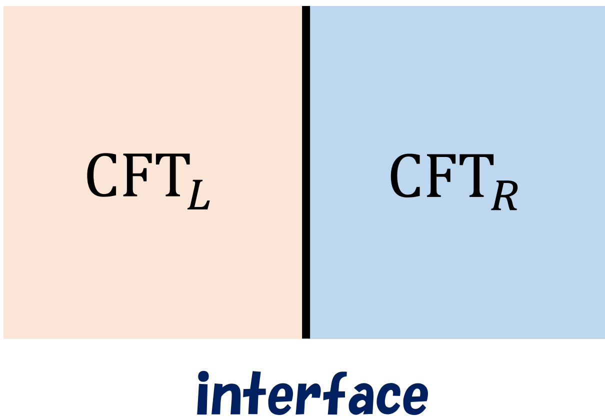


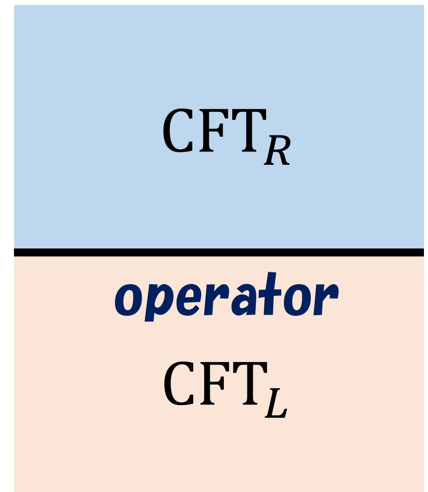
<u>Interface</u> glues two (possibly different) theories

including boundary & defect

⇒predict the effect of boundaries in real systems

Why Interface?



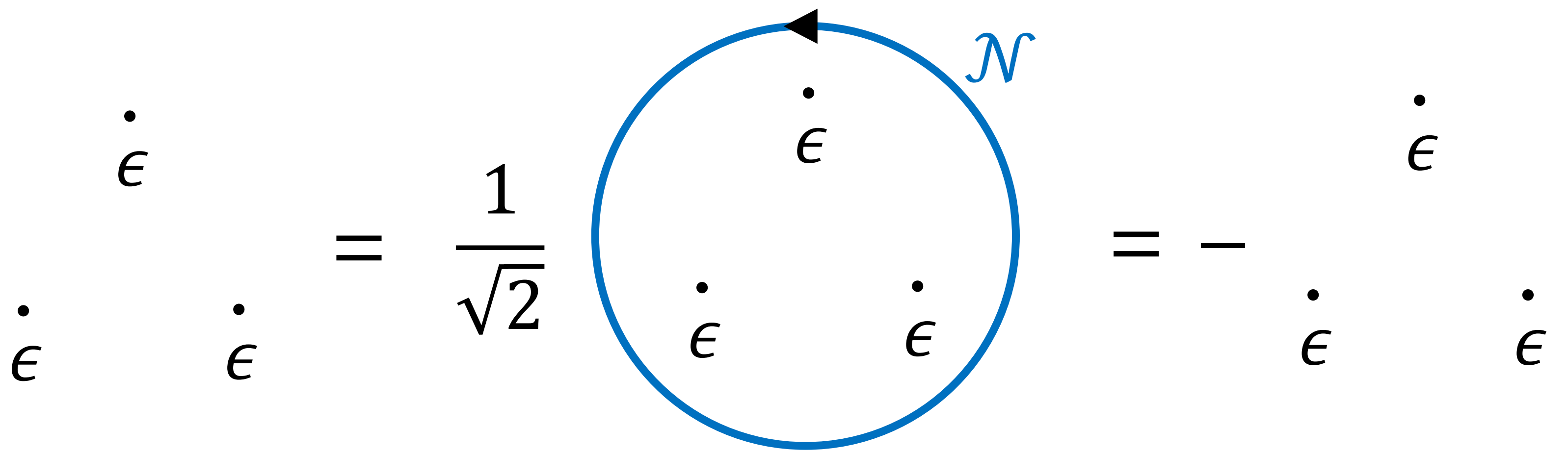


<u>Interface</u> glues two (possibly different) theories.

viewed as an **<u>operator</u>** acting on Hilbert space

⇒Non-local probe, such as Entanglement Entropy

Why Interface?



(Generalized) Ward-Takahashi identity

⇒**<u>new selection rule</u>** for correlation functions

$$\langle \epsilon\epsilon\epsilon \cdots \rangle = 0 \qquad \text{if \# of } \epsilon \text{ is odd}$$

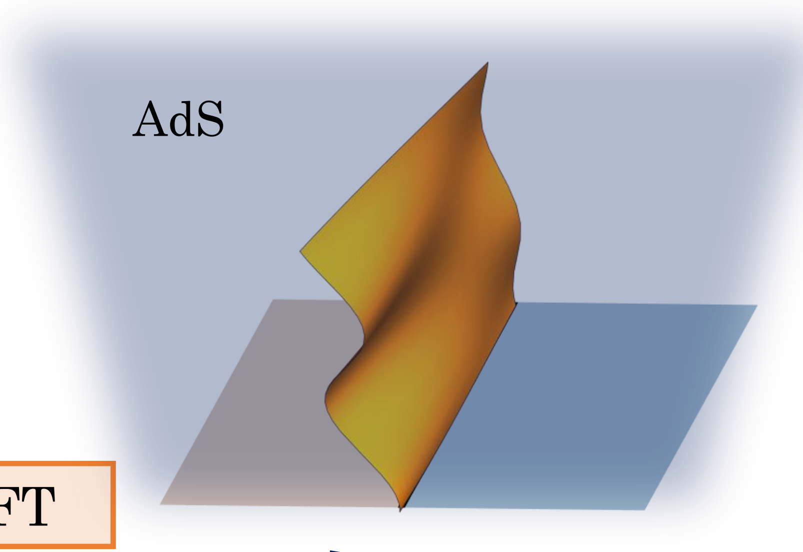
developed as **<u>Generalized symmetry</u>** [Gaiotto]

Why Interface?

interface

=

AdS/CFT



brane

Holographic dual of ICFT involves **brane**

⇒ holographic approach to quantum gravity

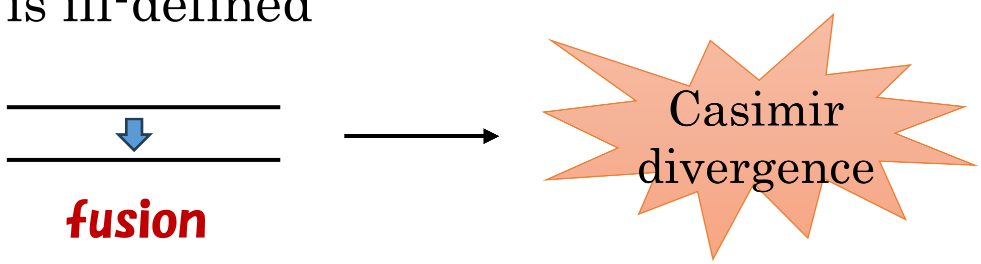
- understanding braneworld holography
- providing tractable model of QG (with brane)
  (i.e., island model)

Difficulty in Interface

- Interface breaks a symmetry

  ⇒ hard to calculate correlation functions
  ⇒ classification is not well-understood at all

- Fusion is ill-defined



  ⇒ algebraic approach is also hard

> Remark:
> **Topological** interface is an exception, since it has no Casimir divergence.
> ⇒ nice structure, i.e., fusion category

Difficulty in Interface

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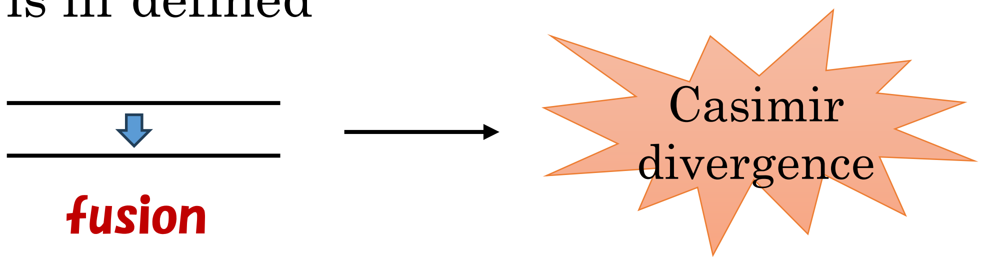

  ⇒ algebraic approach is also hard

- Solvable concrete examples are very limited

  ⇒ hard to conjecture universal formula from concrete examples

Introduction

Summary of current situation:

Current studies of interfaces really utilize some **specialized** properties, such as topological, free, etc.

Our goal in this talk:

Find **model-independent** methods for studying interfaces

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Goal in the future

- New classification of interfaces
- New analytic tools?
  Topological ⇒generalized symmetry

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Current studies of interfaces really utilize some **specialized** properties, such as topological, free, etc.

Our goal in this talk:

Find **model-independent** methods for studying interfaces

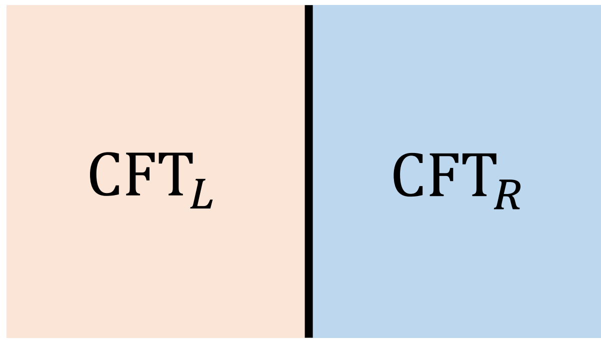
Goal in the future

- New classification of interfaces

- New analytic tools?
  Topological ⇒generalized symmetry
  Non-topological ⇒ “***generalized***” generalized symmetry

🤣

Conformal Interface



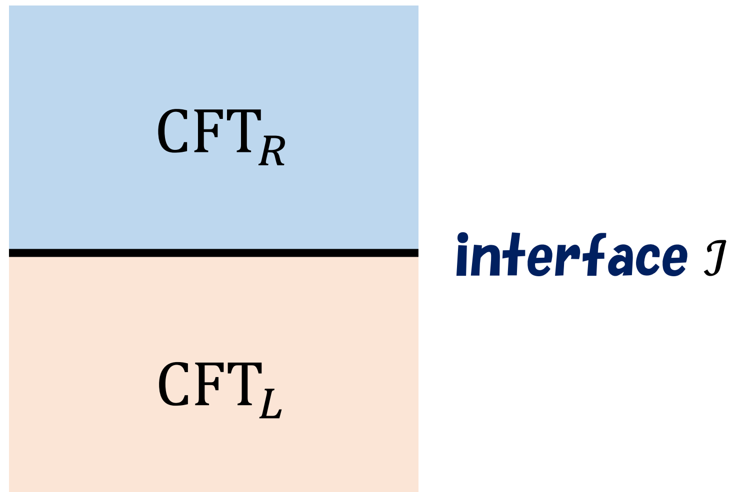
interface $\mathcal{J}$

Assumption

No absorption & emission of energy on $\mathcal{J}$:

$$T^{(L)} - \bar{T}^{(L)} = T^{(R)} - \bar{T}^{(R)} \quad \text{on } \mathcal{J}$$

Conformal Interface

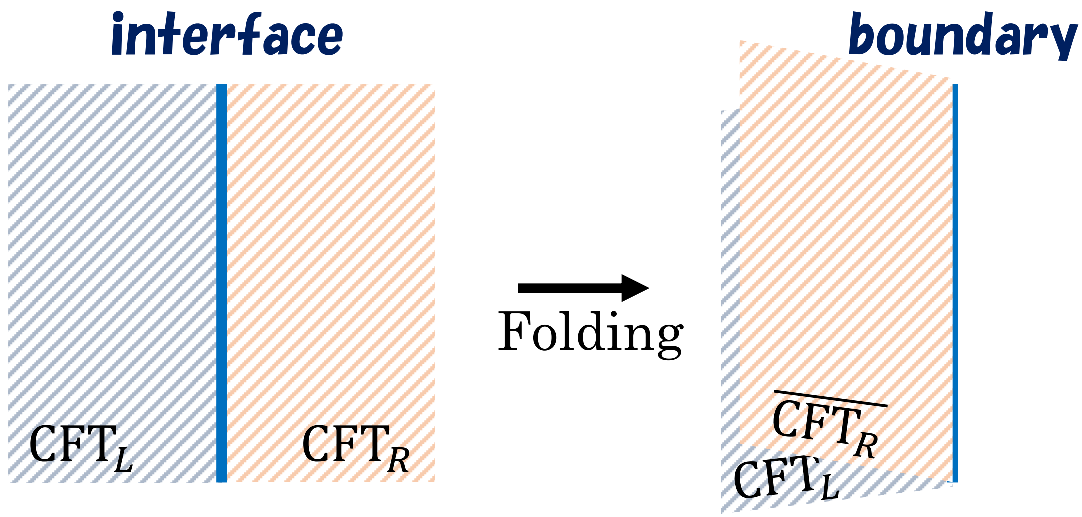


<u>Assumption</u>(Operator formalism)

Map from states in CFT_L to states in CFT_R, satisfying

$$\left(L_n^{(L)} - \bar{L}_{-n}{}^{(L)}\right)\mathcal{J} = \mathcal{J}\left(L_n^{(R)} - \bar{L}_{-n}{}^{(R)}\right)$$

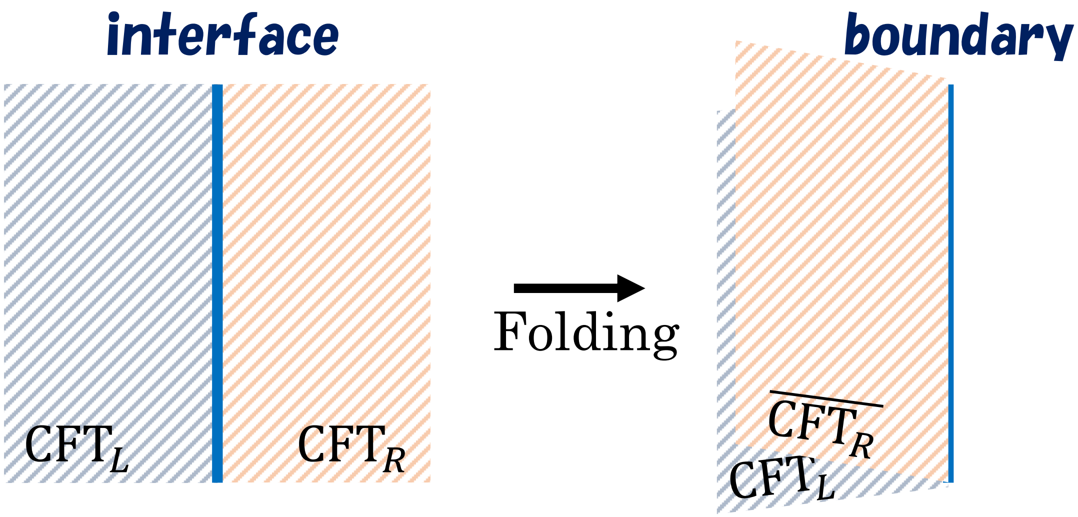
$\mathrm{ICFT} \to \mathrm{BCFT}$



<u>Alternative Interpretation of Assumption</u>

$$T^{(L)} - \bar{T}^{(L)} = T^{(R)} - \bar{T}^{(R)}$$
$$\Updownarrow$$
$$T^{(L)} + \bar{T}^{(R)} = \bar{T}^{(L)} + T^{(R)}$$

ICFT → BCFT

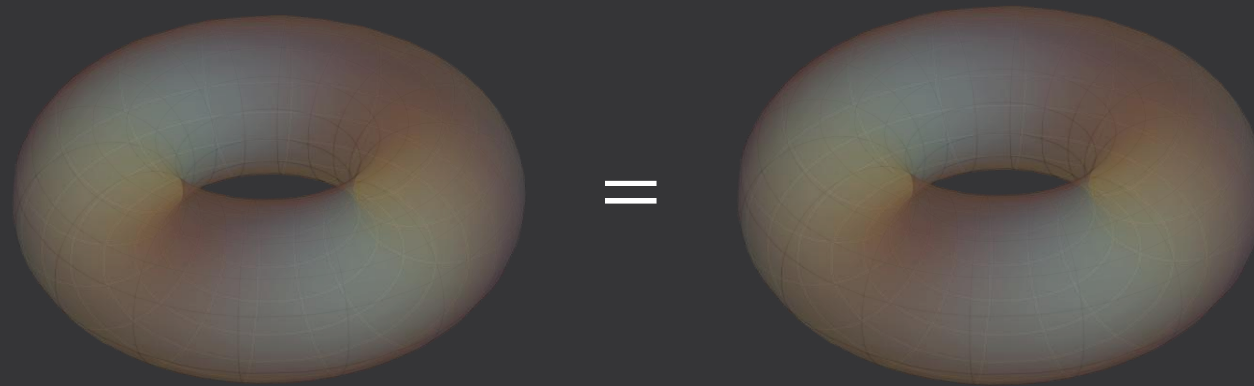


Alternative Interpretation of Assumption

Define $T_{\rm tot}$ as stress tensor of product CFT, then

$$T_{\rm tot} = \overline{T_{\rm tot}} \quad \text{on } \mathcal{I}$$

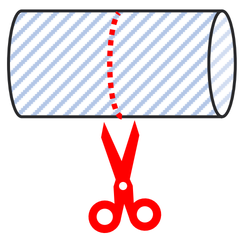
⇒ **Conformal boundary condition**



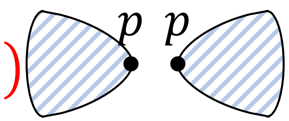
Bootstrapping $BCFT_2$

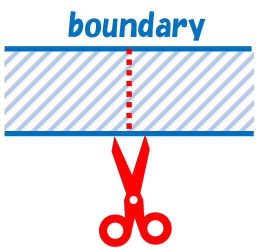
BCFT data

- Spectrum

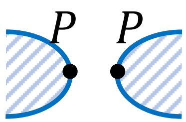


$$= \int d\Delta_p \, \rho_{\mathrm{bulk}}(\Delta_p)$$

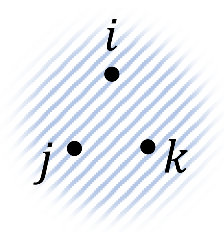




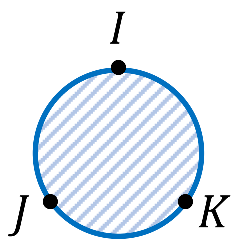
$$= \int d\Delta_P \, \rho_{\mathrm{bdy}}(\Delta_P)$$



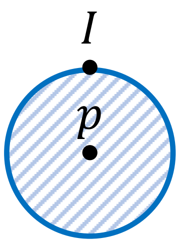
- OPE coef.



C_{ijk}



C_{IJK}

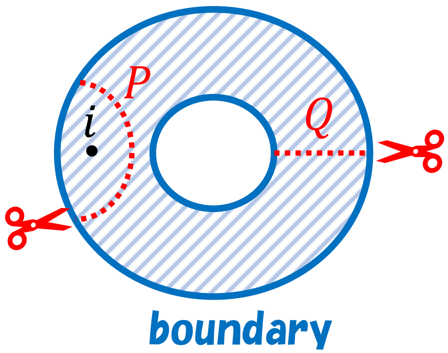


C_{pI}

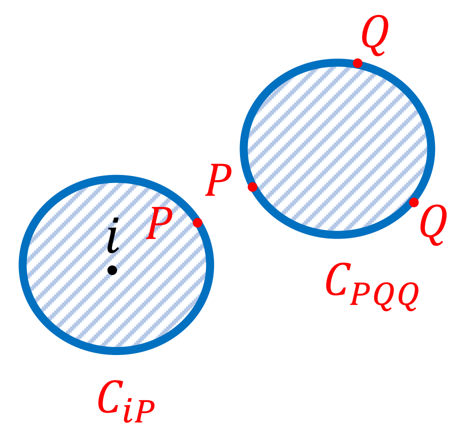
BCFT data

With $\{\rho_{\text{bulk}}(\Delta), \rho_{\text{bdy}}(\Delta), C_{ijk}, C_{iJ}, C_{IJK}\}$,
<u>any</u> correlation function can be calculated

Example:



$$= \int d\Delta_P\, \rho_{\text{bdy}}(\Delta_P) \int d\Delta_Q\, \rho_{\text{bdy}}(\Delta_Q)$$



Solve CFT = Determine $\{\rho_{\text{bulk}}(\Delta), \rho_{\text{bdy}}(\Delta), C_{ijk}, C_{iJ}, C_{IJK}\}$

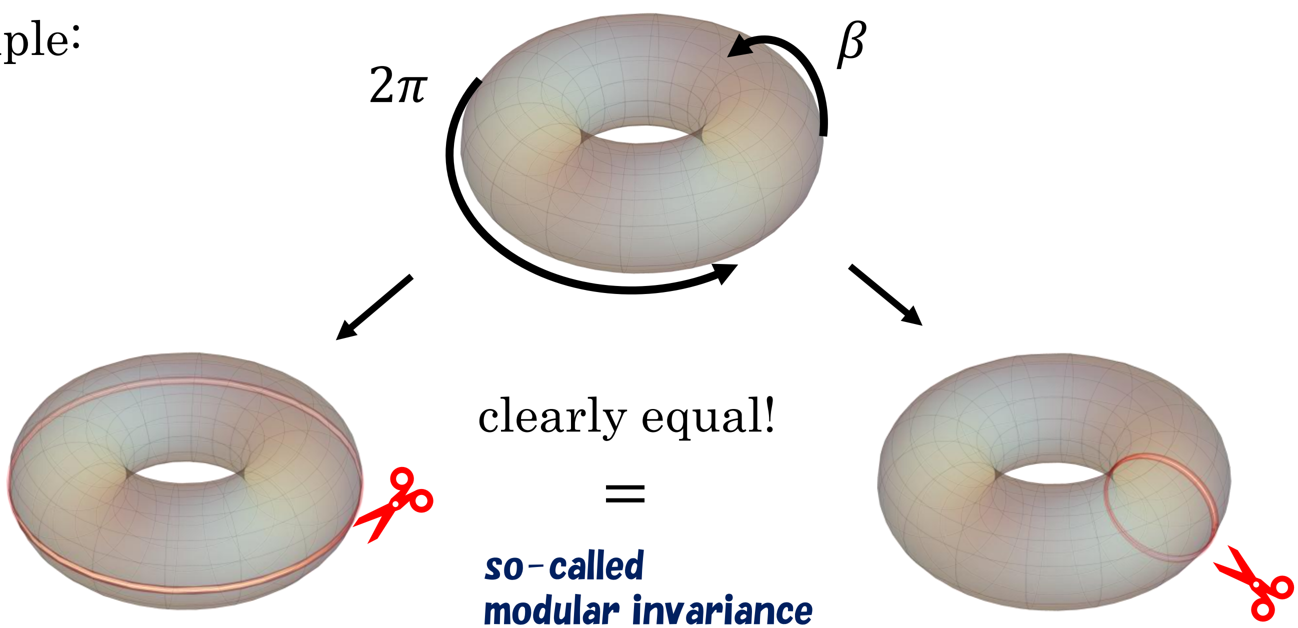
But how?

Conformal Bootstrap

Idea

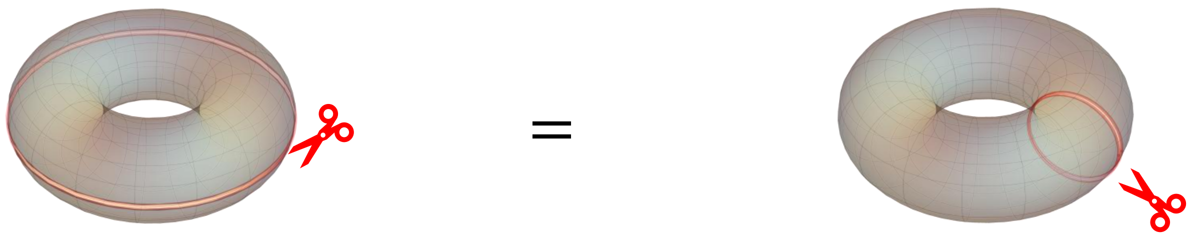
Equation between two different cuts

Example:



$\mathrm{Tr}\, e^{-\beta H}$ $=$ $\mathrm{Tr}\, e^{-\frac{(2\pi)^2}{\beta} H}$

Conformal Bootstrap



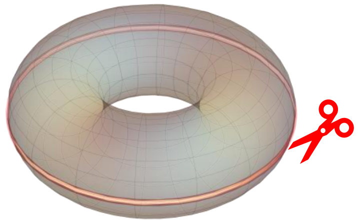
How to solve?

To solve it, we generally need **some assumptions**, such as low-energy spectrum, number of irreps, etc.

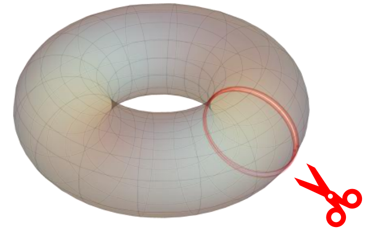
But in particular region, one can solve it **without such assumptions**,
which leads to **universal formula** in CFT.

Let us see this.

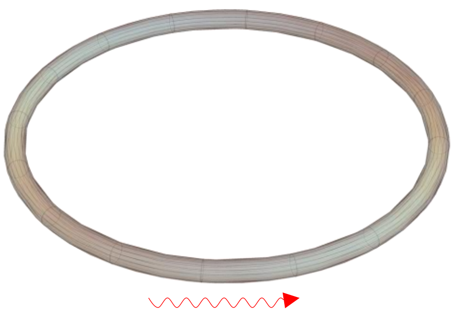
Conformal Bootstrap



$=$



$\downarrow \ \beta \to 0$

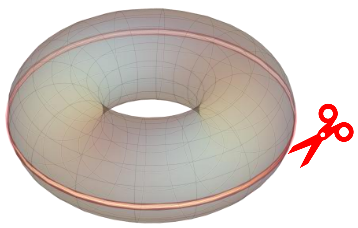


only **vacuum** propagates

$$\mathrm{Tr}\, \mathrm{e}^{-\frac{(2\pi)^2}{\beta} H} \simeq \mathrm{e}^{-\frac{(2\pi)^2}{\beta} E_0}$$

where $E_0 = -\frac{c}{12}$

Conformal Bootstrap



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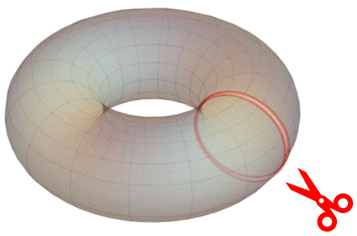


$\parallel$

$\downarrow \; \beta \to 0$

$$\int d\Delta\, \rho_{\text{bulk}}(\Delta) e^{-\beta\left(\Delta - \frac{c}{12}\right)} \simeq \mathrm{e}^{\frac{(2\pi)^2}{\beta}\frac{c}{12}}$$

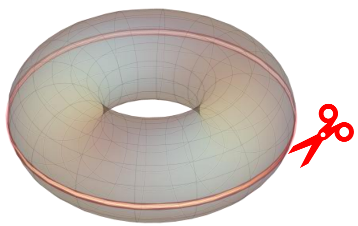
By the inverse Laplace trans., we obtain **universal formula**,

Cardy formula

$$\rho_{\text{bulk}}(\Delta) \simeq \exp\left(2\pi\sqrt{\frac{c}{3}\Delta}\right)$$

Conformal Bootstrap

① different cuts



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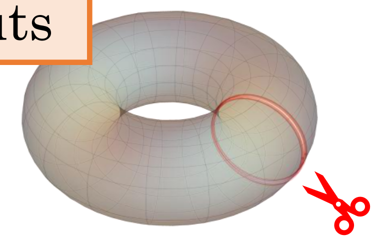

② vacuum approximation

$\|$

$$\int d\Delta\, \rho_{\mathrm{bulk}}(\Delta)\, e^{-\beta\left(\Delta-\frac{c}{12}\right)} \simeq \mathrm{e}^{\frac{(2\pi)^2}{\beta}\frac{c}{12}}$$

③ inverse Laplace transformation

$$\rho_{\mathrm{bulk}}(\Delta) \simeq \exp\left(2\pi\sqrt{\frac{c}{3}\Delta}\right)$$

Conformal Bootstrap

Algorithm to obtain **universal formula**

① equation between two cuts
② vacuum approximation
③ inverse Laplace transformation

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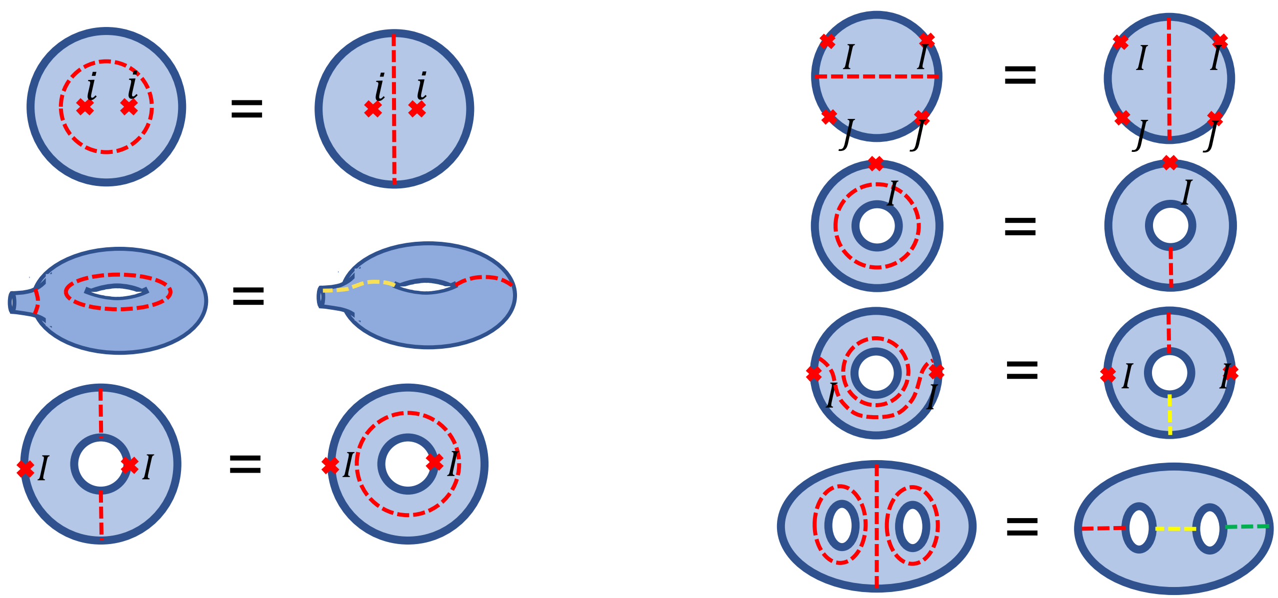
This algorithm can be generalized



Conformal Bootstrap

Algorithm to obtain **universal formula**

① equation between two cuts
② vacuum approximation
③ inverse Laplace transformation

This algorithm can be generalized

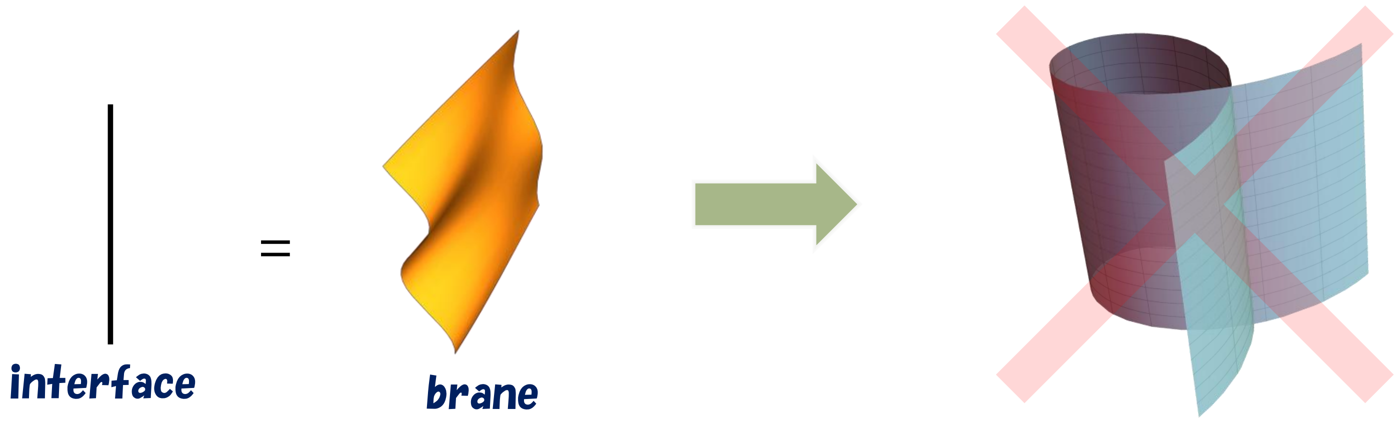
Result [YK], [Numasawa-Tsiares]

In the limit $\max\{h_i\}, \{h_I\} \gg c$,
C_{ijk}, C_{iJ}, C_{IJK} are asymptotically universal

which is generalization of Cardy formula

Application

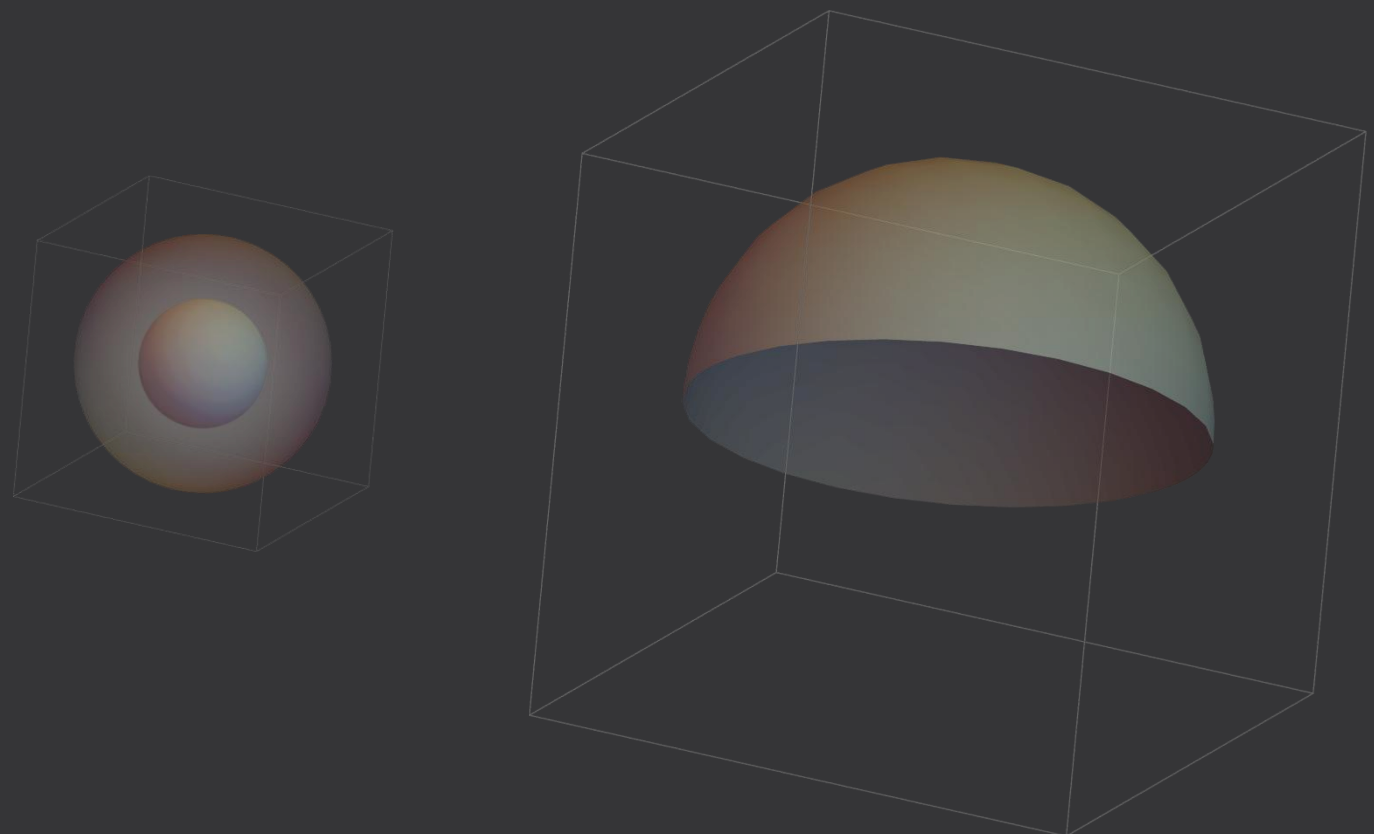
[YK], [YK, Wei]



Implication from **<u>universal formula</u>**

Self-interacted brane is impossible

When attempting to create self-interaction,
black hole is always formed



Bootstrapping BCFT_d

Modular bootstrap in higher d?

$\mathbb{S}^1 \times \mathbb{S}^1 \longrightarrow \mathbb{S}^{d-1} \times \mathbb{S}^1$

modular invariance $\cancel{\longrightarrow}$

In $2d$, the modular invariance gives us the universality,

$$Z(\beta) = Z\left(\frac{(2\pi)^2}{\beta}\right) \simeq e^{\frac{(2\pi)^2}{\beta}\frac{c}{12}}$$

On the other hand, $\mathbb{S}^{d-1} \times \mathbb{S}^1$ dues not have such invariance.
$\Rightarrow$ No similar universality ?

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Hint

One can still find a low-temperature description by

Thermal CFT$_d$ on $\Sigma_{d-1} \times \mathbb{S}^1_\beta$	$\simeq$	Effective gapped QFT on Σ_{d-1} with $m_{gap} \propto 1/\beta$

Modular bootstrap in higher d?

Thermal CFT_d on $\Sigma_{d-1} \times \mathbb{S}^1_\beta$

$\simeq$

Effective gapped QFT on Σ_{d-1} with $m_{gap} \propto 1/\beta$

KK ansatz for background metric on $\Sigma_{d-1} \times \mathbb{S}^1_\beta$,

$$\begin{aligned} ds^2 &= g_{ij}(\vec{x}) dx^i dx^j + e^{2\phi(\vec{x})} d\tau^2 \\ &= e^{2\phi(\vec{x})} \left[\hat{g}_{ij}(\vec{x}) dx^i dx^j + d\tau^2 \right] \end{aligned}$$

$\hat{g} \coloneqq e^{-2\phi}$, and $e^{-2\phi} = \beta^{-2}$

In the $\beta \to 0$ limit, gapped $\Rightarrow$ depend only on background fields

$$Z\left[\Sigma_{d-1} \times \mathbb{S}^1_\beta\right] \to e^{-S_{\mathrm{eff}}[\hat{g}]}$$

Weyl inv. $Z[g, \phi] = Z[\hat{g}, 0]$

Weyl invariance constraints the form,

$$S_{\mathrm{eff}}[\hat{g}] = \int d^{d-1}x \sqrt{\hat{g}} \left(-f + c_1 \hat{R} + \cdots \right)$$

Modular bootstrap in higher d?

Thermal CFT_d on $\Sigma_{d-1} \times \mathbb{S}^1_\beta$

$\simeq$

Effective gapped QFT on Σ_{d-1} with $m_{gap} \propto 1/\beta$

Higher derivative terms are suppressed

$$S_{\mathrm{eff}}[\hat{g}] = \int d^{d-1}x \sqrt{\hat{g}}(-f + c_1 \hat{R} + \cdots)$$

Note: $\hat{g} = e^{-2\phi} g$, and $e^{-2\phi} = \beta^{-2}$

$\hat{R}$ is dimensionless.
If making mass dimension explicit,

$$\hat{R} = \beta^2 R$$

$\Rightarrow$ suppressed by β^2

$\Rightarrow$ only a few parameters f,

Note: Wilson coefficients ar

$\Rightarrow$ **<u>universality</u>**

Modular bootstrap in higher d?

Thermal CFT_d on $\Sigma_{d-1} \times \mathbb{S}^1_\beta$ $\simeq$ Effective gapped QFT on Σ_{d-1} with $m_{gap} \propto 1/\beta$

Higher derivative terms are suppressed

$$S_{\mathrm{eff}}[\hat{g}] = \int d^{d-1}x \sqrt{\hat{g}}\left(-f + c_1 \hat{R} + \cdots\right)$$

Note: $\hat{g} = e^{-2\phi} g$, and $e^{-2\phi} = \beta^{-2}$

$\Rightarrow$ only a few parameters $f, c_1, \cdots$ characterize $Z\left[\Sigma_{d-1} \times \mathbb{S}^1_\beta\right]$

Note: Wilson coefficients are independent of background metric

$\Rightarrow$ **<u>universality</u>**

Modular bootstrap in higher d?

Alternative derivation of Cardy formula

$$S_{\text{eff}}[\hat{g}] \simeq -\frac{f}{\beta^{d-1}}\text{vol}(\Sigma_{d-1})$$

In $2d$,

$$f = \frac{2\pi c}{12}, \qquad c_1, \cdots = 0$$

Thus, we obtain

$$\int d\Delta\, \rho(\Delta)\, e^{-\beta\Delta} \simeq e^{-S_{\text{eff}}[\hat{g}]}$$

Modular bootstrap in higher d?

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In $2d$,

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Thus, we obtain

$$\int d\Delta\, \rho(\Delta)\, e^{-\beta\Delta} \simeq \exp\left(\frac{2\pi}{\beta}\frac{2\pi c}{12}\right)$$

By the inverse Laplace transformation,

$$\log \rho(\Delta) \simeq 2\pi\sqrt{\frac{c}{3}\Delta}$$

Cardy formula in CFT$_d$

[Benjamin, Lee, Ooguri, Simmons-Duffin]

Consider CFT$_d$ on $\mathbb{S}^{d-1} \times \mathbb{S}^1_\beta$, we have

$$S_{\text{eff}}[\hat{g}] \simeq -\frac{f}{\beta^{d-1}} \text{vol}\left(\mathbb{S}^{d-1}\right)$$

<u>EFT approach</u> remains valid even in higher dimensions,

$$\int d\Delta\, \rho(\Delta)\, e^{-\beta\Delta} \simeq e^{-S_{\text{eff}}[\hat{g}]}$$

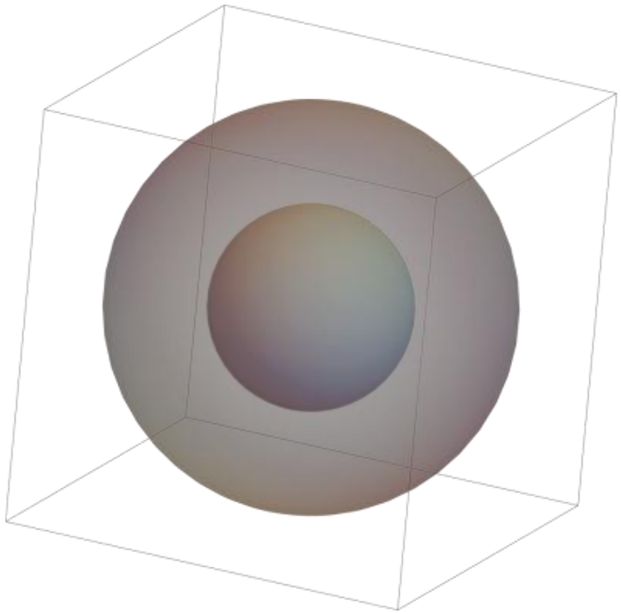
By the inverse Laplace transformation,

$$\log \rho(\Delta) \simeq f^{\frac{1}{d}} \frac{d}{d-1} \left((d-1)\text{vol}\left(\mathbb{S}^{d-1}\right)\right)^{\frac{1}{d}} \; \Delta^{\frac{d-1}{d}}$$

Note: The sub-leading term is suppressed by $\Delta^{-\frac{2}{d}}$,
and its coefficient is determined by f and c_1.

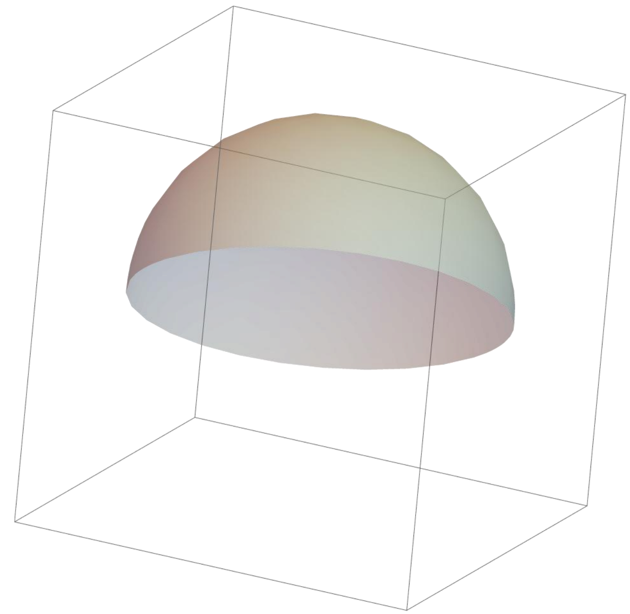
Cardy formula in BCFT$_d$

Two ways to place boundaries on $\mathbb{S}^{d-1} \times \mathbb{S}^1_\beta$



[Diatlyk, Khanchandani, Popov, Wang]

sandwiched by $\mathbb{S}^{d-1}$ boundaries

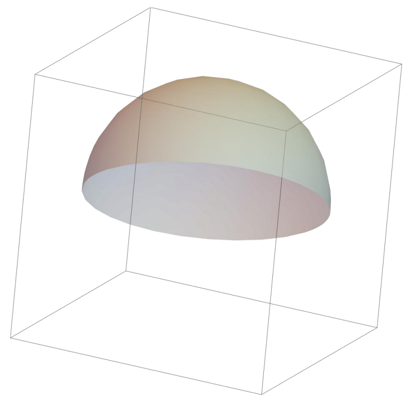


[Kusuki, Ooguri, Pal]

$\mathbb{S}^{d-1}$ with $\mathbb{S}^{d-2}$ boundary

Cardy formula in BCFT$_d$

[Kusuki, Ooguri, Pal]



Consider BCFT$_d$ on $\mathbb{S}_+^{d-1} \times \mathbb{S}_\beta^1$

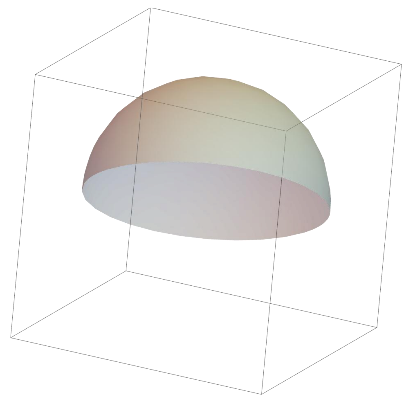
$$S_{\mathrm{eff}}[\hat{g}] = \int d^{d-1}x \sqrt{\hat{g}}\left(-f + c_1 \hat{R} + \cdots\right)$$
$$+ \int d^{d-2}x \sqrt{\hat{\sigma}}\left(-g + b_1 \hat{K} + \cdots\right)$$

boundary contribution

σ: induced metric on the boundary
K: trace of extrinsic curvature $K_{ab} \coloneqq \nabla_a n_b$
n: unit vector normal to the boundary

Cardy formula in BCFT$_d$

[Kusuki, Ooguri, Pal]



β expansion of $S_{\text{eff}}[\hat{g}]$ includes

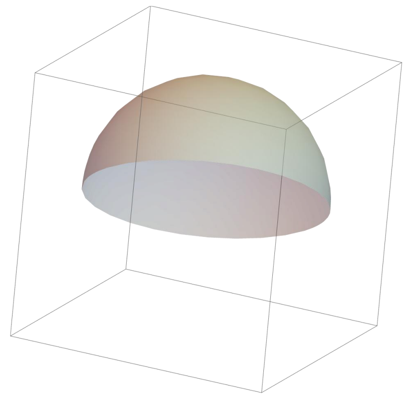
- Bulk leading: $-f\,\dfrac{\mathrm{vol}(\mathbb{S}^{d-1})}{2\beta^{d-1}}$
- Bulk sub-leading: $c_1\,\dfrac{(d-2)(d-1)\mathrm{vol}(\mathbb{S}^{d-1})}{2\beta^{d-3}}$
- Boundary leading: $-g\,\dfrac{\mathrm{vol}(\mathbb{S}^{d-2})}{\beta^{d-2}}$

related to g-function in $2d$

$\Rightarrow$ Next leading comes from the **boundary leading**

Cardy formula in BCFT_d

[Kusuki, Ooguri, Pal]



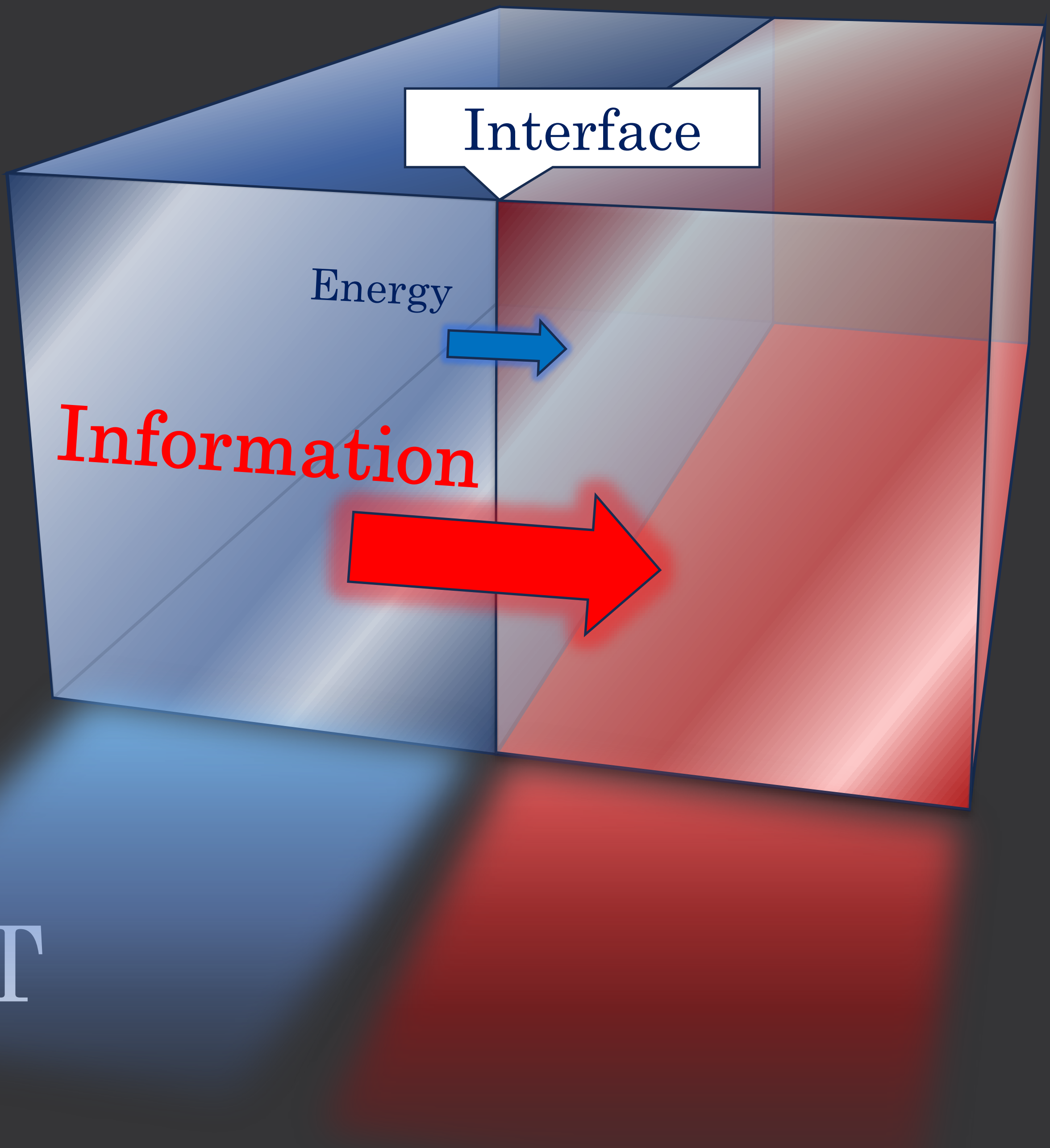
Cardy Formula in BCFT_d

$$\log\rho(\Delta) \simeq f^{\frac{1}{d}}\frac{d}{d-1}\left(\frac{(d-1)\mathrm{vol}\left(\mathbb{S}^{d-1}\right)}{2}\right)^{\frac{1}{d}}\Delta^{\frac{d-1}{d}}$$

$$+f^{\frac{2-d}{d}}g\,\mathrm{vol}\left(\mathbb{S}^{d-2}\right)\left(\frac{(d-1)\mathrm{vol}\left(\mathbb{S}^{d-1}\right)}{2}\right)^{\frac{2-d}{d}}\Delta^{\frac{d-2}{d}}$$

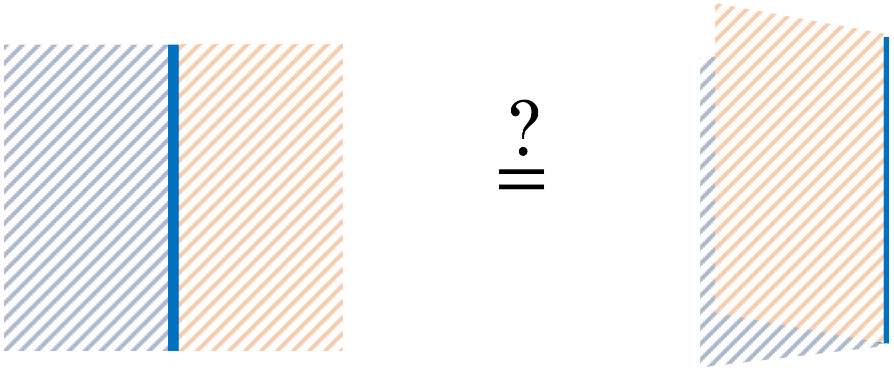
$$+o\left(\Delta^{\frac{d-3}{d}}\right)$$

boundary contribution



AdS/ICFT

$\mathrm{ICFT} \stackrel{?}{=} \mathrm{BCFT}$



BCFT picture obscures unique properties of ICFT

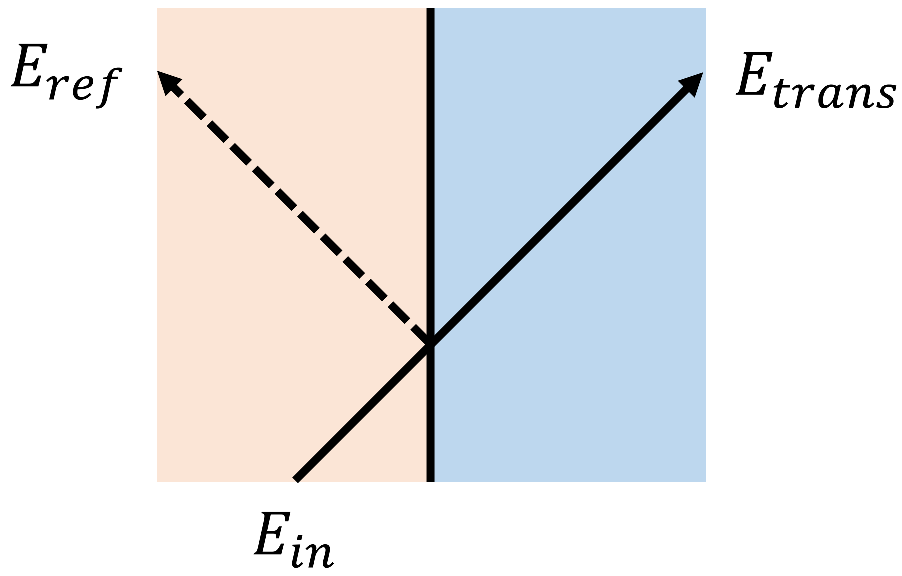
- **Energy transmission** from L to R
- **Entanglement** between L & R

which are encoded in BCFT but make sense only in ICFT

Transmission Coefficient

[Quella, Runkel, Watts]

$$\mathcal{T} := \frac{E_{trans}}{E_{in}}$$



Transmission coef. can be extracted by

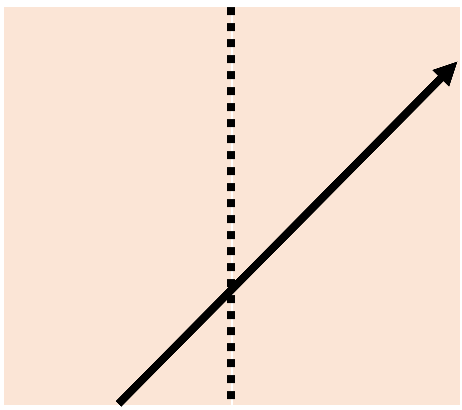
[Meineri, Penedones, Rousset]

$$\mathcal{T} = \frac{c_{LR}}{c_{L/R}}, \qquad \langle T_L(z) T_R(w) \rangle = \frac{c_{LR}}{2(z-w)^4}$$

Transmission Coefficient

[Quella, Runkel, Watts]

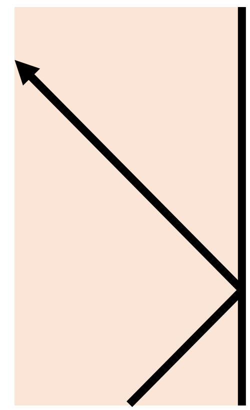
Two trivial cases



topological

$$\mathcal{T} = 1$$

max

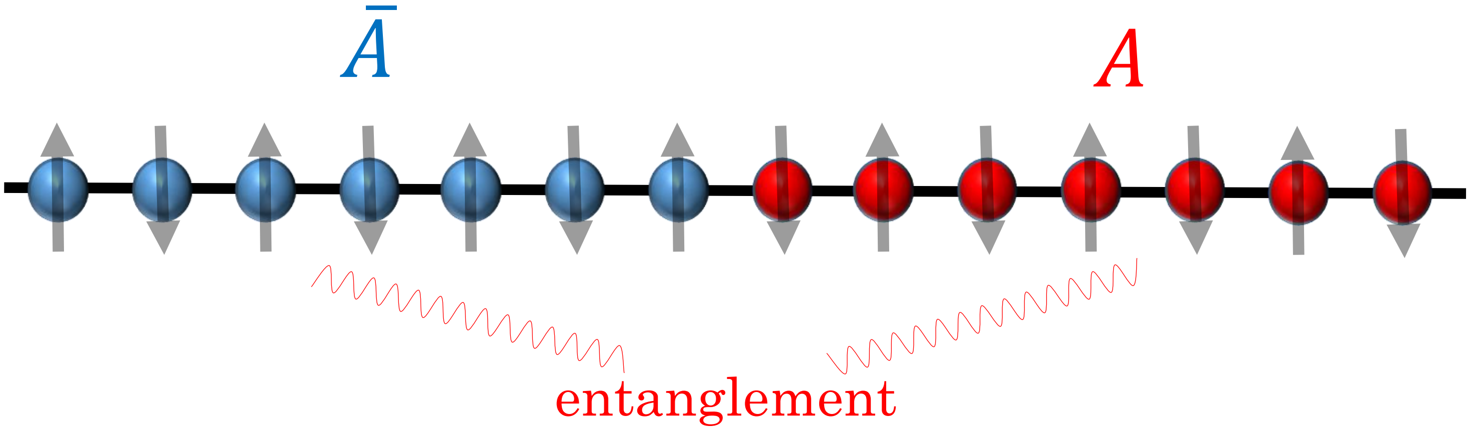


boundary

$$\mathcal{T} = 0$$

min

Entanglement



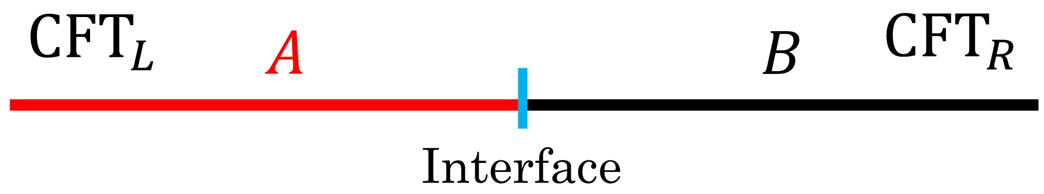
Definition

$$S_A^{(n)} \coloneqq \frac{1}{1-n} \log \mathrm{Tr}\, \rho_A^n$$

$\rho_A \coloneqq \mathrm{Tr}_{\bar{A}}\, \rho$

- Entanglement Entropy $\coloneqq \lim_{n\to 1} S_A^{(n)}$
- Measuring the amount of entanglement (# of EPR pairs)

Entanglement



Important quantity in ICFT

Amount of entanglement (# of EPR pairs) across interface

$$S_A \equiv -\mathrm{tr}\rho_A \log \rho_A = \frac{c_{\mathrm{eff}}}{3} \log \frac{|A|}{\epsilon}$$

[Sakai, Satoh]

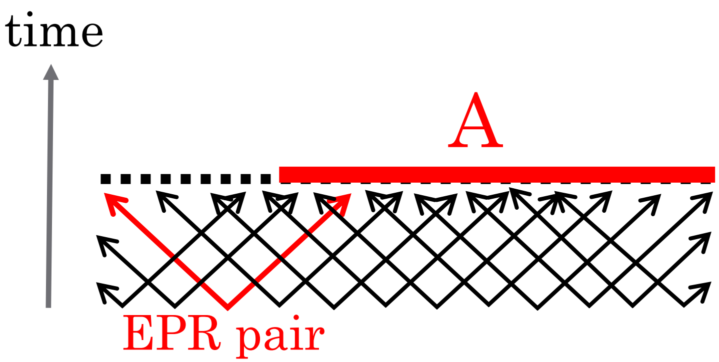
Alternative interpretation

$$\mathcal{T}^{\mathrm{info}}_{L/R} = \frac{\text{transmitted information}}{\text{incident information}} = \frac{c_{\mathrm{eff}}}{c_{L/R}}$$

[Wen, Wang, Ryu]

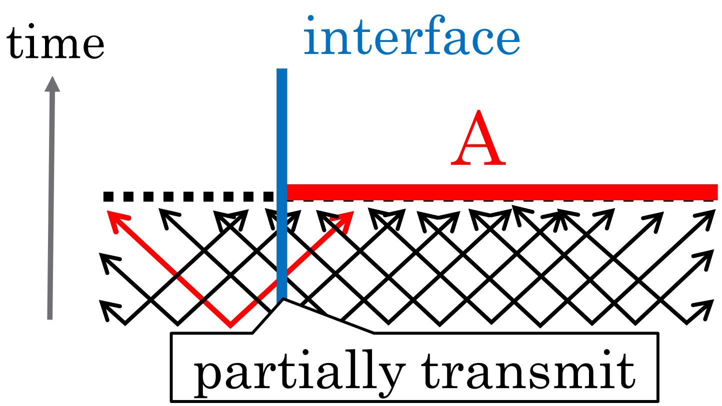
c_{eff} as Information Transmission

[Wen, Wang, Ryu], [Karch, YK, Ooguri, Sun, Wang]



EE under global quench

$$S_A = \#\, c\, t$$



EE under global quench

$$S_A = \#\, c_{\mathrm{eff}}\, t$$

$\Rightarrow \dfrac{c_{\mathrm{eff}}}{c} =$ information transmission

Issue

BCFT picture obscures unique properties of ICFT

- $\mathcal{T}$: **Energy transmission** from L to R
- c_{eff} : **Entanglement** between L & R

which are encoded in BCFT but make sense only in ICFT

Problem ☹

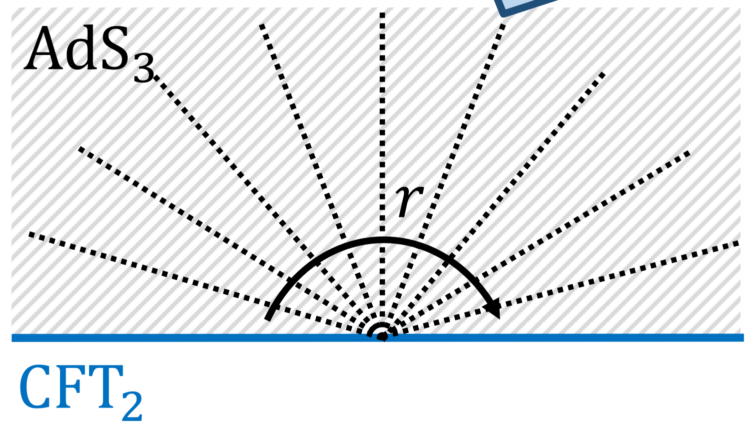
Hard to calculate both $\mathcal{T}$ and c_{eff}

Only computed in a **few examples** by model-specific methods

⇒ How can we overcome this problem?

AdS/ICFT

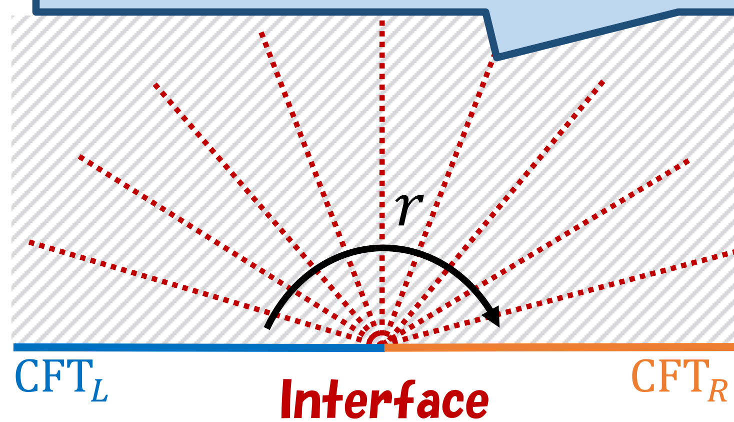
black dotted line = AdS_2 slice



AdS/CFT

$$ds^2 = \cosh r \frac{dx^2 - dt^2}{x^2} + dr^2$$

each AdS_2 slice has a different warp factor $A(r)$



AdS/ICFT

$$ds^2 = e^{2A(r)} \frac{dx^2 - dt^2}{x^2} + dr^2$$

Details of interface is encoded in $A(r)$

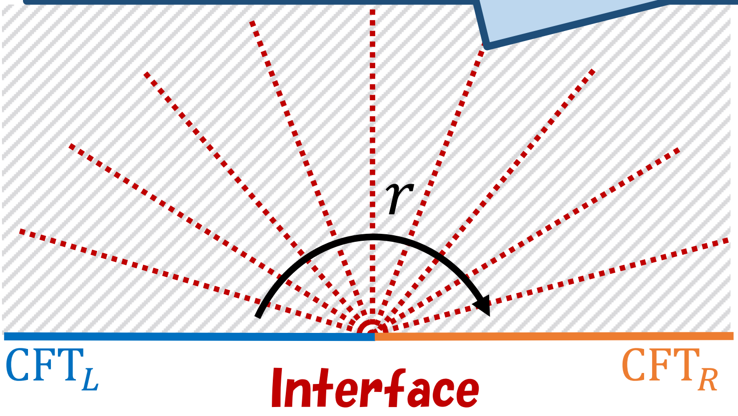
AdS/ICFT

AdS/ICFT

$$ds^2 = e^{2A(r)} \frac{dx^2 - dt^2}{x^2} + dr^2$$

Details of interface is encoded in $A(r)$

each AdS_2 slice has a different warp factor $A(r)$



Example:

- AdS: $e^{2A(r)} = \cosh r$
- Janus: $e^{2A(r)} = \frac{1}{2}\left(1 + \sqrt{1 - 2\gamma^2} \cosh 2r\right)$

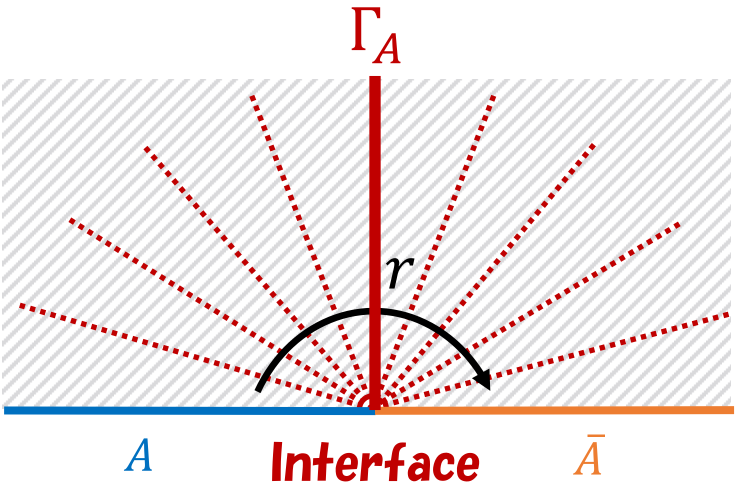
Generic interface is obtained from generic $A(r)$

$\Rightarrow$ universal information of interface is encoded as universal property of $A(r)$

AdS/ICFT

AdS/ICFT

$$ds^2 = e^{2A(r)} \frac{dx^2 - dt^2}{x^2} + dr^2$$



Gravity dual of EE (Ryu-Takayanagi formula)

$$S_A = \min_{\substack{\Gamma_A \\ \partial\Gamma_A = \partial A}} \left(\frac{\mathrm{Area}(\Gamma_A)}{4G_N} \right)$$

still works in AdS/ICFT

Universal Bounds

- Holographic c_{eff}

  [Ryu, Takayanagi]

- Holographic c_{LR}

  [Bachas, Baiguera, Chapman, Policastro, Schwartzman]

Point: Both are determined by $A(r)$

- Generalized holographic c theorem (NEC)

$$A''(r) \leq e^{-2A(r)}$$

$\Rightarrow$ upper bound on c_{eff} [Karch, YK, Ooguri, Sun, Wang '23]

- Comparison between holographic c_{eff} & c_{LR}

$\Rightarrow$ upper bound on c_{LR} [Karch, YK, Ooguri, Sun, Wang '24]

Universal Bounds

[Karch, YK, Ooguri, Sun, Wang]× 2

- Holographic c_{eff}
  [Ryu, Takayanagi]
- Holographic c_{LR}
  [Bachas, Baiguera, Chapman, Policastro, Schwartzman]

Point: Both are determined by $A(r)$

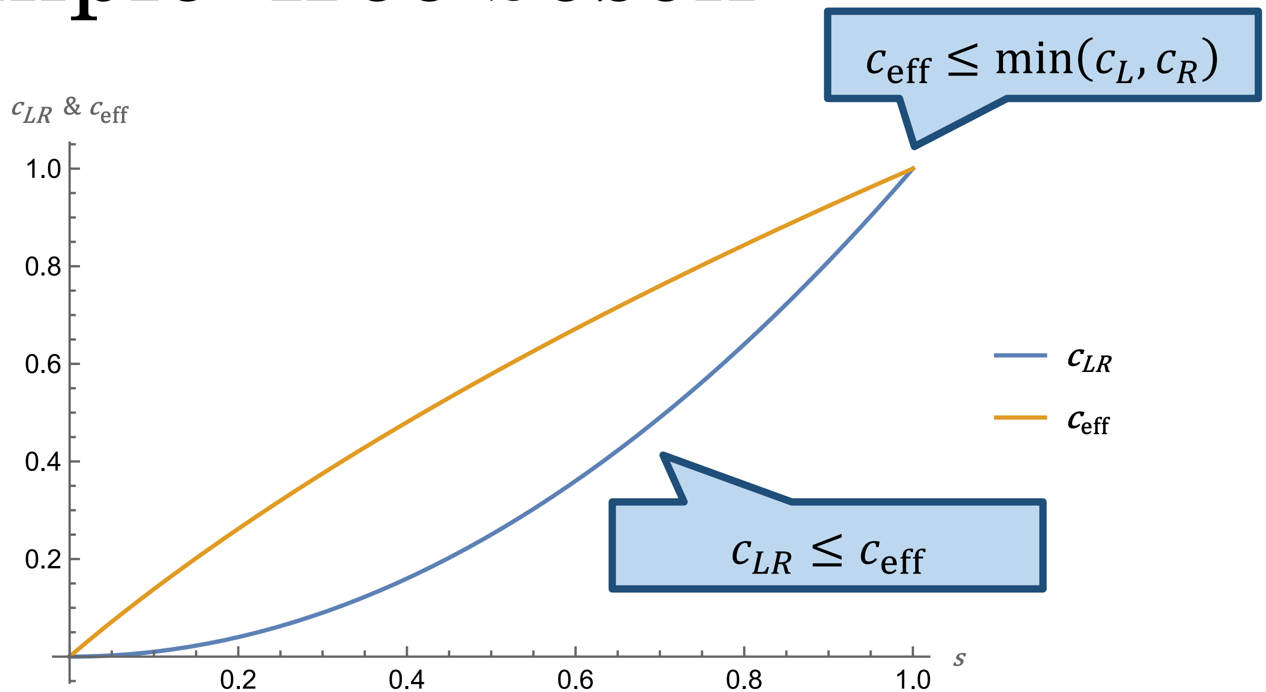
Result

$$0 \leq c_{LR} \underset{①}{\leq} c_{\text{eff}} \underset{②}{\leq} \min(c_L, c_R)$$

① The amount of energy transmission can never exceed the amount of information transmission

② Any interface decreases entanglement

Example: free boson

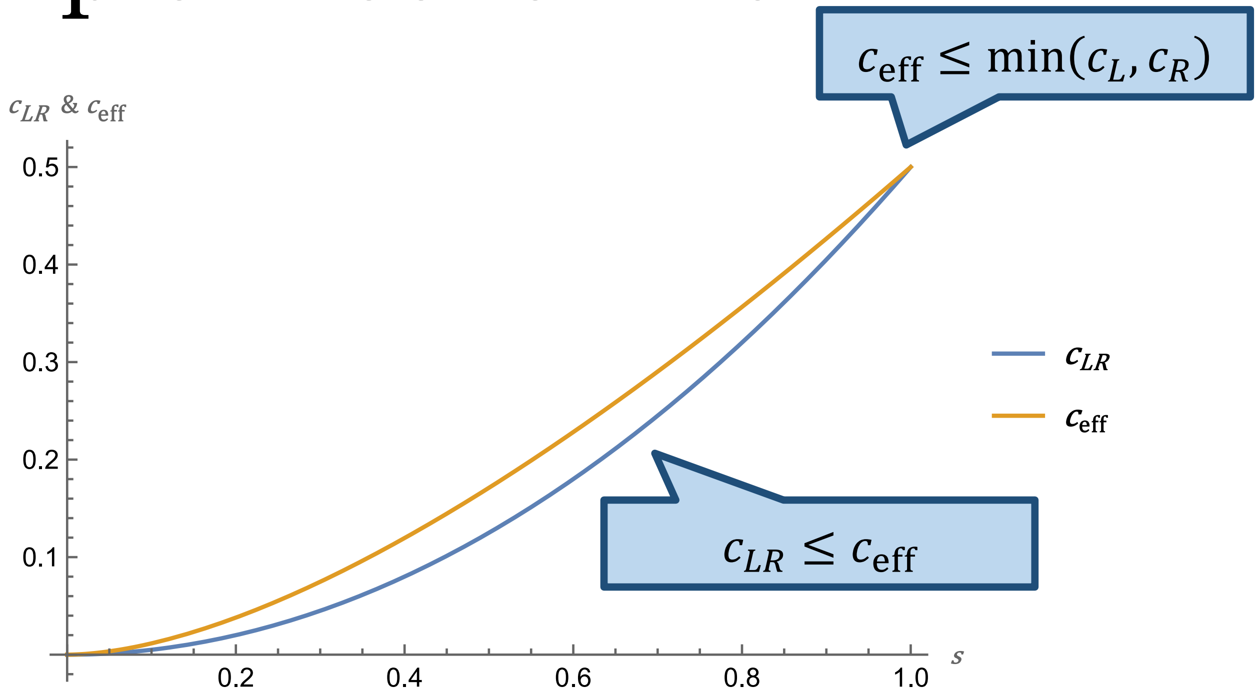


c_{LR} & c_{eff} are parametrized by one parameter $s \in (0,1)$;

$c_{LR} = s^2$

$$c_{\text{eff}} = \frac{1}{2} + s + \frac{3}{\pi^2}\left((s+1)\log(s+1)\log s \; + (s-1)\mathrm{Li}_2(1-s) + (s+1)\mathrm{Li}_2(-s)\right)$$

Example: free fermion

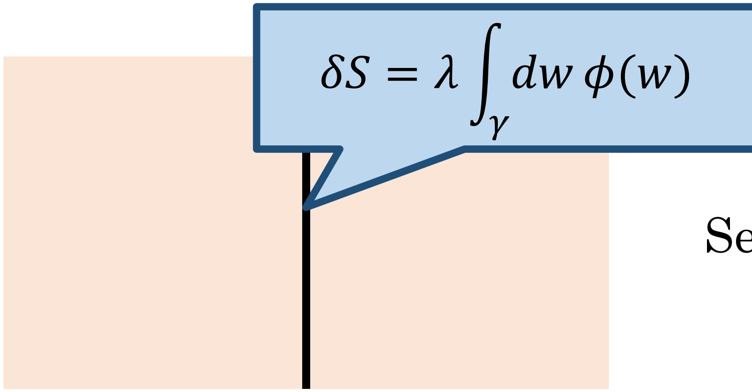


c_{LR} & c_{eff} are parametrized by one parameter $s \in (0,1)$;

$$c_{LR} = \frac{s^2}{2}$$

$$c_{\text{eff}} = \frac{s-1}{2} - \frac{3}{\pi^2}\big((s+1)\log(s+1)\log s \; + (s-1)\text{Li}_2(1-s) + (s+1)\text{Li}_2(-s)\big)$$

Example: defect perturbation



Setup: general CFT

$$\mathrm{CFT}_L = \mathrm{CFT}_R$$

Perturbation by defect field ϕ

$$\delta S = \lambda \int_\gamma dw\, \phi(w)$$

It has been shown in [Brehm],

$$c_{\mathrm{eff}} = c\left(\mathcal{T} + \frac{1}{4}(1 - \mathcal{T})\right) + O(\lambda^2)$$

This is consistent with λ expansion of c_{eff} in free fermion.

Since $c_{LR} = c\mathcal{T}$ and $0 \leq \mathcal{T} \leq 1$,

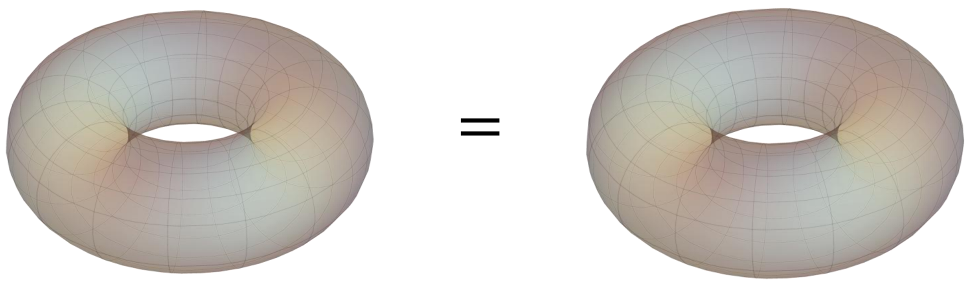
$$c_{LR} \leq c_{\mathrm{eff}}$$

Discussion

Summary

Our Goal

Model-independent methods in ICFT



Conformal Bootstrap

Equation between two cuts ⇒ Asymptotic formula

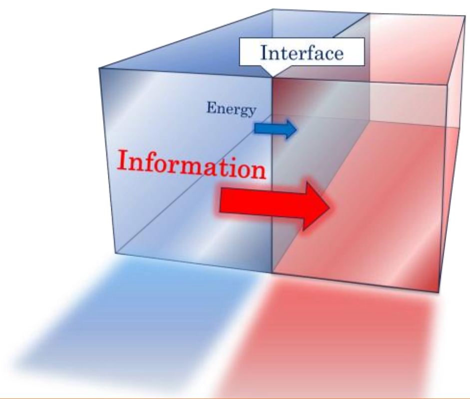
This approach can be applied to any CFT 😎

Resulting universal formula has important implications for quantum gravity

Summary

Our Goal

Model-independent methods in ICFT



AdS/CFT

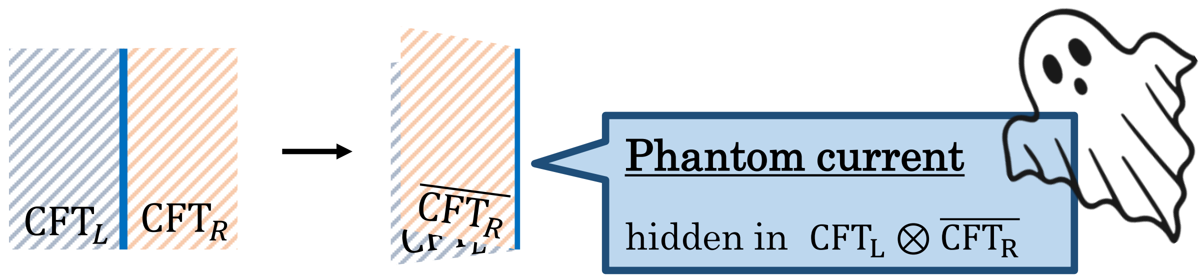
Known constraints on gravity ⇒ Nontrivial constraints on CFT

Example:

(Energy transmission) ≤ (Information transmission)

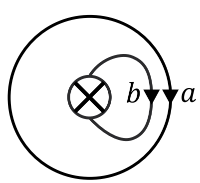
Other topics in ICFT

- New model-independent method [Furuta, YK, Onagi]

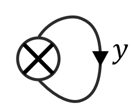


Focusing on the hidden structure—the phantom current—shared by many interfaces enables a model-independent computation of the transmission coefficient.

- Parity action on interface [Harada, Kaidi, YK, Liu]



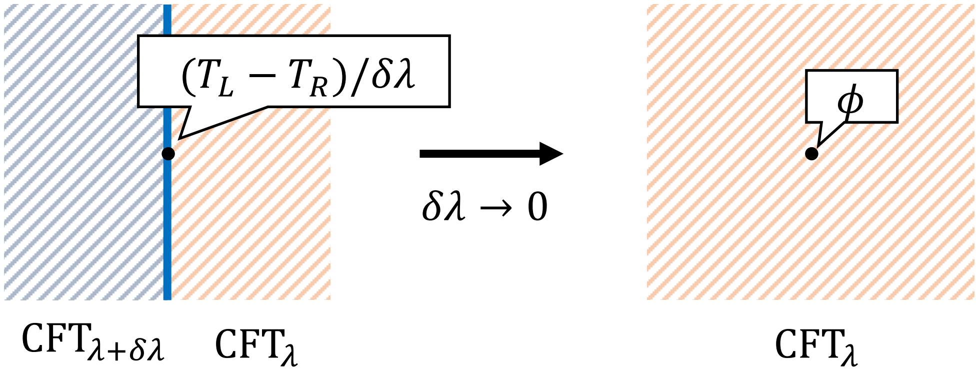
$$= \sum_{x,y} \sum_{\mu,\rho} \sum_{\nu,\sigma} (P^x_{ba})_{\rho\mu} (P^y_{xa})_{\nu\sigma} (F^y_{aba})_{x\sigma\rho,x\mu\nu}$$



Other topics in ICFT

- Conformal manifold implies exactly marginal operator?

[Komatsu, YK, Meineri, Ooguri]



The interface between deformed and undeformed CFT is useful to study CFT deformation.
One can show that a conformal manifold implies the existence of exactly marginal operators.

Thank you

Appendix

- Universal Asymptotic Formula
- AdS/BCFT
- Calculation of EE in ICFT
- Universal Bound on c_{eff}
  - Holographic proof
  - Entropic proof
- Universal Bound on c_{LR}
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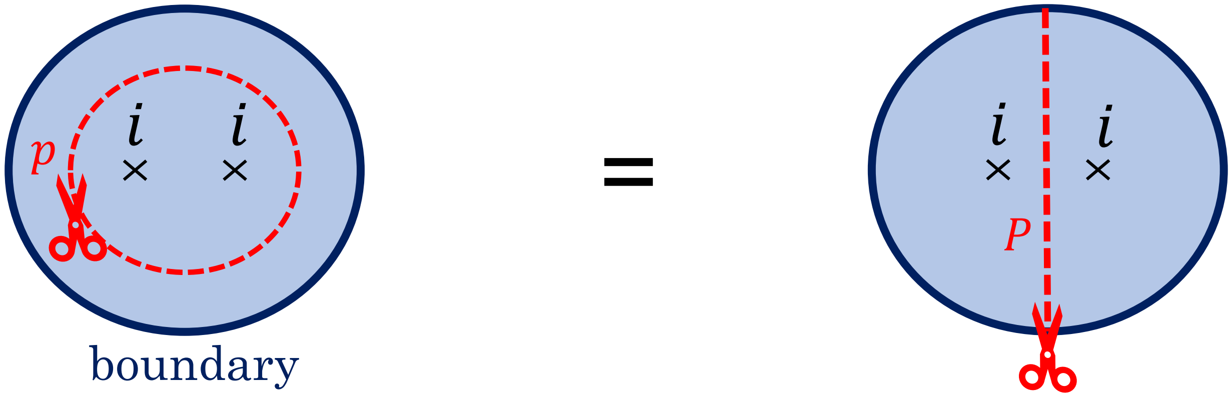
- Higher dimensional generalization

Universal Asymptotic Formula

[YK], [Numasawa-Tsiares]



$$\int \mathrm{d}h_p\ \rho^{\mathrm{bulk}}(h_p, h_p)\overline{C_{iip}C_{p0}}\mathcal{F}_{ii}^{ii}(p|1-z) = \int \mathrm{d}h_P\ \rho^{\mathrm{bdy}}(h_P)\overline{(C_{iP})^2}\mathcal{F}_{ii}^{ii}(P|z)$$

$1 - z \ll 1$

$$\mathcal{F}_{ii}^{ii}(0|1-z) \simeq \int \mathrm{d}h_P\ \rho^{bdy}(h_P)\overline{(C_{iP})^2}\mathcal{F}_{ii}^{ii}(P|z)$$

Universal Asymptotic Formula

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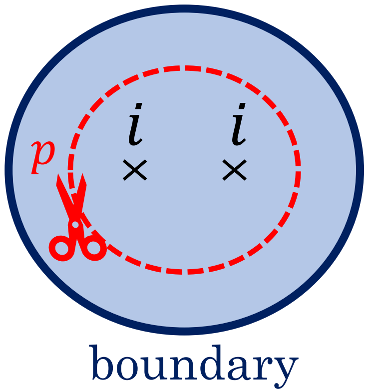

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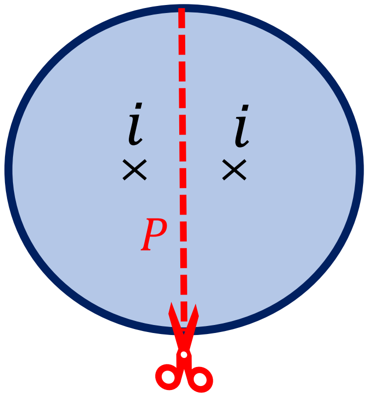

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$$\mathcal{F}_{ii}^{ii}(0|1-z) \simeq \int \mathrm{d}h_P\ \rho^{bdy}(h_P)\overline{(C_{iP})^2}\mathcal{F}_{ii}^{ii}(P|z)$$

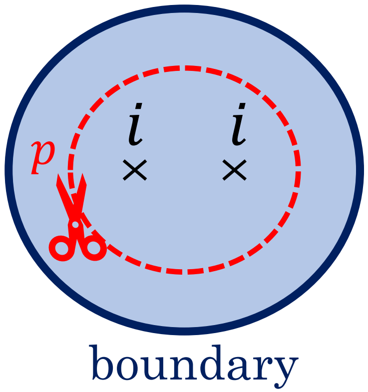
Fusion transformation

$$\mathcal{F}_{ii}^{ii}(0|1-z) = \int \mathrm{d}h_P\, F_{0P}\begin{bmatrix} i & i \\ i & i \end{bmatrix}\mathcal{F}_{ii}^{ii}(P|z)$$

theory-independent

Universal Asymptotic Formula

[YK], [Numasawa-Tsiares]



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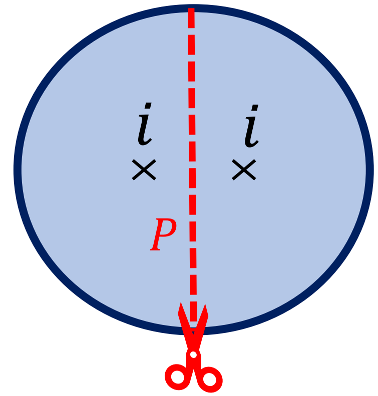

$$\int \mathrm{d}h_p \;\rho^{\mathrm{bulk}}(h_p, h_p)\overline{C_{iip}C_{p0}}\mathcal{F}_{ii}^{ii}(p|1-z) = \int \mathrm{d}h_P \;\rho^{\mathrm{bdy}}(h_P)\overline{(C_{iP})^2}\mathcal{F}_{ii}^{ii}(P|z)$$

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$$\wr\!|$$

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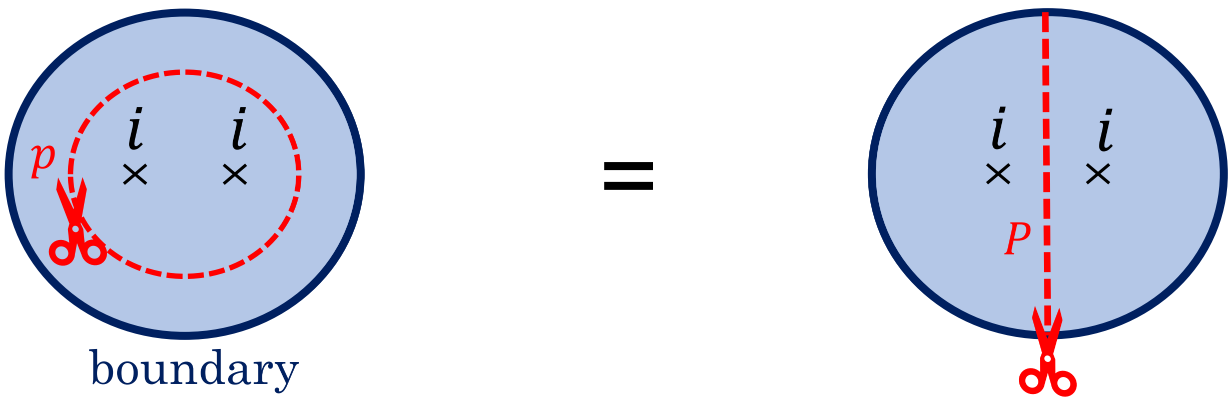

$$\int \mathrm{d}h_p\ \rho^{\mathrm{bulk}}(h_p, h_p)\overline{C_{iip}C_{p0}}\mathcal{F}^{ii}_{ii}(p|1-z) = \int \mathrm{d}h_P\ \rho^{\mathrm{bdy}}(h_P)\overline{(C_{iP})^2}\mathcal{F}^{ii}_{ii}(P|z)$$

$1 - z \ll 1$

$$\mathcal{F}^{ii}_{ii}(0|1-z) \simeq \int \mathrm{d}h_P\ \rho^{bdy}(h_P)\overline{(C_{iP})^2}\mathcal{F}^{ii}_{ii}(P|z)$$

$$\wr\wr \quad h_P \gg c$$

Fusion transformation

$$\mathcal{F}^{ii}_{ii}(0|1-z) = \int \mathrm{d}h_P\, F_{0P}\begin{bmatrix} i & i \\ i & i \end{bmatrix}\mathcal{F}^{ii}_{ii}(P|z)$$

theory-independent

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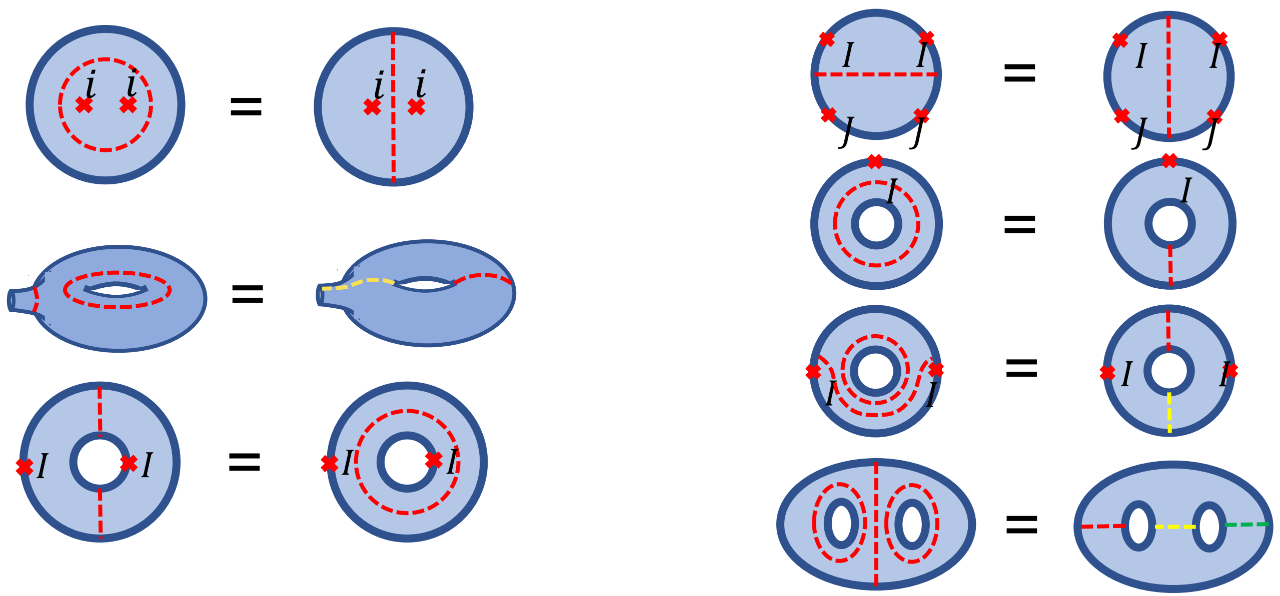

Result

C_{ijk}, C_{iJ}, C_{IJK} = fusion matrix, if $\max\{h_i\}, \{h_I\} \gg$ c

Note: fusion matrix does not depend on theory

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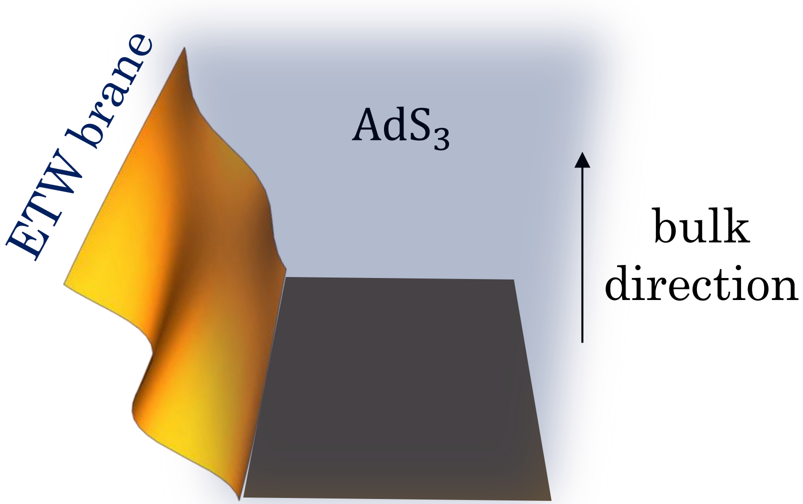
Application

[YK], [YK, Wei]

AdS/BCFT

$$I_{grav} = -\frac{1}{16\pi G_N}\int_M d^3x\,\sqrt{g}(R-2\Lambda) + \sum_i m_i \int dl_i - \frac{1}{8\pi G_N}\int_Q d^2x\,\sqrt{h}(K-T)$$

E-H action　　massive particle　　ETW brane



Asymptotic bdy = BCFT$_2$

Application

[YK], [YK, Wei]

AdS/BCFT

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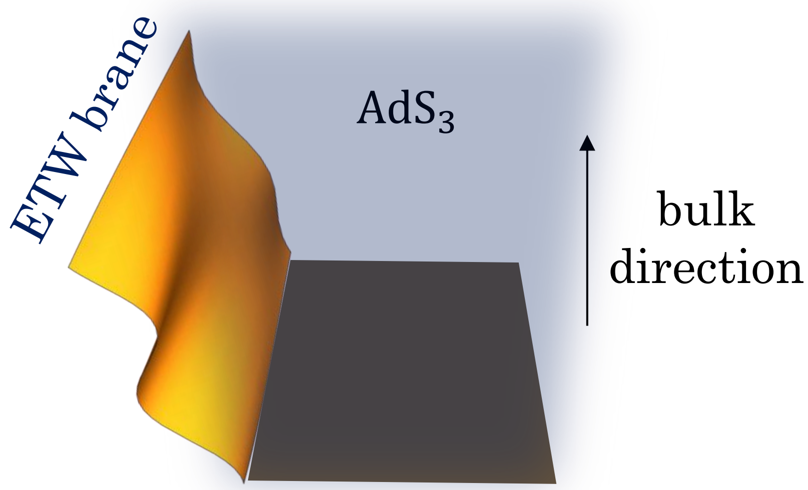
E-H action　　massive particle　　ETW brane



Asymptotic bdy = BCFT$_2$

<u>Extra assumption</u>

$$C_{p\mathbb{I}} = \delta_{p\mathbb{I}}$$

⇒ **<u>extension</u>** of validity regime of asymptotic formula

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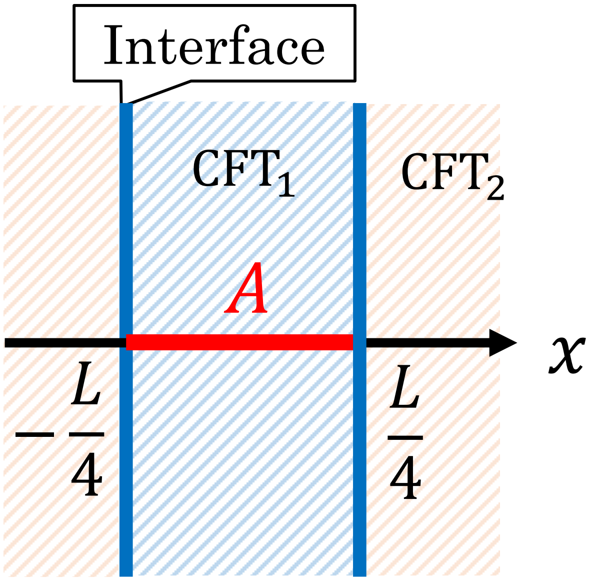
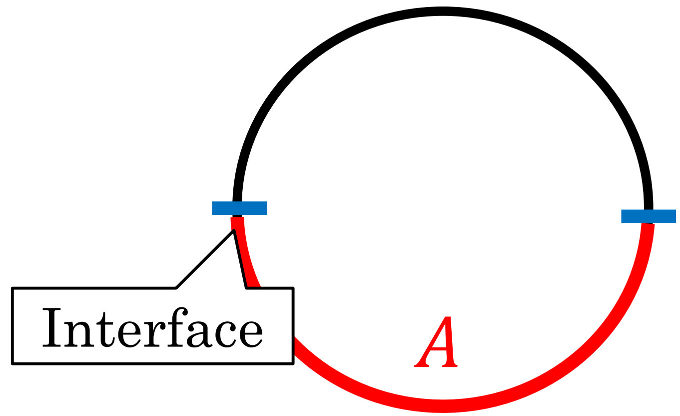
Calculation of EE

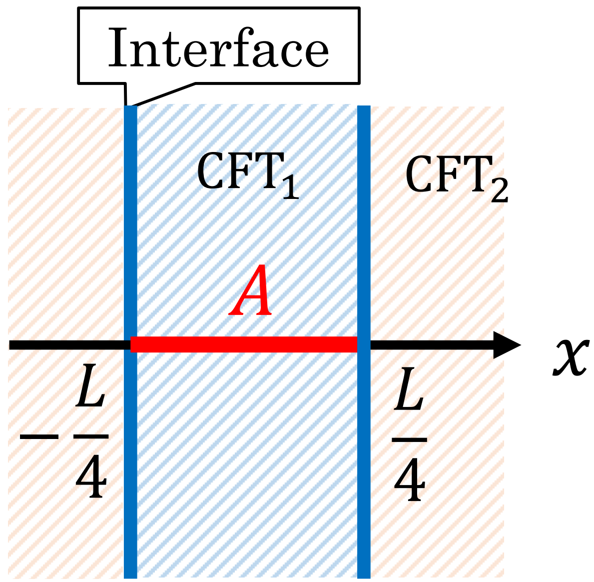
Our setup

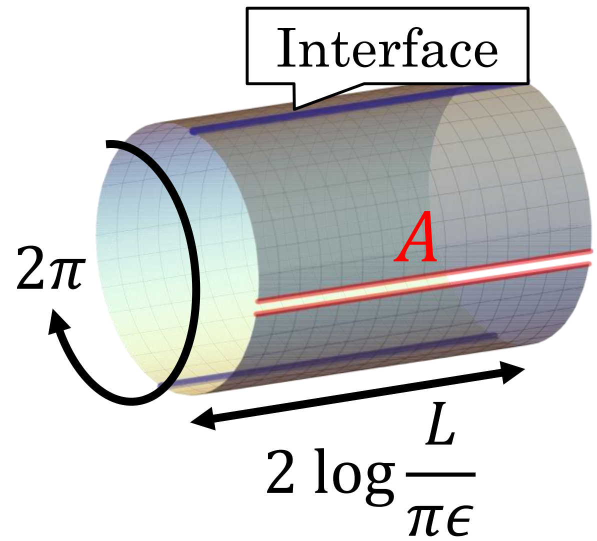




Calculation of EE

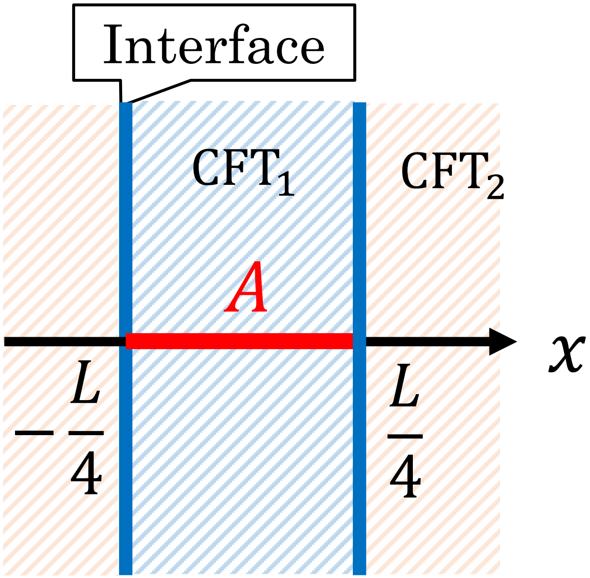
Our setup

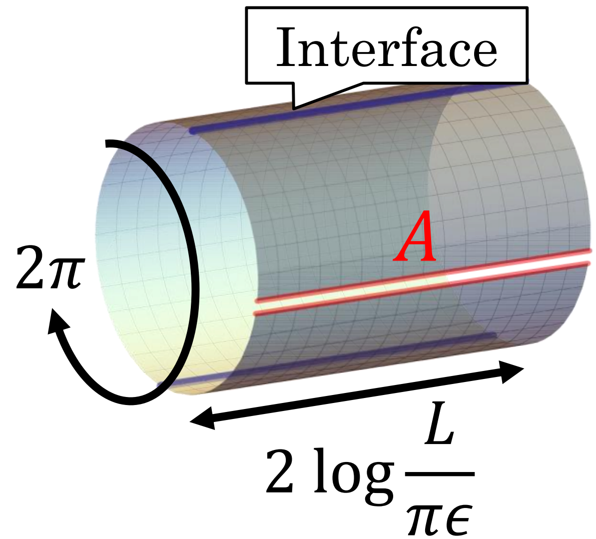




Conformal map
$z \to \log z$

Calculation of EE





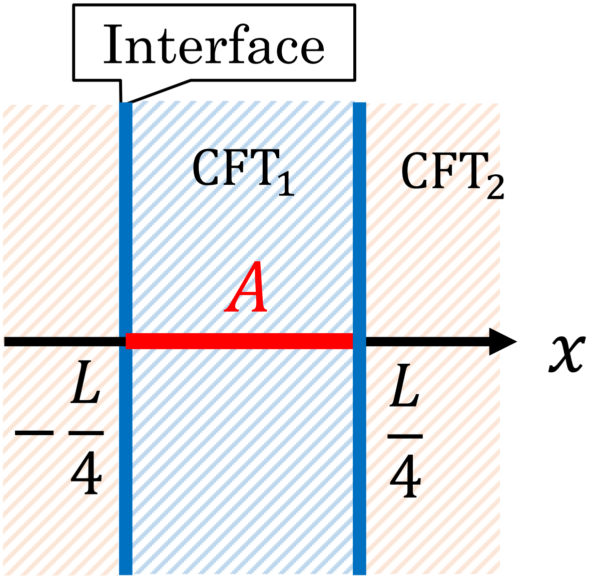
Replica trick

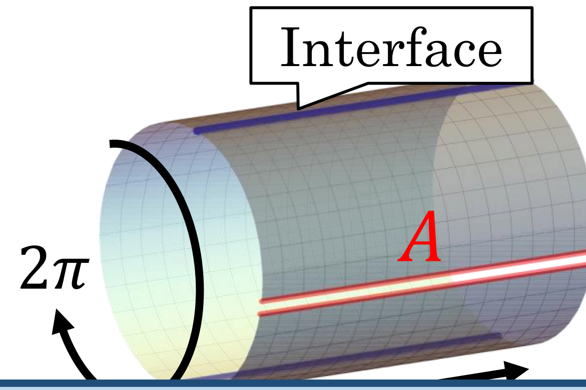
$$S_A = \lim_{n\to 1} S_A^{(n)}, \qquad S_A^{(n)} \equiv \frac{1}{1-n}\log\frac{Z_n}{(Z_1)^n}$$

where the replica partition function is

$$Z_n = \mathrm{tr}\left(e^{-\frac{\beta}{2}H^{(1)}}\,\mathcal{I}\,e^{-\frac{\beta}{2}H^{(2)}}\mathcal{I}^\dagger\right)^n, \qquad \beta \equiv \frac{\pi^2}{\log\frac{L}{\pi\epsilon}}, \qquad H^{(i)} \equiv L^{(i)} - \frac{c_i}{24}$$

Calculation of EE





Hamiltonian in BCFT is conventionally defined on $[0, \pi]$,

$$H^{(i)}_{\mathrm{conv}} = \pi\left(L^{(i)} - \frac{c_i}{24}\right)$$

But for convenance, we use

$$\beta H^{(i)} = \frac{2\pi}{2\log\frac{L}{\pi\epsilon}} H^{(i)}_{\mathrm{conv}}$$

Replica trick

$$S_A = \lim_{n\to 1} S_A^{(n)},$$

where the replica partition function is

$$Z_n = \mathrm{tr}\left(e^{-\frac{\beta}{2}H^{(1)}}\,\mathcal{I}\,e^{-\frac{\beta}{2}H^{(2)}}\mathcal{I}^\dagger\right)^n, \quad \beta \equiv \frac{\pi^2}{\log\frac{L}{\pi\epsilon}}, \quad H^{(i)} \equiv L^{(i)} - \frac{c_i}{24}$$

Calculation of EE

Open-closed duality

$$Z_n = \mathrm{tr}\left(e^{-\frac{\beta}{2}H^{(1)}}\,\mathcal{I}\,e^{-\frac{\beta}{2}H^{(2)}}\mathcal{I}^{\dagger}\right)^n = \langle B|e^{-\frac{(2\pi)^2}{2n\beta}H_n}|B\rangle$$

where H_n is the Hamiltonian of the theory with $2n$ interfaces.
(B is some boundary condition imposed on entangling surface.)

In the high-temperature limit $\beta = \frac{\pi^2}{\log\frac{L}{\pi\epsilon}} \xrightarrow[\epsilon\to 0]{} 0$,
only the vacuum ($\equiv \Delta_n^0$)propagates through the cylinder,

$$Z_n \sim e^{-\frac{(2\pi)^2}{2n\beta}\Delta_n^0}$$

Thus, we have

$$S_A^{(n)} = \frac{1}{1-n}\log\frac{Z_n}{(Z_1)^n} = \frac{2}{1-n}\left(\frac{\Delta_n^0}{n} - n\Delta_1^0\right)\log\frac{l}{\epsilon}$$

Calculation of EE

Just a formal expression
No simple formula is known

Open-closed duality

$$Z_n = \mathrm{tr}\left(e^{-\frac{\beta}{2}H^{(1)}}\,\mathcal{I}\,e^{-\frac{\beta}{2}H^{(2)}}\mathcal{I}^\dagger\right)^n = \langle B|e^{-\frac{(2\pi)^2}{2n\beta}H_n}|B\rangle$$

where H_n is the Hamiltonian of the theory with $2n$ interfaces.

(Def) Effective central charge

$$c_{\mathrm{eff}} = \lim_{n\to 1}\frac{12n}{1-n^2}\left(n\Delta_1^0 - \frac{\Delta_n^0}{n}\right),$$

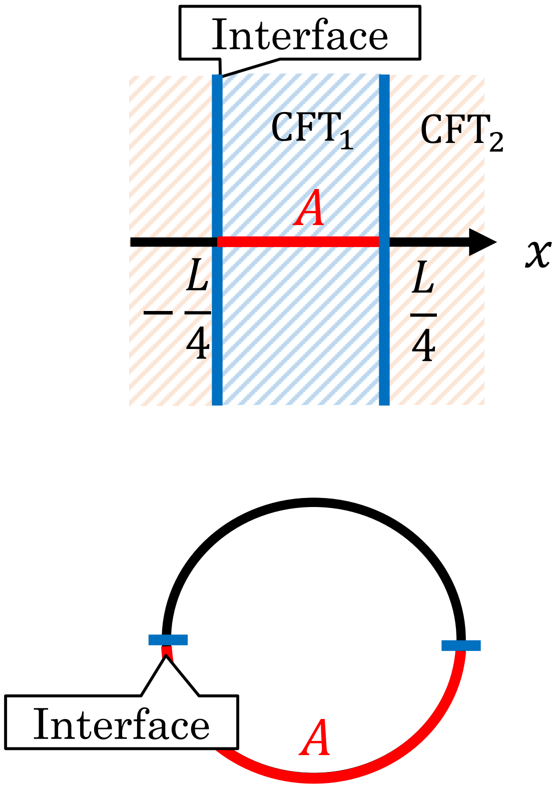
where Δ_n^0 is the vacuum energy in the $2n$-interface Hilbert space.

⇒Is this useful to give insights into general interfaces?

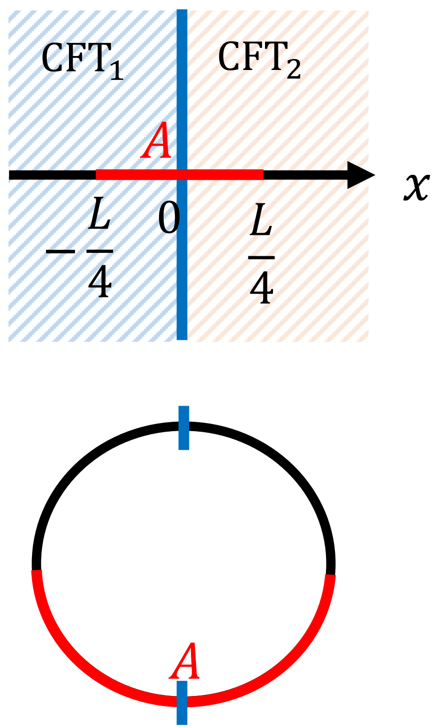
No nice property is known for Δ_n^0.

EE in various setups

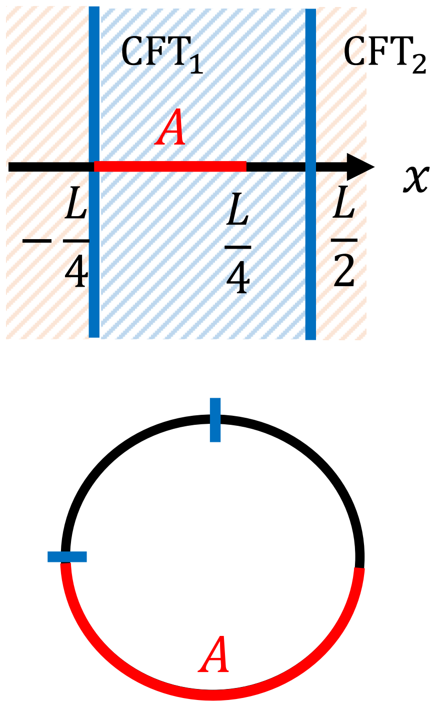
(i) one-side



(ii) symmetric

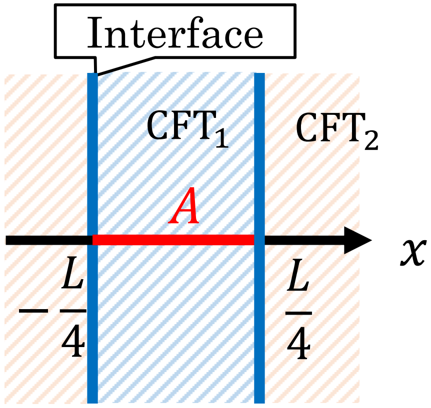


(iii) asymmetric



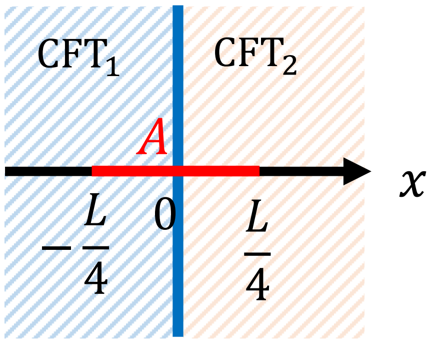
EE in various setups

(i) one-side



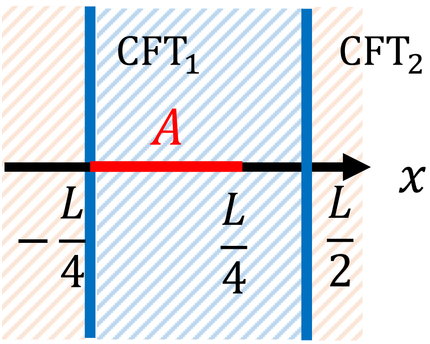
$$S_A = \frac{c_{\mathrm{eff}}}{3}\log\frac{L}{\pi\epsilon}$$

(ii) symmetric



$$S_A = \frac{c_1 + c_2}{6}\log\frac{L}{\pi\epsilon}$$

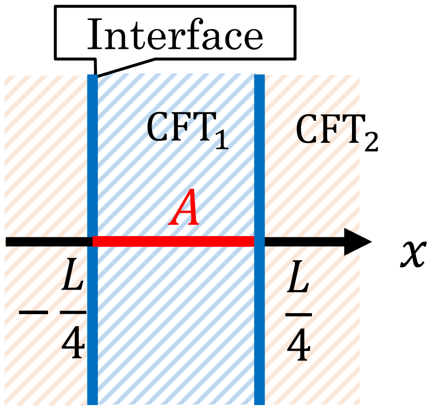
(iii) asymmetric



$$S_A = ?$$

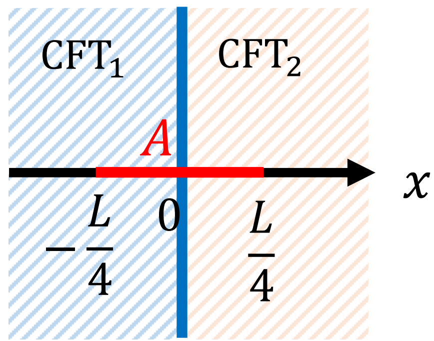
EE in various setups

(i) one-side



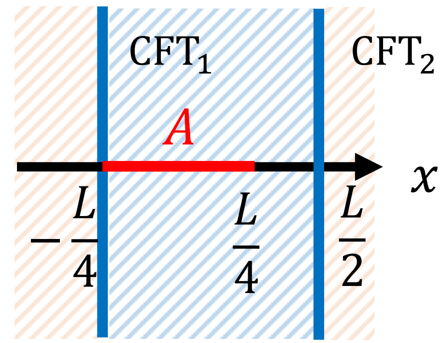
$$S_A = \frac{c_{\text{eff}}}{3} \log \frac{L}{\pi\epsilon}$$

(ii) symmetric



$$S_A = \frac{c_1 + c_2}{6} \log \frac{L}{\pi\epsilon}$$

(iii) asymmetric

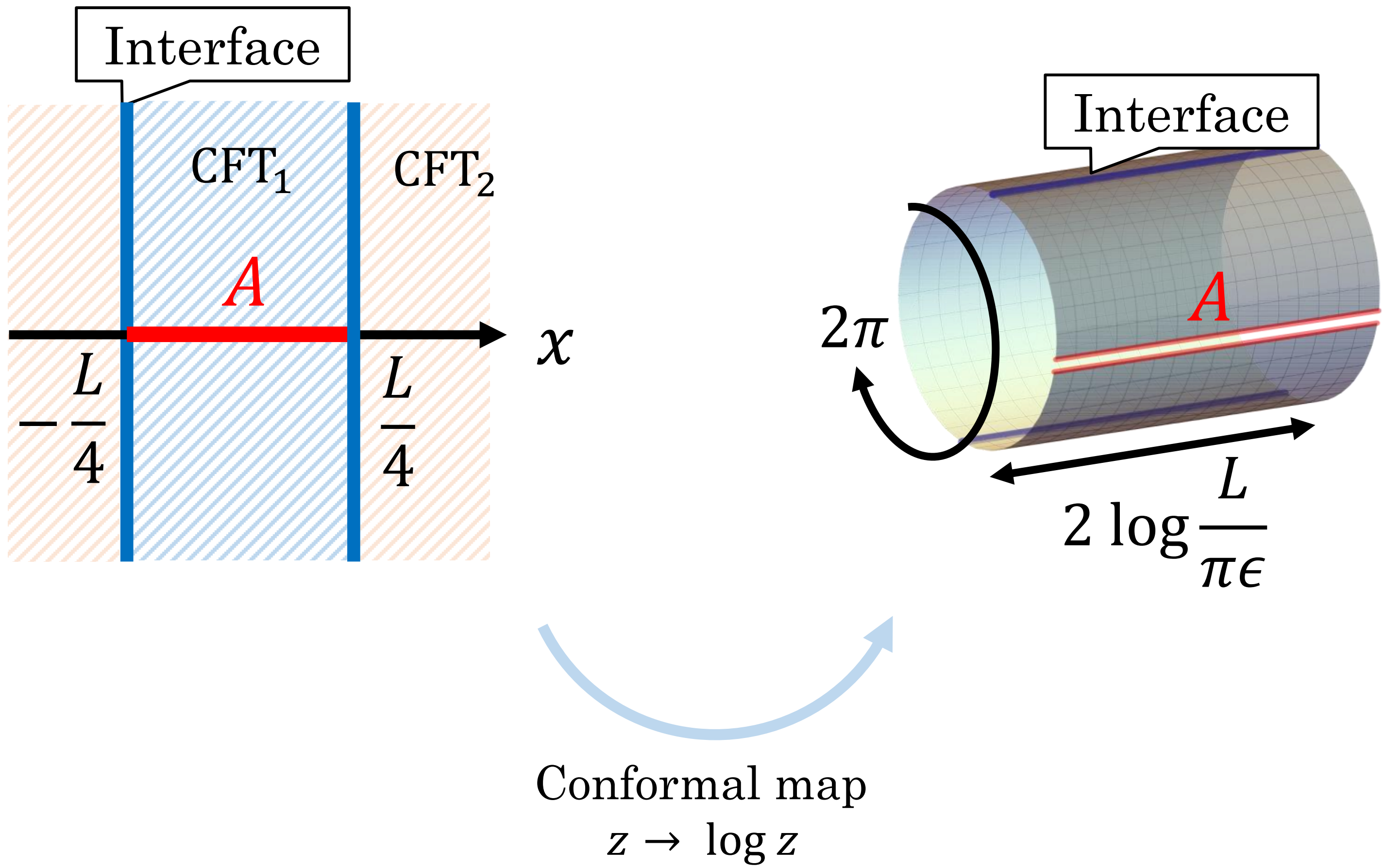


$$S_A = \frac{c_1 + c_{\text{eff}}}{6} \log \frac{L}{\pi\epsilon}$$

Conjectured from the gravity side [Karch, Luo, Sun] & [Karch, Wang]

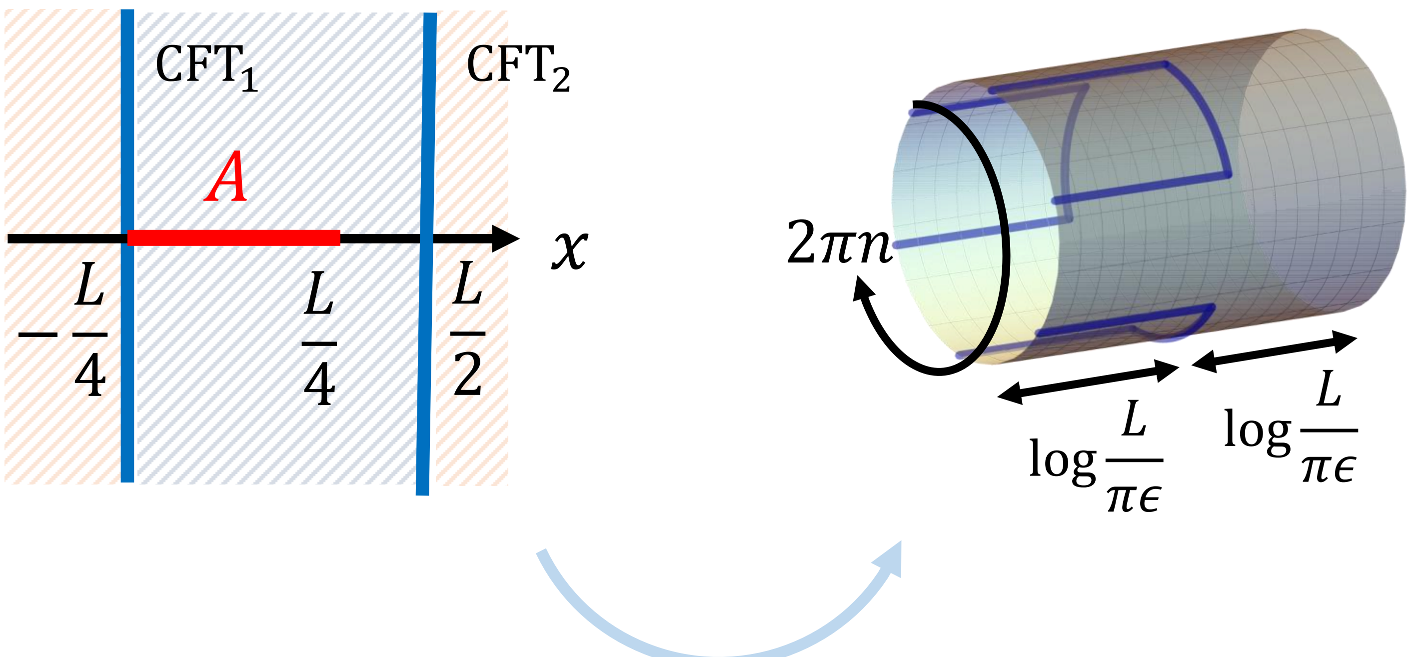
EE in one-side setup

Our setup



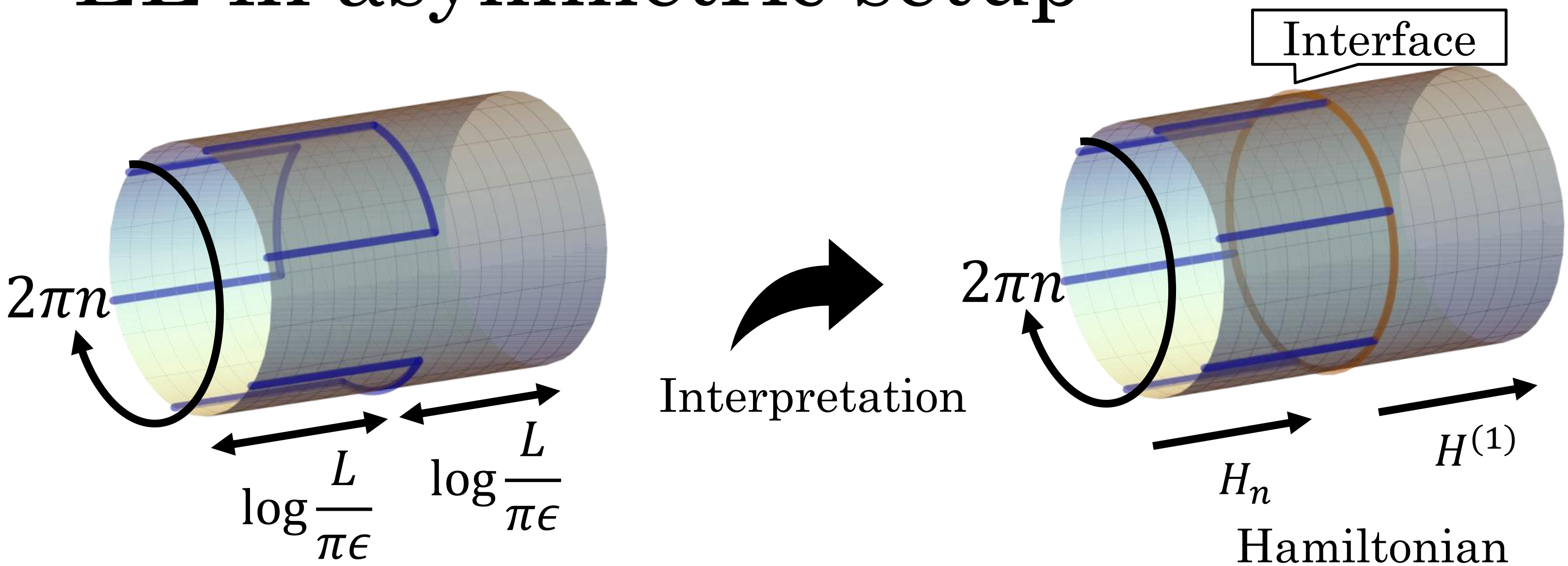
EE in asymmetric setup

Our setup



Conformal map
$z \to \log z$

EE in asymmetric setup



(Result)

$$Z_n = \langle B| e^{-\frac{1}{n}\log\frac{L}{\pi\epsilon}H_n}\, \mathcal{I}\, e^{-\frac{1}{n}\log\frac{L}{\pi\epsilon}H^{(1)}} |B\rangle$$

$$\sim\ e^{-\frac{1}{n}\log\frac{L}{\pi\epsilon}\Delta_n^0} \times e^{-\frac{1}{n}\log\frac{L}{\pi\epsilon}\Delta_{CFT_1}^0}$$

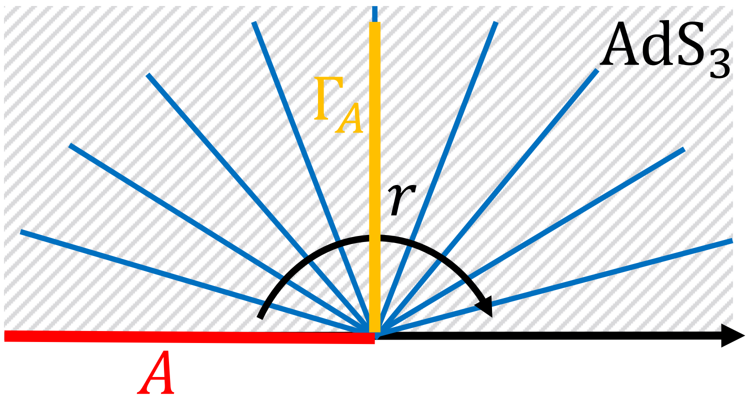
$$\Rightarrow \quad S_A = \frac{c_1 + c_{\mathrm{eff}}}{6}\log\frac{L}{\pi\epsilon}$$

Conjecture from gravity is now proven!

Appendix

- Universal Asymptotic Formula
- AdS/BCFT
- Calculation of EE in ICFT
- Universal Bound on c_{eff}
    - Holographic proof
    - Entropic proof
- Universal Bound on c_{LR}
    - Holographic proof
    - Check in non-holographic CFTs
- Higher dimensional generalization

Holographic EE in ICFT



$$ds^2 = e^{2A(r)}\frac{dx^2 - dt^2}{x^2} + dr^2$$

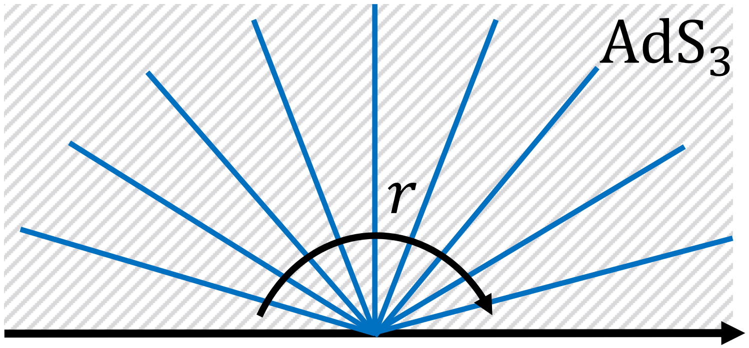
Relation between c and $A(r)$:

$$c_L = \frac{3}{2G_N}\frac{1}{A'(\infty)}$$
$$c_R = -\frac{3}{2G_N}\frac{1}{A'(-\infty)}$$
$$c_{\mathrm{eff}} = \frac{3}{2G_N}e^{A(r_{\mathrm{min}})}$$

R-T formula

where r_{min} is r that leads to the minimum value of A(r).

Holographic EE in ICFT



$$ds^2 = e^{2A(r)} \frac{dx^2 - dt^2}{x^2} + dr^2$$

Generalization of holographic c-theorem (=NEC):

$$A''(r) \le -k e^{-2A(r)}$$

where k is the sign of the constant curvature of the 2d slice.

Remark:

- $k = 0$ leads to the standard c-theorem.
- $k = -1$ makes the inequality weaker, so nobody is interested in this case. But now we make use of this!

Calculation of EE in ICFT

Define

$$F(r) \equiv A'(r)^2 + e^{-2A(r)}$$

Then,

$$F'(r) = 2\left(A''(r) - e^{-2A(r)}\right)A'(r)$$

Calculation of EE in ICFT

Define

$$F(r) \equiv A'(r)^2 + e^{-2A(r)}$$

Then,

$$F'(r) = 2\left(A''(r) - e^{-2A(r)}\right)A'(r)$$

Proposition 1

When approaching infinity, we reach AdS_3, i.e. $A(r) \approx \cosh(r)$.

$\Rightarrow \quad F(\pm\infty) = A'(\pm\infty)^2$

Calculation of EE in ICFT

Define

$$F(r) \equiv A'(r)^2 + e^{-2A(r)}$$

Then,

$$F'(r) = 2\underbrace{\left(A''(r) - e^{-2A(r)}\right)}_{<0\ (\because \mathrm{NEC})}A'(r)$$

Proposition 1

When approaching infinity, we reach AdS_3, i.e. $A(r) \approx \cosh(r)$.

$\Rightarrow \quad F(\pm\infty) = A'(\pm\infty)^2$

Proposition 2

$\Rightarrow \quad F'(r_{\min}) = A'(r_{\min}) = 0$

$\Rightarrow \quad F(r_{\min}) = e^{-2A(r_{\min})}$

$F(r_{\min})$ is the maximum of $F(r)$

Calculation of EE in ICFT

Proposition 1

$$F(\pm\infty) = A'(\pm\infty)^2$$

Proposition 2

$$F(r_{\min}) = e^{-2A(r_{\min})}$$

$F(r_{\min})$ is the maximum of $F(r)$

Proposition 3

$$c_L = \frac{3}{2G_N}\frac{1}{A'(\infty)}$$

$$c_R = -\frac{3}{2G_N}\frac{1}{A'(-\infty)}$$

$$c_{\text{eff}} = \frac{3}{2G_N}e^{A(r_{\min})}$$

(Result) Universal upper bound

$$F(r_{\min}) \geq F(\pm\infty)$$

$$\Rightarrow \quad e^{-2A(r_{\min})} \geq A'(\pm\infty)^2$$

$$\Rightarrow \quad c_{\text{eff}} \leq \min(c_L, c_R)$$

AdS version of holographic c-theorem

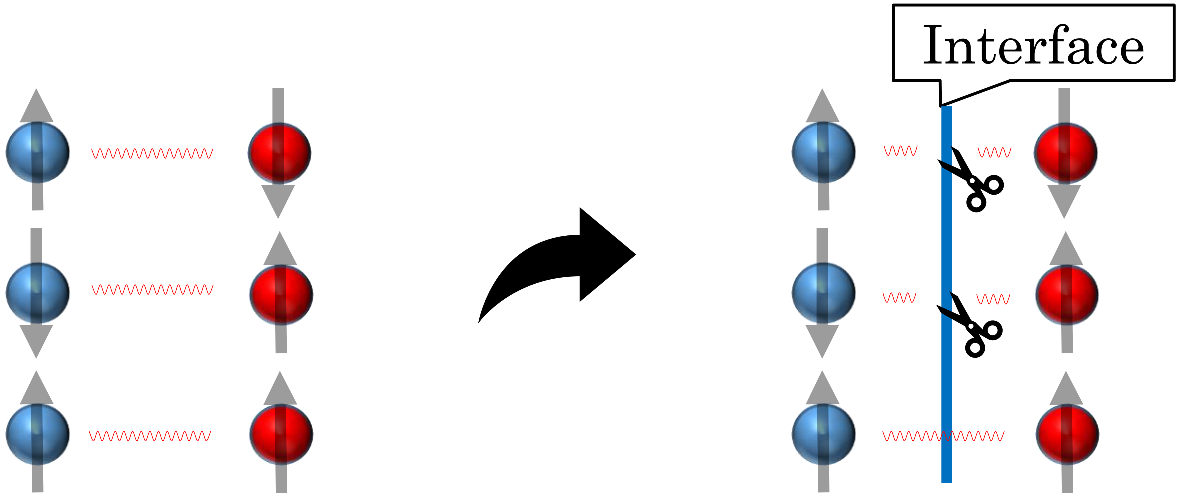
Result

(Result) Universal upper bound

$$c_{\text{eff}} \leq \min(c_L, c_R)$$

(Interpretation)

<u>Any interface decreases entanglement.</u>



Result

(Result) Universal upper bound

$$c_{\mathrm{eff}} \leq \min(c_L, c_R)$$

(Interpretation)

Any interface decreases entanglement.

Proof without AdS/CFT?

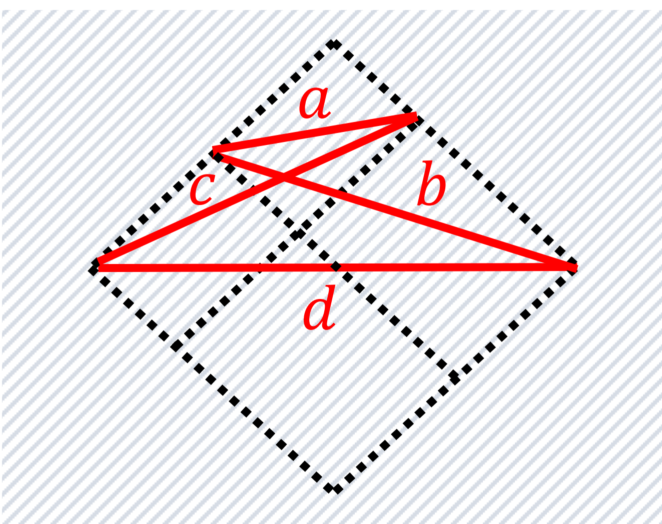
$\Rightarrow$ Entropic c-theorem (see details in our paper or App.)

Upper bound from CFT

How can we show the upper bound on c_{eff} on the CFT side?

On the gravity side, the **<u>holographic c-theorem</u>** is the key!

⇒The c-theorem may be useful on the CFT side.

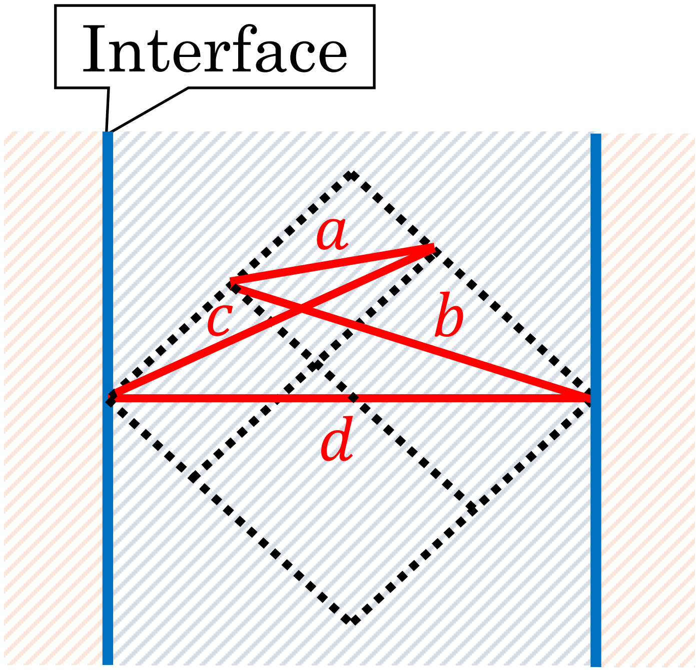


Key tools to derive entropic c-theorem:

(1) $S(b) + S(c) \geq S(a) + S(d)$

(2) $|a||d| = |b||c|$

Entropic c-theorem



Key tools to derive entropic c-theorem:

(1) $S(b) + S(c) \geq S(a) + S(d)$

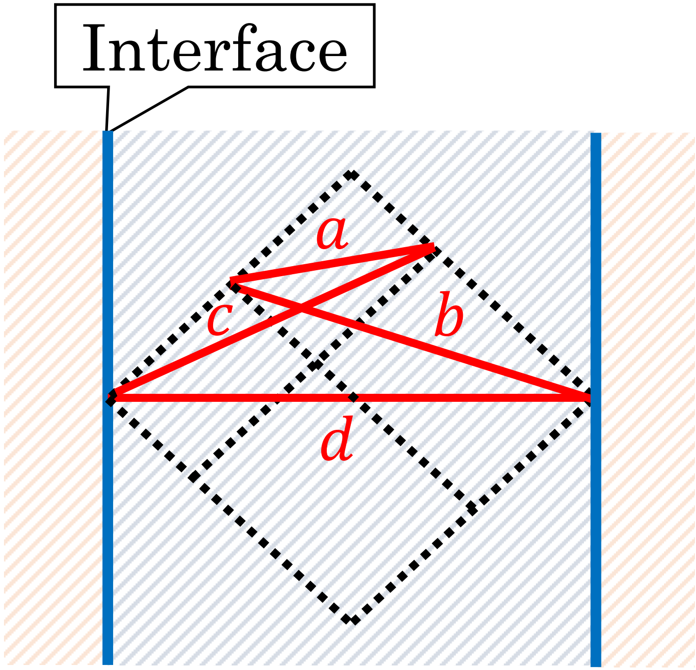
(2) $|a||d| = |b||c|$

Remark:

These two still work in ICFT.

$S(b)$ and $S(c)$ are evaluated using the formula (iii).

Entropic c-theorem



Key tools to derive entropic c-theorem:

(1) $S(b) + S(c) \geq S(a) + S(d)$

(2) $|a||d| = |b||c|$

(Result)

$$\frac{c_1 - c_{\text{eff}}}{6} \log \frac{|d|}{|a|} \geq 0, \qquad |d| \gg |a|$$

$$\Rightarrow \quad c_{\text{eff}} \leq \min(c_1, c_2)$$

CFT version of the proof

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  - Holographic proof
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Holographic c_{LR}

- metric

$$ds^2 = \Omega(\theta)^2\left(\frac{dx^2 - dt^2}{x^2} + d\theta^2\right), \qquad \theta \in (-\pi/2, \pi/2)$$

- effective AdS radius

$$l(\theta) \equiv \frac{\Omega(\theta)^2}{\sqrt{\Omega(\theta)^2 + \Omega'(\theta)^2}}$$

- effective brane tension ($\equiv \sigma(\theta)$)

$$8\pi G_N \frac{d\sigma}{d\theta} = \frac{\Omega(\theta)|l'(\theta)|}{l(\theta)^2\sqrt{\Omega(\theta)^2 - l(\theta)^2}}$$

Gravity calculation of c_{LR}

$$c_{LR} = \frac{3}{G_N}\left(\frac{1}{l_L} + \frac{1}{l_R} + 8\pi G_N \sigma\right)^{-1}$$

Simple bound [Karch, YK, Ooguri, Sun, Wang]

Gravity calculation of c_{LR}

$$c_{LR} = \frac{3}{G_N}\left(\frac{1}{l_L} + \frac{1}{l_R} + 8\pi G_N \sigma\right)^{-1}$$

σ is complicated but has simple bound;

$$8\pi G_N \int d\sigma = \int_{-\pi/2}^{\pi/2} d\theta \frac{\Omega(\theta)|l'(\theta)|}{l(\theta)^2\sqrt{\Omega(\theta)^2 - l(\theta)^2}}$$

$$\geq \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \frac{|l'(\theta)|}{l(\theta)^2} = \int \frac{|dl|}{l^2} \geq \left|\frac{1}{l_L} - \frac{1}{l_{\min}}\right| + \left|\frac{1}{l_{\min}} - \frac{1}{l_R}\right|$$

Thus, we obtain

$$c_{LR} \leq \frac{3 l_{\min}}{2 G_N} = c_{\mathrm{eff}}$$

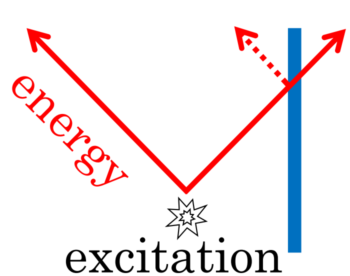
Result

(Result) Universal lower bound

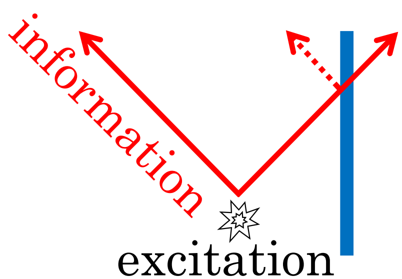
$$c_{LR} \leq c_{\mathrm{eff}}$$

(Interpretation)

The amount of energy transmission can never exceed the amount of information transmission.



$\leq$



Amount of transmitted energy

Amount of transmitted information
(= # of transmitted EPR pairs)

Result

(Result) Universal lower bound

$$c_{LR} \leq c_{\mathrm{eff}}$$

(Interpretation)

The amount of energy transmission can never exceed the amount of information transmission.

Is the inequality sharp?

Only two possibilities:

- $c_{LR} = c_{\mathrm{eff}} = 0$ (⇔ totally reflective)
- $c_{LR} = c_{\mathrm{eff}} = c_L = c_R$ (⇔ totally transmissive)

Result

(Result) Universal lower bound

$$c_{LR} \leq c_{\text{eff}}$$

(Interpretation)

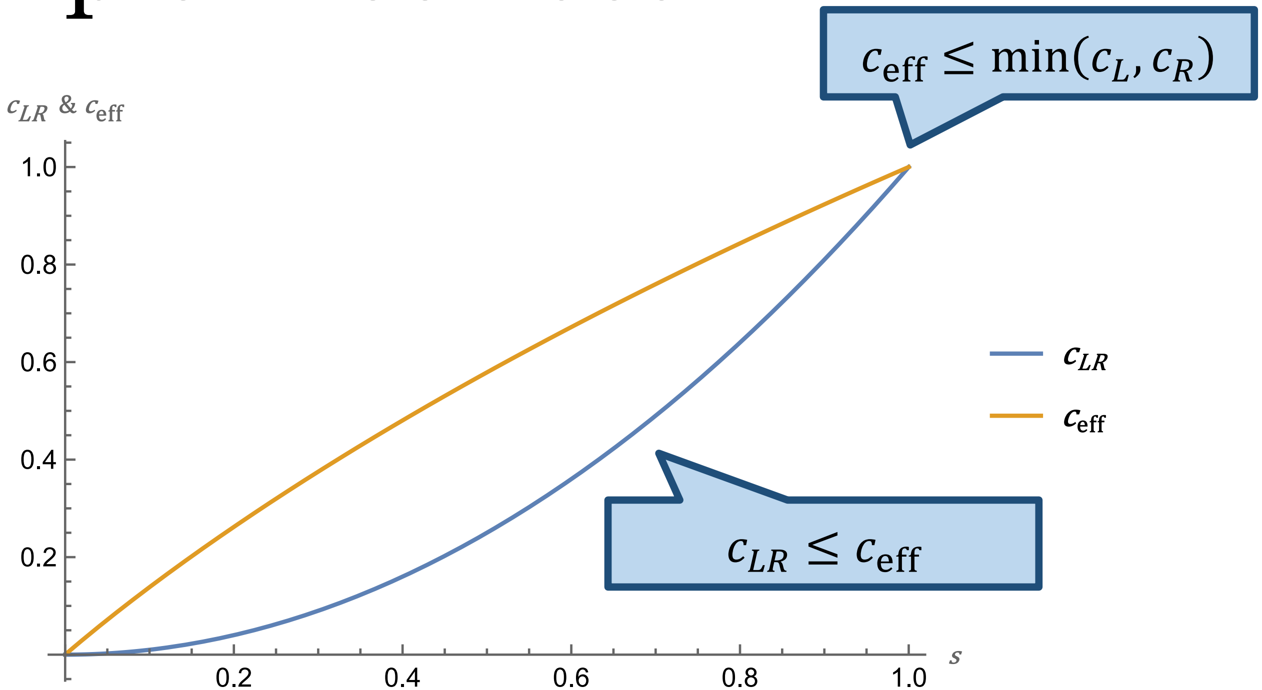
The amount of energy transmission can never exceed the amount of information transmission.

Beyond holography?

⇒ One can check in some examples:

- free boson
- free fermion
- defect perturbation

Example: free boson



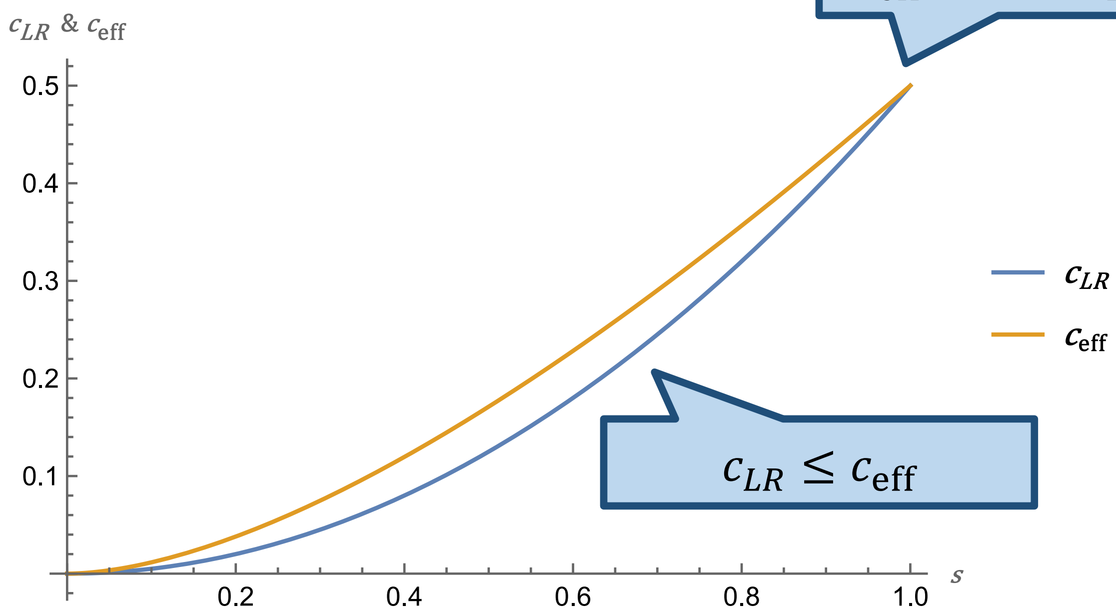
c_{LR} & c_{eff} are parametrized by one parameter $s \in (0,1)$;

$c_{LR} = s^2$

$$c_{\text{eff}} = \frac{1}{2} + s + \frac{3}{\pi^2}\left((s+1)\log(s+1)\log s + (s-1)\mathrm{Li}_2(1-s) + (s+1)\mathrm{Li}_2(-s)\right)$$

Example: free fermion

$c_{\text{eff}} \leq \min(c_L, c_R)$



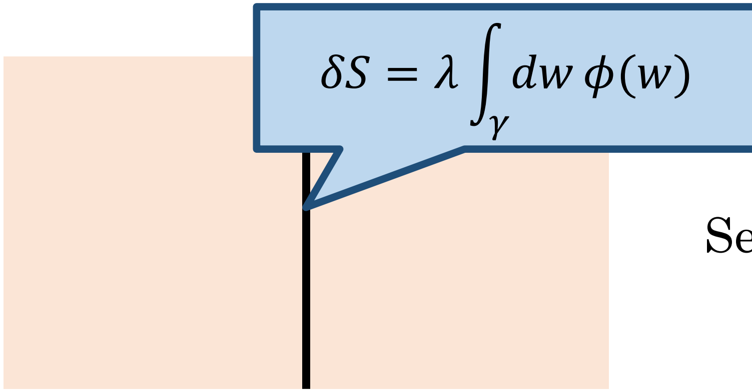
$c_{LR} \leq c_{\text{eff}}$

c_{LR} & c_{eff} are parametrized by one parameter $s \in (0,1)$;

$$c_{LR} = \frac{s^2}{2}$$

$$c_{\text{eff}} = \frac{s-1}{2} - \frac{3}{\pi^2}\big((s+1)\log(s+1)\log s \; + (s-1)\mathrm{Li}_2(1-s) + (s+1)\mathrm{Li}_2(-s)\big)$$

Example: defect perturbation



Setup: general CFT

$$\mathrm{CFT}_L = \mathrm{CFT}_R$$

Perturbation by defect field ϕ

$$\delta S = \lambda \int_\gamma dw\, \phi(w)$$

It has been shown in [Brehm],

$$c_{\mathrm{eff}} = c\left(\mathcal{T} + \frac{1}{4}(1-\mathcal{T})\right) + O(\lambda^2)$$

This is consistent with λ expansion of c_{eff} in free fermion.

Since $c_{LR} = c\mathcal{T}$ and $0 \leq \mathcal{T} \leq 1$,

$$c_{LR} \leq c_{\mathrm{eff}}$$

Comment

In three examples, c_{eff} is a function of c_{LR},
which means that c_{eff} and c_{LR} have the same information.

Is this true?

😟 Answer is NO.

While c_{LR} is given by an integration over the **entire region**, c_{eff} depends only on the **minimal value** of $A(r)$.

$\Rightarrow$ Relation between c_{LR} & c_{eff} is highly nontrivial.

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c_{eff} in higher d

Question:

What is a higher-dimensional analog of c_{eff}?

Assume d is even (few modifications in odd d)

General form of EE

$$S_A = s_{d-2}\left(\frac{l}{\epsilon}\right)^{d-2} + s_{d-4}\left(\frac{l}{\epsilon}\right)^{d-4} + \cdots + C \log\frac{l}{\epsilon} + s_0$$

Candidate

c_{eff} in higher d

Setup:

CFT on $\mathbb{R} \times S^{d-1}$ with interface along equator $\mathbb{R} \times S^{d-2}$

Holographic ICFT:

$$ds^2 = e^{2A(r)} ds^2_{\text{AdS}_d} + dr^2$$

Holographic EE:

$$S_A = \cdots + (-1)^{\frac{d}{2}-1} 4\, c_{\text{eff}} \log \frac{2}{\epsilon} + s_0$$

where

$$c_{\text{eff}} = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \frac{e^{(d-1)A(r_{\min})}}{4\pi G_N^{(d-1)}}$$

c_{eff} in higher d

Proposition 1

$$c_L = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \frac{\left(A'(\infty)\right)^{1-d}}{4\pi G_N^{(d-1)}}$$

$$c_R = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \frac{\left(A'(-\infty)\right)^{1-d}}{4\pi G_N^{(d-1)}}$$

$$c_{\text{eff}} = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \frac{e^{(d-1)A(r_{\min})}}{4\pi G_N^{(d-1)}}$$

Proposition 2

NEC holds in **any dimension**

$$\Rightarrow \quad e^{-2A(r_{\min})} \geq A'(\pm\infty)^2$$

(Result) Universal upper bound in **any dimension**

$$c_{\text{eff}} \leq \min(c_L, c_R)$$

Higher-dimension

- Higher dimensional analog of universal formula

$$S_A = \frac{c_L + c_{\mathrm{eff}}}{6} \log \frac{L}{\pi \epsilon}$$ ✓ True

- Higher dimensional analog of

$$c_{\mathrm{eff}} \leq \min(c_L, c_R)$$ ✓ True

Our candidate for c_{eff} satisfies all higher dimensional analogs of universal formulae.

⇒higher dimensional analog of c_{eff} is defined by

$$S_A = \cdots + (-1)^{\frac{d}{2}-1} 4\, c_{\mathrm{eff}} \log \frac{2}{\epsilon} + s_0$$

c_{eff} in odd d

Setup:

CFT on $\mathbb{R} \times S^{d-1}$ with interface along equator $\mathbb{R} \times S^{d-2}$

Holographic ICFT:

$$ds^2 = e^{2A(r)} ds^2_{\mathrm{AdS}_d} + dr^2$$

Holographic EE:

Not log term

$$S_A = \cdots + (-1)^{\frac{d-1}{2}} 2\pi \, c_{\text{eff}}$$

Same as even d

where

$$c_{\text{eff}} = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \frac{e^{(d-1)A(r_{\min})}}{4\pi G_N^{(d-1)}}$$