Towards a Hamiltonian Framework for String-Inspired Nonlocality

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Stringy Nonlocality

- In **string field theory**, fundamental interactions (splitting and joining of strings) are nonlocal. [Witten ('85); Zwiebach ('92)]
- This nonlocality in the worldsheet moduli space manifests as infinitely many spacetime derivatives $e^{\ell^2 \partial^2/2}$ in the vertices. ($\ell \sim$ string length $\sqrt{\alpha'}$)

$$S[\phi_a] = \int d^D x \left[\frac{1}{2} \sum_a \phi_a \left(\partial^2 - m_a^2\right) \phi_a - \sum_{\{a_i\}} \lambda_{a_1 \cdots a_n} \tilde{\phi}_{a_1} \cdots \tilde{\phi}_{a_n} \right]$$

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or

$$\tilde{\phi}_a \equiv \exp(\ell^2 \partial^2/2)\, \phi_a$$

$$S[\tilde{\phi}_a] = \int d^D x \left[\frac{1}{2} \sum\nolimits_a \tilde{\phi}_a \left(\partial^2 - m_a^2 \right) e^{-\ell^2 \partial^2} \tilde{\phi}_a - \sum\nolimits_{\{a_i\}} \lambda_{a_1 \cdots a_n} \tilde{\phi}_{a_1} \cdots \tilde{\phi}_{a_n} \right]$$

- Much progress has been made in understanding the S-matrices of such theories within the **path-integral formalism**.

[Sen ('16); Pius, Sen ('16, '18); de Lacroix, Erbin, Sen ('18), ...]

- On the other hand, they apparently lack a well-defined **Hamiltonian formalism** due to nonlocality.

THE PROBLEM OF NONLOCALITY IN STRING THEORY

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Is there a consistent Hamiltonian formalism ?

To properly treat $e^{-\ell^2 \partial^2}$, **analytic continuation** is necessary.

1. Euclideanized momenta (via *generalized Wick rotation*) [Pius, Sen ('16)]

   Employed to establish Cutkosky rules [Pius, Sen ('16, '18)], unitarity [Sen ('16)], analyticity, crossing symmetry [de Lacroix, Erbin, Sen ('18)] ...

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(Scattering amplitudes are analytic in $\ell^2 p_i \cdot p_j$.)

We attempt to construct a Hamiltonian formalism for the *analytically-continued* theory with $\ell^2 \to i\ell_E^2$.

(Continuation ${\ell_E}^2 \to -i\ell^2$ should be carried out in the end.)

In this talk, I will illustrate our approach using a **2D toy model**:

$$S_{2\mathrm{D}}[\tilde{\phi}] = \int d^2x \left[\frac{1}{2}\,\tilde{\phi}\,\partial^2\, e^{-\ell^2\partial^2}\,\tilde{\phi} + 2\lambda\,\widetilde{B}(v)\,\tilde{\phi}^2\right]$$

$$\hookrightarrow \int d^2x \left[\frac{1}{2}\,\tilde{\phi}\,\partial^2\, e^{-i\ell_E^2\partial^2}\,\tilde{\phi} + 2\lambda\,\widetilde{B}(v)\,\tilde{\phi}^2\right] \quad (\ell^2 \hookrightarrow i\ell_E^2)$$

background field

$\tilde{B}(v) = B(v)$

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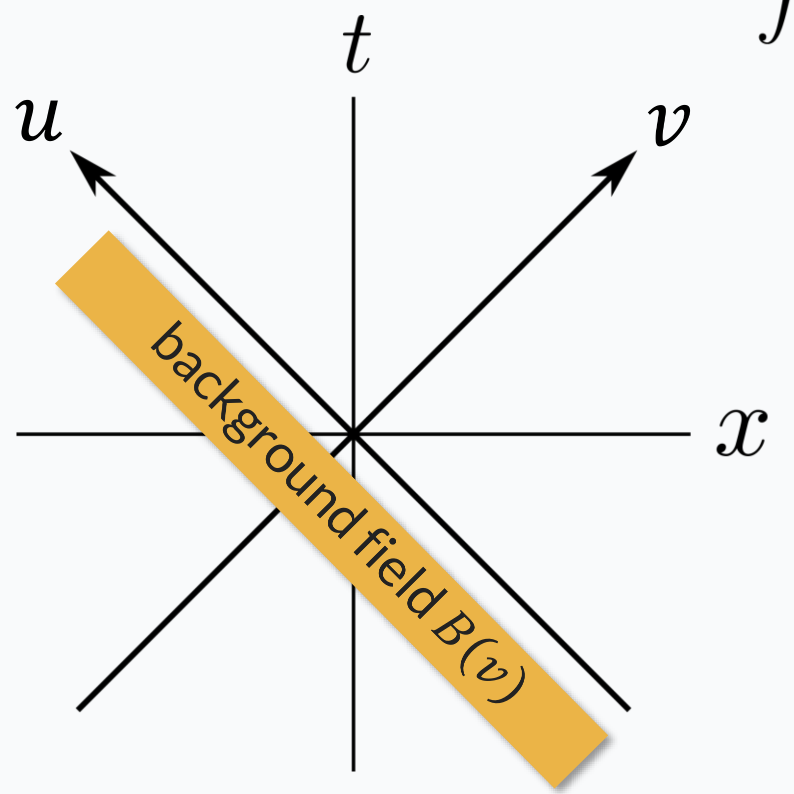

In the **light-cone frame** (u, v),

$$\partial^2 = -\partial_t^2 + \partial_x^2 = -4\,\partial_u\partial_v$$

$$e^{-i\ell_E^2\,\partial^2} = \exp(4i\ell_E^2\,\partial_u\partial_v)$$

milder nonlocality

[Gross, Erler ('04)]

Simplification in the light-cone frame

Light-cone (*outgoing*) mode expansion:

$$\tilde{\phi}(u,v) = \int_0^\infty \frac{dP_u}{\sqrt{4\pi P_u}} \left[\tilde{a}_{P_u}(v)\, e^{-iP_u u} + \tilde{a}^\dagger_{P_u}(v)\, e^{iP_u u}\right]$$

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⇒ In Fourier space,

$$\begin{aligned} S_{2\mathrm{D}}[\tilde{\phi}] &= \int du\, dv \left[-\tilde{\phi}\, \partial_u \partial_v \exp\!\left(4i\ell_E^2\, \partial_u \partial_v\right) \tilde{\phi} + \lambda\, B(v)\, \tilde{\phi}^2\right] \\ &= \int dV \int_0^\infty dP_u \Big[i\, \tilde{a}^\dagger_{P_u}(v - L(P_u))\, \partial_v\, \tilde{a}_{P_u}(v) \\ &\qquad + \frac{\lambda}{P_u} B(v)\, \tilde{a}_{P_u}(v)\, \tilde{a}^\dagger_{P_u}(v)\Big] \end{aligned}$$

$$L(P_u) \equiv 4\,\ell_E^2 P_u$$

Simplification in the light-cone frame

After replacing

$$\left\{\tilde{a}_{P_u}(v)\,,\tilde{a}^\dagger_{P_u}(v)\right\} \quad\to\quad \{\tilde{a}(t)\,,\tilde{a}^\dagger(t)\}$$

$$B(v)/P_u \quad\to\quad b(t)$$

$$L(P_u) \equiv 4\ell_E^2 P_u \quad\to\quad L$$

the 2D model is equivalent to a **1D model:**

$$\boxed{S_{1\mathrm{D}}[\tilde{a},\tilde{a}^\dagger] = \int dt\left[i\,\tilde{a}^\dagger(t-L)\,\partial_t\,\tilde{a}(t) + \lambda\, b(t)\,\tilde{a}(t)\,\tilde{a}^\dagger(t)\right]}$$

with a shift nonlocality on the scale L .

Path-Integral Formalism for 1D Model

$$S_{1\mathrm{D}}[\tilde{a}, \tilde{a}^\dagger] = \int dt\, \left[i\, \tilde{a}^\dagger(t - L)\, \partial_t\, \tilde{a}(t) + \lambda\, b(t)\, \tilde{a}(t)\, \tilde{a}^\dagger(t)\right]$$

Higher-pt correlation functions are fixed by the **2-pt correlator**:

$$\langle \tilde{a}(t)\, \tilde{a}^\dagger(t') \rangle_0 = i \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \frac{e^{-i\omega(t-t'-L)}}{\omega + i\epsilon} = \boxed{\Theta(t - t' - L)}$$

order of λ

$$\langle \tilde{a}(t)\, \tilde{a}^\dagger(t') \rangle_1 = i\lambda\, \Theta(t - t' - 2L) \int_{t'+L}^{t-L} b(t'')\, dt''$$

⋮

(perturbative in λ,
non-perturbative in L)

Operator Formalism for 1D Model

- We construct an operator formalism for the **nonlocal 1D model** by demanding the correspondence

$$\langle 0|\, \mathcal{T}\{\hat{\tilde{a}}(t)\, \hat{\tilde{a}}^\dagger(t')\}\, |0\rangle = \langle \tilde{a}(t)\, \tilde{a}^\dagger(t') \rangle \quad \text{——} \quad \bigstar$$

between **time-ordered** VEV and the path-integral correlator.

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We assume the perturbative expansion:

$$\hat{\tilde{a}}(t) = \hat{\tilde{a}}_0(t) + \hat{\tilde{a}}_1(t) + \cdots + \hat{\tilde{a}}_n(t) \ + \cdots$$

↑ order of λ

and solve for the operators *order by order* using $\bigstar$.

Operator algebra from path integral

Define the vacuum state $|0\rangle$ using $\hat{\tilde{a}}_0(t)|0\rangle = 0$.

- **Free theory:**

$$\begin{aligned}&\Theta(t-t')\,\langle 0|\hat{\tilde{a}}_0(t)\,\hat{\tilde{a}}_0^\dagger(t')|0\rangle \\ &+\Theta(t'-t)\,\langle 0|\hat{\tilde{a}}_0^\dagger(t')\,\hat{\tilde{a}}_0(t)|0\rangle\end{aligned} = \langle \tilde{a}(t)\,\tilde{a}^\dagger(t')\rangle_0 = \Theta(t-t'-L)$$

$$\Rightarrow \boxed{[\hat{\tilde{a}}_0(t)\,,\hat{\tilde{a}}_0^\dagger(t')] = \Theta(|t\ -\ t'|\ -\ L)}$$

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$$\Rightarrow \boxed{[\hat{\tilde{a}}_0(t)\,,\hat{\tilde{a}}_0^\dagger(t')] = \Theta(|t\ -\ t'|\ -\ L)}$$

- $\mathcal{O}(\lambda^n)$ **in interacting theory:**

$$\hat{\tilde{a}}_n(t) = i\lambda\int_{-\infty}^{\infty} dt''\,\Theta(t-t''-L)\,b(t'')\,\hat{\tilde{a}}_{n-1}(t'') \qquad \forall\, n\geq 1$$

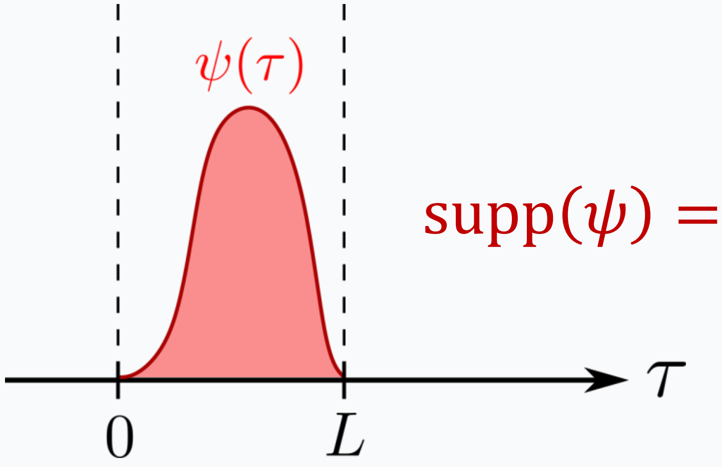
No “particle-like” states

A generic 1-particle state in the *naïve* Fock space:

$$|1_\psi\rangle \equiv \int_{-\infty}^{\infty} d\tau\, \psi(\tau)\, \hat{\tilde{a}}_0^\dagger(\tau)\, |0\rangle$$

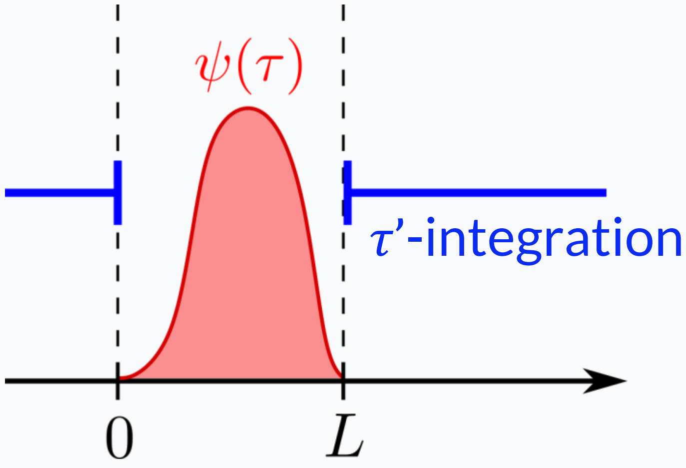
↑ “state function”

Such a state is “*particle-like*” if $\psi(\tau)$ has compact support within the nonlocality scale L .



$\text{supp}(\psi) = [0, L]$

“Particle-like” states have **vanishing norm:**



$$\langle 1_\psi \,|\, 1_\psi \rangle = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \, \psi^*(\tau) \, \psi(\tau') \, [\hat{\tilde{a}}_0(\tau) \, , \hat{\tilde{a}}_0^\dagger(\tau')]$$

$$= \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \, \psi^*(\tau) \, \psi(\tau') \, \Theta(|\tau - \tau'| - L)$$

$$= \int_{-\infty}^{\infty} d\tau \, \psi^*(\tau) \left[\int_{-\infty}^{\tau - L} d\tau' \, \psi(\tau') + \int_{\tau + L}^{\infty} d\tau' \, \psi(\tau') \right] = 0$$

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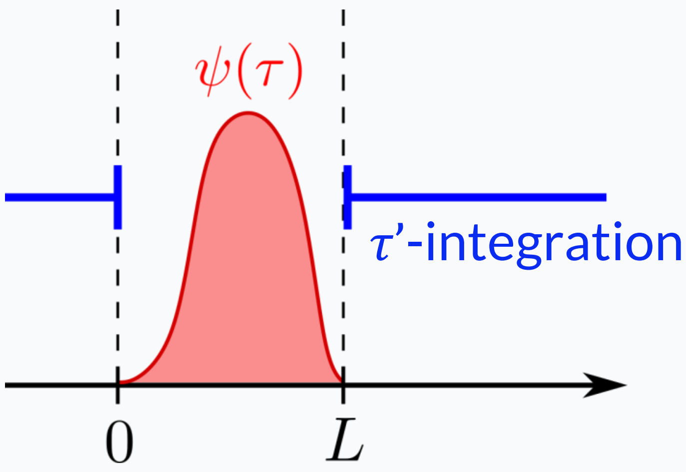

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⇒ **Physical states** have to be defined over a time interval $|\Delta\tau| > L$.

In the original 2D theory, this translates into:

$$|\Delta v| > L(P_u) \equiv 4\ell_E^2 P_u \quad \Rightarrow \quad \boxed{\Delta u\, \Delta v \gtrsim 4\ell_E^2}$$

(space-time uncertainty relation) [Yoneya ('87, '89, '97, '00)]

1. Time dependence of $\hat{\tilde{a}}_0$ and $\hat{\tilde{a}}_0^\dagger$

The free classical equations of motion (EoMs)

$$i\partial_t \,\hat{\tilde{a}}_0(t+L) = 0 \qquad \text{and} \qquad -i\partial_t \,\hat{\tilde{a}}_0^\dagger(t-L) = 0$$

are **incompatible** with the desired operator algebra $\left[\hat{\tilde{a}}_0(t), \hat{\tilde{a}}_0^\dagger(t')\right] = \Theta(|t-t'|-L)$.

Q: How can the EoMs be incorporated ?

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2. Enlarged Fock space ?

The naïve Fock space $\mathrm{span}\left\{\Pi_{i=1}^{n}\,\hat{\tilde{a}}_0^\dagger(t_i)|0\rangle\right\}$ contains unphysical DoFs.

Q: How can they be removed ?

EoMs as Physical-State Constraints

Classical EoMs:

$$i\,\partial_t\,\tilde{a}(t+L) + \lambda\, b(t)\,\tilde{a}(t) = 0\,, \qquad -i\,\partial_t\,\tilde{a}^\dagger(t-L) + \lambda\, b(t)\,\tilde{a}^\dagger(t) = 0$$

We impose them as constraints on the **physical-state space**:

$$\mathcal{H}_{\text{phys}} \equiv \left\{ |\psi\rangle \in \text{span}\left\{ \Pi_{i=1}^{n}\, \hat{\tilde{a}}_0^\dagger(\tau_i)|0\rangle \right\} \,\middle|\, \langle\psi|(\text{EoMs})|\psi\rangle = 0 \right\}$$

⇔ Physical states are those that obey:

$$\begin{cases} \left[i\,\partial_t\,\hat{\tilde{a}}(t+L) + \lambda\, b(t)\,\hat{\tilde{a}}(t)\right]|\psi\rangle = 0 \\ \langle\psi|\left[-i\,\partial_t\,\hat{\tilde{a}}^\dagger(t-L) + \lambda\, b(t)\,\hat{\tilde{a}}^\dagger(t)\right] = 0 \end{cases}$$

Physical-state condition on $\psi(\tau)$

In the free theory,

$$\begin{aligned} 0 = i\,\partial_t\,\hat{\tilde{a}}_0(t+L)\,|1_\psi\rangle &= \int_{-\infty}^{\infty} d\tau\,\psi(\tau)\,i\,\partial_t\,[\hat{\tilde{a}}_0(t+L)\,,\hat{\tilde{a}}_0^\dagger(\tau)]\,|0\rangle \\ &= \int_{-\infty}^{\infty} d\tau\,\psi(\tau)\,i\,\partial_t\,\Theta(|t-\tau+L|-L)\,|0\rangle \\ &= i\,[\psi(t)-\psi(t+2L)]\,|0\rangle \end{aligned}$$

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⇒ 1-particle state functions must satisfy:

$$\boxed{\psi(\tau+2L)-\psi(\tau)=0 \quad \forall\,\tau}$$ (2L-periodic)

* $\mathcal{O}(\lambda^n)$ $(n \geq 1)$ corrections to ψ can be fixed in a similar way.

Removal of negative-norm states

Let ψ_i $(i = 1, 2)$ be physical state functions.

$$|1_{\psi_i}\rangle := \lim_{N\to\infty} \frac{1}{2NL} \int_{-NL}^{NL} d\tau\, \psi_i(\tau)\, \hat{\tilde{a}}_0^\dagger(\tau)\, |0\rangle$$

(2L-periodic)

$$\Rightarrow \boxed{\langle 1_{\psi_1} \,|\, 1_{\psi_2}\rangle = \overline{\psi}_1^*\, \overline{\psi}_2}$$

where $$\overline{\psi} \equiv \frac{1}{2NL}\int_{-NL}^{NL} \psi(t)\; dt = \frac{1}{2L}\int_{-L}^{L} \psi(t)\; dt$$ ("zero mode")

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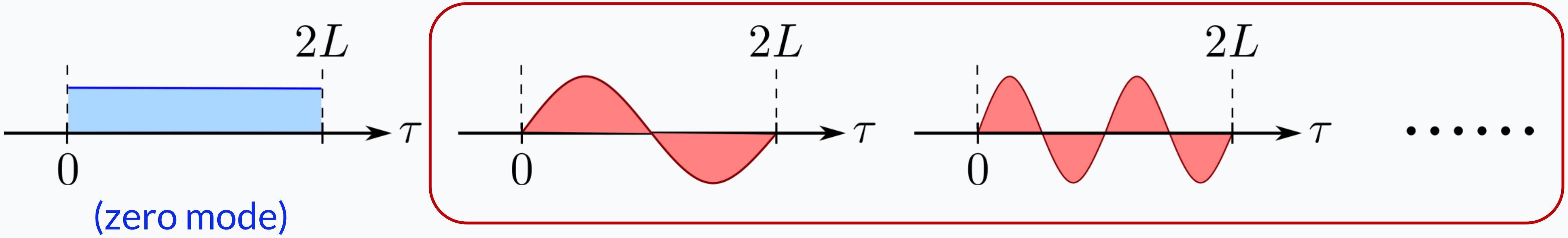
$\Rightarrow$ Negative-norm states are **absent** in $\mathcal{H}_{\text{phys}}$:

$$\left| |1_{\psi_1}\rangle + |1_{\psi_2}\rangle \right|^2 = |\bar{\psi}_1|^2 + |\bar{\psi}_2|^2 + 2\,\mathrm{Re}\left\{\bar{\psi}_1^*\, \bar{\psi}_2\right\} = |\bar{\psi}_1 + \bar{\psi}_2|^2 \geq 0$$

Zero-norm states in $\mathcal{H}_{\text{phys}}$

A general physical state function $\psi(\tau)$ (2L-periodic) can be decomposed as:

$$\psi(\tau) = \text{constant} + \sum_{n\in\mathbb{Z}\setminus\{0\}} C_n \exp(i\pi n\tau/L)$$



$$\psi_{\text{sp}}(\tau) = \exp(i\pi n\tau/L)$$

⇒ **zero-norm states:** $|1_{\text{sp}}\rangle := \lim_{N\to\infty} \frac{1}{2NL} \int_{-NL}^{NL} d\tau\, \psi_{\text{sp}}(\tau)\, \hat{\tilde{a}}_0^\dagger(\tau)\, |0\rangle$

Zero-norm states $|1_{\mathrm{sp}}\rangle$ are spurious

In the interacting theory, $\hat{\tilde{a}}$ has a perturbative expansion in λ that is **linear** in $\hat{\tilde{a}}_0$.

⇒ Any linear functional $L[\hat{\tilde{a}}]$ can be expressed schematically as:

$$L[\hat{\tilde{a}}] = \int \cdots \int (\cdots)\, \hat{\tilde{a}}_0(t+c)$$

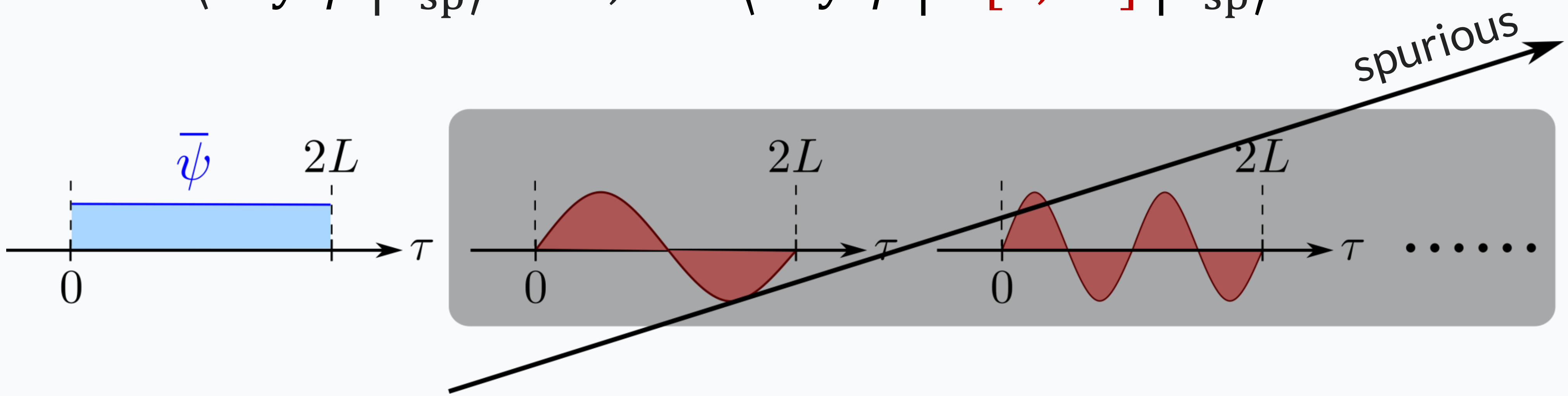
⇒ Zero-norm states are annihilated by $L[\hat{\tilde{a}}]$:

$$L[\hat{\tilde{a}}]|1_{\mathrm{sp}}\rangle = \int \cdots \int (\cdots) \int_{-\infty}^{\infty} d\tau\, \psi_{\mathrm{sp}}(\tau)\, [\hat{\tilde{a}}_0(t+c)\,, \hat{\tilde{a}}_0^{\dagger}(\tau)]\, |0\rangle$$

$$= \int \cdots \int (\cdots) \left[\int_{-\infty}^{\infty} d\tau\, e^{in\pi\tau/L} - \int_{t+c-L}^{t-c+L} d\tau\, e^{in\pi\tau/L}\right] |0\rangle = 0$$

Zero-norm states $|1_{\rm sp}\rangle$ decouple **to all orders** in λ :

$$\Big\langle \text{any}\ \psi\ \Big|1_{\rm sp}\Big\rangle = 0\,, \qquad \Big\langle \text{any}\ \psi\ \Big|\ F[\hat{\tilde{a}},\hat{\tilde{a}}^\dagger]\ \Big|1_{\rm sp}\Big\rangle = 0$$



Only the zero mode $\overline{\psi}$ contributes to physical observables.

$$\mathcal{H}_{\rm phys} = \operatorname{span}\Big\{\big(\hat{\tilde{A}}^\dagger\big)^n|0\rangle\Big\}\,,\quad \text{where}\quad \hat{\tilde{A}}^\dagger \equiv \lim_{N\to\infty}\frac{1}{2NL}\int_{-NL}^{NL}d\tau\,\hat{\tilde{a}}_0^\dagger(\tau)$$

$\dim(\mathcal{H}_{\rm phys})$ is the **same as the local theory** $(L=0)$.

Summary

- We put forward an operator formalism for stringy nonlocality $e^{-\ell^2\partial^2}$.
- Consistency with the path-integral formalism is automatic.
- Classical EoMs are realized as physical-state constraints.

  ⇒ Negative-norm states **eliminated**; zero-norm states **decoupled**.

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Future prospects:

1) Possibility of an *interaction picture* with covariant time ordering T^* ?
2) Extension to treat multiple fields and higher-point interactions.