Twist Field Deformation in Closed Superstring Field Theory

from $\mathbb{C}^2/\mathbb{Z}_2$ to Eguchi-Hanson space

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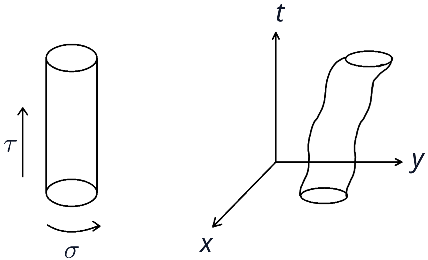
Motivation

Closed Strings on Orbifolds

String theory is defined by a CFT on the worldsheet

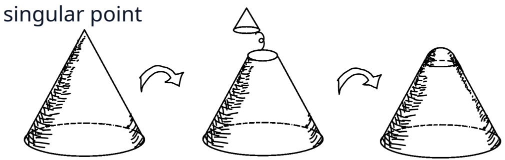


As a gravity theory, the excitations of the theory also **deforms the spacetime**

Closed Strings on Orbifolds

Deforming/Resolving an **orbifold:**

$\mathbb{C}^2/\mathbb{Z}_2 : z_1, z_2 \in \mathbb{C}, (z_1, z_2) \sim (-z_1, -z_2)$



Mathematical description:
Eguchi-Hanson space $\cong T^*S^2$

The metric is known as one of the **gravitational instantons**

Question:

What's the metric determined by string dynamics on the new space?

We use string field theory (SFT) to tackle this problem!

Why String Field Theory?

Worldsheet (CFT)

- Exactly marginal deformations
- Fixed background
- On-shell: scattering amplitudes $\mathcal{A}_{g,n}$

"Second Quantization" à la Batalin-Vilkovisky

$\rightarrow$

VS

SFT

- Solutions to the classical EOM
- Background independent
- Off-shell: action principle $S[\Psi]$

SFT is often just a cumbersome path to worldsheet results! Unless you need...

Applications:

1. RR-flux background (CFT is non-local)
2. LSZ reduction formula and mass renormalization (Off-shell)
3. Non-perturbativity (D-instanton, Tachyon condensation)

X. Yin,
A. Sen,
M. Schnabl,
...

Results from SFT

Perturbative Setup: Type IIB on $\mathbb{C}^2/\mathbb{Z}_2$

Hilbert space where the string field Ψ lives

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_\theta$$

$\mathcal{H}_\theta$: twisted sector $\sigma^i|0\rangle$, $i = 1, 2, 3, 4$

Marginal deformation:

$$V_{twist} = c w_\alpha S^\alpha \Delta e^{-\phi} \otimes \text{right-moving part},$$

where $\Delta = \sigma^1\sigma^2\sigma^3\sigma^4$.

BRST language:

$$QV_{twist} = 0$$

Perturbative Setup: Type IIB on $\mathbb{C}^2/\mathbb{Z}_2$

Non-polynomial theory (L_∞ algebra)

$$S[\Psi] = \frac{1}{2}\omega(\Psi, Q\Psi) + \sum_{n=2}^{\infty} \frac{1}{(n+1)!}\omega(\Psi, L_n(\Psi^n))$$

$$\underbrace{0 = Q\Psi}_{\textit{Linearized}} + \frac{1}{2!}L_2(\Psi^2) + \frac{1}{3!}L_3(\Psi^3) + \cdots$$

We expand the string field Ψ in the deformation parameter λ:

$$\Psi = \sum_{n=1}^{\infty} \lambda^n \Psi^{(n)}, \quad \Psi^{(1)} = V_{twist}$$

Perturbative Setup: Type IIB on $\mathbb{C}^2/\mathbb{Z}_2$

- **Order 1 (Linearized):** $Q\Psi^{(1)} = 0$.
  Solution: $\Psi^{(1)} = V_{twist}$ (The marginal blow-up modes $w_\alpha, \bar{w}_\beta$).
- **Order 2:** $Q\Psi^{(2)} = -\frac{1}{2}L_2(\Psi^{(1)}, \Psi^{(1)})$.
  Backreaction of the deformation on the metric.
- **Cohomological obstruction:** The source must be Q-exact.
  [Trivial cohomology]

Order 2: Recovering the Metric

To see the geometry change, we contract the second-order field with the **Graviton** state:

$$G_{ij}^{(2)} \sim \langle V_G, \Psi^{(2)} \rangle$$

This is an **off-shell** calculation

SFT correctly captures the resolution of the singularity!

Moduli $w_\alpha = w_\beta = (1/\sqrt{2}, 1/\sqrt{2})$ reproduces the leading correction of the **Eguchi-Hanson Instantons**:

$$H_{I\bar{J}} = (1 + \frac{\rho^4}{r^4})^{1/2} \left[\delta_{I\bar{J}} - \frac{\rho^4 \bar{z}_I z_{\bar{J}}}{r^2(\rho^4 + r^4)} \right]$$

Order 3: The Obstruction

Do higher-order corrections destroy the solution? At 3rd order, we face a potential obstruction:

$$\mathcal{O}^{(3)} \sim P_0[L_2(\Psi^{(1)}, \Psi^{(2)}) + \frac{1}{3!}L_3(\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)})]$$

By carefully evaluating the contact terms in the large Hilbert space, we prove:

$$\mathcal{O}^{(3)} = 0$$

Conclusion:

The deformation is **unobstructed** to this order
No extra constraints needed

Summary & Outlook

What we have:

- Used **Closed SFT** to dynamically describe the resolution of $\mathbb{C}^2/\mathbb{Z}_2$
- **Order 2:** Recovered the Eguchi-Hanson metric (and more)
- **Order 3:** Proved obstruction vanishes

Demonstrates SFT as a practical tool for extracting new vacuum information.

Possible extensions...

- All-order background. e.g. p-p wave background argument from charge conservation
- Non-Kähler background