Bridging IKKT and the polarized IKKT

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In colloboration with

Tin-Long Chau, Chien-Yu Chou, Jun Nishimura

based on :

Chien-Yu Chou, Jun Nishimura, CTW *(Phys.Rev.Lett*. 135 (2025) 22, 221601)

Tin-Long Chau, Chien-Yu Chou, Jun Nishimura, CTW (to appear)

Review : IKKT and the polarzied IKKT

IKKT matrix model

$\begin{cases} A_\mu : SO(10) \text{ vector bosons} \\ \Psi_\alpha : \text{SO(10) MW fermions} \end{cases}$

$\rightarrow N \times N$ Hermitian traceless matrices

$A_\mu \rightarrow U A_\mu U^\dagger \,,\, \Psi_\alpha \rightarrow U \Psi_\alpha U^\dagger$

$$\begin{cases} \delta_{\text{SUSY}}\, A_\mu &= -\Psi_\alpha \left(\mathcal{C}\Gamma^\mu\right)_{\alpha\beta} \epsilon_\beta \,, \\ \delta_{\text{SUSY}}\, \Psi_\alpha &= \frac{i}{2} \left[A_\mu, A_\nu\right] \Gamma^{\mu\nu}_{\alpha\beta} \epsilon_\beta \,. \end{cases}$$

$$S_{\text{IKKT}} = \text{tr} \left\{ -\frac{1}{4} \left[A_\mu, A_\nu\right]^2 - \frac{i}{2} \Psi_\alpha (\mathcal{C}\Gamma^\mu)_{\alpha\beta} [A_\mu, \Psi_\beta] \right\}$$

A promising non-perturbatvie definition of superstring theory.

Ishibashi-Kawai-Kitazawa-Tsuchiya 96

No pregeometry

→ Spacetime dynamically emerges from matrices

Aoki-Iso-Kawai-Kitazawa-Tada 98

Hotta-Nishimura-Tsuchiya 98

Q : How our (3+1) spacetime emerges from the (9+1) superstring as the dominant eigenvalues distribution.

e.g. : *First principle numerical simulation*

Kim-Nishimura-Tsuchiya 12

Ito-Nishimura-Tsuchiya 15

Aoki-Hirasawa-Ito-Nishimura-Tsuchiya 19

Anagnostopoulos-Azuma-Hatakeyama-Hirasawa-Ito-Nishimura-Papadoudis-Tsuchiya 22

Euclidean Polarized IKKT

$$\begin{cases} \delta_{\mathrm{SUSY}} A_\mu &= -\Psi_\alpha \left(\mathcal{C}\Gamma^\mu\right)_{\alpha\beta} \epsilon_\beta \,, \\ \delta_{\mathrm{SUSY}} \Psi_\alpha &= \frac{i}{2} \left[A_\mu, A_\nu\right] \Gamma^{\mu\nu}_{\alpha\beta} \epsilon_\beta + \frac{\Omega}{8} \Gamma^{123}_{\alpha\beta} \left(3A_a\Gamma^a + A_I\Gamma^I\right)_{\beta\gamma} \epsilon_\gamma \end{cases}$$

$$S = S_{\mathrm{IKKT}} + S_\Omega \,, \quad S_{\mathrm{IKKT}} = \mathrm{tr}\left\{-\frac{1}{4}\left[A_\mu, A_\nu\right]^2 - \frac{i}{2}\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]\right\}$$

$$S_\Omega = \mathrm{Tr}\left\{\frac{\Omega^2}{8^2}\left(3A_a^2 + A_I^2\right) + i\,\Omega[A_1, A_2]A_3 - \frac{\Omega}{8}\Psi_\alpha(\mathcal{C}\Gamma^{123})_{\alpha\beta}\Psi_\beta\right\}$$

$a = 1, 2, 3$ and $I = 4, \cdots, 10$

Bonelli 02

uniquely preserves 16 supercharges but breaks $SO(10)$ to $SO(3) \times SO(7)$

$\Longrightarrow$ (Euclidean) holographic dual in IIB SUGRA is identified

Hartnoll-Liu 24
Komatsu-Martina-Penedones-Vuignier-Zhao 24

Q : What is the spacetime picture in polarized IKKT?

$\Longrightarrow$ *First principle numerical simulation*

Chou-Nishimura-CTW 25
Chau-Chou-Nishimura-CTW (to appear)

Large Ω

Hartnoll-Liu 24
Komatsu-Martina-Penedones-Vuignier-Zhao 24

$$S_{\rm IKKT} = {\rm tr}\left\{-\frac{1}{4}[A_\mu, A_\nu]^2 - \frac{i}{2}\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]\right\}$$

$$S_\Omega = {\rm Tr}\left\{\frac{\Omega^2}{8^2}(3A_a^2 + A_I^2) + i\,\Omega[A_1, A_2]A_3 - \frac{\Omega}{8}\Psi_\alpha(\mathcal{C}\Gamma^{123})_{\alpha\beta}\Psi_\beta\right\}$$

S_Ω introduces a dimension to the system, e.g. $[A_\mu] = \Omega\,, [\Psi_\alpha] = \Omega^{3/2}$

e.o.m.

$$[A_\mu, [A_\mu, A_a]] + \frac{3\Omega^2}{32}A_a + i\Omega\epsilon_{abc}A_bA_c = 0\,,$$
$$[A_\mu, [A_\mu, A_I]] + \frac{\Omega^2}{32}A_I = 0\,.$$

soln.

$$A_a = 0\,, \frac{1}{8}\Omega J_a\,, \frac{3}{8}\Omega J_a \qquad [J_a, J_b] = i\epsilon_{abc}J_c$$
$$A_I = 0$$

su(2) representation.

$\Longrightarrow$ polarized IKKT classicalizes into the maximal fuzzy sphere in large Ω

$\Longrightarrow$ D instantons get polarized into D1-brane (Myer's effect)

$\Omega \to 0$ divergence

$\Omega \to 0$ divergence

Via SUSY localization, it is found that the partition function diverges when $\Omega \to 0$,

Komatsu-Martina-Penedones-Vuignier-Zhao 24

$$Z(\Omega \to 0) \propto \left(\frac{1}{\Omega}\right)^{2(N-1)}$$

The polarized IKKT at $\Omega \to 0$ limit doesn't reduce back to IKKT,

$$Z(\Omega \to 0) = \lim_{\Omega \to 0} \int dA\, d\Psi\, e^{-S_{\mathrm{IKKT}} + S_{\Omega}} \neq \int dA\, d\Psi\, \lim_{\Omega \to 0} e^{-S_{\mathrm{IKKT}} + S_{\Omega}} = Z_{\mathrm{IKKT}}$$

divergent

finite

Austing 01
Komatsu-Martina-Penedones-Vuignier-Zhao 24
Chou-Nishimura-CTW 25

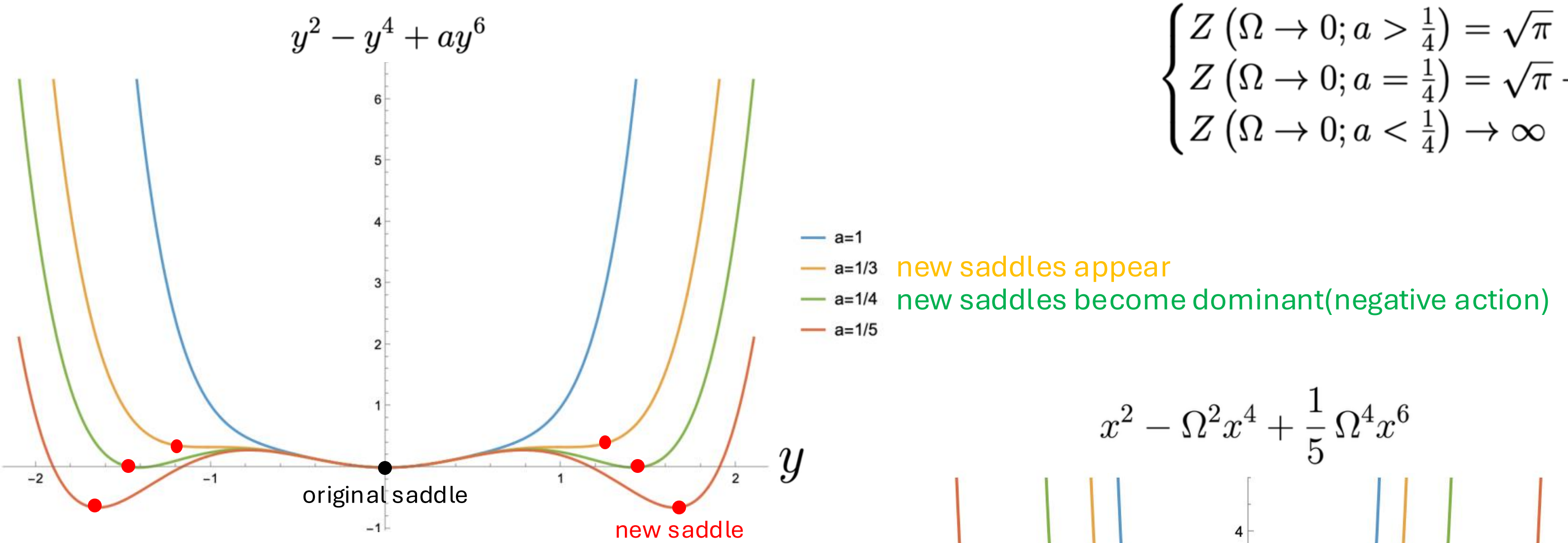
Moore-Nekrasov-Shatashvili 98
Krauth-Nicolai-Staudacher 98
Austing-Wheater 01

Let's first check a toy model.

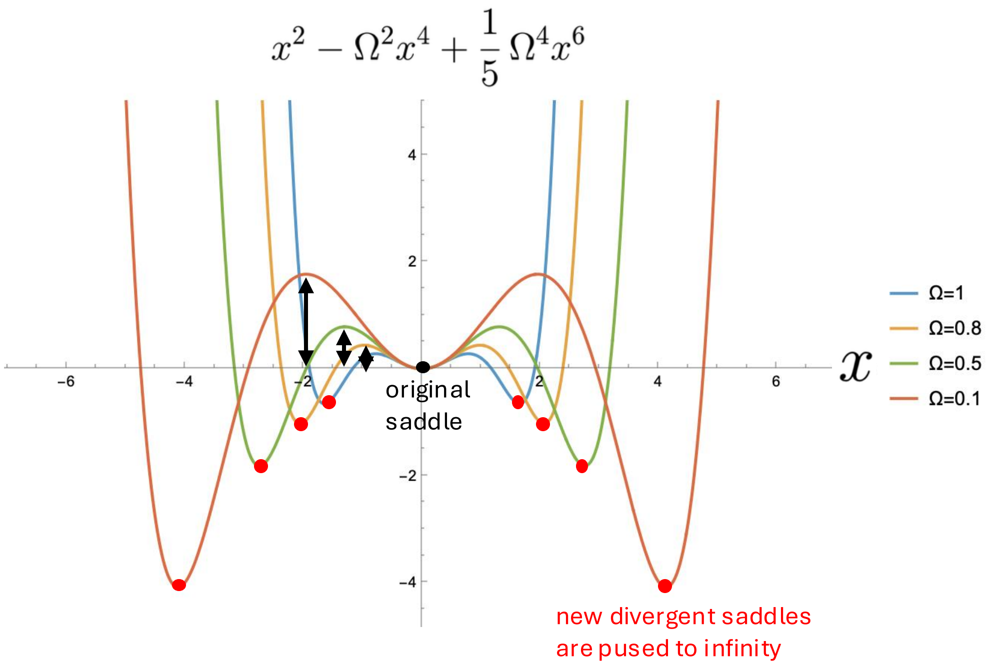
$\Omega \to 0$ divergence(toy model)

$$Z(\Omega;a)=\int_{-\infty}^{\infty}\mathrm{d}x\; e^{-x^2+\Omega^2x^4-a\,\Omega^4x^6}=\frac{1}{\Omega}\int_{-\infty}^{\infty}dy\, e^{-\frac{1}{\Omega^2}(y^2-y^4+ay^6)}$$

$$\begin{cases} Z\left(\Omega\to 0; a>\frac{1}{4}\right)=\sqrt{\pi} \\ Z\left(\Omega\to 0; a=\frac{1}{4}\right)=\sqrt{\pi}+2\sqrt{\pi/4}=2\sqrt{\pi} \\ Z\left(\Omega\to 0; a<\frac{1}{4}\right)\to\infty \end{cases}$$



new divergent saddle cannot be accessed by the original saddle through perturbation



$\Omega = 0$ convergence

Aoki-Iso-Kawai-Kitazawa-Tada 98
Hotta-Nishimura-Tsuchiya 98

$$S_{\rm IKKT} = {\rm tr}\left\{-\frac{1}{4}[A_\mu, A_\nu]^2 - \frac{i}{2}\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]\right\}$$

The original IKKT vacuum moduli, $A_\mu = {\rm diag}\left(x_\mu^{(1)}, x_\mu^{(2)}, \cdots, x_\mu^{(N)}\right)$

Decompose matrices into space + force carrier,

$$A = x + a = \begin{pmatrix} x^{(1)} & a_{12} & a_{13} & \dots & a_{1N} \\ a_{21} & x^{(2)} & a_{23} & \dots & a_{2N} \\ a_{31} & a_{32} & x^{(3)} & \dots & a_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & a_{N3} & \dots & x^{(N)} \end{pmatrix} \quad \Psi = \xi + \psi = \begin{pmatrix} \xi^{(1)} & \psi_{12} & \psi_{13} & \dots & \psi_{1N} \\ \psi_{21} & \xi^{(2)} & \psi_{23} & \dots & \psi_{2N} \\ \psi_{31} & \psi_{32} & \xi^{(3)} & \dots & \psi_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \psi_{N1} & \psi_{N2} & \psi_{N3} & \dots & \xi^{(N)} \end{pmatrix}$$

$S_{\rm space}$ from integrating out a and ψ,

$$\langle aa\rangle \sim \frac{1}{\Delta^2}, \quad \langle\psi\psi\rangle \sim \frac{1}{\Delta}$$

$$Z = \int dx da\, d\xi d\psi\, e^{-S(A=x+a, \Psi=\xi+\psi)}$$
$$= \int dx_\mu^{(i)} d\xi_\alpha^{(i)} e^{-S_{\rm ss.}\left(x_\mu^{(i)}, \xi_\alpha^{(i)}\right)} = \int dx_\mu^{(i)}\, e^{-S_{\rm space}\left(x_\mu^{(i)}\right)}$$

off-diagonal one-loop cancel
due to SUSY
c.f. super YM

$$S_{\rm ss.} \sim \mathcal{O}\left(\Xi\frac{1}{\Delta^3}\Xi\right)^4 + \mathcal{O}\left(\Xi\frac{1}{\Delta^3}\Xi\right)^8$$

→ A strong attractive force from ξ
→ small emergent spacetime

$$\Xi_{ij} = \xi^{(i)} - \xi^{(j)}, \Delta_{ij} = x^{(i)} - x^{(j)}$$

$\Omega \to 0$ divergence

Austing 01
Komatsu-Martina-Penedones-Vuignier-Zhao 24
Chou-Nishimura-CTW 25

$$S_{\text{IKKT}} = \text{tr}\left\{-\frac{1}{4}[A_\mu, A_\nu]^2 - \frac{i}{2}\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]\right\}$$

$$S_\Omega = \text{Tr}\left\{\frac{\Omega^2}{8^2}(3A_a^2 + A_I^2) + i\,\Omega[A_1, A_2]A_3 - \frac{\Omega}{8}\Psi_\alpha(\mathcal{C}\Gamma^{123})_{\alpha\beta}\Psi_\beta\right\}$$

If Ω is turned on, there appear the classical parts

$$S_{\text{ss.}} \sim \mathcal{O}(\Omega^2 x^2) + \mathcal{O}(\Omega\xi\xi) + \mathcal{O}\left(\Xi\frac{1}{\Delta^3}\Xi\right)^4 + \mathcal{O}\left(\Xi\frac{1}{\Delta^3}\Xi\right)^8$$

$x \sim 1/\Omega$

$\sim O(\Omega^3\xi\xi)$

and the classical parts dominate the superspace effective action.

$$S_{\text{ss.}} \sim \mathcal{O}(\Omega^2 x^2) + \mathcal{O}(\Omega\xi\xi)$$

$\to \xi$ decouple from x

$$Z_{\text{1-loop}} = \Omega^{8(N-1)} \int dx \exp\left\{-\frac{\Omega^2}{2^6}\left(3\left(x_a^{(i)}\right)^2 + \left(x_I^{(i)}\right)^2\right)\right\}$$

the exact divergece behavior predicted from localization.

$\to$ large (anisotropic) spacetime emerges

Let's see check this from saddle point analysis

saddle point analysis

Chou-Nishimura-CTW 25
Chau-Chou-Nishimura-CTW (to appear)

saddle point equation

$$Z(\Omega) = \int dA d\Psi e^{-S(A,\Psi;\Omega)} = \int dA\, \mathrm{Pf}(M_\Omega(A))\, e^{-S_b(A;\Omega)} = \int dA\, e^{-S_{\mathrm{eff.}}(A;\Omega)}$$

effective action : $S_{\mathrm{eff.}}(A;\Omega) = S_b(A;\Omega) - \ln(\mathrm{Pf}(M_\Omega(A)))$

MW fermion kernel : $M_\Omega(A) = i[A_\mu\,,\,\cdot\,] \otimes \Gamma^\mu + \underline{\frac{\Omega}{4} \mathbb{1}_{\mathrm{adj}} \otimes \Gamma^{123}}$

fermion mass regulates
the zero modes of Dirac operator

saddle point equation :

$$0 = \frac{dS_{\mathrm{eff.}}}{dA} = \frac{dS_b}{dA} \underline{- \frac{1}{2} \mathrm{Tr}\left(M_\Omega^{-1} \frac{dM_\Omega}{dA} \right)}$$

⇕ fermion loop back reaction ~ O(1/A)

$$[A_\mu, [A_\mu, A_a]] + \frac{3\Omega^2}{32} A_a + i\Omega\epsilon_{abc} A_b A_c + \underline{\langle i \left(\mathcal{C}\Gamma^a\right)_{\alpha\beta} \Psi_\alpha \Psi_\beta \rangle} = 0\,,$$

$$[A_\mu, [A_\mu, A_I]] + \frac{\Omega^2}{32} A_I + \underline{\langle i \left(\mathcal{C}\Gamma^I\right)_{\alpha\beta} \Psi_\alpha \Psi_\beta \rangle} = 0$$

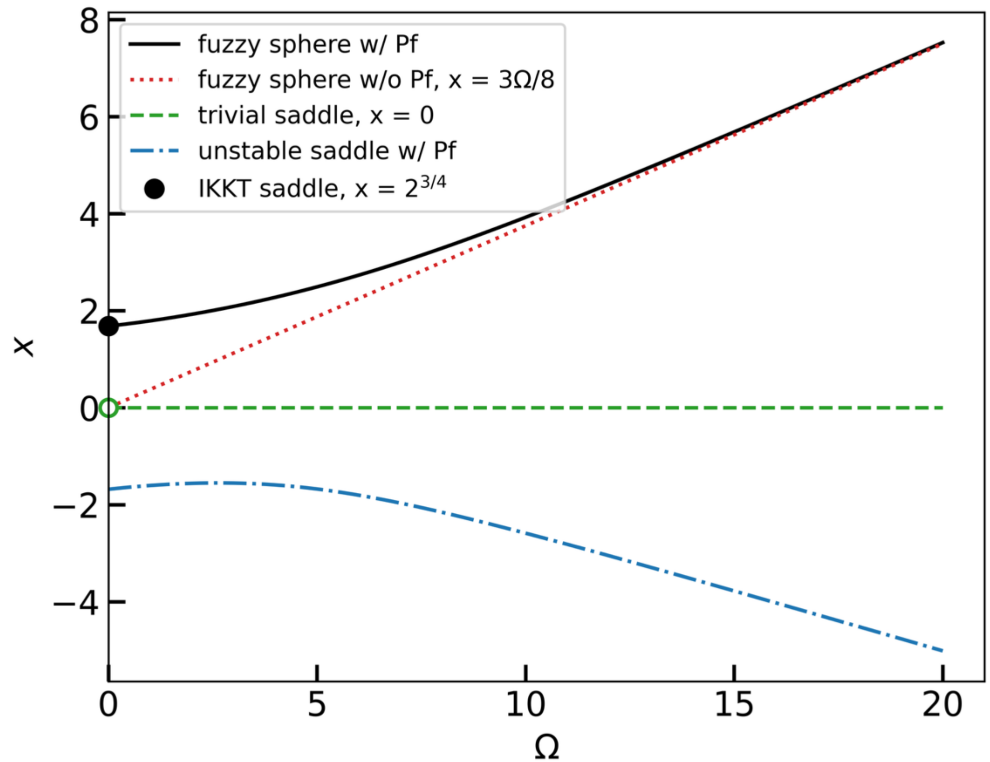
Let’s see how fuzzy sphere get corrected.

fuzzy sphere saddle

$\underline{N=2}$

ansatz : $A_a = xJ_a\,,\ A_I = 0$

e.o.m : $\frac{3}{64}x(8x-3\Omega)(8x-\Omega) - 64\left(\frac{1}{8x-\Omega} + \frac{1}{4x+\Omega}\right) = 0$



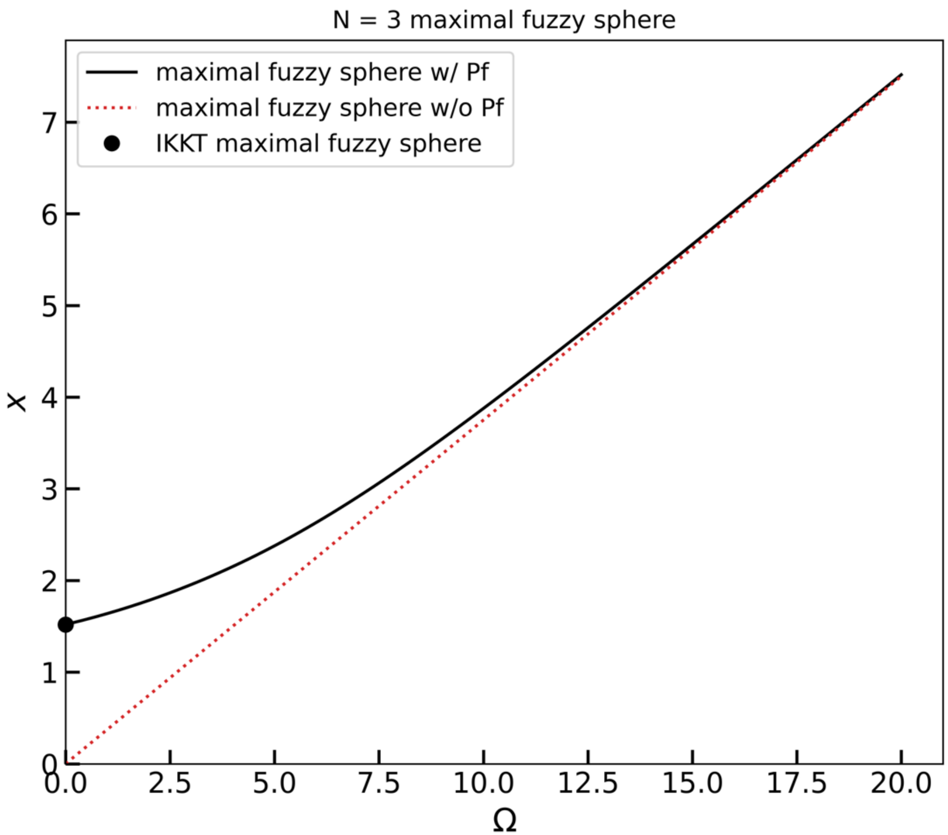
remakrs :

1. IKKT saddle contains fuzzy sphere
2. polarized IKKT saddle is smoothly connected to IKKT saddle

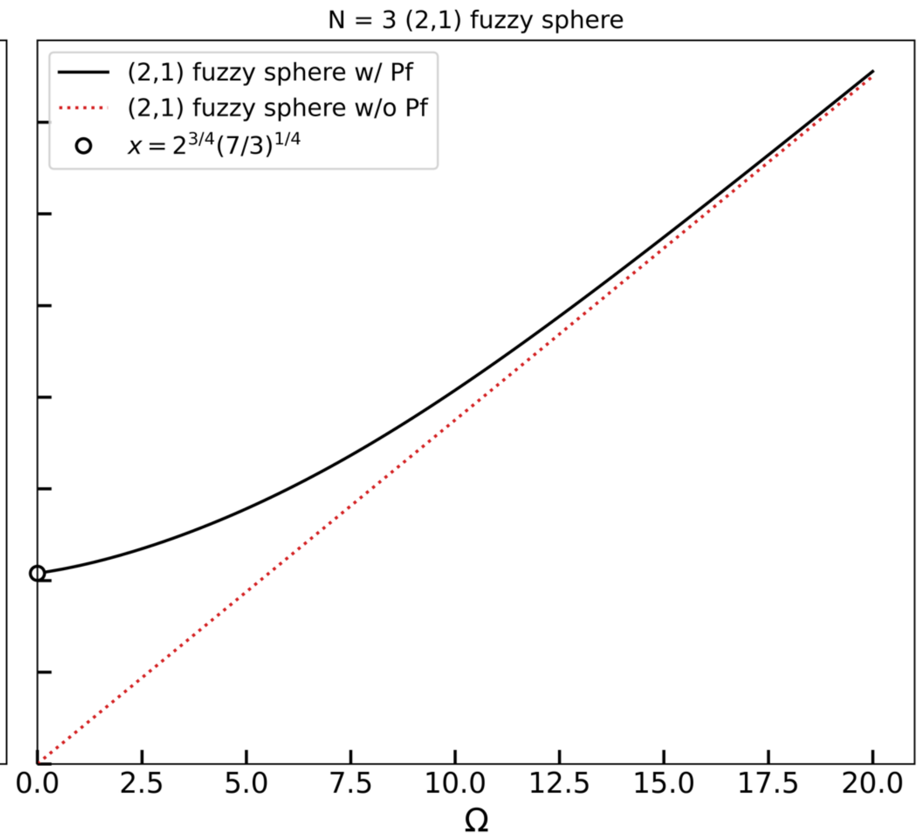
fuzzy sphere saddle

$\underline{N = 3}$

$$J_3 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix}$$



$$J_3 = \begin{pmatrix} \frac{1}{2} & & \\ & -\frac{1}{2} & \\ & & 0 \end{pmatrix}$$



remakrs :

1. The result also holds for $N = 3$
2. The reducible representation has finite radius but divergent $S_{\text{eff.}}$

due to the zero mode in the fermion kernel

commuting-diverging saddle

schemetically, the saddle point equation reads

$$\mathcal{O}(A)^3 + \Omega\,\mathcal{O}(A)^2 + \Omega^2\,\mathcal{O}(A) + \mathcal{O}(1/A) = 0$$

YM CS mass pfaffian

motivated by toy model, we try an ansatz with $A \sim \Omega^{-P}$

$$\mathcal{O}\left(\Omega^{-3P}\right) + \mathcal{O}\left(\Omega^{-2P+1}\right) + \mathcal{O}\left(\Omega^{-P+2}\right) + \mathcal{O}\left(\Omega^{P}\right) = 0$$

YM CS mass pfaffian

mass $\propto A$

pfaffian must exist, otherwise reduced back to cl. eom

The contribution from Yang-Mills and Chern-Simons must vanish!

The diverging saddle must be commuting and come from mass term and pfaffian.

commuting-diverging saddle

$\underline{N = 2}$

general ansatz : $A_3 = x\dfrac{\sigma_3}{2}\,,\ A_{10} = y\dfrac{\sigma_3}{2}\,,\ \text{others} = 0$

soln. : $(x, y) = \left(\dfrac{1}{4\Omega}\sqrt{2^{14}/3 + \Omega^4}\,,\,0\right), \left(0\,,\ \dfrac{1}{4\Omega}\sqrt{2^{14} - \Omega^4}\right)$

$\underline{N = 3}$

general ansatz : $A^3_\mu = (x_1, 0, 0, x_2, 0, 0, 0, 0, 0, 0)$

$A^8_\mu = (x_3, x_4, 0, x_5, x_6, 0, 0, 0, 0, 0)$

soln. : many

e.g. ansatz :

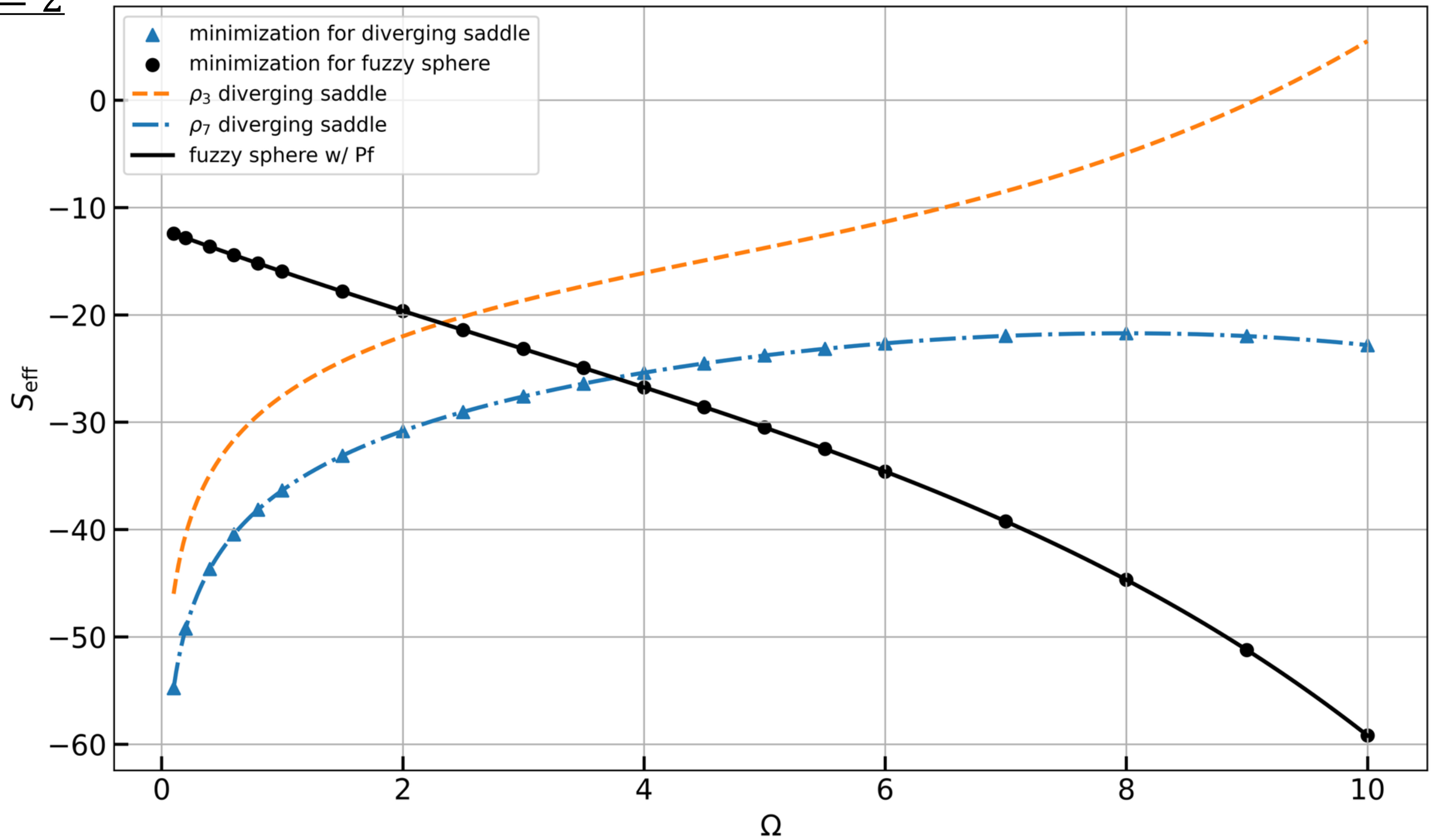
$x_2 = x\,, x_6 = y\,,\ \text{others} = 0$

soln. : $(x, y) = \left(0, \dfrac{\sqrt{24576 - \Omega^4}}{2\sqrt{3}\Omega}\right)\,, \quad \left(\dfrac{\sqrt{-5\Omega^2 + \frac{49152}{\Omega^2} \pm \frac{\sqrt{3}\sqrt{3\Omega^8 - 32768\Omega^4 + 805306368}}{\Omega^2}}}{4\sqrt{2}}, 0\right)$

$\left(\dfrac{\sqrt{24576 - \Omega^4}}{4\Omega}, \dfrac{\sqrt{24576 - \Omega^4}}{4\Omega}\right)\,,$

minimization

$\underline{N = 2}$

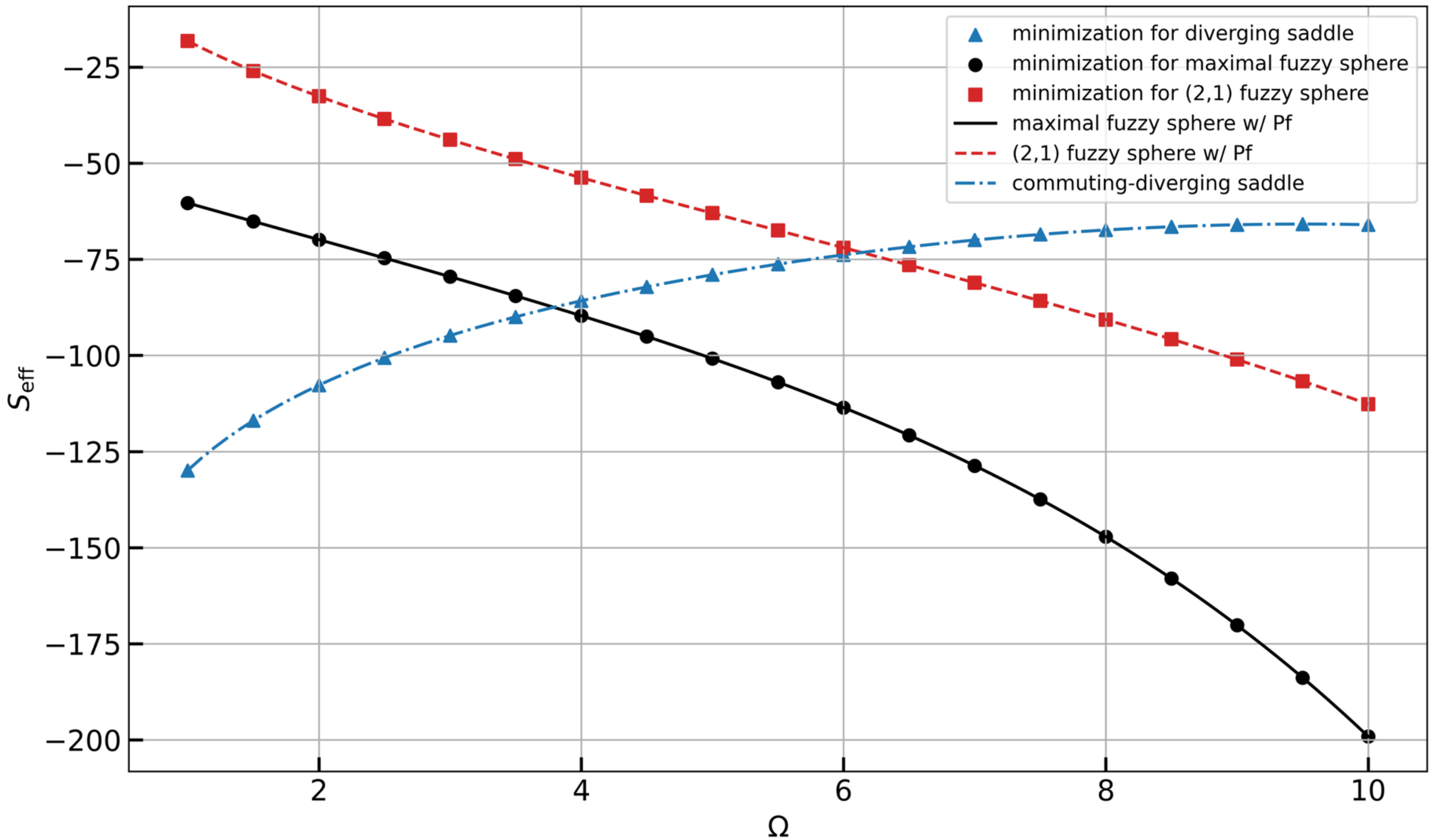


remakrs :

1. fuzzy sphere saddle dominates at large Ω
2. commuting-diverging saddle dominates at small Ω

minimization

$\underline{N = 3}$



remakrs :

1. maximal fuzzy sphere saddle dominates at large Ω
2. commuting-diverging saddle dominates at small Ω
3. (2,1) fuzzy sphere never dominates

Monte Carlo results

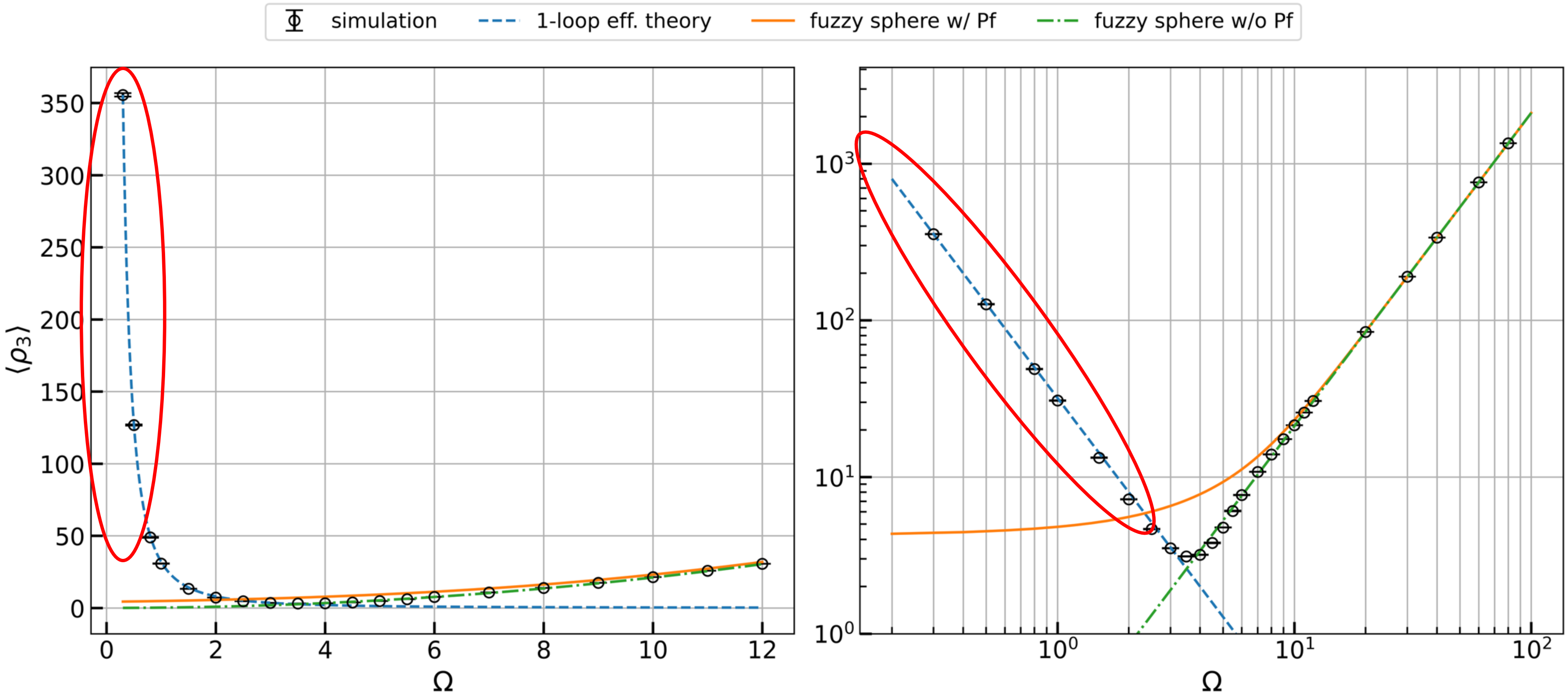
Chou-Nishimura-CTW 25
Chau-Chou-Nishimura-CTW (to appear)

large spacetime emergence

spacetime extent - polarized

$$\rho_3 = \mathrm{Tr}\, A_a^2$$

$a = 1, 2, 3$



remakrs :

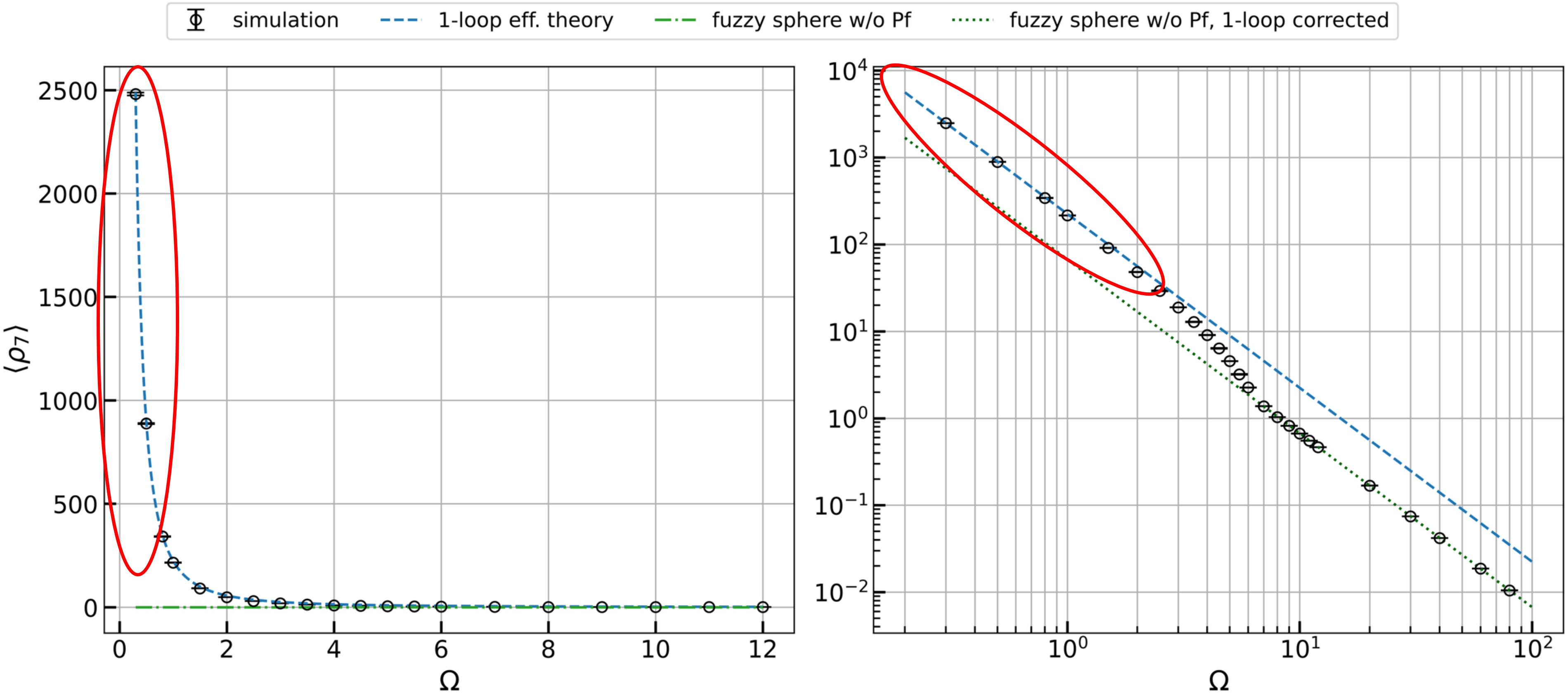
1. $\Omega \to 0$ divergence (well described by 1-loop eff. theory)
2. fuzzy sphere saddle is not favored in small Ω (though it exists)

spacetime extent - unpolarized

large spacetime emergence

$$\rho_7 = \mathrm{Tr}\, A_I^2$$

$I = 4, \cdots, 10$



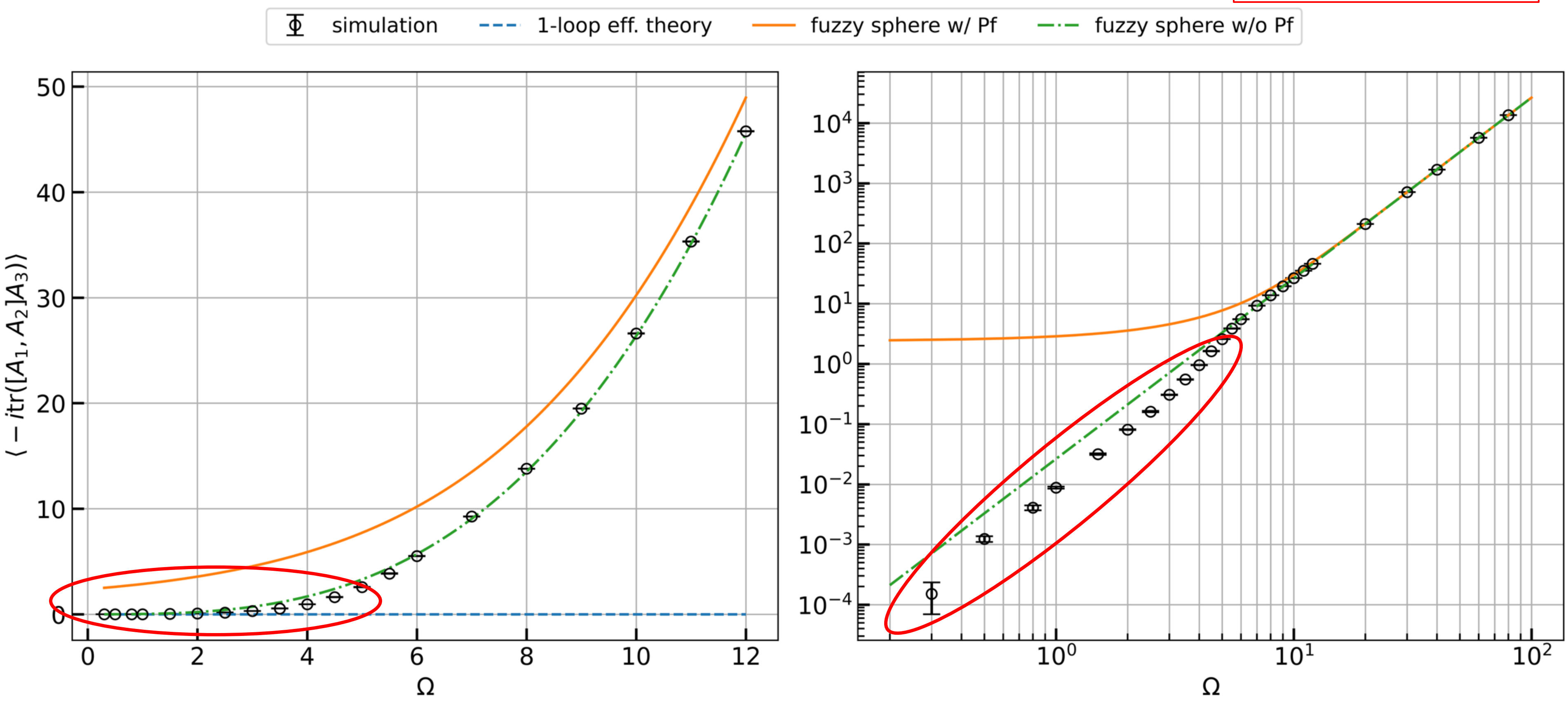
remakrs :

$\Omega \to 0$ divergence (well described by 1-loop eff. theory)

spacetime non-commutativity

Chern-Simons observable

$-i\,\mathrm{Tr}([A_1, A_2]A_3)$

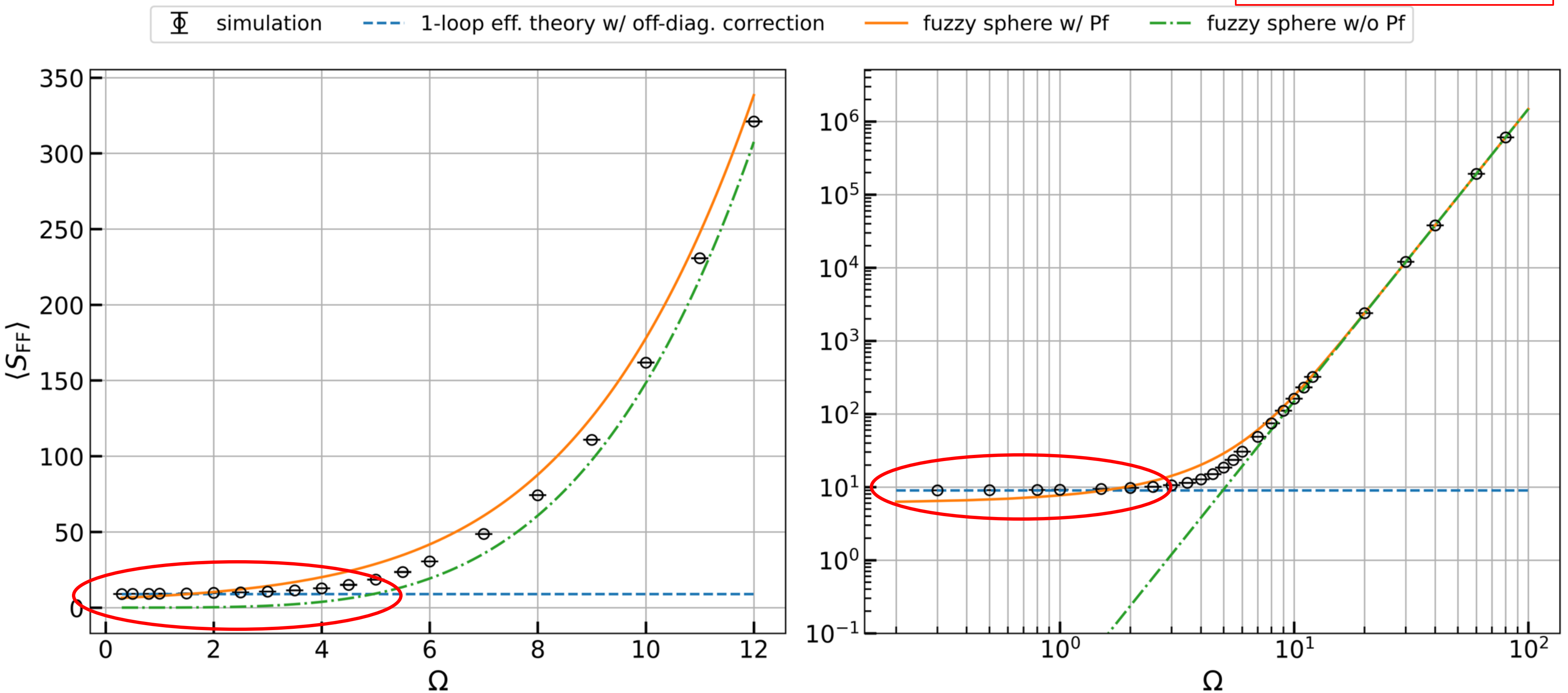


remakrs :

Chern-Simons observable decays to 0 in small Ω

spacetime non-commutativity

Yang-Mills observable $-\frac{1}{4}\mathrm{Tr}[A_\mu, A_\nu]^2$



remakrs :

Yang-Mills observable asymptotes to a constant, $\frac{9}{2}(\mathrm{N}^2 - \mathrm{N})$, in small Ω

(account for the off-diagonal modes kinetic energy)

Summary & Outlook

Summary

- $\Omega \to 0$ divergence $\leftarrow$ fermion zero modes decouple

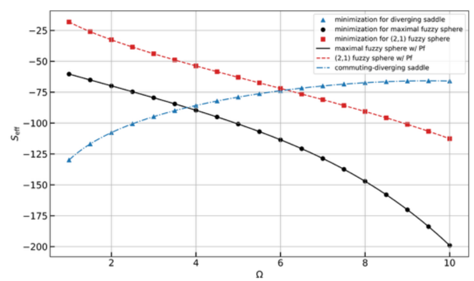
  spacetime 1-loop. eff. theory : a simple Gaussian

$$Z_{1\text{-loop}} = \Omega^{8(N-1)} \int dx \exp\left\{ -\frac{\Omega^2}{2^6} \left(3 \left(x_a^{(i)} \right)^2 + \left(x_I^{(i)} \right)^2 \right) \right\}$$

- identify all the relevant saddle points

  small Ω commuting phase $\leftrightarrow$ 1-loop. eff. theory (diverging saddle)

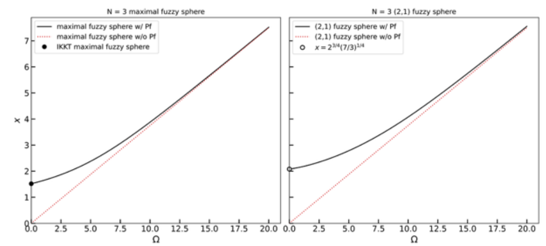
  large Ω fuzzy sphere phase $\leftrightarrow$ fuzzy sphere



- A way to retreat to IKKT

  the fuzzy sphere saddles

  interpolating the BPS and the IKKT fuzzy sphere



Outlook

- Lorentzian polarized IKKT

  Does such gap and divergence still happen?

  A different mechanism of emergence spacetime?

  Hirasawa-Anagnostopoulos-Azuma-Hatakeyama-Nishimura-Papadoudis-Tsuchiya 24

  Chou-Nishimura-Tripathi 25

- SSB in the original Euclidean IKKT

  Can the concentric fuzzy sphere at $\Omega = 0$ explain

  the $\mathrm{SO}(10) \rightarrow \mathrm{SO}(3)$?

  Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis 02

- A natrual realization of Eguchi-Kawai reduction

  $\mathrm{U}(1)^{\mathrm{D}}$ restoration due to the decoupling of fermionic zero mode

  no need of quenching

  Eguchi-Kawai 82

  Gross-Kitazawa 82

Thank you & Happy Christmas 🎄