No-boundary Proposal and Hořava-Lifshitz Gravity

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[H. Matsui, and S. Mukohyama, Phys. Rev. D 109, 023504 (2024)]

Basic Idea of Our Work

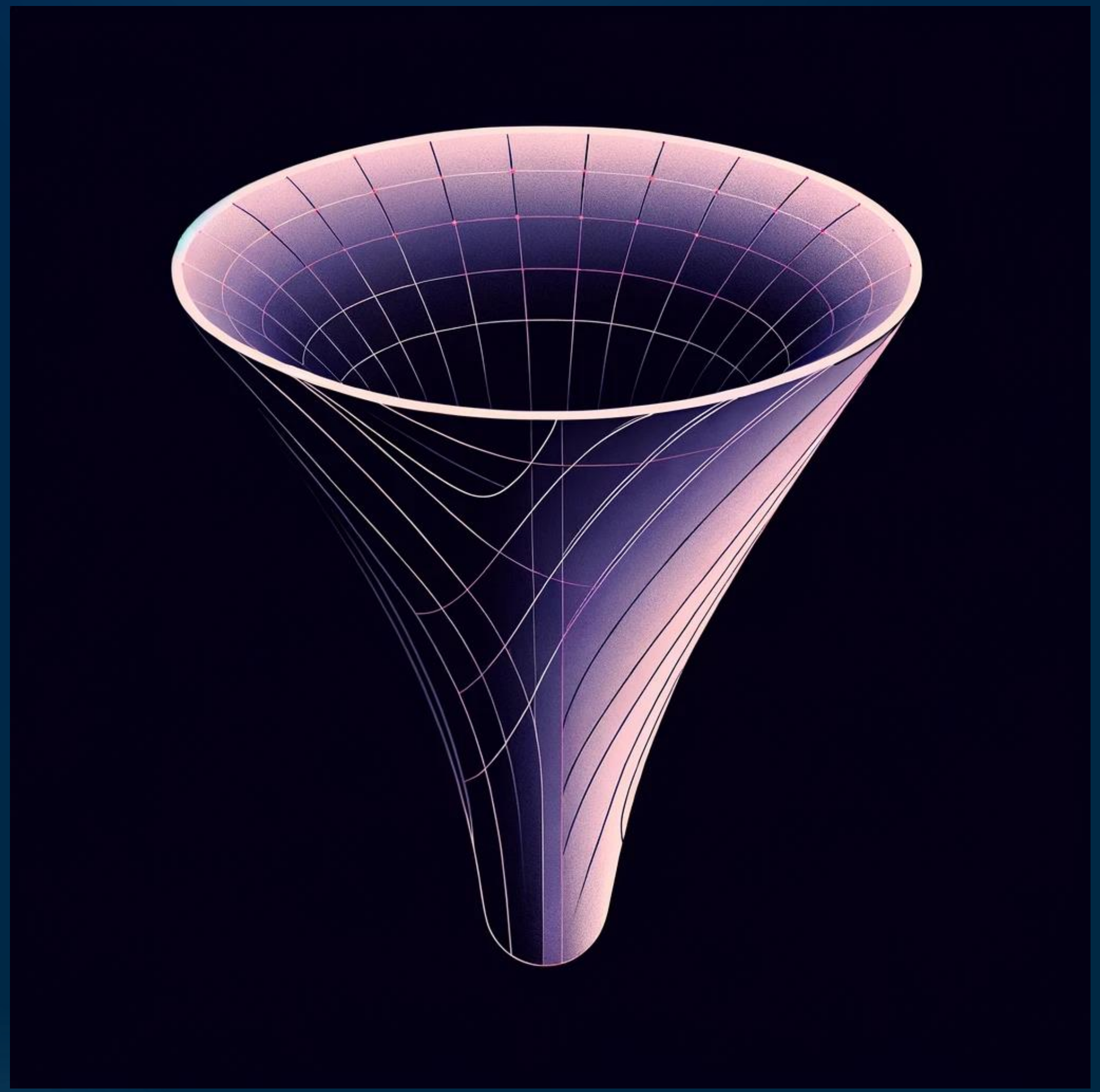
No-boundary/Tunneling Proposal

which describes the quantum creation of the universe

✖

Horava-Lifshitz Gravity

Gravity theories satisfying renormalizability/unitarity.



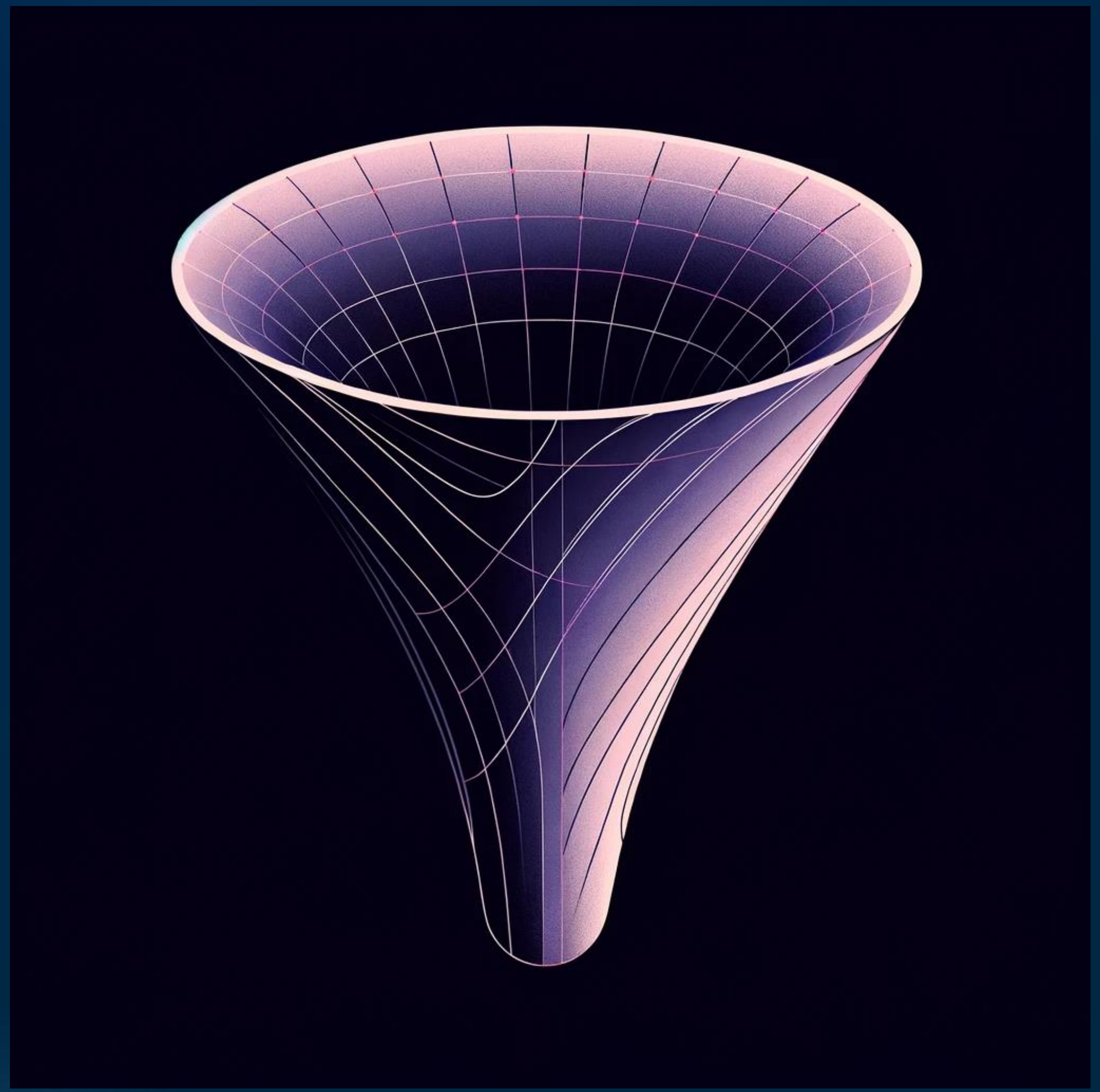
Outline of Our Work

No-boundary/Tunneling Proposal ✖ Horava-Lifshitz Gravity

$$G\left[q(t_f);q(t_i)\right]=\sqrt{\frac{i\mathcal{V}}{2\pi\hbar}}\int\frac{\mathrm{d}N}{N^{1/2}}\exp\left(\frac{iS_{\mathrm{on-shell}}[q(t_i),q(t_f),N]}{\hbar}\right)$$

H. Matsui, and S. Mukohyama,
Phys. Rev. D 109, 023504 (2024)

$q(t_i)\to 0$ ∞

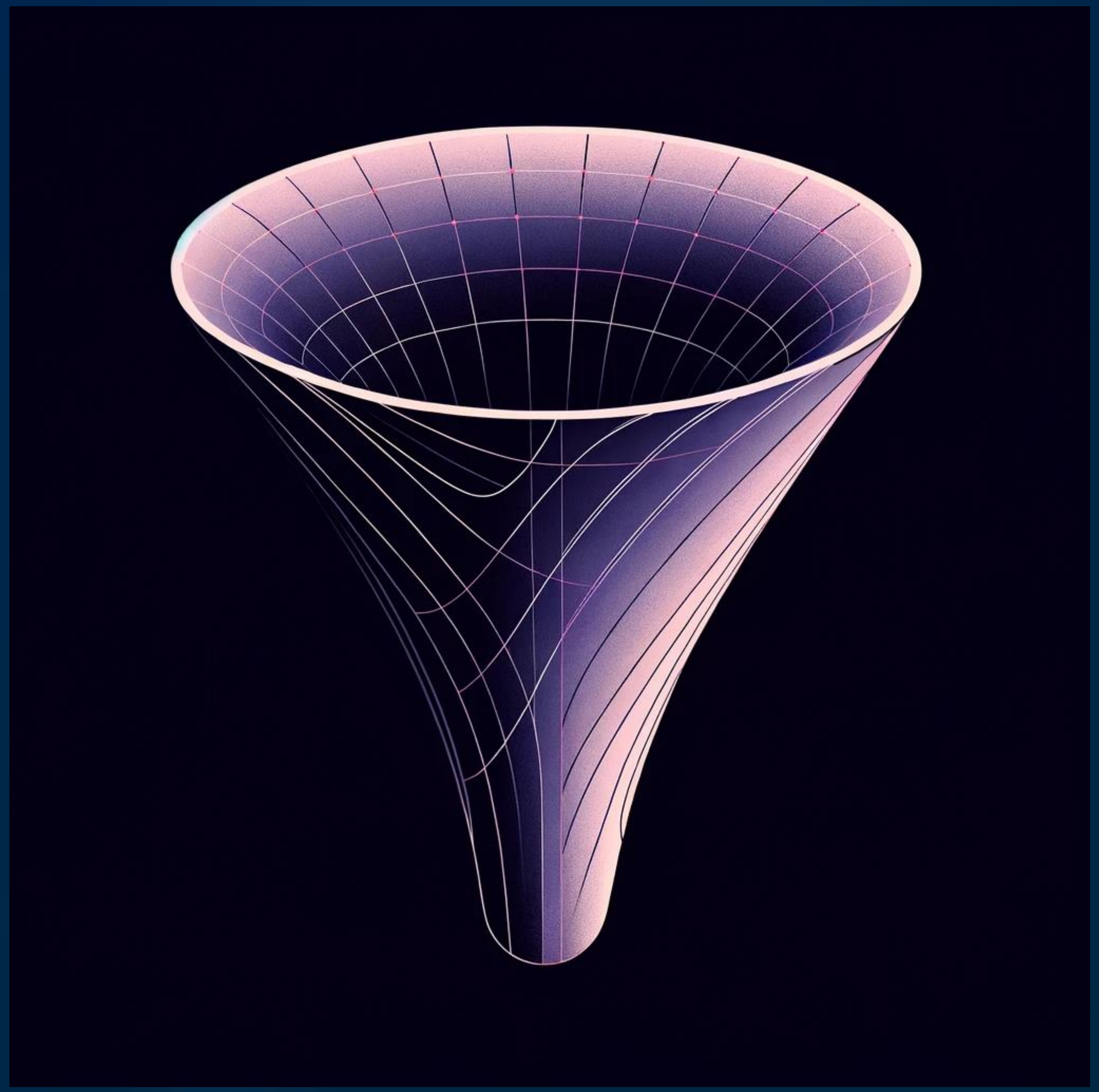


1. Renormalization group (RG) flow of Gravity
2. Neumann or Robin boundary conditions

Talk plan

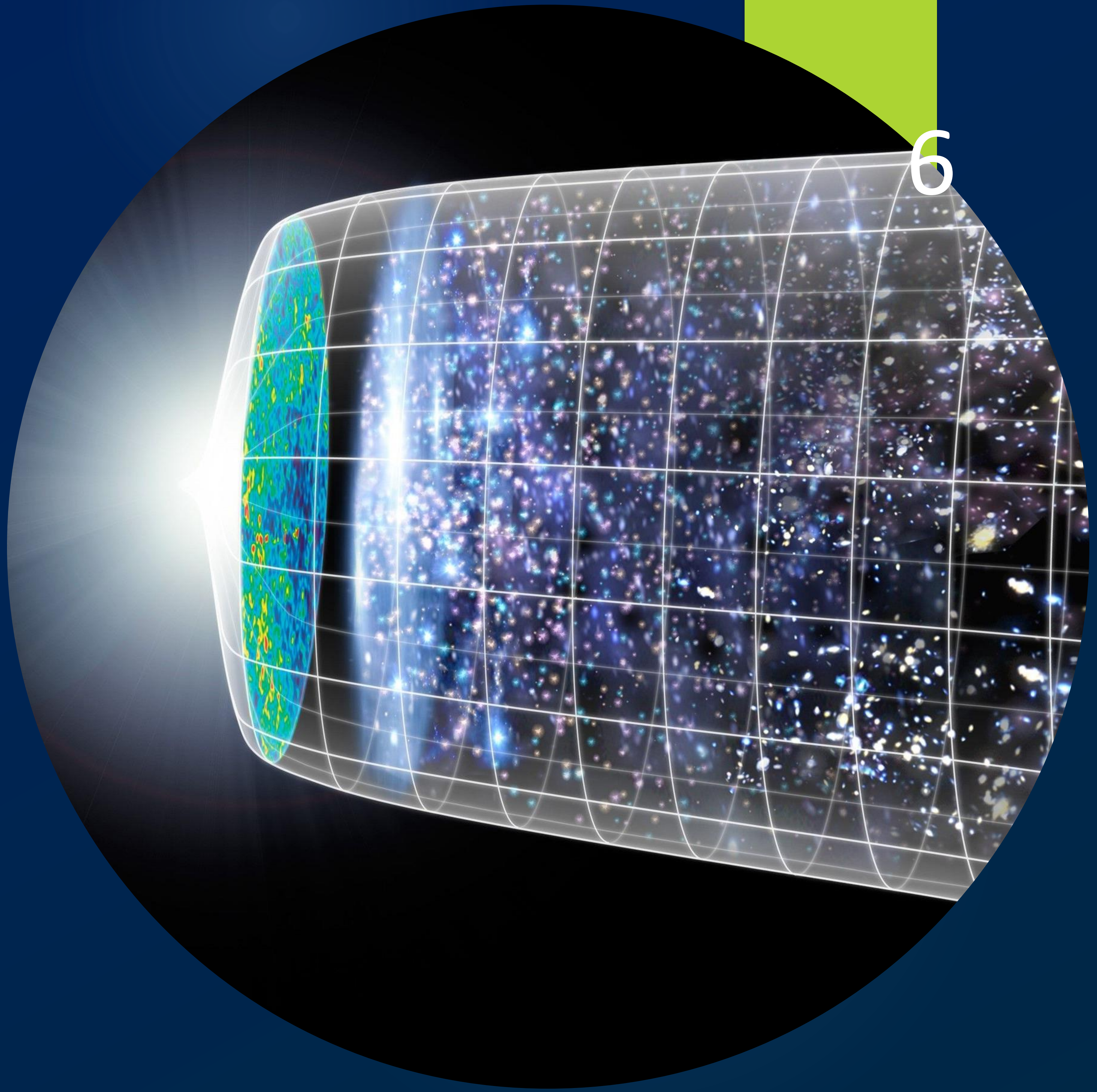
In this talk, I will discuss the application of Horava–Lifshitz gravity — a candidate quantum gravity theory — to the no-boundary and tunneling proposal, which describe the quantum creation of the universe.

1. Review of Quantum Cosmology
2. Review of Horava–Lifshitz gravity
3. Application of HL gravity to no-boundary and tunneling proposal
4. Problems on HL no-boundary proposal: RG flow of Gravity, Robin boundary condition



Quantum Cosmology

Quantum Cosmology



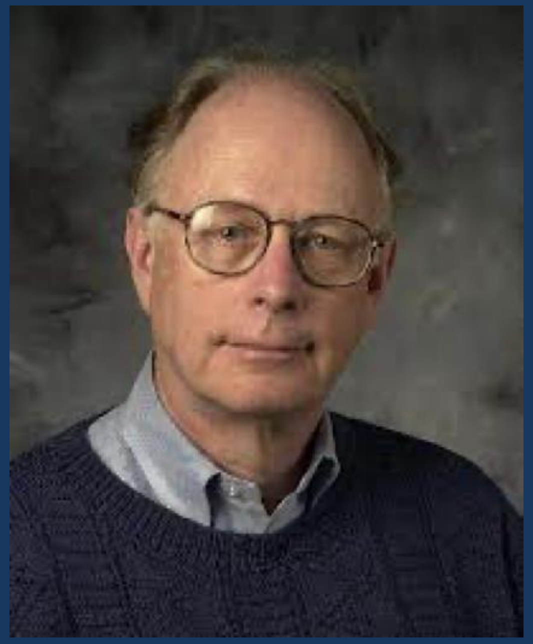
Quantum cosmology analyses how the universe was created and evolved in quantum gravity theory. It introduces the wave function of the universe.

Wave function of the Universe

$$\mathcal{H}\Psi = \left[-16\pi G_N \mathcal{G}_{ijkl}\frac{\partial^2}{\partial g_{ij}\partial g_{kl}} + \frac{\sqrt{g}}{16\pi G_N}(-R+2\Lambda)\right]\Psi = 0$$

$$\Psi(g,\phi) = \int^{(g,\phi)} \mathcal{D}g_{\mu\nu}\mathcal{D}\phi e^{iS[g_{\mu\nu},\phi]/\hbar}$$

Hartle-Hawking no-boundary proposal



Stephen Hawking **James B. Hartle**

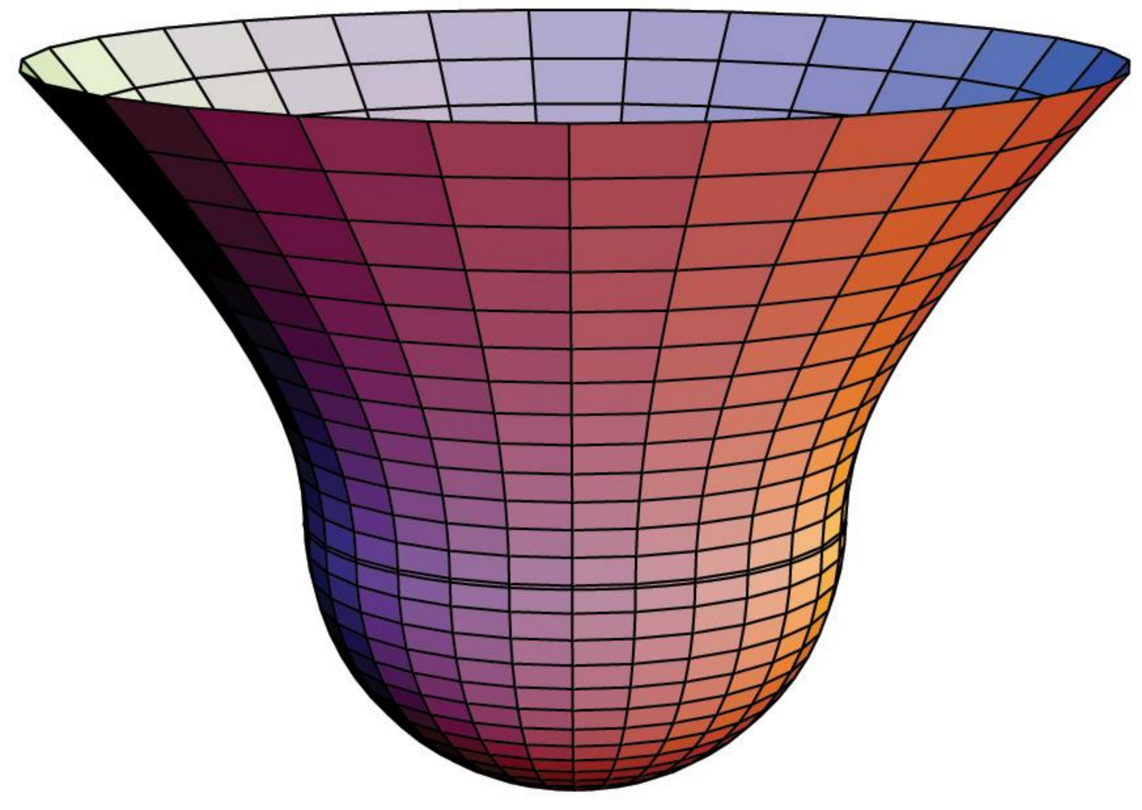
No-boundary wave function
which describes the universe created from nothing

Phys. Rev. D 28 (1983) 2960-2975

$\Longleftrightarrow$

Euclidean Path Integral of Gravity
which integrates the compact and regular Euclidean metric

$$\Psi_{\mathrm{HH}} = \int_{\text{no-boundary}}^{(g,\phi)} \mathcal{D}g_{\mu\nu}\mathcal{D}\phi\; e^{-S_E[g_{\mu\nu},\phi]/\hbar}$$



Conformal factor problem

$$S_E[g_{\mu\nu},\phi] < 0$$

Euclidean action

$$\int \mathcal{D}a\mathcal{D}\phi\, e^{-S_E[a,\phi]/\hbar} \to +\infty$$

Lorentzian Quantum Cosmology

J. Feldbrugge

J. Lehners

N. Turok

Phys. Rev. D 95 (2017) 103508

No-boundary wave function which describes the universe created from nothing

$$\Psi_{\mathrm{HH}} = \int_{\text{no-boundary}}^{(g,\phi)} \mathcal{D}g_{\mu\nu}\mathcal{D}\phi \; e^{-S_E[g_{\mu\nu},\phi]/\hbar}$$

Lorentzian Path Integral of Gravity

which integrates real and Lorentzian metric

Lorentzian Quantum Cosmology

$$G[q_f] = \int \mathrm{d}N \int_{q(t=t_i)=0}^{q(t=t_f)=q_f} \mathcal{D}q \exp\left(iS_{\mathrm{GR}}[N,q]/\hbar\right)$$

Euclidean → Lorentzian

Horava-Lifshitz Gravity

Horava-Lifshitz Gravity

Higher derivative gravity theory (proposed by P. Horava in 2009), a candidate quantum gravity theory satisfying renormalizability and unitarity

Anisotropic scaling between space and time

Phys. Rev. D 79 (2009) 084008

$$\begin{cases} t \to b^{z} t \\ \vec{x} \to b\vec{x} \end{cases} \implies \text{Lorentz-invariance is broken at UV (for } z \neq 1)$$

Power counting renormalizability at UV (z=3)

$$\frac{1}{2}\int dt d^3x \left[\dot{h}^2 - h\left(-\Delta\right)^z h\right] \implies h \to b^{\frac{z-3}{2}} h, \ (z=3)$$

Higher derivative terms in spatial direction

$$c_1 \nabla_i R_{jk} \nabla^i R^{jk} + c_2 \nabla_i R \nabla^i R + c_3 R_i^j R_j^k R_k^i + c_4 R R_i^j R_j^i + c_5 R^3) + (c_6 R_i^j R_j^i + c_7 R^2)$$

Projectable Horava-Lifshitz Gravity

3 + 1 dimensional metric as ADM formalism

$$\mathrm{d}s^2 = N^2\mathrm{d}t^2 + g_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$$

Lapse function   Spatial metric   Shift vector

Projectable HL gravity

$$N = N(t)\ , \quad N^i = N^i(t,\mathbf{x})\ , \quad g_{ij} = g_{ij}(t,\mathbf{x})$$

Non-projectable HL gravity

$$N = N(t,\mathbf{x})\ , \quad N^i = N^i(t,\mathbf{x})\ , \quad g_{ij} = g_{ij}(t,\mathbf{x})\ .$$

3 + 1 dimensional action of projectable HL gravity

$$S = \frac{\mathcal{M}_{\mathrm{HL}}^2}{2}\int \mathrm{d}t\mathrm{d}^3x N\sqrt{g}\left(K^{ij}K_{ij} - \lambda K^2 + c_g^2 R - 2\Lambda + \mathcal{O}_{z>1}\right)$$

Higher dimensional operators at UV (z=3)

$$\frac{\mathcal{O}_{z>1}}{2} = c_1\nabla_i R_{jk}\nabla^i R^{jk} + c_2\nabla_i R\nabla^i R + c_3 R_i^j R_j^k R_k^i + c_4 R R_i^j R_j^i + c_5 R^3 + c_6 R_i^j R_j^i + c_7 R^2$$

Anisotropic scaling

$$t \to b^z t\ , \quad \vec{x} \to b\vec{x}$$

Global Hamiltonian Constraint

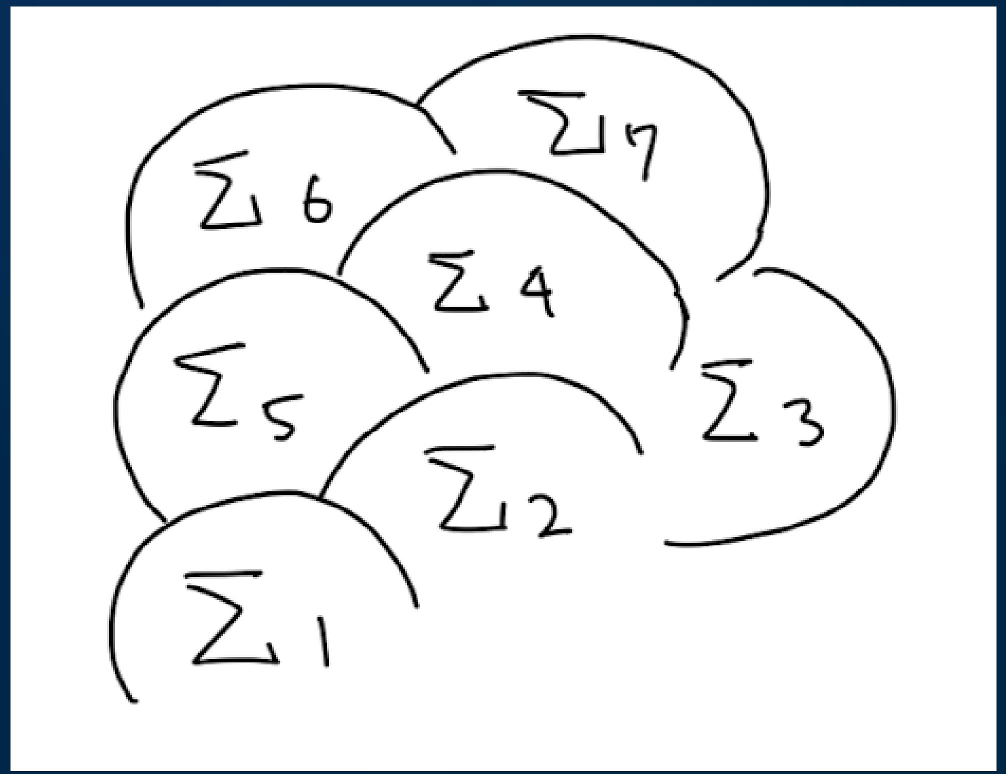
Projectable HL gravity

$$N = N(t)\ , \quad N^i = N^i(t,\mathbf{x})\ , \quad g_{ij} = g_{ij}(t,\mathbf{x})$$

Counting all local 3-dimensional universe $\Sigma\ \alpha$, the global Hamiltonian constraint must be imposed.

Global Hamiltonian constraint

$$\sum_\alpha \int_{\Sigma_\alpha} \mathrm{d}^3x\,\mathcal{H}_{g\perp} = 0 \qquad \mathcal{H}_{g\perp} = \frac{\mathcal{M}_{\mathrm{HL}}^2}{2}\sqrt{g}(K^{ij}K_{ij} - \lambda K^2 - c_g^2 R + 2\Lambda - \mathcal{O}_{z>1})$$



Local closed FLRW universe

$$N_\alpha^i = 0\,, \quad g_{ij}^\alpha = a_\alpha^2(t)\,[\Omega_{ij}(\mathbf{x})]$$

$$S\,[\{a_\alpha\}, N] = \int_{t_i}^{t_f} \mathrm{d}t \sum_\alpha V(p_{a_\alpha}\dot{a}_\alpha - N\mathcal{H}(\{a_\alpha, p_{a_\alpha}\}))$$

Local Hamiltonian constraint is non-zero

$$\int_{\{\Sigma_\alpha\}} d^3x\,\mathcal{H}_{g\perp} \neq 0$$

Lorentzian Quantum Cosmology

Lorentzian path integral

$$G[\{a_\alpha\}] = \int \mathrm{d}N(t_f - t_i) \int \prod_\alpha \mathcal{D}a_\alpha \mathcal{D}p_{a_\alpha} \exp\left(i \int_{t_i}^{t_f} \mathrm{d}t \sum_\alpha V \left(p_{a_\alpha} \dot{a}_\alpha - N\mathcal{H} \right) / \hbar \right)$$

$$= \int \mathrm{d}N(t_f - t_i) \int \prod_\alpha \mathcal{D}a_\alpha \exp\left(iS[\{a_\alpha\}, N]/\hbar \right)$$

Projectable HL action

$$S\left[\{a_\alpha\}, N\right] = \sum_\alpha V \int_{t_i}^{t_f} \mathrm{d}t \, \left(N a_\alpha^3 \right) \left[\frac{3(1-3\lambda)}{2} \left(\frac{\dot{a}_\alpha}{N a_\alpha} \right)^2 + \frac{\alpha_3}{a_\alpha^6} + \frac{\alpha_2}{a_\alpha^4} + c_{\mathrm{g}}^2 \frac{3}{a_\alpha^2} - \Lambda \right]$$

$$g_K = 3c_{\mathrm{g}}^2 \left(\frac{9}{4} \right)^{\frac{1}{3}} \frac{1}{3(3\lambda - 1)}, \quad g_\Lambda = \Lambda \left(\frac{9}{4} \right) \frac{1}{3(3\lambda - 1)}, \qquad q_\alpha(t) = \frac{2}{3} a_\alpha(t)^{\frac{3}{2}}$$

$$g_2 = \alpha_2 \left(\frac{4}{9} \right)^{\frac{1}{3}} \frac{1}{3(3\lambda - 1)}, \quad g_3 = \alpha_3 \left(\frac{4}{9} \right) \frac{1}{3(3\lambda - 1)}, \quad \mathcal{V} = 3(3\lambda - 1)V,$$

Lorentzian Quantum Cosmology

$$S\left[\{q_\alpha\},N\right]=\sum_\alpha \mathcal{V}\int_{t_i=0}^{t_f=1} N\,dt\left[-\frac{\dot{q}_\alpha^2}{2N^2}+g_K q_\alpha^{\frac{2}{3}}-g_\Lambda q_\alpha^2+g_2 q_\alpha^{-\frac{2}{3}}+g_3 q_\alpha^{-2}\right]+S_B\,.$$

Equation of motion

$$\frac{\ddot{q}_\alpha}{N^2}=-\frac{2}{3}g_K q_\alpha^{-\frac{1}{3}}+2g_\Lambda q_\alpha+\frac{2}{3}g_2 q_\alpha^{-\frac{5}{3}}+2g_3 q_\alpha^{-3}$$

Projectable HL action

$$\frac{\ddot{q}_\alpha}{N^2}-\frac{2g_3}{q_\alpha^3}=0 \qquad q_\alpha(t)=\pm\frac{\sqrt{b_{\alpha1}^2(b_{\alpha2}+t)^2+2g_3N^2}}{\sqrt{b_{\alpha1}}}$$

Lorentzian path integral

$$G\left[\{q_\alpha(t_f)\};\{q_\alpha(t_i)\}\right]=\sqrt{\frac{i\mathcal{V}}{2\pi\hbar}}\int_{0,-\infty}^{\infty}\frac{\mathrm{d}N}{N^{1/2}}\exp\left(\frac{iS_{\text{on-shell}}[\{q_\alpha(t_i),q_\alpha(t_f)\},N]}{\hbar}\right)$$

$$S_{\text{on-shell}}[\{q_\alpha(t)\},N]=\sum_\alpha\mathcal{V}\left\{\sqrt{2g_3}\left(\tan^{-1}\left[\frac{b_{\alpha1}(b_{\alpha2}+1)}{\sqrt{2g_3}N}\right]-\tan^{-1}\left[\frac{b_{\alpha1}b_{\alpha2}}{\sqrt{2g_3}N}\right]\right)-\frac{b_{\alpha1}}{2N}\right\}$$

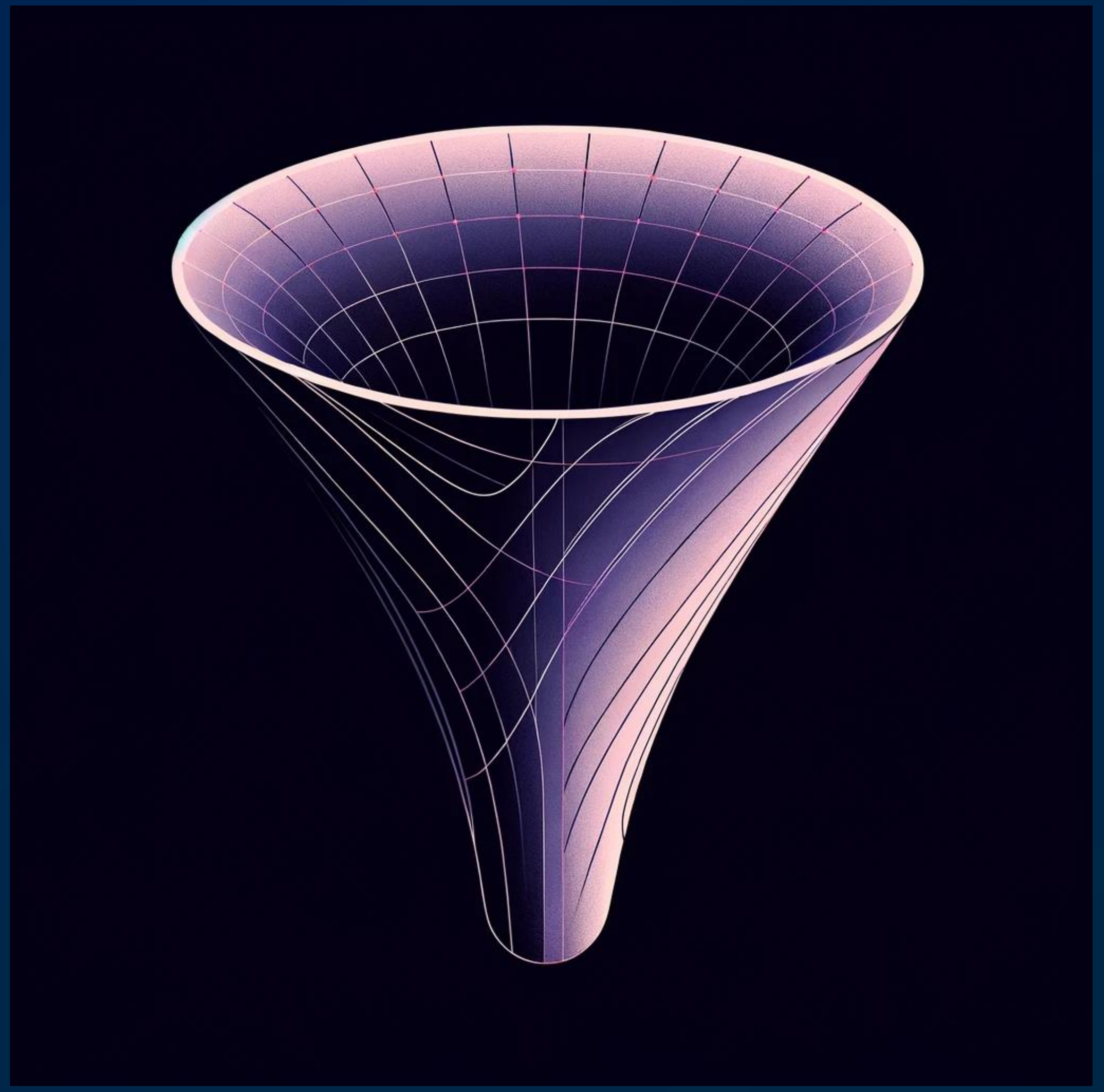
No-boundary/Tunneling proposal

Dirichlet boundary condition

$$q_\alpha \,\Big|_{t_i=0}^{t_f=1} = \text{fixed} \implies -\frac{\mathcal{V}}{N}\dot{q}_\alpha\,\delta q_\alpha\,\Big|_{t_i=0}^{t_f=1} = 0$$

$$q_\alpha(t_i=0) = q_{\alpha i}, \quad q_\alpha(t_f=1) = q_{\alpha f}$$

$q_\alpha(t_f=1) = q_{\alpha f}$



$q_\alpha(t_i=0) = q_{\alpha i}$

On-shell action

$$S_{\text{on-shell}}[\{q_{\alpha i}, q_{\alpha f}\}, N] = \sum_\alpha \mathcal{V}\Bigg\{\mp\sqrt{2g_3}\Bigg(\tan^{-1}\left[\frac{\mp q_{\alpha i}^2 + \sqrt{q_{\alpha i}^2 q_{\alpha f}^2 - 2g_3 N^2}}{\sqrt{2g_3}N}\right] + \tan^{-1}\left[\frac{\mp q_{\alpha f}^2 + \sqrt{q_{\alpha i}^2 q_{\alpha f}^2 - 2g_3 N^2}}{\sqrt{2g_3}N}\right]\Bigg) - \frac{q_{\alpha i}^2 + q_{\alpha f}^2 \mp 2\sqrt{q_{\alpha i}^2 q_{\alpha f}^2 - 2g_3 N^2}}{2N}\Bigg\}$$

Divergent on-shell action

$$\lim_{\epsilon\to 0} S_{\text{on-shell}}[\{\epsilon, q_{\alpha f}\}, N] = \sum_\alpha \mathcal{V}\Bigg\{\sqrt{2g_3}\left(\mp i\tanh^{-1}\left(1 \pm \frac{i q_{\alpha f}^2}{\sqrt{2g_3}N}\right) \mp i\tanh^{-1}(1) \pm i\right) - \frac{q_{\alpha f}^2}{2N}\Bigg\}$$

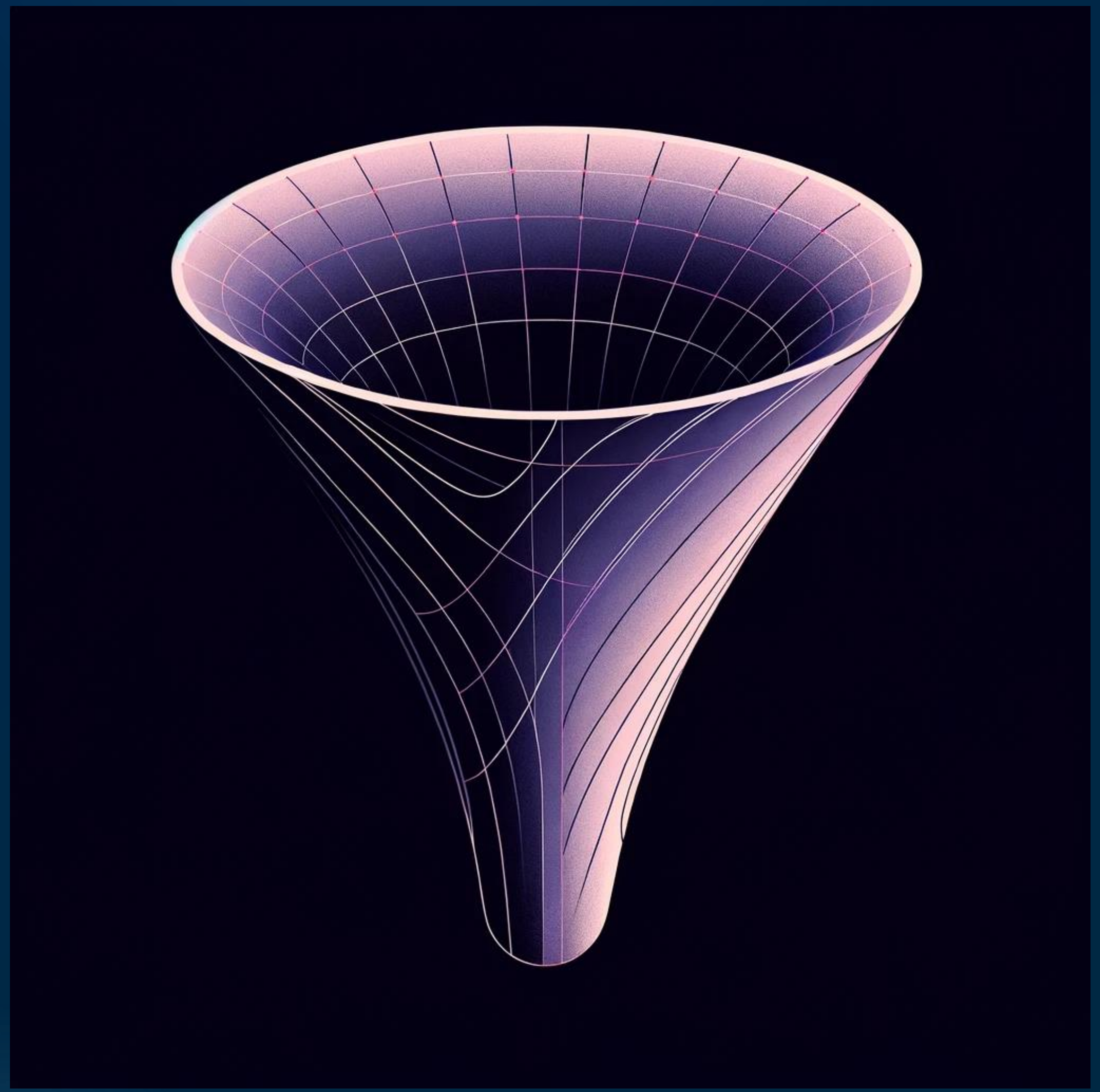
Problems on On-shell Action

No-boundary/Tunneling proposal ✖ **Horava-Lifshitz Gravity**

$$G\left[q(t_f);q(t_i)\right]=\sqrt{\frac{i\,\mathcal{V}}{2\pi\hbar}}\int\frac{\mathrm{d}N}{N^{1/2}}\exp\left(\underbrace{\frac{iS_{\text{on-shell}}[q(t_i),q(t_f),N]}{\hbar}}_{\infty}\right)$$

$$q(t_i)\to 0$$

$$\lim_{\epsilon\to 0}S_{\text{on-shell}}[\{\epsilon,q_{\alpha f}\},N]=\sum_{\alpha}\mathcal{V}\left\{\sqrt{2g_3}\left(\mp i\tanh^{-1}\left(1\pm\frac{iq_{\alpha f}^2}{\sqrt{2g_3}N}\right)\underbrace{\mp i\tanh^{-1}(1)}_{\infty}\pm i\right)-\frac{q_{\alpha f}^2}{2N}\right\}$$



1: Renormalization group flow

No-boundary/Tunneling proposal ✖ Horava-Lifshitz Gravity

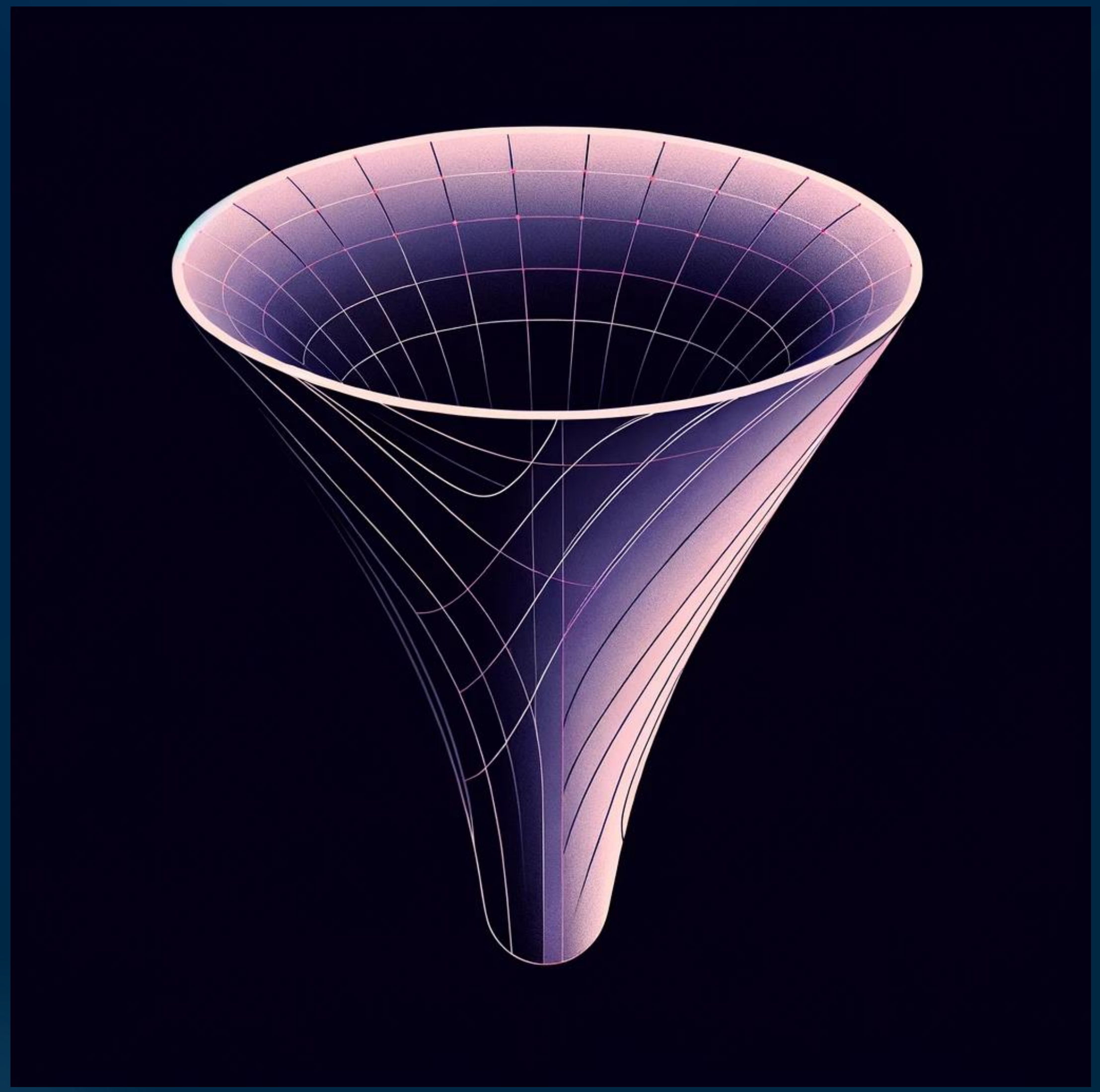
$$S[\{a_\alpha\}, N] = \sum_\alpha V \int_{t_i}^{t_f} \mathrm{d}t \, (N a_\alpha^3) \left[\underline{\frac{3(1-3\lambda)}{2} \left(\frac{\dot{a}_\alpha}{N a_\alpha} \right)^2} + \underline{\frac{\alpha_3}{a_\alpha^6}} + \frac{\alpha_2}{a_\alpha^4} + c_{\mathrm{g}}^2 \frac{3}{a_\alpha^2} - \Lambda \right]$$

$\lambda \to \infty \qquad a \to 0 \qquad f(a) \propto a^{-2\alpha} \qquad h(a) \propto a^{2\beta}$

A. E. Gumrukcuoglu and S. Mukohyama, Phys. Rev. D 83 (2011) 12403,

J. I. Radkovski and S. M. Sibiryakov, Phys. Rev. D 108 (2023) 046017

$$L = -\frac{2h(a)}{a^3} \propto \tau^{-1+\frac{\beta-\alpha}{3-\alpha-\beta}} \longrightarrow 0 < \alpha < \beta < 3 - \alpha$$



2: Robin boundary condition

Robin boundary term

$$\frac{1}{N} q_\alpha^2 \delta \left(\frac{\dot{q}_\alpha}{q_\alpha} \right) = 0 \implies \frac{1}{N} \frac{\dot{q}_\alpha}{q_\alpha} \Big|_{t_i=0}^{t_f=1} = \text{fixed}$$

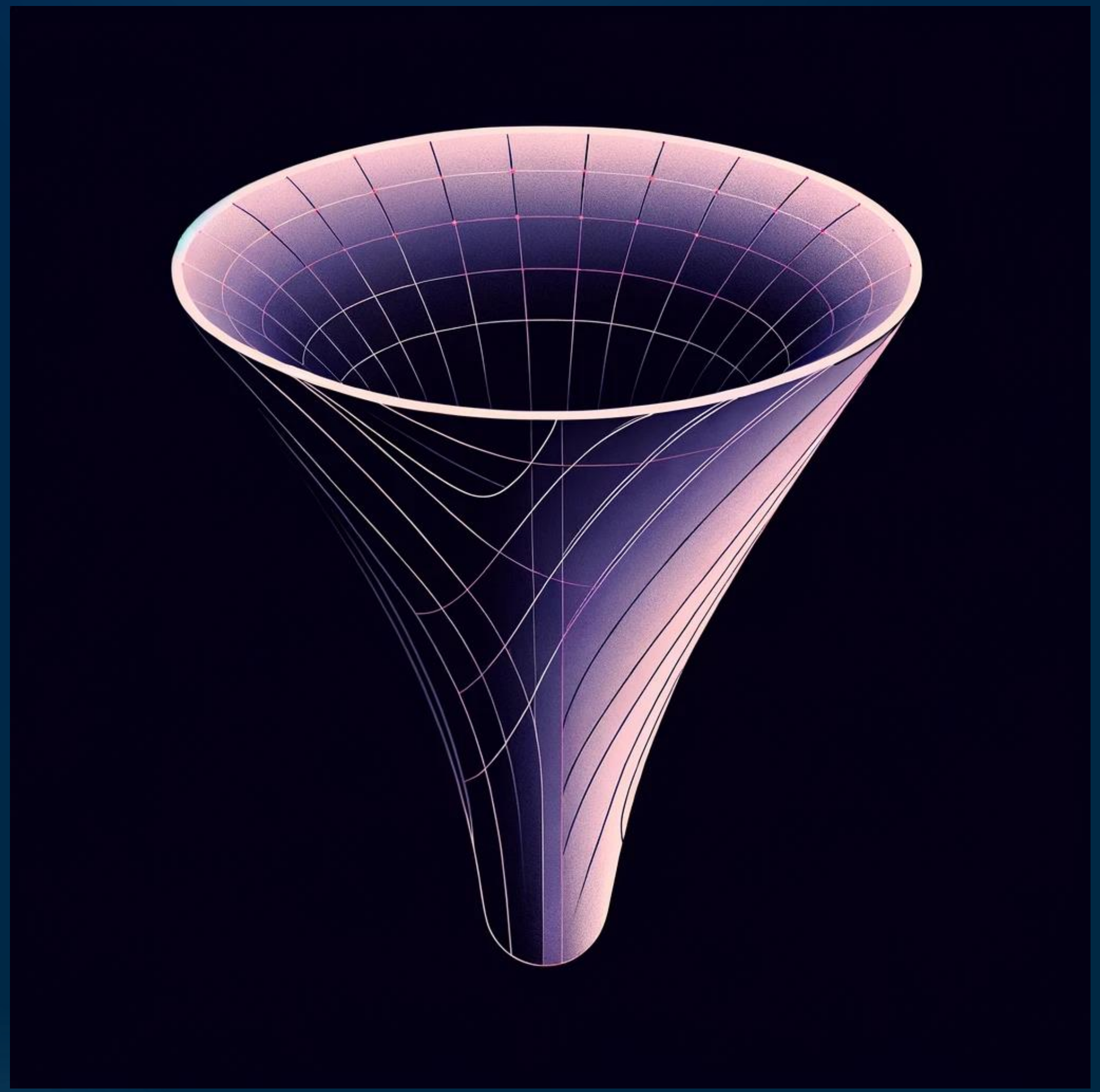
$$S_B = \frac{1}{2\zeta} \sum_\alpha \int_{\Sigma_\alpha} d^3x \sqrt{\gamma} K = \sum_\alpha \frac{\mathcal{V}}{2N} q_\alpha \dot{q}_\alpha \Big|_{t_i=0}^{t_f=1} = \sum_\alpha \frac{\mathcal{V}}{2N} q_\alpha^2 \frac{\dot{q}_\alpha}{q_\alpha} \Big|_{t_i=0}^{t_f=1}$$

Robin boundary condition

$$\frac{1}{2N} \frac{\dot{q}_\alpha(t_i = 0)}{q_\alpha(t_i = 0)} = \frac{1}{\xi_{\alpha i}}, \quad \frac{1}{2N} \frac{\dot{q}_\alpha(t_f = 1)}{q_\alpha(t_f = 1)} = \frac{1}{\xi_{\alpha f}}$$

Hubble parameter

$$H_\alpha = \frac{1}{N a_\alpha} \frac{\mathrm{d}a_\alpha}{\mathrm{d}t} = \frac{2}{3N} \frac{\dot{q}_\alpha}{q_\alpha}$$



On-shell action

$$S_{\text{on-shell}}[\{\xi_\alpha\}, N] = \sum_{\alpha=1}^{n} \mathcal{V} \Bigg\{ i\sqrt{2g_3} \Bigg(\tanh^{-1} \left(\frac{\mp\sqrt{2N}\,(2N - \xi_{\alpha f})}{\sqrt{(\xi_{\alpha i} + 2N)\,(2N - \xi_{\alpha f})\,(\xi_{\alpha i} - \xi_{\alpha f} + 2N)}} \right) \mp \tanh^{-1} \left(\frac{\sqrt{2N}\,(\xi_{\alpha i} + 2N)}{\sqrt{(\xi_{\alpha i} + 2N)\,(2N - \xi_{\alpha f})\,(\xi_{\alpha i} - \xi_{\alpha f} + 2N)}} \right) \Bigg) \Bigg\}$$

2: Imaginary Robin boundary

Robin boundary condition

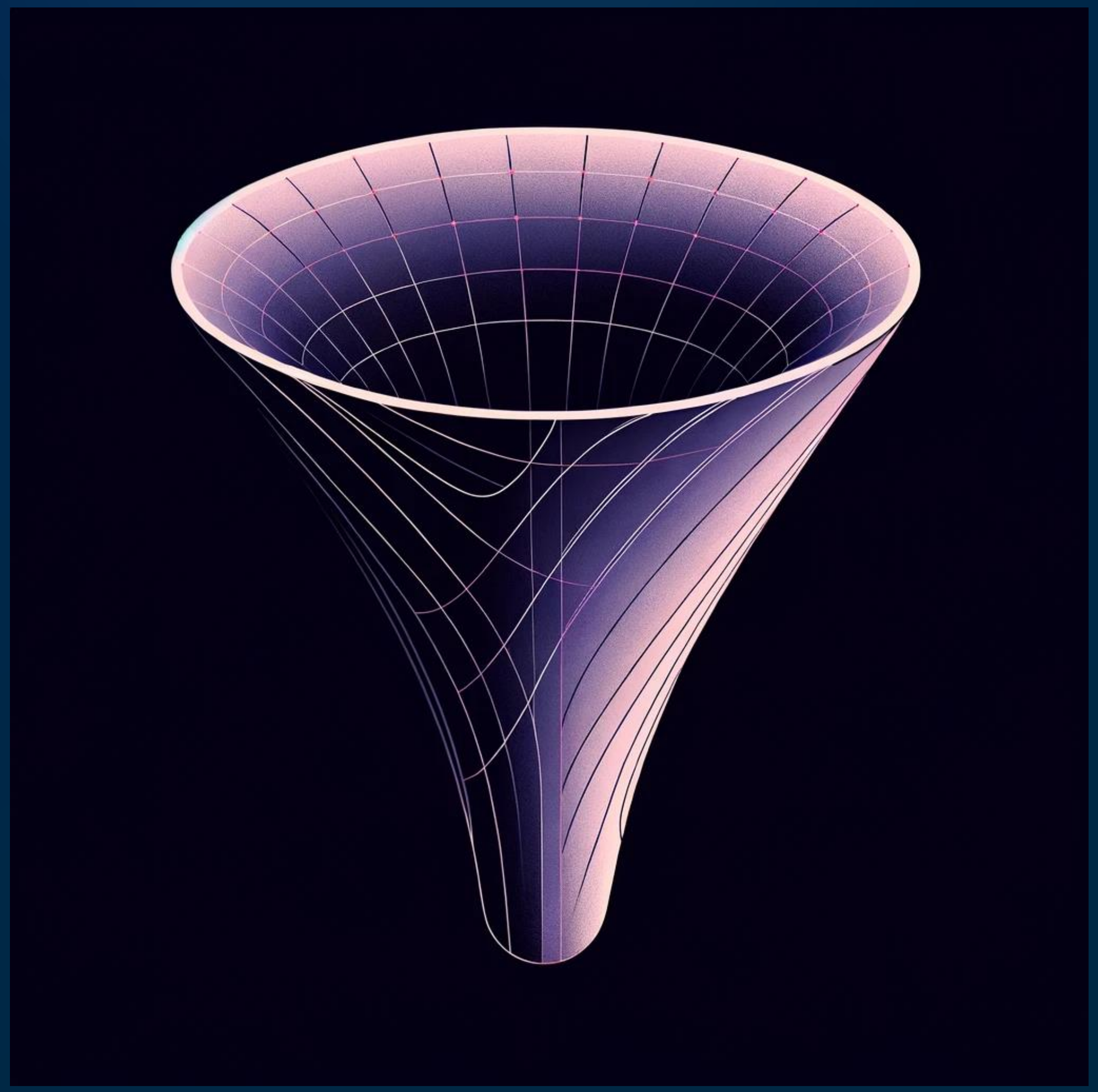
$$\xi_{\alpha i} = -i, \quad \xi_{\alpha f} = \text{const.}$$

A. Di Tucci and J. L. Lehners, Phys. Rev. Lett. 122 (2019) 201302

A. Di Tucci, J. L. Lehners and L. Sberna, Phys. Rev. D 100 (2019) 123543

On-shell action

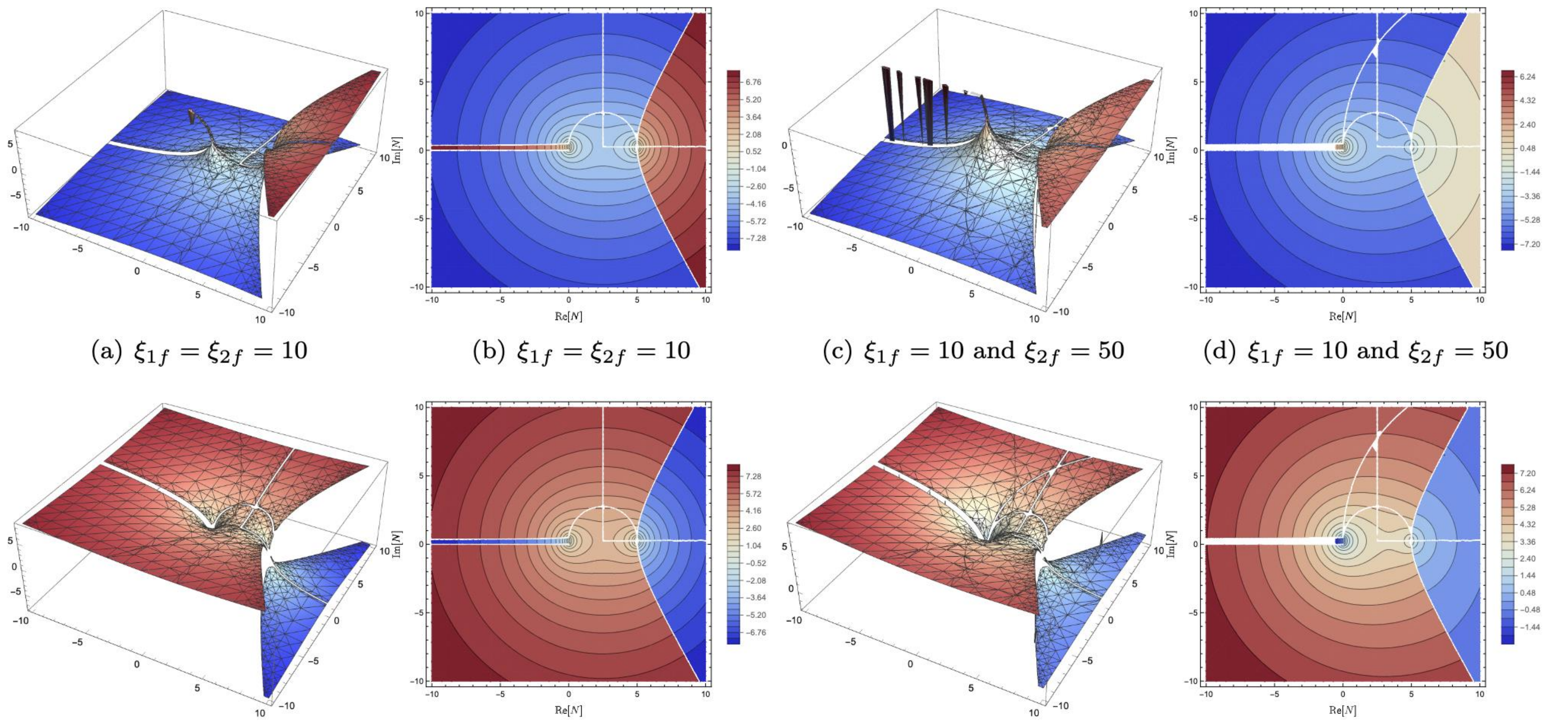
$$S_{\text{on-shell}}[\{\xi_\alpha\}, N] = \sum_{\alpha=1}^{n} \mathcal{V} \left\{ i\sqrt{2g_3} \left(\tanh^{-1} \left(\frac{\mp\sqrt{2N}\,(2N - \xi_{\alpha f})}{\sqrt{(\xi_{\alpha i} + 2N)\,(2N - \xi_{\alpha f})\,(\xi_{\alpha i} - \xi_{\alpha f} + 2N)}} \right) \mp \tanh^{-1} \left(\frac{\sqrt{2N}\,(\xi_{\alpha i} + 2N)}{\sqrt{(\xi_{\alpha i} + 2N)\,(2N - \xi_{\alpha f})\,(\xi_{\alpha i} - \xi_{\alpha f} + 2N)}} \right) \right) \right\}$$



Lorentzian path integral

$$G\left[\{q_\alpha(t_f)\}; \{q_\alpha(t_i)\}\right] = \sqrt{\frac{i\mathcal{V}}{2\pi\hbar}} \int_{0,-\infty}^{\infty} \frac{\mathrm{d}N}{N^{1/2}} \exp\left(\frac{iS_{\text{on-shell}}[\{q_\alpha(t_i), q_\alpha(t_f)\}, N]}{\hbar} \right)$$

2: Imaginary Robin boundary



(a) $\xi_{1f} = \xi_{2f} = 10$ (b) $\xi_{1f} = \xi_{2f} = 10$ (c) $\xi_{1f} = 10$ and $\xi_{2f} = 50$ (d) $\xi_{1f} = 10$ and $\xi_{2f} = 50$

(e) $\xi_{1f} = \xi_{2f} = 10$ (f) $\xi_{1f} = \xi_{2f} = 10$ (g) $\xi_{1f} = 10$ and $\xi_{2f} = 50$ (h) $\xi_{1f} = 10$ and $\xi_{2f} = 50$

Summary

1. We analyzed a formulation of the Hartle-Hawking no-boundary proposal based on Horava-Lifshitz gravity
2. The Hartle-Hawking wave function cannot be formulated due to the divergence of the on-shell action caused by the higher-order derivative term of curvature.
3. Solution 1: Renormalization group flow of gravity.
4. Solution 2: Robin boundary conditions