Holography by conformal smearing

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Based on the work in collaboration with

Janos Balog (Wigner Institute), **Kengo Shimada** (YITP, RIKEN)

S. Aoki, J. Balog and K. Shimada,
"Derivation of the GKP-Witten relation by symmetry without Lagrangian",
Phys. Lett. B868(2025) 139757 (arXiv.2411.16269).

New results from collaboration with

Daichi Takeda (RIKEN)

in preparation

I. Introduction

AdS/CFT correspondences

Maldacena 1997

Conformal symmetry in d-dimensions $\Longleftrightarrow$ $\mathrm{SO}(d,2)$ $\Longleftrightarrow$ AdS Isometry in (d+1) dimensions

GKP-Witten relation (in the large N)

Gubser-Klebanov-Polyakov 1998, Witten 1998

$$(\Box_{\mathrm{ADS}} - m^2)\Phi_{\mathrm{cl}} = 0$$ EOM for free theory

$$\lim_{z\to 0} \Phi_{\mathrm{cl}}(z,x) = z^{d-\Delta_S} J(x)$$ boundary condition

$$R_{\mathrm{AdS}}^2 m^2 = \Delta_S(\Delta_S - d)$$

solution $$\lim_{z\to 0} \Phi_{\mathrm{cl}}(z,x) = z^{d-\Delta_S} J(x) + z^{\Delta_S}\langle S(x)\rangle_J + \cdots$$

$$\langle S(x)\rangle_J := \langle 0|S(x)\exp[\int d^d y\, S(y)J(y)]|0\rangle = \int d^d y\, \underline{\langle 0|S(x)S(y)|0\rangle} J(y) + O(J^2)$$

boundary scalar operator

$$= \frac{1}{|x-y|^{2\Delta_S}}$$

Δ_S : conformal dimension of S

CFT 2-pt function for scalar

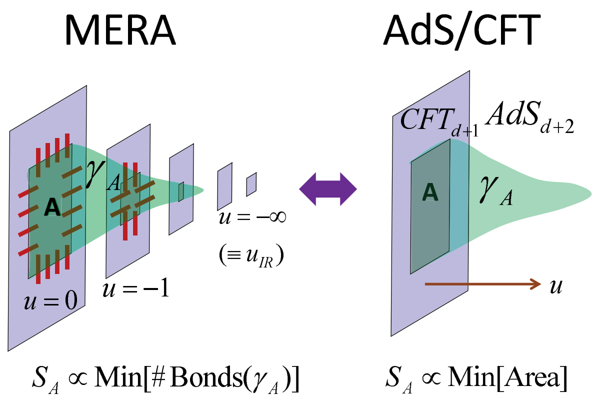
A different but more general point of view for holography (AdS/CFT)

"An extra (holographic) dimension = energy scale of the boundary theory"

Example

Nozaki-Ryu-Takayanagi, "Holographic geometry of entanglement renormalization in quantum field theories", JHEP10 (2012)193

energy scale = scale of renormalization group (RG)



A bulk geometry is not assumed but is determined by Information metric.

A bulk AdS spacetime is emergent !

Applicable to non-holographic CFT or even to non-CFT.

This talk

1. We introduce the “conformal smearing approach” to construct/understand AdS/CFT correspondence.

2. Employing the conformal smearing, we derive the GKP-Witten for an arbitrary spin at all order in J via symmetry without bulk Lagrangian or large N expansion.

3. As an explicit example, a bulk 2-rank tensor - 2 boundary scalars is considered. This correlation function reproduces the EMT(EnergyMomentumTensor)-scalar-scalar in CFT near boundary. Contact terms in Ward-Takahashi identities are also derived.

Hereafter we work on an Euclidian AdS or CFT.

II. Conformal smearing

Aoki-Balog-Onogi-Yokoyama, PTEP **2023**(2023) 013B03.

Our approach to holography

We consider a scalar CFT in d dimensional Euclidean space, whose primary field satisfies

$$\langle 0|\hat{\varphi}(x)\hat{\varphi}(y)|0\rangle = \frac{C_0}{|x-y|^{2\Delta}}$$

We then smear this field as

$$\hat{\sigma}(X) = \int d^d y\, h(z, x-y)\hat{\varphi}(y)$$

smearing kernel

field in $d+1$ dimensions

$$X := (x, z)$$

z is an extra direction, which corresponds to an energy scale of CFT.

$$z = 0 \text{ (UV) and } z = \infty \text{ (IR)}$$

QFT(CFT) in d-dimension + energy scale → d+1 dimensional bulk space

"Holography"

A choice of kernel : conformal smearing

Conformal smearing

$$\hat{\sigma}(X) = \int d^d y\, h(z, x-y)\hat{\varphi}(y) \qquad h(z,x) = \Sigma_0 \left(\frac{z}{x^2+z^2}\right)^{d-\Delta}, \qquad \Delta < \frac{d}{2}$$

1. A conformal transformation on $\hat{\varphi}(x)$ generates an AdS isometry on $\hat{\sigma}(X)$.

conformal transformation $\quad y \to \tilde{y} \qquad U\hat{\varphi}(x)U^\dagger = h(x)^{\Delta_\varphi}\hat{\varphi}(\tilde{x})$

↓

AdS isometry $\quad X \to \tilde{X} \qquad U\hat{\sigma}(X)U^\dagger = \hat{\sigma}(\tilde{X})$ **"AdS/CFT correspondence"**

2. **BDHM relation** is reproduced as

Banks-Douglas-Horowitz-Martinec, hep-th/980816.

$$\lim_{z\to 0} z^{-\Delta}\hat{\sigma}(X) = \lim_{z\to 0}\int d^d y\, z^{-\Delta} h(z, x-y)\hat{\varphi}(y) = \frac{\Sigma_0}{\Lambda}\hat{\varphi}(x)$$

for $\Delta < d/2$

3. $\hat{\sigma}(X)$ satisfies EOM of a free scalar field on AdS as

$$(\Box_{\mathrm{AdS}} - m^2)\hat{\sigma}(X) = 0$$

where $m^2 = (\Delta - d)\Delta < 0$.

$R_{\mathrm{AdS}} = 1$

III. GKP-Witten relation

S. Aoki, J. Balog and K. Shimada, Phys. Lett. B868(2025) 139757 (arXiv.2411.16269).

GKP-Witten relation in conformal smearing

VEV of the bulk spin L field with sources J to CFT operators

$$\Phi^{1}_{s_1}(X_1) := \frac{1}{Z(J)} \langle 0 | \underline{G^{1}_{s_1}(X_1)} \exp\left[\sum_{p,s} \int d^d y\, \underline{J^{q,s}(y)}\, \underline{O^{p}_{s}(y)}\right] |0\rangle$$

bulk spin L operator — source — CFT primary operator

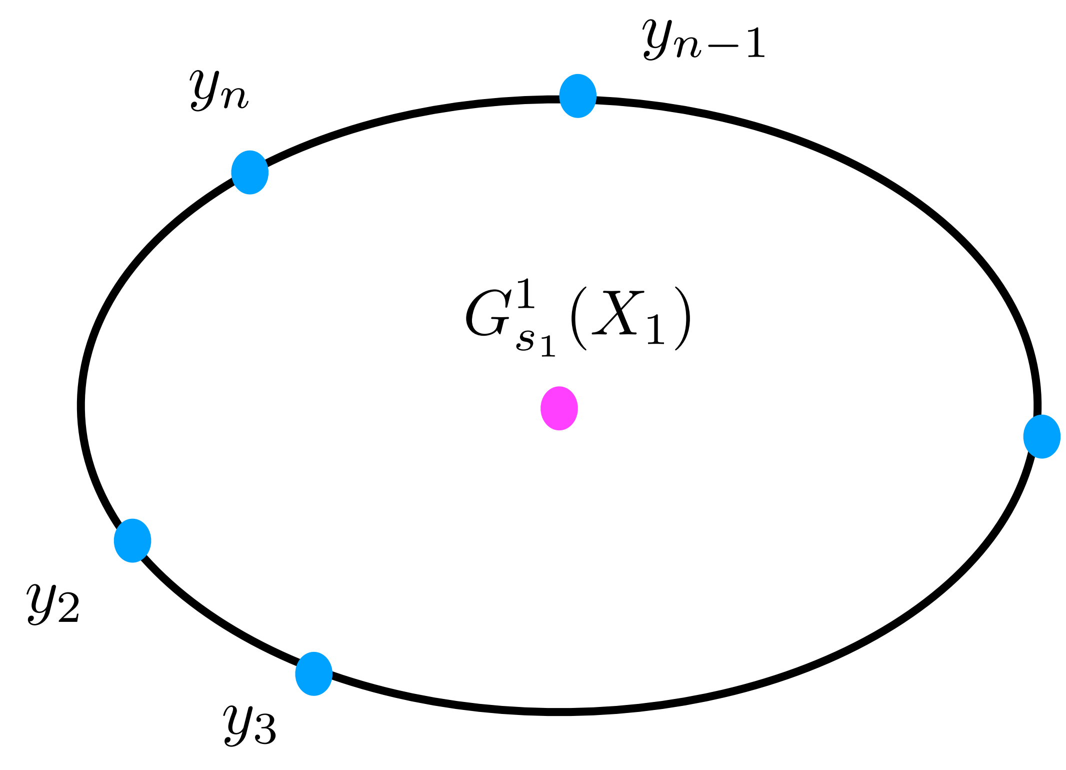
GKP-Witten relation

$$\lim_{z_1 \to 0} \Phi^{1}_{s_1}(X_1) = \frac{z_1^{d-\Delta_{p_1}-L_{p_1}}}{\Lambda_{p_1}} \underline{J^{p_1,s_1}(x_1)} + \cdots + z_1^{\Delta_{p_1}-L_{p_1}} \underline{\langle 0 | O^{p_1}_{s_1}(x_1) \exp\left[\sum_{p,s} \int d^d y\, J^{p,s}(y) O^{p}_{s}(y)\right] |0\rangle_c}$$

source term

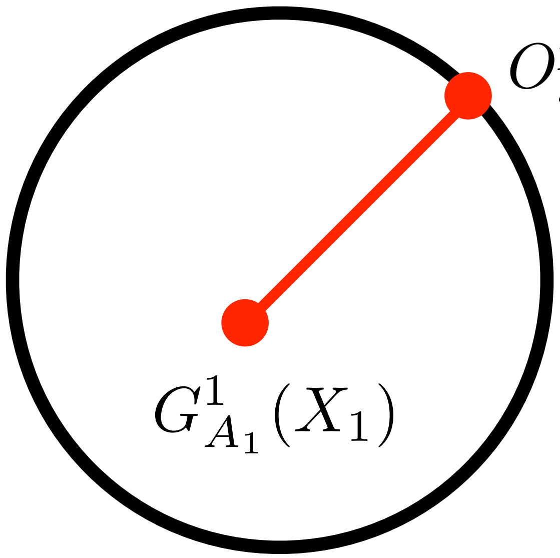
$$= \langle O^{p_i}_{s_1}(x_1) \rangle_J$$

We show the GKP-Witten relation in the expansion of J .



Step 1. Bulk-boundary 2-pt function

The bulk-boundary 2-pt function is defined as



$$G^{1,p_2}_{A_1,s_2}(X_1,x_2) = \langle 0|G^{1}_{A_1}(X_1)O^{p_2}_{s_2}(x_2)|0\rangle$$

A_1 : bulk tensor index

p_2 : conformal dimension

s_2: spin index

Symmetry uniquely determines the 2-pt function.

$z_1 \to 0$ limit

s_1, s_2 :symmetric and traceless in d-dimensions

$$\lim_{z_1\to 0} G^{1,p_2}_{s_1,s_2}(X_1,x_2) = z_1^{\Delta_{p_1}-L_{p_1}}\langle 0|O^{p_1}_{s_1}(x_1)O^{p_2}_{s_2}(x_2)|0\rangle\delta^{p_1,p_2} + \frac{1}{\Lambda_{\Delta_{p_1}}} z_1^{d-\Delta_{p_1}-L_{p_1}}\delta_{s_1,s_2}\delta^{p_1,p_2}\delta^{(d)}(x_{12})$$

$$\frac{1}{\Lambda_\alpha} := \int d^d x \frac{1}{(1+x^2)^\alpha} \qquad \alpha > \frac{d}{2}$$

Ex. Scalar 2-pt $\quad G(X_1,x_2) = \left(\frac{z_1}{z_1^2+x_{12}^2}\right)^{\Delta_p} \longrightarrow \frac{z_1^{\Delta_p}}{|x_1-x_2|^{2\Delta_p}} + \frac{z_1^{d-\Delta_p}}{\Lambda_{\Delta_p}}\delta^{(d)}(x_1-x_2)$

Step 2. Bulk-boundary-boundary 3-pt function

The bulk-boundary-boundary 3-pt function is given by

$$\langle 0|G^1_{s_1}(X_1)O^{p_2}_{s_2}(x_2)O^{p_3}_{s_3}(x_3)|0\rangle = H_{s_1 s_2 s_3}(X_1,x_2,x_3)\left(\frac{z_1}{z_1^2+(x_1-x_2)^2}\right)^{\Delta_{p_2}+L_{p_2}}\left(\frac{z_1}{z_1^2+(x_1-x_3)^2}\right)^{\Delta_{p_3}+L_{p_3}}$$

spin factor

$$\times F\left(\frac{z_1^2(x_2-x_3)^2}{[z_1^2+(x_1-x_2)^2][z_1^2+(x_1-x_3)^2]}\right)$$

$$\lim_{z_1\to 0} H_{s_1 s_2 s_3} = z_1^{-2L_{p_1}} h_{s_1 s_2 s_3}$$

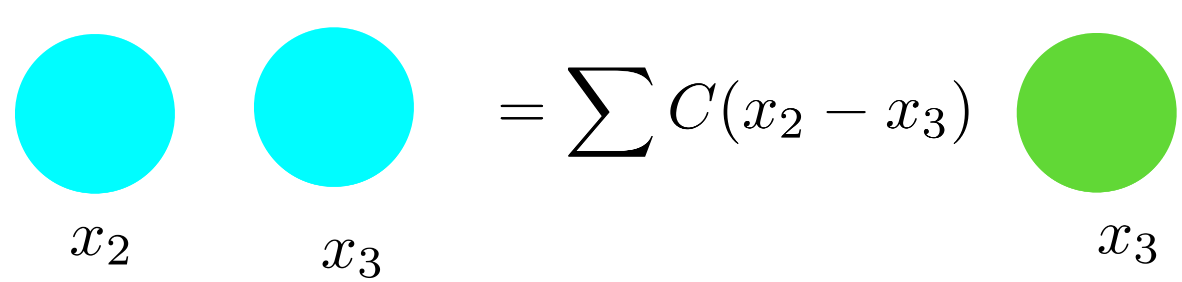
spin-factor in CFT

arbitrary function

We will fix the small x behavior of $F(x)$ by the operator product expansion.

Operator product expansion (OPE)

$$O^{p_2}_{s_2}(x_2)O^{p_3}_{s_3}(x_3) = \sum_{p,s} C^{p_2 p_3 p}_{s_2 s_3 s}(x_2-x_3,\partial^{x_3})O^p_s(x_3)$$



assume $\quad |x_1| > |x_2|, |x_3| \quad$ or $\quad |x_1| < |x_2|, |x_3|$

$$\lim_{z_1 \to 0} \langle 0 | G^1_{s_1}(X_1) O^{p_2}_{s_2}(x_2) O^{p_3}_{s_3}(x_3) | 0 \rangle = \lim_{z_1 \to 0} \sum_{p,s} C^{p_2 p_3 p}_{s_1 s_2 s}(x_2 - x_3, \partial^{x_3}) \underline{\langle 0 | G^1_{s_1}(X_1) O^p_s(x_3) | 0 \rangle}$$

OPE

2-pt function

$$= z_1^{\Delta_{p_1} - L_{p_1}} \sum_{p,s} C^{p_2 p_3 p_1}_{s_1 s_2 s}(x_2 - x_3, \partial^{x_3}) \underline{\langle 0 | O^{p_1}_{s_1}(x_1) O^{p_i}_s(x_3) | 0 \rangle} + \cdots$$

$$= z_1^{\Delta_{p_1} - L_{p_1}} \langle 0 | O^{p_1}_{s_1}(x_1) \underline{O^{p_2}_{s_2}(x_2) O^{p_3}_{s_3}(x_3) | 0 \rangle} + \cdots \quad ★$$

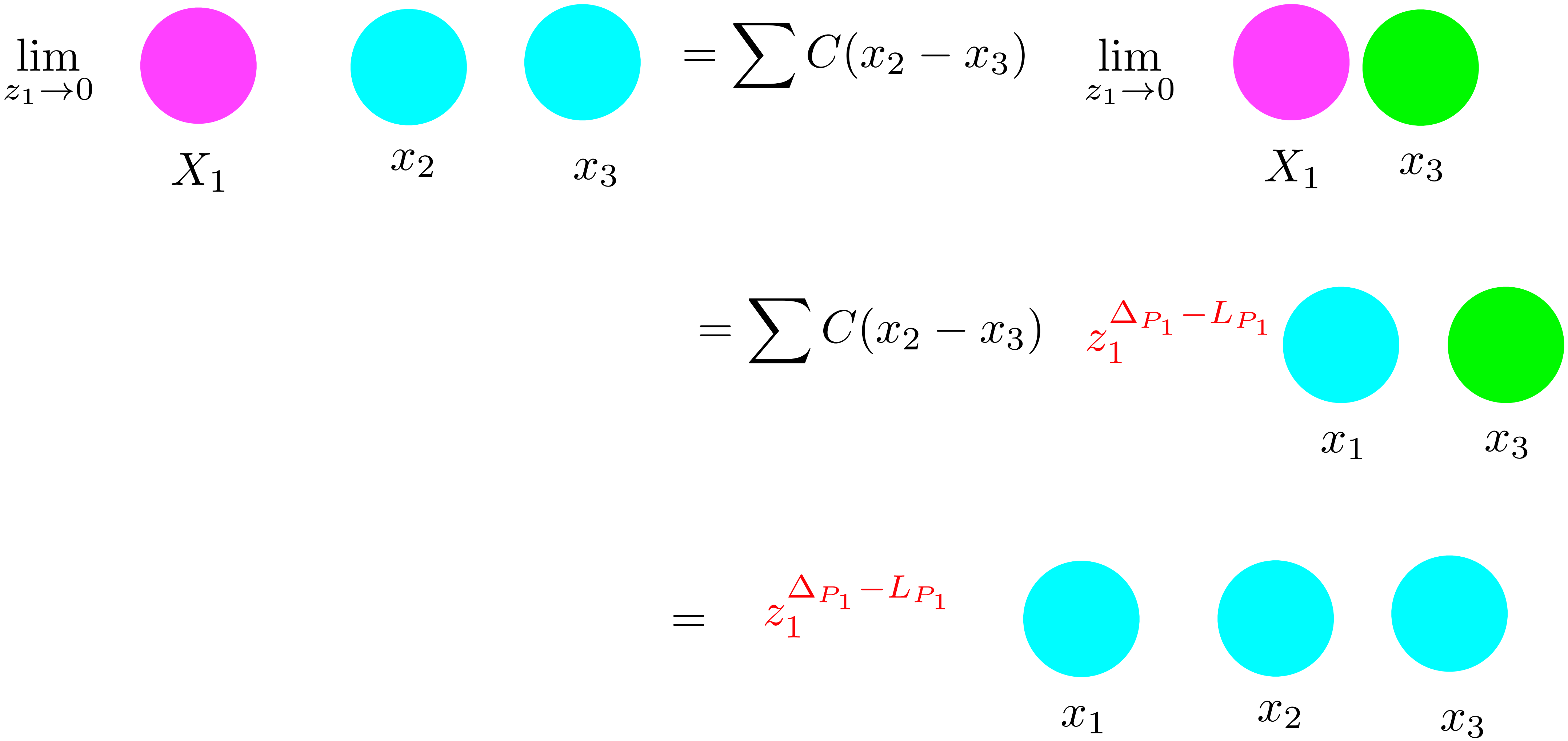
OPE back

contributions from delta-functions

Therefore $F(x)$ behaves for small x as

$$\lim_{x \to 0} F(x) \propto x^\alpha \qquad 2\alpha = \Delta_{p_2} + L_{p_2} + \Delta_{p_3} + L_{p_3} - \Delta_{p_1} - L_{p_1}$$

This behaves leads to ★ for all operator orderings.



$\lim_{z_1 \to 0}$
X_1
x_2
x_3
$= \sum C(x_2 - x_3)$
$\lim_{z_1 \to 0}$
X_1
x_3
$= \sum C(x_2 - x_3)$
$z_1^{\Delta_{P_1} - L_{P_1}}$
x_1
x_3
$=$
$z_1^{\Delta_{P_1} - L_{P_1}}$
x_1
x_2
x_3

Step 3. Bulk-boundary n-pt function

$$G^{1,p_2,\cdots,p_n}_{s_1,s_2,\cdots,s_n}(X_1,x_2,\cdots,x_n) := \langle 0|G^1_{s_1}(X_1)O^{p_2}_{s_2}(x_2)\cdots O^{p_n}_{s_n}(x_n)|0\rangle \qquad n\geq 3$$

We will show the following equality by the mathematical induction.

★ $$\lim_{z_1\to 0} G^{1,p_2,\cdots,p_n}_{s_1,s_2,\cdots,s_n}(X_1,x_2,\cdots,x_n) = z_1^{\Delta_{p_1}-L_{p_1}}\langle 0|O^{p_1}_{s_1}(x_1)O^{p_2}_{s_2}(x_2)\cdots O^{p_n}_{s_n}(x_n)|0\rangle + \cdots$$

(1) It is correct at $n=3$.

(2) We assume that it holds at n . Then it holds at $n+1$ as

n-pt function

OPE $$G^{1,p_2,\cdots,p_{n+1}}_{s_1,s_2,\cdots,s_{n+1}}(X_1,x_2,\cdots,x_n,x_{n+1}) = \sum_{p,s} C^{p_n p_{n+1} p}_{s_n s_{n+1} s}(x_n-x_{n+1},\partial^{x_{n+1}})G^{1,p_2,\cdots,p_{n-1},p}_{s_1,s_2,\cdots,s_{n-1},s}(X_1,x_2,\cdots,x_{n-1},x_{n+1})$$

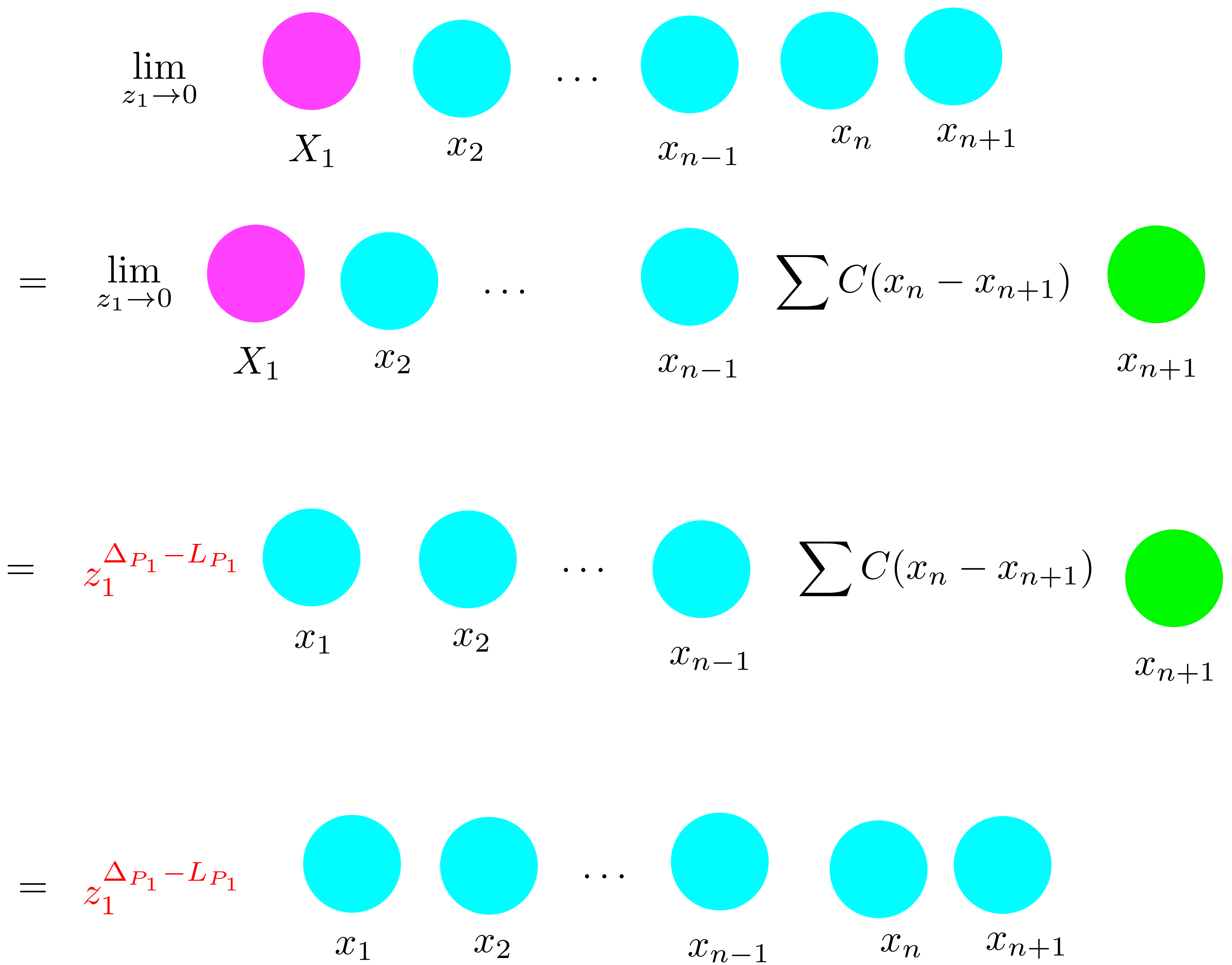
$$\longrightarrow \lim_{z_1\to 0} G^{1,p_2,\cdots,p_{n+1}}_{s_1,s_2,\cdots,s_{n+1}}(X_1,x_2,\cdots,x_n,x_{n+1}) = z_1^{\Delta_{p_1}-L_{p_1}}\sum_{p,s} C^{p_n p_{n+1} p}_{s_n s_{n+1} s}(x_n-x_{n+1},\partial^{x_{n+1}})$$

$$\times\langle 0|O^{p_1}_{s_1}(x_1)O^{p_2}_{s_2}(x_2)\cdots O^{p_{n-1}}_{s_{n-1}}(x_{n-1})O^{p}_{s}(x_{n+1})|0\rangle$$

$$= z_1^{\Delta_{p_1}-L_{p_1}}\langle 0|O^{p_1}_{s_1}(x_1)O^{p_2}_{s_2}(x_2)\cdots O^{p_{n+1}}_{s_{n+1}}(x_{n+1})|0\rangle$$

OPE back

★ is correct.



$\lim_{z_1 \to 0}$
X_1 x_2 $\cdots$ x_{n-1} x_n x_{n+1}
$= \lim_{z_1 \to 0}$
X_1 x_2 $\cdots$ x_{n-1} $\sum C(x_n - x_{n+1})$ x_{n+1}
$= z_1^{\Delta_{P_1} - L_{P_1}}$
x_1 x_2 $\cdots$ x_{n-1} $\sum C(x_n - x_{n+1})$ x_{n+1}
$= z_1^{\Delta_{P_1} - L_{P_1}}$
x_1 x_2 $\cdots$ x_{n-1} x_n x_{n+1}

Step 4. GKP-Witten relation

$$\Phi^1_{s_1}(X_1) = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \int d^d x_2 \cdots d^d x_n \langle 0 | G^1_{s_1}(X_1) O^{p_2}_{s_2}(x_2) \cdots O^{p_n}_{s_n}(x_n) | 0 \rangle_c J^{p_2,s_2}(x_2) \cdots J^{p_n,s_n}(x_n)$$

$$\begin{aligned}\lim_{z_1 \to 0} \Phi^1_{s_1}(X_1) &= \frac{z_1^{d-\Delta_{p_i}-L_{p_i}}}{\Lambda_{p_1}} J^{p_i,s_1}(x_1) + \cdots \\ &+ \sum_{n=1}^{\infty} \frac{z_1^{\Delta_{p_i}-L_{p_i}}}{(n-1)!} \int d^d x_2 \cdots d^d x_n \langle 0 | O^{p_1}_{s_1}(x_1) O^{p_2}_{s_2}(x_2) \cdots O^{p_n}_{s_n} | 0 \rangle_c J^{p_2,s_2}(x_2) \cdots J^{p_n,s_n}(x_n) \\ &= \frac{z_1^{d-\Delta_{p_1}-L_{p_1}}}{\Lambda_{p_1}} J^{p_1,s_1}(x_1) + \cdots + z_1^{\Delta_{p_1}-L_{p_1}} \langle 0 | O^{p_1}_{s_1}(x_1) | 0 \rangle_J\end{aligned}$$

$$\langle 0 | O^{p_1}_{s_1}(x_1) | 0 \rangle_J = \langle 0 | O^{p_1}_{s_1}(x_1) \exp\left[\sum_{p,s} \int d^d y \, J^{p,s}(y) O^p_s(y) \right] | 0 \rangle_c$$

GKP-Witten relation is established for an arbitrary primary operators at all order in sources J coupled to primaries.

$+ \cdots +$ contributions ? The next section

IV. 3-pt function of EMT-scalar-scalar

Aoki-Takeda, in preparation.

One bulk tensor - two boundary scalars

As an explicit example, let us consider the following 3-pt function.

$$G_{\mu\nu}(x_1,x_2,X_3) := \langle O_1(x_1)O_2(x_2)g_{\mu\nu}(X_3)\rangle$$

$O_i(x_i)$: boundary scalar with the conformal dimension Δ_i

$g_{AB}(X_3)$: bulk symmetric tensor

Condition 1 In the $z \to 0$ limit, $G_{\mu\nu}$ should become traceless.

$$G_{\mu\nu}(x_1,x_2,X_3) = \frac{z^{\Delta_3-2}\, C_{12T}}{(x_{13}^2+z^2)^{\frac{\alpha_{13}}{2}}(x_{23}^2+z^2)^{\frac{\alpha_{23}}{2}}|x_{12}|^{\alpha_{12}}}\left[\frac{Z_\mu(x_1,x_2,X_3)Z_\nu(x_1,x_2,X_3)}{\tilde{Z}^2(x_1,x_2,X_3)} - \frac{\delta_{\mu\nu}}{d}\right]$$

$$+O(z^{\Delta_3})$$

C_{12T}: unknown 3-pt coupling

Δ_3: conformal dimension of a boundary operator coupled to $g_{\mu\nu}(X_3)$

$$\alpha_{13} = \Delta_1 + \Delta_3 - \Delta_2$$

$$\alpha_{23} = \Delta_2 + \Delta_3 - \Delta_1$$

$$\alpha_{12} = \Delta_1 + \Delta_2 - \Delta_3$$

$$Z_\mu(x_1,x_2,X_3) := \frac{x_{13,\mu}}{x_{13}^2+z^2} - \frac{x_{23,\mu}}{x_{23}^2+z^2}$$

$$\tilde{Z}^2(x_1,x_2,X_3) := \frac{x_{12}^2}{(x_{13}^2+z^2)(x_{23}^2+z^2)}$$

Condition 2

$$\lim_{z\to 0} z^{-(\Delta_3-2)}\partial_\mu^{x_3} G^\mu{}_\nu(x_1,x_2,X_3)=0 \qquad (x_{12}\neq 0, x_{13}\neq 0, x_{23}\neq 0)$$

$\longrightarrow$ $\Delta_3 = d,\ \Delta_1=\Delta_2 := \Delta$

$g_{\mu\nu}(X_3)$ couples to the EMT in the $z\to 0$ limit

$$G_{\mu\nu}(x_1,x_2,X_3)=\frac{z^{d-2}\,C_{12T}}{(x_{13}^2+z^2)^{\frac{d}{2}}(x_{23}^2+z^2)^{\frac{d}{2}}|x_{12}|^{2\Delta-d}}\left[\frac{Z_\mu(x_1,x_2,X_3)Z_\nu(x_1,x_2,X_3)}{\tilde{Z}^2(x_1,x_2,X_3)}-\frac{\delta_{\mu\nu}}{d}\right]+O(z^d)$$

As $z\to 0$, this reproduces the EMT-scalar-scalar correlation function in CFT.

Ward-Takahashi (WT) identities

If delta function contributions are included, we have

$$\lim_{z\to 0}\partial^3_\mu G^\mu{}_\nu(x_1,x_2,X_3) = z^{d-2}\frac{S_d}{d}\left[\frac{d-1}{\Delta}\left\{\delta^{(d)}(x_{31})+\delta^{(d)}(x_{32})\right\} - \partial^3_\nu\delta^{(d)}(x_{31}) - \partial^3_\nu\delta^{(d)}(x_{32})\right]\frac{1}{|x_{12}|^{2\Delta}}$$

$$S_d = 2\pi^{d/2}\Gamma(d/2)$$

This does not reproduce the WT identity for diffeomorphism invariance.

Let us consider the higher order term as

$$\frac{z^d\,\delta_{\mu\nu}C}{(x_{13}^2+z^2)^{\frac{d}{2}+1}(x_{23}^2+z^2)^{\frac{d}{2}+1}|x_{12}|^{\Delta-\frac{d}{2}-1}} \longrightarrow O(z^d) \qquad (x_{12}\neq 0, x_{13}\neq 0, x_{23}\neq 0)$$

This term does not contribute to the 3-pt function itself.

Thus we have neglected before.

Including this term, the WT identity is recovered:

$$\longrightarrow \lim_{z\to 0} \partial^3_\mu \langle O(x_1)O(x_2)G^\mu{}_\nu(X_3)\rangle = z^{d-2}\left[\partial^3_\nu \delta^{(d)}(x_{31})\langle O(x_3)O(x_2)\rangle + \partial^3_\nu \delta^{(d)}(x_{32})\langle O(x_3)O(x_1)\rangle\right]$$

$$C_{12T} = -g_{12}\frac{d\Delta}{(d-1)S_d}$$

CFT relation between 2-pt and 3-pt functions.

Osborn-Petkos (Textbook by Nakayama-san)

$$\langle O(x_1)O(x_2)\rangle := \frac{g_{12}}{|x_1-x_2|^{2\Delta}}$$

$$C = \frac{\Delta - d + 1}{\Delta}$$

This value of C also leads to the WT identity for the scale invariance as

$$\lim_{z\to 0} \langle O(x_1)O(x_2)G^\mu{}_\mu(X_3)\rangle = z^{d-2}(d-\Delta)\left[\delta^{(d)}(x_{31})\langle O(x_3)O(x_2)\rangle + \delta^{(d)}(x_{32})\langle O(x_3)O(x_1)\rangle\right]$$

trace

The bulk 3-pt function $\langle O(x_1)O(x_2)G_{\mu\nu}(X_3)\rangle$ in the $z \to 0$ reproduces not only the form of $\langle O(x_1)O(x_2)T_{\mu\nu}(x_3)\rangle$ in CFT but also the corresponding WT identities.

V. Conclusion

Conformal smearing approach

The conformal smearing generates the bulk field from the CFT field, where the conformal symmetry turns into the bulk AdS isometry.

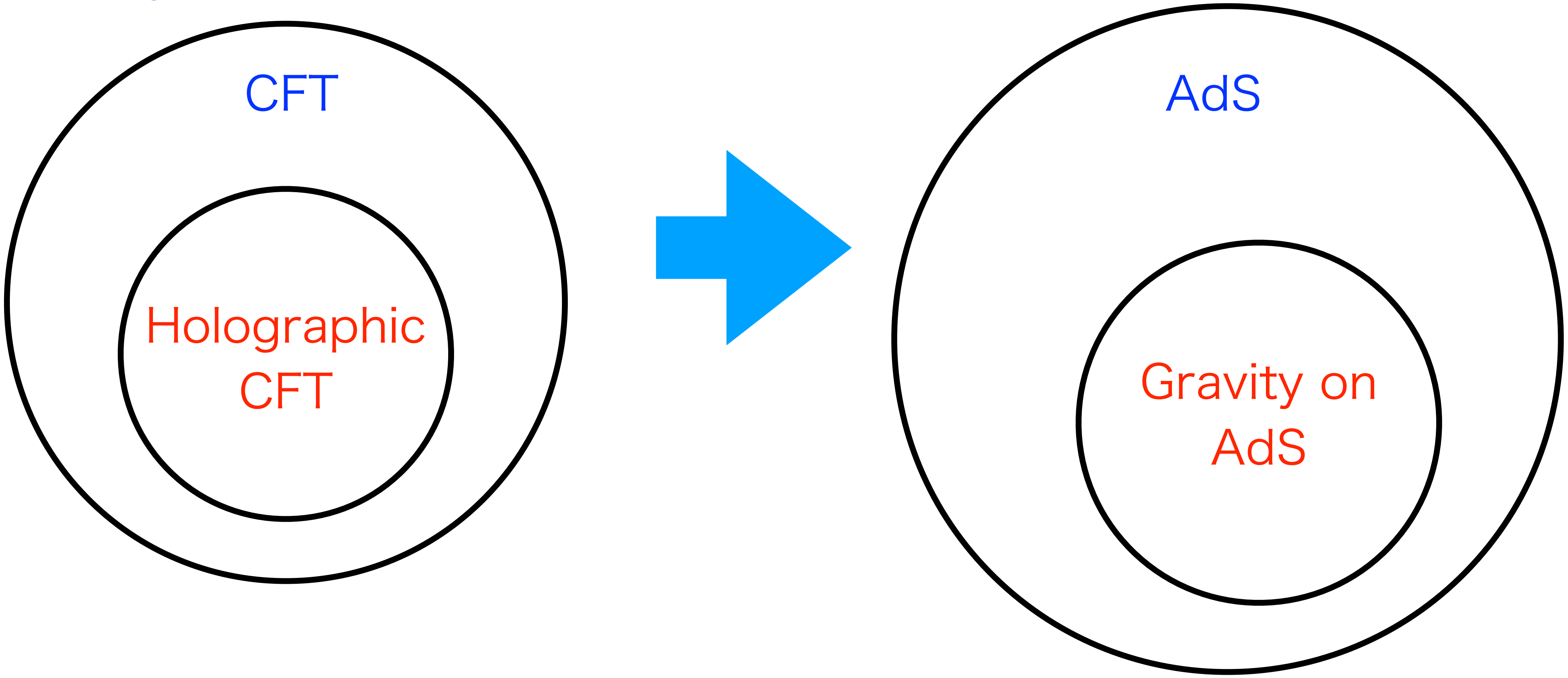
The GKP-Witten relation in terms of correlation functions can be derived using only symmetries without bulk Lagrangian for a generic CFT.

a. an arbitrary primary operator
b. all order in sources J
c. fully quantum without 1/N expansion
d. without AdS metric -> definition of “metric” (work in progress with Takeda)

What is (an origin of) AdS/CFT correspondence ?

Conformal smearing approach

A arbitrary CFT seems to have a bulk dual.



What are distinct properties of the constructed bulk space, which tell us whether CFT is holographic or not ?

Our results indicate that GKP-Witten relation cannot distinguish the two.

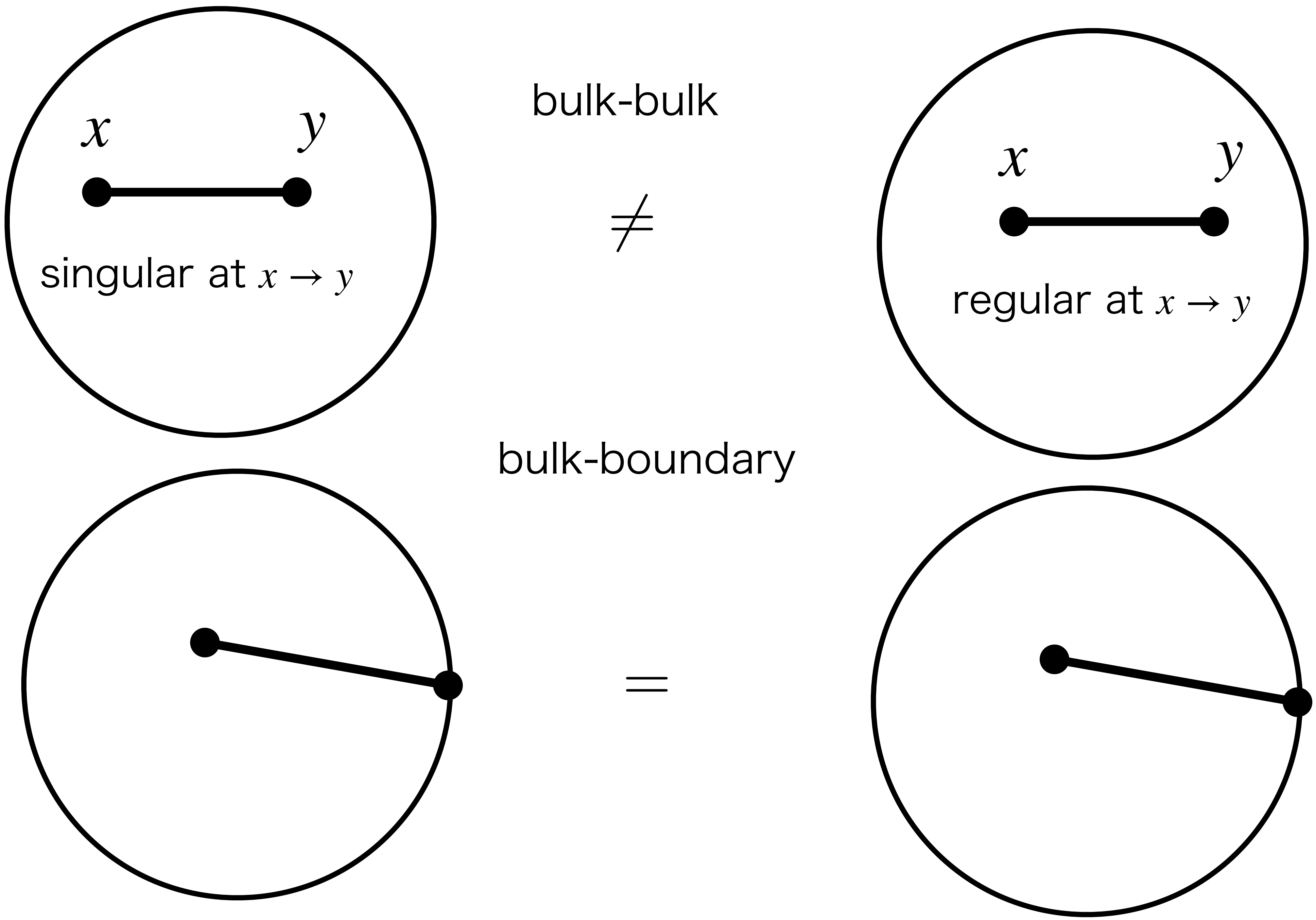
Thank you for your attention !

Bulk locality ?

Since the bulk theory dual to the holographic CFT has its Lagrangian, the bulk theory must be local.

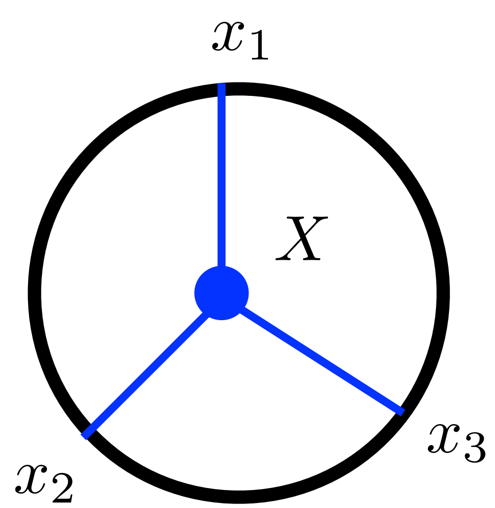
holographic dual **2-pt function** generic CFT

However



Bulk-boundary 2-pt function cannot distinguish the two.

3-pt function (scalar)



Witten diagram

Feynman diagram language

$$\langle 0|O^{p_1}(x_1)O^{p_2}(x_2)O^{p_3}(x_3)|0\rangle_{\rm CFT} \stackrel{?}{=} \int d^{d+1}X\, g_{ijk}G^{ip_1}(X,x_1)G^{jp_2}(X,x_2)G^{kp_3}(X,x_3)$$

Operator language

$$= \langle 0|O^{p_1}(x_1)O^{p_2}(x_2)O^{p_3}(x_3)\int d^{d+1}X\, g_{ijk}G^{i}(X)G^{j}(X)G^{k}(X)|0\rangle$$

The right-hand side in the 2nd line and the left-hand side in the 1st line have the same symmetric property in CFT.

↓

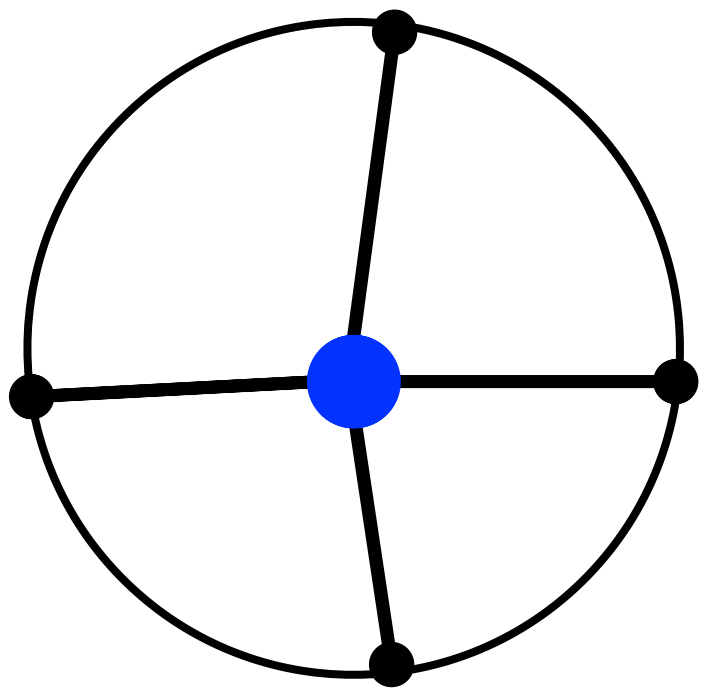
Both agree up to normalization, since the scalar 3-pt function is unique in CFT.

3-pt function (scalar) also cannot distinguish the two.

4-pt function

Heemskerk, Penedones, Polchinski, Sully, JHEP10(2009) 079

Locality of 4-pt vertices restricts properties of the boundary CFT.



We will work on 4-pt functions in detail in future studies.

S. Aoki, K. Shimada, J. Balog and K. Kawana,
“Bulk modified gravity from a thermal CFT by the conformal flow”,
Phys. Rev. D109 (2024)4, 046006.

S. Aoki, K. Kawana and K. Shimada,
“AdS/CFT correspondence for the O(N) invariant ϕ^4 model in 3-dimension by the conformal smearing”, JHEP10 (2024)111.

Back up

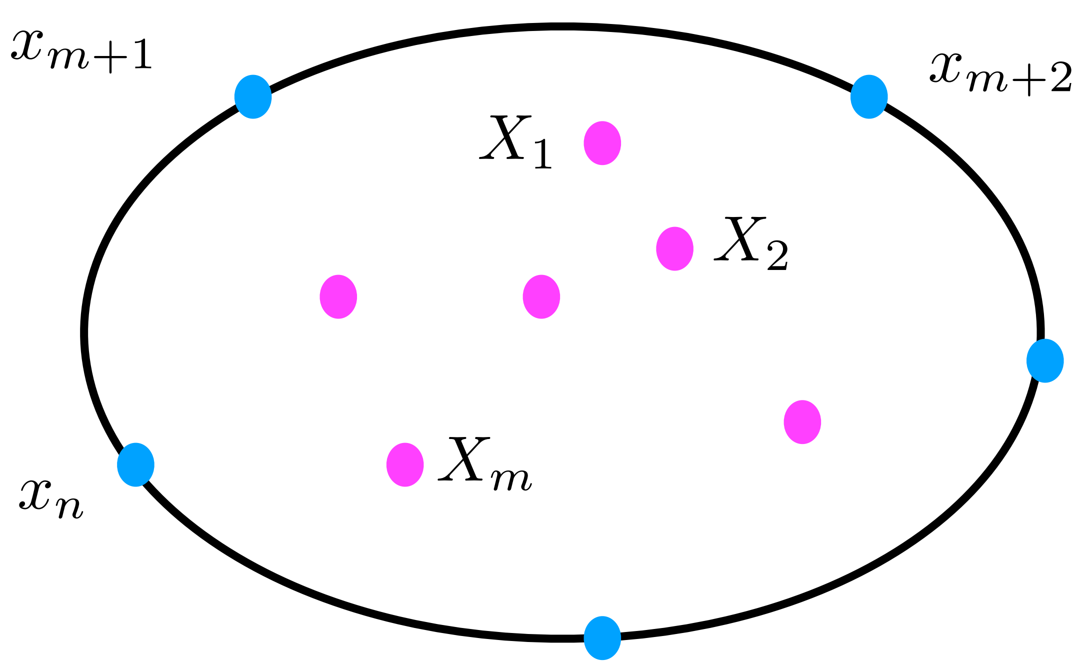
Constraints by symmetries

bulk operator (with a spin L) $G_A(X) := G_{A_1\cdots A_L}(X)$ $L = |A| = |s|$

boundary primary operator (with Δ_p and a spin L) $O^p_s(x) := O^p_{s_1\cdots s_L}(x)$

$$\langle 0| \prod_{i=1}^{m} G^i_{\tilde{A}_i}(\tilde{X}_i) \prod_{j=m+1}^{n} O^{p_j}_{\tilde{s}_j}(\tilde{x}_j)|0\rangle = \prod_{i=1}^{m} \frac{\partial X_i^{A_i}}{\partial \tilde{X}_i^{\tilde{A}_i}} \prod_{j=m+1}^{n} h(x_j)^{-\Delta_{p_j}} \frac{\partial x_j^{s_j}}{\partial \tilde{x}_j^{\tilde{s}_j}}$$

$$\times \langle 0| \prod_{i=1}^{m} G^i_{A_i}(X_i) \prod_{j=m+1}^{n} O^{p_j}_{s_j}(x_j)|0\rangle$$



ex. symmetric tensor

$$G_{AB}(X) := \partial_A\hat{\sigma}(X)\partial_B\hat{\sigma}(X)$$

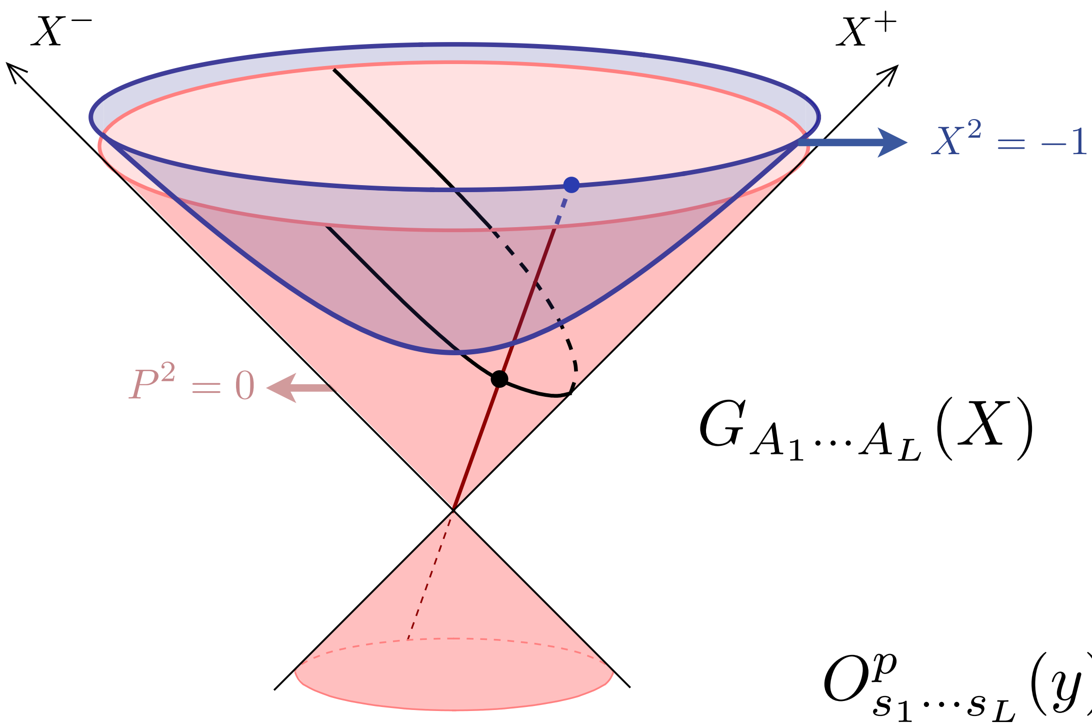
$$O_{\mu\nu}(y) := \partial_\mu\hat{\varphi}(y)\partial_\nu\hat{\varphi}(y)$$

Correlation functions including all quantum corrections satisfy these constraints, which are easily solved in the Embedding space $\mathbb{R}^{d+1,1}$.

$$\mathrm{SO}(d+1,1) = \text{Conformal symmetry} = \text{AdS isometry}$$

Costa-Penedones-Poland-Rychkov 2011, Costa-Goncalves-Penedones 2014

Embedding Formalism



Embedding space = $\mathbb{R}^{d+1,1}$

$SO(d+1,1)$ = Lorentz transformation in $\mathbb{R}^{d+1,1}$

AdS space is defined by $X^2 = -1,\ X^0 > 0$

$G_{A_1 \cdots A_L}(X)$ $\quad X^A := (X^+, X^-, X^\mu) = \frac{1}{z}(1, z^2 + x^2, x^\mu)$ bulk

Boundary is defined by $P^2 = 0,\ P := \lambda P,\ \lambda \in \mathbb{R}$

$O^p_{s_1 \cdots s_L}(y)$ $\quad P = (1, y^2, y^\mu)$ boundary

Symmetric tensor indices are handled by W^A, Z^A which satisfy $X \cdot W = Z \cdot P = 0$.

Transverse condition for the tensor on $P^2 = 0$ implies invariance under $Z \to Z + {}^\forall\alpha P$.

$SO(d+1,1)$ invariant function: $\quad G(\{X_i, W_i\}_{i=1,\cdots,m}; \{P_j, Z_j\}_{j=m+1,\cdots,n})$

conditions

conformal dimension

spin

$$G(\{X_i, \alpha_i W_i\}; \{\lambda_j P_j, \beta_j Z_j + \gamma_j P_j\}) = \prod_{i=1}^{m} \alpha_i^{L_i} \prod_{j=m+1}^{n} \beta_j^{L_j} \lambda^{-\Delta_{p_j}} G(\{X_i, W_i\}; \{P_j, Z_j\})$$

spin