Dyonic lines in 3d monopole semiclassics

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KEK theory workshop
16–18 Dec 2025

in collaboration with Yuya Tanizaki [in preparation]

Want to know all possible phases of $SU(N)$ gauge (+adj.) theories

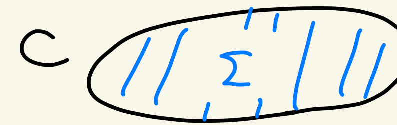
Motivation : Wilson-'t Hooft classification

['t Hooft '78, ... ; cf. Nguyen-Tanizaki-Ünsal '23]

$$\boxed{\begin{array}{l}\text{Gapped phases (w/ } \mathbb{Z}_N^{[1]} \text{ symmetry) can be classified by} \\ \text{area/perimeter law of dyonic lines } W_{(n,m)}(C;\Sigma) = W^n_{(C)} H^m_{(C;\Sigma)}\end{array}}$$

$$\begin{cases} W(C) : \text{Wilson loop} \\ H(C,\Sigma) : \text{(non-genuine) 't Hooft loop} \end{cases}$$

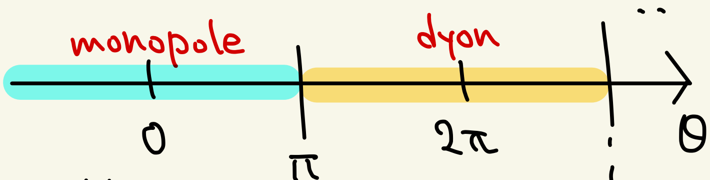
$$= \Big(\underbrace{\mathbb{Z}_N^{[1]} \text{ defect}}_{\sim \text{ 'magnetic flux of } \frac{2\pi}{N}\text{'}} \text{ on } \Sigma \Big)$$



$(\partial\Sigma = C)$

'non-genuine' but important! (string order parameter for SPT)

e.g. : SU(2) YM + θ term



0	2π
$\langle H \rangle \neq 0$	$\langle HW \rangle \neq 0$
$\langle W \rangle = 0$	$\langle W \rangle = 0$
$\langle HW \rangle = 0$	$\langle H \rangle = 0$

& if adj. higgs(es) added,
$\langle W \rangle \neq 0$, $\left.\begin{matrix} \langle H \rangle = 0 \\ \langle HW \rangle = 0 \end{matrix}\right\}$ deconf.

$$\left(\begin{matrix} \langle \bigcirc \rangle = 0 \iff \text{area law} \\ \langle \bigcirc \rangle \neq 0 \iff \text{perimeter law} \end{matrix} \right)$$

$\boxed{\text{In this talk,}}$

- We employ 3d monopole semiclassics
  'calculable (weakly-coupled) confinement'

  & aim to understand Wilson-'t Hooft classification

  observing dyonic lines in this framework.

3d monopole semiclassics

Weakly coupled confining theory:

[SYM: Davies-Hollowood-Khoze-Mattis '99, …]
[non-susy: Ünsal '07, Ünsal-Yaffe '08, …]

Example: $\boxed{SU(2)\ \text{YM}\ (+\text{massive adjoint fermions})\ \text{on}\ \mathbb{R}^3\times S^1_L}$ ⟵ Periodic B.C

3d effective theory at small S^1_L : $(e^{i\sigma}, e^{i\phi}) \in \dfrac{U(1)_\sigma \times U(1)_\phi}{\mathbb{Z}_2}$

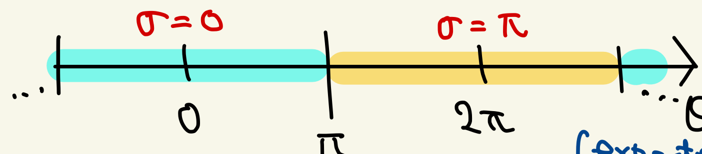
residual gauge $(\sigma,\phi)\sim(-\sigma,-\phi)$

d.o.f. $\begin{cases} \sigma: \text{dual photon}, \\ \phi: \text{holonomy}, \end{cases}$ $f^3 = \# * d\sigma$, $P = \begin{pmatrix} e^{i\phi} & \\ & e^{-i\phi} \end{pmatrix}$ in 'Polyakov gauge'

$$\mathcal{L}_{3d,\text{eff}}(\sigma,\phi) = g^2\left(\partial\sigma + \frac{\theta}{\pi}\partial\phi\right)^2 + \frac{1}{g^2}(\partial\phi)^2$$
$$- e^{-\frac{8\pi^2}{2g^2}}\left[\cos\left(\sigma + \frac{\theta}{2}\right) + \cos\left(-\sigma + \frac{\theta}{2}\right)\right] \Longleftarrow \text{(monopole dilute gas)}$$
$$+ V_{\text{eff}}(\phi) \longleftarrow \text{potential favoring center-symmetric point } (\phi = \pm\pi/2)$$

min.

$$\boxed{\sigma = \begin{cases} 0 & (-\pi<\theta<\pi, \cdots) \\ \pi & (\pi<\theta<3\pi, \cdots) \end{cases}} \Longrightarrow$$

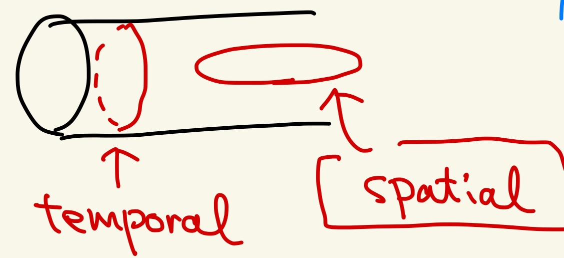


(expected vacuum structure)

't Hooft loop on $\mathbb{R}^3 \times S^1$

$\mathbb{R}^3 \times S^1$

$L^{4/9}$



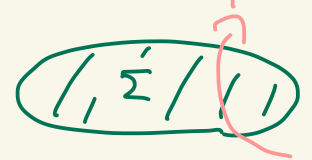
- $\begin{cases} \text{temporal loop (along } S^1) \\ \text{spatial loop (on } \mathbb{R}^3) \end{cases}$

- Here, we focus on spatial 't Hooft loop $H(C;\Sigma]$

$$\boxed{\text{Spatial 't Hooft loop} \overset{?}{=} \text{bdy. of } \underbrace{\pm\pi\text{-shift of } \phi}_{\text{center sym. defect.}}}$$

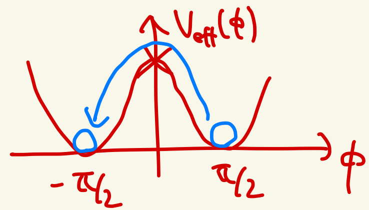
$P = \begin{pmatrix} e^{i\phi} & \\ & e^{-i\phi} \end{pmatrix}$

$\phi \mapsto \phi + \pi$



Problem: This 't Hooft loop is "bad"

$\begin{cases} \times \text{ looks area law} \\ \times \text{ hits } \underbrace{\text{singularity}}_{\text{beyond low-energy EFT.}} \ (\phi = 0) \end{cases}$



Idea: Screening

UV theory: $SU(2)$ YM$_{(+\cdots)}$ on $\mathbb{R}^3 \times S^1$

$H_{UV}(C;\Sigma)$ = bdy. of $(\mathbb{Z}_N^{[0]})_{3d}$ defect ← [works well on the lattice $(\theta=0)$ [De Forcrand - D'Elia - Pepe '00]]

↓ integrating out heavy d.o.f.

IR theory: $[(U(1)_\sigma \times U(1)_\phi)/\mathbb{Z}_2]$ scalar

$H_{IR}(C;\Sigma)$ = bdy. of $(\phi \mapsto \phi + \pi)$

$$H_{UV}(C;\Sigma) \rightsquigarrow \underbrace{H_{IR}(C;\Sigma)}_{\text{always area (or hits singularity)}} + \boxed{(\text{"screening"})}$$

area/perimeter of $H_{UV}(C;\Sigma)$ can be affected by this.

e.g.) Perimeter law of charge-2 Wilson loop

Screening by twisted vortices

$$H_{UV}(C;\Sigma) \rightsquigarrow \underbrace{H_{IR}(C;\Sigma)}_{\text{"area law"}} + \boxed{\left(\text{"screening"}\right)}$$

(← worldlines of heavy d.o.f.)

- In particular, we focus on 'twisted vortex' (or Alice string)

(It's non-dynamical in IR theory, but dynamical in UV theory)

[:= bdy. of S_2 gauge (Gukov - Witten defect)

S_2 gauge transf. $(\sigma, \phi) \mapsto (-\sigma, -\phi)$]

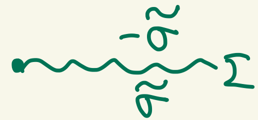
$\boxed{\text{Claim}}$: Screening by twisted vortex

$$H_{UV(C;\Sigma)} \supset e^{-\#|C|} H_{IR}(C;\Sigma) \underbrace{T(C)}_{\text{twisted vortex}} \xrightarrow{\text{fix } \phi=\pi/2} e^{-\#|C|} \text{"}\underbrace{C(\Sigma)}_{\substack{\text{charge conjugation in } \sigma \\ \sigma \mapsto -\sigma}}\text{"}$$

is lightest term after fixing $\phi = \pi/2$

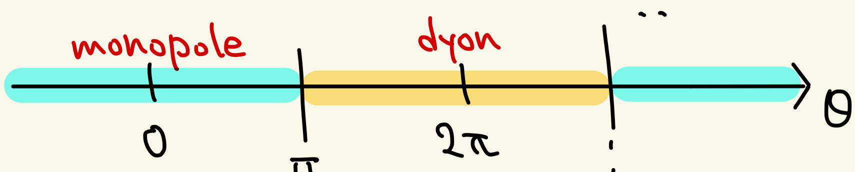
Dyonic lines in confining phases

This screened 't Hooft loop behaves as expected!

$H(C;\Sigma) =$ "$C(\Sigma)$" 

($\tilde{\sigma}$: a lift of σ to $\mathbb{R}$)

SU(2) YM + θ term



$\langle H \rangle \neq 0$	$\langle HW \rangle \neq 0$	$\langle H \rangle \neq 0$	
$\langle W \rangle = 0$	$\langle W \rangle = 0$	$\langle W \rangle = 0$	...
$\langle HW \rangle = 0$	$\langle H \rangle = 0$	$\langle HW \rangle = 0$	
① $\sigma = 0$	② $\sigma = \pi$	① $\sigma = 0$	

$\langle \circledcirc \rangle = 0$ $\Leftrightarrow$ area

$\langle \circledcirc \rangle \neq 0$ $\Leftrightarrow$ perimeter

3d EFT (at $\phi = \pi/2$): $\mathcal{L}_{3d} = |d\sigma|^2 - \#\left(\cos(\sigma + \theta/2) + \cos(-\sigma + \theta/2)\right)$

Dyonic lines in confining phases

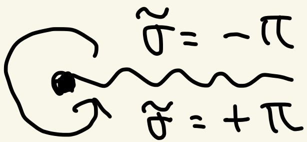
In both phases, $\langle W(c)\rangle \sim e^{-\#(\text{area})}$

$(\sigma = 0, \pi)$ (need kink for $\sigma \sim \sigma + 2\pi$)

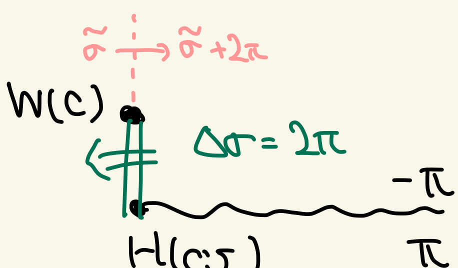
① $\boxed{-\pi < \theta < +\pi : \ \text{min. at } \sigma = 0}$

$H(c;\Sigma)$: $\langle H(c;\Sigma)\rangle_{\sigma=0} \neq 0$ (perimeter)

② $\boxed{\pi < \theta < 3\pi : \ \text{min. at } \sigma = \pi.}$

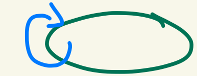
$H(c;\Sigma)$: 

need kink $\Rightarrow$ area law

$H(c;\Sigma)W(c)$: 

kink doesn't span the area
$\Rightarrow$ perimeter law.

Wilson loop $W(c)$

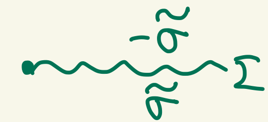


$\sigma \sim \sigma + 2\pi$
monodromy defect

$\Leftrightarrow$ change of lift $\tilde{\sigma} \mapsto \tilde{\sigma} + 2\pi$ on Σ

't Hooft loop

$H(c;\Sigma) =$ "$C(\Sigma)$"

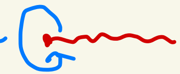


for a lift to $\mathbb{R}$: $\tilde{\sigma}$

Summary

$\boxed{\text{We've considered (non-genuine) dyonic lines in } SU(2) \text{ YM (+ massive adjoint fermions) on } \mathbb{R}^3 \times S^1_L \text{ at small } L.}$

3d IR EFT : $\dfrac{U(1)_\phi \times U(1)_\sigma}{S_2}$

$\int d\phi = \pi$ 

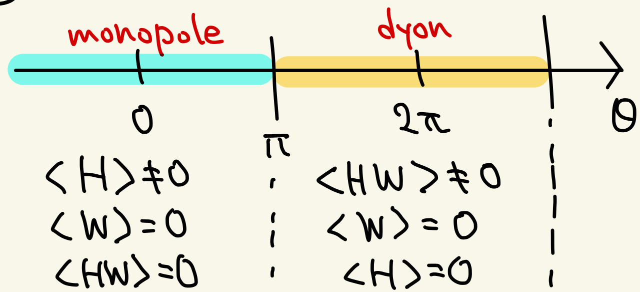
① Naive spatial 't Hooft loop is heavy ;

We propose "screening by twisted vortex" ($\leftarrow$ dynamical in UV)

$\Rightarrow$ light object! [at $\phi = \pi/2$, $H(c;\Sigma) =$ "$C(\Sigma)$"]

charge-conjugation in σ

② With this definition, we obtain



$\langle \circ \rangle = 0 \Leftrightarrow$ area law

$\langle \circ \rangle \neq 0 \Leftrightarrow$ perimeter law

Things I skipped

- Deconfined phase.
- Generalization to SU(N) → $\dfrac{U(1)_\sigma^{N-1} \times U(1)_\phi^{N-1}}{S_N}$. [root charge, W boson, ...
- temporal 't Hooft loop
- More on θ-dependence of dual string tension (fun w/ ~~CP~~ deconf. phase)
- Perimeter law of spatial Wilson loop in Higgs phase.
- Exotic phase in SU(4) YM.

(realizable in $\mathcal{N}=1^*$)