Higher derivative gravity, HD Dirac, HD SUSY, HD CFT

ー Construction of new types of quantum field theory

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Remember : HD = higher-derivative ($\neq$ higher dimensions)

Résumé

Higher-derivative means $\Box^2$ → Field Eq : $(\Box + m_1^2)(\Box + m_2^2)\,\phi = 0$.

Higher-derivative **(HD)** field theory has some history.
Solves **gauge hierarchy problem** (Lee-Wick), **dark matter** (Woodard), ⋯, but it is ***beyond*** Newtonian mechanics !

$d^2\mathbf{x}/dt^2 = \mathbf{F}/m \rightarrow d^4\mathbf{x}/dt^4 = \cdots$

Dirac (1942), T D Lee-Wick (1955, 1970), Woodard (2007), ⋯, Holdom,

i SM + HD quantum gravity ○

ii Exploit new types of HD theories

a) Scattering in HD Dirac theory ○

b) HD supersymmetry ○/△ (Ref: Ferrara-Zumino 1978, Fujimori et al,)

Conformal SUGRA contains HD and susy → existence proof

iii Is HD Liouville theory (mass deformed ***ghost CFT***) doable? ？

(Ref: Makeenko 2023, Fukusumi and T Kawamoto 2025)

Content

1. Introduction — HD field theories in new directions
2. **Higher-derivative Dirac theory**
3. **Higher-derivative supersymmetry**
4. Other theories are being studied
   Ghost CFT → mass deformed → higher-derivative CFT
5. Reflexion: Is 4th derivative operator $\Box^2$ not in good accord with mathematics?

Goal only partially obtained.

1. Introduction - HD field theories in new directions

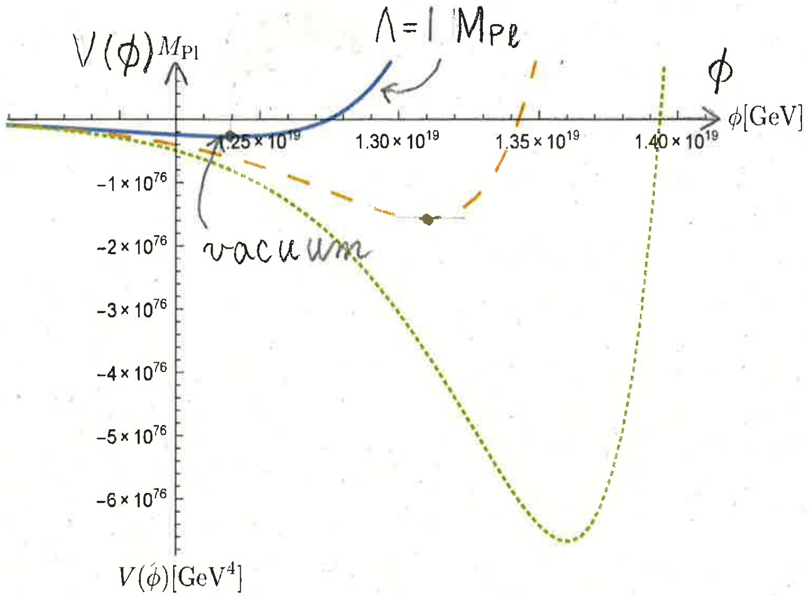
Study higher-derivative field theories in new perspectives.

i Phenomenology ー Incorporate ***HD quantum gravity*** in SM

Quantum correc to Higgs potential V(ϕ) in *quadratic gravity*

$\lambda\ \phi^4 + a\,\phi^4 \ln\phi$ + gravity correc.

ー Affects stability of the SM vacuum at Planck energies.



Correc in Einstein gravity is Λ dependent.

∵ non-renotrmalizable

→ ***renormalizable*** quantum gravity

ー quadratic gravity K Stelle 1976

$(R_{\mu\nu})^2 = h_{\alpha\beta}\, P^{(\alpha\beta,\mu\nu)}\, (\Box^2 + \Box)\, h_{\mu\nu}$

ii Exploit new types of HD theories

HD scalar field theory

$$S= \int d^4x -1/2 \ \phi (\Box + m_1^2)(\Box + m_2^2) \phi + \text{interactions}$$

a) HD Dirac theory

b) HD supersymmetry (HD susy)

iii Is HD Liouville theory (mass deformed ***ghost CFT***) doable?

Zamolodchikov (05) Makkenko (21, 23) “HD CFT and minimal models”

Why these problems may possibly be interestig?

1. Question : Does quantum gravity have effects in the SM.

2. SUSY is a beautiful theory, but has found little solid use in physics.

Other use of SUSY in new types of SUSY field theory is worth studying.

3. String is a beautiful theory, supposed to contain SM particles.
Unfortunately, string theories are formulated in ***d= 26 (d=10).***

Try to invent a new kind of string theory with ***other d (d=4).***
Construction of such CFT may open the way. But, is HD CFT doable?
Makkenko (21, 23)

HD Liouville theory (mass deformed ghost CFT) is still under study.
In this talk I focus on HD Dirac theory and HD SUSY (only partly finished).

2. Higher-derivative Dirac theory

2.1 HD scalar field theory

$S=\int d^4x\, -1/2\ \phi(\Box + m_1^2)(\Box + m_2^2)\phi$ +interaction.

Ex: $L_{int} = \lambda\{(\partial_\mu \phi)^2\}^2 \quad \because [\phi]=0$

Rewritten in the 2nd derivative way.

$S=\int d^4x\ 1/2\ [-\phi_1(\Box + m_1^2)\phi_1 + \phi_2(\Box + m_2^2)\phi_2]$ + interaction.

ϕ_2 is ***negative norm*** (ghost)

Roughly,

4-th derivative theory of ϕ = (+ norm) + (− norm)

$\phi_2 = (\Box + m_1^2)\phi/\sqrt{(m_2^2 - m_1^2)}$, $\phi_1 = \ldots$, $[\phi_2]=[\phi_1]=1$

Canonical quantization of ϕ_2 in two different ways!

$[a_2(\mathbf{q}), a_2^\dagger(\mathbf{p})] = +$ or $- \delta^3(\mathbf{q}-\mathbf{p})$ (+ for the usual QFT)

$+ \rightarrow$ negative energy, Hawking et al (2002), ⋯, Mukohyama et al,

$- \rightarrow$ negative norm (ghost) Lee-Wick (1955), Woodard (2007), Holdom, ⋯

Abe, T I, Izumi take the second view, have studied S-matrix unitarity

$S^\dagger S = 1$

in graviton scattering (2023).

Issue of ***unitarity*** in HD theory of Lee-Wick was studied by a few people,

T D Lee-Wick unitarity ○

N Nakanishi (1971) unitarity ○, but Lorentz symmetry ×

Kubo and Kugo (2023, 2024) unitarity ×

Myself, not being an expert in QFT, has nothing to add to this issue.

2. 2 Few studies of 2nd D Dirac theory

We conjecture mimicking HD scalar theory

$L_{2nd} = -1/2\bar{\psi}(i\gamma\cdot\partial - m_1)(i\gamma\cdot\partial - m_2)\psi$ +int. $[\psi]=1$

Rewritten in the 1st D way.

$L_{1st} = +1/2\bar{\psi}_1(i\gamma\cdot\partial - m_1)\psi_1 - 1/2\bar{\psi}_2(i\gamma\cdot\partial - m_2)\psi_2$

$\psi_2 = (i\gamma\cdot\partial - m_1)\psi/\sqrt{(m_2 - m_1)}$ negative norm fermion?

Interacting 2nd D Dirac theory can be constructed.

$L_{int} = f(\bar{\psi}\Gamma\psi)(\bar{\psi}\Gamma\psi)$, $\Gamma=1,\ \gamma_\mu,\ \cdots$ $[f]=0$ $\because [\psi]=1$,

・Canonical quantizaion in two alternative ways!

$\{\psi_1(q),\ \psi_1^\dagger(p)\} = +\delta^3(q-p)$, $\{\chi_1(q),\ \chi_1^\dagger(p)\} = +\delta^3(q-p)$ (usual theory)

$\{\psi_2(q),\ \psi_2^\dagger(p)\} = \pm\delta^3(q-p)$, $\{\chi_2(q),\ \chi_2^\dagger(p)\} = \pm\delta^3(q-p)$.

$+ \to$ negative energy ?

$- \to$ negative norm (fermionic ghost?)

Questions :

- Does concept of Dirac sea makes sense in HD Dirac theory?

  It's physical consequences yet to be studied.

- Show renormalizability of 4−fermi theory: $L_{2nd} + L_{int}$.

  ∵ coupling const $[f] = 0$

- Find other types of interactions

3. Higher(4th)-derivative supersymmetry

Aim at HD Wess-Zumino model (without F and G).

scalar (A, B) + Majorana ψ

- Kinetic terms borrowed from Sec 2.1 and 2.2

$$L_0 = -\tfrac{1}{2}\left[A(\Box+m_1^2)(\Box+m_2^2)A+B(\Box+m_1^2)(\Box+m_2^2)B\right]$$
$$-\tfrac{1}{2}\bar{\psi}(i\gamma\cdot\partial-m_1)(i\gamma\cdot\partial-m_2)\psi .$$

- Hint for interactions : Fermion part is four-Fermi : $(\bar{\psi}\,\Gamma\,\psi)(\bar{\psi}\,\Gamma\,\psi)$

2nd/1st derivative formalism is easy to work out.

We add the two sectors, $L_{+norm}(\phi_1, \psi_1)$ and $L_{-norm}(\phi_2, \psi_2)$.

$$L_0(\phi_1,\phi_2,\psi_1,\psi_2) = -1/2[A_1(\Box+m_1^2)A_1 + B_1(\Box+m_1^2)B_1]$$
$$+1/2[A_2(\Box+m_2^2)A_2 + B_2(\Box+m_2^2)B_2]$$
$$+1/2\,\bar{\psi}_1(i\gamma\cdot\partial-m_1)\psi_1 - 1/2\bar{\psi}_2(i\gamma\cdot\partial-m_2)\psi_2 .$$

Remark 1: Negative norm fermions have not been studied before (?), and I am not sure whether the ψ_2 term with − sign is the correct definition.

Remark 2: A few related papers on HD susy:
Conformal SUGRA contains HD and susy, HD susy may be realized.
Ferrara and Zumino (1977),
Fujimori et al (2016, 2017) try to construct HD susy by eliminating ghost, different from our spirit, but useful hints.

A few questions remain to be answered.

・Quantization

$[a_1(\mathbf{q}), a_1^\dagger(\mathbf{p})] = +\delta^3(\mathbf{q}-\mathbf{p})$

$\{\psi_1(q), \psi_1^\dagger(p)\} = +\delta^3(q-p)$, $\{\chi_1(q), \chi_1^\dagger(p)\} = +\delta^3(q-p)$.

Two different ways in − norm sector

$[a_2(\mathbf{q}), a_2^\dagger(\mathbf{p})] = +$ or $-\delta^3(\mathbf{q}-\mathbf{p})$

$\{\psi_2(q), \psi_2^\dagger(p)\} = \pm\delta^3(q-p)$, $\{\chi_2(q), \chi_2^\dagger(p)\} = \pm\delta^3(q-p)$.

Two (four?) different alternatives give ***different physics law***, which we have not yet fully understood.

Questions : Do old susy theorems hold in HD SUSY?

・***positive energy theorem***

・Concept of ***F term*** Fujimori et al (2016, 2017)

・***Non-renormalization theorem***

・Does HD superfield formalism work?

Superfield $\Phi(x^{\mu}, \theta) \to$ HD superfield $\Phi((x^{\mu}, \partial_{\nu} ?, \theta)$

Two superfield $\Phi_1(x^{\mu}, \theta)$, $\Phi_2(x^{\mu}, \theta)$ are easily constructed. The question of how to relate Φ_1 and Φ_2 to $\Phi(x^{\mu}, \theta)$ is under study.

4. Other theories are being studied

- HD CFT May be constructed by

  = CFT + ghost CFT.

- ghost Liouville gravity by Al Zamolodchikov (05).

- The latter has to be massive (mass-deformed?).

What kind of deformation?

Al Zamolodchikov (05), Cardy (2018), ⋯

Can one write Lagrangian?

Deformed WZW Q-H Park (1984)

5. Reflexion (not well thought):

Is 4th derivative operator $\Box^2$ not in good accord with mathematics?

- Is ghost field theory allowed? Zamolodchikov / Kubo and Kugo
- Mathematicians like $\Box$ rather than $\Box^2$.

  Cauchy problem concerns $\Box$, but not $\Box^2$.

  Can we formulate Cauchy problem with $\Box^2$.
- Does HD Dirac operator have meaning in K theory?

$\Box^2$ を2つの$\Box$ に分けて考えればよいか.

伝播子$=[(\Box+ m_1^2)(\Box+ m_2^2)]^{-1} \longrightarrow (\Box+ m_1^2)^{-1} - (\Box+ m_2^2)^{-1}$

After all you are beyond Newtonian mechanics!

Perhaps N Ohta san has some opinion on HD gravity, on HD susy.