On anomalies in family unification models

Hiroki Wada (Tohoku University)

Faculty of Science
Graduate School of Science
TOHOKU UNIVERSITY

Based on the ongoing work with T. Sugeno (Tohoku University)

Plan

1. Introduction

2. Formulation of fermion anomaly

3. Family unification model

4. Anomalies in family unification model

5. Summary

Introduction

☆ Recent progress: Formulation of anomalies

[Witten '15], [Witten, Yonekura '19], …

[Kapustin, Thorngren, Turzillo, Wang '14], [Freed, Hopkins '16], [Yonekura '18], …

➡ Applications to various areas

e. g.) String theory, Condensed matter physics, Particle physics, …

◎ Implications for Beyond the Standard Model (BSM)

・ A candidate of BSM is ruled out, if the theory has an anomaly!

➡ Additional degrees of freedom are needed to cancel the anomaly.

・ Anomalies have been studied for SM and candidates of BSM.

e.g.) [García-Etxebarria, Montero '18], [Davighi, Gripaios, Lohitsiri '19], [Wang '20],…

Family unification model

[Büchmuller, Peccei Yanagida '83], [Ong '83], [Kugo, Yanagida '83]

☆ Family unification model:

- A model based on a non-linear sigma model with Kähler manifold G/H
- The model includes three generations of quark and lepton for suitable G/H.

e. g.) $G/H = E_7/\left(\mathrm{SU}(5) \times \mathrm{SU}(3) \times \mathrm{U}(1)/\mathbb{Z}_{15}\right)$

◎ σ-model anomaly:

- For the family unification models to be consistent, not only gauge anomalies but also σ-model anomalies must vanish.

[Moore, Nelson '84 '85]

- We study these anomalies from modern perspective.

➡ In particular, we calculate global anomalies in these models.

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Witten-Yonekura construction

[Witten '15], [Witten, Yonekura '19]

◎ Goal: Defining a partition function of fermions

- Suppose fermions coupled to background fields on four-manifold X.

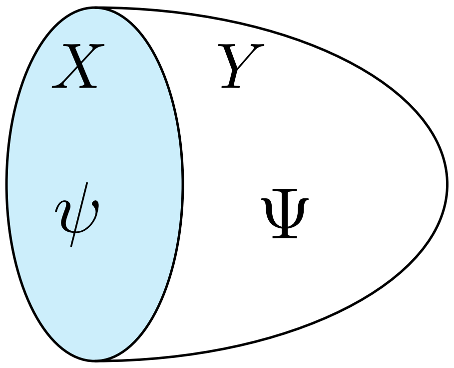
  e. g.) metric, gauge field, σ-model map

- The naive determinant is problematic.

$$Z = \int \mathcal{D}\overline{\psi}\mathcal{D}\psi \, \exp\left(-\int_X \mathrm{i}\overline{\psi}\not{D}\psi\right) \overset{?}{\sim} \det \mathrm{i}\not{D}$$

◎ Witten-Yonekura construction:

The fermions ψ are realized as boundary modes of heavy fermions Ψ on five-manifold Y.



➡ Define a partition function of ψ as that of Ψ:

$$Z_Y = \left|\det \mathrm{i}\not{D}\right| \mathrm{e}^{-2\pi \mathrm{i}\eta(Y)}$$

Eta-invariant

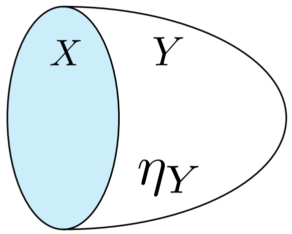
Dai-Freed theorem and anomaly

[Witten '15], [Witten, Yonekura '19]

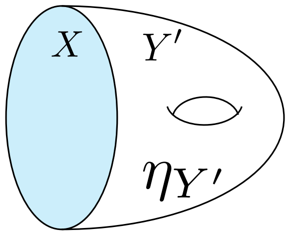
☆ Anomaly: Dependence on a five-manifold

- Suppose two partition functions:

Z_Y :



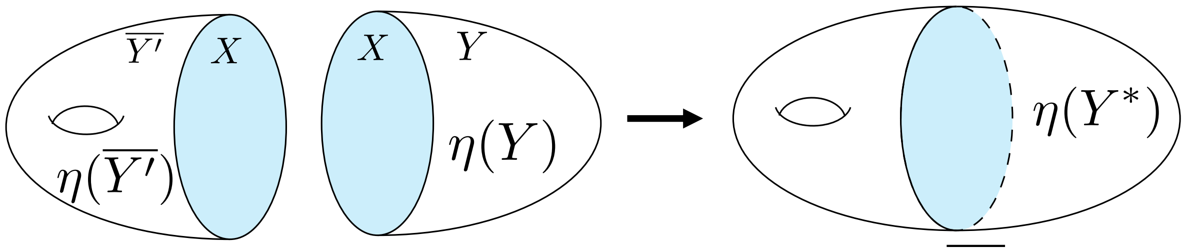
$Z_{Y'}$:



- The ratio is an eta-invariant on Y^* via the Dai-Freed theorem. [Dai, Freed '94], [Yonekura '16]

$$\frac{Z_Y}{Z_{Y'}} = \frac{\left|\det \mathrm{i}\not{D}\right| \mathrm{e}^{2\pi \mathrm{i}\eta(Y)}}{\left|\det \mathrm{i}\not{D}\right| \mathrm{e}^{2\pi \mathrm{i}\eta(Y')}} = \mathrm{e}^{2\pi \mathrm{i}\eta(Y^*)}$$

Anomaly



$Y^* = \overline{Y'} \cup Y$

- This phase is regarded as a partition function of Symmetry Protected Topological (SPT) phase on Y^*.

➡ **An anomaly of 4d theory is characterized by SPT phase in 5d.**

Classification of SPT phase

[Kapustin, Thorngren, Turzillo, Wang '14], [Freed, Hopkins '16], [Yonekura '18], [Yamashita, Yonekura '21]

☆ **SPT phases in 5d are classified by the group $(I_{\mathbb{Z}}\Omega^{\mathrm{Spin}})^6(M)$.**

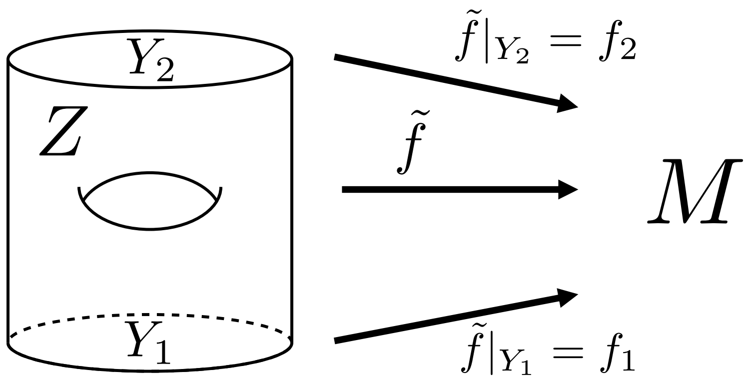
- Nature of the group $(I_{\mathbb{Z}}\Omega^{\mathrm{Spin}})^6(M)$:

$$0 \to \Omega_5^{\mathrm{Spin}}(M)_{\mathrm{tor}} \to (I_{\mathbb{Z}}\Omega^{\mathrm{Spin}})^6(M) \to \Omega_6^{\mathrm{Spin}}(M)_{\mathrm{free}} \to 0$$

~Global anomaly ~Perturbative anomaly

- Bordism group: $\Omega_d^{\mathrm{Spin}}(M) = \{(Y, f)\,|\,Y : d\text{-dim spin manifold}, f : Y \to M\}/\sim$

$$(Y_1, f_1) \sim (Y_2, f_2) \iff \exists (Z, \tilde{f})$$ s. t.



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Family unification model

◎ Setup:

[Buchmuller, Peccei Yanagida '83], [Ong '83], [Kugo, Yanagida '83]

- G/H: Kähler manifold
- Begin with supersymmetric non-linear sigma model with a target space G/H

$$\phi : X \to G/H \qquad \psi : \text{Superpartner of } \phi$$

- Gauge a subgroup of H

◎ Examples including three generations:

- $G/H = E_7/\left((\mathrm{SU}(5)\times\mathrm{SU}(3)\times\mathrm{U}(1))/\mathbb{Z}_{15}\right)$ model [Kugo, Yanagida '83]

  Representations of $\mathfrak{su}(5)\oplus\mathfrak{su}(3)$ $\qquad (\mathbf{10},\overline{\mathbf{3}})\oplus(\overline{\mathbf{5}},\mathbf{3})\oplus(\mathbf{5},\mathbf{1})$

- $G/H = E_7/\left(\left(\mathrm{SU}(5)\times\mathrm{U}(1)^3\right)/\mathbb{Z}_{15}\right)$ model [Yanagida, Yasui '85], [Sato, Yanagida '97]

  Representations of $\mathfrak{su}(5)$ $\qquad 3\times(\mathbf{10}\oplus\overline{\mathbf{5}})\oplus\mathbf{5}$

- There are also models relating with $\mathrm{Spin}(10)$-GUT. [Ong '85], [Irie, Yasui '85], [Itoh, Kugo, Kunitomo '85],…

Family unification model

◎ Advantages of $E_7/\left(\left(\mathrm{SU}(5)\times\mathrm{U}(1)^3\right)/\mathbb{Z}_{15}\right)$ model:

- Three generations are included. $3\times(\mathbf{10}\oplus\overline{\mathbf{5}})\oplus\mathbf{5}$
- Useful to realize an asymmetric structure among three generations [Sato, Yanagida '97]
- This model may be realized as an F-theory background. [Mizoguchi '14], …

➡ We study anomalies of this model.

◎ Our strategy

- It is technically difficult to calculate anomalies of the $E_7/\left(\left(\mathrm{SU}(5)\times\mathrm{U}(1)^3\right)/\mathbb{Z}_{15}\right)$ model.
- It is helpful to study anomalies of the $E_7/\left(\left(\mathrm{SU}(5)\times\mathrm{SU}(3)\times\mathrm{U}(1)\right)/\mathbb{Z}_{15}\right)$ model.

➡ We also focus on the $E_7/\left(\left(\mathrm{SU}(5)\times\mathrm{SU}(3)\times\mathrm{U}(1)\right)/\mathbb{Z}_{15}\right)$ model.

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σ-model anomaly

- In family unification models, fermions ψ are coupled to a tangent bundle of G/H through $\phi : X \to G/H$.

➡ The theories may be ill-defined due to σ-model anomalies.

[Moore, Nelson '84 '85]

- Classification of σ-model anomaly: $(I_{\mathbb{Z}}\Omega^{\mathrm{Spin}})^6(G/H)$ [Yamashita, Yonekura '21]

$$0 \to \underline{\Omega_5^{\mathrm{Spin}}(G/H)_{\mathrm{tor}}} \to (I_{\mathbb{Z}}\Omega^{\mathrm{Spin}})^6(G/H) \to \underline{\Omega_6^{\mathrm{Spin}}(G/H)_{\mathrm{free}}} \to 0$$

Possibility of an additional anomaly

Studied in [Moore, Nelson '84 '85], [Yanagida, Yasui '85]

- G/H: Kähler manifold ➡ No additional anomaly!

Argument similar to [Yonekura '22]

- $\Omega_5^{\mathrm{Spin}}(E_7/\left((\mathrm{SU}(5)\times\mathrm{SU}(3)\times\mathrm{U}(1))/\mathbb{Z}_{15}\right)) \cong 0$

Anomaly after gauging

- To obtain a gauge theory, we gauge a subgroup of H .

➡ An anomaly may arise after this gauging procedure.

- One choice: Gauging entire H [Sato, Yanagida '97]
- Classification of anomaly: $(I_{\mathbb{Z}}\Omega^{\mathrm{Spin}})^6(BH \times_H G/H)$ [Freed, Hopkins '19]

$$0 \to \Omega_5^{\mathrm{Spin}}(BH \times_H G/H)_{\mathrm{tor}} \to (I_{\mathbb{Z}}\Omega^{\mathrm{Spin}})^6(BH \times_H G/H) \to \Omega_6^{\mathrm{Spin}}(BH \times_H G/H)_{\mathrm{free}} \to 0$$

Possibility of a subtle anomaly

Essentially studied in [Moore, Nelson '84 '85]

- For the $E_7/\left((\mathrm{SU}(5)\times\mathrm{SU}(3)\times\mathrm{U}(1))/\mathbb{Z}_{15}\right)$ model, $\Omega_5^{\mathrm{Spin}}(BH \times_H G/H) \cong 0$

➡ No subtle anomaly!

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Summary

- We study anomalies of family unification models.
- We show that an additional σ-model anomaly is absent.
- For the $E_7/\left((\mathrm{SU}(5)\times\mathrm{SU}(3)\times\mathrm{U}(1))/\mathbb{Z}_{15}\right)$ model, gauging H does not lead to any global anomaly.

◎ Future work

- Calculate anomaly of $E_7/\left(\left(\mathrm{SU}(5)\times\mathrm{U}(1)^3\right)/\mathbb{Z}_{15}\right)$ model that may arise in Sato-Yanagida's setup [Sato, Yanagida '97]
- Implication for the realization by F-theory? [Mizoguchi '14], …

Backup

Example

- Suppose $\mathrm{SU}(2)$ gauge theories in 4d.
- Classification of gauge anomaly: $(I_{\mathbb{Z}}\Omega^{\mathrm{Spin}})^6(B\mathrm{SU}(2))$

$B\mathrm{SU}(2)$: Classifying space

Global anomaly

Perturbative anomaly

$$0 \to \Omega_5^{\mathrm{Spin}}(B\mathrm{SU}(2))_{\mathrm{tor}} \to (I_{\mathbb{Z}}\Omega^{\mathrm{Spin}})^6(B\mathrm{SU}(2)) \to \Omega_6^{\mathrm{Spin}}(B\mathrm{SU}(2))_{\mathrm{free}} \to 0$$

$\Omega_5^{\mathrm{Spin}}(B\mathrm{SU}(2))_{\mathrm{tor}} \cong \mathbb{Z}_2$ ⟵ Witten anomaly

$\Omega_6^{\mathrm{Spin}}(B\mathrm{SU}(2))_{\mathrm{free}} \cong 0$