Lattice QCD for Neutrino Oscillation Experiments

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October 27, 2025

Neutrino Interactions for Current and Future Experiments — KEK

This work is supported in part by:
Lawrence Livermore National Security, LLC #DE-AC52-07NA27344,
Neutrino Theory Network Program Grant #DE-AC02-07CHI11359,
U.S. Department of Energy Award #DE-SC0020250.

LLNL-PRES-2012633

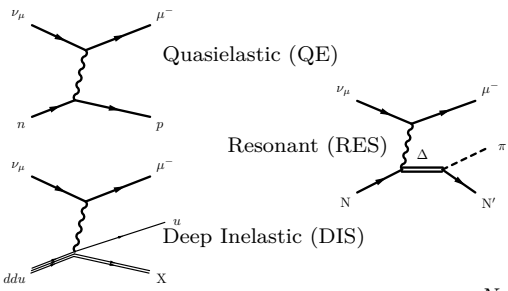
Outline

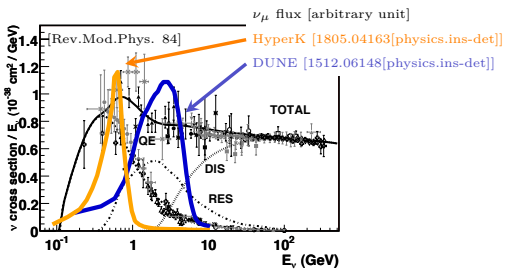
- Cross Sections in Accelerator Neutrino Experiments
- Quasielastic Scattering from Experiments
- Lattice QCD Introduction
- LQCD Calculations for Quasielastic Scattering
- Implications of LQCD Results for Oscillation Experiments
- The Future of LQCD for Neutrinos

Note: all references in online slides are hyperlinked

Neutrino Cross Sections

Using neutrino-nucleon to learn about neutrino-nucleus





N_{events} =flux · cross_section · efficiency

Precise, theoretically robust *nucleon* inputs

$\Longrightarrow$ quantitative & informative *nuclear* uncertainties

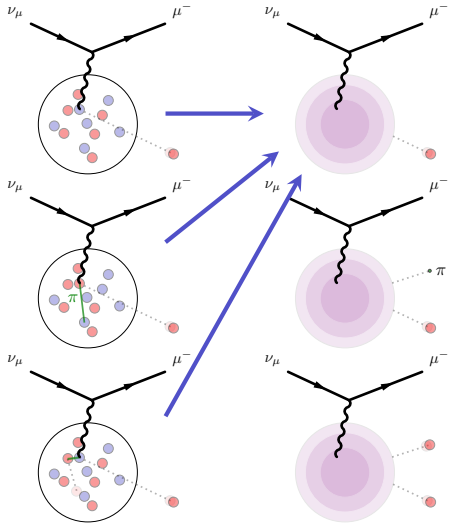
Interactions are hidden within a nuclear target

Nuclear environment complicates measurements:

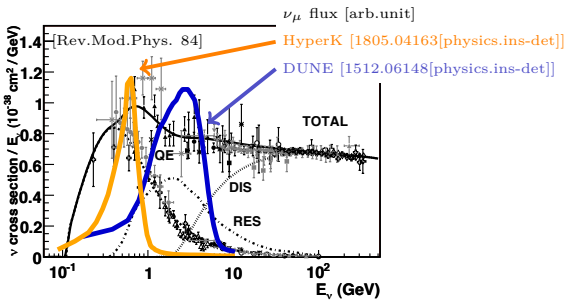
- Many allowed kinematic channels
- Reinteractions within nucleus
- Only final state particles are observable

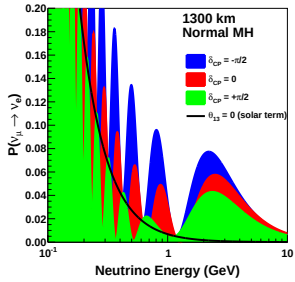
Isolating interactions depends on precise theory knowledge

$\Longrightarrow$ difficult, especially for axial contribution



Neutrino cross sections from elementary targets





Question:

How well do we know nucleon cross sections from elementary target sources?

For quasielastic, three main sources:

- Deuterium scattering
- Hydrogen scattering
- Lattice QCD

Quasielastic Scattering

z expansion parameterization

Model independent parameterization: z expansion [Phys.Rev.D 84 (2011)] —

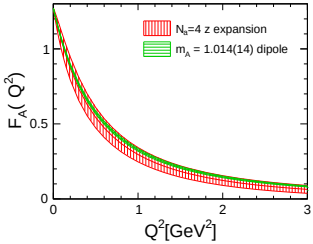
$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\text{cut}}) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} - t_0}} \qquad t_{\text{cut}} \leq (3M_\pi)^2$$

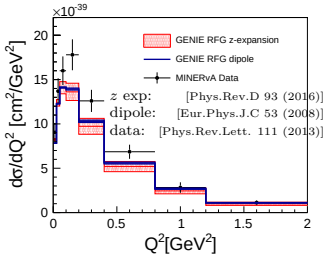
- Rapidly converging expansion
- Controlled procedure for introducing new parameters
- Sum rule constraints to regulate large-Q^2 behavior

Neutrino-nucleon uncertainties are not negligible in experiments

Previous conclusions from [Phys.Rev.D 93 (2016)]

- Older bubble chamber results
- Dipole overconstrained by data
  underestimated uncertainty $\times 10$
- Discrepancies in nuclear cross sections
  could be nucleon and/or nuclear origins





Additional modern experimental datasets

Previous datasets:

- ANL, BNL, FNAL deuterium bubble chamber event distributions

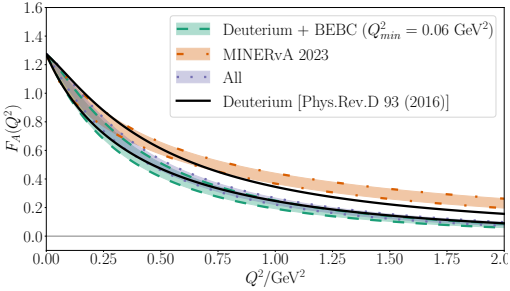
Two additional datasets:

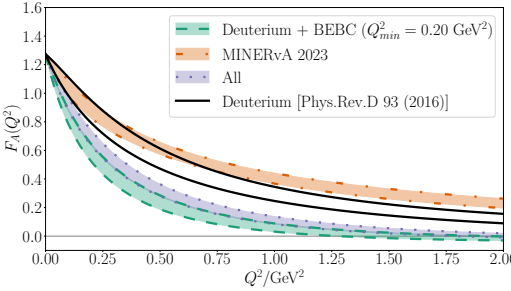
- MINERνA 2023 $\bar{\nu}_\mu p \to \mu^+ n$ [Nature 614 (2023)]
  Special thanks: Tejin Cai, Kevin McFarland, Miriam Moore
- BEBC 1990 $\nu_\mu D \to \mu^- pp$ [Nucl.Phys.B 343]
  Special thanks: Clarence Wret, NUISANCE

Updated vector form factors:

- Use Borah *et al.* z expansion [Phys.Rev.D 102 (2020)]

Deuterium & MINERvA are not consistent





Apparent improvement to p value at $Q^2_{\text{min}} = 0.06$ GeV^2 only from poorly-described data at low Q^2

Remnant systematic effects in deuterium data?

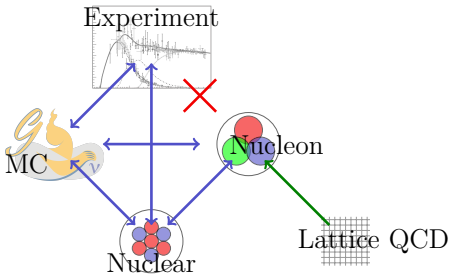
	$Q^2_{\text{min}} = 0.06$ GeV^2		$Q^2_{\text{min}} = 0.20$ GeV^2	
fit	χ^2_{data}/DoF	$p_{\Delta\chi^2}$	χ^2_{data}/DoF	$p_{\Delta\chi^2}$
All Deuterium	116.1/111		100.4/105	
MINERvA	9.1/ 16		9.1/ 16	
All	129.5/125		120.9/119	
$\Delta\chi^2$	4.3/ 1	0.04	11.3/ 1	8×10^{-4}

Lattice QCD Basics

What is Lattice Quantum Chromodynamics (LQCD)?

LQCD is the **only known mathematically rigorous method**
to compute properties of hadrons in nonperturbative QCD

- ✓ First principles calculations of QCD amplitudes
- ✓ Controlled nuclear effects
- ✓ Realistic, robust uncertainty quantification
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive



First principles predictions of QCD from the lattice

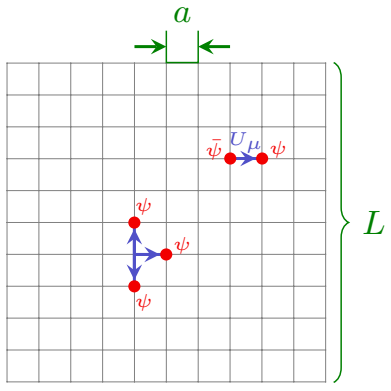
Numerical evaluation of path integral

Quark, gluon degrees of freedom —

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}U \, \exp(-S) \, \mathcal{O}[\psi, \overline{\psi}, U]$$

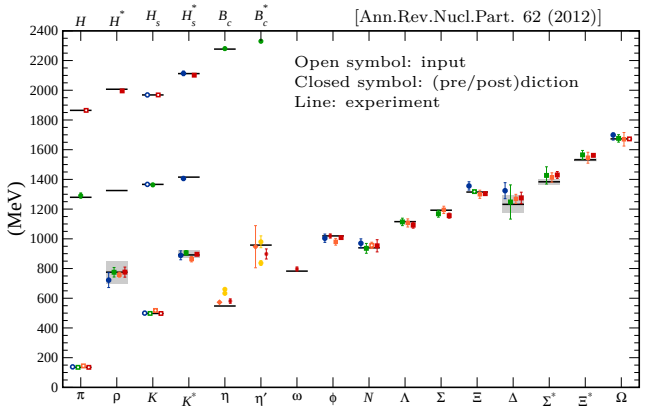
Parameters: $am_{(u,d),\text{bare}}$, $am_{s,\text{bare}}$, $\beta = 6/g^2_{\text{bare}}$

Matching: e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, M_Ω
1 per parameter



Results — amplitudes with gluon+$q\bar{q}$ corrections to all orders

LQCD is extremely predictive



- (Very) few inputs, (very) many outputs
- Heavily constrained by SM
- Widely used in flavor physics (CKM matrix elements)

“Physical Point” results have extrapolated away systematics

At a minimum, a complete error budget includes:

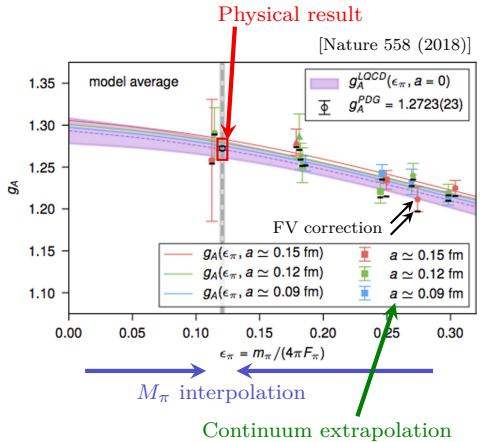
- $a \to 0$ Continuum extrapolation
- $L \to \infty$ Finite volume extrapolation
- $M_\pi \to M_{\pi,\text{phys}}$ “Chiral” extra(inter)polation

Every point in extrapolation is a *separate ensemble*

$\implies$ very expensive!

Incomplete results will get reported,
updated later with more ensembles

Always compare to extrapolated values or use due caution!

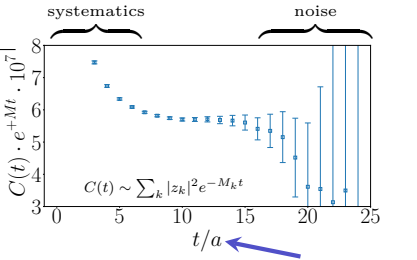


LQCD correlation functions in Euclidean time

Statistical uncertainties obtained from sampling **gauge configurations** on an **ensemble**

Signal	Noise^2
$\langle C(t) \rangle$	$\sigma^2_{C(t)} = \langle \lvert C(t)\rvert^2 \rangle - \lvert \langle C(t) \rangle \rvert^2 \leq \langle \lvert C(t)\rvert^2 \rangle$
$\langle$  $\rangle$	$\langle \lvert$  $\rvert^2 \rangle = \langle$  $\rangle$
$\sim e^{-M_n t}$	$\sim e^{-3M_\pi t}$

Signal/Noise $\sim e^{\left(\frac{3}{2}M_\pi - M_N\right)t}$



Fit to full Euclidean time dependence of correlation function

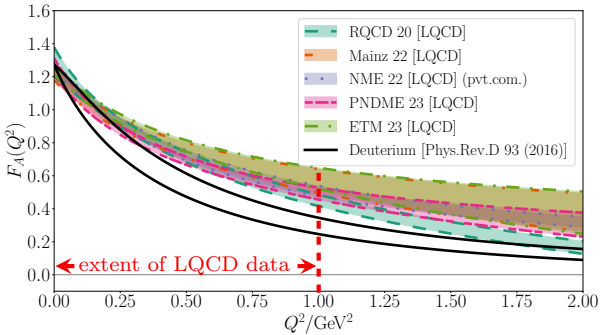
Exponential degredation of signal/noise with time

Samples on the same ensemble are **highly correlated**:

- different Euclidean times from same correlation function
- correlation functions computed on same configurations with different operators

LQCD in Quasielastic Scattering

LQCD predicts slow falloff with Q^2



LQCD results maturing:

- Many results, complete error budgets
- Small systematic effects observed (expectation: largest at $Q^2 \to 0$)
- Nontrivial consistency checks from PCAC

LQCD prediction of slow Q^2 falloff **predates MINERvA hydrogen**

Averaging strategy

Want same z expansion parameterization used in experiment fitting:

- g_A fixed to exact experimental value $g_A = 1.2754$
- 4 sum rules regulate large-Q^2 behavior
- no priors imposed

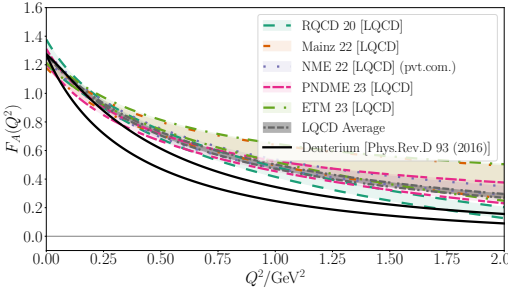
Procedure for matching LQCD results with different F_A parameterizations:

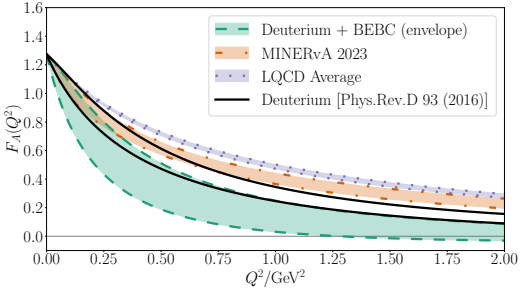
1. fit to derivatives $(d/dQ^2)^n F_A(Q^2)$ for $n \in \{0, ..., n_{\max}\}$ with $n_{\max} \leq k_{\max} - 4$
2. derivatives evaluated at $Q^2 = \max\left[\, |-t_0|,\, 0.05\ \text{GeV}^2 \,\right]$

One result uses 3-parameter Padé parameterization

$\implies$ treated same as a 3-parameter z expansion with $t_0 = 0$

MINERvA hints at consistency with LQCD average





LQCD dominates joint fit with MINERvA

99% confidence upper bound allows for consistency

fit	χ^2/DoF	$p_{\Delta\chi^2}$
LQCD	6.7/ 8	
MINERvA	9.1/16	
Combined	18.6/22	
$\Delta\chi^2$	2.7/ 1	0.10

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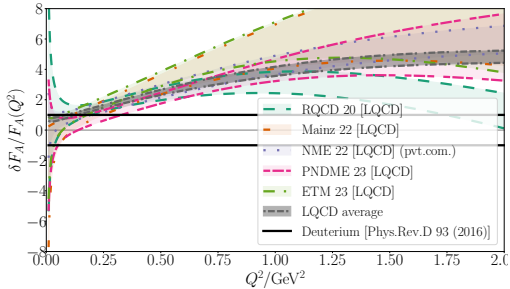

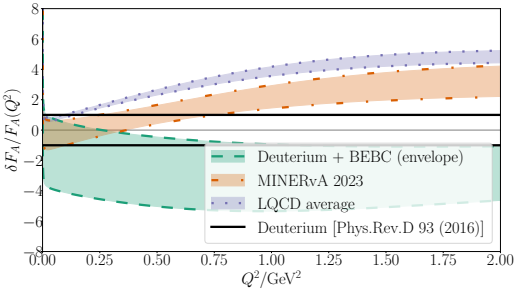

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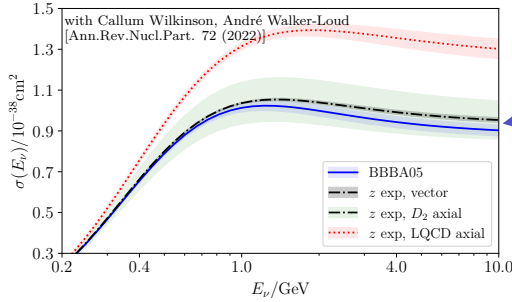
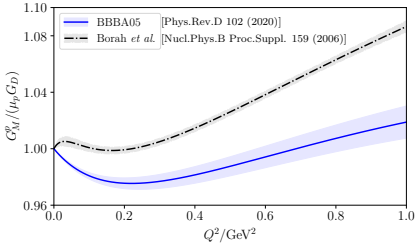
Implications of LQCD Results

Fits suggest enhanced nucleon quasielastic cross section





LQCD prefers 30-40% enhancement of ν_μ CCQE cross section

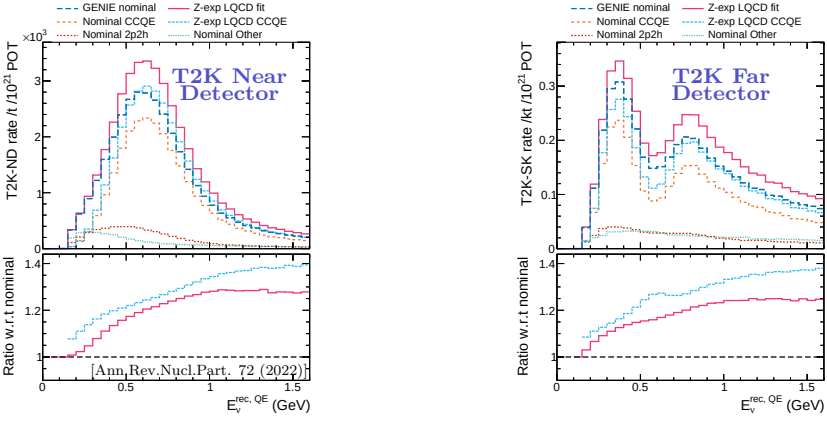
Recent Monte Carlo tunes require 20% enhancement of QE

[Phys.Rev.D 105 (2022)] [Phys.Rev.D 106 (2022)]

With improved precision, sensitive to

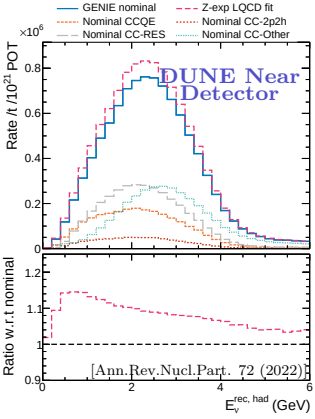
- BBBA vs z expansion vector form factor difference
- isospin-breaking corrections? [Phys.Rev.Lett. 129 (2022)]

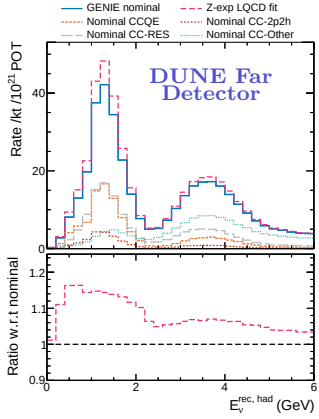
E_ν-dependent neutrino event rates (T2K)



- Dashed dark blue (GENIE nominal) vs solid magenta (z exp LQCD fit)
- QE enhancements produce 10–20% event rate enhancement, E_ν-dependent
- Monte Carlo tuning invalidates more sophisticated comparisons

E_ν-dependent neutrino event rates (DUNE)

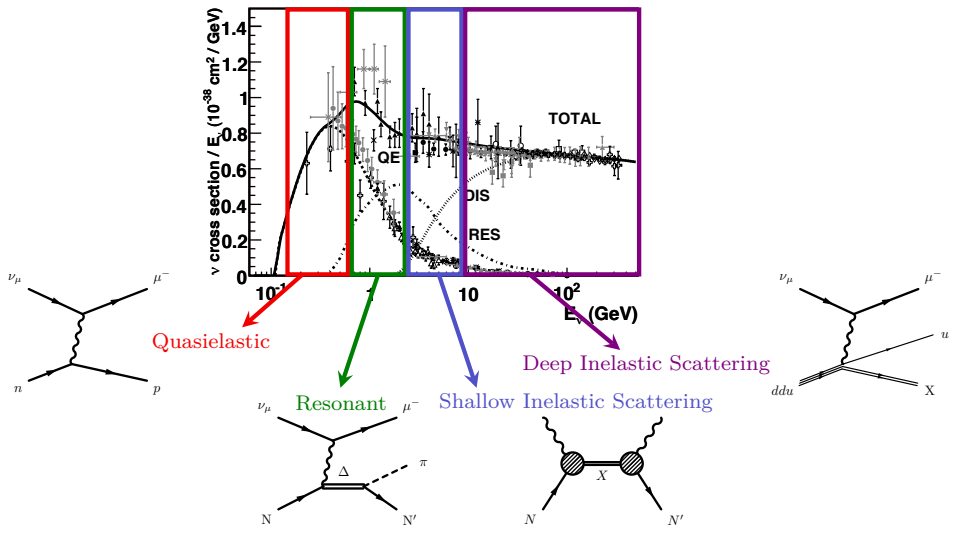




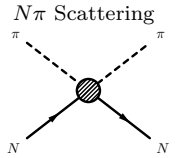
- Solid dark blue (GENIE nominal) vs dashed magenta (z exp LQCD fit)
- QE enhancements produce 10–20% event rate enhancement, E_ν-dependent
- Monte Carlo tuning invalidates more sophisticated comparisons

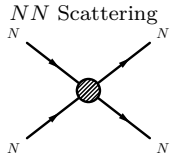
Concluding Remarks

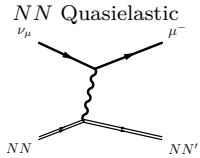
LQCD covers energy ranges relevant for accelerator oscillations



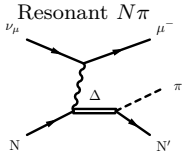
LQCD can probe many interaction mechanisms

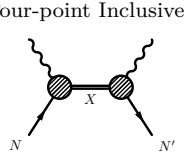


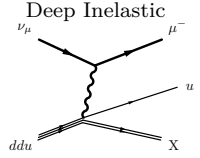


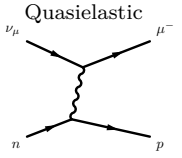


(incomplete list!)





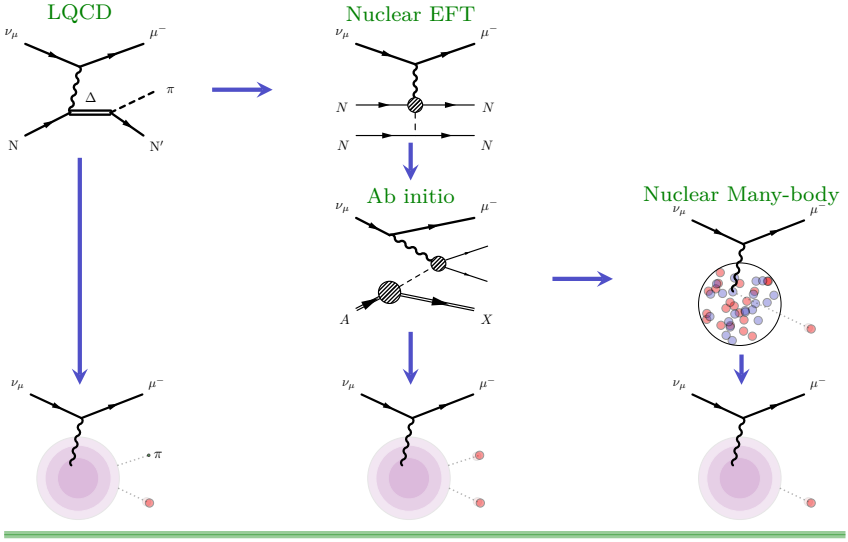




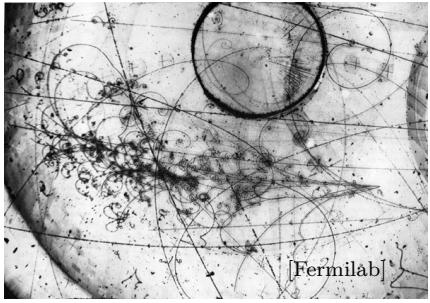
Nuclear

Nucleon

Roadmap connecting LQCD to neutrino cross sections



Outlook



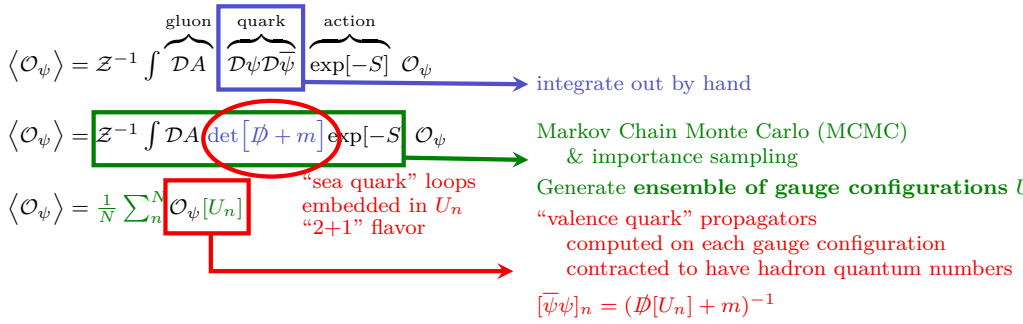
- LQCD is a disruptive technology, complementary to experiment
- LQCD produces first-principles predictions of hadronic amplitudes
  $\Longrightarrow$ strong constraints on interferences!
- Nucleon axial form factor uncertainty historically underestimated
- Evidence that QE cross section central value underestimated
  $\Longrightarrow$ LQCD *pre*diction + experimental evidence
- Many developments from LQCD applicable to accelerator neutrino cross sections
  $\Longrightarrow$ exciting times ahead!

Thank you for your attention!

Backup

Dissecting a LQCD Computation

In QCD, the path integral is an expectation value of a combination of operators $\mathcal{O}_\psi$:



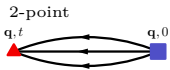
Computation is "just" an average of correlation functions computed on sampled gauge snapshots

LQCD Computation Anatomy

Write correlation functions with complete set of states $\mathbb{1} = \sum_n |n\rangle\langle n|$,
including ground state and **excited states**

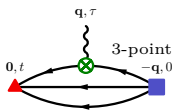
2-point function

$$\langle \blacktriangle(t)\, \blacksquare\, (0)\rangle = \sum_n \underbrace{\langle 0|\blacktriangle|n\rangle}_{\text{“sink”}} \underbrace{\langle n|\blacksquare|0\rangle}_{\text{“source”}} e^{-E_n t}$$



3-point function

$$\langle \blacktriangle(t) \otimes(\tau)\, \blacksquare\, (0)\rangle = \sum_{mn} \langle 0|\blacktriangle|n\rangle\langle n|\otimes|m\rangle\langle m|\blacksquare|0\rangle e^{-E_n(t-\tau)-E_m\tau}$$



Extract mass/energy from 2-point, matrix elements from 3-point

Source/sink operators ($\blacksquare$, $\blacktriangle$) have different **overlaps** onto states m, n

Source and sink operators are *instructions* on how to contract quark propagators:

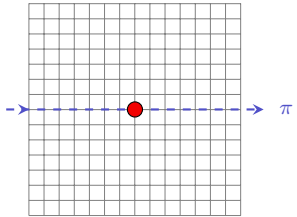
$$\mathcal{O} = \bar{q}\gamma_5 q,\ \not{D}^{-1}(t,0) = q(t)\bar{q}(0) \implies \left\langle \mathcal{O}(t)\mathcal{O}(0)\right\rangle = \mathrm{Tr}\left[\not{D}^{-1}(t,0)\gamma_5 \not{D}^{-1}(0,t)\gamma_5\right]$$

Finite Volume Effects

Finite volume effects for hadrons via virtual particle exchange with themselves through periodic boundary

Like time dependence, falls off $\propto \exp[-EL]$

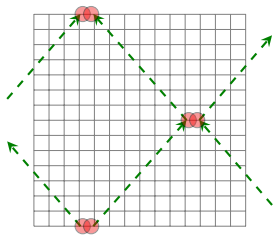
For typical ensembles, $M_\pi L \sim 4 \implies$ only π nonnegligible



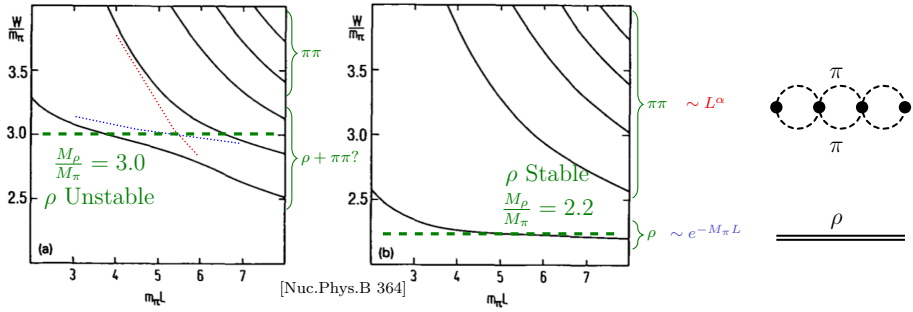
No analogue using QFT definition of asymptotic separation particles in multiparticle states cannot be isolated

Presence of other nearby particles has large effects on spectrum

Strong spectrum modifications $\propto L^\alpha$



Resonances in Finite Volume



Toy example above: $\pi\pi$ scattering with ρ state

- (Left) Resonances, avoided level crossings: FV corrections $\propto L^{\alpha}$
- (Right) Single-particle states: FV corrections $\propto \exp\{-M_\pi L\}$

$\Longrightarrow$ Particle count is *not* a good quantum number

Continuum Limit

[Lepage Proceedings]

Reminder of the link definition: $U_\mu(x) = e^{-ig\int_x^{x+a\hat{\mu}} A_\mu(x')dx'} =$ 

Define a plaquette:
$$P_{\mu\nu}(x) = \tfrac{1}{3}\mathrm{Re}\Big[\mathrm{Tr}\Big[U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)\Big]\Big]$$
$$= \tfrac{1}{3}\mathrm{Re}\Big[\mathrm{Tr}\Big[e^{-ig\oint_\square A\cdot dx'}\Big]\Big] = \tfrac{1}{3}\mathrm{Re}\Big[\mathrm{Tr}\Big[$$  $$\Big]\Big]$$

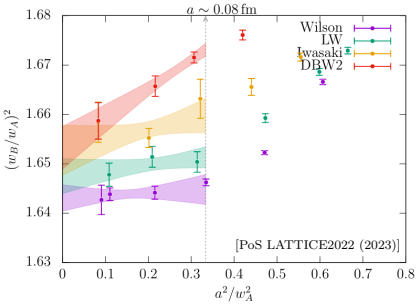
Taylor expand:
$$P_{\mu\nu}(x) = \tfrac{1}{3}\mathrm{Re}\Big[\mathrm{Tr}\Big[1 - ig\Big(\oint_\square A\cdot dx'\Big) - \tfrac{1}{2}g^2\Big(\oint_\square A\cdot dx'\Big)^2 + ...\Big]\Big]$$

Stokes' theorem
$$\oint_\square A\cdot dx' = \int_x^{x+a\hat{\mu}}\int_x^{x+a\hat{\nu}} dx'_\mu dx'_\nu\Big(\partial_\mu A_\nu(x') - \partial_\nu A_\mu(x')\Big)$$
$$= a^2F_{\mu\nu}(\bar{x}) + \tfrac{a^4}{24}\Big(D_\mu^2 + D_\nu^2\Big)F_{\mu\nu}(\bar{x}) + ... \qquad \bar{x} = x + \tfrac{a}{2}\Big(\hat{\mu}+\hat{\nu}\Big)$$

The Wilson action:
$$S_W = \beta\sum_{x,\mu>\nu}\Big(1 - P_{\mu\nu}(x)\Big)$$
$$= \int d^4x\sum_{\mu\nu}\Big\{\tfrac{1}{2}\mathrm{Tr}F_{\mu\nu}^2 + \tfrac{a^2}{24}\mathrm{Tr}F_{\mu\nu}\Big(D_\mu^2 + D_\nu^2\Big)F_{\mu\nu} + ...\Big\}$$

Lattice action same as physical continuum action up to order a^2 corrections...

Continuum Limit Universality

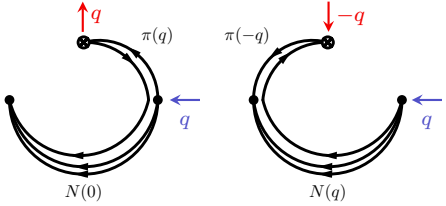


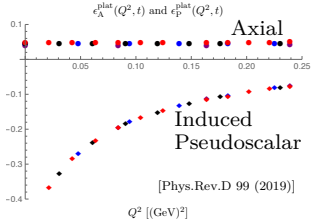
Different actions can have different approach $a \to 0$

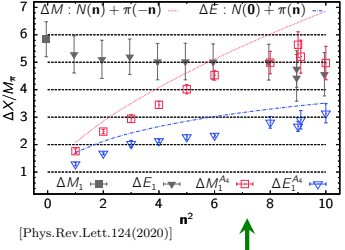
Usually $O(a^2)$, some improved to e.g. $O(a^2\alpha_s)$
$\implies$ additional terms added to action

All actions should give same result in continuum

Example — χPT and $N\pi$ in Excited States







NOTE: expect only approx agreement between data/curves

Excited state uncertainties are another critical systematic

(Opinion: most challenging in nucleon computations)

χPT: Contamination in $F_A(Q^2)$ primarily from enhanced $N\pi$, mostly from induced pseudoscalar

Empirical: two $N\pi$ states dominate contamination at nonzero Q^2

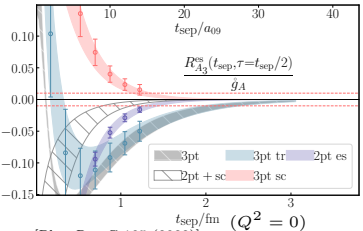
LQCD references

Reference	N_{ens}	$N_{\text{ens}}^{\text{phys}}$	quark action	fit method	parameterization
[RQCD 2020]	37	2	clover	χPT-inspired	z^2, ~~sum rules~~
[NME 2022]	7	0	clover	Bayesian prior exponential	Padé[0/2]
[Mainz 2022]	14	1	clover	summation + z expansion	z^2, ~~SR~~
[PNDME 2023]	13	2	clover (on HISQ)	Bayesian prior exponential	z^2, ~~SR~~
[ETM 2023]	3	3	twisted mass	Bayesian prior exponential	z^3, ~~SR~~

Notes:

- $g_A \sim a_0$ is an output for LQCD
  fixed to g_A from experiment in these fit (not in input LQCD parameterizations)
- these fits will use sum rules (input LQCD parameterizations do not)
- RQCD, NME, Mainz results highly correlated with unknown correlations
  (use covariance derating [Phys.Rev.D 111 (2025)])

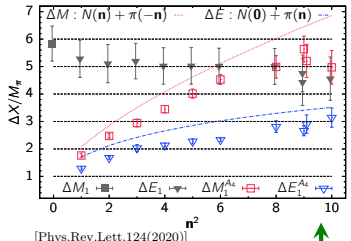
LQCD Excited States — Empirical



$(Q^2 = 0)$

[Phys.Rev.C 105 (2022)]

$(Q^2 \geq 0)$



[Phys.Rev.Lett.124(2020)]

NOTE: expect only approx agreement between data/curves

Compare fit to correlator data ratio

Contamination dominated by "transition" states $(0 \to n$, blue)

Typically signal below $\lesssim 1$ fm, contamination $\gtrsim 2$ fm

Excited states present in practically-achievable large time limit

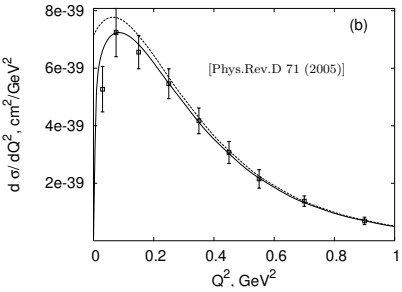
NME collab:

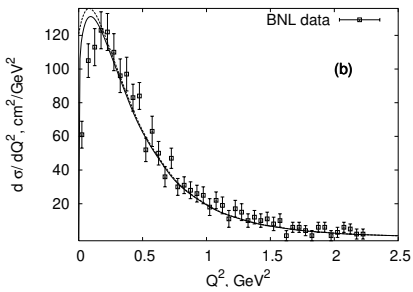
Q^2 contamination from $N \to N\pi$

Dominant contribution agrees with χPT expectation

$N\pi$ is important for $F_A(Q^2)$

Resonance Production - $N \to \Delta$





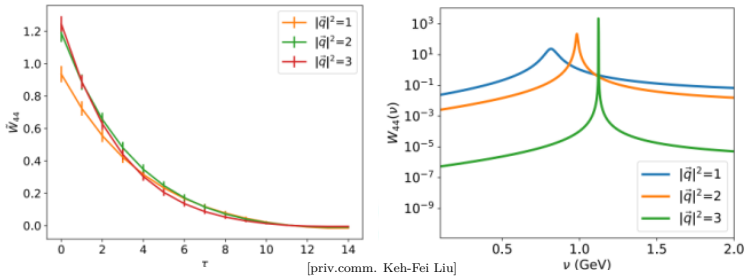
1π cross section known to 30% [Phys.Rev.C 88 (2013)]
DUNE error budget $\lesssim$10% precision [2002.03005 [hep-ex]]

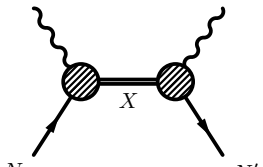
Unconstrained axial form factors in $J^P = 3/2^-$ channels
$\implies$ 100% uncertainties from $V - A$, $A - A$ interference terms

[Phys.Rev.D 74 (2006)]

Previous work by ETM: [Phys.Rev.D 83 (2011)] [Phys.Rev.Lett. 98 (2007)]

Resonance Production - $N \to N^*$





Hadronic tensor methods for addressing SIS (1.4 GeV $\leq W \leq$ 2.0 GeV)
See also: [Phys.Rev.D 101 (2020)]

$\langle \mathcal{O}(0)\mathcal{J}_4(-q)\mathcal{J}_4(q)\bar{\mathcal{O}}(0)\rangle$, $M_\pi \sim 370$ MeV, removed elastic contribution
Large $N\pi$, $N\pi\pi$ contributions

Previously no practical $Q^2 \neq 0$ data in this region [S.Nakamura - NuSTEC S&DIS]
As of 2025, MINERvA collaboration: [2503.20043[hep-ex]]