Electroweak responses of light nuclei and towards describing neutrino-nucleus reactions

Neutrino Interactions for Current and Future Experiments
KEK Tsukuba campus, Tsukuba, Japan
2025.10.27-29

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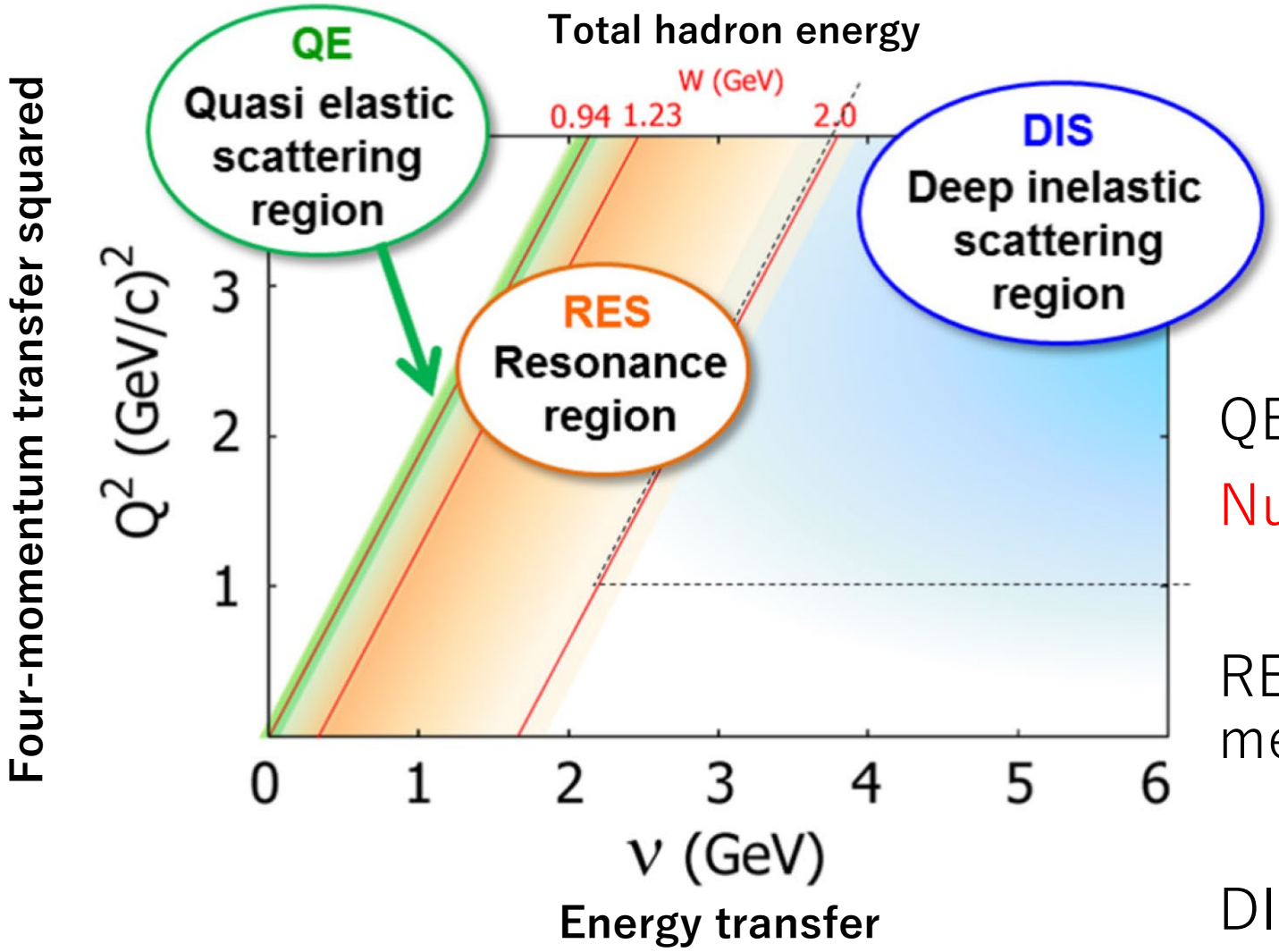
Neutrino-nucleus reactions (ν vs Q^2)



QE: **Nucleon degrees of freedom**

Nuclear structure theory can be used

RES: Nucleon is excited, inducing meson-baryon production

DIS: quarks and gluons

S.X. Nakamura et al., Rep. Prog. Phys. 80, 056301 (2017)

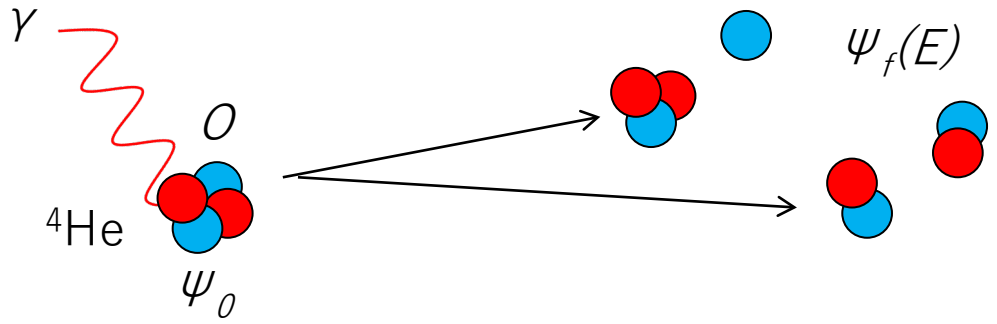
Electroweak responses of nuclei

- Key quantity: Strength (response) function

$$S(E) = \mathcal{S}_{f\mu} \left| \langle \Psi_f | \mathcal{O}_{\lambda\mu} | \Psi_0 \rangle \right|^2 \delta(E_f - E_0 - E)$$

Related to structure of
  - Excited states, resonant, continuum states
  - Ground state

- Electroweak response function of light nuclei
  - Electromagnetic interaction
    - Electromagnetic excitation, decay (E1, M1, E2, etc.)
    - Photoabsorption X(γ,b)Y ⇔ Radiative capture X(a,γ)Y
    - Electron scattering X(e,e')Y
  - "Weak" interaction
    - Beta decay X→Y+e+ ν
    - Electron (muon) capture X+e(μ)→Y+ ν
    - Neutrino-nucleus reaction X(ν,ν')Y, X(ν,eν')Y



- Reliable theory → Prediction of reaction rates that are hard to observe

  Example: $^{12}C+\alpha \rightarrow {}^{16}O+\gamma$ radiative capture, neutrino-nucleus reactions

Electroweak excitation of ^{4}He

- Low-energy photoabsorption reactions
  - Electric dipole (E1) excitation
  - Photoabsorption cross-section data of ^{4}He

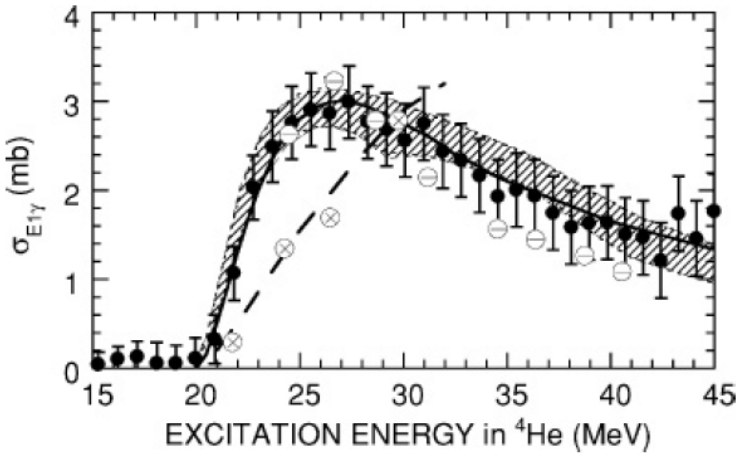
    S. Nakayama et al., PRC 76, 021305(2007)

    Peak ～27MeV

    T. Shima et al., PRC 72, 044004 (2005)

    Peak ～30 MeV

Taken from S. Nakayama et al. PRC 76, 021305 (2007).



- "Weak" excitation of ^{4}He
  - Induced by ν -^{4}He reaction (Gamow-Teller, Spin-dipole type, etc.)

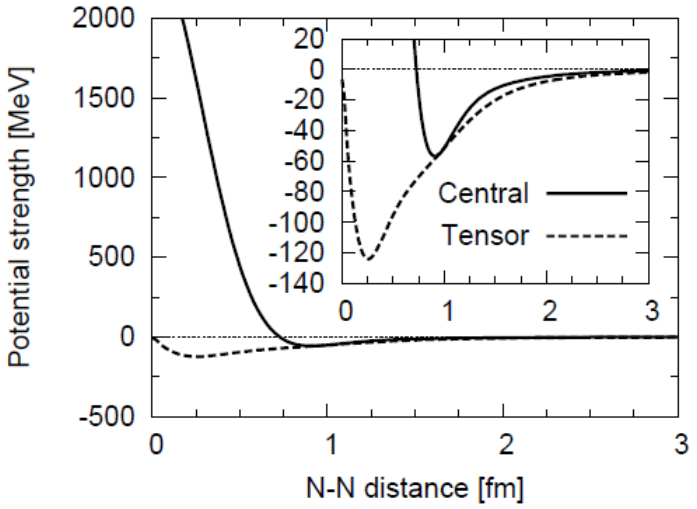
    → Impact on a supernova explosion

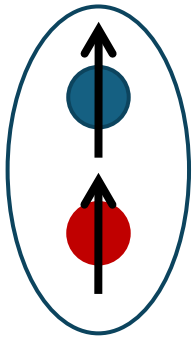
Reliable theoretical investigation → *ab initio* calculation

- Nucleon degrees of freedom
- Realistic nucleon-nucleon int. (N-N scattering, properties of ^{2}H)
- No specific model assumption

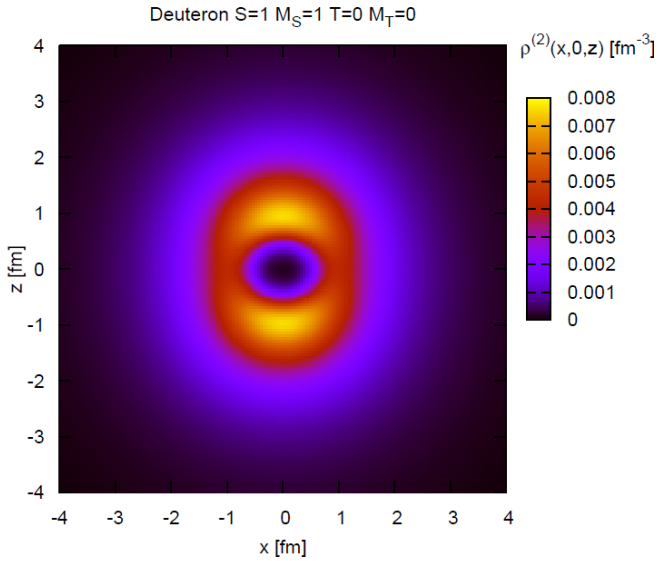
Nuclear force

- Strongly depends on spin and isospin
  - Central short-range repulsion
  - Non-central tensor force





Density of deuteron



- Explicitly correlated wave function approach
  - Correlated Gaussian basis + global vector representation (Hign-precision few-body method)

Review: “Theory and application of explicitly correlated Gaussians”
J. Mitroy, S. Bubin, WH et al, Rev. Mod. Phys. 85, 693 (2013)

Variational calculation for many-body systems

- Solving many-body Schroedinger equation

  ⇔ Eigenvalue problem of Hamiltonian matrix

  $$H\Psi = E\Psi$$

- Variational principle $<\Psi|H|\Psi> = E \geqq E_0$ (Exact energy)

  (Equal holds if the exact wf is used)

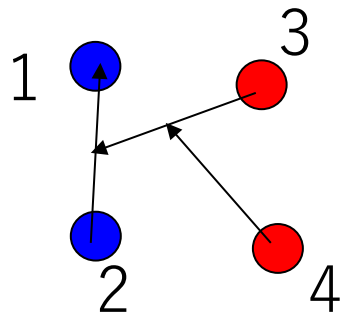
- Superposition of correlated Gaussian function

  Parameter β_{ij} express explicit correlations

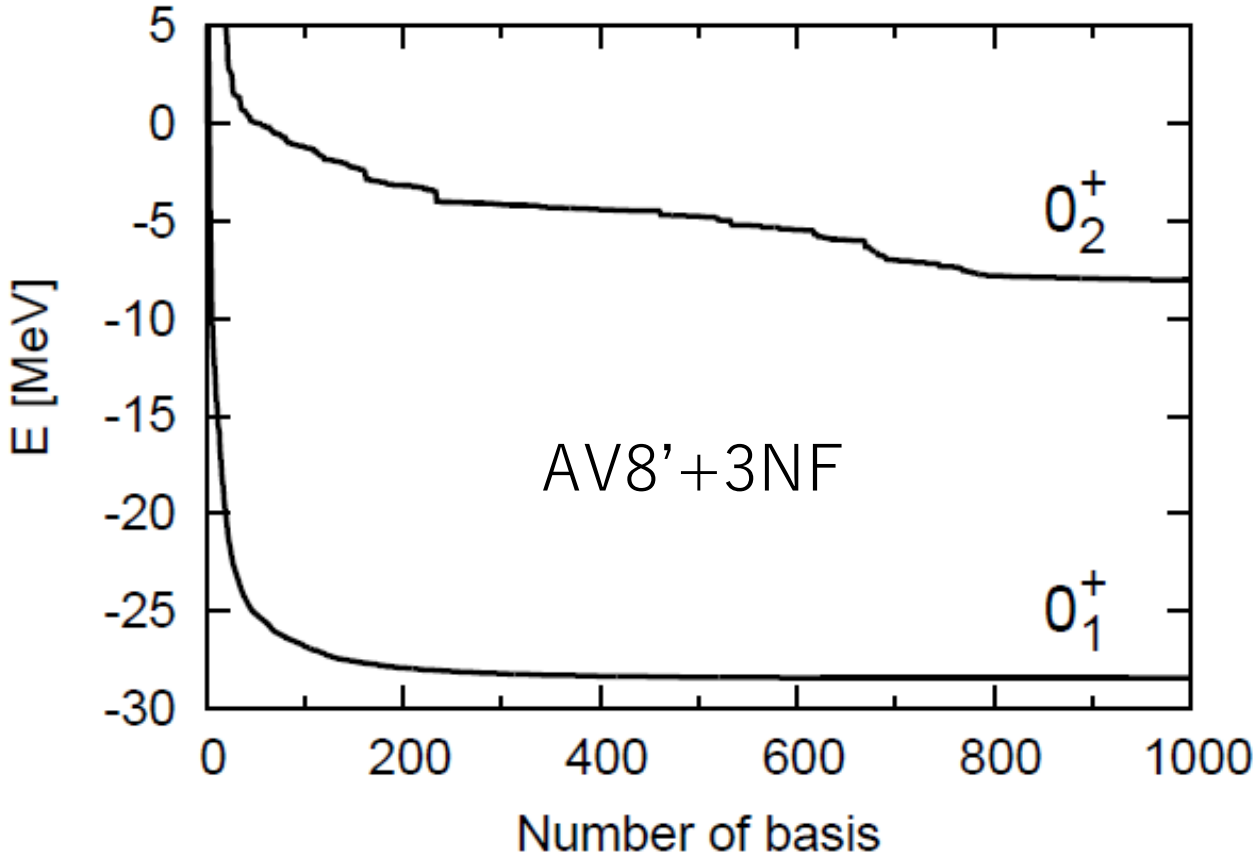
  - Stochastic optimization
    1. Generate several candidates randomly
    2. Calculate the energy of each candidate
    3. Select the one that gives the lowest energy
    4. Increase the number of basis
    5. Return to 1., till the energy is converged

    → Precise solution with small number of basis

$$\Psi = \sum_{k=1}^{K} C_k \exp\left(-\sum_{i<j=1} \beta_{ij}^k (\boldsymbol{r}_i - \boldsymbol{r}_j)^2\right)$$



Energy convergence of ^{4}He

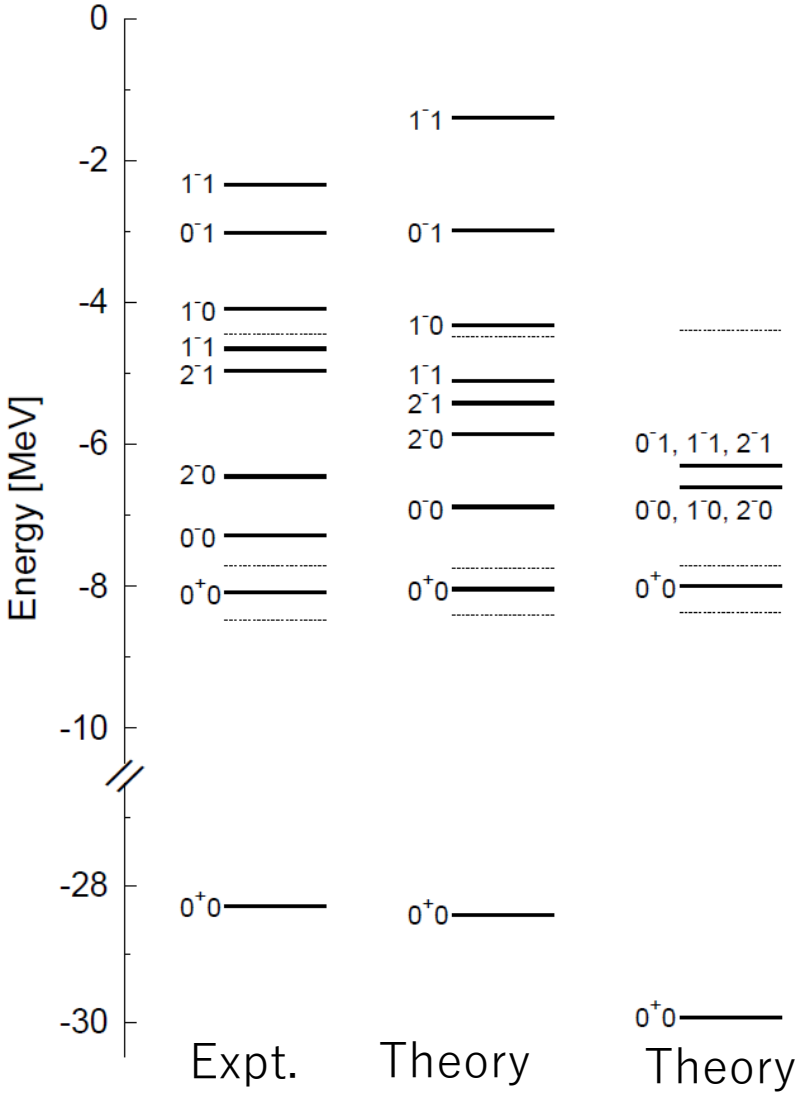


Good agreement with other *ab initio* methods (FY, GFMC, NCSM,…)

H. Kamada et al., PRC64, 044001 (2001)

Energy spectrum of ^{4}He

WH and Y. Suzuki, Phys. Rev. C 78, 034305 (2008).
WH and Y. Suzuki, Few-Body Syst. 54, 2407-2418 (2013).



- Input: nuclear force, no adjustable parameter
- Good agreement with experimental data

  *Nuclear state is specified by quantum number $J^{\pi}T$

  J: Total angular momentum

  π : Parity

  $P\Psi \rightarrow \pi\Psi$ P: Spatial inversion operator

  T: Total isospin

- Hard to reproduce the data only with the central force

Description of final states in continuum

- No bound state in the excited states of ^{4}He
  - Gaussian (square integrable function) does not satisfy the scattering boundary condition
  - Hard to quantitative evaluation in the bound-state approximation
    - Cf. Decay width from a few to a few tens of MeV
  - Many-body decay channels open
    - Two-body (3N+N) and three-body (d+p+n), four-body thresholds may open

Response function in the complex scaling method

Y. K. Ho, Phys. Rep. 99, 1 (1983)
N. Moiseyev, Phys. Rep. 302, 211 (1998)
S. Aoyama et al., Prog. Theor. Phys. 116, 1 (2006)

$$S(p,\lambda,E) = \mathcal{S}_{f\mu} |\langle \Psi_f | \mathcal{O}^p_{\lambda\mu} | \Psi_0 \rangle|^2 \delta(E_f - E_0 - E)$$

$$= -\frac{1}{\pi} \mathrm{Im} \sum_\mu \langle \Psi_0 | \mathcal{O}^{p\dagger}_{\lambda\mu} \frac{1}{E + E_0 - H + i\epsilon} \mathcal{O}^p_{\lambda\mu} | \Psi_0 \rangle$$

Coordinate and momentum are rotated in the complex plane

$$\boldsymbol{r}_j \to \boldsymbol{r}_j e^{i\theta}, \quad \boldsymbol{p}_j \to \boldsymbol{p}_j e^{-i\theta}$$

→ Outgoing-wave boundary condition

$$= -\frac{1}{\pi} \mathrm{Im} \sum_\mu \langle \Psi_0 | \mathcal{O}^{p\dagger}_{\lambda\mu} U^{-1}(\theta) R(\theta) U(\theta) \mathcal{O}^p_{\lambda\mu} | \Psi_0 \rangle$$

Complex-scaled resolvent $R(\theta) = U(\theta) R U^{-1}(\theta) = \dfrac{1}{E + E_0 - H(\theta) + i\epsilon}$

Eigenvalue problem in complex symmetric matrix $H(\theta) = U(\theta) H U^{-1}(\theta)$

$$\Psi_\nu(\theta) = \sum_i C^\nu_i(\theta) \Phi_i(\boldsymbol{x}) \qquad H(\theta)\Psi_\nu(\theta) = E_\nu(\theta)\Psi_\nu(\theta)$$

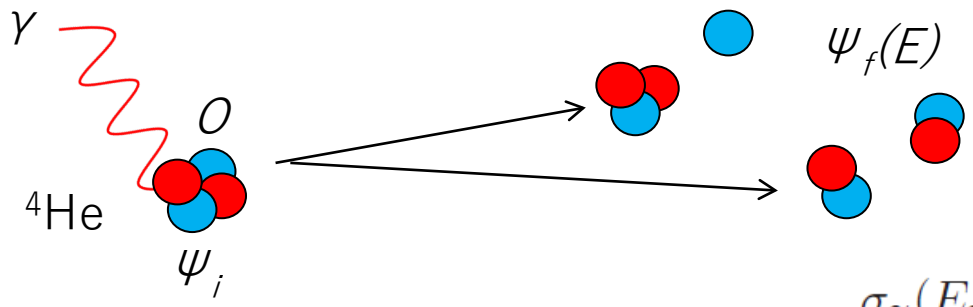
$$S(p,\lambda,E) = -\frac{1}{\pi} \sum_{\nu,\mu} \mathrm{Im} \frac{\tilde{\mathcal{D}}^{p,\nu}_{\lambda\mu}(\theta) \mathcal{D}^{p,\nu}_{\lambda\mu}(\theta)}{E + E_0 - E_\nu(\theta) + i\epsilon}$$

$$\mathcal{D}^{p,\nu}_{\lambda\mu} = \langle (\Psi_\nu(\theta))^* | \mathcal{O}^p_{\lambda\mu}(\theta) | U(\theta)\Psi_0 \rangle$$

$$\tilde{\mathcal{D}}^{p,\nu}_{\lambda\mu} = \langle (U(\theta)\Psi_0)^* | \tilde{\mathcal{O}}^p_{\lambda\mu}(\theta) | \Psi_\nu(\theta) \rangle$$

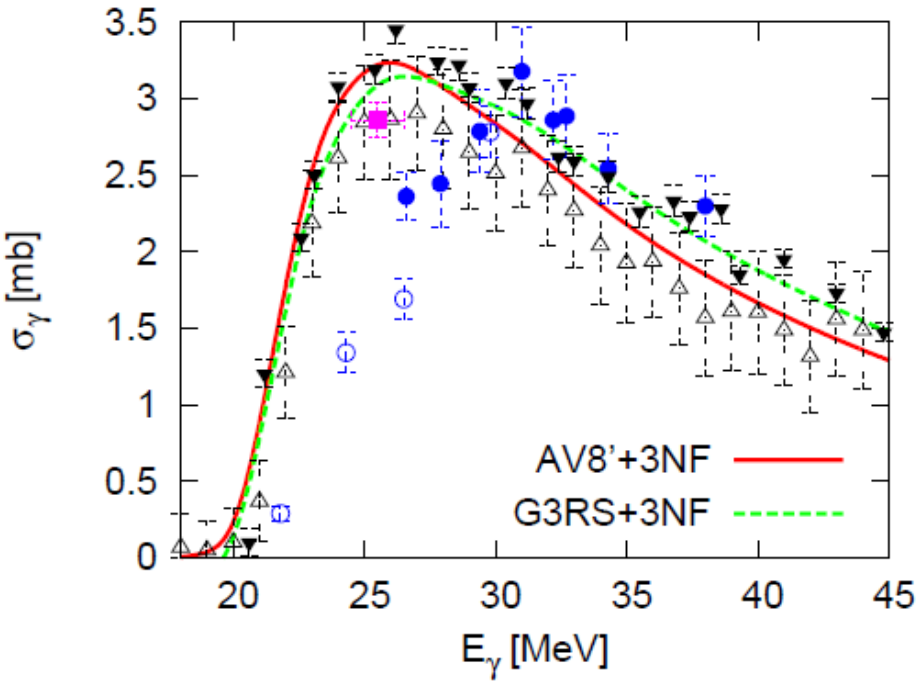
$$\mathcal{O}^p_{\lambda\mu}(\theta) = \mathcal{O}^p_{\lambda\mu} e^{i\theta}, \quad \tilde{\mathcal{O}}^p_{\lambda\mu}(\theta) = \mathcal{O}^{p\dagger}_{\lambda\mu} e^{i\theta}$$

Ab initio description of photodisintegration of ^{4}He

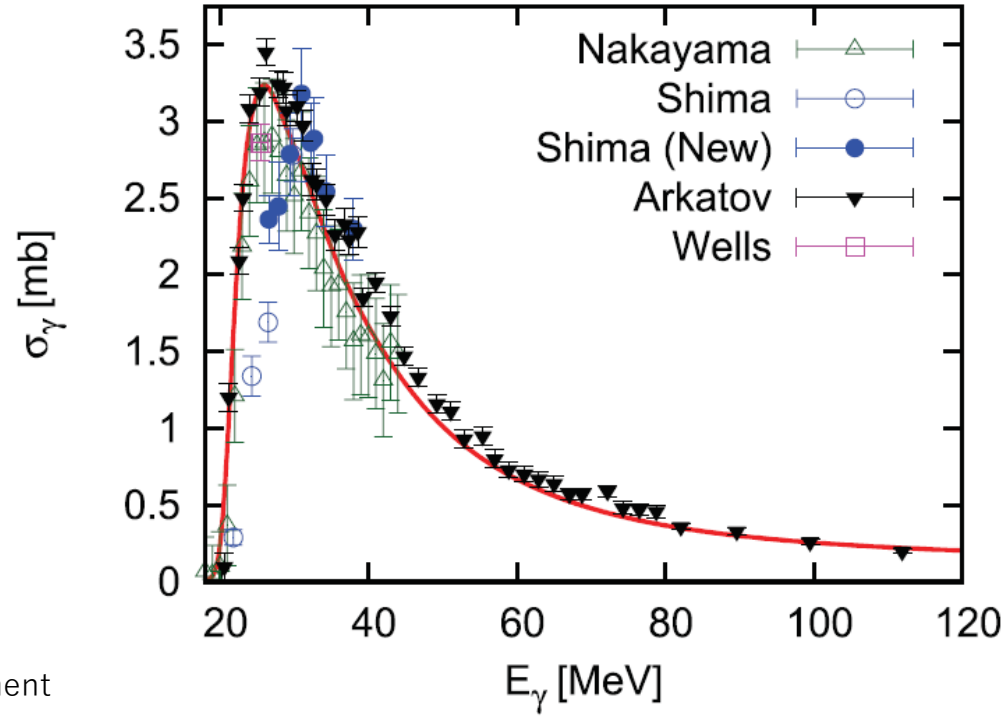


WH, Y. Suzuki, K. Arai, Phys. Rev. C 85, 054002 (2012)

$$\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$



△ S. Nakayama et al., (2007)
■ D.P. Wells et al. (1992)
▼ Y. M. Arkatov et al.,(1974).
○ T. Shima et al., (2005).
● T. Shima et al., new measurement



- Nucleon degrees of freedom, no specific model assumption (First principles in nuclear physics)
- Unified description of bound and many-body scattering states
- Good agreement with the experimental data
- Predictive power → **Neutrino-nucleus reaction**

Cross section and sumrule

$$m_\kappa = \int_0^\infty E_\gamma^\kappa \sigma_\gamma(E_\gamma) dE_\gamma$$

- κ =-1; Bremstrahlungs (non-energy weighted) sumrule

$$m_{-1} = \mathcal{G}\left(Z^2\langle r_p^2\rangle - \frac{Z(Z-1)}{2}\langle r_{pp}^2\rangle\right) \qquad \mathcal{G} = 4\pi^2 e^2/3\hbar c$$

Proton radius and p-p distance

99.6%

- κ =0; Thomas-Reiche-Kuhn (TRK) sumrule

$$m_0 = \mathcal{G}\frac{3NZ\hbar^2}{2Am_N}(1+K) \qquad K\text{: Enhancement factor}$$

$$K = \sum_{q=1}^{8} K_q \qquad V_q = \sum_{i<j} v^{(q)}(r_{ij})\mathcal{O}_{ij}^{(q)}$$

$$K_q = \frac{2Am_N}{3NZ\hbar^2 e^2}\frac{1}{2}\sum_\mu \langle\Psi_0|[\mathcal{M}_{1\mu}^\dagger,[V_q,\mathcal{M}_{1\mu}]]|\Psi_0\rangle$$

Calculable only with the ground state wave function

Importance of pion exchange force on K

$$V_{OPEP} = \frac{1}{3}\frac{f^2}{\hbar c}m_\pi c^2(\tau_1\cdot\tau_2)\left\{(\sigma_1\cdot\sigma_2) + \left(1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2}\right)S_{12}\right\}\frac{e^{-\mu r}}{\mu r}$$

		AV8′+3NF	
q		$\langle V_q\rangle$	K_q
1	1	17.39	0
2	$\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$	−9.59	0
3	$\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$	−5.22	0.011
4	$\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$	−59.42	0.460
		(−12.51)	(0.187)
5	S_{ij}	0.75	0
6	$S_{ij}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$	−70.93	0.574
		(−68.65)	(0.667)
7	$(\boldsymbol{L}\cdot\boldsymbol{S})_{ij}$	11.09	0
8	$(\boldsymbol{L}\cdot\boldsymbol{S})_{ij}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$	−15.93	0.061
	Total	−131.9	1.11

Dipole-type operator and the spectrum of ^{4}He

First forbidden transition

Electric dipole (E1) $J^\pi T=0^+0 \rightarrow 1^-1$

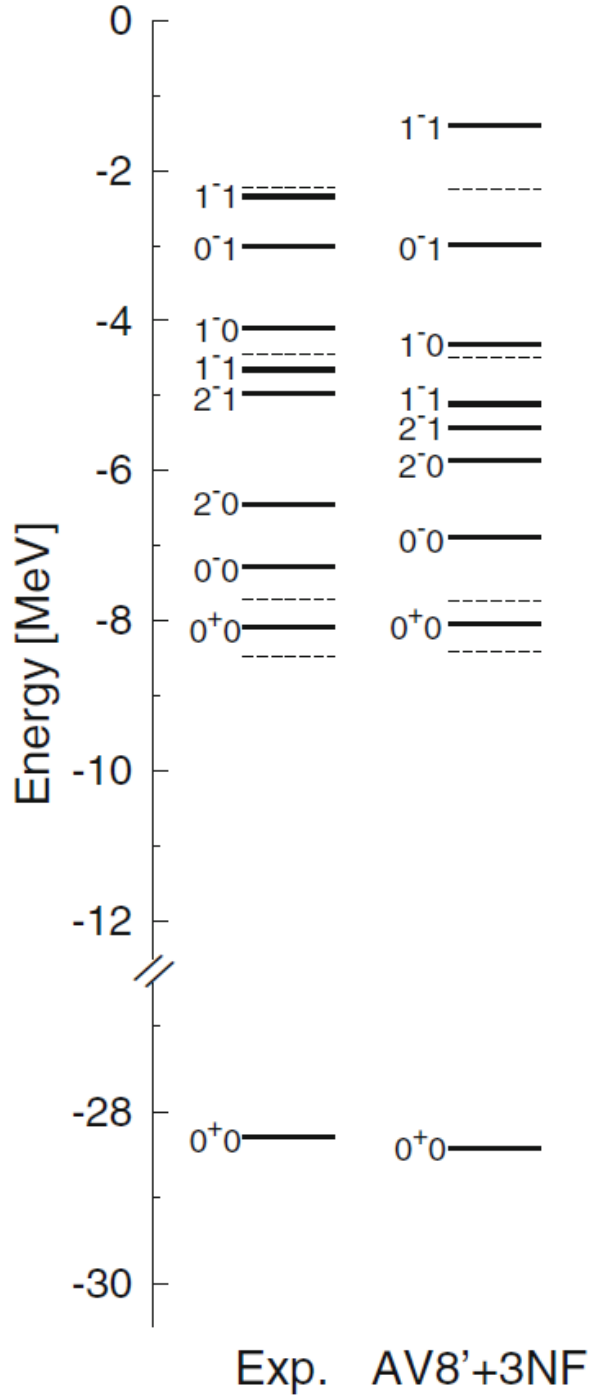
$$\mathcal{M}_{1\mu} = \sum_{i=1}^{4} \frac{e}{2}(1-\tau_{3_i})(\boldsymbol{r}_i - \boldsymbol{x}_4)_\mu$$

Spin-dipole (SD) $J^\pi T=0^+0 \rightarrow \lambda^- 0,\ \lambda^- 1$

$$\mathcal{O}^p_{\lambda\mu} = \sum_{i=1}^{4} [(\boldsymbol{r}_i - \boldsymbol{x}_4) \times \boldsymbol{\sigma}_i]_{\lambda\mu}\, T^p_i$$

$$T_i^{\mathrm{IS}} = 1, \quad T_i^{\mathrm{IV0}} = \tau_z(i), \quad T_i^{\mathrm{IV}\pm} = t_\pm(i)$$

- ^{4}He: 7 negative parity states
  J^π T= 0^-1, 1^-1, 2^-1, 0^-0, 1^-0, 2^-0



Spin-dipole response functions

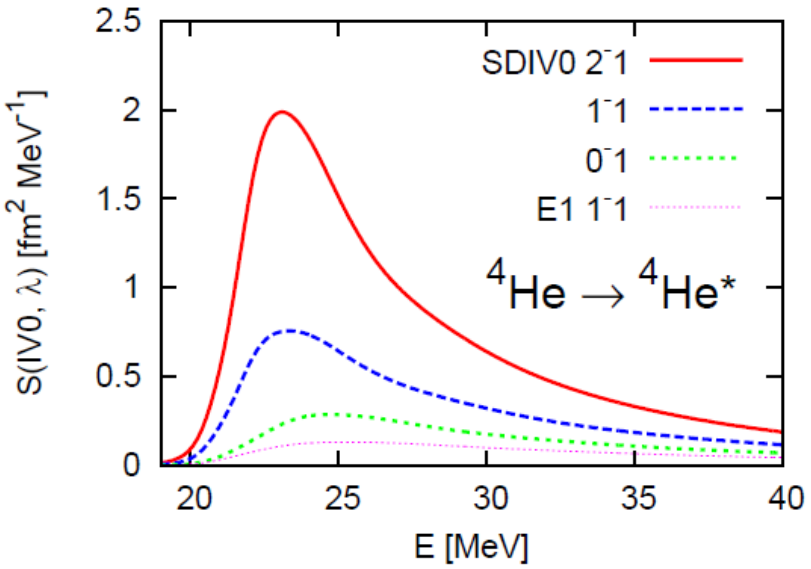
WH, Y. Suzuki, Phys. Rev. C 87, 034001 (2013)

Spin-dipole operator

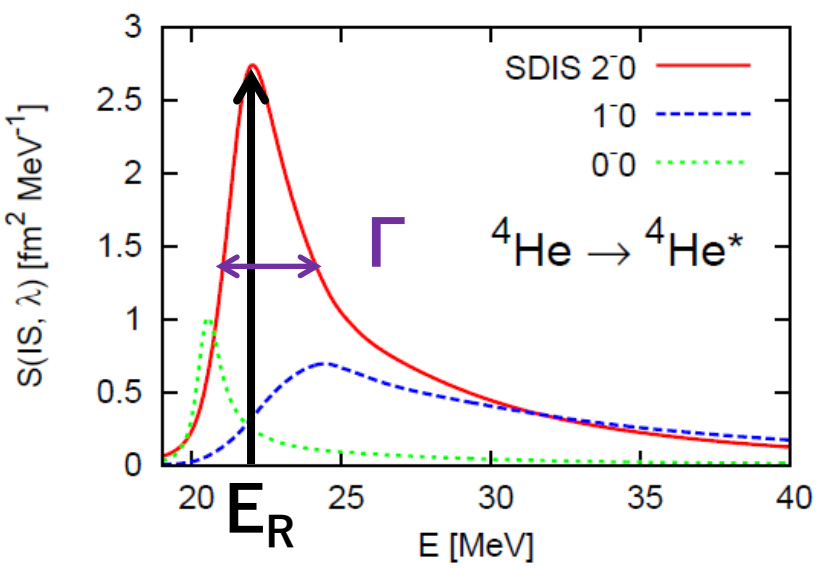
$$\mathcal{O}^p_{\lambda\mu} = \sum^4 [(\boldsymbol{r}_i - \boldsymbol{x}_4) \times \boldsymbol{\sigma}_i]_{\lambda\mu} \tau^p_{3i}$$

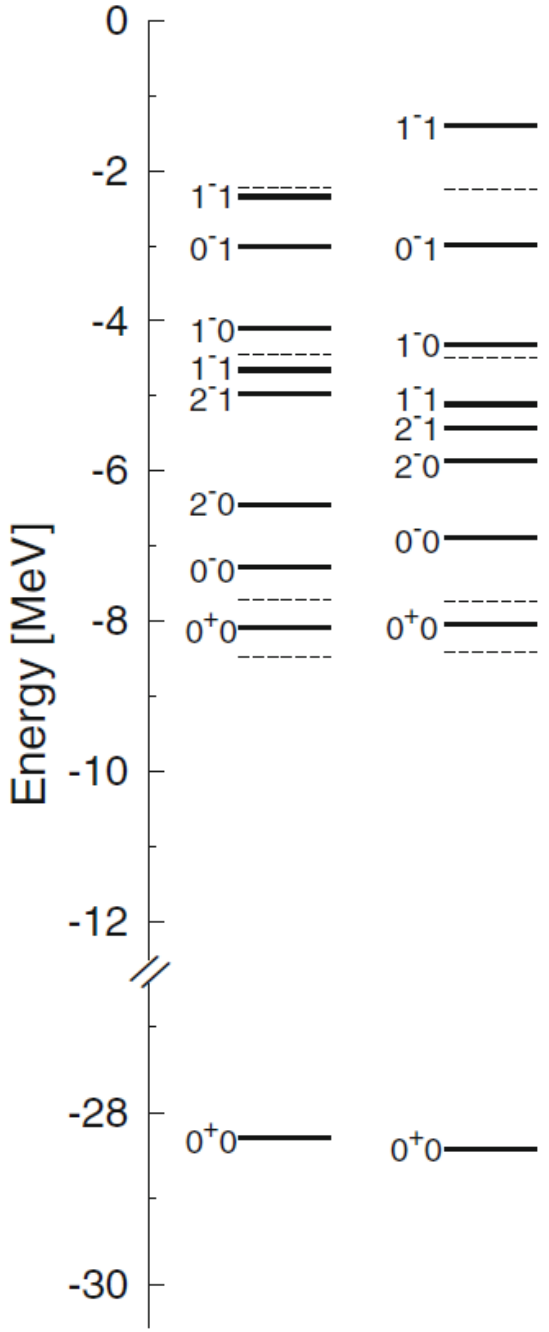
$$S(p,\lambda,E) = \mathcal{S}_{f\mu} |\langle \Psi_f | \mathcal{O}^p_{\lambda\mu} | \Psi_0 \rangle|^2 \delta(E_f - E_0 - E)$$

Isovector type (IV0)



Isoscalar type (IS)





- Resonant structure (7 negative parity states) ⇔ Response functions
- Relatively smaller decay width 0^-0, 2^-0 (0.84, 2.01 MeV)
- Average errors in E_R 0.4 MeV (Cf. Decay width 0.5-13 MeV)

Spin-dipole sumrule

$$m_0(p,\lambda) = \int_0^\infty S(p,\lambda,E)dE = \sum_{\kappa=0}^{2} U_{\lambda\kappa}\langle \mathcal{Q}^p_{(\kappa)0}\rangle$$

$$\mathcal{Q}^p_{(\kappa)0} = \sum_{i,j=1}^{A} \left([\boldsymbol{\rho}_i \times \boldsymbol{\rho}_j]_\kappa \cdot [\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]_\kappa\right) T_i^{p\dagger} T_j^p,$$

$$(U_{\lambda\kappa}) = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ \frac{5}{3} & \frac{5}{6} & \frac{1}{6} \end{pmatrix}$$

$$\boldsymbol{\rho}_i = \boldsymbol{r}_i - \boldsymbol{x}_N$$

	IV0	
	$m_0(p,\lambda)$	SR
0-1	4.59	4.59
1-1	9.35	9.36
2-1	18.36	18.38

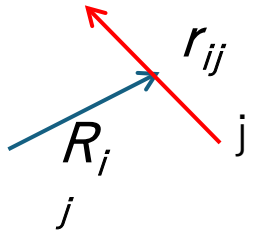
Sumrule ～100%

$(0s)^4$ (κ =0 contribution only) : Ratio of sumrule 1 : 3 : 5
Realistic calculation (IV0): 1 : 2.0 : 4.0 (κ =2 contribution)

cf. $$m_0(p,2) = \frac{5}{3}m_0(p,0) + \frac{1}{2}(5\langle \mathcal{Q}^p_{(1)0}\rangle - 3\langle \mathcal{Q}^p_{(2)0}\rangle)$$
$$\approx \frac{5}{3}m_0(p,0) - \frac{3}{2}\langle \mathcal{Q}^p_{(2)0}\rangle.$$

Relative coordinate $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j, \quad \boldsymbol{R}_{ij} = \frac{\boldsymbol{r}_i + \boldsymbol{r}_j}{2} - \boldsymbol{x}_N$

$$[\boldsymbol{\rho}_i \times \boldsymbol{\rho}_j]_{\kappa\mu} = -\frac{1}{4}[\boldsymbol{r}_{ij} \times \boldsymbol{r}_{ij}]_{\kappa\mu} + [\boldsymbol{R}_{ij} \times \boldsymbol{R}_{ij}]_{\kappa\mu}$$
$$+ \frac{1}{2}(1-(-1)^\kappa)[\boldsymbol{r}_{ij} \times \boldsymbol{R}_{ij}]_{\kappa\mu},$$

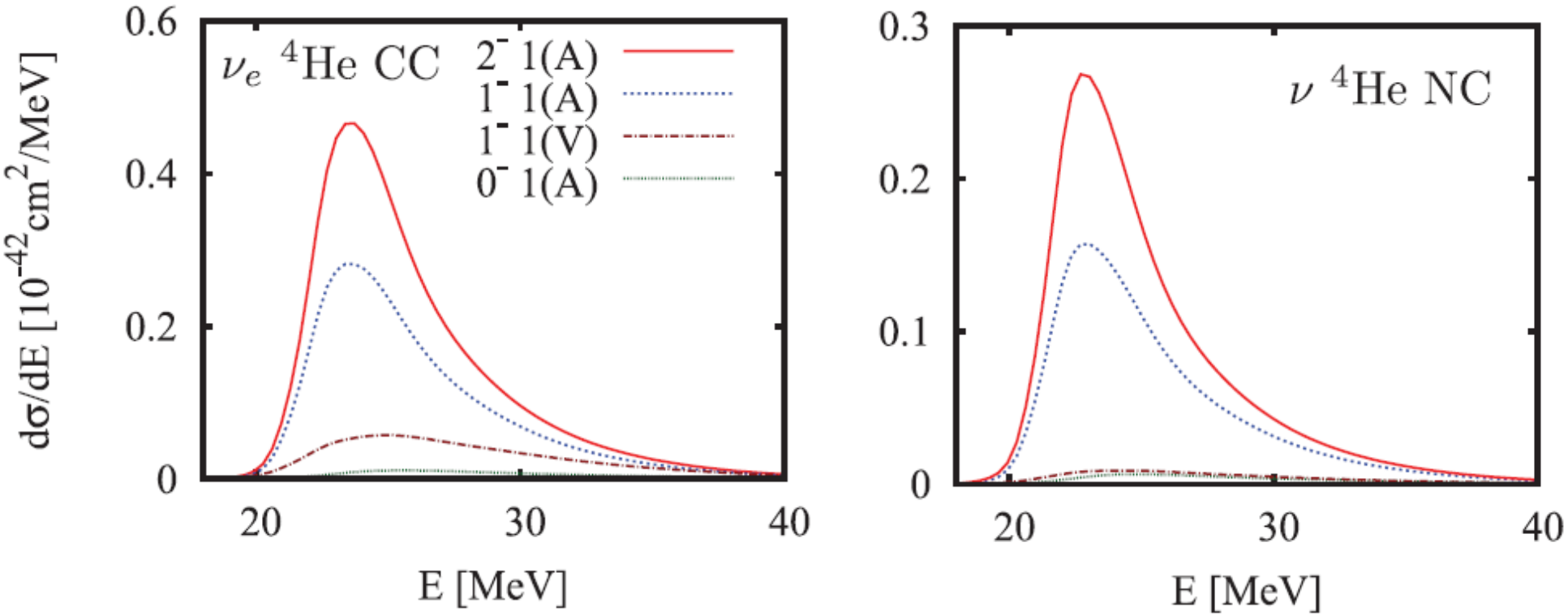


κ =2: tensor operator $\left([\boldsymbol{r}_{ij} \times \boldsymbol{r}_{ij}]_2 \cdot [\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]_2\right)$

Evidence for tensor correlations

$$\langle \mathcal{Q}^p_{(\kappa)0}\rangle = \sum_{\lambda=0}^{2} U^{-1}_{\kappa\lambda} m_0(p,\lambda)$$

Response functions
→ Neutrino-nucleus reaction cross sections



E: Excitation energy of ^{4}He
Cross section: Response function multiplied by phase-space factors The largest 2^- contribution

Neutrino-nucleus reaction cross sections

Neutrino-nucleus reaction (Gamow-Teller, dipole, Spin-dipole operators)

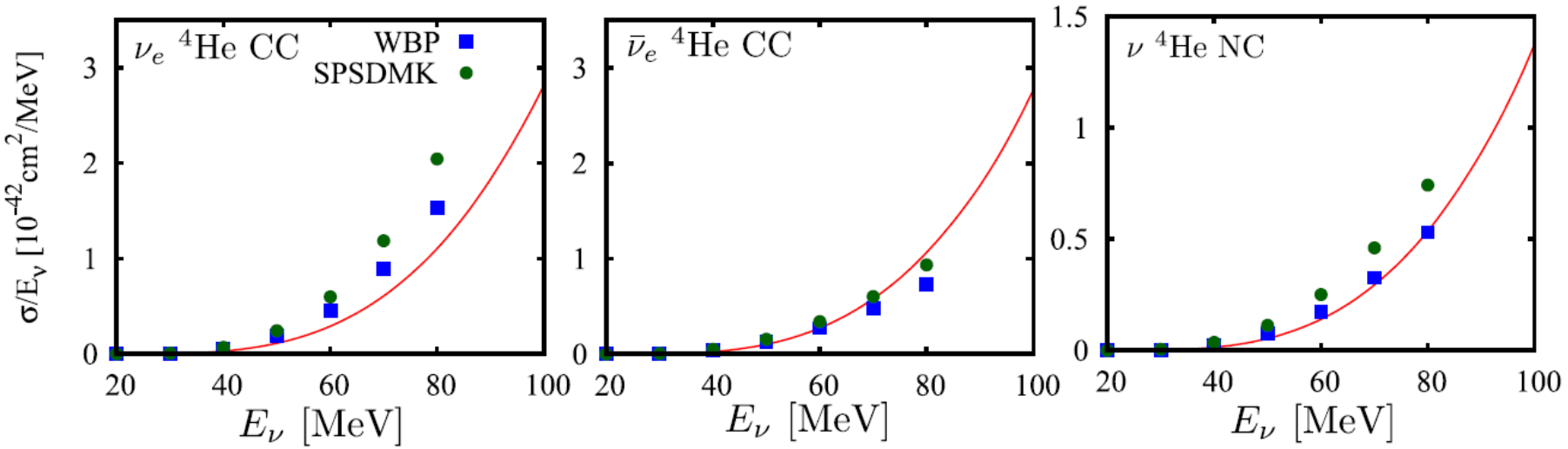
→ Impact on supernova explosion scenario

too small neutrino-nucleus cross sections (∼fb)

Reliable theoretical evaluation

Inclusive neutrino-^{4}He reactions

$$\nu_e + {}^4\mathrm{He} \to e^- + 3p + n,\ e^- + 2p + d,\ e^- + p + {}^3\mathrm{He}$$
$$\bar{\nu}_e + {}^4\mathrm{He} \to e^+ + p + 3n,\ e^+ + 2n + d,\ e^+ + n + {}^3\mathrm{H}$$
$$\nu + {}^4\mathrm{He} \to \nu + 2p + 2n,\ \nu + p + n + d,\ \nu + n + {}^3\mathrm{He},\ \nu + p + {}^3\mathrm{H},\ \nu + 2d$$



S.X. Nakamura et al., Rep. Prog. Phys. 80, 056301 (2017)

Towards describing weak responses: Beta-decay formulas revisited

PTEP

Prog. Theor. Exp. Phys. **2021**, 103D03 (23 pages)
DOI: 10.1093/ptep/ptab069

Electron wave functions in beta-decay formulas revisited (I): Gamow–Teller and spin-dipole contributions to allowed and first-forbidden transitions

Wataru Horiuchi[1], Toru Sato[2,3,*], Yuichi Uesaka[4], and Kenichi Yoshida[5]

Nuclear matrix element of the weak interaction

$$\langle e^-(p_e)\bar{\nu}_e(p_\nu)f\,|\,H_{\rm eff}\,|i\rangle = \frac{G_F V_{ud}}{\sqrt{2}}\int d\boldsymbol{x}\bar{\psi}^{(-)}_{e^-,\boldsymbol{p}_e,s_e}(\boldsymbol{x})\gamma^\mu(1-\gamma_5)v_{s_\nu}(\boldsymbol{p}_\nu)e^{-ip_\nu\cdot x}\langle f\,|\,J_\mu(\boldsymbol{x})\,|i\rangle\,,$$

Decay rate $$\Gamma = \frac{(G_F V_{ud})^2}{2\pi^3}\int_{m_e}^{E_0} dE_e p_e E_e (E_0-E_e)^2 F(Z,E_e)C(E_e).$$

Fermi function, shape correction factor

$$F(Z,E_e)C(E_e) = \sum_{J,L,\kappa_e,\kappa_\nu}\frac{2\pi}{2J_i+1}|\,\langle f\,\|\,\Xi_{JL}(\kappa_e,\kappa_\nu)\,\|i\rangle\,|^2,$$

$$\langle f\,\|\,\Xi_{JL}(\kappa_e,\kappa_\nu)\,\|i\rangle = \langle f\,\|\int d\boldsymbol{r}[Y_L(\hat{r})\otimes\boldsymbol{A}(\boldsymbol{r})]_J[c_g G_{\kappa_e}(r)g_{\kappa_\nu}(r)+c_f F_{\kappa_e}(r)f_{\kappa_\nu}(r)]\,\|i\rangle$$

$$= g_{\rm A}\int_0^\infty dr r^2\rho_{JL}(r)[c_g G_{\kappa_e}(r)g_{\kappa_\nu}(r)+c_f F_{\kappa_e}(r)f_{\kappa_\nu}(r)].$$

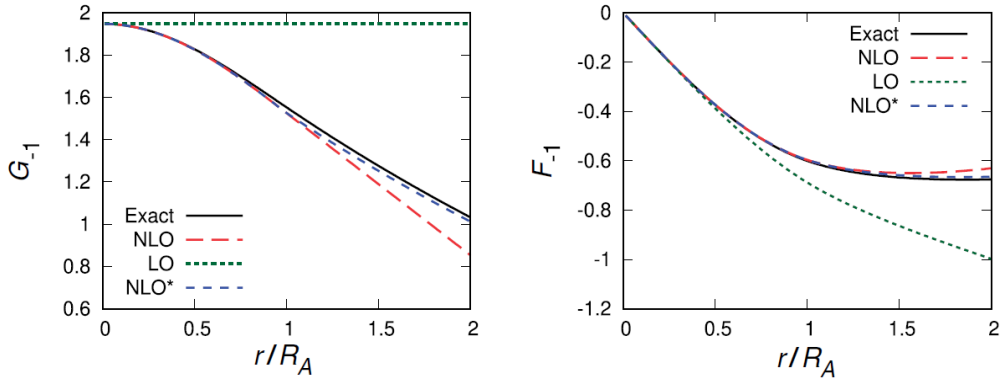
Nuclear, electron, Neutrino wfs.

Electron Coulomb wf. (Dirac eq.)

$$[\boldsymbol{\alpha}\cdot\boldsymbol{p}_e+\beta m_e+V_C(r)]\psi_e(\boldsymbol{r}) = E_e\psi_e(\boldsymbol{r}).$$

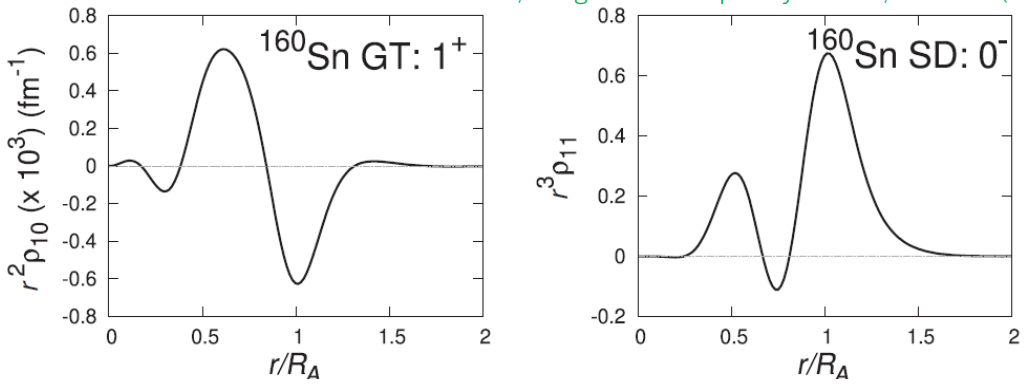
$$\frac{dG_\kappa}{dr}+\frac{1+\kappa}{r}G_\kappa-(m_e+E_e-V_C)F_\kappa = 0,$$

$$\frac{dF_\kappa}{dr}+\frac{1-\kappa}{r}F_\kappa-(m_e-E_e+V_C)G_\kappa = 0.$$



DFT: Transition densities by QRPA

K. Yoshida, Prog. Theor. Exp. Phys. 2013, 113D02 (2013)



J^π	^{80}Ni			^{160}Sn		
	$t_{1/2}$(s)	LO	NLO	$t_{1/2}$(s)	LO	NLO
1^+	3.50×10^{-1}	0.962	1.00	2.39×10^{-3}	0.874	1.00
0^-	1.08	0.928	1.00	1.34×10^{-2}	0.874	1.00
1^-	3.02	0.943	1.00	5.18×10^{-2}	0.895	1.00
2^-	2.18	0.942	1.00	1.42×10^{-1}	0.857	1.00

Towards describing the weak responses: Beta-decay formulas revisited II (ν dependent terms)

PTEP

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Electron wave functions in beta-decay formulas revisited (II): Completion including recoil-order and induced currents

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We present complete formulas of the allowed and first-forbidden transitions of nuclear beta decay taking into account the recoil-order and induced currents up to the next-to-leading order (NLO). The longitudinal part of the vector current is cleared away by the use of the conservation of the vector current for the multipole operators of the natural-parity transitions, which makes the effect of the meson exchange current for the vector current as small as possible. The formula is transparent enough to be applied to various beta-decay processes. As a numerical demonstration, we apply our formulas to the beta decay of a neutron-rich nucleus ^{160}Sn. We find that the NLO corrections amount to 10–20 percent of the total decay rate, whereas the induced currents alter the rate at most by 5 percent.

$$\Gamma = \frac{(G_F V_{ud})^2}{\pi^2} \int_{m_e}^{E_0} dE_e\, p_e E_e\, (E_0 - E_e)^2 \sum_{J,\kappa_e,\kappa_\nu} \frac{1}{2J_i+1} \left| \sum_L \langle f || \Xi_{JL}(\kappa_e, \kappa_\nu) || i \rangle \right|^2 ,$$

$$\langle f || \sum_L \Xi_{JL}(\kappa_e, \kappa_\nu) || i \rangle = S_{\kappa_e} \int_0^\infty dr\, r^2 \big[\rho^{\sigma}_{J-1J}(r)\phi_a(r) + \rho^{\sigma}_{J+1J}(r)\phi_b(r) + \rho^{\sigma\nabla}_{J}(r)\phi_c(r) + \rho^{\nabla}_{JJ}(r)\phi_d(r) \big],$$

$$\langle f || \sum_L \Xi_{JL}(\kappa_e, \kappa_\nu) || i \rangle = S_{\kappa_e} \int_0^\infty dr\, r^2 \left[\rho_J(r)\phi_A(r) + \rho^{\sigma}_{JJ}(r)\phi_B(r) + \rho^{\nabla}_{J+1J}(r)\phi_C(r) \right],$$

$$\rho_J(r) = \langle f || \sum_{j=1}^{A} \int d\Omega_r \delta(r - r_j) \tau_j^{\pm} Y_J(\hat{r}) || i \rangle ,$$

$$\rho^{\sigma}_{LJ}(r) = \langle f || \sum_{j=1}^{A} \int d\Omega_r \delta(r - r_j) \tau_j^{\pm} [Y_L(\hat{r}) \otimes \sigma_j]_J || i \rangle ,$$

$$\rho^{\nabla}_{LJ}(r) = \langle f || \sum_{j=1}^{A} \int d\Omega_r \delta(r - r_j) \tau_j^{\pm} [Y_L(\hat{r}) \otimes \nabla_j]_J || i \rangle ,$$

$$\rho^{\sigma\nabla}_{J}(r) = \langle f || \sum_{j=1}^{A} \int d\Omega_r \delta(r - r_j) \tau_j^{\pm} Y_J(\hat{r}) \sigma_j \cdot \nabla_j || i \rangle .$$

Inputs from structure theory

Extension towards applying to neutrino-nucleus reactions **underway**

Summary

- Electroweak responses of light nuclei

  Example: Electroweak excitations and neutrino reaction of ^{4}He

  WH, Y. Suzuki, K. Arai, Phys. Rev. C 85, 054002 (2012) S.X. Nakamura et al., Rep. Prog. Phys. 80, 056301 (2017)
  WH, Y. Suzuki, Phys. Rev. C 87, 034001 (2013)

- Nuclear matrix elements of the weak interaction
  - Beta-decay formula revisited
    - No long-wave-length approximation
    - Accurate treatment of electron Coulomb wavefunctions

    WH, T. Sato, Y. Uesaka, K. Yoshida, Prog. Theor. Exp. Phys. **2021**, 103D03, ibid **2023** 073D02

- Towards describing neutrino-nucleus reactions for heavier nuclei
  - Fully microscopic evaluation
    - Input: transition densities
    - Application to lepton-nucleus reactions