Bound muons as probes of new physics

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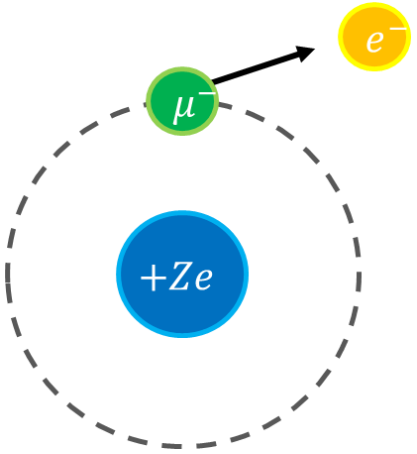

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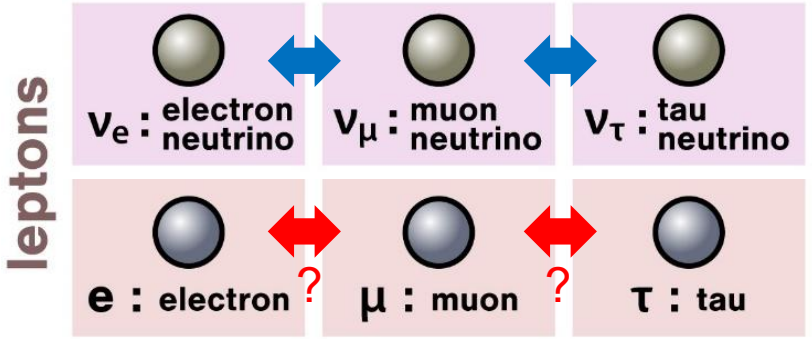
1. Introduction

- Neutrino oscillations are known.
- How about charged leptons?



Charged Lepton Flavor Violation (CLFV)

- lepton flavor # (L_e, L_μ, L_τ)

cf. lepton #, $L = L_e + L_\mu + L_\tau$

	e^-	μ^-	τ^-	ν_e	ν_μ	ν_τ	e^+	μ^+	τ^+	$\overline{\nu_e}$	$\overline{\nu_\mu}$	$\overline{\nu_\tau}$	others
L_e	+1	0	0	+1	0	0	-1	0	0	-1	0	0	0
L_μ	0	+1	0	0	+1	0	0	-1	0	0	-1	0	0
L_τ	0	0	+1	0	0	+1	0	0	-1	0	0	-1	0

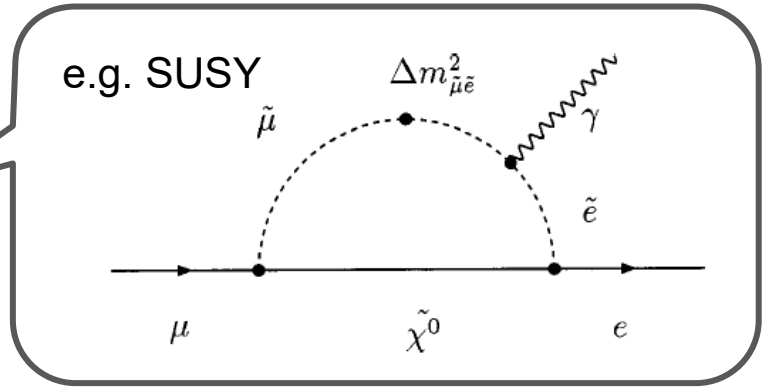
◆ The lepton flavors are conserved in the SM. e.g. $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$, $\pi^+ \to \mu^+ \nu_\mu$

◆ lepton flavor violation in charged lepton sector = CLFV

e.g. $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^-e^+$, $\mu^- N \to e^- N$, $\tau^+ \to \mu^+\gamma$, etc.

CLFV is

- forbidden in SM
- predicted in many models beyond SM



CLFV tells us information on BSM!!

Why are lepton flavors conserved in SM?

(The lepton flavor conservation is not included in the guiding principle of the SM.)

Lepton fields in SM

- SU(2) doublet ×3
- singlet ×3

$$L_f = \begin{pmatrix} \nu_{fL} \\ e_{fL} \end{pmatrix} \qquad e_{fR} \qquad f = 1,2,3$$

✓ Gauge interaction does not distinguish flavors:

$$g \sum_{f=1,2,3} W_i^{\alpha} \overline{L_f} \gamma_{\alpha} \sigma_i L_f$$

$L_f = \begin{pmatrix} \nu_{fL} \\ e_{fL} \end{pmatrix}$

(There are no rules to determine e, μ, τ.)

✓ Yukawa interaction to the higgs doublet distinguishes flavors:

$$\sum_{f,f'=1,2,3} \Phi \overline{L_f} Y_{ff'} e_{Rf'} \quad \xrightarrow{\text{diagonalization}} \quad \sum_{f=e,\mu,\tau} \Phi \overline{L_f} \tilde{Y}_f e_{Rf}$$

(The eigenstates of Y define e, μ, τ.)

Why are lepton flavors conserved in SM?

◆ electroweak part of SM Lagrangian ($\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{em}}$)

$$-\mathcal{L} \supset \sum_{f=e,\mu,\tau} \left\{ \frac{m_f}{v} \bar{\ell}_f \ell_f + h \bar{\ell}_f \ell_f + \frac{g}{\sqrt{2}} \left(W^\alpha \overline{\nu_f} \gamma_\alpha P_L \ell_f + \mathrm{H.c.} \right) + e A^\alpha \overline{\ell_f} \gamma_\alpha \ell_f \right.$$
$$\left. + \frac{g}{2\cos\theta_W} Z^\alpha \left[\bar{\nu}_f \gamma_\alpha \nu_f - \overline{\ell_f} \gamma_\alpha \left\{ \left(1 - 2\sin^2\theta_W\right) P_L - 2\sin^2\theta_W P_R \right\} \ell_f \right] \right\}$$

✓ Global $U(1)_e \times U(1)_\mu \times U(1)_\tau$ symmetry

- $U(1)_e$
  $\ell_e \to e^{i\theta_e} \ell_e$
  $\nu_e \to e^{i\theta_e} \nu_e$
- $U(1)_\mu$
  $\ell_\mu \to e^{i\theta_\mu} \ell_\mu$
  $\nu_\mu \to e^{i\theta_\mu} \nu_\mu$
- $U(1)_\tau$
  $\ell_\tau \to e^{i\theta_\tau} \ell_\tau$
  $\nu_\tau \to e^{i\theta_\tau} \nu_\tau$

→ conservation of lepton flavors

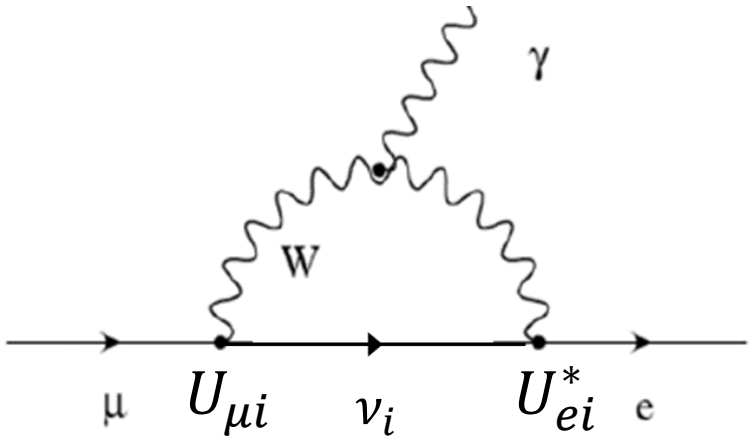
cf. Quark flavors are not conserved because both up-type & down-type have mass terms.

✓ However, the symmetry is easily broken if you add another source to mix flavors.

e.g. neutrino mass, new particle interacting with leptons, …

CLFV by neutrino masses

✓ Neutrino mass induces $\mu \to e\gamma$ by loop diagrams.



U_{fi}: MNS matrix

→ However, the rate is strongly suppressed by the (leptonic) GIM mechanism.

transition amplitude

$$\mathcal{M} \propto \sum_i \frac{U_{\mu i}U_{ei}^*}{k^2 - m_i^2} = \underbrace{\sum_i \frac{U_{\mu i}U_{ei}^*}{k^2}}_{=0 \text{ due to unitarity}} + \sum_i \frac{U_{\mu i}U_{ei}^*}{k^2}\left(\frac{m_i^2}{k^2}\right) + \mathcal{O}\left(\frac{m_i^4}{k^4}\right)$$

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U_{\mu i}U_{ei}^* \frac{\Delta m_{i1}^2}{m_W^2}\right|^2 \simeq 10^{-54}$$

cf. current limit
$\mathrm{Br} < 1.5 \times 10^{-13}$

✓ far too small to be observed

CLFV as a probe of new physics

✓ Neutrino oscillation is already known.

- There is no fundamental reason for the lepton flavor conservation.
- Rather, we need a seed to induce the neutrino mass and mixing.

✓ If the SM is extended, the lepton flavors are easily violated.

(unless you assume any special reasons to suppress CLFV)

⇨ CLFV provides strong constraints
and valuable hints about how to extend the SM.

✓ Sensitive even to heavy particles that cannot be produced at colliders

The current $\mu \to e\gamma$ search corresponds to a collider search for $\mathcal{O}(10^5)$ TeV.

(assuming coupling $\sim\mathcal{O}(1)$)

Muons as probes of CLFV

- High intensity beams are available. (~10^8 /s)
- long lifetime (~2.2 μs) → Bound states can also be utilized.

Free muons

1) $\mu^+ \to e^+\gamma$

$\mathrm{BR} < 1.5 \times 10^{-13}$ by MEG II

arXiv:2504.15711.

2) $\mu^+ \to e^+e^-e^+$

$\mathrm{BR} < 1.0 \times 10^{-12}$ by SINDRUM

Nucl. Phys. B 299 (1988) 1.

Muonic atoms

3) $\mu^- N \to e^- N$

$\mathrm{BR} < 7 \times 10^{-13}$ by SINDRUM II

Eur. Phys. J. C 47 (2006) 337.

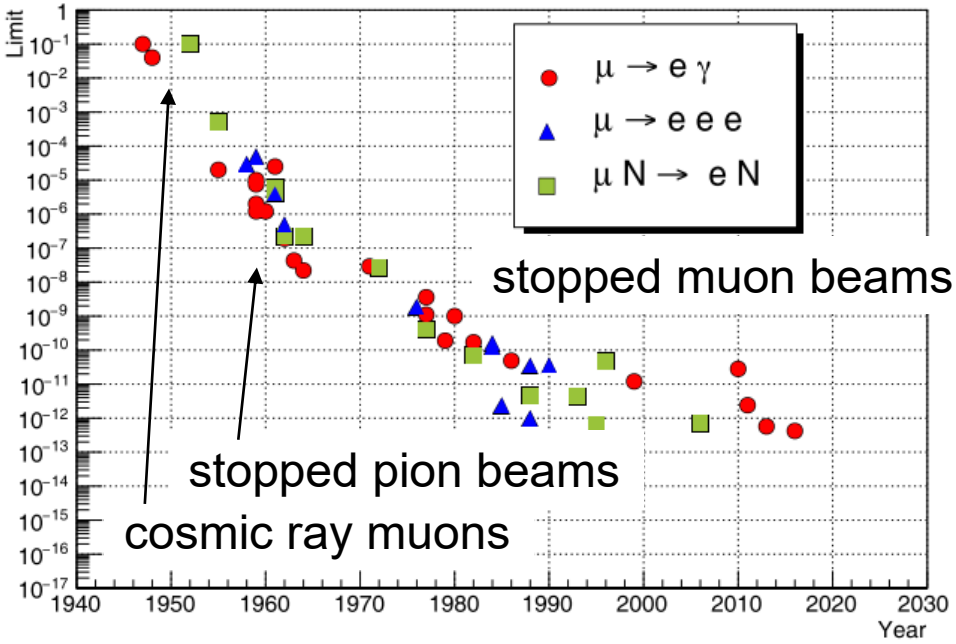
Muoniums

4) $\mu^+e^- \to \mu^-e^+$

$P < 8.3 \times 10^{-11}$ by MACS

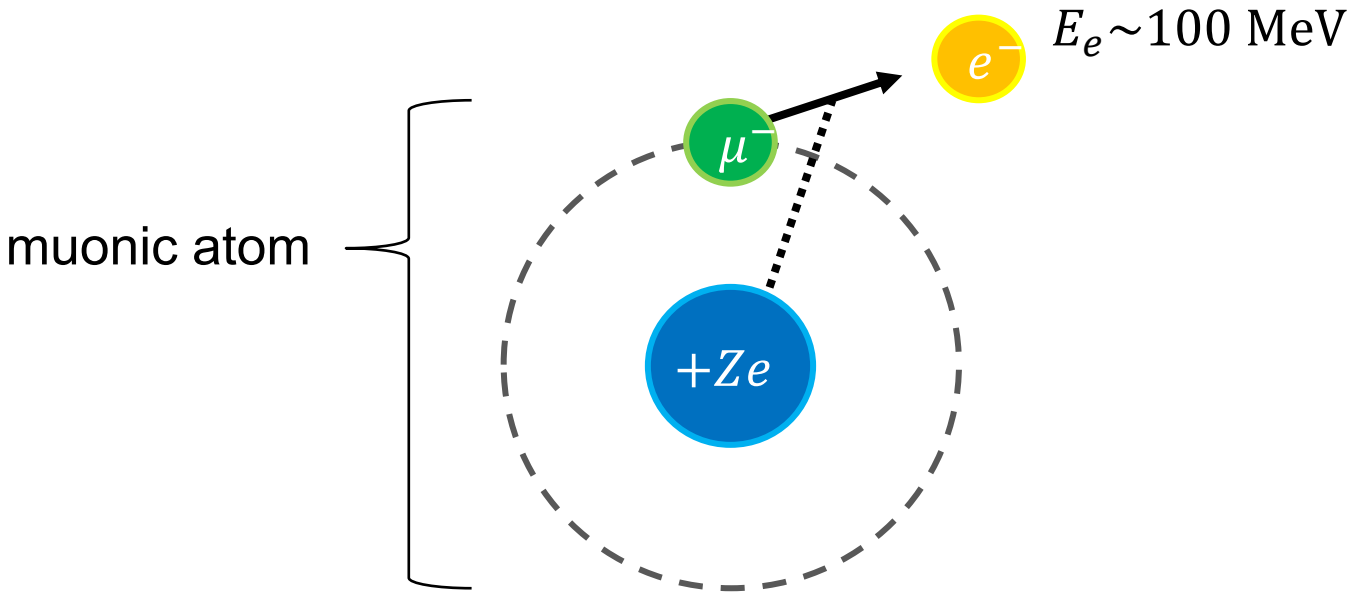
(Magnetic field ~ 0.1 T) Phys. Rev. Lett. **82**, 49 (1999).

History of muonic LFV



L. Calibbi & G. Signorelli, Riv. Nuovo Cim. **41**, no. 2, 1 (2018).

(Coherent) $\mu^- \rightarrow e^-$ conversion

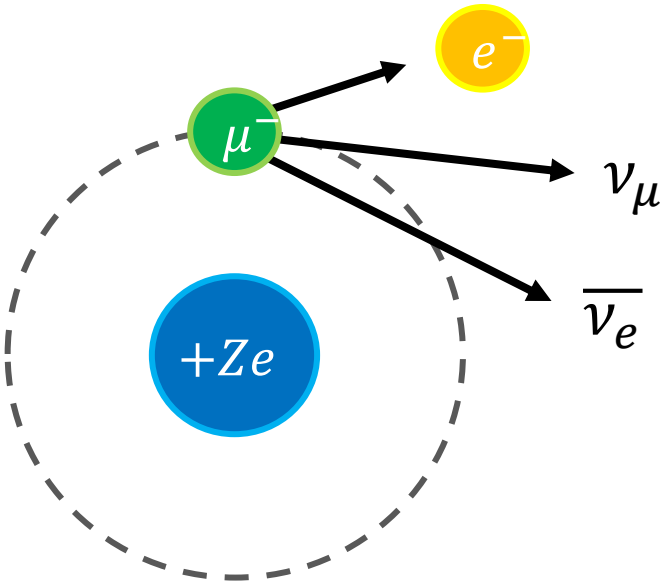


- one of CLFV processes
- monochromatic energy of e^- : $E_e = m_\mu - E_b - E_{\mathrm{recoil}} \sim 105$ MeV

E_b : binding energy of muon ~ 0.5 MeV
E_{recoil} : nuclear recoil energy ~ 0.2 MeV
for Al

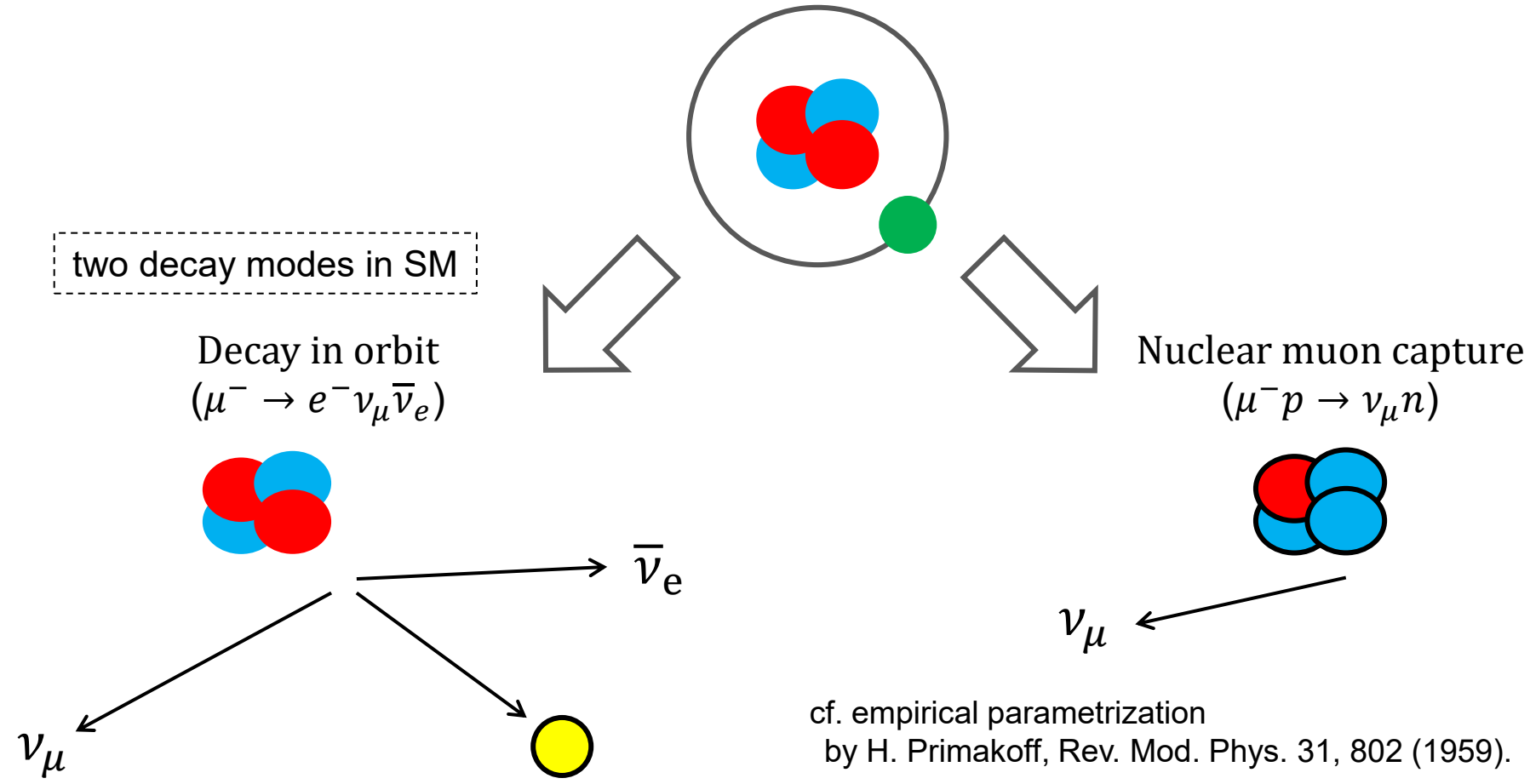
- coming experiments: COMET (J-PARC), Mu2e (Fermilab)
- The main background comes from "$\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$" in orbit.

2. $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$ in a muonic atom



Decays of a bound muon in orbit

- If μ^- stops in material, it is bound in atomic orbit and forms a *muonic atom*.



cf. empirical parametrization
by H. Primakoff, Rev. Mod. Phys. 31, 802 (1959).

$$\Gamma_c \simeq Z_{\text{eff}}^4 X_1 \left(1 - X_2 \left(\frac{A-Z}{2A}\right)\right)$$

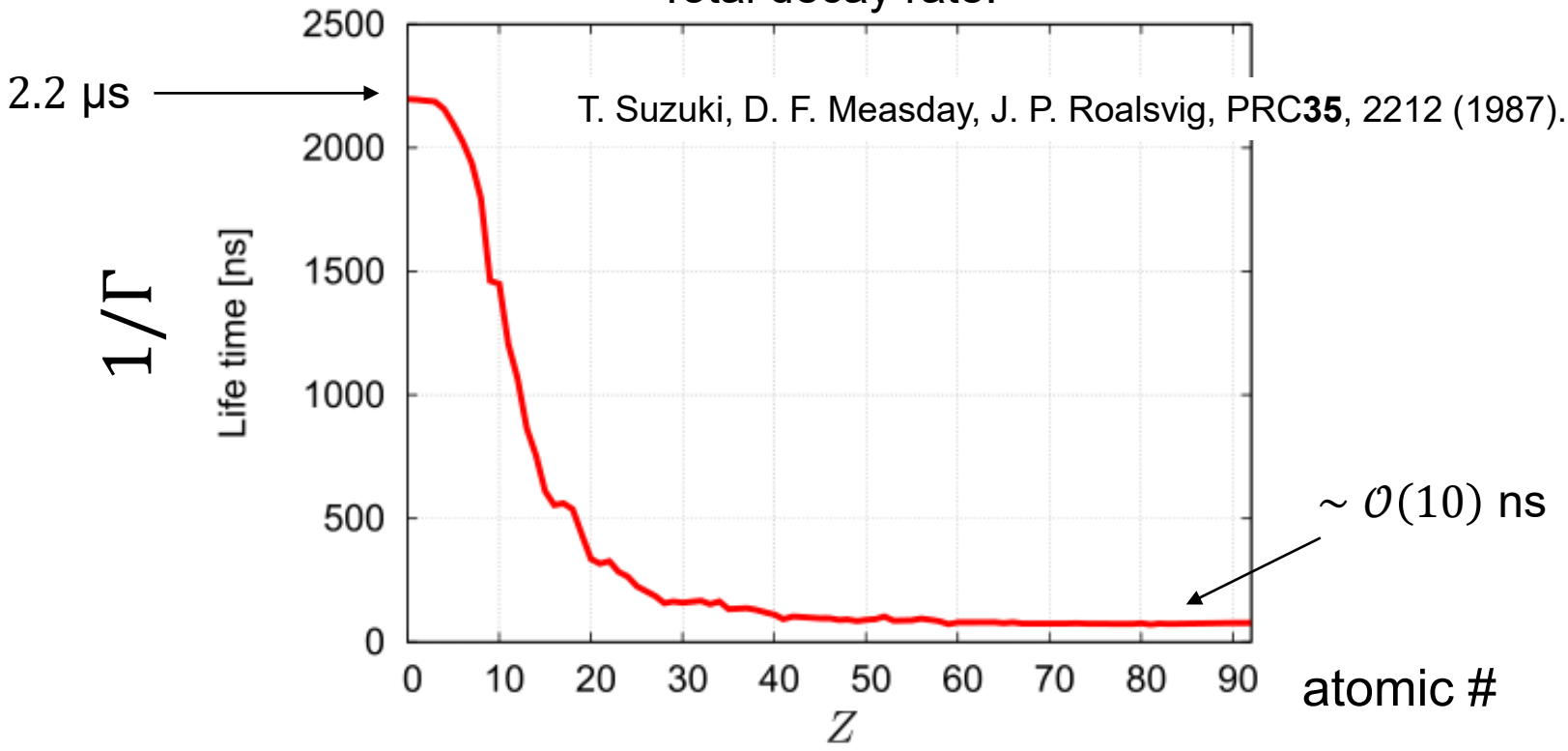
Decays of a bound muon in orbit

The "standard" decay width of muonic atoms is given by
the sum of the DIO width and the nuclear capture width:

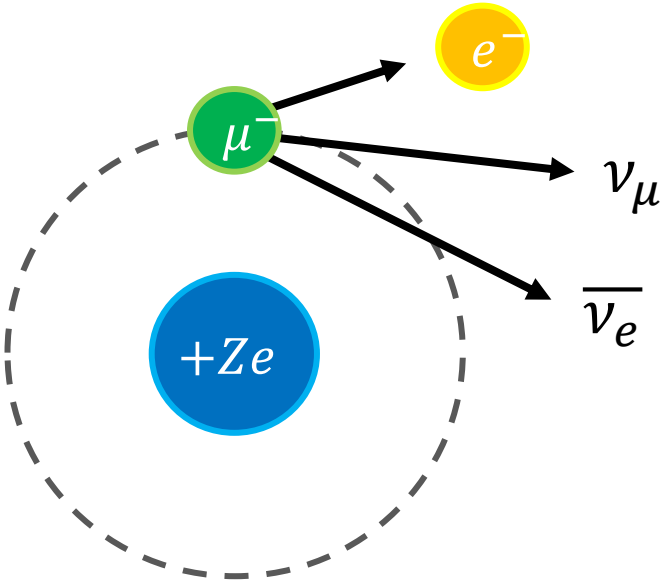
$$\Gamma = Q\Gamma_d + \Gamma_c$$

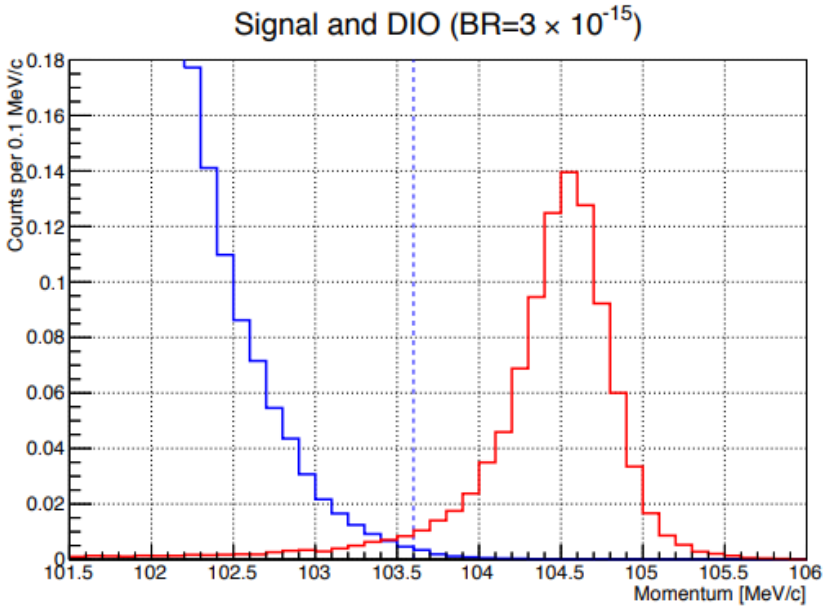
$\Gamma_d = 1/(2.2\ \mu\text{s})$ Q: Huff factor (< 1)

Total decay rate:



Decay in orbit (DIO)





COMET technical design report, arXiv:1812.09018.

- the "standard" decay of a muon, $\mu^- \to e^- \nu_\mu \bar{\nu}_e$
- electron energy : $E_e < m_\mu - E_b - E_N$

cf. For free muons, $E_e \leq m_\mu/2$

due to the energy-momentum conservation of a two-body decay.

For muonic atoms, the nuclear recoil makes the tail of the spectrum.

Bound muon & scattering electron

(also related to the beta decay formula in Horiuchi-san's talk yesterday)

For calculating the $\mu^- \to e^-$ transition,
we need wave functions of the bound μ^- & the scattering e^-
to obtain the overlap integral.

(e.g., $\int d^3r \rho(r) \bar{\psi}_e(r) O \psi_\mu(r)$ for coherent μ-e conv.)

✓ The emitted e^- w.f. is distorted
by the nuclear Coulomb potential. (not a plane wave)

The formula for the decay rate gets complicated
because the electron w.f. should be expanded by its angular momenta.

✓ In heavy nuclei, a bound muon is located near the nucleus.

Bohr radius: $\sim (m_\mu Z\alpha)^{-1} \sim 2 \times \frac{137}{Z}$ fm

Nuclear radius: $\sim 1.2 \times A^{1/3}$ fm ↔ comparable

It is important to take into account the finite nuclear size.

Dirac equation with Coulomb potential

$$\left[i\partial_\mu \gamma^\mu - m + eA_\mu \gamma^\mu\right]\psi(\boldsymbol{r}) = 0$$

replacing A_0 with the nuclear Coulomb potential

$$\Rightarrow \quad E\psi(\boldsymbol{r}) = [-i\boldsymbol{\alpha}\cdot\nabla + m\beta - V(\boldsymbol{r})]\psi(\boldsymbol{r})$$

Assuming a spherical potential $V(\boldsymbol{r}) = V(r)$, we obtain the radial Dirac equation

$$\begin{cases} \dfrac{dg_\kappa(r)}{dr} + \dfrac{1+\kappa}{r} g_\kappa(r) - \big(E + m - V(r)\big) f_\kappa(r) = 0 \\ \dfrac{df_\kappa(r)}{dr} + \dfrac{1-\kappa}{r} f_\kappa(r) + \big(E - m - V(r)\big) g_\kappa(r) = 0 \end{cases} \qquad \psi_\kappa(r) = \begin{pmatrix} g_\kappa(r)\chi_\kappa \\ i f_\kappa(r)\chi_{-\kappa} \end{pmatrix}$$

κ is the index of angular momenta: $\kappa = \begin{cases} -(l+1) & (j = l + 1/2) \\ l & (j = l - 1/2) \end{cases}$

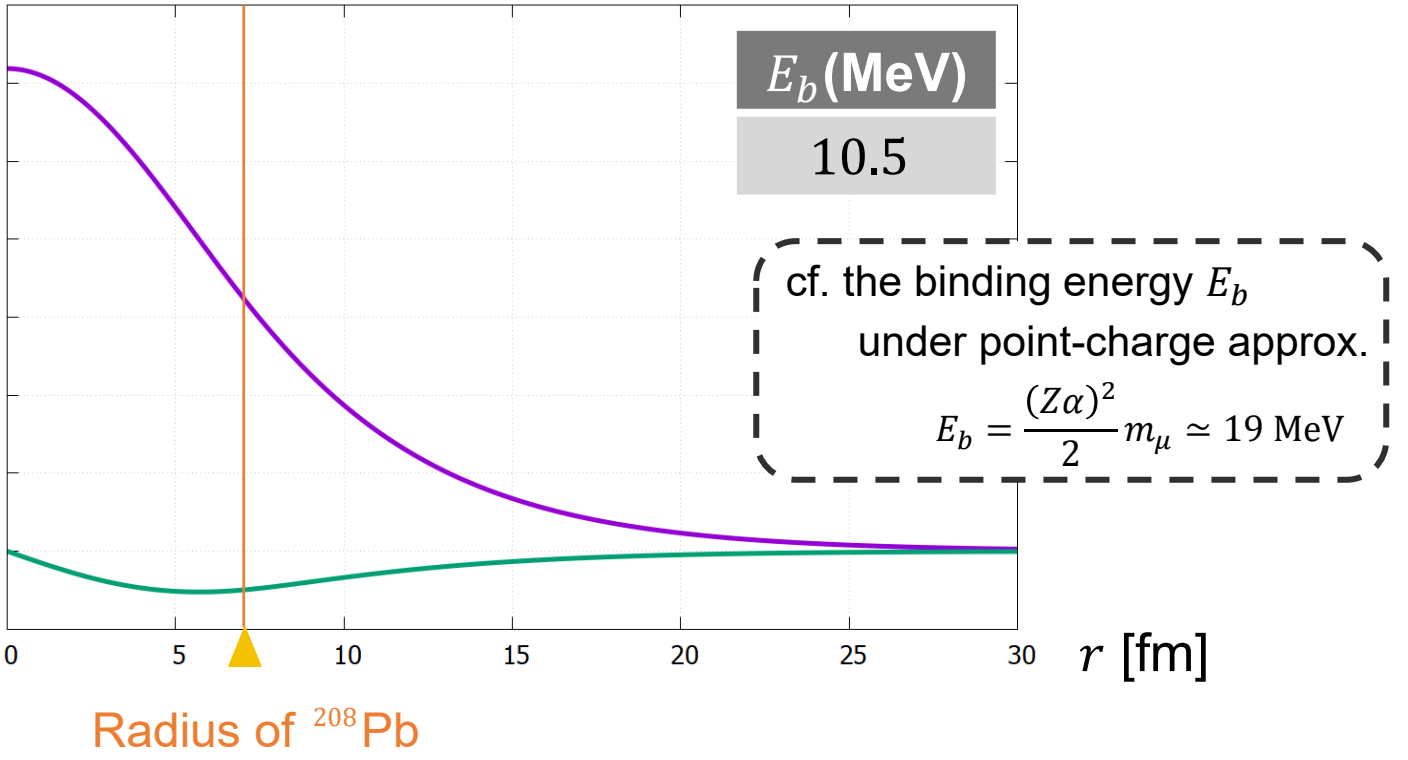
l: orbital angular momentum $\quad$ j: total angular momentum

e.g., s-wave corresponds to $\kappa = -1$.

Bound wave functions

for $^{208}\mathrm{Pb}$ $(Z = 82, A = 208)$

$g(r)$ & $f(r)$ of bound μ^- (1s)

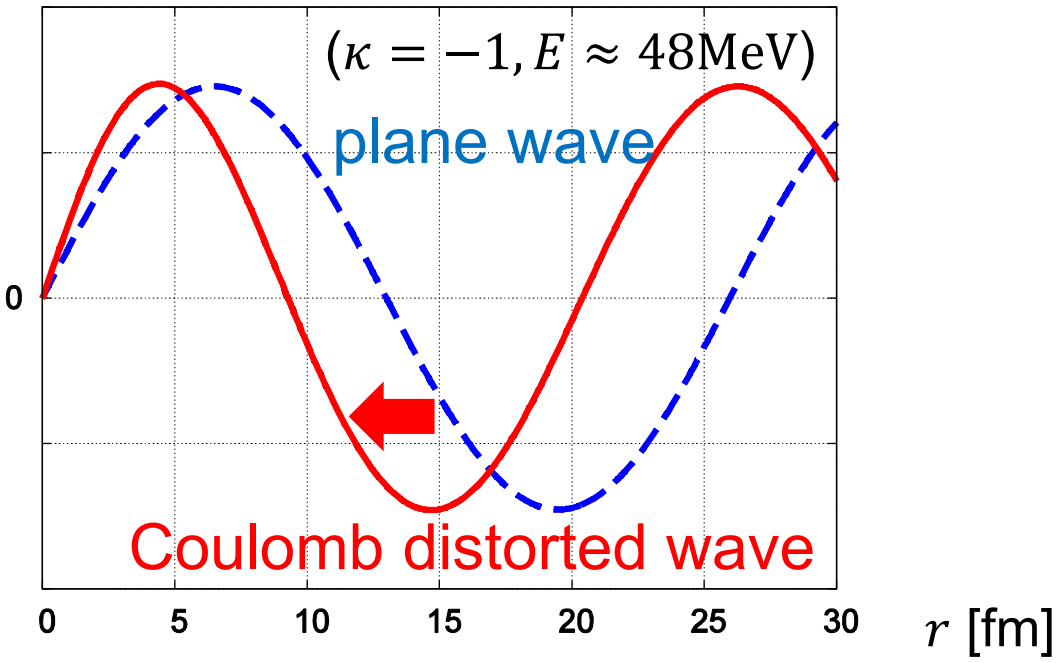


✓ Because the bound muon is close to the nucleus,
it is necessary to consider the nuclear finite size.

Scattering wave functions

for $^{208}\mathrm{Pb}$ $(Z = 82, A = 208)$

$rg(r)$ of scattering e^-



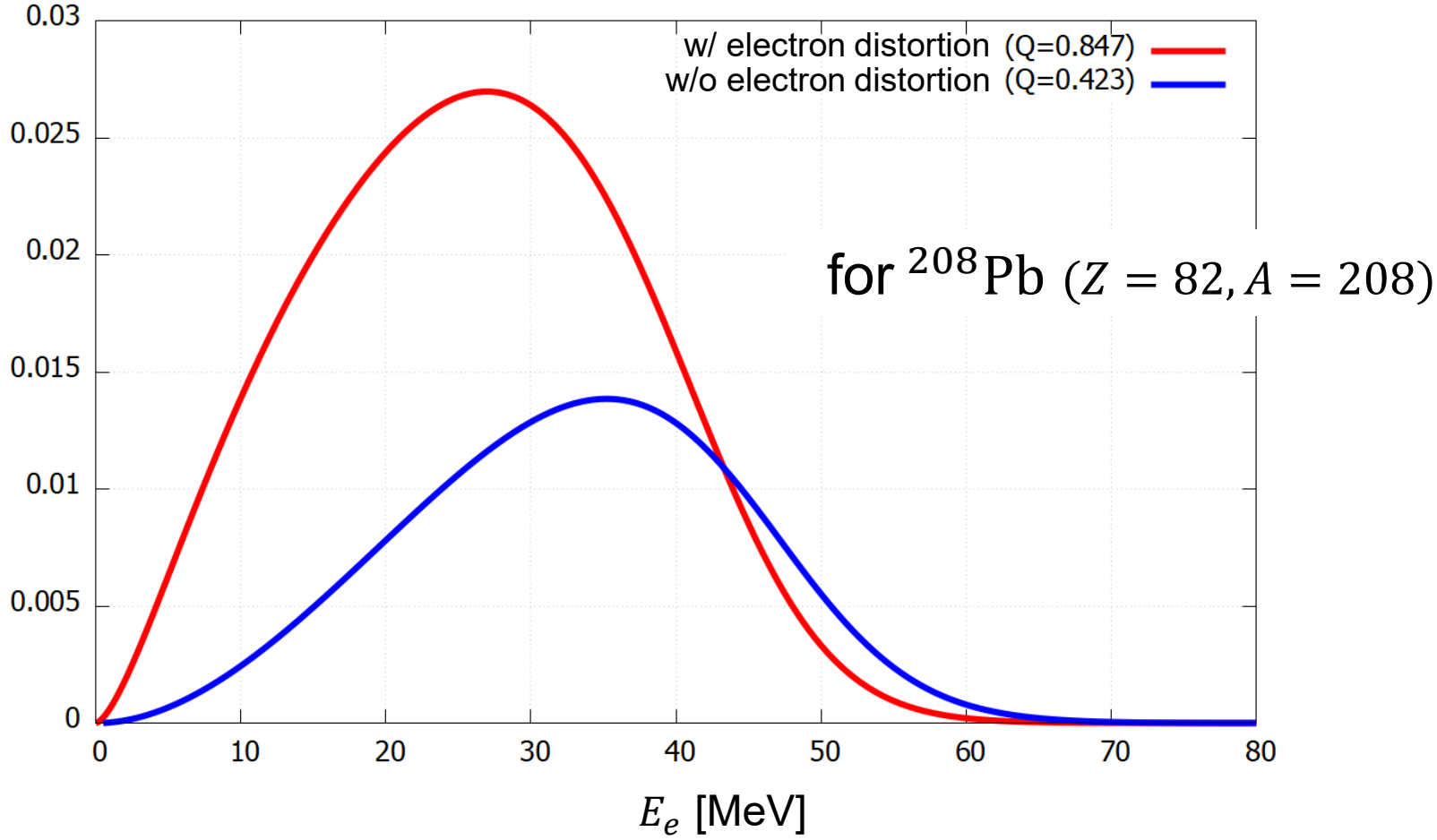
Distortion effects make...

① The value near $r = 0$ gets larger, to enhance the overlap with the bound muon.

② The effective momentum (wave number) gets larger,
to suppress the overlap with the bound muon.

DIO spectrum

dQ/dE_e [1/MeV]



for $^{208}\mathrm{Pb}$ $(Z = 82, A = 208)$

✓ The e^- distortion enhances the value of the low energy region & suppresses that of the high energy region.

(The total integrated value is enhanced by the distortion effect as seen in the previous slide.)

Behavior near the endpoint

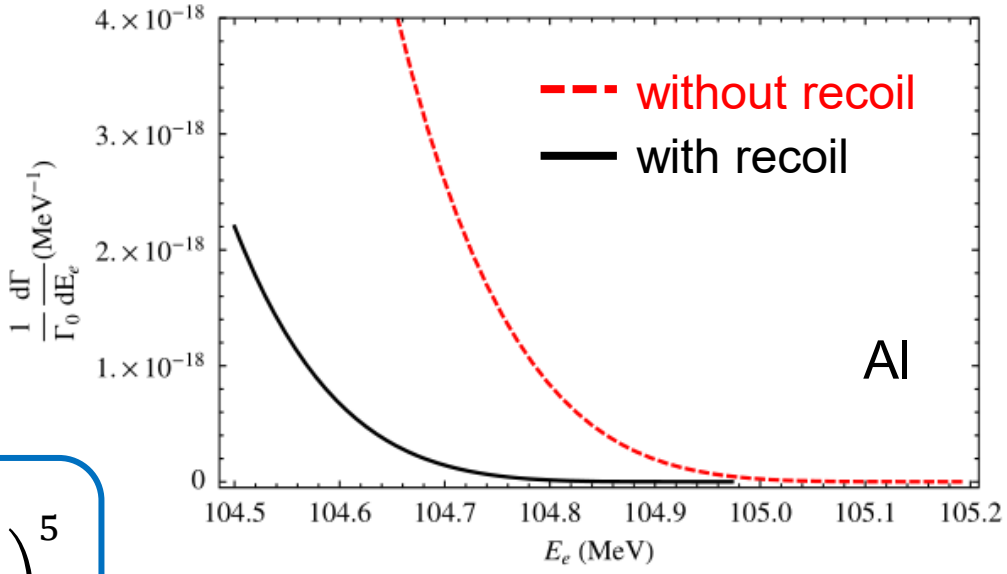
important to estimate BG of $\mu^- \to e^-$ conv.

- nuclear recoil O. Shanker, PRD**25**, 1847 (1981). A. Czarnecki *et al.*, PRD**84**, 013006 (2011).

$$E_e \to E_e - \frac{E_e^2}{2m_N}$$

$$\Rightarrow E_{\text{end}} = m_\mu - E_b - E_{\text{recoil}}$$

$$E_{\text{recoil}} = \frac{(m_\mu - E_b)^2}{2m_N}$$



expansion near the endpoint

$$\left.\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_e}\right|_{E_e \simeq E_{\text{end}}} = B E_{\text{end}}^5 \left(1 - \frac{E_e}{E_{\text{end}}}\right)^5$$

$$B = \frac{64}{5\pi m_\mu^5}\left(p_1^2 + \frac{s_1^2}{3} + \frac{2}{3}r_2^2\right)$$

$$p_\kappa = \langle g_{-\kappa} G \rangle$$
$$s_\kappa = \langle f_{-\kappa} F \rangle$$
$$r_\kappa = \langle g_{-\kappa} F \rangle$$

g, f: electron w.f.

G, F: muon w.f.

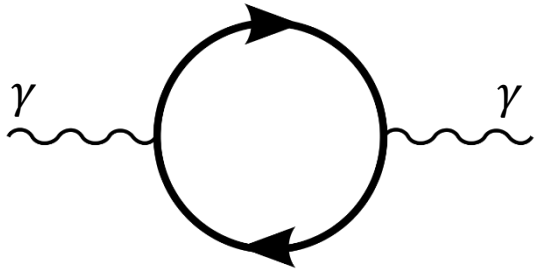
$$\langle \cdots \rangle \equiv \int_0^\infty \cdots r^2 dr$$

κ: index of angular mom.

Radiative corrections

R. Szafron & A. Czarnecki, PRD**94**, 051301 (2016).

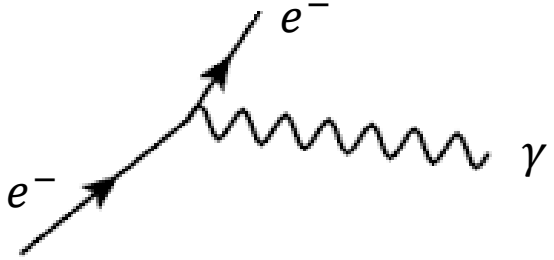
- vacuum polarization



⇨ modify the nuclear potential

⇨ modify the endpoint via E_b

- soft photon radiation



⇨ modify the shape near endpoint

$$\left.\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_e}\right|_{E_e \simeq E_{\text{end}}} = B E_{\text{end}}^5 \left(1 - \frac{E_e}{E_{\text{end}}}\right)^5 \Rightarrow \left.\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_e}\right|_{E_e \simeq E'_{\text{end}}} = B E'^5_{\text{end}} \left(1 - \frac{E_e}{E'_{\text{end}}}\right)^{5+\delta}$$

$$E'_{\text{end}} = E_{\text{end}} + \frac{\alpha m_\mu (Z\alpha)^2}{\pi}\left(\frac{11}{9} - \frac{2}{3}\log\left[\frac{2m_\mu Z\alpha}{m_e}\right]\right) \qquad \delta = \frac{2\alpha}{\pi}\left(\log\left[\frac{2m_\mu}{m_e}\right] - 1\right) \simeq 0.023$$

Nuclear charge distribution

- measured by electron scattering

W. Boeglin *et al.*, Nucl. Phys. A **477** (1988) 399.
J. Wesseling *et al.*, PRC **55** (1997) 2773.
H. De Vries *et al.*, Atom. Data Nucl. Data Tabl. **36** (1987) 495.
G. Fricke *et al.*, Atom. Data Nucl. Data Tabl. **60** (1995) 177.
A. A. Kabir, PhD thesis, Kent State Univ., US (2015).

- Different "fitting functions" are used:

Examples of fitting funtions

1. three(two)-parameter Fermi (3pF, 2pF)

$$\rho(r) = \frac{\rho_0}{1 + \exp\frac{r-c}{z}}\left(1 + \omega\frac{r^2}{c^2}\right) \qquad \omega = 0$$

2. there-parameter Gaussian (3pG)

$$\rho(r) = \frac{\rho_0}{1 + \exp\frac{r^2-c^2}{z^2}}\left(1 + \omega\frac{r^2}{c^2}\right)$$

3. modified-harmonic oscillator (MHO)

$$\rho(r) = \rho_0\left(1 + \omega\frac{r^2}{a^2}\right)\exp\left(-\frac{r^2}{a^2}\right)$$

4. Fourier-Bessel (FB)

5. Sum of Gaussians (SOG)

- There are nuclides which has only data of "radius". ← we assume 1pF dist.

I. Angeli & K. P. Marinova, Atom. Data Nucl. Data Tabl. **99** (2013) 69.

$$\rho(r) = \frac{\rho_0}{1 + \exp\frac{r-c}{0.52\ \mathrm{fm}}}$$

Numerical results

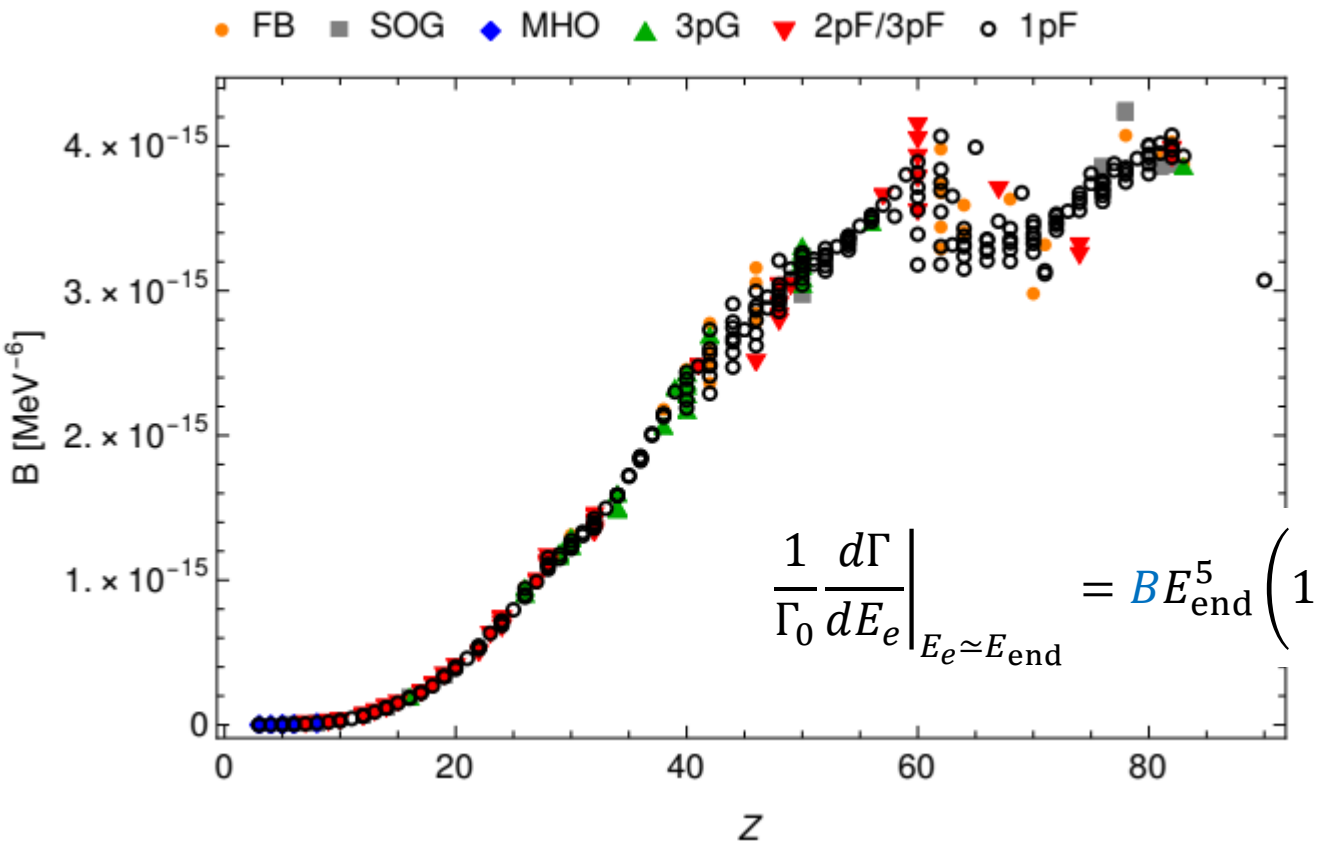
Table of $E'_{\rm end}$ & B for various nuclides and models

	$E'_{\rm end}$/MeV	$B_{\rm 1pF}$/MeV^{-6}	$B_{\rm FB}$/MeV^{-6}	$B_{\rm SOG}$/MeV^{-6}	$B_{\rm MHO}$/MeV^{-6}	$B_{\rm 3pF}$/MeV^{-6}	$B_{\rm 3pG}$/MeV^{-6}
$^{4}_{2}$He	104.150			2.53×10^{-20}			
$^{6}_{3}$Li	104.637	1.20×10^{-19}	1.29×10^{-19}				
$^{7}_{3}$Li	104.779	1.28×10^{-19}			1.31×10^{-19}		
$^{9}_{4}$Be	104.949	4.97×10^{-19}			4.96×10^{-19}		
$^{10}_{5}$B	104.99	1.52×10^{-18}	1.56×10^{-18}		1.50×10^{-18}		
$^{11}_{5}$B	105.044	1.53×10^{-18}			1.52×10^{-18}		
$^{12}_{6}$C	105.059	3.55×10^{-18}	3.53×10^{-18}	3.55×10^{-18}			
$^{13}_{6}$C	105.097	3.56×10^{-18}			3.63×10^{-18}		
$^{14}_{7}$N	105.094	7.04×10^{-18}				7.13×10^{-18}	
$^{16}_{8}$O	105.106	1.22×10^{-17}	1.19×10^{-17}	1.20×10^{-17}	1.19×10^{-17}		
$^{19}_{9}$F	105.118	1.85×10^{-17}				1.87×10^{-17}	
$^{20}_{10}$Ne	105.081	2.79×10^{-17}				2.87×10^{-17}	
$^{22}_{10}$Ne	105.108	2.89×10^{-17}				2.89×10^{-17}	
$^{23}_{11}$Na	105.063	4.35×10^{-17}					
$^{24}_{12}$Mg	105.011	6.17×10^{-17}		6.29×10^{-17}		6.10×10^{-17}	

⋮

J. Heeck, R. Szafron, & YU, PRD105, 053006; arXiv:2110.14667.

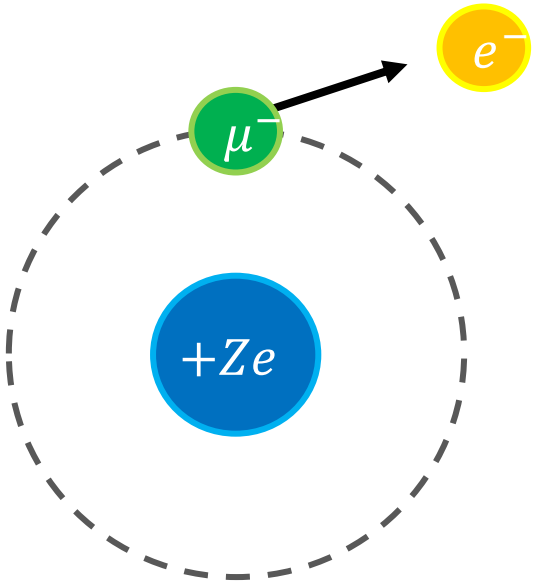
B coefficient



$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_e}\bigg|_{E_e \simeq E_{\mathrm{end}}} = B E_{\mathrm{end}}^5 \left(1 - \frac{E_e}{E_{\mathrm{end}}}\right)^5$$

- The uncertainty from the choice of the fitting function is about 10%.
- The isotope differences are often of the same order or even larger than the uncertainty.

3. $\mu^- \rightarrow e^-$ conversion in muonic atoms



Effective Lagrangian beyond the SM

Effective field theory obtained where heavy fields are integrated out:

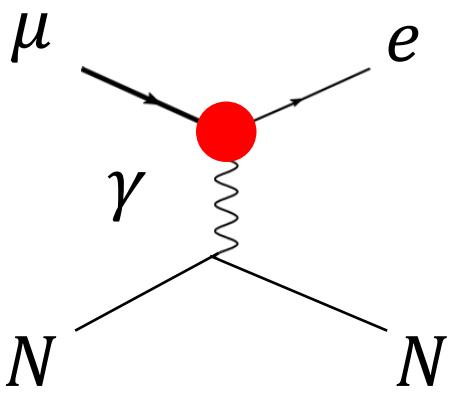
$$\mathcal{L}_{\mu e} = -\frac{4G_F}{\sqrt{2}} \sum_{X=L,R} [m_\mu C_{D,X} \bar{e} \sigma^{\alpha\beta} P_X \mu F_{\alpha\beta}$$

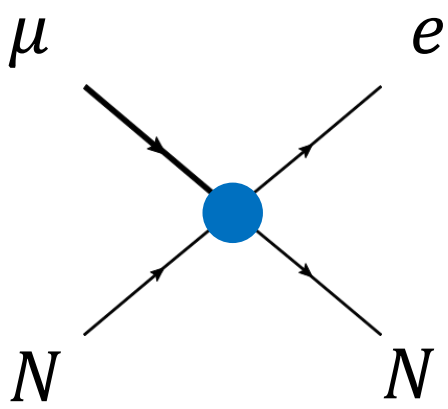
dipole interaction

$$+ \sum_{N=p,n} \left(C_{S,X}^{(N,N)} \bar{e} P_X \mu \bar{N} N + C_{V,X}^{(N,N)} \bar{e} \gamma^\alpha P_X \mu \bar{N} \gamma_\alpha N \right)] + h.c.$$

scalar interaction

vector interaction





Branching ratio

The formulation of $\mu \to e$ conversion for the effective LFV operators was well established by R. Kitano, M. Koike, & Y. Okada, PRD**66**, 096002 (2002).

$$BR_{\rm SI} = \frac{32G_F^2}{\Gamma_{\rm capture}}\left[\left|C_{D,L}\frac{D}{4} + \sum_{N=p,n}\left(C_{S,L}^{NN}S^{(N)} + C_{V,R}^{NN}V^{(N)}\right)\right|^2 + \{L \leftrightarrow R\}\right]$$

- overlap integrals

$$D = \frac{4m_\mu}{\sqrt{2}}\int_0^\infty dr\, r^2[-E(r)]\left(g_e^- f_\mu^- + f_e^- g_\mu^-\right)$$

electric field

$$S^{(p)} = \frac{1}{2\sqrt{2}}\int_0^\infty dr\, r^2 Z\rho^{(p)}\left(g_e^- g_\mu^- - f_e^- f_\mu^-\right)$$

$\rho^{(p)}$: proton density

$$S^{(n)} = \frac{1}{2\sqrt{2}}\int_0^\infty dr\, r^2 N\rho^{(n)}\left(g_e^- g_\mu^- - f_e^- f_\mu^-\right)$$

$\rho^{(n)}$: neutron density

$$V^{(p)} = \frac{1}{2\sqrt{2}}\int_0^\infty dr\, r^2 Z\rho^{(p)}\left(g_e^- g_\mu^- + f_e^- f_\mu^-\right)$$

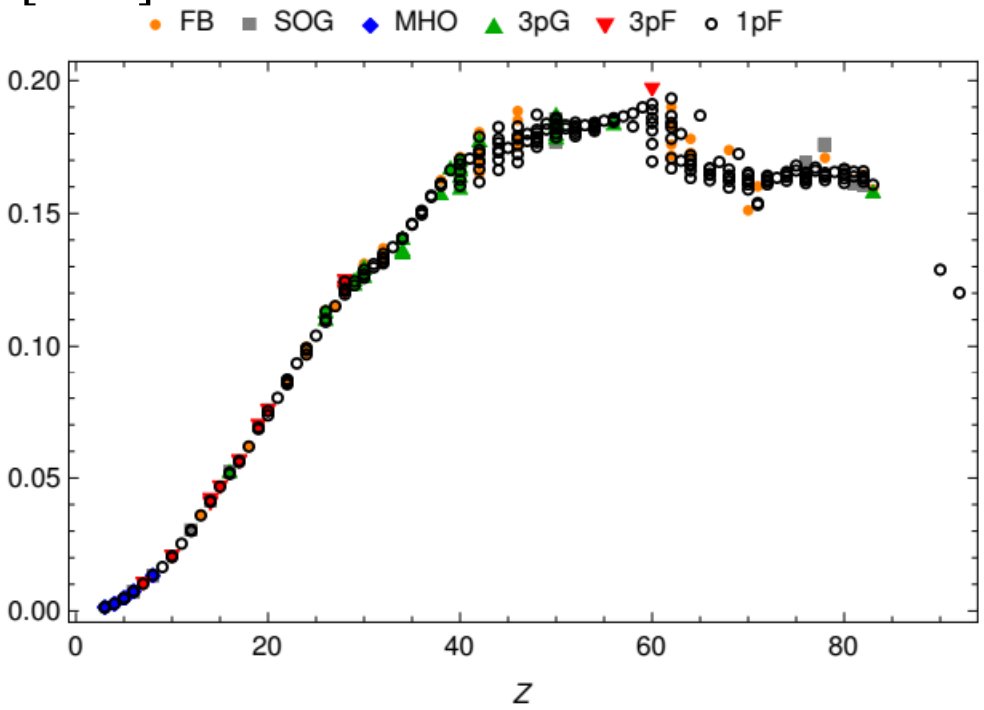
$$V^{(n)} = \frac{1}{2\sqrt{2}}\int_0^\infty dr\, r^2 N\rho^{(p)}\left(g_e^- g_\mu^- + f_e^- f_\mu^-\right)$$

Overlap integrals

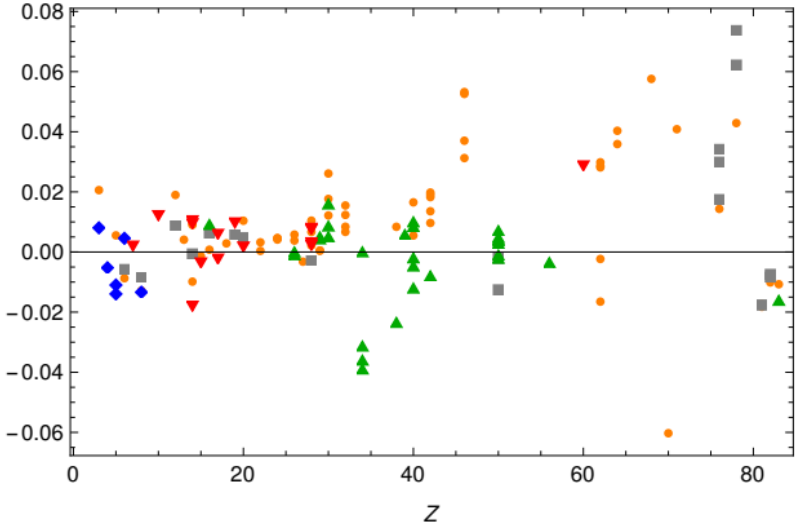
We checked the nuclear dependence of the overlap integrals with the latest nuclear data.

J. Heeck, R. Szafron, & YU, NPB980, 115833.

$D\left[m_\mu^{5/2}\right]$



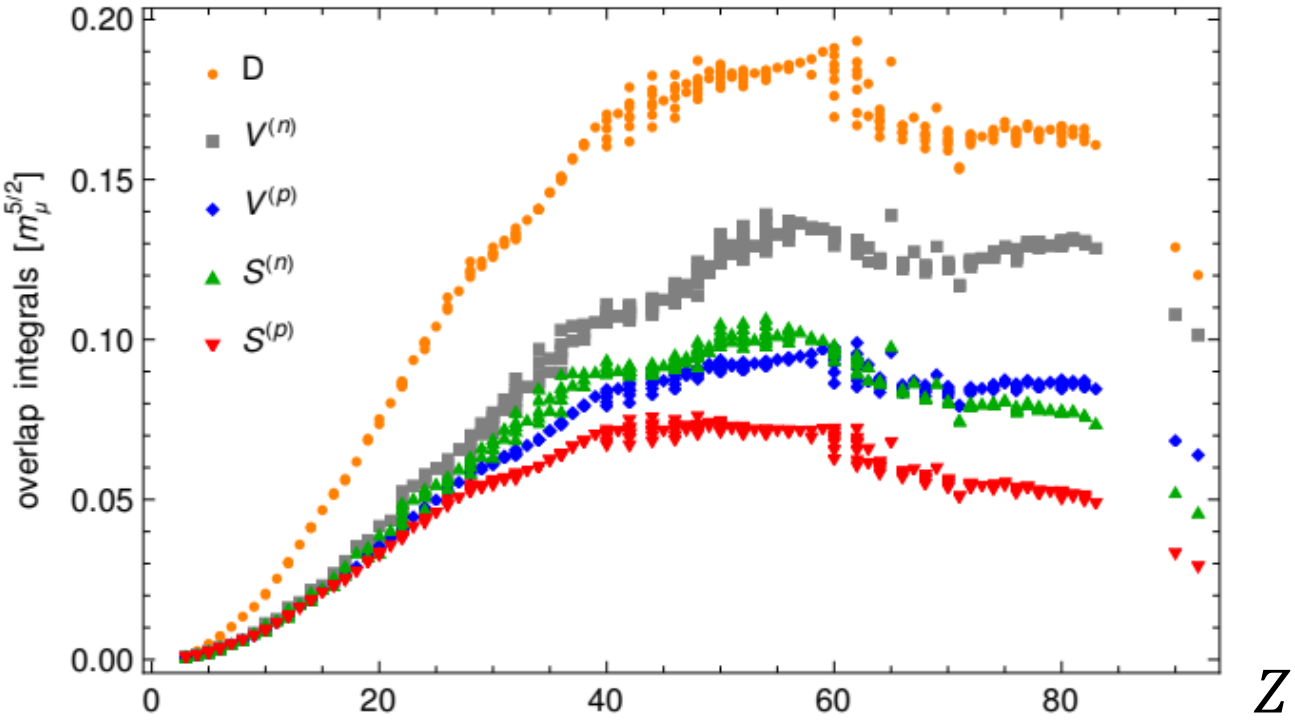
$\frac{D - D(1\text{pF})}{D(1\text{pF})}$ (uncertainty from parametrization of ρ_c)



- For $Z < 30$, the uncertainty due to ρ_c is less than 2%, but grows to 8% for large Z.
- Isotope dependence exceeds the uncertainty at medium Z.

Overlap integrals

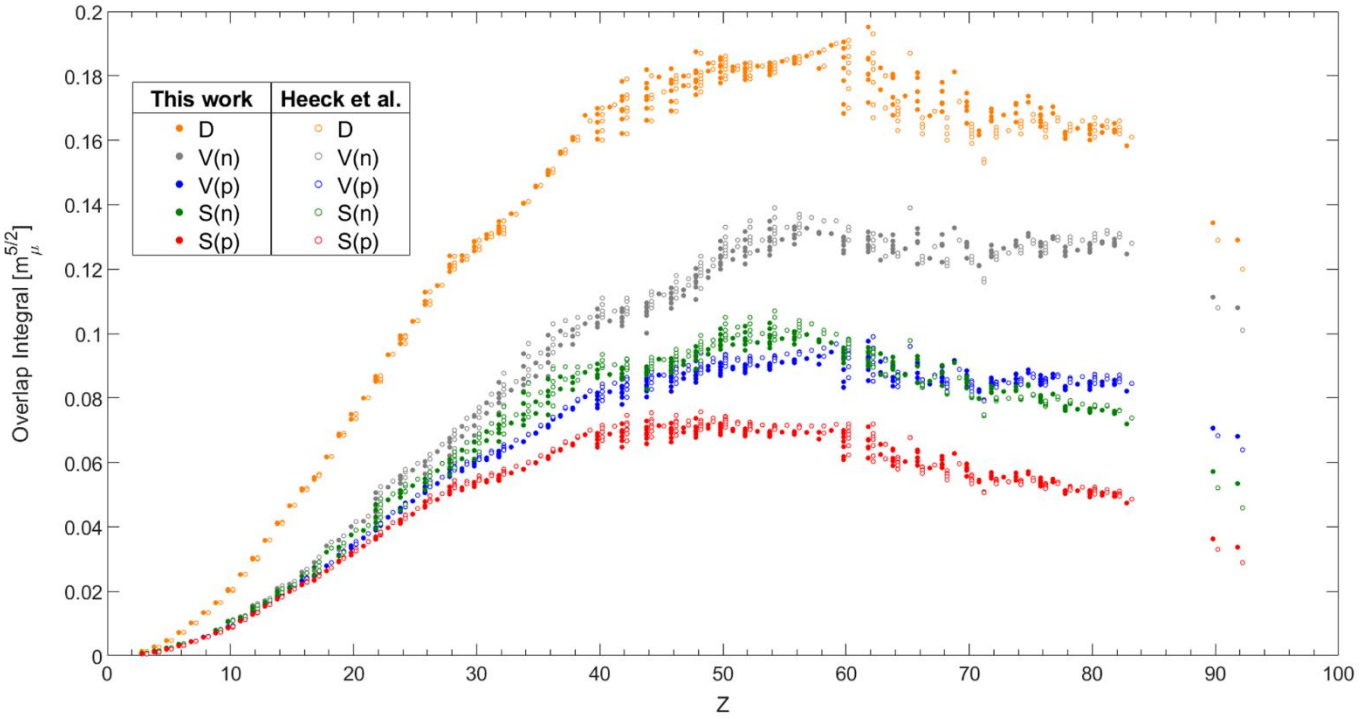
($\rho^{(n)} = \rho^{(p)}$ is assumed.)



- For $S^{(p)}$ and $V^{(p)}$, the uncertainty is estimated to range from 5% at low Z to 10% at high Z.
- For $S^{(n)}$ and $V^{(n)}$, the uncertainty is also 5% at low Z, but more than 10% at high Z.

Updated work: Quadrupole deformation

L. Borrel, D. G. Hitlin, & S. Middleton, NPA**1062**, 123161 (2025).



$$\rho(r,\theta) = \frac{\rho_0}{1+\exp\left[\frac{r-c(1+\beta Y_{20}(\theta))}{t}\right]}$$

We convert this to a spherically-symmetric 2-parameter Fermi distribution by removing the quadrupole deformation β and adjusting the skin thickness t to keep a constant Barrett moment (defined in equation 4). This equivalent 2-parameter Fermi distribution is then used to solve the Dirac equation.

To further improvement...

- A $\mu^- \to e^-$ conversion workshop was held at ECT* this April.

website: https://indico.ectstar.eu/event/232/

organized by

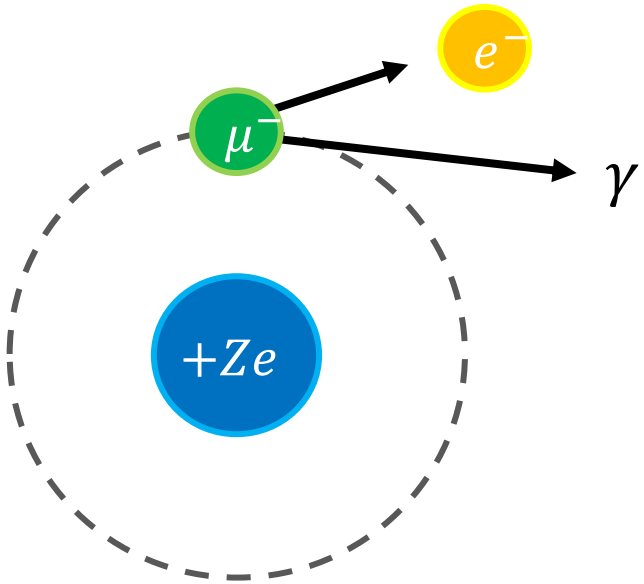
K. Bennaceur

S. Davidson

We need to discuss more...

- Quadrupole deformation
- Incoherent $\mu^- \to e^-$ conversion ($\mu^-(Z,A) \to e^-(Z,A)^*$)
- $\mu^- \to e^+$ conversion ($\mu^-(Z,A) \to e^+(Z-2,A)$)

4. $\mu^- \rightarrow e^- \gamma$ in a muonic atom



CLFV effective interaction with photon fields

➢ Effective Lagrangian

✓ U(1) gauge symmetry:

The photon field A^α can be included only through $F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha$.

- Dipole operator (dim-5)

$$\mathcal{L}_D = \frac{m_\mu}{v^2}\left\{\left[\bar{e}\sigma_{\alpha\beta}(D_L P_L + D_R P_R)\mu\right]F^{\alpha\beta} + \text{H. c.}\right\}$$

strongly constrained by $\mu^+ \to e^+\gamma$

- Diphoton operator (dim-7)

$$\mathcal{L}_{FF} = \frac{1}{v^3}\left\{\left[\bar{e}(C_L P_L + C_R P_R)\mu\right]F_{\alpha\beta}F^{\alpha\beta} + \left[\bar{e}i\gamma_5\left(\tilde{C}_L P_L + \tilde{C}_R P_R\right)\mu\right]F_{\alpha\beta}\tilde{F}^{\alpha\beta} + \text{H. c.}\right\}$$

$BR(\mu^+ \to e^+\gamma\gamma) < 7.2 \times 10^{-11}$ ⇨ $\sqrt{|C_L|^2 + \left|\tilde{C}_L\right|^2 + (L \to R)} < 2.2 \times 10^{-2}$

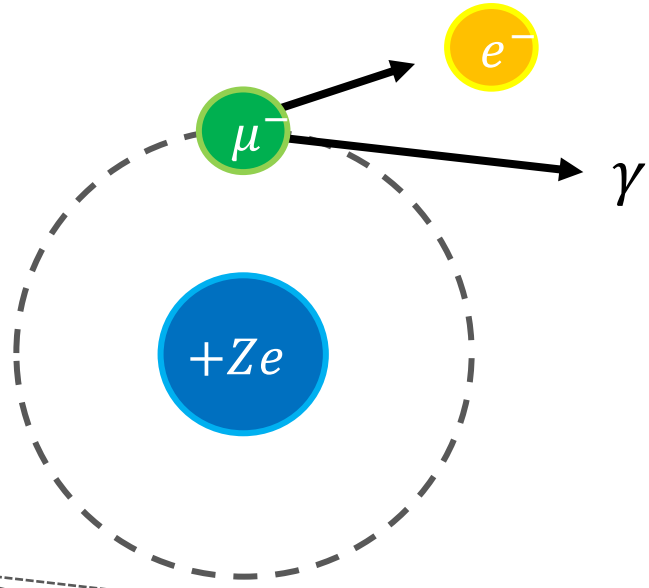
R. D. Bolton *et al.*, PRD**38**, 2077 (1988).

$\mu^- \to e^-\gamma$ in a muonic atom

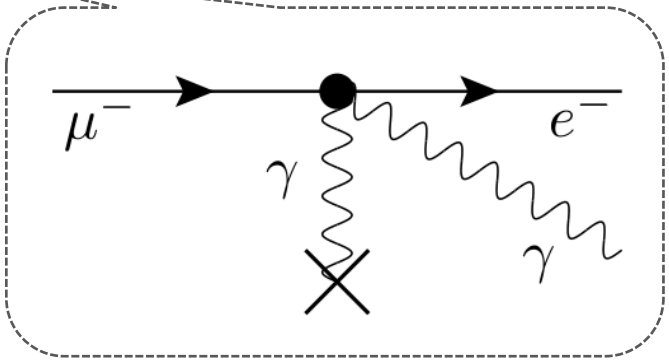
YU, M. Yamanaka & Y. Kuno, PRD111, 035017.

◆ Rare decay of μ^- in orbit (not free μ^+)

- signal: a pair of e^- & γ with $E_e + E_\gamma = m_\mu - E_b$
  (E_b: binding energy)
- The signal e^- & γ are typically emitted back-to-back with $E_e \simeq E_\gamma \simeq 50$ MeV
  - ✓ But it's not a strict two-body decay, the spectrum is smeared due to the nuclear recoil.



- As well as dipole operator, diphoton operator(FF & $F\tilde{F}$) can contribute.
  - ✓ The diphoton ope. can be directly restricted as $\mu^+ \to e^+\gamma\gamma$.



Disadvantages:

- Muonic atoms have shorter lifetime than free muons.
- Invariant mass $m_{e\gamma} \neq m_\mu$;
  Although the energy is (approximately) conserved, the 3-momentum is not in $\mu^- \to e^-\gamma$.

Formulation of the decay rate

- Nucleus is treated as a static Coulomb potential.

$$d\Gamma = \frac{d^3p_e}{(2\pi)^3 2E_e}\frac{d^3p_\gamma}{(2\pi)^3 2E_\gamma}(2\pi)\delta\left(E_e + E_\gamma - E_\mu\right)\frac{1}{2}\sum_{spins}|\mathcal{M}|^2$$

transition amplitude

$$\begin{aligned}\mathcal{M} = &-\frac{2im_\mu}{v^2}\int d^3r\overline{\psi}_e(\boldsymbol{r})\,\sigma_{\alpha\beta}\left(D_LP_L + D_RP_R\right)\psi_\mu^{1s}(\boldsymbol{r})\,p_\gamma^\alpha\epsilon^{s_\gamma *\beta}\exp\left(-i\boldsymbol{p}_\gamma\cdot\boldsymbol{r}\right)\\ &-\frac{4i}{v^3}\int d^3r\overline{\psi}_e(\boldsymbol{r})\left(C_LP_L + C_RP_R\right)\psi_\mu^{1s}(\boldsymbol{r})\,p_\gamma^\alpha\epsilon^{s_\gamma *\beta}\exp\left(-i\boldsymbol{p}_\gamma\cdot\boldsymbol{r}\right)\langle N\,|\,F_{\alpha\beta}\,|\,N\rangle\\ &-\frac{4i}{v^3}\int d^3r\overline{\psi}_e(\boldsymbol{r})\,i\gamma_5\left(\tilde{C}_LP_L + \tilde{C}_RP_R\right)\psi_\mu^{1s}(\boldsymbol{r})\,p_\gamma^\alpha\epsilon^{s_\gamma *\beta}\exp\left(-i\boldsymbol{p}_\gamma\cdot\boldsymbol{r}\right)\left\langle N\left|\tilde{F}_{\alpha\beta}\right|N\right\rangle\end{aligned}$$

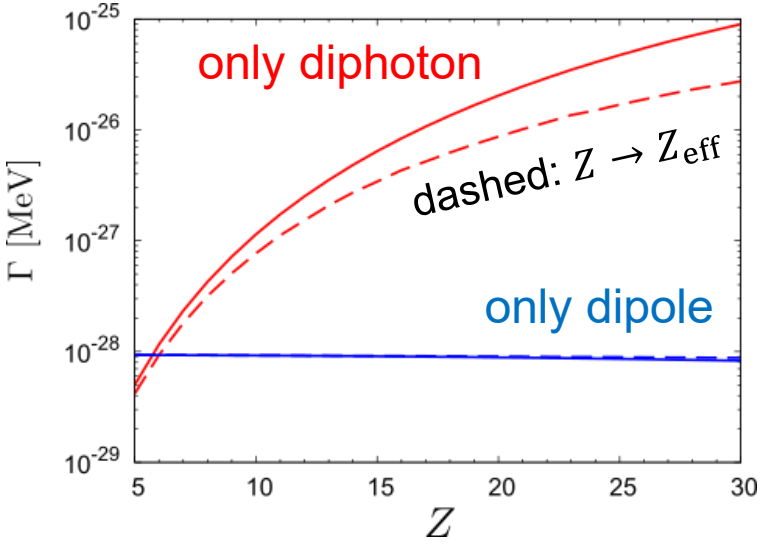
→ The field strength F is replaced with nuclear electric field E.

$$\langle N\,|\,F_{\alpha\beta}\,|\,N\rangle = \begin{cases}-E_i & (\alpha = i, \beta = 0)\\ E_j & (\alpha = 0, \beta = j)\\ 0 & (\alpha = i, \beta = j)\end{cases}\qquad \left\langle N\left|\tilde{F}_{\alpha\beta}\right|N\right\rangle = \begin{cases}-\epsilon_{ijk}E_k & (\alpha = i, \beta = j)\\ 0 & (\alpha = 0 \text{ or } \beta = 0)\end{cases}$$

※ In this calculation, we restrict ourselves to the case that the proton number is small.

→ We assume “point-charge nucleus” & “plane-wave electron”

Z dep. of the decay rate ($Z \leq 30$)



$\Gamma_{FF} \propto Z^5$

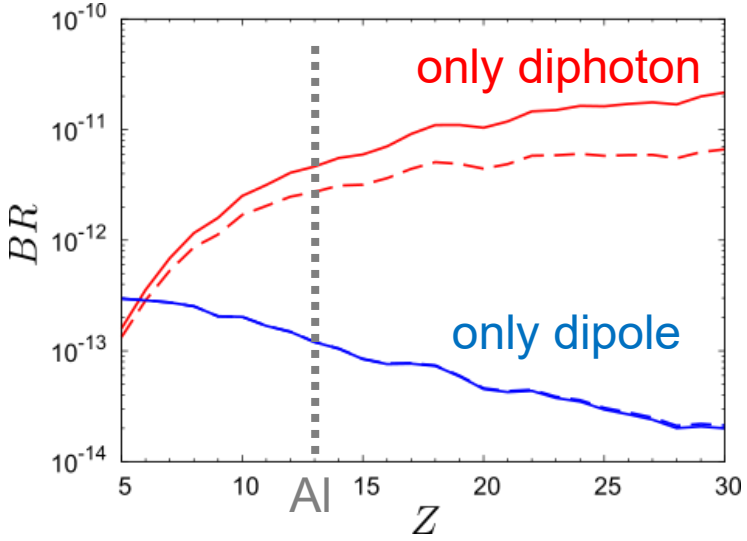
$Z^3 \leftarrow$ muon w.f.

$Z^2 \leftarrow (E)^2$

$\Gamma_D \propto 1$

(moderately decreasing)

- Upper limits of BR from the past experiments



$BR(\mu^+ \to e^+\gamma\gamma) < 7.2 \times 10^{-11}$

$BR < 4.6 \times 10^{-12}$ for Al

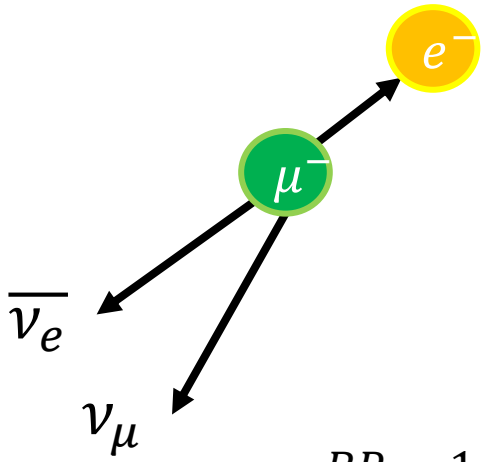
$BR(\mu^+ \to e^+\gamma) < 3.1 \times 10^{-13}$

$BR < 1.2 \times 10^{-13}$ for Al

Backgrounds

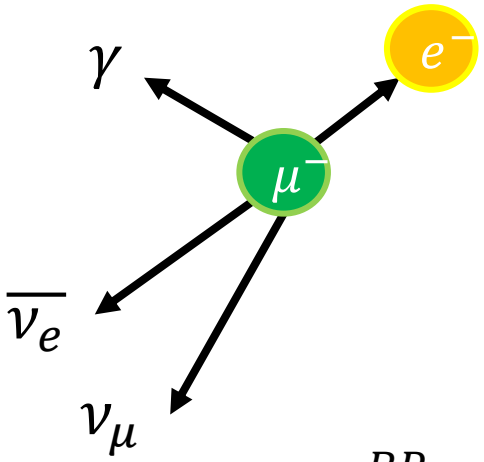
✓ Electrons & photons are also produced in the ordinary muon decays:

1. $\mu^- \to e^-\nu_\mu\overline{\nu}_e$



$BR \sim 1$

2. $\mu^- \to e^-\nu_\mu\overline{\nu}_e\gamma$



$BR \sim 10^{-7}$

➢ BG1: accidental BG, where the energy sum of e^- & γ from different μ^-s accidentally matches the energy of the signal event.

$$E_e + E_\gamma = m_\mu - E_b$$

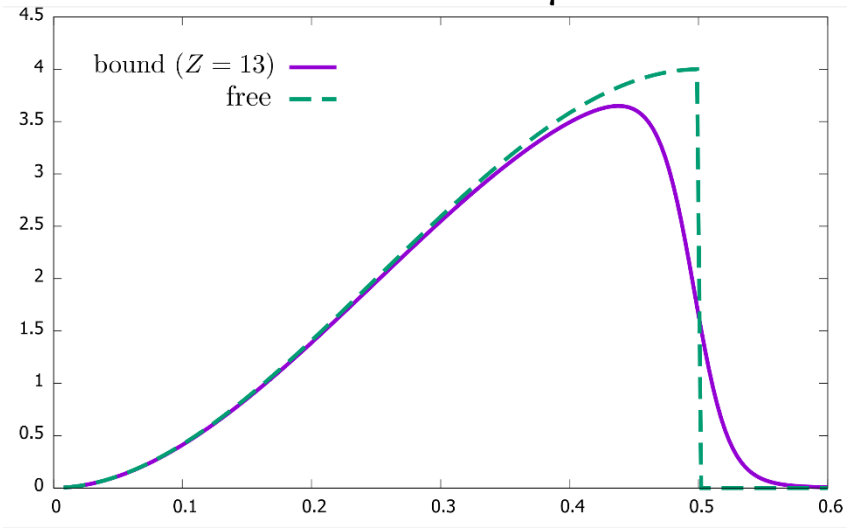
➢ BG2: physics BG, where the neutrino energies are too small in $\mu^- \to e^-\nu_\mu\overline{\nu}_e\gamma$.

(but it is negligible.)

BG e^- & γ spectra

E_e distribution

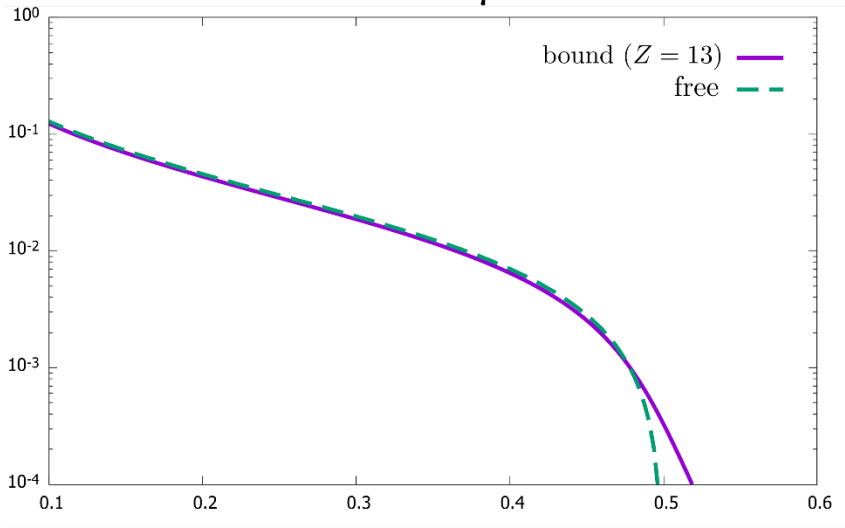
$$\mu^- \to e^-\nu_\mu\bar{\nu}_e$$



E_e/m_μ

E_γ distribution

$$\mu^- \to e^-\nu_\mu\bar{\nu}_e\gamma$$



E_γ/m_μ

The value at $E_\gamma = m_\mu/2$ is 0 in free muon,
but it is nonzero in bound muon…

Effective branching ratio

Benchmark : $R_\mu = 10^7$ /s $\quad \Delta t_{e\gamma} = 78$ ps $\quad \Delta_{x+y} = 0.01$ (energy resolution)

Effective BR

$$B_{acc} = R_\mu \Delta t_{e\gamma} \frac{\Delta\Omega_{e\gamma}}{4\pi} f_{acc}$$

R_μ: produced μ^-/ time

$\Delta t_{e\gamma}$: time resolution

$\Delta\Omega_{e\gamma}$: angular resolution

f_{acc}: the rate that BG e^- & γ satisfy the energy condition

	Al ($Z = 13$)	Zn ($Z = 30$)
Signal (diphoton)	4.6×10^{-12}	2.2×10^{-11}
Signal (dipole)	1.2×10^{-13}	2.0×10^{-14}
accidental BG	1.2×10^{-10}	1.2×10^{-11}
BG ($\mu^- \to e^-\nu\bar{\nu}\gamma$)	1.5×10^{-15}	5.7×10^{-16}

(negligible)

large BG events

constraint of $\theta_{e\gamma}$ to optimize the diphoton signal

$$\theta_{e\gamma}^{peak} - 0.01 < \theta_{e\gamma} < \theta_{e\gamma}^{peak} + 0.01$$

$\theta_{e\gamma}^{peak} = 2.39$ for Al

$\theta_{e\gamma}^{peak} = 2.13$ for Zn

	Al ($Z = 13$)	Zn ($Z = 30$)
Signal (diphoton)	7.1×10^{-14}	3.3×10^{-13}
Signal (dipole)	1.2×10^{-17}	1.6×10^{-17}
accidental BG	8.5×10^{-13}	1.1×10^{-13}
BG ($\mu^- \to e^-\nu\bar{\nu}\gamma$)	2.6×10^{-19}	6.1×10^{-19}

Signal is larger than BG !

(There may still be room for further optimization.)

5. Summary

Summary

- Lepton flavor violation: a powerful probe of models beyond the SM
  - Muons are particularly interesting as a CLFV probes, as their bound states are available.
- To calculate the CLFV processes and backgrounds, we need lepton wave functions in a nuclear Coulomb potential.

◆ high-energy spectrum from DIO

- behavior near the endpoint

$$\left.\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_e}\right|_{E_e \simeq E'_{\mathrm{end}}} = B E'^{5}_{\mathrm{end}} \left(1 - \frac{E_e}{E'_{\mathrm{end}}}\right)^{5+\delta}$$

We made the table of B & E_{end} for available nuclear data.

J. Heeck, R. Szafron, & YU, PRD105, 053006.

◆ $\mu^- \to e^-$ conversion

- overlap integrals, D, S, & V

J. Heeck, R. Szafron, & YU, NPB980, 115833.

◆ $\mu^- \to e^-\gamma$ conversion

YU, M. Yamanaka & Y. Kuno, PRD111, 035017.