Exploring Lepton Number Violation through Same-Sign Lepton Processes at Future Colliders

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Neutrino masses

- Neutrino mass squared differences have been measured precisely by oscillation experiments

Normal Hierarchy(NH)

$$\begin{cases} \Delta m_{21}^2 = 7.49 \times 10^{-5} \mathrm{eV}^2 \\ \Delta m_{31}^2 = 2.513 \times 10^{-3} \mathrm{eV}^2 \end{cases}$$

Inverted Hierarchy(IH)

$$\begin{cases} \Delta m_{21}^2 = 7.49 \times 10^{-5} \mathrm{eV}^2 \\ \Delta m_{32}^2 = -2.484 \times 10^{-3} \mathrm{eV}^2 \end{cases}$$

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(NuFIT 6.0 (2024), www.nu-fit.org.

- Neutrinos are much lighter than charged leptons

$$m_\nu \sim 10^{-11}\mathrm{GeV}, \quad m_e \sim 10^{-4}\mathrm{GeV}$$

The seesaw mechanism can naturally account for neutrino masses and their smallness

Seesaw mechanism

[Minkowski(1977), T.Yanagida(1979), Gell-Mann, Ramond, Slansky(1979), Glashow(1979)]

- Standard Model + 2 right-handed neutrinos (RHNs)

$$\mathcal{L} = \mathcal{L}_{SM} + \overline{N_{RI}} i\partial_\mu \gamma^\mu N_{RI} - \left(Y_{\alpha I} \overline{L_{L\alpha}} H N_{RI} + \frac{M_M}{2} \underline{\overline{N^c_{RI}} N_{RI}} + h.c. \right)$$

$Y_{\alpha I}$:Yukawa matrix H:Higgs N_I:RHNs ($I = 1,2$)

	N_{RI}	$\overline{N^c_{RI}}$
L	1	1

Lepton number is violated at the Lagrangian level

- Neutrino mixing in the charged current interaction

$$\nu_{L\alpha} = \sum_i^3 U_{\alpha i} \nu_i + \sum_I^2 \Theta_{\alpha I} N^c_I$$

Contributions of HNLs

$U_{\alpha i}$: PMNS matrix ν_i : Active neutrino

N_I : Heavy neutral lepton (HNL) $\Theta_{\alpha I}$: Mixing elements

Seesaw mechanism

- Mixing elements of heavy neutral leptons

$$\Theta_{\alpha I} = \frac{[M_D]_{\alpha I}}{M_I} = \frac{vY_{\alpha I}}{M_I}$$

- Parametrization of Yukawa coupling [Casas,Ibarra(2001)]

$$Y = \frac{i}{v} U D_{\sqrt{m}} \Omega D_{\sqrt{M}}$$

$$D_{\sqrt{m}} = \mathrm{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$$

$$D_{\sqrt{M}} = \mathrm{diag}(\sqrt{M_1}, \sqrt{M_2})$$

$$\Omega = \begin{pmatrix} 0 & 0 \\ \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \text{For NH}$$

$$\Omega = \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \\ 0 & 0 \end{pmatrix} \text{For IH}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\eta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{ij} = \sin\theta_{ij}, \quad c_{ij} = \cos\theta_{ij}$$

ω:complex free parameter

η :Majorana phase

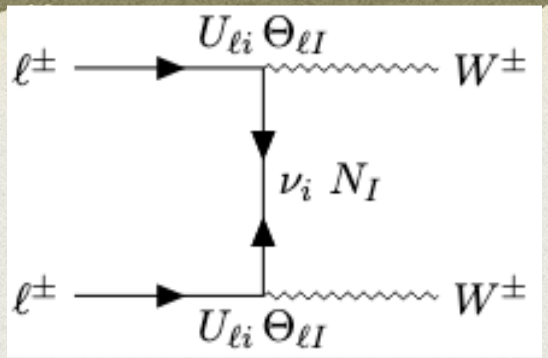
Lepton number violating processes

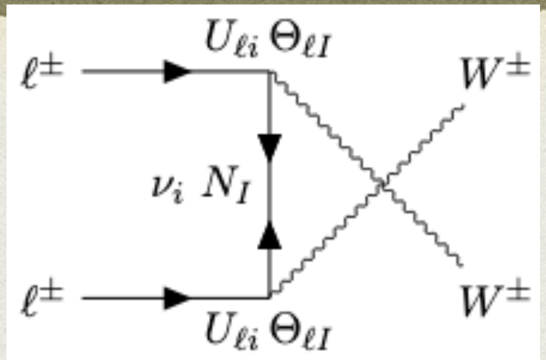
- A typical LNV processes
  - Neutrinoless double beta ($0\nu2\beta$) decay [(A,Z) →(A,Z+2) + 2e$^-$]
  - Rare meson decays [$K^+ \to \ell^+\ell^+\pi^-$…]
  - proton-proton collision [$pp \to \ell^\pm\ell^\pm$ + jets…]
  - $e^\pm e^\pm \to W^\pm W^\pm$, $\mu^\pm\mu^\pm \to W^\pm W^\pm$

We focus on these processes

- Features of $e^\pm e^\pm \to W^\pm W^\pm$, $\mu^\pm\mu^\pm \to W^\pm W^\pm$
  - Avoid uncertainties associated with nuclear matrix element
  - Complementary to $0\nu2\beta$ decay

$e^{\pm}e^{\pm} \to W^{\pm}W^{\pm}$ and $\mu^{\pm}\mu^{\pm} \to W^{\pm}W^{\pm}$

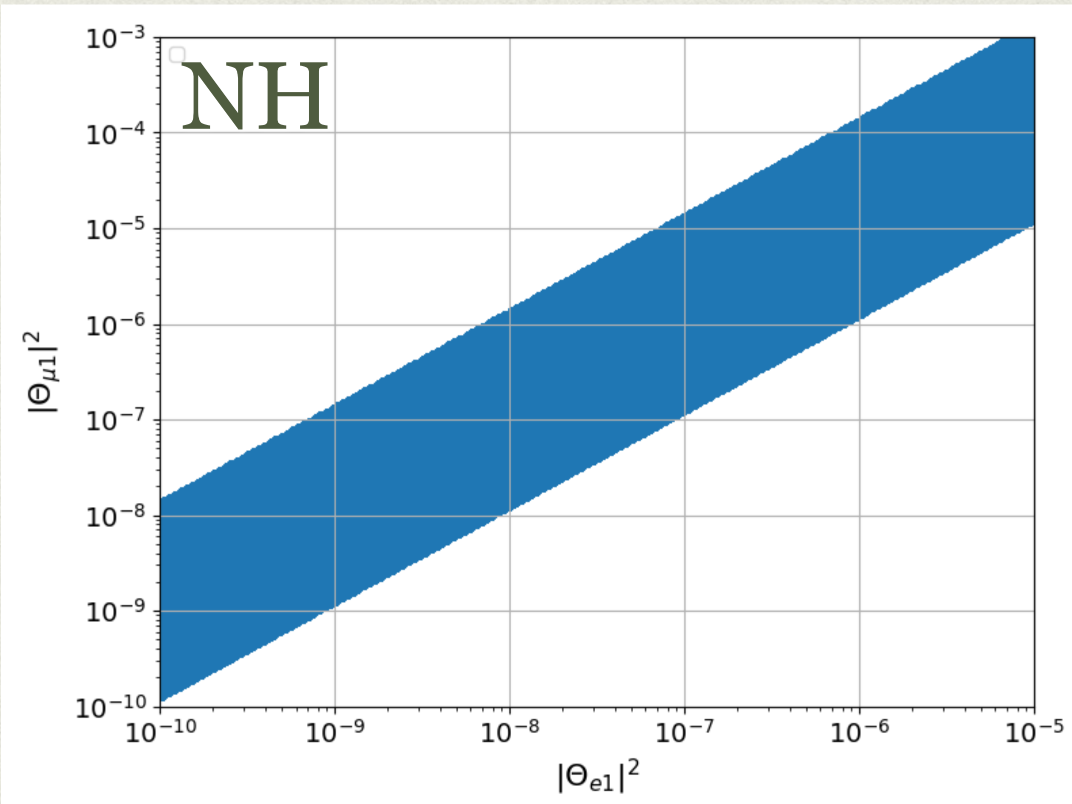


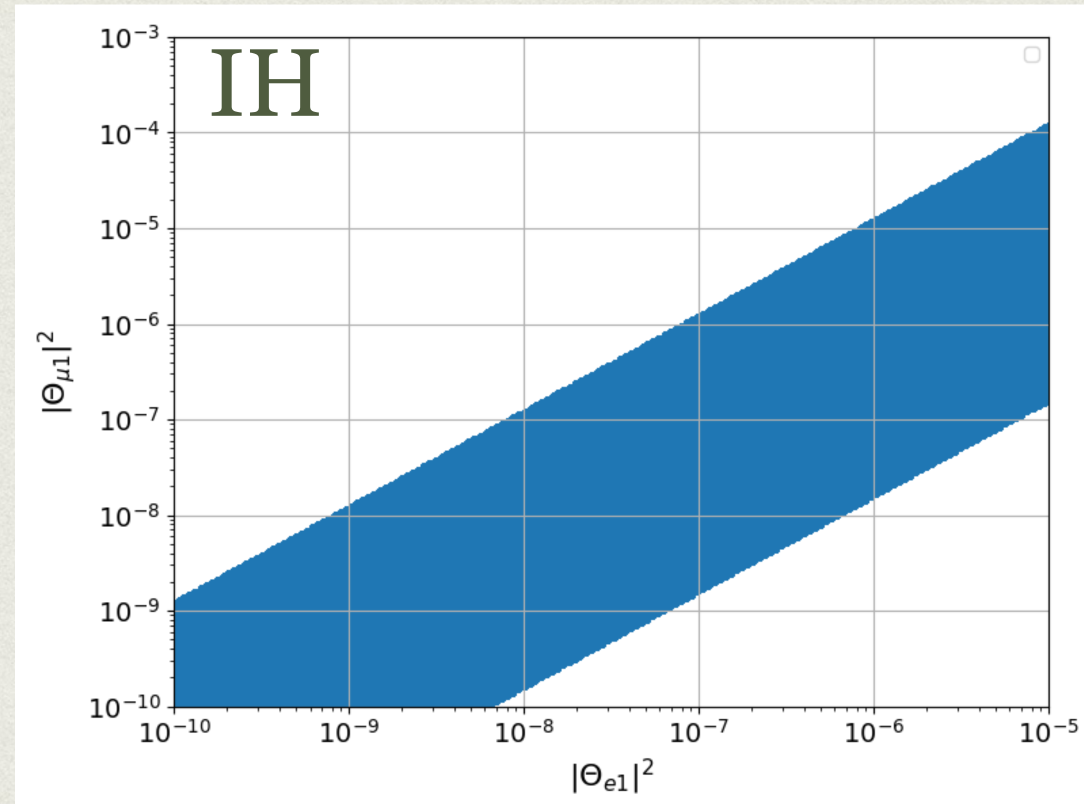


- $\Delta L = \pm 2$ processes
- The main contribution is proportional to $|\Theta_\ell|^4 (\ell = e, \mu)$
- $e^{\pm}e^{\pm} \to W^{\pm}W^{\pm}$ is restricted by stringent constraint on Θ_e from $0\nu 2\beta$ decay [Asaka, Tsuyuki(2015)]
- As for $\mu^{\pm}\mu^{\pm} \to W^{\pm}W^{\pm}$, such a constraint to Θ_μ might be avoided and cross section might be large
- However, Θ_e and Θ_μ are not independent free parameters in the seesaw mechanism

Correlation between Θ_e and Θ_μ

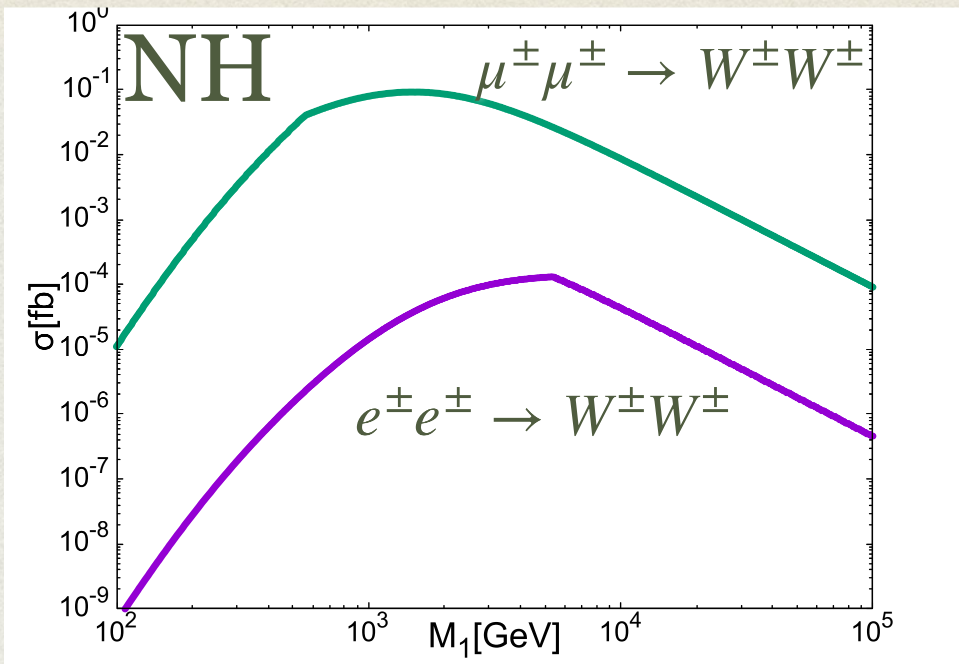
- Assume: $M_1 \ll M_2$

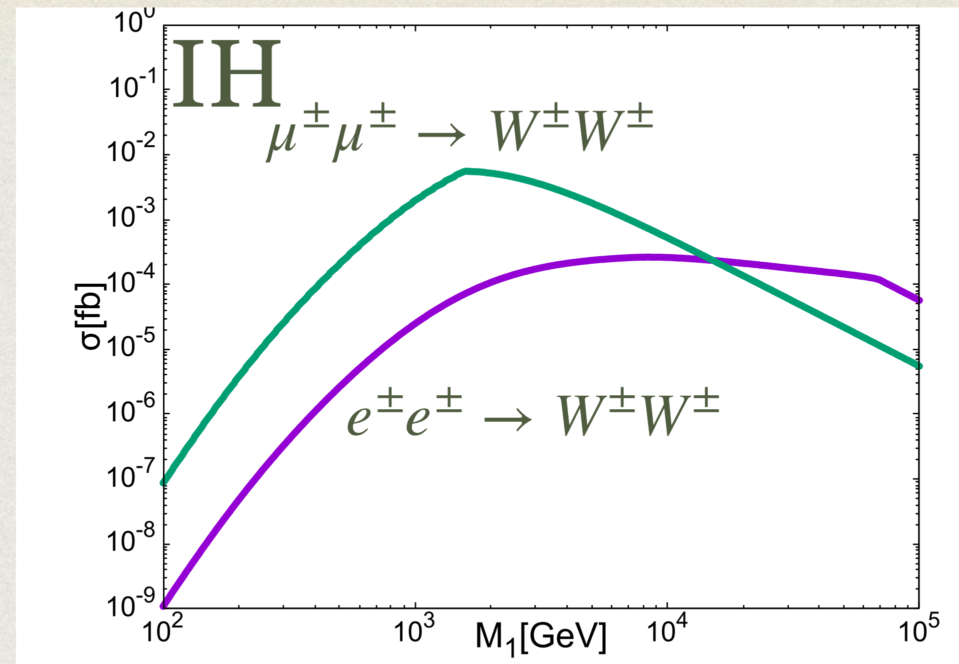




- In the seesaw mechanism, Θ_e and Θ_μ are related to each other
- In the NH, $|\Theta_{\mu 1}|^2/|\Theta_{e1}|^2 \sim 1 - 10^2$
- In the IH, $|\Theta_{\mu 1}|^2/|\Theta_{e1}|^2 \sim 10^{-2} - 10$

Upper bounds on cross sections





Constraints

1. $0\nu2\beta$ decay : $m_{\rm eff} < 122$meV [KamLAND-Zen(2024)]
2. EWPM : upper bounds on $|\Theta_{\alpha I}\Theta^*_{\beta I}|$ [M.Blennow et al.(2023)]

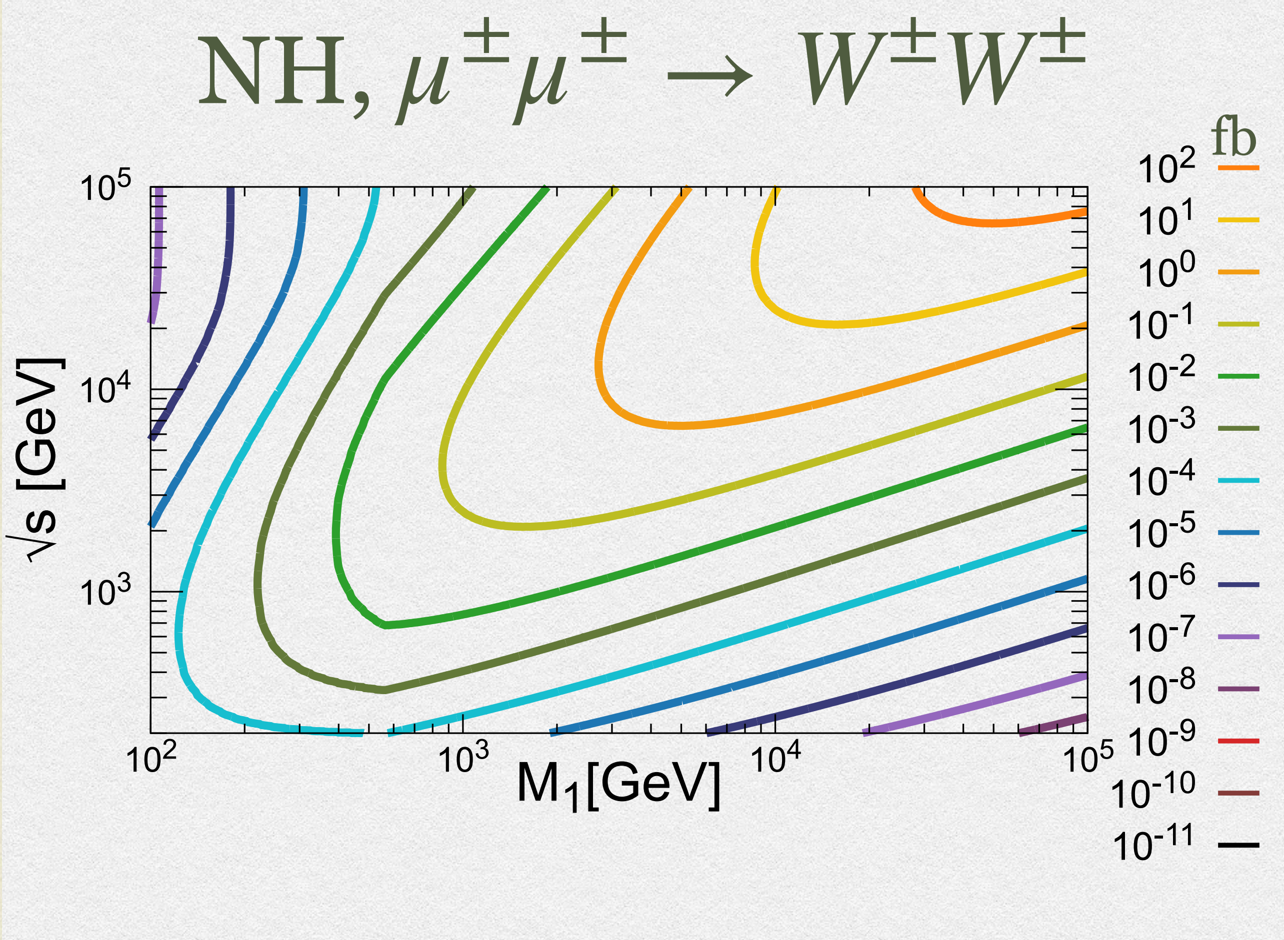
Conditions

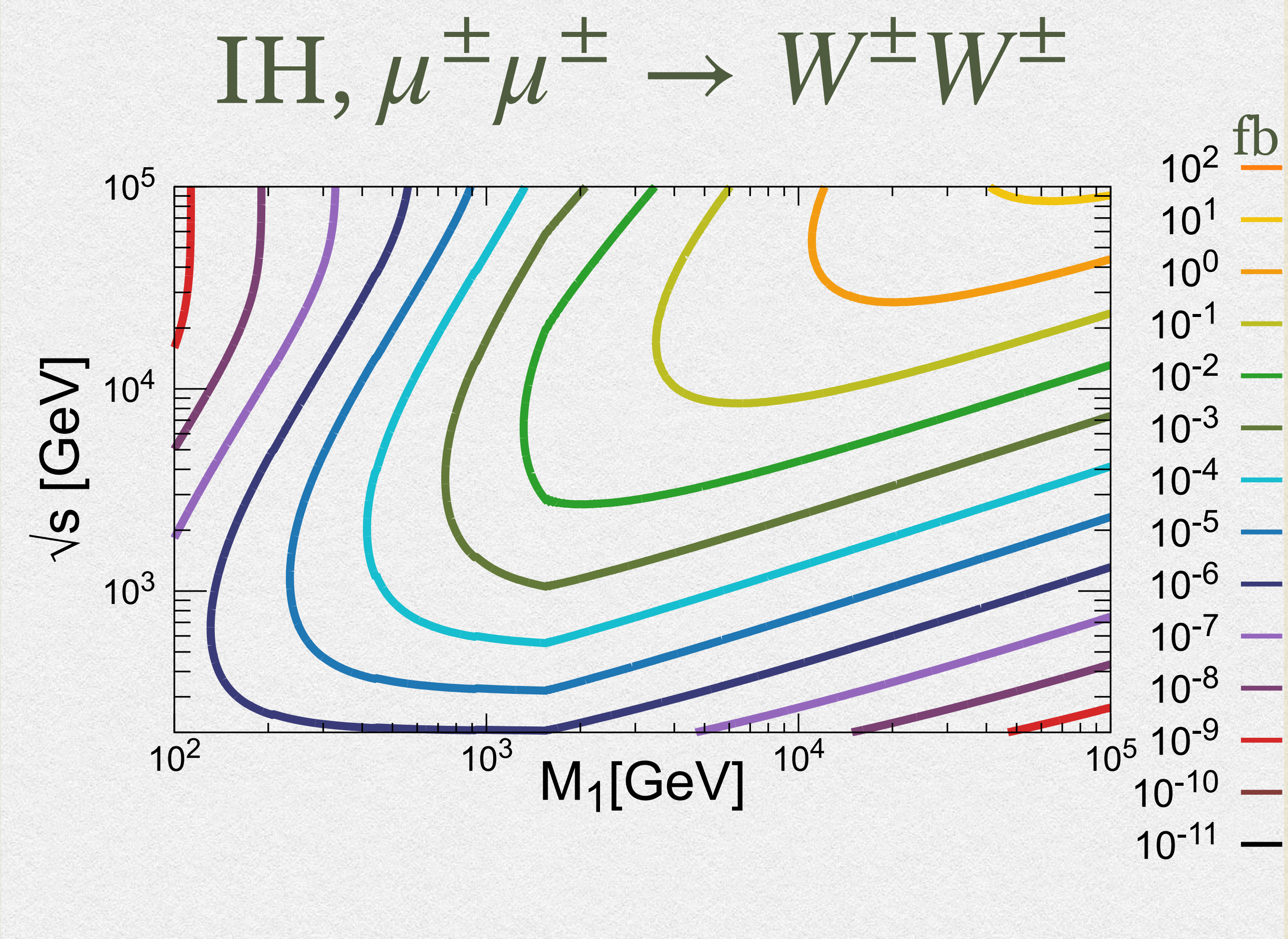
$\sqrt{s} = 2$TeV

$M_2 = 10M_1$

- Light M_1 region: $0\nu2\beta$ decay constraint is important
- Heavy M_1 region: EWPM constraints are important

Upper bounds on cross section





- Large cross section can be obtained for large $\sqrt{s}$ and M_1 with $\sqrt{s} \sim M_1$

Summary

- We considered the seesaw mechanism with two RHNs and investigated $e^{\pm}e^{\pm} \to W^{\pm}W^{\pm}$, and $\mu^{\pm}\mu^{\pm} \to W^{\pm}W^{\pm}$ when RHNs have TeV-scale masses

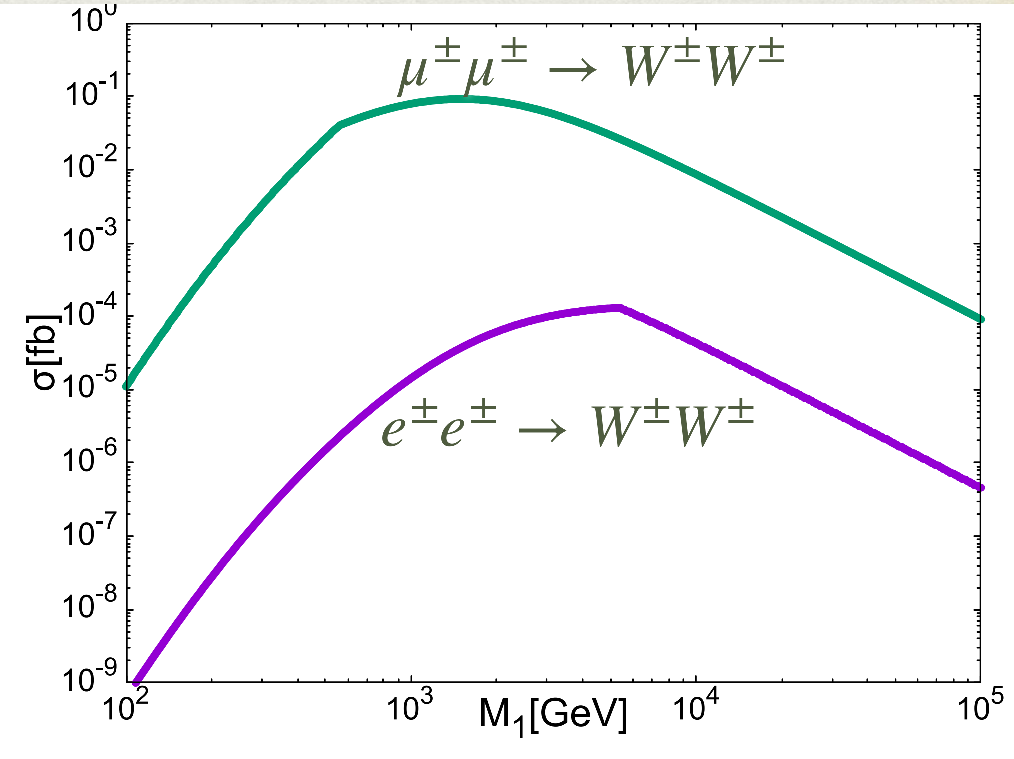
- We found

1. Not only Θ_e but also Θ_μ is constrained by $0\nu2\beta$ decay in the seesaw mechanism

2. The ratio $R_{\mu e} = \dfrac{\sigma(\mu^{\pm}\mu^{\pm} \to W^{\pm}W^{\pm})}{\sigma(e^{\pm}e^{\pm} \to W^{\pm}W^{\pm})}$ can be much lager then unitary and $\sigma(\mu^{\pm}\mu^{\pm} \to W^{\pm}W^{\pm})$ in NH can be larger than that in IH

Summary

	Center of mass energy	Luminosity
IMCC($\mu^+\mu^-$) [IMCC(2026)]	10 TeV	$1\text{ab}^{-1}/y$
μTRISTAN($\mu^+\mu^+$) [Y.Hamada et al.(2022)]	2 TeV	$0.02\text{ab}^{-1}/y$
ILC(e^+e^-) [K.Fujii et al.(2018)]	500 GeV	$0.12\text{ab}^{-1}/y$
CLIC(e^+e^-) [A.Robson et al.(2025)]	3 TeV	$0.7\text{ab}^{-1}/y$



$\mu^\pm\mu^\pm \to W^\pm W^\pm$ is a good target for probing TeV-scale RHNs in the seesaw mechanism

APPENDIX

The EWPM constraints

2N-SS	Normal Ordering		Inverted Ordering	
	68%CL	95%CL	68%CL	95%CL
$\eta_{ee} = \frac{\lvert\theta_e\rvert^2}{2}$	$6.4 \cdot 10^{-6}$	$9.4 \cdot 10^{-6}$	$[0.98, 4.4] \cdot 10^{-4}$	$5.5 \cdot 10^{-4}$
$\eta_{\mu\mu} = \frac{\lvert\theta_\mu\rvert^2}{2}$	$6.9 \cdot 10^{-5}$	$1.3 \cdot 10^{-4}$	$[0.20, 1.0] \cdot 10^{-6}$	$3.2 \cdot 10^{-5}$
$\eta_{\tau\tau} = \frac{\lvert\theta_\tau\rvert^2}{2}$	$8.6 \cdot 10^{-5}$	$2.1 \cdot 10^{-4}$	$[0.94, 2.8] \cdot 10^{-5}$	$4.5 \cdot 10^{-5}$
$\mathrm{Tr}\,[\eta] = \frac{\lvert\theta\rvert^2}{2}$	$1.6 \cdot 10^{-4}$	$2.9 \cdot 10^{-4}$	$[1.1, 4.8] \cdot 10^{-4}$	$6.0 \cdot 10^{-4}$
$\lvert\eta_{e\mu}\rvert = \frac{\lvert\theta_e\theta_\mu^*\rvert}{2}$	$8.3 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$	$[0.37, 1.0] \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$
$\lvert\eta_{e\tau}\rvert = \frac{\lvert\theta_e\theta_\tau^*\rvert}{2}$	$1.5 \cdot 10^{-5}$	$2.2 \cdot 10^{-5}$	$[0.25, 1.2] \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$
$\lvert\eta_{\mu\tau}\rvert = \frac{\lvert\theta_\mu\theta_\tau^*\rvert}{2}$	$7.2 \cdot 10^{-5}$	$1.3 \cdot 10^{-4}$	$[0.38, 3.0] \cdot 10^{-6}$	$3.5 \cdot 10^{-5}$

[M.Blennow et al.(2023)]

- The seesaw relation

$$0 = \sum_i U_{ei}^2 m_i + \sum_I \Theta_{\alpha I}^2 M_I$$

- The effective mass

$$m_{\rm eff} = \sum_i U_{ei}^2 m_i + \sum_I \Theta_{\alpha I}^2 M_I f_I(M_I)$$

$$f(M_I) \sim \frac{< p^2 >}{< p^2 > + M_I} \text{:Suppression factor, } \sqrt{< p^2 >} \sim \mathcal{O}(10^2 \text{MeV})$$

The cross section

- $$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2\beta_W}{32\pi}\Big[\,|A_t|^2 B_t + |A_u|^2 B_u + (A_t A_u^* + A_t^* A_u) B_{tu}\Big]$$

$$A_t = \sum_{i=1}^{3} \frac{U_{\ell i}^2\, m_i}{t - m_i^2} + \sum_{I=1}^{2} \frac{\Theta_{\ell I}^2\, M_I}{t - M_I^2},\; A_u = A_t\Big|_{t\to u}$$

$$B_t = (1 - 4\, r_W)\, t^2 - 4\, r_W (1 - 2\, r_W)\, s\, t + \, 4(1 - r_W)\, r_W^2\, s^2$$

$$B_u = B_t\Big|_{t\to u},\; B_{tu} = (1 - 4\, r_W)\, t\, u + 4\, r_W^3\, s^2$$

$$r_W = \frac{m_W^2}{s}$$