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Top-Yukawa coupling at muon collider

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The University of Osaka

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LHC searches and constraints

$$\mathcal{L}_{ttH} = -gH\bar{t}(\cos\xi + i\gamma_5 \sin\xi)t$$

g is real and positive, $-\pi < \xi < \pi$

When $g = g_{SM} = m_t/v$, $\xi = 0$ ⟶ SM

- X. Zhang, S. K. Lee, K. Whisnant, and B. L. Young, "Phenomenology of a nonstandard top quark Yukawa coupling," Phys. Rev. D 50 (1994) 7042–7047, arXiv:hep-ph/9407259.
- H. Bahl, E. Fuchs, S. Heinemeyer, J. Katzy, M. Menen, K. Peters, M. Saimpert, and G. Weiglein, "Constraining the CP structure of Higgs-fermion couplings with a global LHC fit, the electron EDM and baryogenesis," Eur. Phys. J. C 82 (2022) no. 7, 604, arXiv:2202.11753 [hep-ph].
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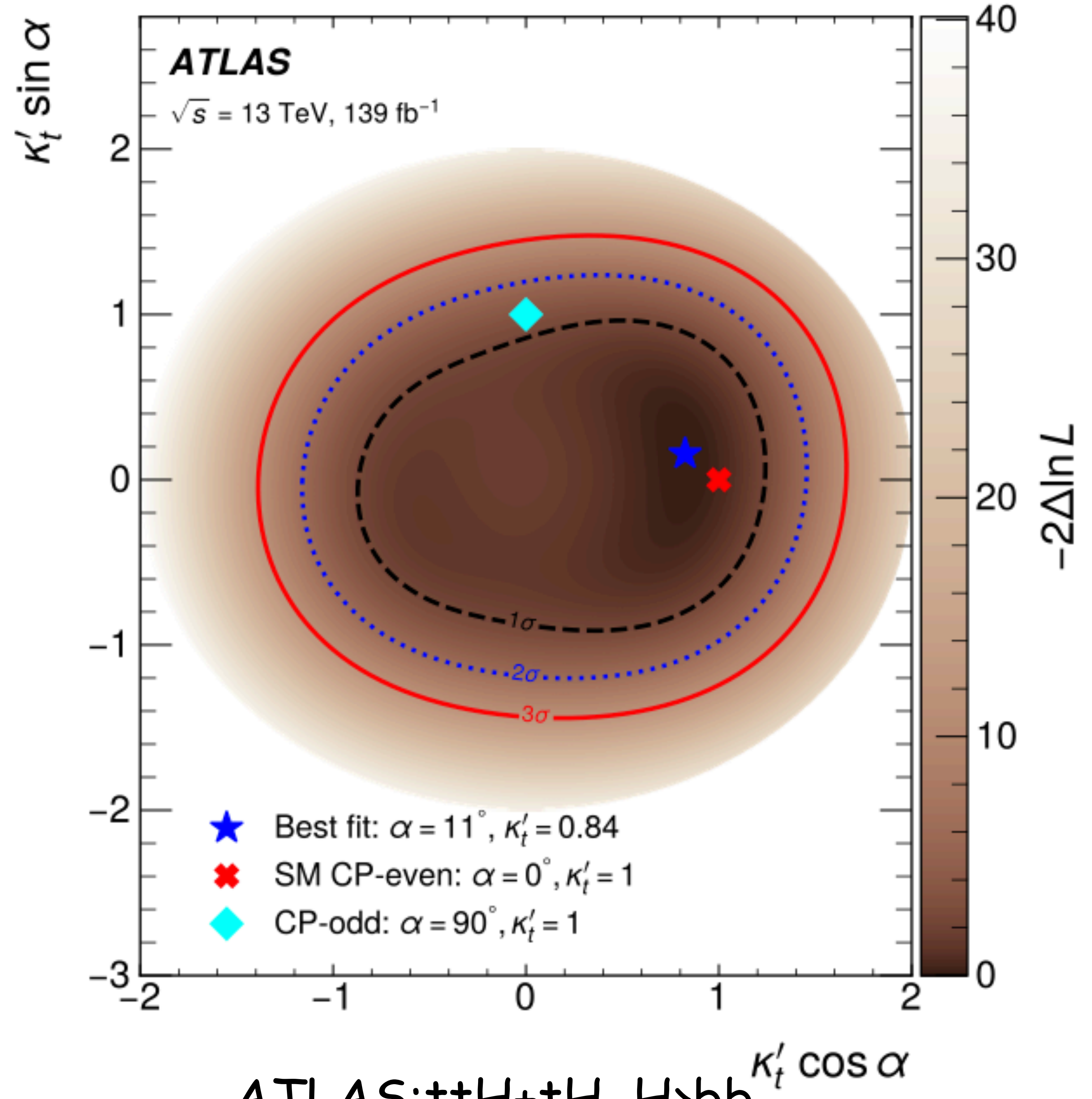
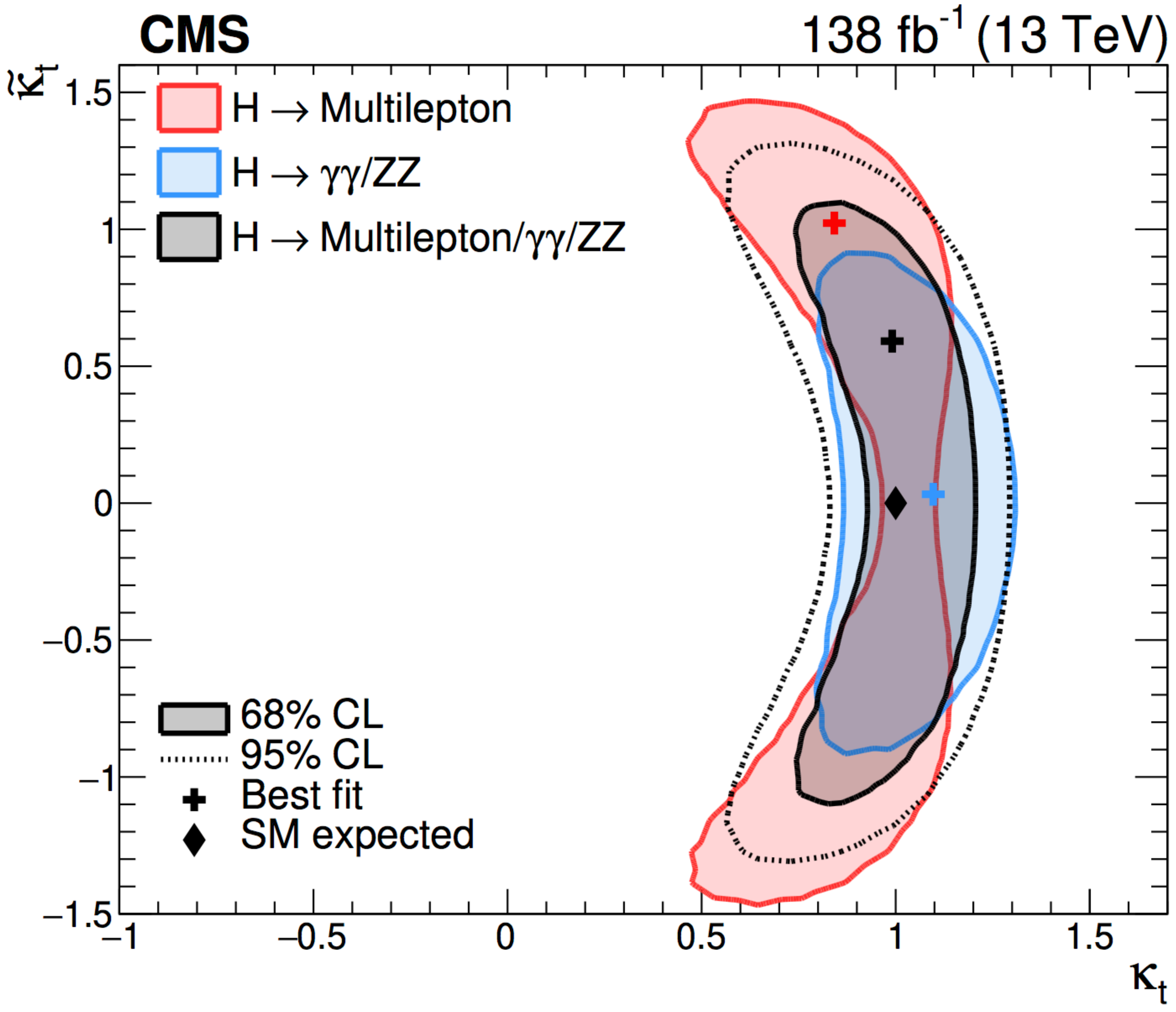


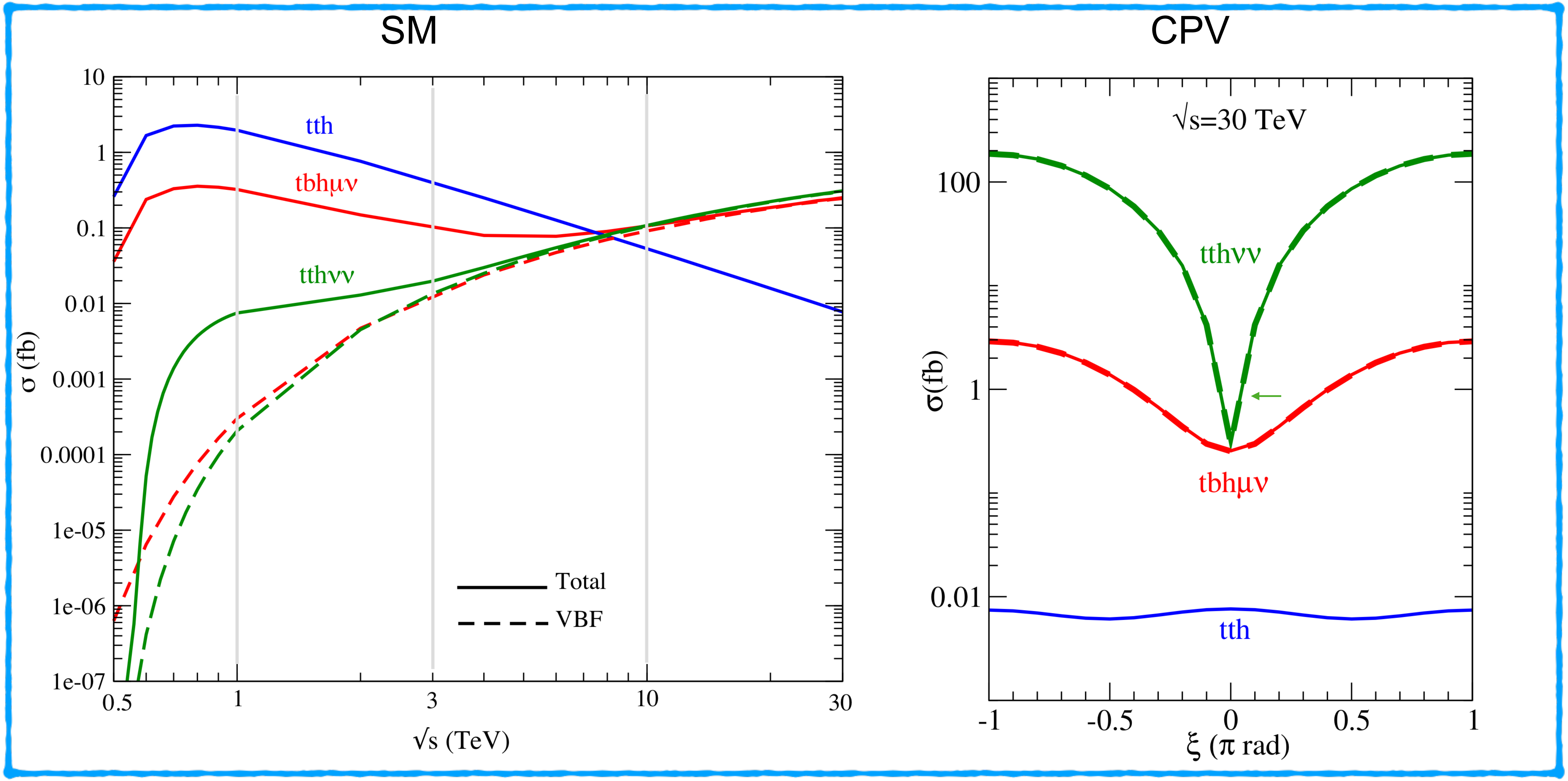
ATLAS:ttH+tH, H>bb
arXiv:2303.05974



CMS:ttH+tH, H>ls/rr/ZZ
JHEP07(2023)092

LHC direct searches:
ATLAS best fit:11+52-73 degree

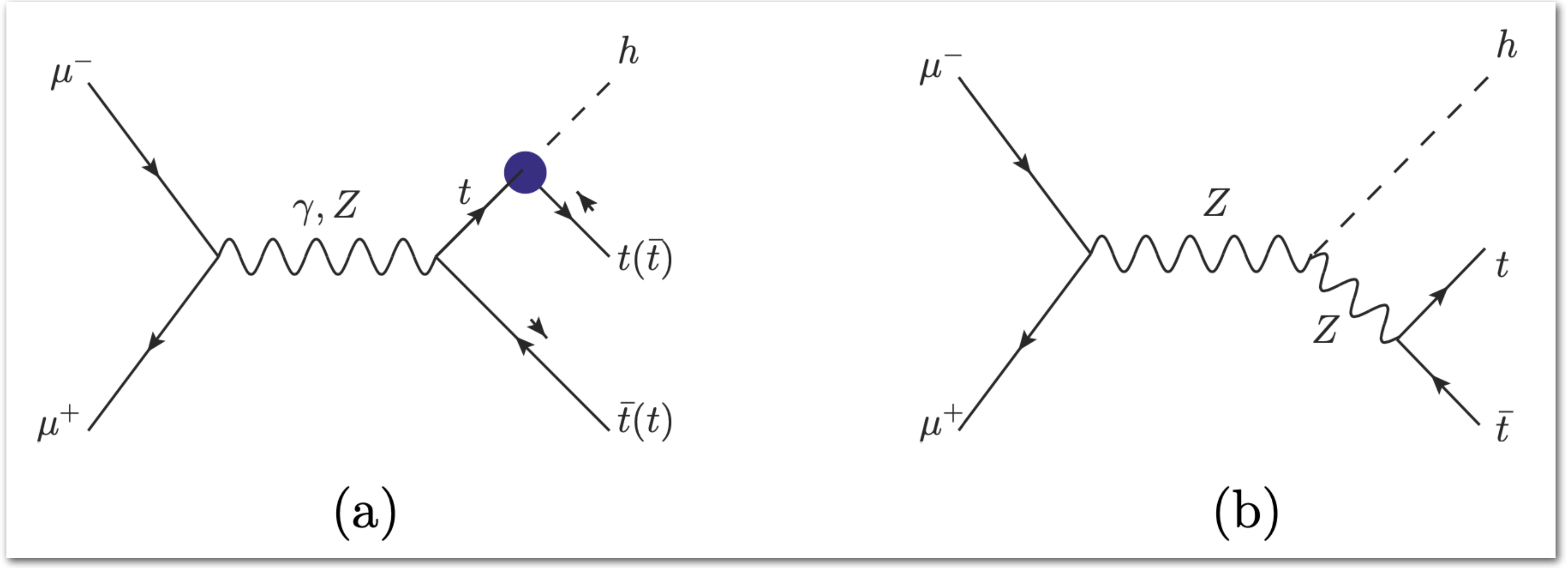
Top Yukawa processes at muon collider



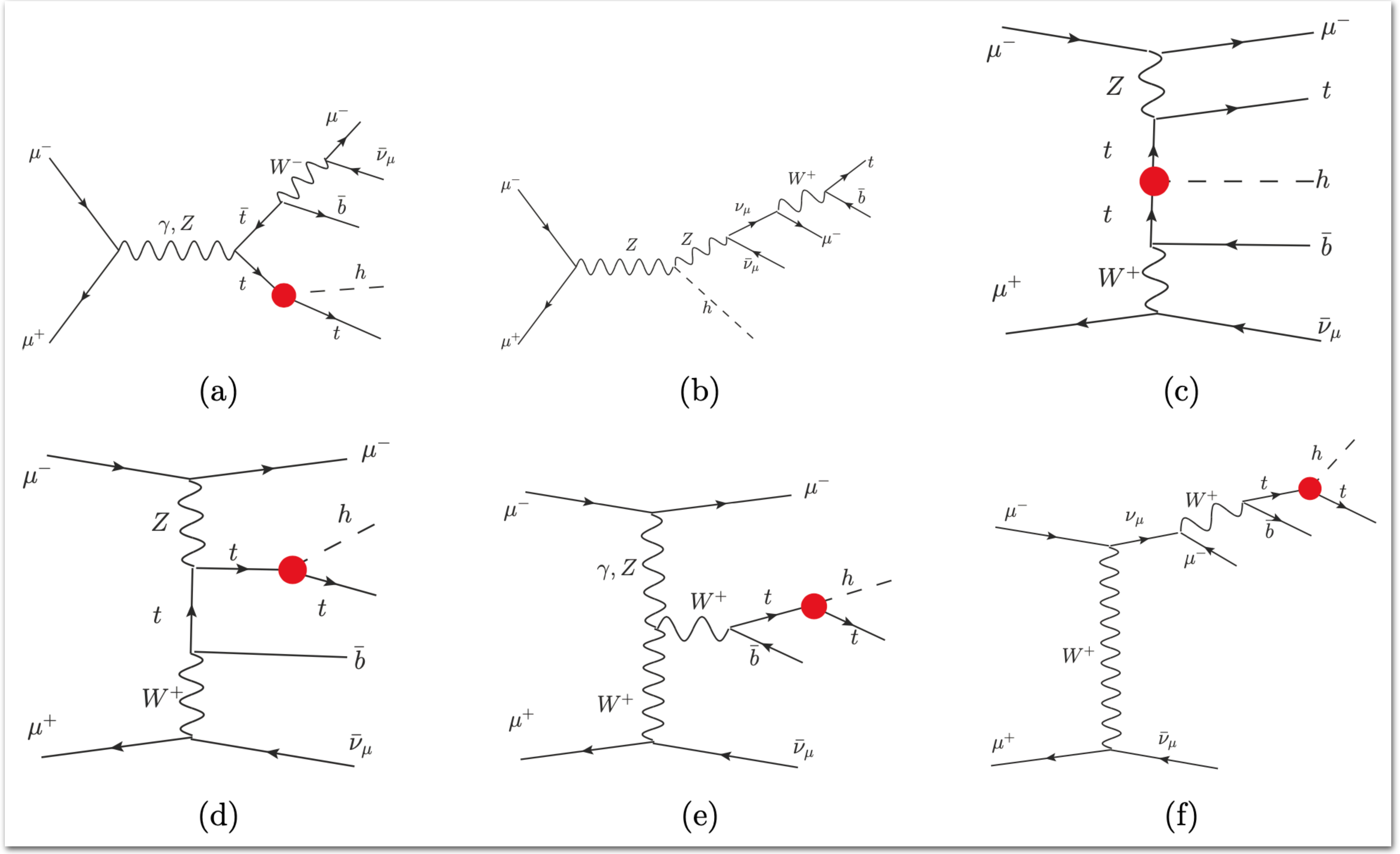
M.Cassidy, Z.Dong, KC.Kong, I. Lewis, Y.Zhang and YJZ, JHEP 05 (2024) 176.

Representative Feynman diagrams

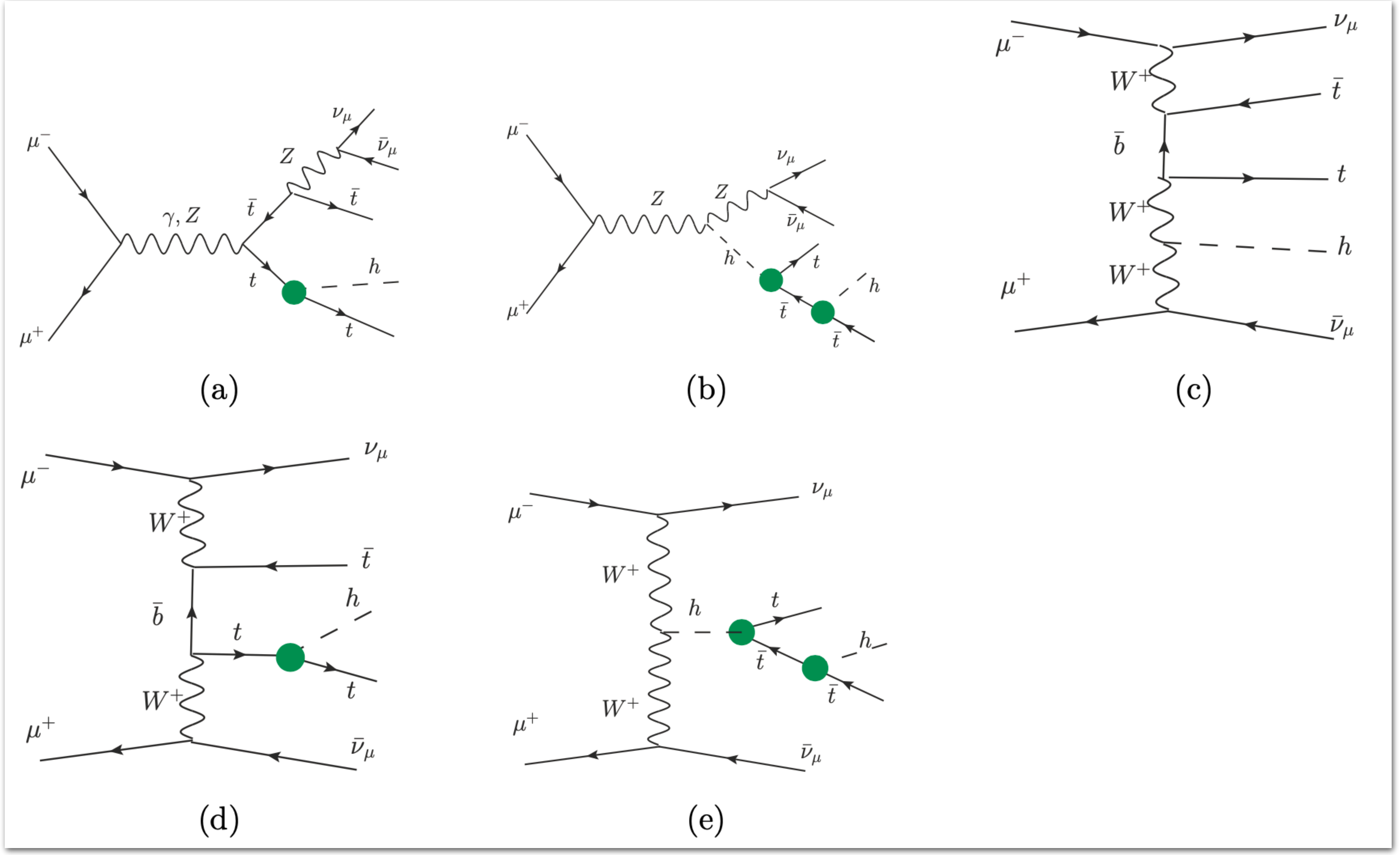
tth



tbhμν

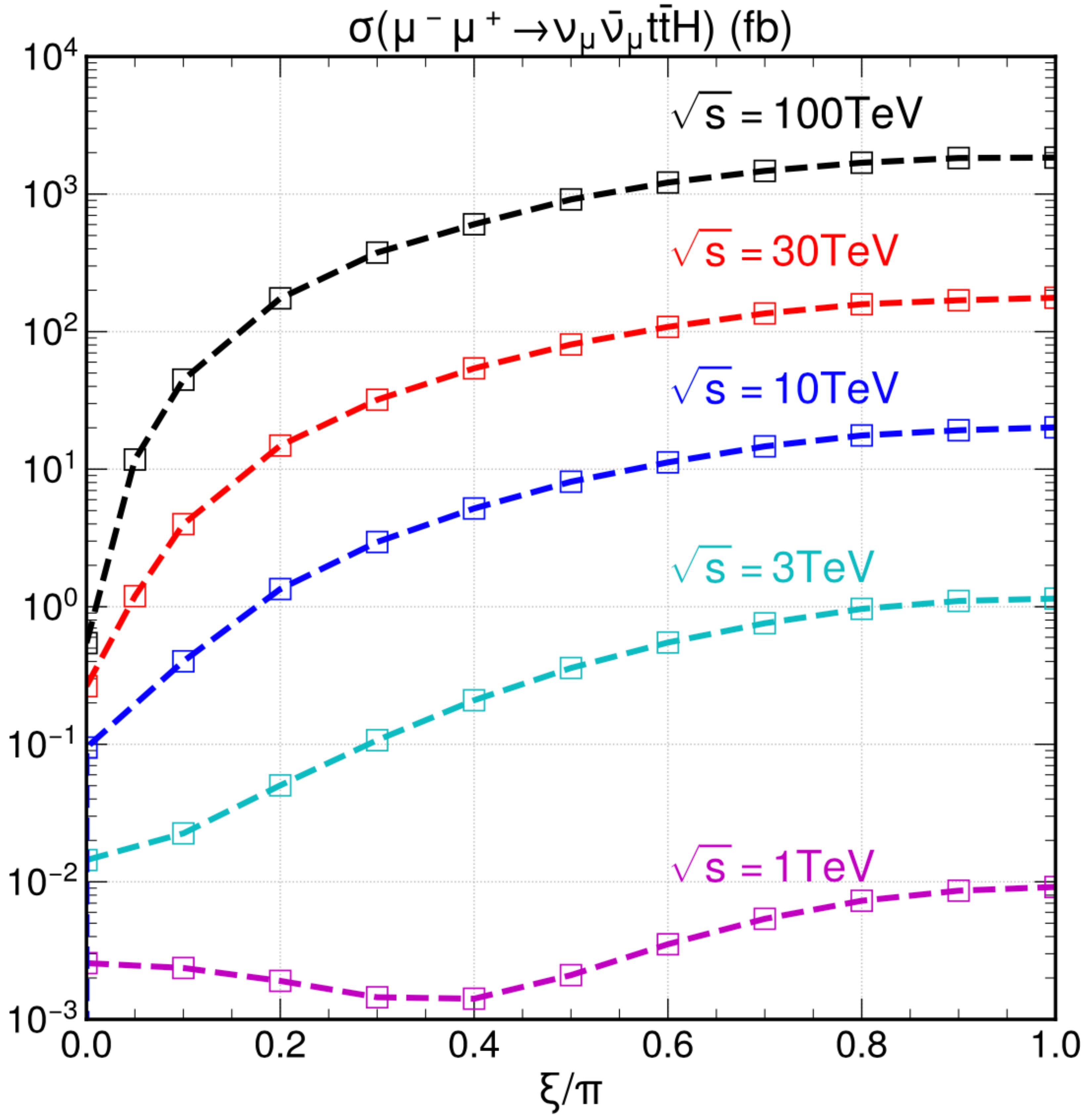


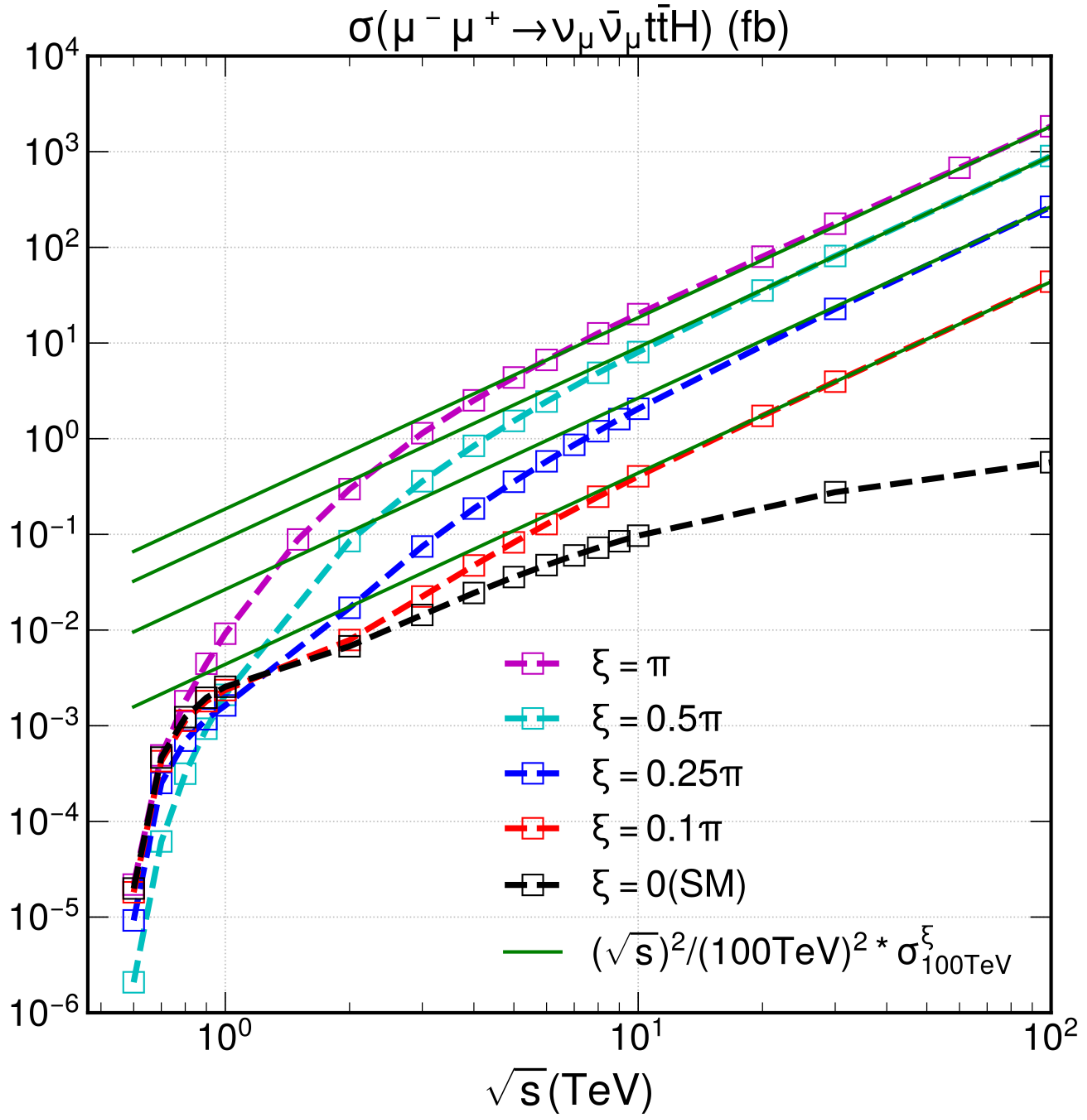
tthνν

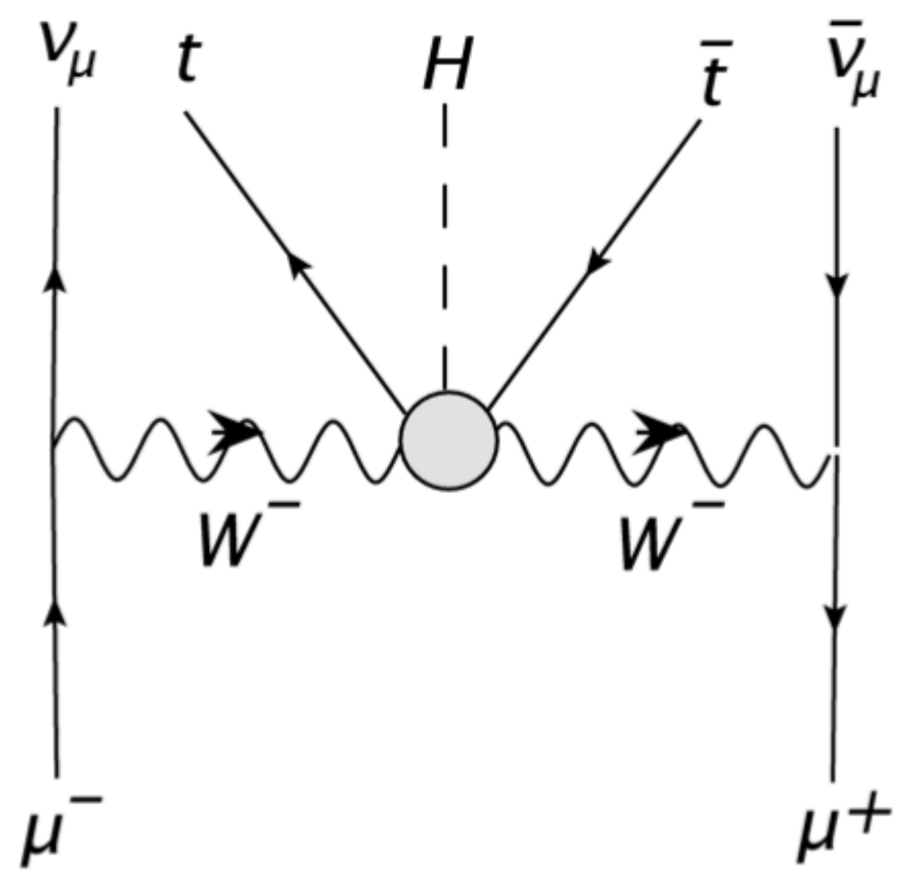


M.Cassidy, Z.Dong, KC.Kong, I. Lewis, Y.Zhang and YJZ, JHEP 05 (2024) 176.

$\nu_\mu \bar{\nu}_\mu t\bar{t}H$ at future muon collider







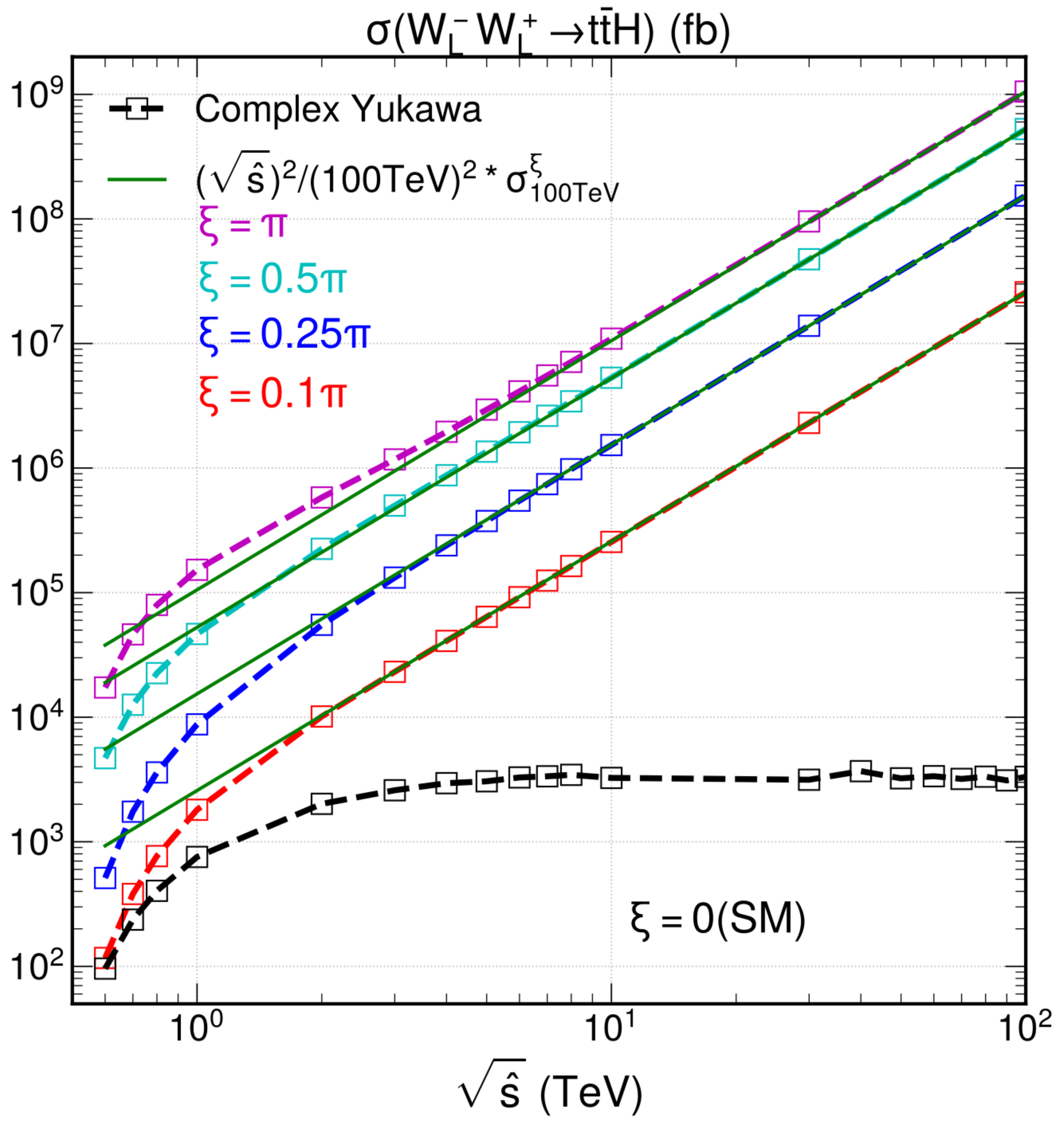
88 diagrams generated from Madgraph5, 20 diagrams are Vector Boson Fusions

ξ dependence:
at low energy: s-channel diagrams contribute.
at high energy: t-channel (VBF) dominates.

Energy dependence at high energy:
BSM: quadratic growth
SM: logarithmically growth

Vector Boson Fusion

Dominate sub-diagrams $W_L^- W_L^+ \to t\bar{t}H$



$$W^-(q, h=0)\ W^+(\bar{q}, \bar{h}=0) \to t\bar{t}H$$

$\xi \neq 0$ **quadratic energy growth** from the longitudinally polarized weak boson wave functions **E/mW**.

$\xi = 0$ In the SM, **E/mW** from individual diagram **cancels** after summing up, leading to the **Goldstone boson equivalence theorem** (GBET) as a manifestation of gauge invariance.

$$\mathcal{L}_{ttH} = -gH\bar{t}(\cos\xi + i\gamma_5 \sin\xi)t$$

Gauge invariant formulation: Models like two Higgs doublet models,etc

Energy dependence at high energy:
BSM: quadratic growth from (**E/mW)², ** with E=$\sqrt{\hat{s}}/2$
SM: constant

SMEFT

A gauge invariant top Yukawa sector

Dimension-6 operator

$$\mathcal{L} = -y_{\rm SM} Q^\dagger \phi t_R + \frac{\lambda}{\Lambda^2} Q^\dagger \phi t_R \left(\phi^\dagger \phi - \frac{v^2}{2} \right) + \text{h.c.}$$

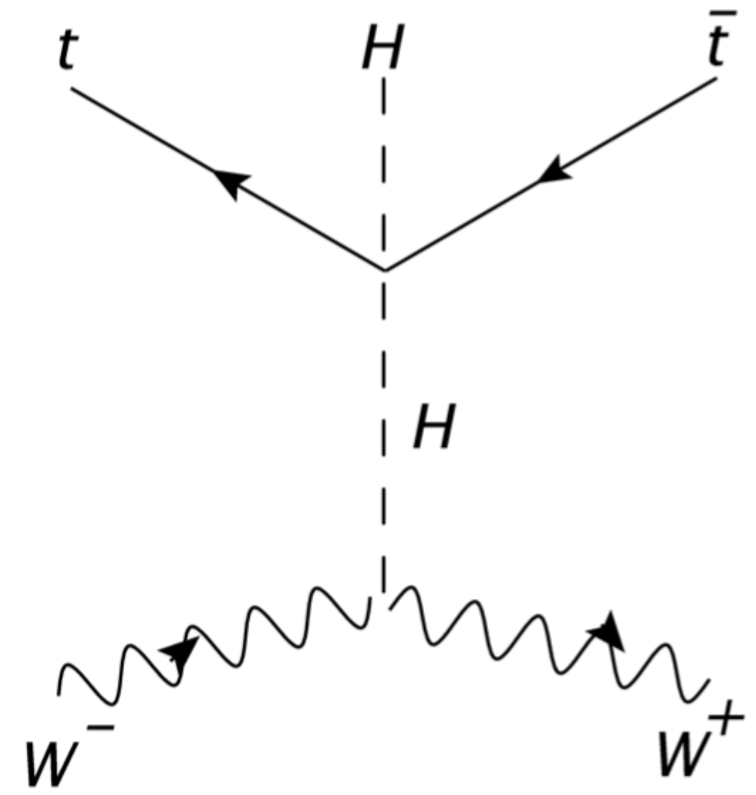
$$Q = (t_L, b_L)^T$$

$$\phi = ((v + H + i\pi^0)/\sqrt{2}, i\pi^-)^T$$

$$\begin{aligned}\mathcal{L}_{ttH}^{\rm SMEFT} = & -m_t t_L^\dagger t_R - g_{\rm SM} \left[(H + i\pi^0) t_L^\dagger + i\sqrt{2}\pi^- b_L^\dagger \right] t_R \\ & -(g e^{i\xi} - g_{\rm SM}) \left\{ H t_L^\dagger t_R + \frac{H}{v} \left[(H + i\pi^0) t_L^\dagger + i\sqrt{2}\pi^- b_L^\dagger \right] t_R \right\} \\ & -(g e^{i\xi} - g_{\rm SM}) \left\{ \left[\frac{H^2 + (\pi^0)^2}{2v} + \frac{\pi^+\pi^-}{v} \right] t_L^\dagger t_R \right. \\ & \left. + \frac{H^2 + (\pi^0)^2 + 2\pi^+\pi^-}{2v^2} \left[(H + i\pi^0) t_L^\dagger + i\sqrt{2}\pi^- b_L^\dagger \right] t_R \right\} + h.c.,\end{aligned}$$

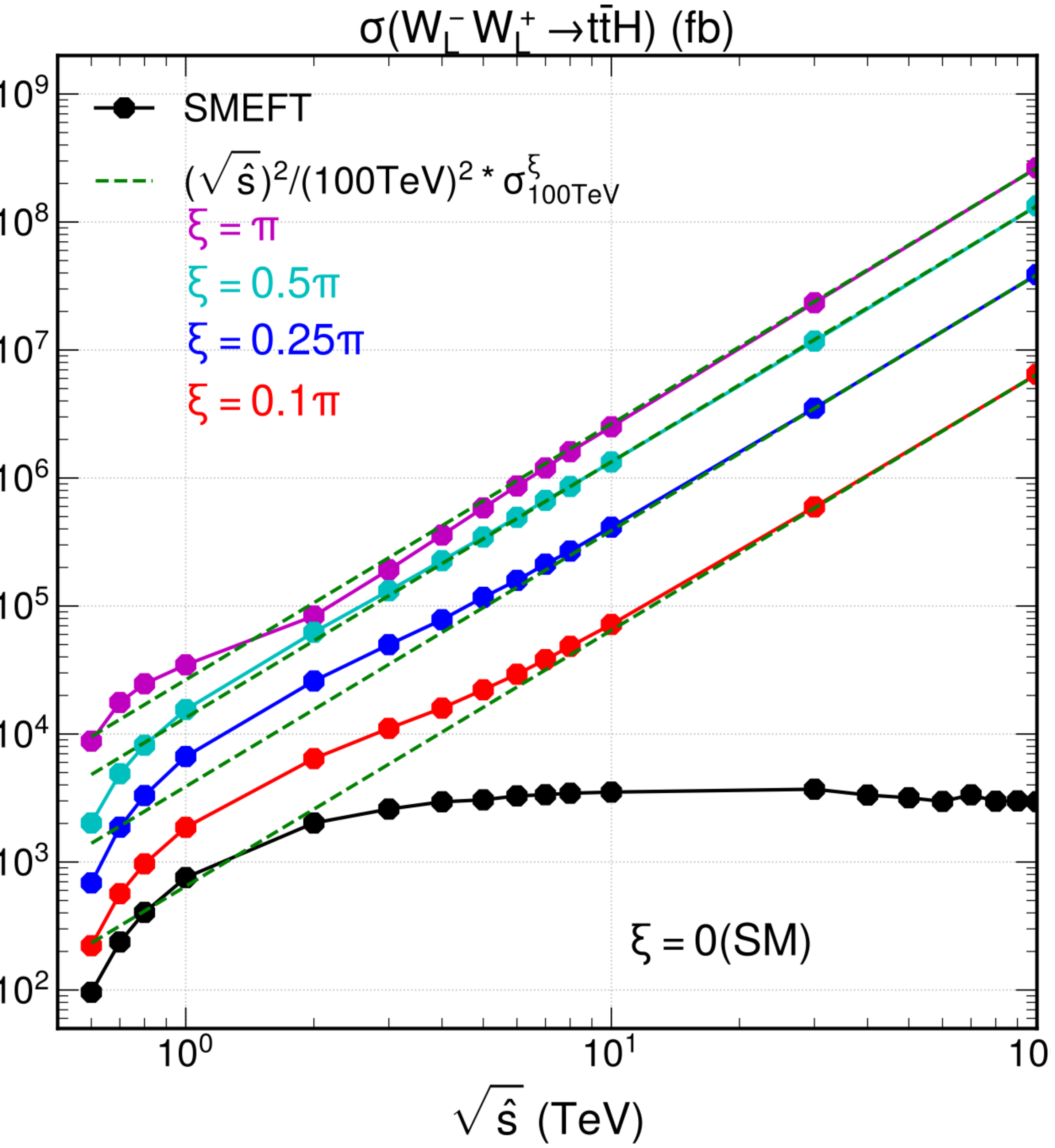
$$g_{\rm SM} = \frac{y_{\rm SM}}{\sqrt{2}} = \frac{m_t}{v} \qquad \frac{g_{\rm SM} - g e^{i\xi}}{v^2} = \frac{\lambda}{\Lambda^2}$$

Additional ttHH and ttHHH coupling

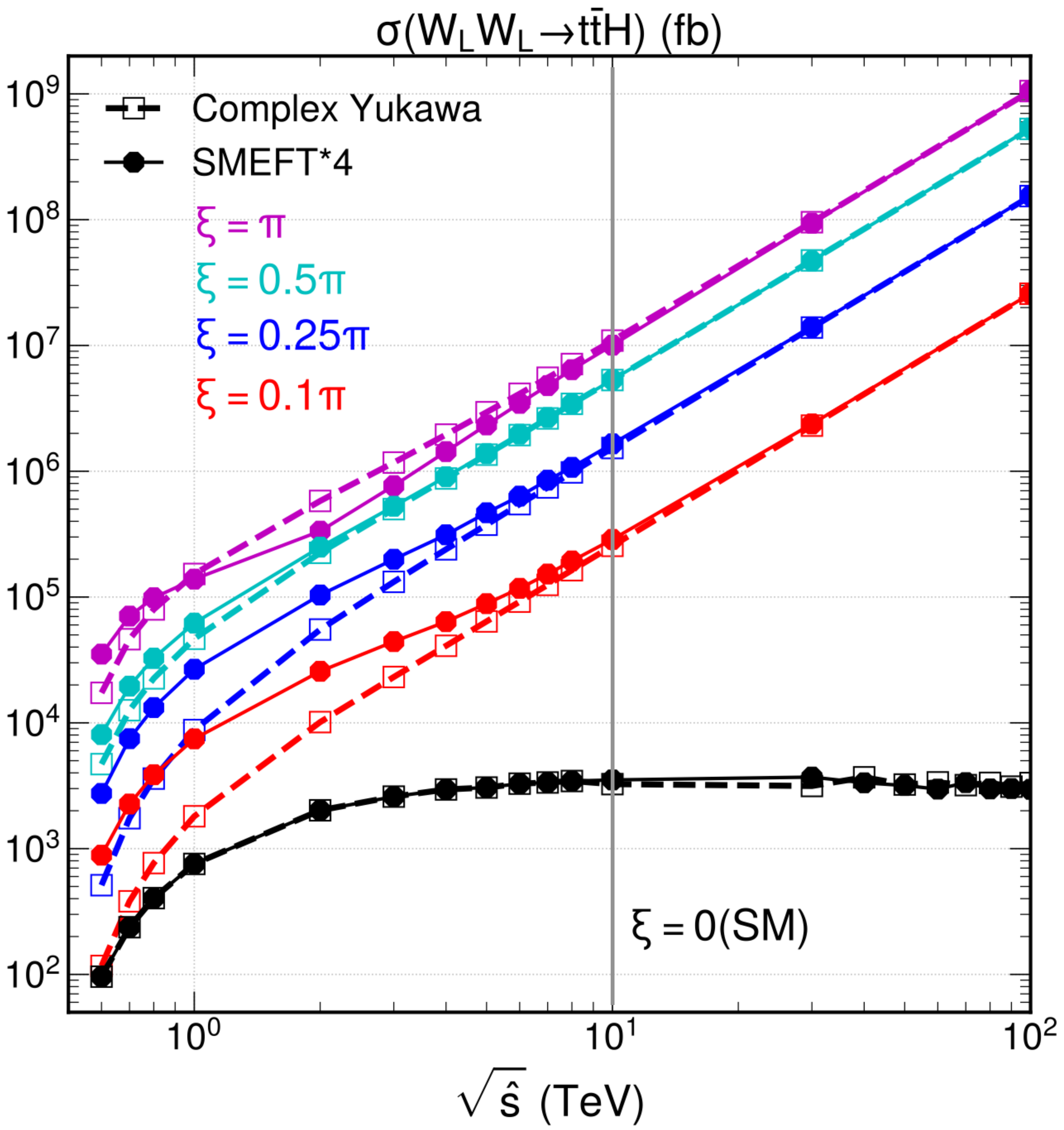


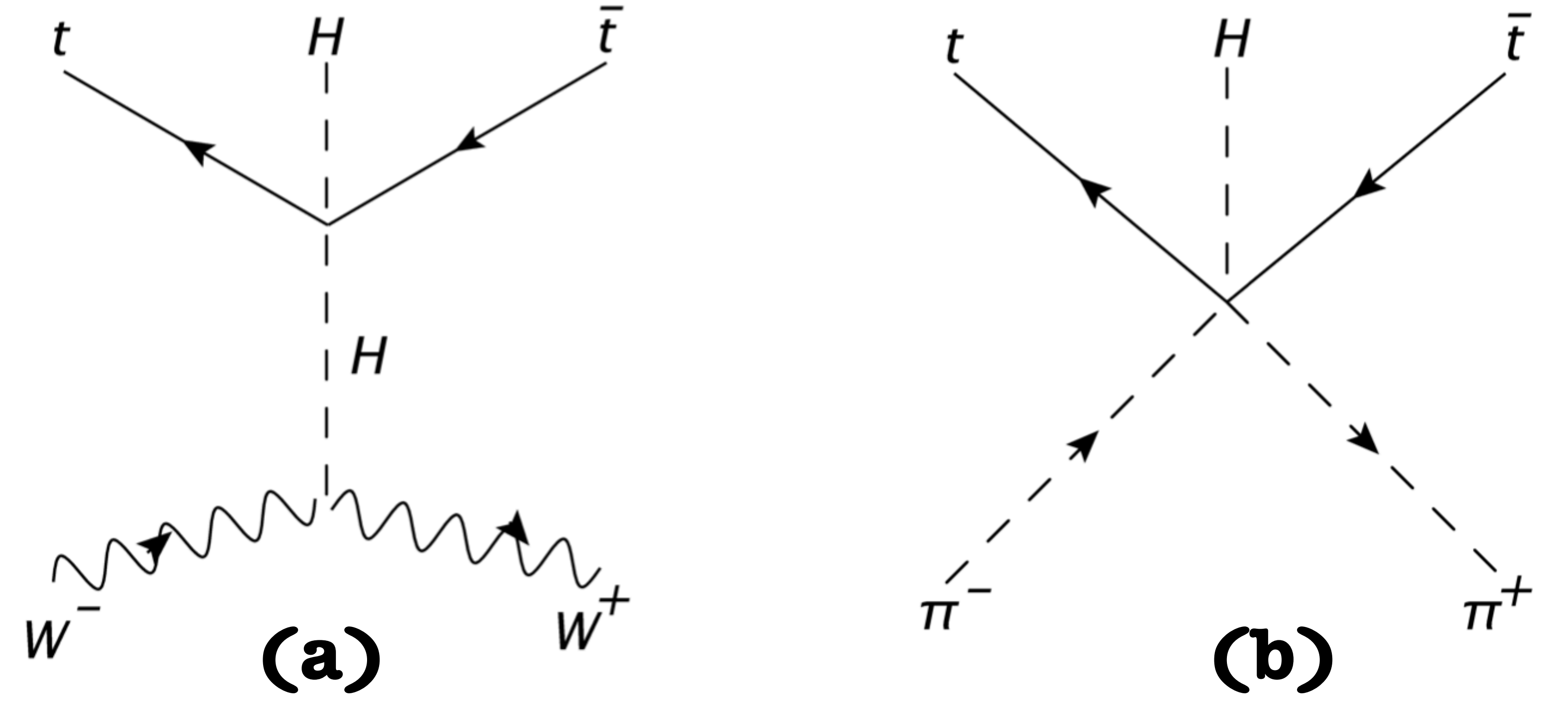
$$\mathcal{L}_{ttHH}^{\rm SMEFT} = \frac{3(g_{\rm SM} - g e^{i\xi})}{v} \frac{H^2}{2} t_L^\dagger t_R + \text{h.c.}$$

$W_L^- W_L^+ \to t\bar{t}H$



above 10 TeV





Complete amplitude in SMEFT:

$$\mathcal{M}(W_L^- W_L^+ \to t\bar{t}H)_{\rm SMEFT} = \sum_{k=1}^{20} \mathcal{M}_k + \mathcal{M}_{\text{Fig.}\,\mathbf{a}}$$

$$\mathcal{M}^{\pm\pm}_{\text{Fig.}\,\mathbf{b}} = \frac{1}{v^2}\left[\mp 2p_t\left(g_{\rm SM} - g\cos\xi\right) - i m_{tt}\left(g\sin\xi\right)\right]$$

$$\mathcal{M}^{\pm\pm}_{\text{Fig.}\,\mathbf{a}} = \frac{3}{v^2}\left[\mp 2p_t(g_{\rm SM} - g\cos\xi) - i m_{tt}(g\sin\xi)\right]\frac{(\hat{s} - 2m_W^2)}{(\hat{s} - m_H^2)};$$

$$\mathcal{M}^{\pm\pm}_{\text{Fig.}\,\mathbf{a}} = 3\mathcal{M}^{\pm\pm}_{\text{Fig.}\,\mathbf{b}} \cdot \left\{1 + \mathcal{O}\left(\frac{1}{\hat{s}}\right)\right\}$$

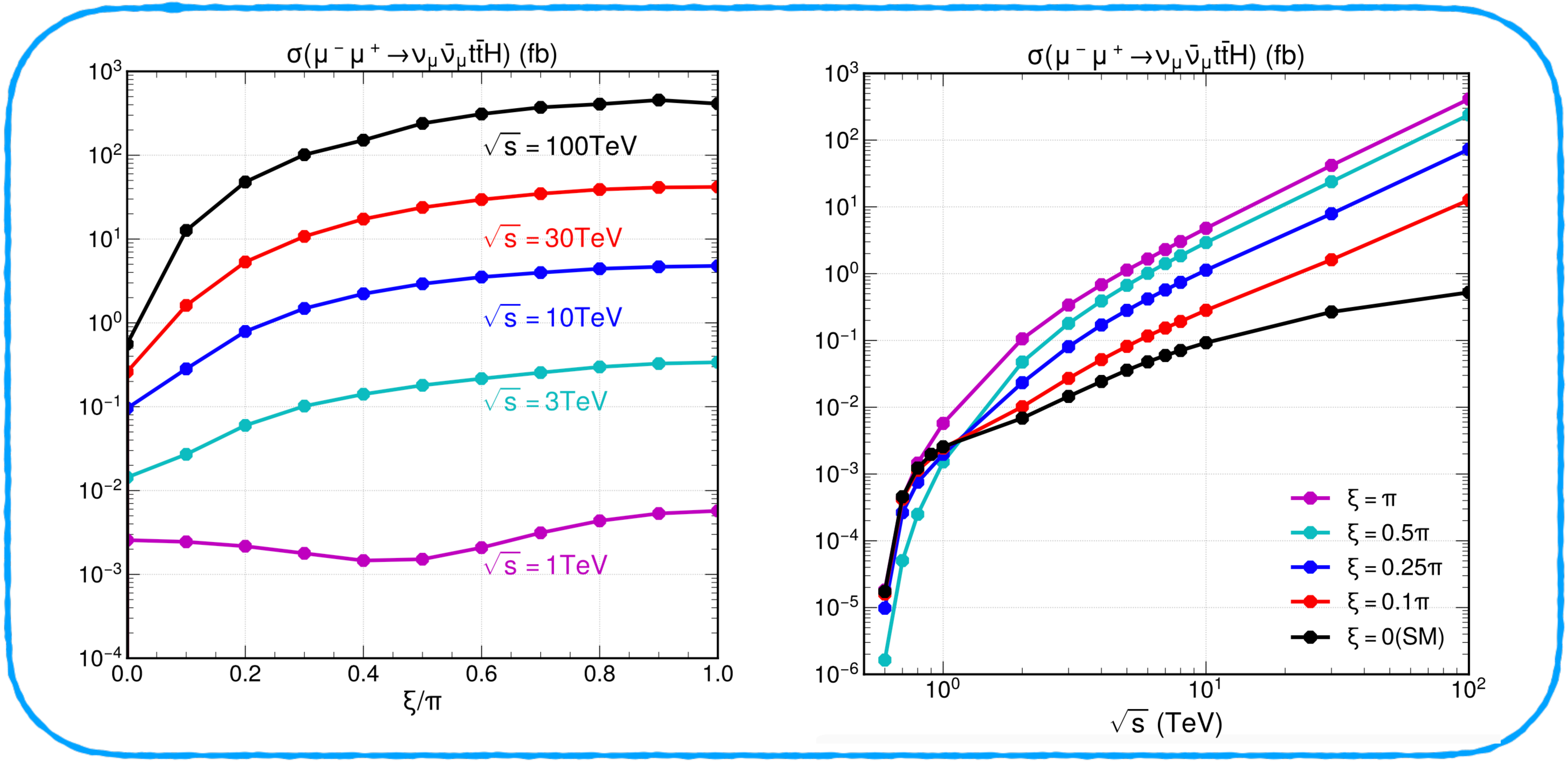
GBET tells:

$$\sum_{k=1}^{20} \mathcal{M}_k + \mathcal{M}_{\text{Fig.}\,\mathbf{a}} = \mathcal{M}_{\text{Fig.}\,\mathbf{b}} \cdot \left\{1 + \mathcal{O}\left(\frac{1}{\hat{s}}\right)\right\}$$

$$\sum_{k=1}^{20} \mathcal{M}_k \approx -2\mathcal{M}_{\text{Fig.}\,\mathbf{b}}$$

$$\sigma_{\rm tot}(W_L^- W_L^+ \to ttH)_{\rm SMEFT} \approx \frac{1}{4}\sigma_{\rm tot}(W_L^- W_L^+ \to ttH)_{\text{complex Yukawa}}$$

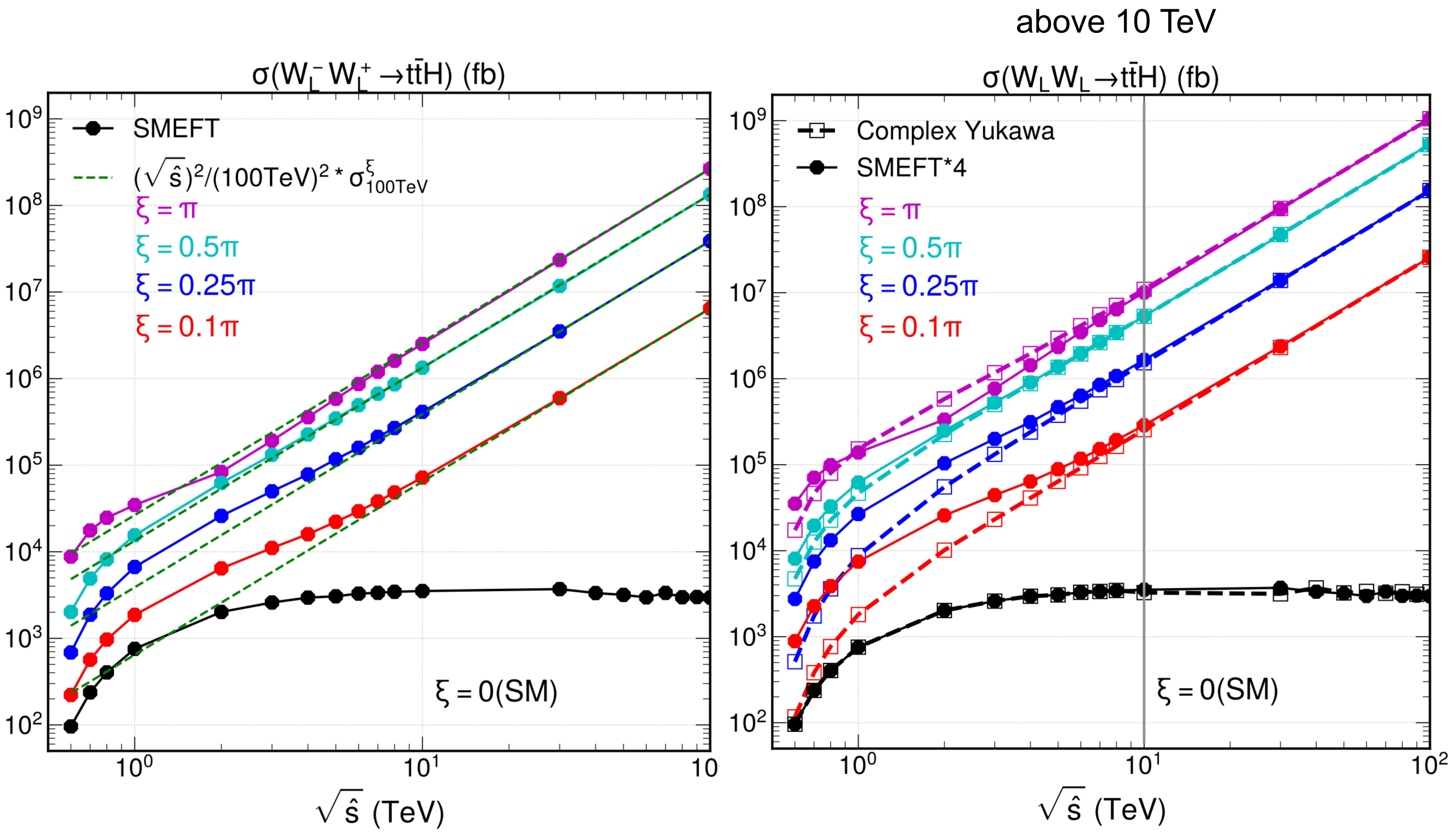
$\mu^-\mu^+ \to \nu_\mu \bar{\nu}_\mu t\bar{t}H$ in SMEFT



V.Barger, K.Hagiwara, YJZ, PLB850 (2024) 138547.

Questions Unanswered

Why the high energy behavior of the WW→Htt cross section in the original complex Yukawa model is 4 times that of the SMEFT?



There may be a gauge invariant representation of the complex Yukawa coupling with 2 times the π+π-Htt GB coupling of SMEFT.

Going to dimension-8?

$$\mathcal{L}_{eff}^{ttH} = Q^\dagger \phi t_R \left\{ -y_{\rm SM} + \frac{\lambda_6}{\Lambda^2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right) + \frac{\lambda_8}{\Lambda^4}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 + \ldots \right\} + h.c.$$

$$= -\frac{\mathbf{v} y_{\rm SM}}{\sqrt{2}} t_L^\dagger t_R - \left(\frac{y_{\rm SM}}{\sqrt{2}} - \frac{\lambda_6 v^2}{\sqrt{2}\Lambda^2}\right) H t_L^\dagger t_R$$

$$+ \left(\frac{3\lambda_6 v}{2\sqrt{2}\Lambda^2} + \frac{\lambda_8 v^3}{\sqrt{2}\Lambda^4}\right) H^2 t_L^\dagger t_R + \cdots \quad = 0 \rightarrow \textbf{Scenario 2}$$

$$+ \frac{\lambda_6 v}{\sqrt{2}\Lambda^2}\left(\pi^+\pi^- + \frac{(\pi^0)^2}{2}\right) t_L^\dagger t_R + \cdots$$

$$+ \left(\frac{\lambda_6}{\sqrt{2}\Lambda^2} + \frac{2\lambda_8 v^2}{\sqrt{2}\Lambda^4}\right) \pi^+\pi^- H t_L^\dagger t_R + \cdots + h.c. \quad = 0 \rightarrow \textbf{Scenario 3}$$

dim. of SMEFT operator	Dim-4	Dim-6	Dim-8	
Scenario 1	$y_{\rm SM}$	$y_{\rm SM} - y e^{i\xi}$	Negelible	SMEFT
Scenario 2	$y_{\rm SM}$	$y_{\rm SM} - y e^{i\xi}$	$-3/2(y_{\rm SM} - y e^{i\xi})$	HEFT
Scenario 3	$y_{\rm SM}$	$y_{\rm SM} - y e^{i\xi}$	$-1/2(y_{\rm SM} - y e^{i\xi})$	No E^2 energy growth

SMEFT expects the couplings from each operator follows |dim4|>>|dim6|>>|dim8|.
Violation of the above condition tells that the new physics scale is not much above the SM scale.

V.Barger, K.Hagiwara, J.Kanzaki, YJZ, in preparation.

In order to learn how the signal process $\mu\mu \to \nu\bar{\nu}Ht\bar{t}$ distinguishes scenarios that gives the same non-standard Yukawa coupling at processes like

$$\mu\mu \to Ht\bar{t}$$
$$\mu\mu \to \nu\bar{\nu}t\bar{t}$$
$$\mu\mu \to \mu\bar{\nu}Ht\bar{b}$$

we are now studying all the above processes as functions of

$$\frac{\lambda_6 v^2}{\Lambda^2} \text{ and } \frac{\lambda_8 v^4}{\Lambda^4}.$$

V.Barger, K.Hagiwara, J.Kanzaki, YJZ, in preparation.

However, we find that the MC integration via MG5 is too inefficient for lepton collider processes with t-channel photon exchange. We developed a new MC integration method, which evaluates the total cross section accurately in the presence of collinear singularities regularized only by a tiny electron or muon mass.

Multi-Channel Phase Space (MCPS) integral with single diagram weighting

$$\sigma = \int d\Phi \; \left| \sum_{k=1}^{86} M_k \right|^2$$

insert $\quad 1 = \sum_{j=1}^{86} \frac{|M_j|^2}{\sum_l |M_l|^2}$

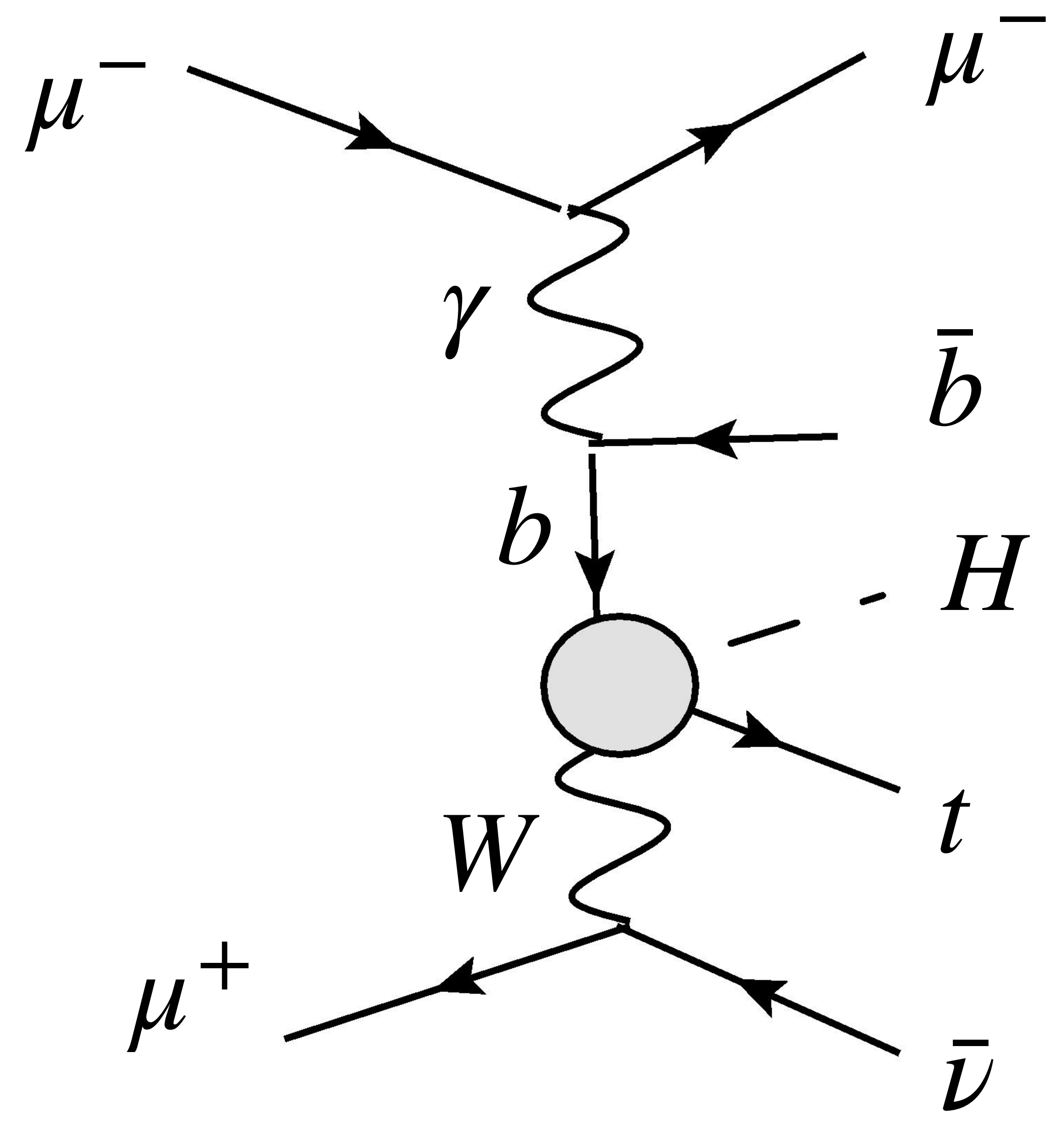
$$= \sum_{j=1}^{86} \int d\Phi \, |M_j|^2 \, \frac{\left| \sum_{k=1}^{86} M_k \right|^2}{\sum_l |M_l|^2}$$

86 channels

Efficient integrals with $|M|^2$ weight

This ratio is O(1) and moderate in the FD gauge.

86 diagrams for $\mu^- \mu^+ \to t\bar{b}H\mu^-\bar{\nu}_\mu$

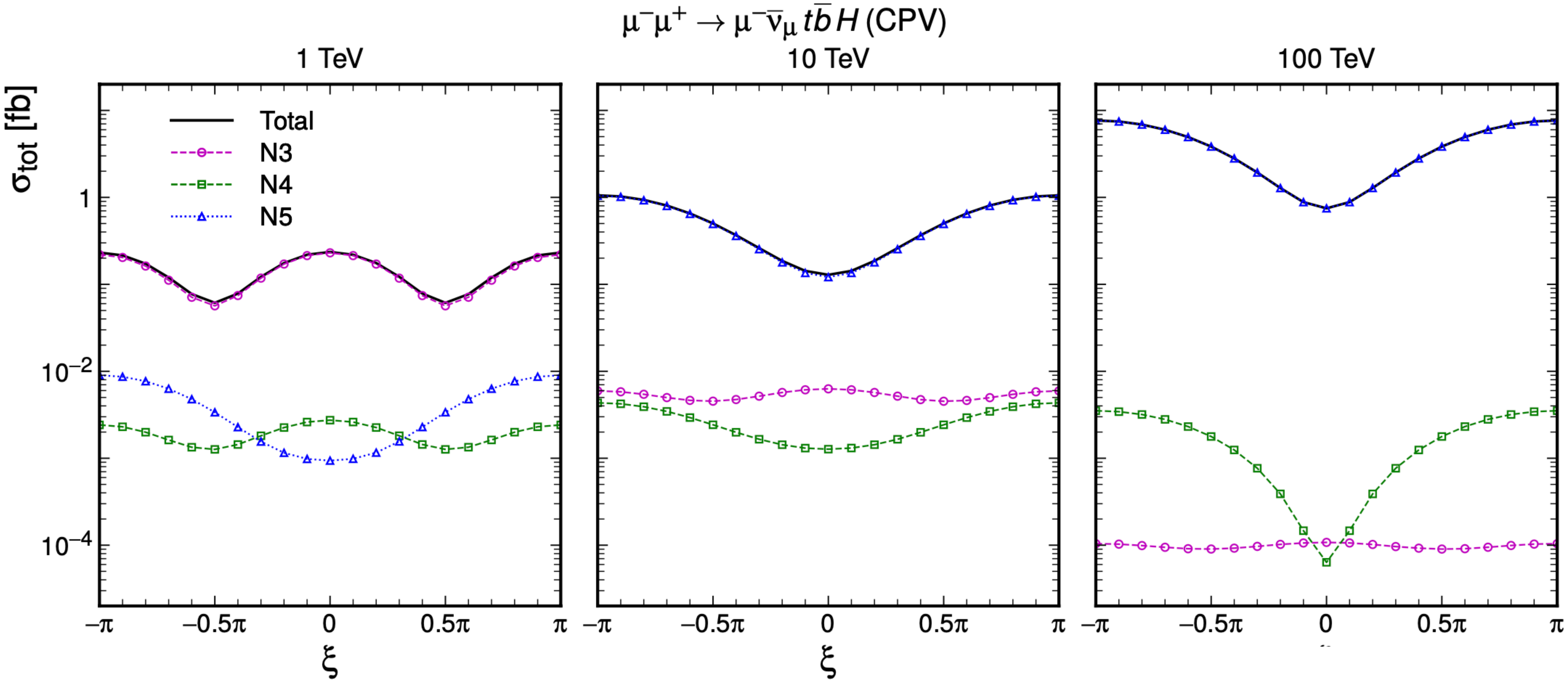


[1] K.Hagiwara, J.Kanzaki, K.Mawatari, EPJC 80 (2020) 6,584.
[2] J.Chen, K.Hagiwara, J.Kanzaki, and K.Mawatari, EPJC83(2023).
[3] J.Chen, K.Hagiwara, J.Kanzaki, K.Mawatari, YJZ, EPJPlus 139 (2024).
[4] K.Hagiwara, J.Kanzaki,O.Mattelear, K.Mawatari, YJZ, PRD110(2024) 5, 056024.

K.Hagiwara, J.Kanzaki, F.Maltoni, K. Mawatari and YJZ, arXiv:2603.01139 (hep-ph).

$\mu^-\mu^+ \to t\bar{b}H\mu^-\bar{\nu}_\mu$

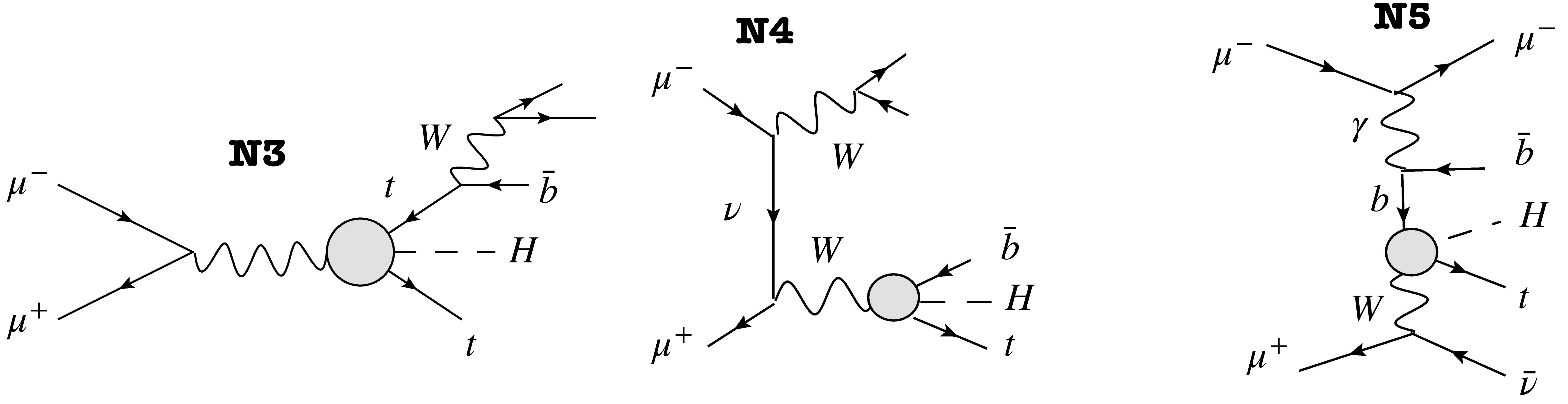
Fig.17 of K.Hagiwara, J.Kanzaki, F.Maltoni, K.Mawatari, YJZ, arXiv:2603.01139 (hep-ph).



$$\mathrm{N3}: l\bar{l} \to Ht\bar{t}\ (\bar{t} \to W^-\bar{b}, W^- \to l\bar{\nu}),$$

$$\mathrm{N4}: l\bar{l} \to Ht\bar{b}W^-\ (W^- \to l\bar{\nu}),$$

$$\mathrm{N5}: l\bar{l} \to Ht\bar{b}l\bar{\nu},$$



Summary

- We identify the cause of a power law increase of the $\mu^-\mu^+ \to \nu\nu t t H$ cross section when the top Yukawa coupling is complex as due to the power law increase of the weak boson fusion subprocess ($WW \to ttH$)cross section.
- We identify the dimension-six SMEFT operator which gives a gauge invariant description of the complex Yukawa coupling and confirm that the total cross section for $W\,W \to ttH$ satisfies the Goldstone Boson Equivalence Theorem.

  V.Barger, K.Hagiwara, YJZ, PLB850 (2024) 138547.

- The mysterious high energy behavior of $\mu^-\mu^+ \to \nu\nu t t H$ cross section in the complex top Yukawa HEFT (4 times larger than the SMEFT) can be reproduced in SMEFT by including both dim-6 and dim-8 operators.
- We are now studying all lepton collider processes sensitive to the top Yukawa coupling by allowing both dim-6 and dim-8 operators to contribute significantly. The aim is to find a clue to estimate the scale of new physics.

  V. Barger, K.Hagiwara, J. Kanzaki and YJZ, in preparation.

- We developed an efficient MC integration method which allows us to calculate the cross sections with collinear singularities sensitive to the lepton mass (m_e or m_μ).

  K.Hagiwara, J.Kanzaki, F.Maltoni, K. Mawatari and YJZ, arXiv:2603.01139 (hep-ph).