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On the origin of lepton masses and dark matter

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Outline

- Introduction
- Z_4 symmetry for neutrino masses and dark matter
- Lepton portals for charged lepton masses and dark matter
- Conclusions

Origin of flavors

Three copies of fermions: the same gauge quantum #'s, but, how come different masses and flavor mixings?

Quarks:
$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} = m_t \begin{pmatrix} 10^{-5} & 0 & 0 \\ 0 & 10^{-3} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} = m_b \begin{pmatrix} 10^{-3} & 0 & 0 \\ 0 & 10^{-2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.225 & 0.0036 \\ 0.225 & 0.974 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{pmatrix}$$

Leptons:
$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} = m_\tau \begin{pmatrix} 10^{-3} & 0 & 0 \\ 0 & 10^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} |\Delta m^2_{21}| &= 7.37 \times 10^{-5}\,{\rm eV}^2, \\ |\Delta m^2_{23}| &= 2.54 \times 10^{-3}\,{\rm eV}^2 \end{aligned}$$

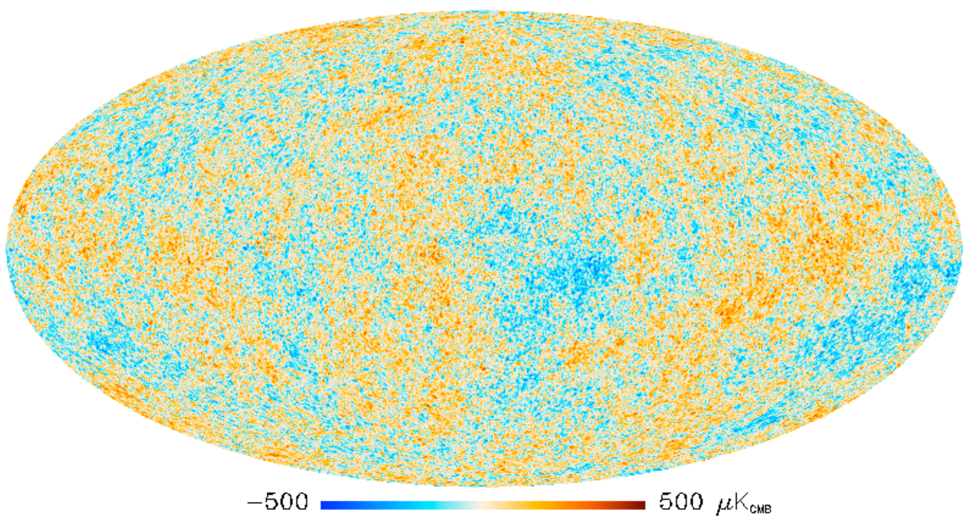
$$U_{\rm PMNS} = \begin{pmatrix} 0.829 & 0.539 & 0.147e^{-i\delta} \\ -0.413 - 0.0801e^{i\delta} & 0.636 - 0.0521e^{i\delta} & 0.645 \\ 0.355 - 0.0932e^{i\delta} & -0.547 - 0.0606e^{i\delta} & 0.750 \end{pmatrix}, \quad \delta/\pi = 1.38$$

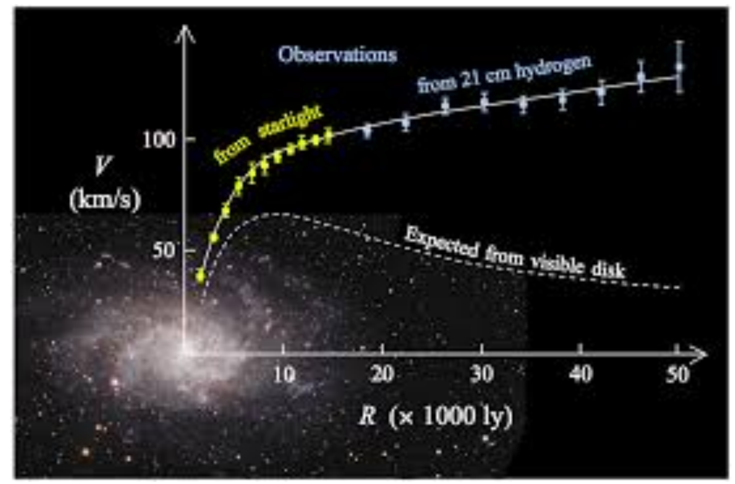
(normal hierarchy)

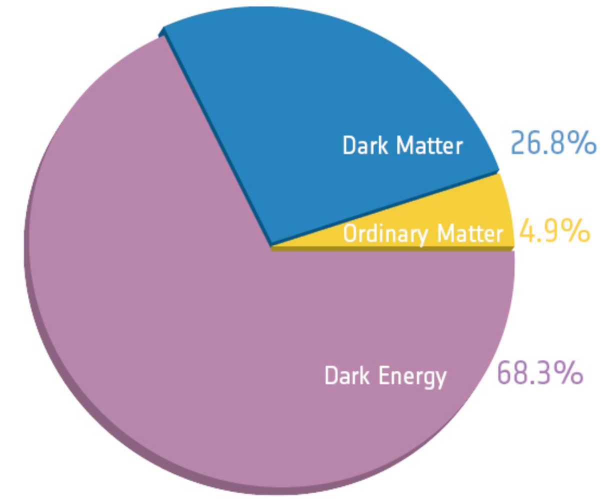
$$m_{\bar\nu_e} = 0.8\,{\rm eV}$$ [KATRIN, 2022]

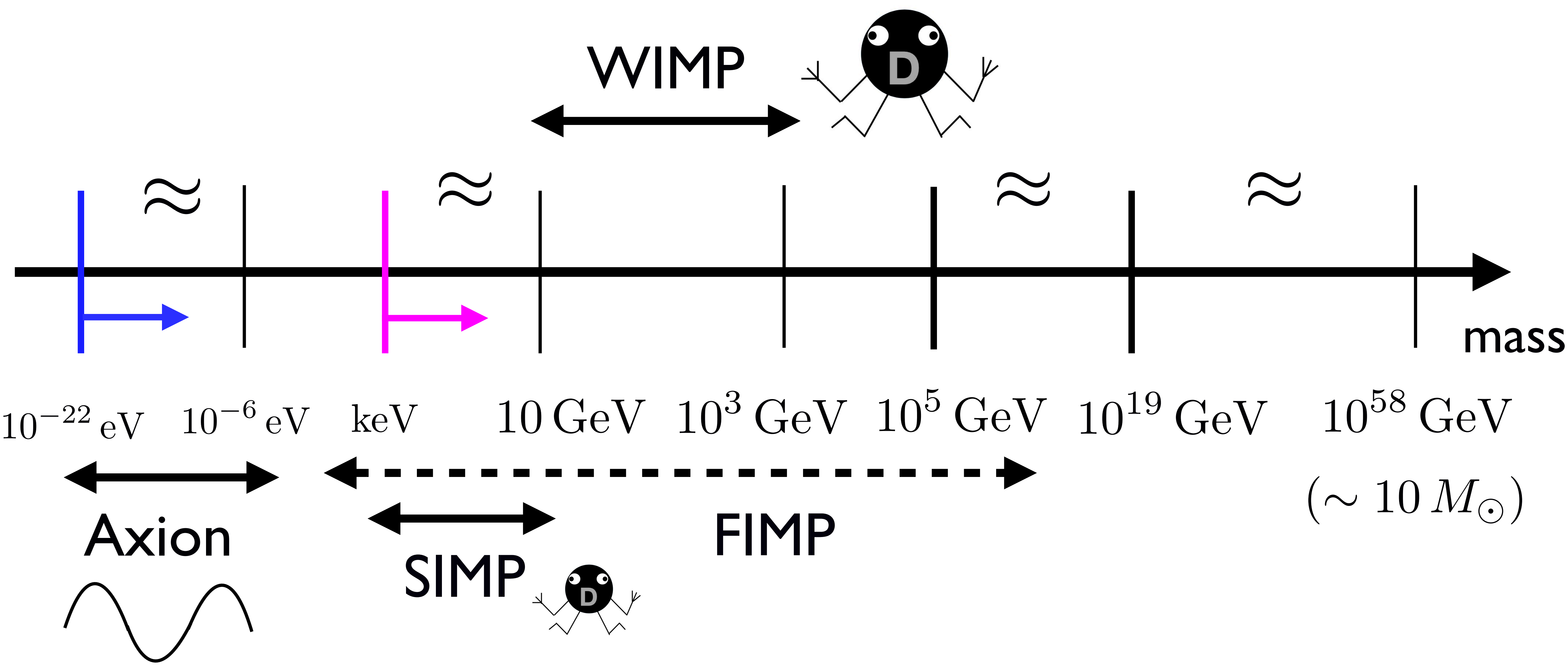
$$\sum_i m_{\nu_i} < 0.12\,{\rm eV}$$ [Planck + BAO, 2018]

Origin of dark matter









Radiative neutrino masses

Seesaw mechanism with right-handed neutrinos:

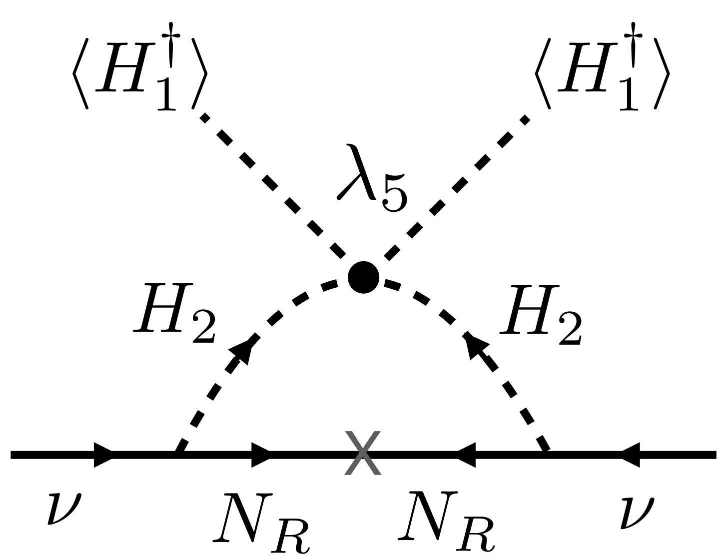
$$\begin{array}{cc} & \begin{array}{cc} \nu_L & \nu_R^c \end{array} \\ M_\nu = & \begin{pmatrix} 0 & y_N v \\ y_N v & M_N \end{pmatrix} \end{array}$$

$$|m_\nu| = \frac{y_N^2 v^2}{M_N} \ll M_N$$

Mixing with RH neutrinos → Neutrino masses at tree level: heavy RH neutrinos for y_N=O(1).

[P. Minkowski, 1977; T. Yanagida, 1979; Gell-Mann et al, 1979; R. N. Mohapatra et al, 1980]

Seesaw mechanism at loops:



$$m_\nu \sim \frac{\lambda_5 y_N^2 v^2}{16\pi^2 M_{N_R}} \ln \frac{M_{N_R}^2}{m_{H_2}^2}$$

[E. Ma, 2006]

Loop-suppressed neutrino masses.

Either N_R or neutral H_2 : dark matter

[Review: Y. Cai et al, 1706.08524]

Seesaw for leptons

Seesaw mechanism for neutrinos:

$$\begin{array}{cc} & \begin{array}{cc} \nu_L & \nu_R^c \end{array} \\ M_\nu = & \begin{pmatrix} 0 & y_\nu v \\ y_\nu v & M_N \end{pmatrix} \end{array} \qquad |m_\nu| = \frac{y_\nu^2 v^2}{M_N} \ll M_N$$

Approximate L # conservation → Small neutrino masses

Seesaw mechanism for charged leptons with VLL:

[HML, J.-S. Song, K. Yamashita, 2021, 2022]

$$\begin{array}{cc} & \begin{array}{cc} l & E \end{array} \\ M_l = & \begin{pmatrix} 0 & m_L = y_E v_2 \\ m_R = \lambda_E v_\phi & M_E \end{pmatrix} \end{array} \qquad |m_l| = \frac{m_L m_R}{M_E} \ll M_E$$

U(1)' breaking at weak scale → Small charged lepton masses

Small bare lepton mass from flavor symmetry

→ shifted seesaw mass: $-m_l \to m_0 - m_l$

Z_4 symmetry for neutrino masses and dark matter

Models with discrete Z_4

Scalar sector:

Extra doublet H_2 + complex singlet S + spurion φ

Fermion sector: Three right-handed neutrinos N_R

	S	φ	H_1	H_2	N_R
Q_X	$+1$	$+2$	0	$+1$	$+1$
$U(1)_X \supset Z_4$	i	-1	1	i	i
$U(1)_X \supset Z_2$	-1	$+1$	$+1$	-1	-1

[J. Kim et al, 2407.13595]

Z_4 symmetry $\longrightarrow$ $\frac{\lambda_5}{2}\left(\left(H_1^\dagger H_2\right)^2 + \left(H_2^\dagger H_1\right)^2\right)$ forbidden.

Neutrino masses vanish for unbroken Z_4.

$\langle\varphi\rangle \neq 0$ $\longrightarrow$ Z_4 is broken to Z_2 :

Neutrino masses and stability for dark matter.

Models with discrete Z4

Z4 invariant Lagrangian:

$$\mathcal{L}_\nu = -y_N \bar{l}\widetilde{H}_2 N_R - \underline{\frac{1}{2}\lambda_N \varphi^* \overline{N_R^c} N_R} + \text{h.c.}$$

$\langle\varphi\rangle \neq 0 \longrightarrow M_{N_R} = \lambda_N v_\varphi$: RH neutrino masses

Mixing between S and neutral components of H_2 :

$$-\mathcal{L}_1 = m_1^2|H_1|^2 + m_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4\left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right)$$
$$+ \underline{\sqrt{2}\kappa S^\dagger H_1^\dagger H_2 + \lambda'_{S\varphi}\varphi^\dagger S H_1^\dagger H_2 + \sqrt{2}\mu\,\varphi^\dagger S^2 + \text{h.c.}}$$

$\langle H_1\rangle \neq 0, \langle H_2\rangle = 0, \langle\varphi\rangle \neq 0$

$\longrightarrow$ General mixed dark matter of singlet and doublet.

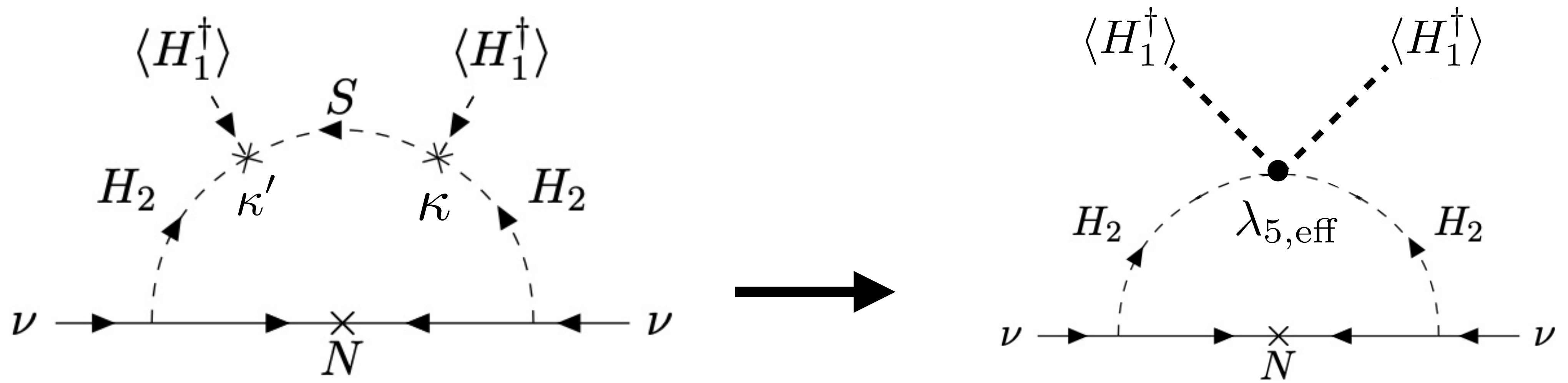
[J. Kim et al, 2025]

$$-\mathcal{L}_2 = m_S^2|S|^2 + \lambda_S|S|^4 + m_\varphi^2|\varphi|^2 + \lambda_\varphi|\varphi|^4$$
$$+ \sum_{i=1,2}\underline{\left[\lambda_{H_iS}\left(H_i^\dagger H_i\right)|S|^2 + \lambda_{H_i\varphi}\left(H_i^\dagger H_i\right)|\varphi|^2\right]} + \lambda_{S\varphi}|S|^2|\varphi|^2$$

"2HDM Higgs-portal"

Neutrino masses

Decoupled singlet S and neutrino masses at loops:



Integrating out S above v_φ leads to $\lambda_{5,\mathrm{eff}} = \frac{2\hat{m}_S^2(\kappa^2+\kappa'^2)}{m_S^4} - \frac{4\kappa\kappa'}{m_S^2}$

$$\kappa' = \frac{1}{2}\lambda'_{S\varphi} v_\varphi, \sqrt{|\hat{m}_S^2|} = \sqrt{2|\mu| v_\varphi} \ll m_S \;,\; |\kappa| \sim m_S \longrightarrow |\lambda_{5,\mathrm{eff}}| \ll 1$$

[J. Kim et al, 2024]

Neutrino masses: $m_\nu \sim \frac{\lambda_{5,\mathrm{eff}} y_N^2 v^2}{16\pi^2 M_{N_R}} \ln \frac{M_{N_R}^2}{m_{H_2}^2}$

Either N_R or neutral H_2 can be light enough without small parameters.

Inert doublet scalar masses

In the decoupling limit of the singlet S:

$m_{H_2} \lesssim M_{N_R}$, lightest neutral scalar in H_2 $\longrightarrow$ dark matter

cf. $m_{H_2} \gtrsim M_{N_R}$, RH neutrino DM [A. Ibarra et al, 2016]

Mass spectrum within H_2: $H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H_0 + iA_0) \end{pmatrix}$

$$m_{H^\pm}^2 = m_2^2 + \frac{1}{2}\lambda_3 v_H^2,$$
$$m_{H_0}^2 = m_2^2 + \frac{1}{2}(\lambda_L + \lambda_{5,\mathrm{eff}})\, v_H^2,$$
$$m_{A_0}^2 = m_2^2 + \frac{1}{2}(\lambda_L - \lambda_{5,\mathrm{eff}})\, v_H^2,$$

$$\lambda_L \equiv \lambda_3 + \lambda_4$$

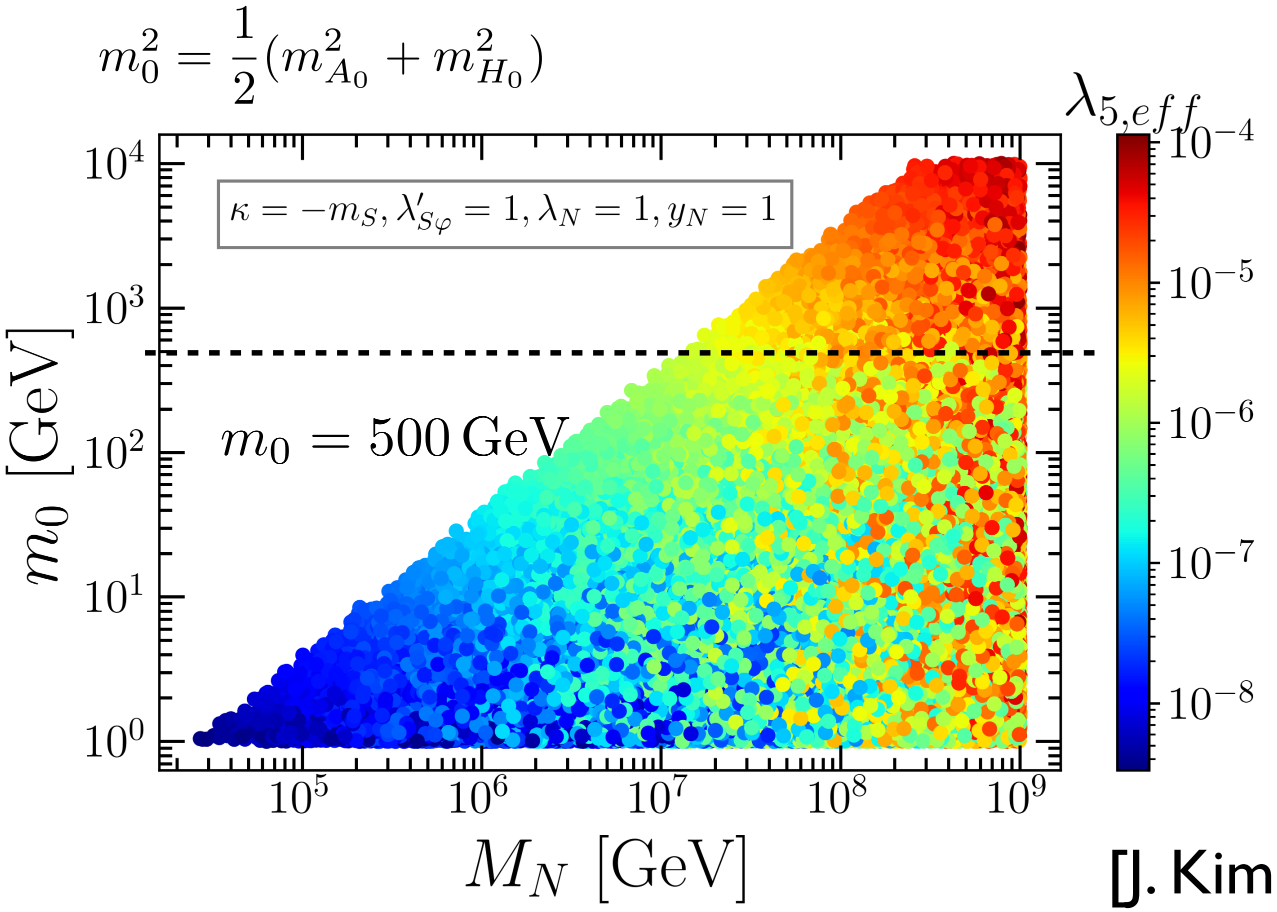
[R. Barbieri et al, 2006;
L. Lopez Honorez et al, 2006]

$\lambda_{5,\mathrm{eff}} > 0:$ A_0 dark matter, $\lambda_{5,\mathrm{eff}} < 0:$ H_0 dark matter,

DM direct detection with Z boson exchange $\longrightarrow$ $\left|\Delta \equiv m_{H_0} - m_{A_0} \simeq \frac{\lambda_{5,\mathrm{eff}} v_H^2}{2m_0}\right| \gtrsim 100\,\mathrm{keV}$

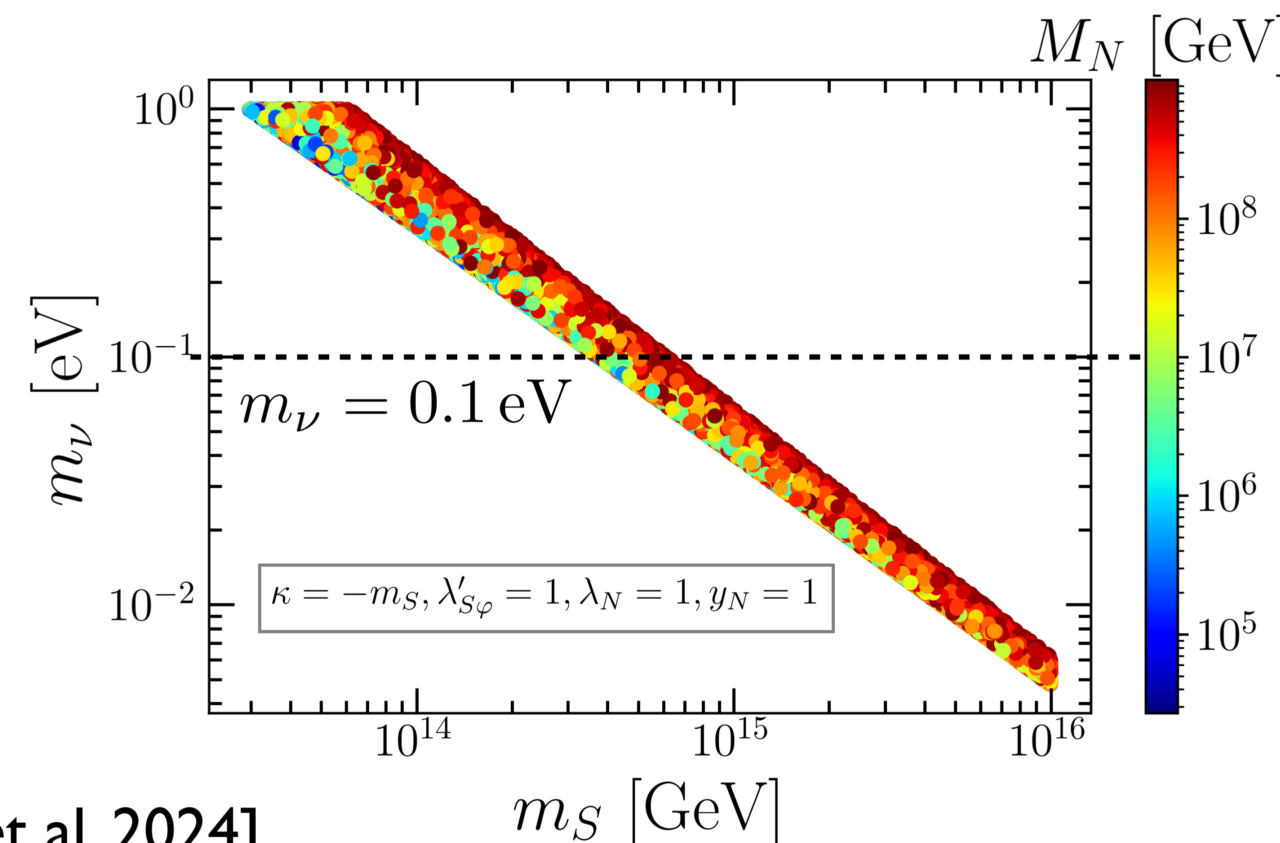
: sizable mass splitting

Neutrino mass vs DD



[J. Kim et al, 2024]

Lightest RH neutrino



Heavy singlet scalar mass

Heaviest neutrino mass

fixed to $m_\nu = 0.1\,\mathrm{eV}$

Smaller Yukawa couplings

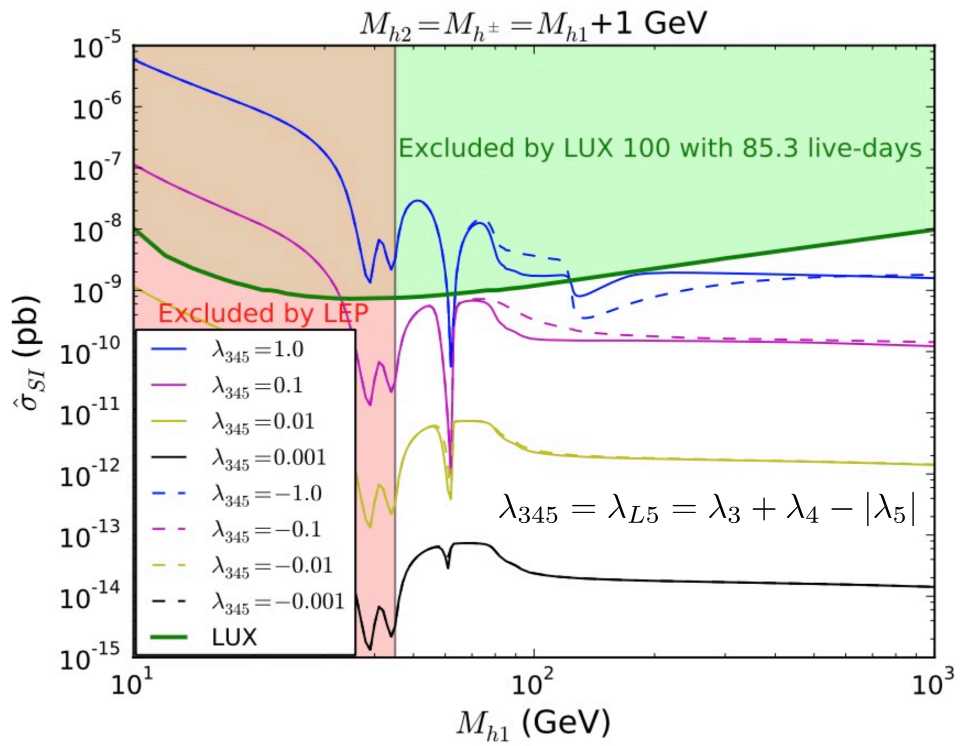
=> smaller RH neutrino masses

Direct detection(DD) $|\Delta| \gtrsim 100\,\mathrm{keV}$

Perturbativity $|\lambda_{5,\mathrm{eff}}| < 1$

=> m_S ~ up to 10^{15} GeV

Inert double-like DM



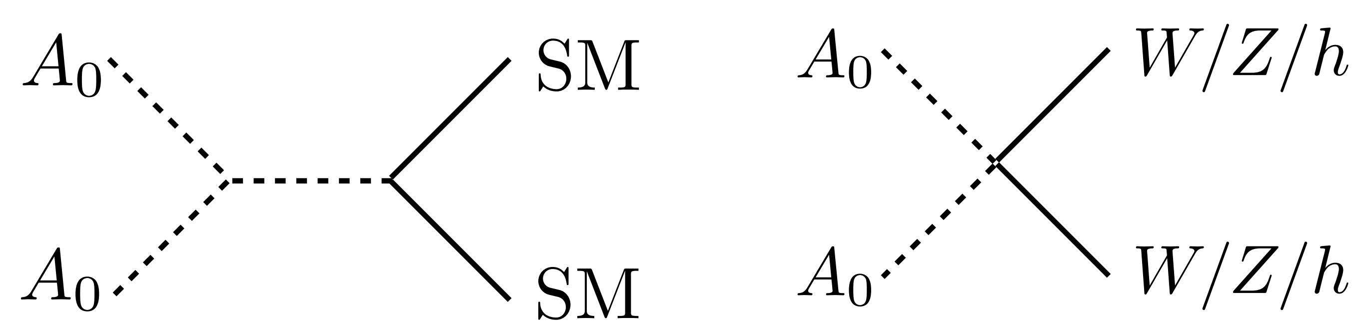
[A. Belyaev et al, 1612.00512]

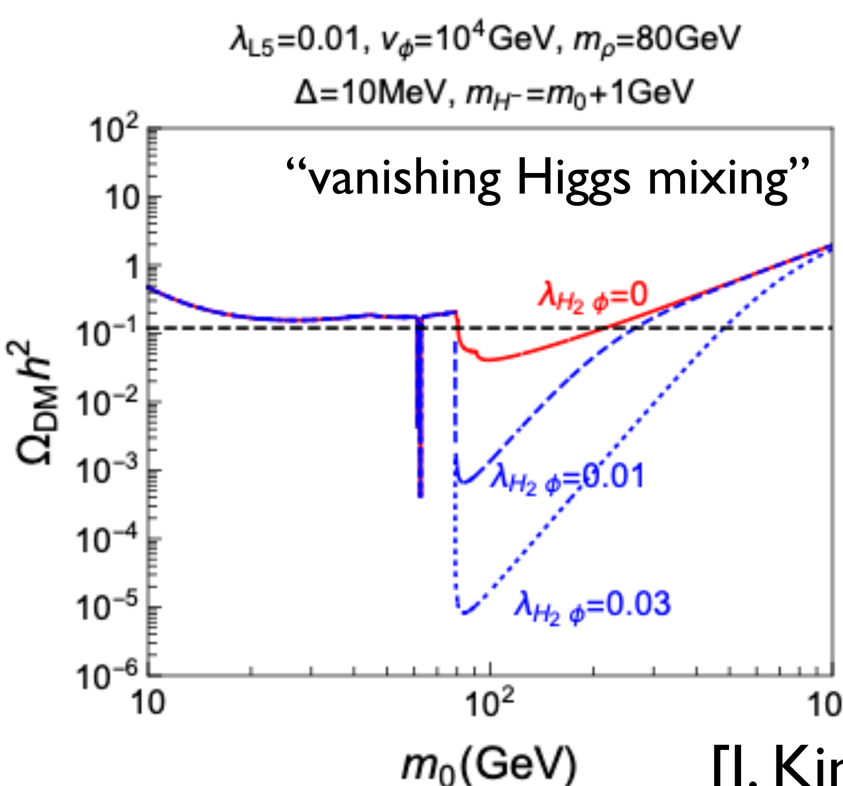
DM-nucleus cross section:

$$\sigma^{\rm SI}_{A_0} = \frac{\lambda^2_{L5}\mu^2_N (Zf_p + (A-Z)f_n)^2}{4\pi m^2_{A_0} A^2}\left(\frac{c^2_\alpha}{m^2_{h_1}} - \frac{s^2_\alpha}{m^2_{h_2}}\right)^2$$

Standard annihilation channels:

$$A_0 A_0 \to f\bar{f}, WW, ZZ, hh$$

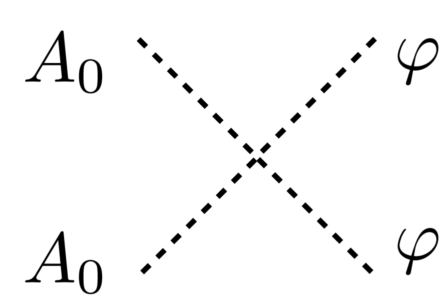




[J. Kim et al, 2024]

New ann. channels into dark scalars:

$$A_0 A_0 \to \varphi\varphi$$



[J. Kim et al, 2024, 2025]

Extended to higher DM masses.

Thermal leptogenesis

RH neutrinos are once in thermal equilibrium.

Assume hierarchical masses for RH neutrinos.

Decays of the lightest RH neutrino:

B-L asymmetry => <u>baryon-to-photon ratio:</u> $\eta_B = -C\varepsilon_1\kappa_1$

<u>Efficiency factor:</u> $\kappa_1(K_1) \simeq \frac{2}{z_B(K_1)K_1}\left(1 - e^{-\frac{1}{2}z_B(K_1)K_1}\right)$ [W. Buchmuller et al, 2004]

$K_1 = \Gamma_1/H(z_1 = 1)$ (decay parameter), $z_B = M_{N,1}/T_B$ (Completion of baryon asymmetry generation)

<u>CP asymmetry:</u> $\varepsilon_1 \lesssim \frac{3\pi}{4\lambda_{5\,\mathrm{eff}} v_H^2}\xi_3(m_h - m_l)M_{N,1}$, $\xi_i = \left(\frac{1}{8}\frac{M_{N,i}^2}{m_{H_0}^2 - m_{A_0}^2}[L(m_{H_0}^2) - L(m_{A_0}^2)]\right)^{-1}$

Loop functions

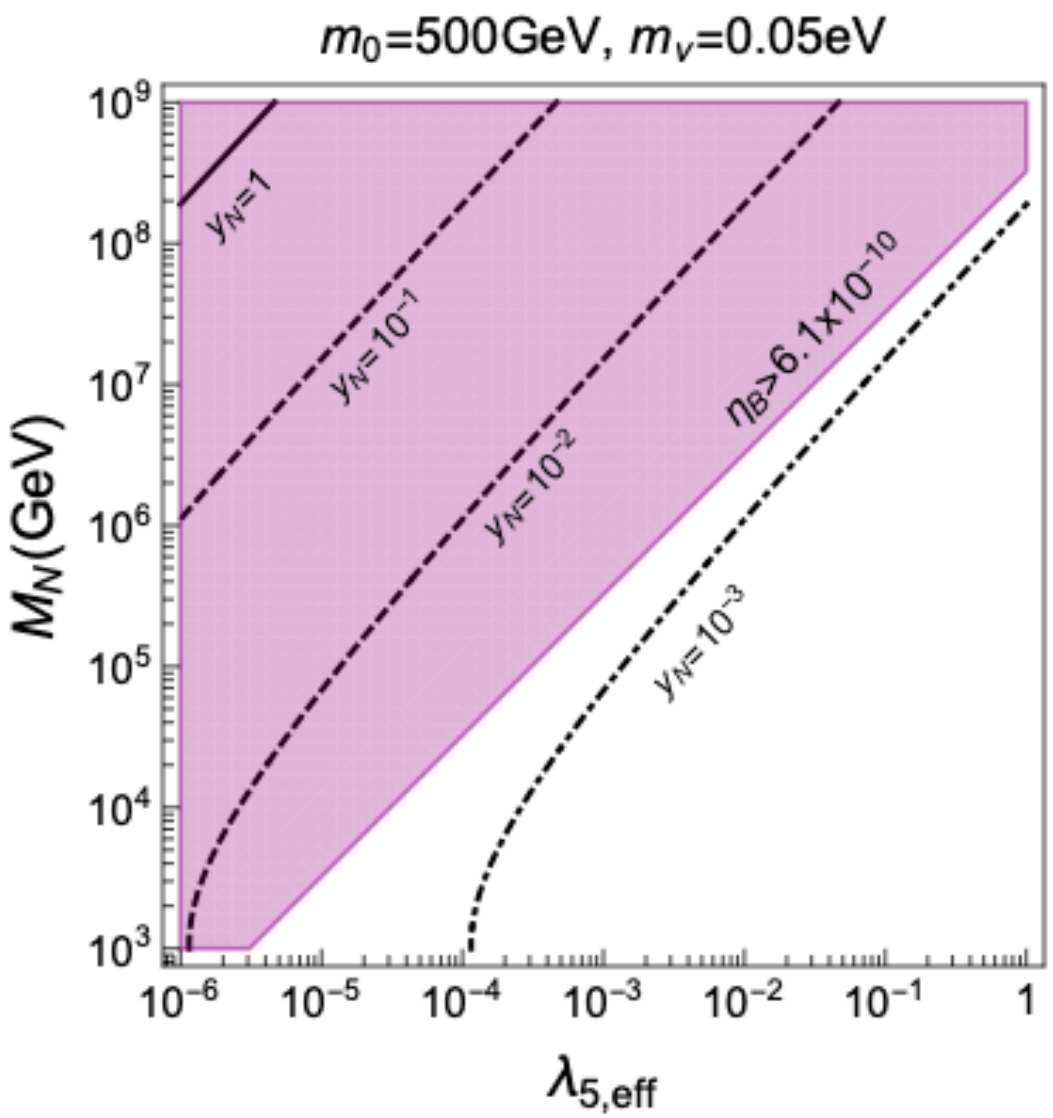
Baryon asymmetry without washout:

[T. Hugle et al, 2018]

$$\eta_B \simeq 3.8 \times 10^{-10}\left(\frac{M_{N,1}}{10^4\,\mathrm{GeV}}\right)\left(\frac{10^{-4}}{\lambda_{5,\mathrm{eff}}}\right)\left(\frac{m_h}{0.1\,\mathrm{eV}}\right),$$

m_h : heaviest ν mass

Thermal leptogenesis



Purple region allowed.

Wash-out processes:

$lH_2 \leftrightarrow \bar{l}\bar{H}_2$, $ll \leftrightarrow \bar{H}_2\bar{H}_2$ (lepton asymmetry)

$H_2\bar{H}_1 \leftrightarrow \bar{H}_2 H_1$ (annihilation of asymmetry in H_2)

[J. D. Clarke et al, 2015; T. Hugle et al, 2018]

Modified efficiency factor:

$$\kappa_{\rm tot} = \kappa_1 \exp(-\int_{z_B}^{\infty} dz \Delta W), \quad \Delta W = \Gamma_{\Delta L=2}/H z_1$$

Inefficient ΔL=2 process: $\int_{z_B}^{\infty} dz \Delta W \lesssim 1$

$\longrightarrow$ $M_{N,1} = 10^4\,\mathrm{GeV}, \quad m_h = 0.05\,\mathrm{eV}$

$\lambda_{5,\mathrm{eff}} \gtrsim 1.7 \times 10^{-6}$

Observed baryon asymmetry can be explained even for relatively light RH neutrino.

Lepton portals for charged lepton masses and dark matter

U(1)' lepton portals

SU(2)-singlet vector-like lepton + local U(1)':

	q_L	u_R	d_R	l_L	e_R	H	H'	E_L	E_R	ϕ
$U(1)'$	0	0	0	0	0	0	$+2$	-2	-2	-2

$$\mathcal{L}_{\text{fermions}} = \sum_{i=\text{SM},E} i\bar{\psi}_i\gamma^\mu D_\mu\psi_i - y_d\bar{q}_L d_R H - y_u\bar{q}_L u_R\tilde{H} - y_l\bar{l}_L e_R H$$
$$-M_E\bar{E}E - \lambda_E\boxed{\phi}\bar{E}_L e_R - y_E\bar{l}_L E_R\boxed{H'} + \text{h.c.}$$

Lepton masses: $\mathcal{M}_L = \begin{pmatrix} m_0 & m_L \\ m_R & M_E \end{pmatrix}$ → $m_{l_1} \simeq m_0 - \dfrac{m_L m_R}{M_E},$

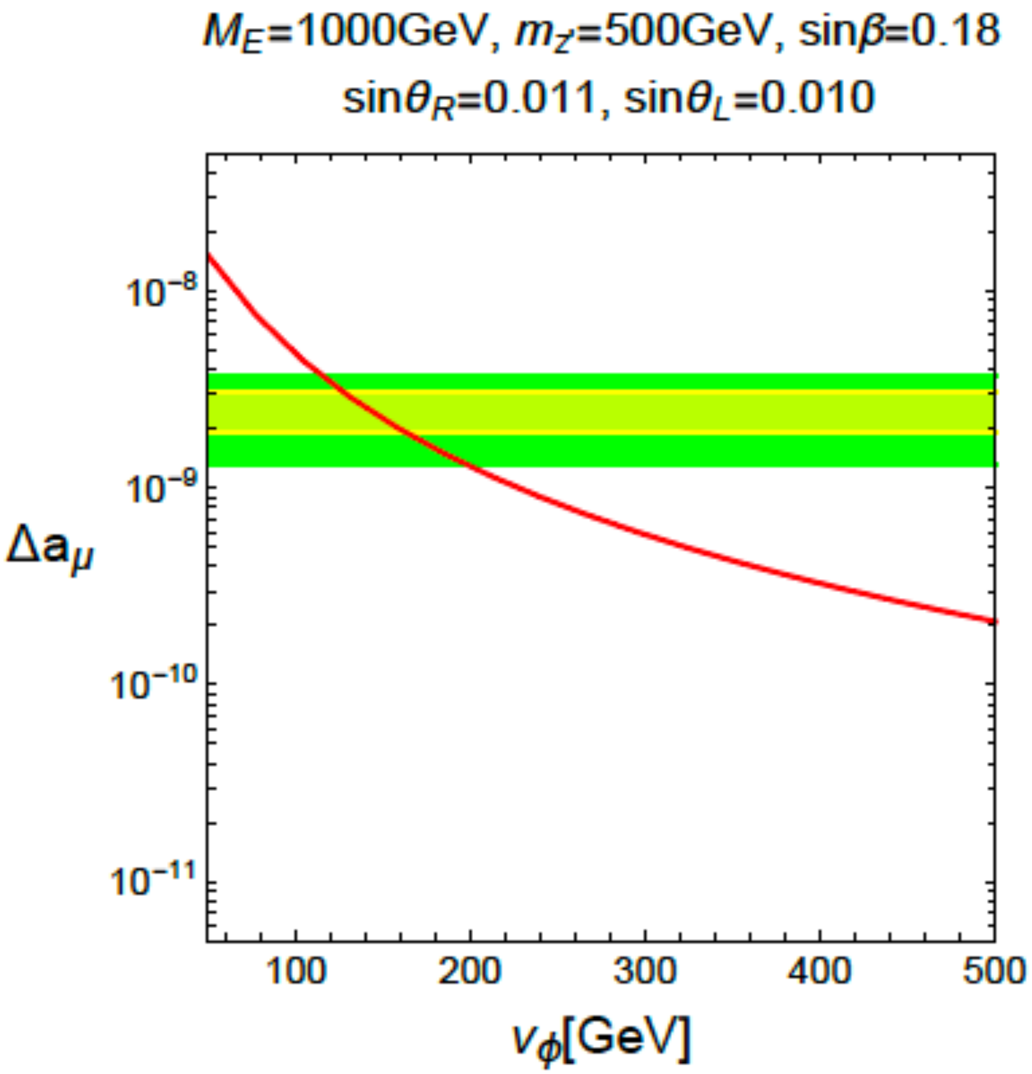
$m_0 = \frac{1}{\sqrt{2}}y_l v_1, \quad m_R = \lambda_E v_\phi, \quad m_L = \frac{1}{\sqrt{2}}y_E v_2$ $\qquad m_{l_2} \simeq M_E$

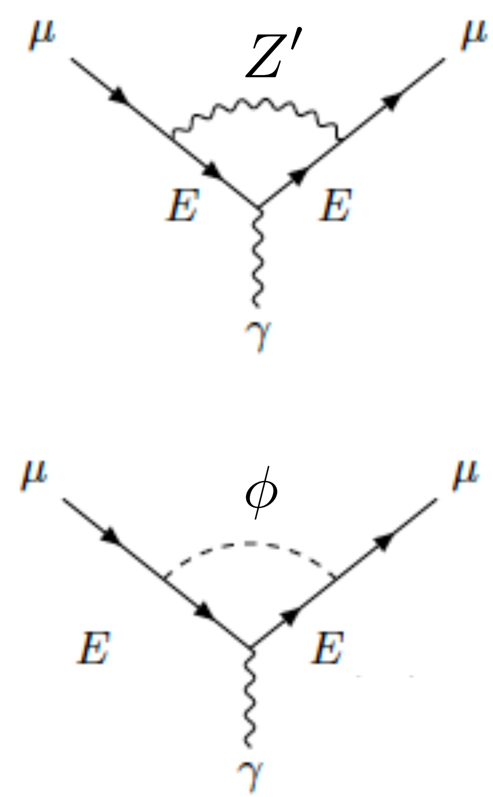
Lepton mixings ⟷ Seesaw masses

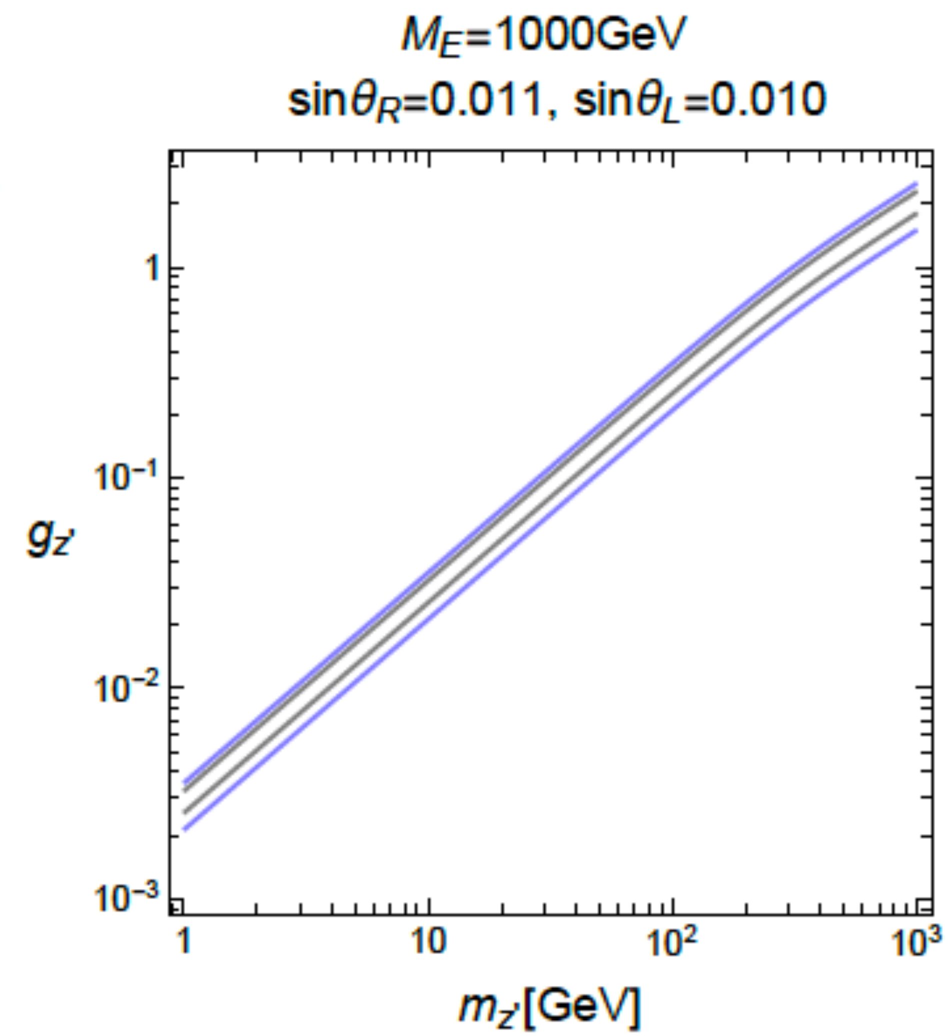
$$\sin 2\theta_{R,L} \simeq \frac{2m_{R,L}}{M_E} \qquad m_{l_1} \sim \frac{m_L m_R}{M_E}, \quad \theta_L\theta_R \sim \frac{m_{l_1}}{M_E}$$

e.g. muon seesaw: $M_E \sim 1\,\text{TeV}$ ⟶ $\theta_L\theta_R \sim 10^{-4}$

Muon g-2







[HML, K.Yamashita, 2022]

Extra scalars decoupled;
Z'-VL loops dominant!

$$\Delta a_\mu \simeq \begin{cases} \frac{g_{Z'}^2 M_E m_\mu}{16\pi^2 m_{Z'}^2}(c_V^2 - c_A^2), & M_E \gg m_{Z'}, \\ \frac{g_{Z'}^2 M_E m_\mu}{4\pi^2 m_{Z'}^2}(c_V^2 - c_A^2), & m_\mu \ll M_E \ll m_{Z'} \end{cases}$$

Seesaw muon mass:

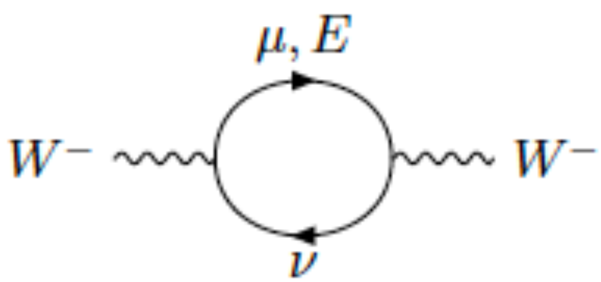
$$c_V^2 - c_A^2 \simeq 4\theta_L\theta_R \simeq \frac{4m_\mu}{M_E}$$

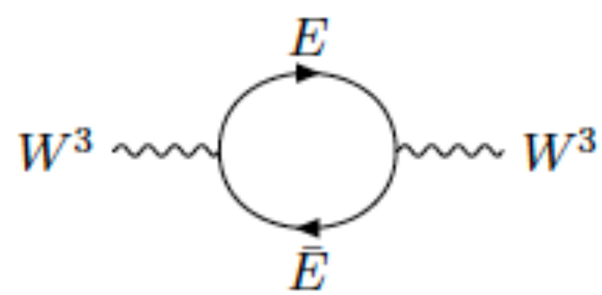
$$\Delta a_\mu \simeq \frac{g_{Z'}^2 m_\mu^2}{4\pi^2 m_{Z'}^2}:$$

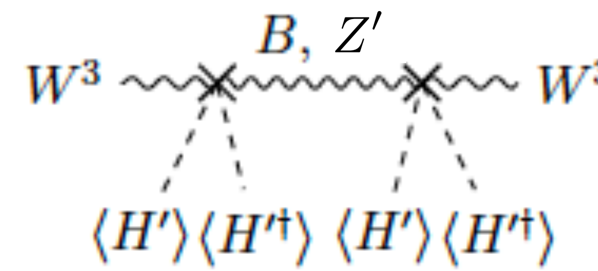
almost independent of VLL mass.

W-boson mass

Weak gauge boson masses modified at tree level and loops:







$$\Delta M_W \simeq \frac{1}{2} M_W \, \frac{c_W^2}{c_W^2 - s_W^2}\, \Delta\rho, \qquad \Delta\rho = \Delta\rho_L + \Delta\rho_H$$

Vector-like leptons in loops

$$\Delta\rho_L = \frac{\alpha}{16\pi s_W^2 c_W^2} \sin^2\theta_L \left[\frac{M_E^2}{M_Z^2} - \frac{m_\mu^2}{M_Z^2} - (\cos^2\theta_L)\theta_+(z_E, z_\mu)\right]$$

$$\Delta\rho_L \simeq \frac{\alpha M_E^2}{16\pi s_W^2 c_W^2 M_Z^2} \sin^4\theta_L$$

[HML, K. Yamashita, 2022]

Z-Z' mass mixing

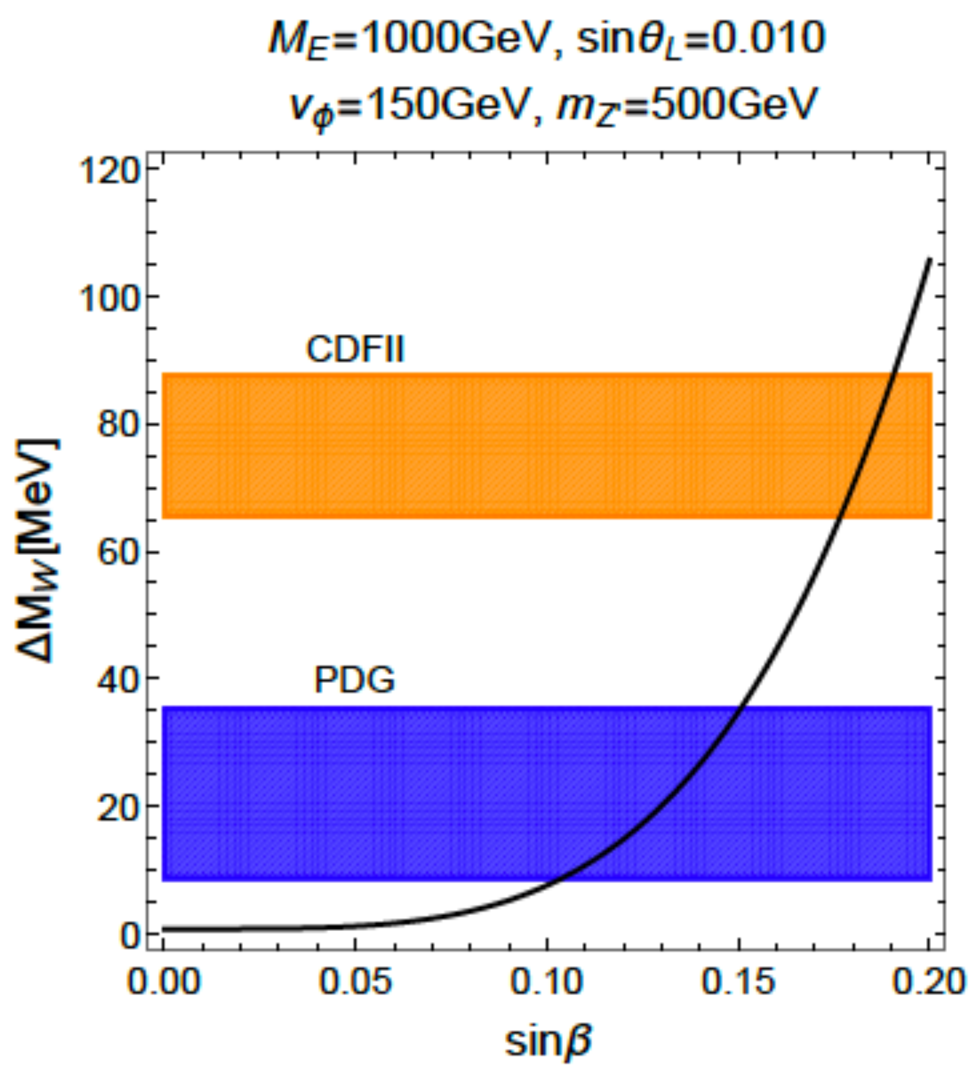
$$\Delta\rho_H = \frac{M_W^2}{M_{Z_1}^2 \cos^2\theta_W} \cos^2\zeta - 1$$

$$\Delta\rho_H \simeq \begin{cases} \frac{16 s_W^2 g_{Z'}^2}{g_Y^2} \frac{M_Z^2}{m_{Z'}^2} \sin^4\beta, & m_{Z'} \gg M_Z, \\ -\frac{16 s_W^2 g_{Z'}^2}{g_Y^2} \sin^4\beta, & m_{Z'} \ll M_Z. \end{cases}$$

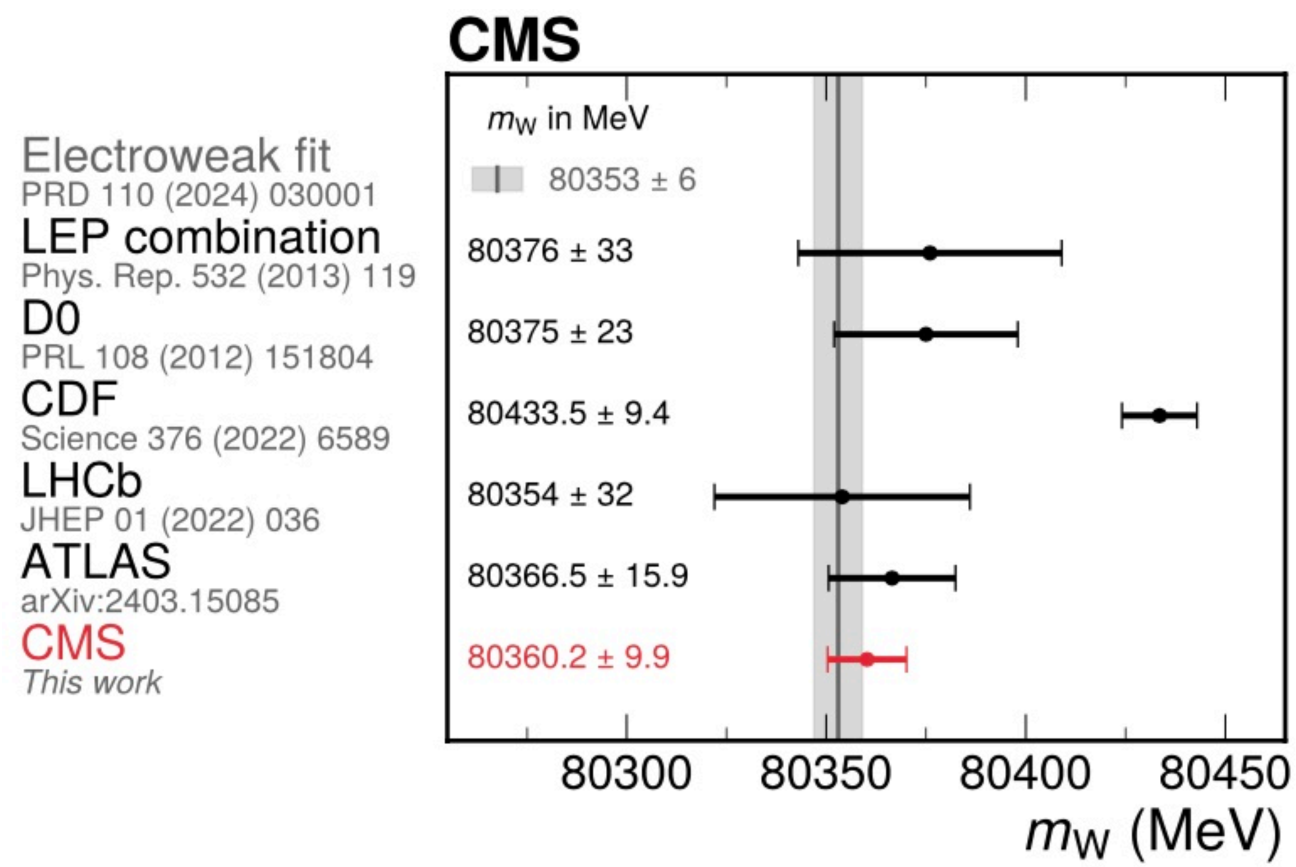
$\sin\beta = \frac{v_2}{v_1}$ [L. Bian, HML, C.B. Park, 2017]

Z-Z' mixing dominates W-boson mass corrections.

W-boson mass



[HML, K. Yamashita, 2022]



[CMS, 2412.13872]

CDFII → $\sin\beta = \dfrac{v_2}{v_1} \gtrsim 0.18(0.20)$ for $m_{Z'} = 500(1000)\,\mathrm{GeV}$

Z-Z' mixing, $\tan 2\zeta \simeq -\dfrac{s_W g_{Z'}}{g_Y}\dfrac{M_Z^2}{m_{Z'}^2}\sin^2\beta$

→ Extra Z' couplings to SM fermions including quarks.

Precision bounds on Z'

Z-boson width: $\Gamma(Z \to \mu\bar{\mu}) = \frac{(v_l + a_l \cos 2\theta_L)^2 + (v_l - a_l)^2}{(v_l + a_l)^2 + (v_l - a_l)^2} \Gamma(Z \to \mu\bar{\mu})_{\rm SM}$

$\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}$ → $\sin\theta_L < 0.114(0.162)$

$$\frac{\mathrm{BR}(Z \to \mu\bar{\mu})}{\mathrm{BR}(Z \to e\bar{e})} = 1.0009 \pm 0.0028,$$ → $\sin\theta_L < 0.030(0.047)$

$$\frac{\mathrm{BR}(Z \to \tau\bar{\tau})}{\mathrm{BR}(Z \to e\bar{e})} = 1.0019 \pm 0.0032.$$

$\Delta\rho_L \longrightarrow \Delta M_W \lesssim 7.04(17.3)\,\mathrm{MeV}$

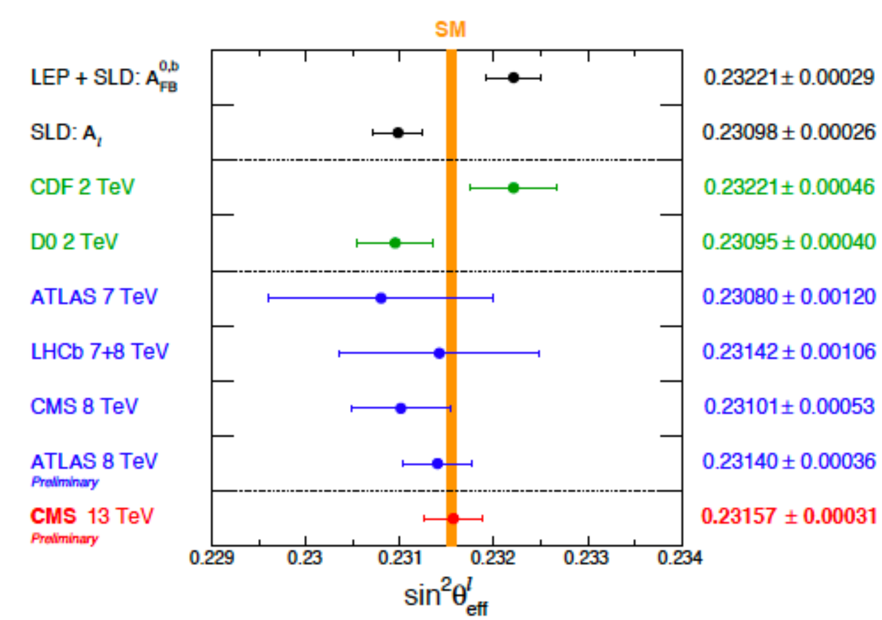
loop effects are limited

Measurement of effective $\sin^2\theta_{\rm eff}^{\rm lept}$:

$\sin^2\theta_{\rm eff}^{\rm lept} = 0.23154 \pm 0.00004$ [SM]

$\sin^2\theta_{\rm eff}^{\rm lept} = 0.23153 \pm 0.00016$ [LEP]

$\sin^2\theta_{\rm eff}^{\rm lept} = 0.23098 \pm 0.00026$ [SLD]



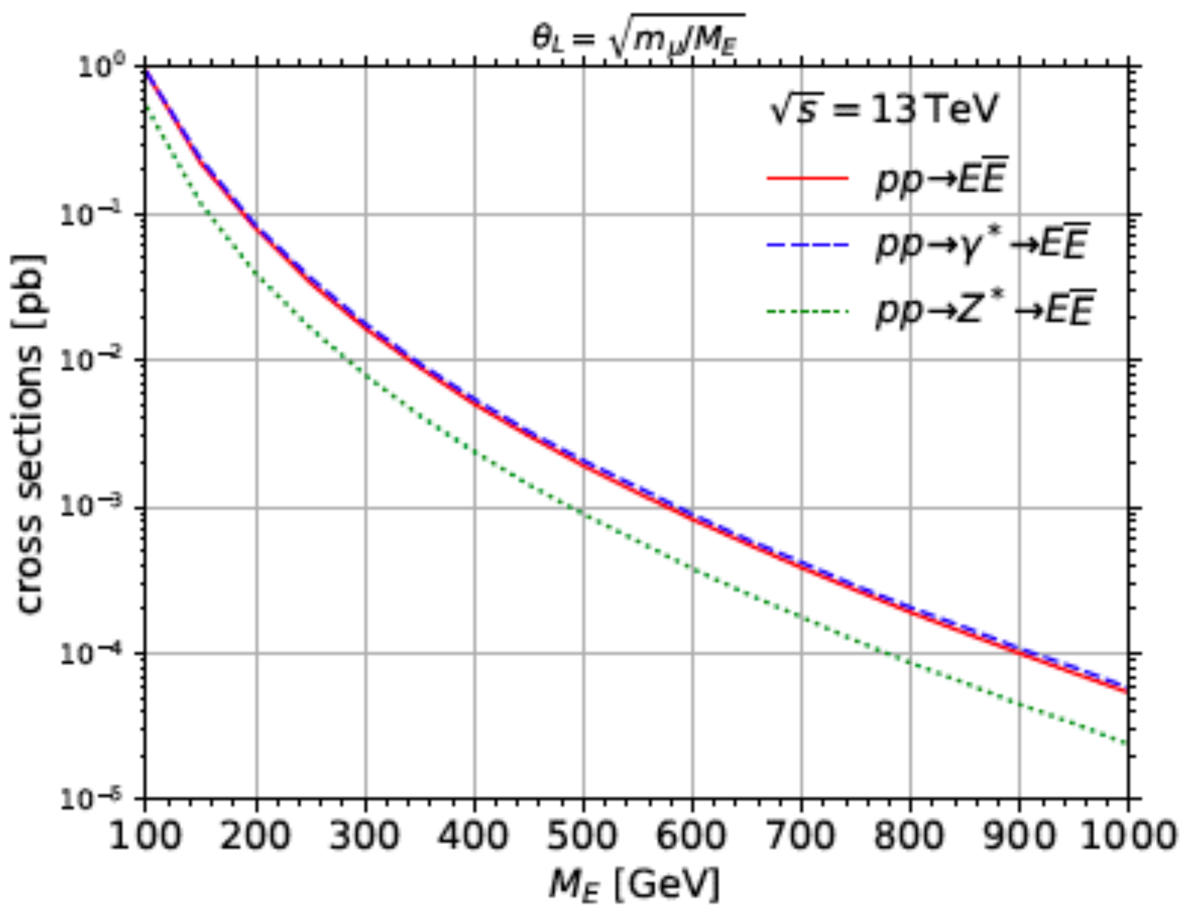
Anti-correlation with W boson mass:

$$\Delta\sin^2\theta_{\rm eff}^{\rm lept} = -\frac{s_W^2 c_W^2}{c_W^2 - s_W^2}\Delta\rho = -0.00043 \quad \text{[CDFII]}$$

→ Consistent with LEP at 2.5σ and world average at 1.2σ

Vector-like leptons at LHC

Production of Vector-like lepton:

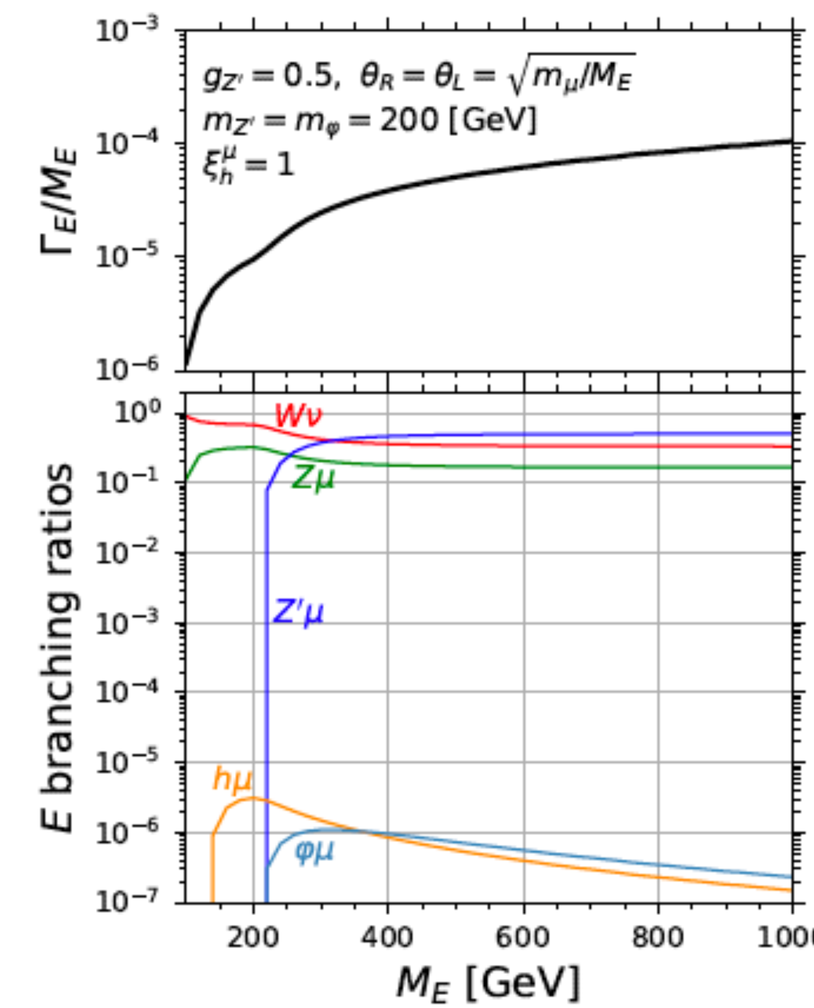


[HML, J.-S. Song K. Yamashita, 2021]

Drell-Yann, mostly SU(2) singlet

σ_{tot}~1fb for $M_E \gtrsim 600\,\mathrm{GeV}$

Decay branching ratios:



E→Z'μ for light Z' + Z'→μμ

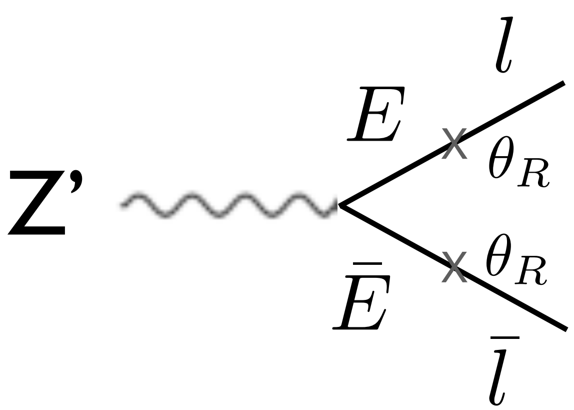
"multi-leptons"

E→Zμ, Wv ">2jets + leptons"

Z’ at LHC

Zero gauge kinetic mixing, $\mathcal{L}_{\rm kin} = -\frac{1}{2}\sin\xi Z'_{\mu\nu}B^{\mu\nu} = 0$

→ dimuons at LEP (or muon colliders)



Evades LEP bound for small mixing with electron or $m_{Z'}$>100GeV.

muon couplings testable at muon colliders.

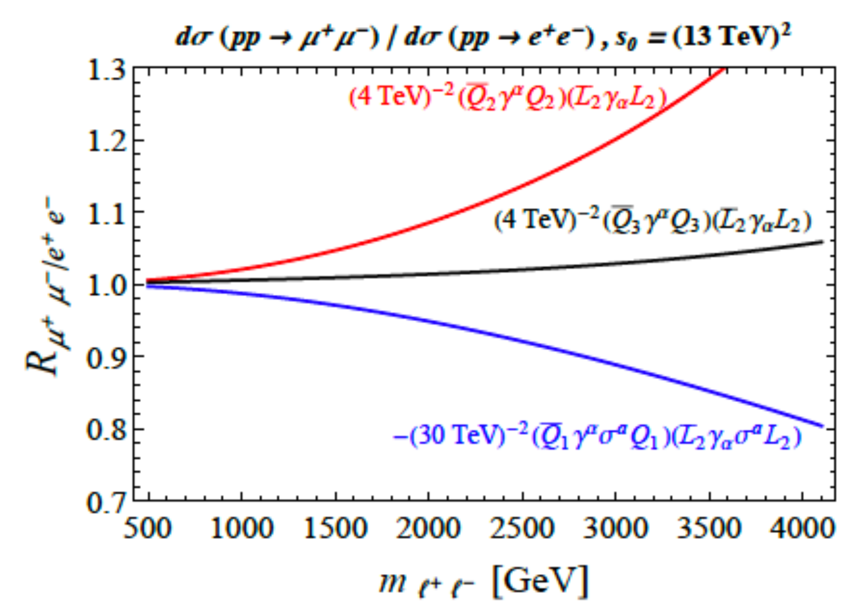
→ dimuon resonances/tails at LHC [HML, K. Yamashita, 2022]

$q\bar{q} \to Z' \to l\bar{l}$ DY suppressed by

$$\sin^4\zeta \sim 10^{-10} - 10^{-9}$$

$$m_{Z'} \gtrsim 1\,\mathrm{TeV},\ \mathcal{L}_{qqll} = \frac{C_{u_L}}{v^2}(\bar{u}_L\gamma^\mu u_L)(\bar{e}_L\gamma^\mu e_L),$$

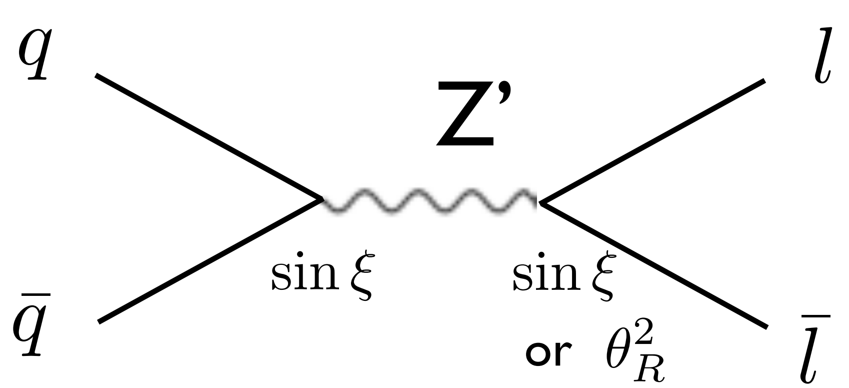
$$C_{u_L} = \frac{v^2}{m_{Z'}^2}\left(\frac{e s_\zeta}{2 s_W c_W}\right)^2 (1-2s_W^2)\left(1-\frac{4}{3}s_W^2\right) \lesssim 10^{-8}$$



consistent with dimuon high pt-tails, $|C_{u_L}| < 10^{-4}$ [A. Greljo, 2017]

Z' at LHC

Nonzero gauge kinetic mixing → dimuons at LEP and LHC

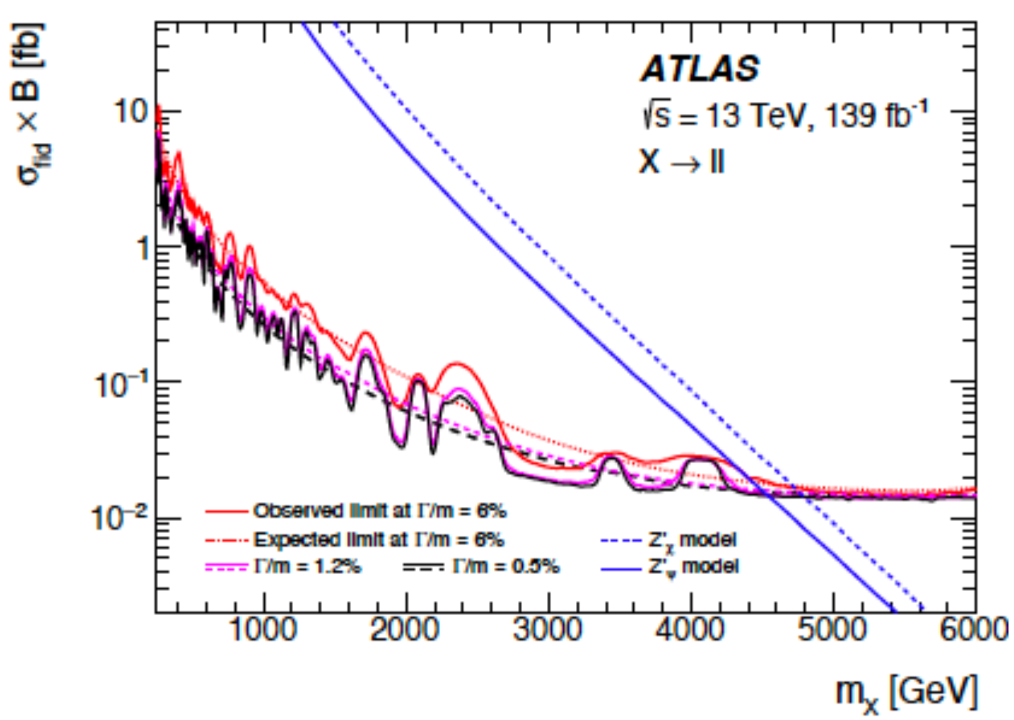


Light vector-like lepton:

BR(Z'→μμ)/BR(Z'→Eμ)~ $\theta_R \sim 10^{-2}$

Heavy vector-like lepton:

Z'→μμ dominant cf. E→Z'μ

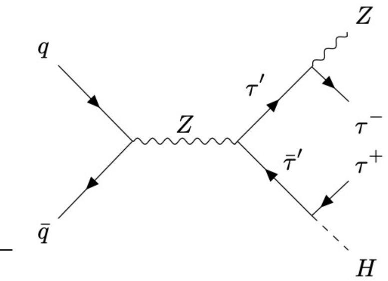


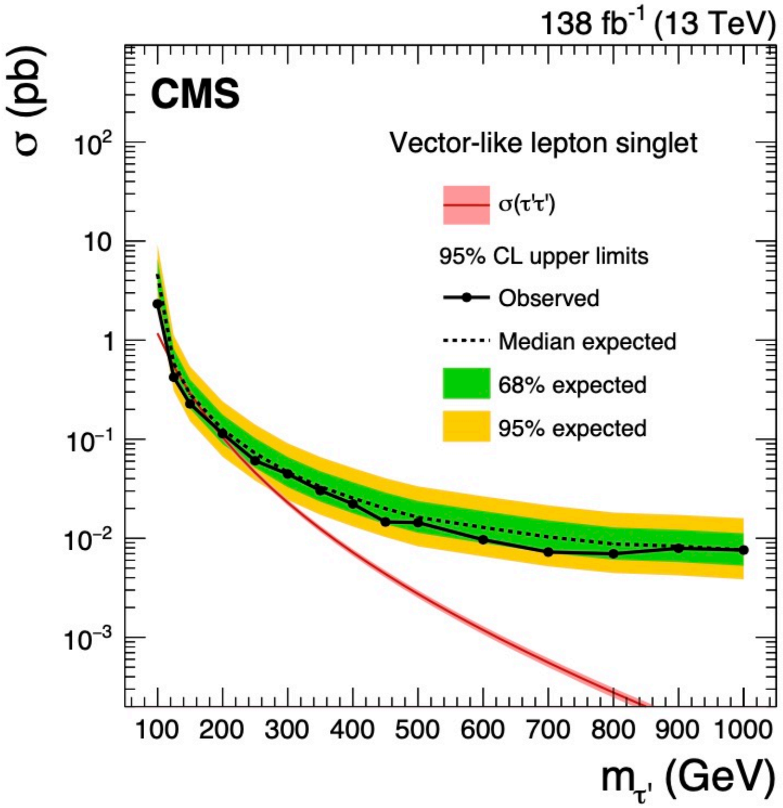
$$\sigma_{\rm tot}(pp \to Z' \to \mu\bar{\mu})$$
$$= 75.3\,{\rm pb}(\sin^2\xi)\,{\rm BR}(Z' \to \mu\bar{\mu})$$
$$@\sqrt{s} = 13\,{\rm TeV}\;({\rm LHC})$$

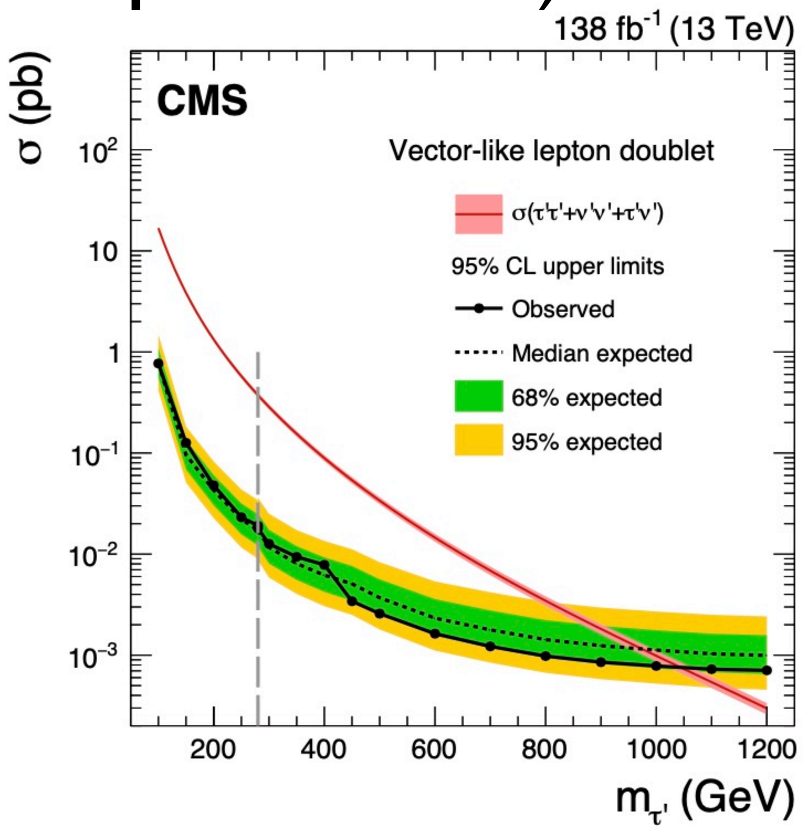
$$\sin\xi \lesssim 0.012,\;\; m_{Z'} \gtrsim 250\,{\rm GeV}$$

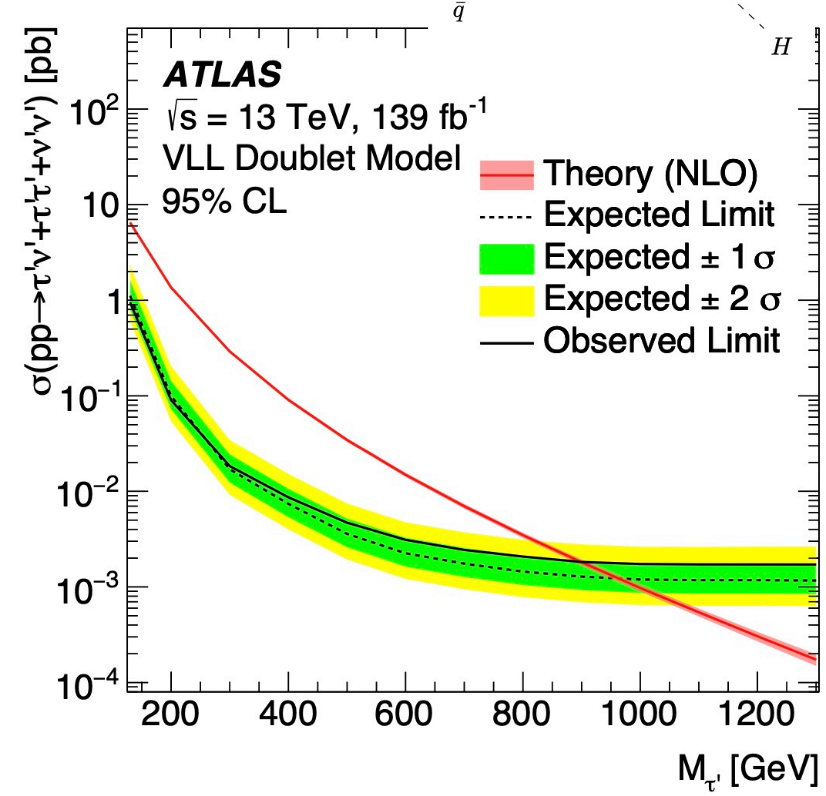
Limits from LHC

Vector-like leptons (coupled to tau)



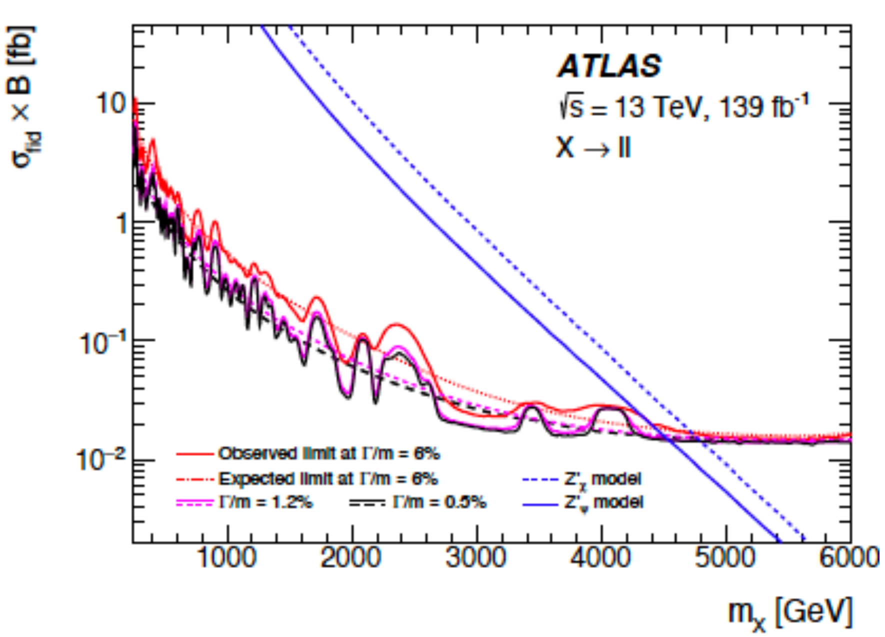






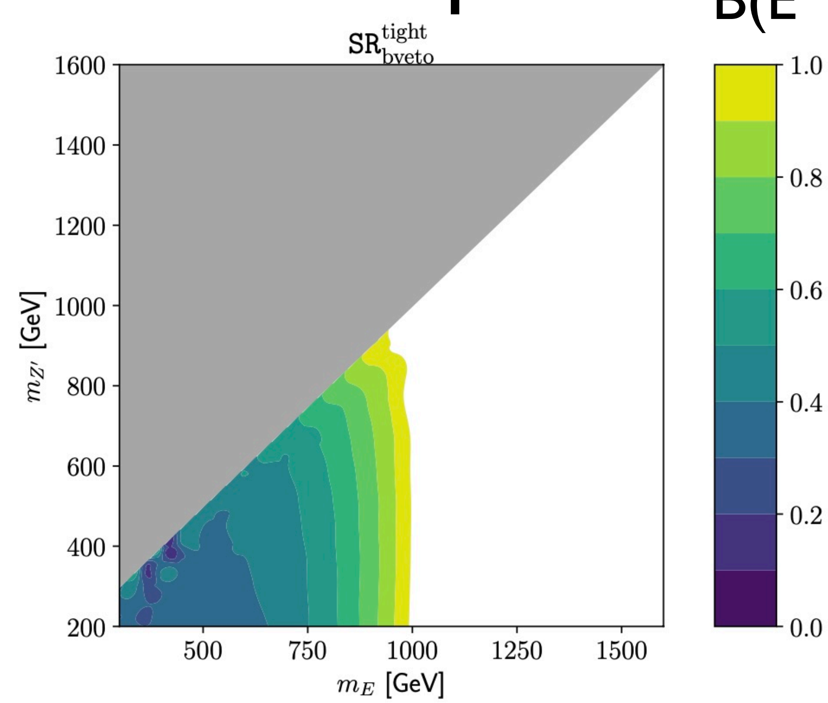
Dimuon for Z' at LHC

Z'



Vector-like leptons

B(E→Z'μ)



[J. Kawamura, S. Raby, 2104.04461]

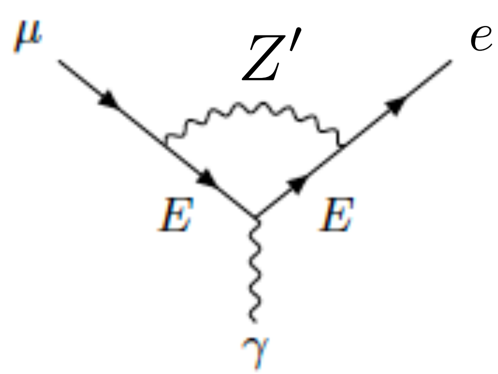
$\geq 4\mu$

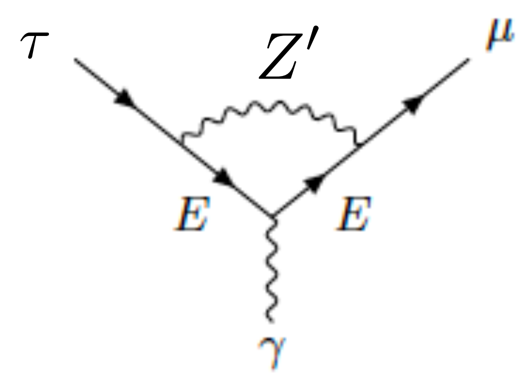
$m_E \gtrsim 1\,\mathrm{TeV}$

Lepton flavor violation

Simultaneous couplings of vector-like leptons to all SM leptons.

→ Flavor violating decays of leptons





$$
\begin{aligned}
\mathrm{BR}(\mu \to e\gamma) &\simeq \tau_\mu \cdot \frac{\alpha m_\mu^3}{256\pi^4}\left(\frac{g_{Z'}^4 M_E^2}{m_{Z'}^4}\right)\left[(\theta_R^\mu \theta_L^e)^2 + (\theta_L^\mu \theta_R^e)^2\right],\\
\mathrm{BR}(\tau \to \mu\gamma) &\simeq \tau_\tau \cdot \frac{\alpha m_\tau^3}{256\pi^4}\left(\frac{g_{Z'}^4 M_E^2}{m_{Z'}^4}\right)\left[(\theta_R^\tau \theta_L^\mu)^2 + (\theta_L^\mu \theta_R^\tau)^2\right],\\
\mathrm{BR}(\tau \to e\gamma) &\simeq \tau_\tau \cdot \frac{\alpha m_\tau^3}{256\pi^4}\left(\frac{g_{Z'}^4 M_E^2}{m_{Z'}^4}\right)\left[(\theta_R^\tau \theta_L^e)^2 + (\theta_L^e \theta_R^\tau)^2\right]
\end{aligned}
$$

Seesaw mass for muon:

$$\theta_L^\mu = \theta_R^\mu = \sqrt{m_\mu/M_E}.$$

Lepton g-2:

$$\Delta a_l \sim g_{Z'}^2 M_E m_l (\theta_L^l \theta_R^l)/m_Z^2$$

Bounds on LFV decays:

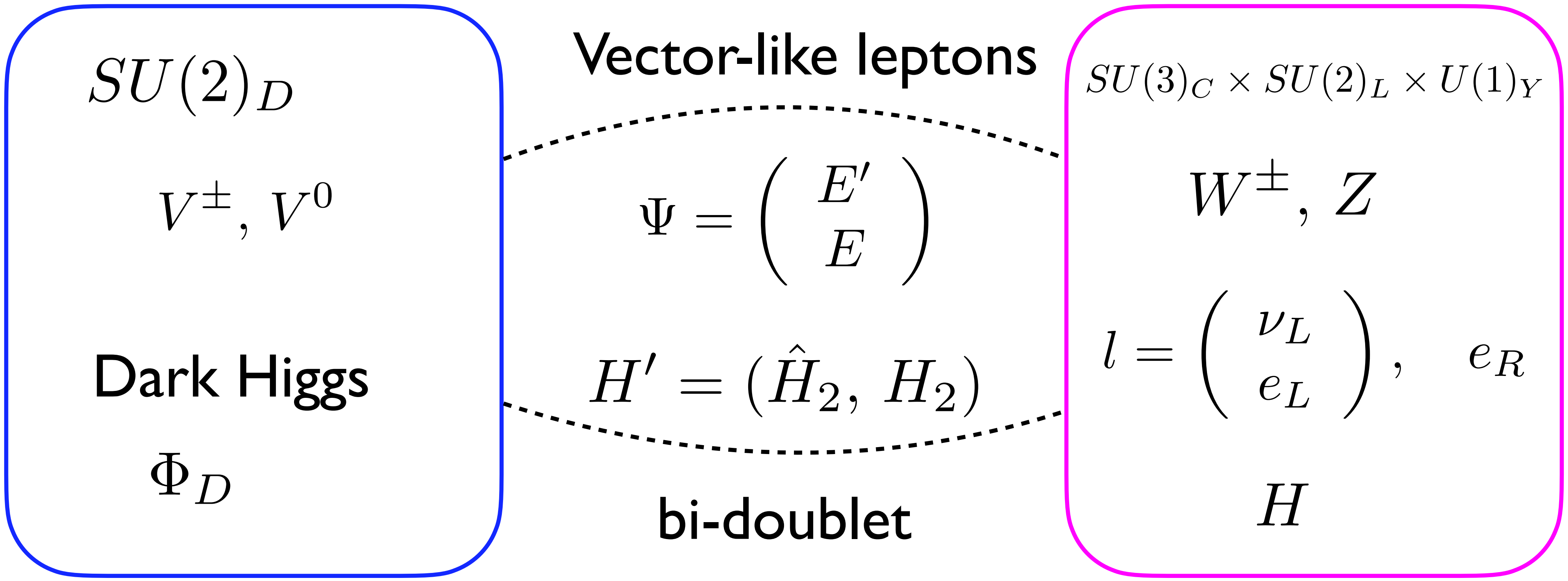
[HML, J.-S. Song K. Yamashita, 2021]

$$
\left.
\begin{aligned}
\mathrm{BR}(\mu \to e\gamma) &< 4.2 \times 10^{-13},\\
\mathrm{BR}(\tau \to \mu\gamma) &< 4.4 \times 10^{-8},\\
\mathrm{BR}(\tau \to e\gamma) &< 3.3 \times 10^{-8}.
\end{aligned}
\right\}
\rightarrow
\begin{aligned}
\theta_R^e = \theta_L^e &\lesssim 2(3) \times 10^{-4}\sqrt{m_e/M_E}\\
\theta_R^\tau = \theta_L^\tau &\lesssim 0.04(0.08)\sqrt{m_\tau/M_E}
\end{aligned}
$$

cf. seesaw relations: $\theta_R^e \simeq \sqrt{m_e/M_E},\ \theta_R^\tau \simeq \sqrt{m_\tau/M_E}$

Extra vector-like leptons → $\boxed{\text{seesaw masses for all SM leptons are compatible with LFV decays.}}$

SU(2) lepton portals



[S.-S. Kim, HML, A. Menkara, K. Yamashita, 2022]

U(1)’ extended to SU(2) + doublet Vector-like leptons.

Seesaw lepton masses via dark Higgs + bi-doublets

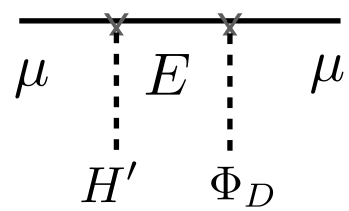
$$\mathcal{L} \supset \; -M_E \bar{\Psi}\Psi - \lambda_E \bar{\Psi}_L \Phi_D e_R - y_E \bar{l}_L H' \Psi_R + \text{h.c.}$$

Z_2 parity: $\quad Z_2 = e^{i\pi(G+I_3^D)} \;, \quad G : U(1)_G \text{ global}$

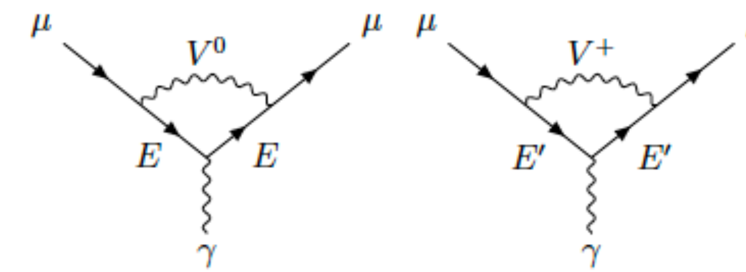
$V^\pm \longrightarrow$ Stability for vector dark matter

Unification with dark matter

Seesaw leptons



Muon g-2



$$\theta_L \theta_R \sim \frac{m_\mu}{M_E}$$

Vector-like leptons

$$\Delta a_\mu \simeq \frac{3 g_D^2 M_E m_\mu}{64 \pi^2 m_{V^0}^2} \, \theta_L \theta_R$$

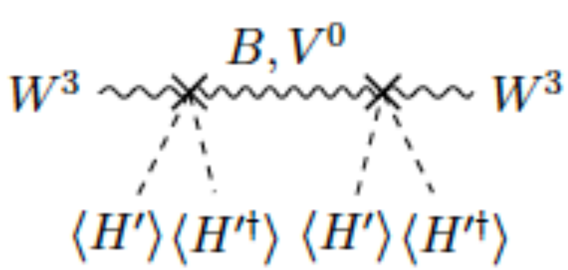
W-boson mass

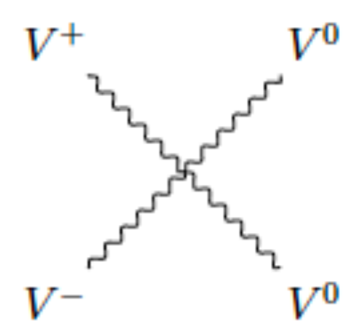
Dark matter

$$\Delta M_W \simeq \frac{1}{2} M_W \, \frac{c_W^2}{(c_W^2 - s_W^2)} \, \Delta\rho_H$$

$Z - V^0$ mixing

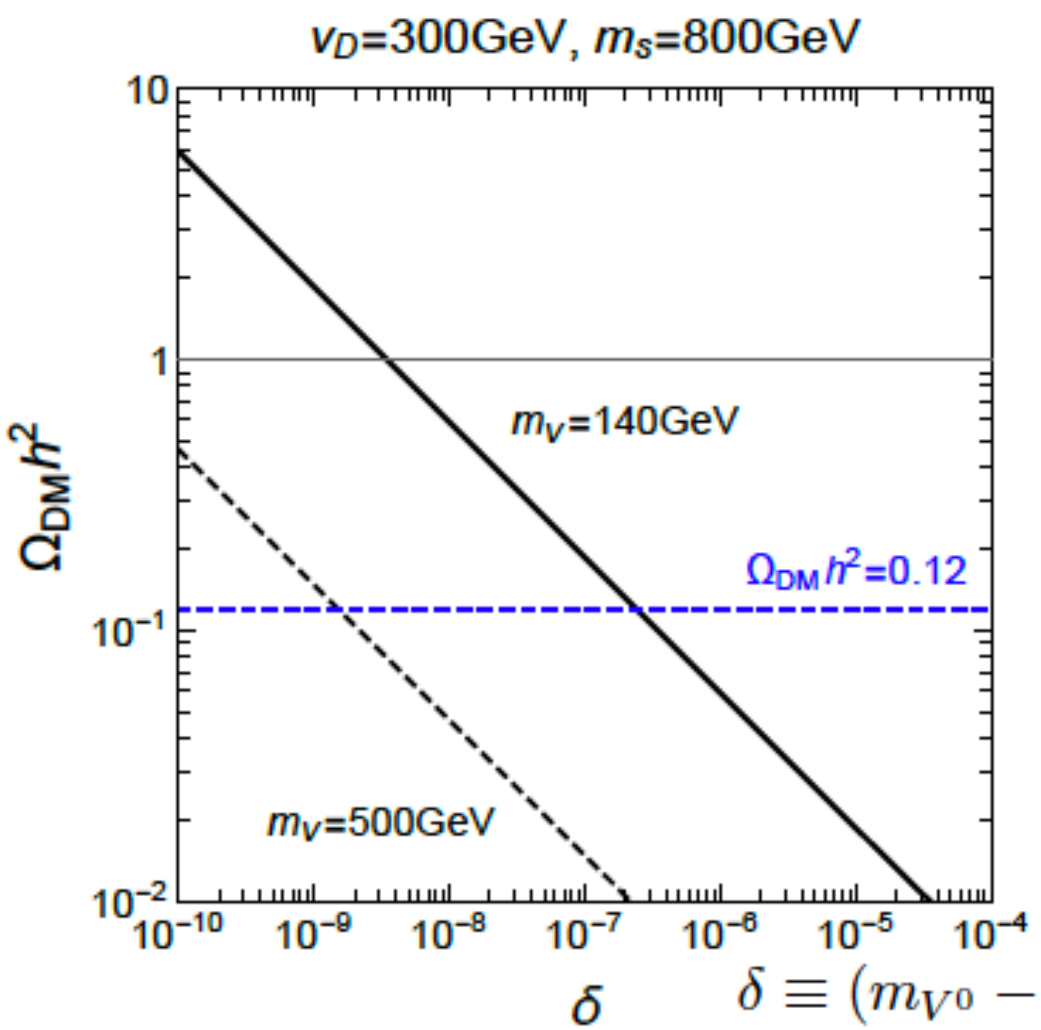
$$m_{V^0} - m_{V^\pm} \simeq \frac{1}{2} \, \frac{M_Z^2}{m_{V^0}} \, \Delta\rho_H$$



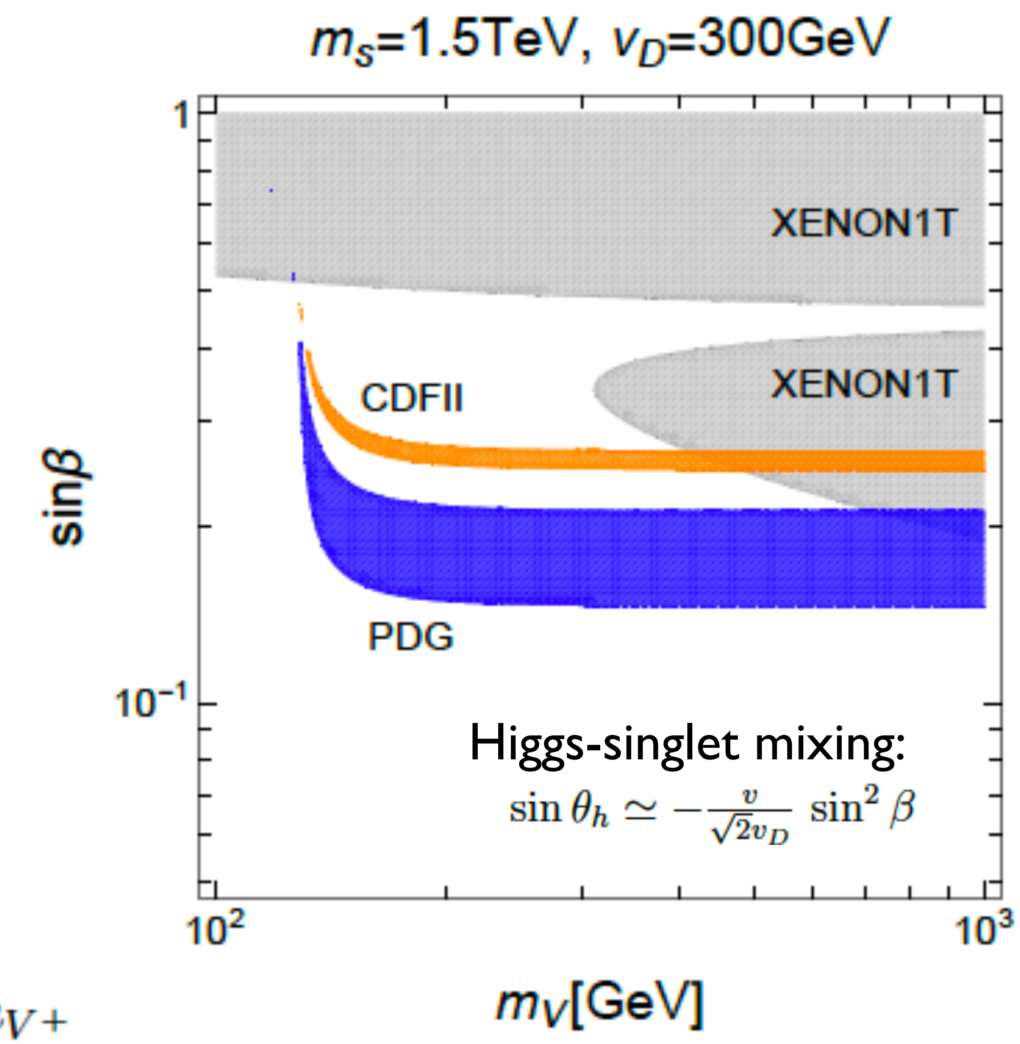


quasi-degenerate
DM masses

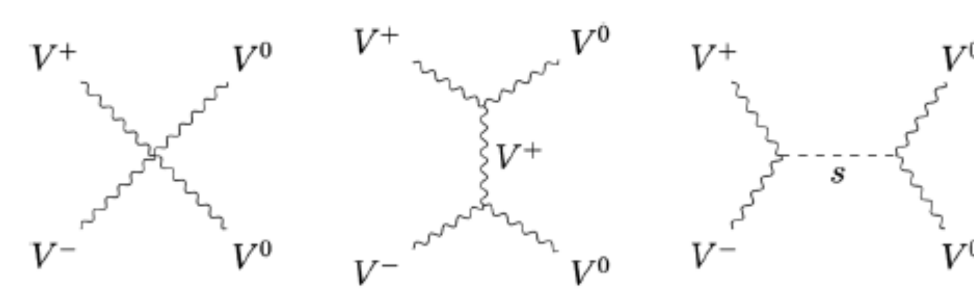
Dark matter and W-boson mass



$\delta \equiv (m_{V^0} - m_{V^+})/m_{V^+}$



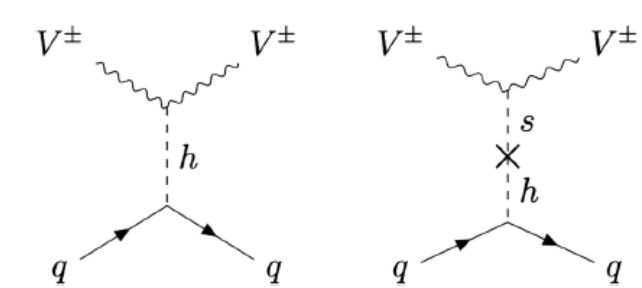
[S.-S. Kim, HML, A. Menkara, K. Yamashita, 2022]



Forbidden annihilations for freeze-out

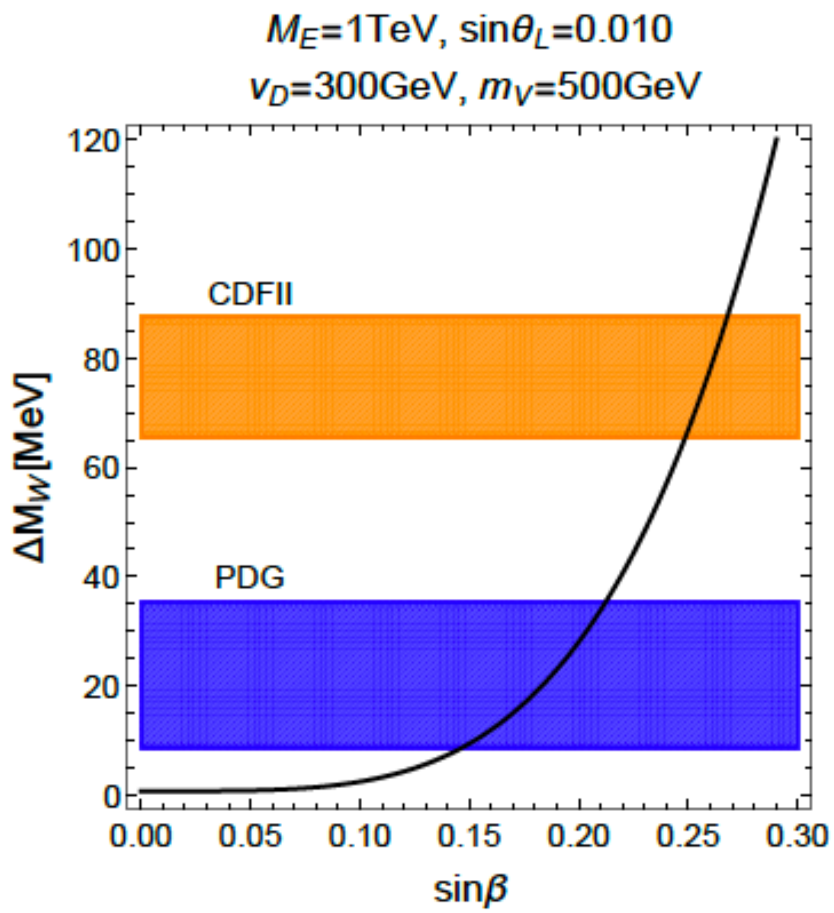
Indirect signal at CMB/galaxies suppressed

Aligned limit for Higgs mixing, $\sin\theta_h \simeq -\frac{v}{\sqrt{2}v_D}\sin^2\beta$



Destructive interference for direct detection

Unification with dark matter



Muon g-2

$$\Delta a_\mu \simeq \frac{3 g_V^2 m_\mu^2}{64\pi^2 m_V^2}$$

Weak-scale SU(2) DM

$$\frac{\delta M_W}{M_W} \simeq \frac{s_W^2 g_D^2 m_Z^2 \sin^4\beta}{2 g_Y^2 m_{V^0}^2}$$

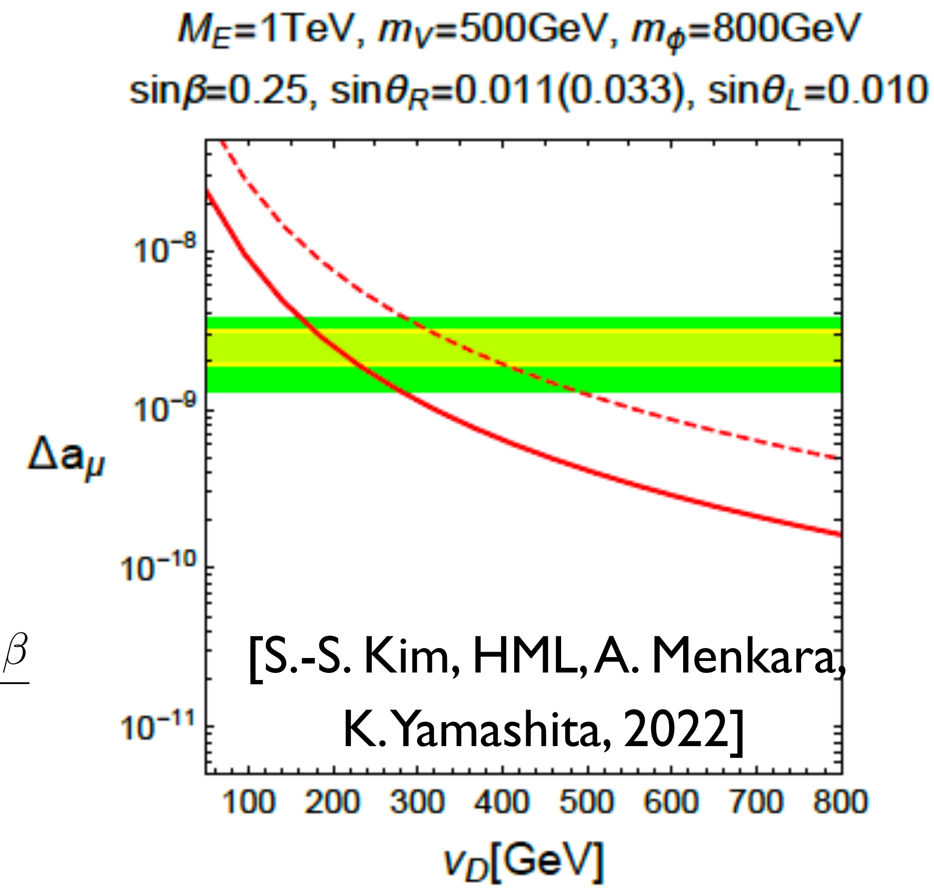
W-boson mass

$$\delta m_V \propto \delta M_W$$

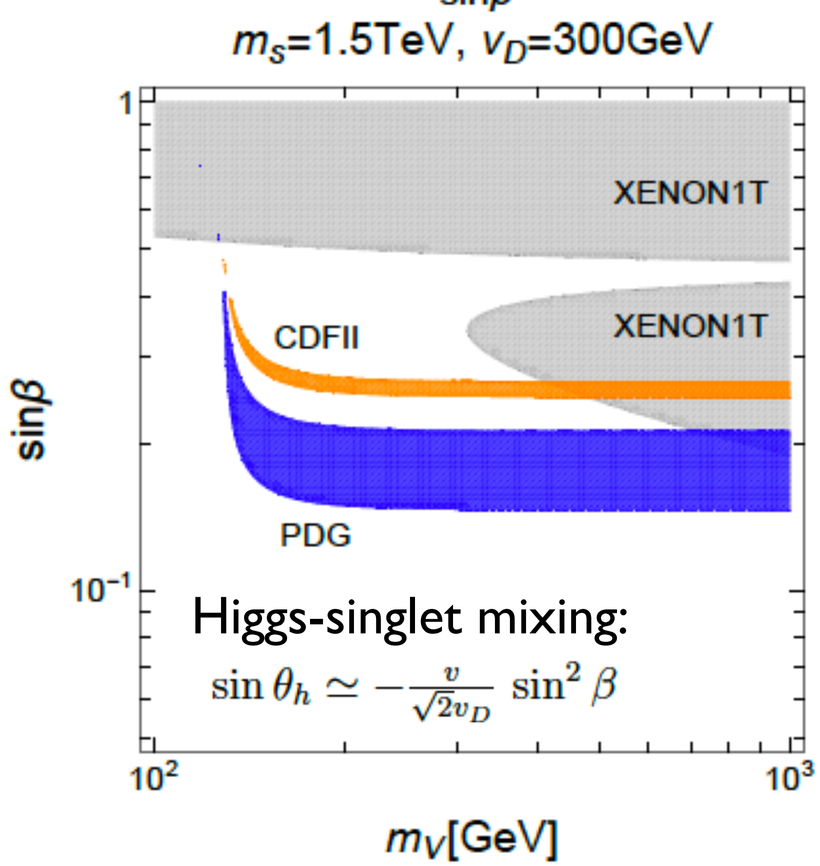
Quasi-degenerate SU(2) DM

Dark matter

$$V^+V^- \to V^0 V^0$$

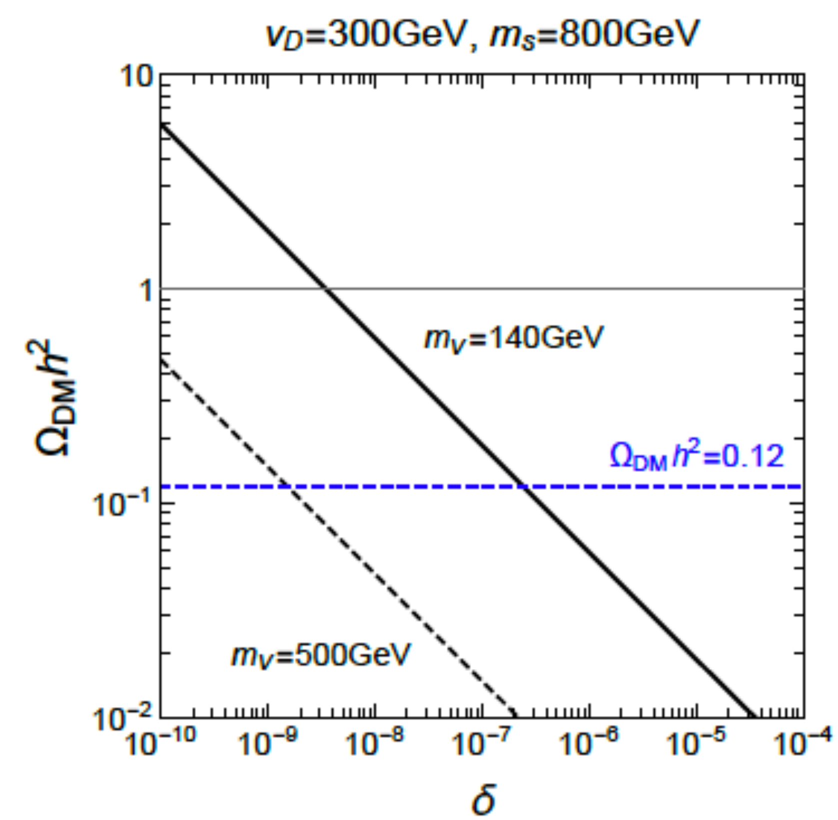


[S.-S. Kim, HML, A. Menkara, K. Yamashita, 2022]



Higgs-singlet mixing:

$$\sin\theta_h \simeq -\frac{v}{\sqrt{2}v_D}\sin^2\beta$$



Conclusions

- Z_4 discrete symmetry ensures small neutrino masses and dark matter stability in radiative seesaw models.

- After Z_4 is broken down to Z_2, dark scalar ϕ provides extra annihilation channels for inert doublet DM, opening up a window for heavy DM.

- Small charged lepton masses can be obtained by seesaw mechanism with vector-like leptons and extra U(1) or SU(2) gauge symmetries.

- SU(2) gauge bosons with nonzero dark isospin can be dark matter candidates as a bonus.