SU(5) GUT with Multi Vector Multiplets

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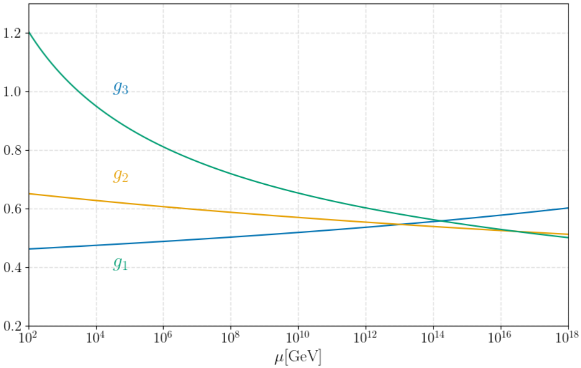
What is Grand Unified Theory?



- **Grand Unified Theory (GUT):** unifies the strong, weak, and electromagnetic interactions at a high energy scale, $10^{13-14}\,\mathrm{GeV}$.
- Quarks and leptons are unified, typically implying proton decay.
- Typical gauge groups are $\mathrm{SU}(5)$, $\mathrm{SO}(10)$, and E_6.

In this work, we focus on $\mathrm{SU}(5)$ GUT.

Minimal SU(5) GUT

$$\underset{\text{GUT}}{\mathrm{SU}(5)} \supset \underset{\text{Standard Model(SM)}}{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y}$$

- **Matter unification**: SM particles embedded into $(\bar{\mathbf{5}} \oplus \mathbf{10}) \times 3$ chiral representation.

$$\bar{\mathbf{5}} = \begin{pmatrix} \bar{d}_R^1 \\ \bar{d}_R^2 \\ \bar{d}_R^3 \\ e_L \\ -\bar{\nu}_L \end{pmatrix}, \quad \mathbf{10} = \begin{pmatrix} 0 & \bar{u}_R^3 & -\bar{u}_R^2 & u_L^1 & d_L^1 \\ -\bar{u}_R^3 & 0 & \bar{u}_R^1 & u_L^2 & d_L^2 \\ \bar{u}_R^2 & -\bar{u}_R^1 & 0 & u_L^3 & d_L^3 \\ -u_L^1 & -u_L^2 & -u_L^3 & 0 & \bar{e}_R \\ -d_L^1 & -d_L^2 & -d_L^3 & -\bar{e}_R & 0 \end{pmatrix}$$

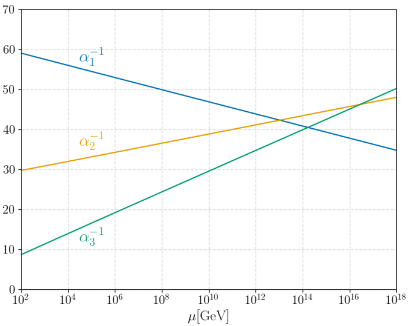
chiral = no mass term

- Only gauge boson X, colored Higgs boson H_c, and 24-dim Higgs boson Σ appear.
- SM emerges from spontaneous breaking of $\mathrm{SU}(5)$ by $\langle \Sigma \rangle = v \times \mathrm{diag}(2, 2, 2, -3, -3)$.

Known Issues in Minimal SU(5) GUT

Two classic problems

- **Gauge coupling non-unification**
- **Too short proton lifetime**



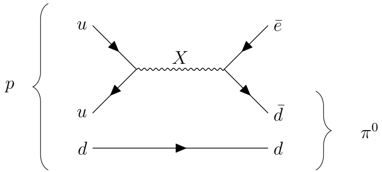
Lifetime estimate:

$$\tau\left(p \to \pi^0 + e^+\right) \simeq 5 \times 10^{26} \left(\frac{M_X/g_5}{10^{14}\,\mathrm{GeV}}\right)^4 \ \mathrm{yr}$$

Current experimental limit(Super Kamiokande):

$$\tau(p \to \pi^0 + e^+) \gtrsim 2.4 \times 10^{34}\ \mathrm{yr}$$

Ref.: A. Takenaka *et al.* (Super-Kamiokande), Phys. Rev. D **102** (2020) 112011



Possible Solutions

- Supersymmetry
- Extra dimension
- **Multiple vector-like fermions** $\Leftarrow$ **New!!**

What is vector-like fermion?

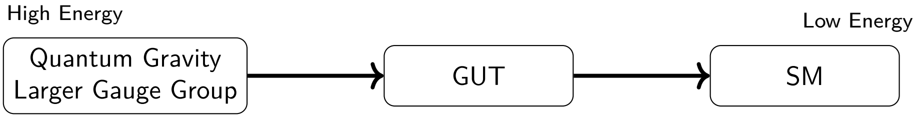
- Dirac fermion with identical left/right gauge transformations

$\longleftrightarrow$ fermion with different left/right gauge transformations

$=$ Chiral fermion (e.g. SM)

- Have a mass

Why Vector-Like Fermions?



The GUT scale is close to fundamental scales such as the string or Planck scale. . .

Many additional particles nearby the GUT scale.

Ref.: H. Georgi, Nucl. Phys. B **156** (1979) 126
R. Barbieri and D. V. Nanopoulos, Phys. Lett. B **91** (1980) 369
T. Watari, Phys. Lett. B **762** (2016) 145

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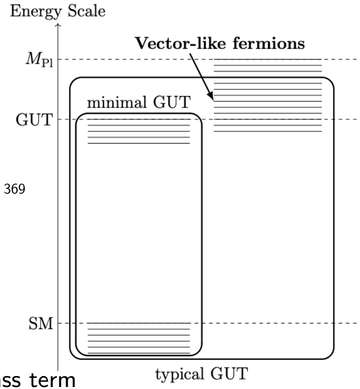
Typical GUT models contain multiple heavy fields.
We focus on heavy fermions, known as
vector-like fermions.

vector-like = have mass term



Model Setup

Matter Content (contain three chiral generations)

$$(\overline{\mathbf{5}},\,\mathbf{10}) \times 3 \;+\; (\mathbf{5},\,\overline{\mathbf{5}}) \times n_5 \;+\; (\mathbf{10},\,\overline{\mathbf{10}}) \times n_{10}$$

Vector-like fermions have mass term(complex matrix),

$$\mathcal{L}_M = -\frac{1}{2}\,\mathrm{tr}[\mathbf{10}\,(M_{10} + Y_{10}\,\Sigma^T)\,\overline{\mathbf{10}}] - \overline{\mathbf{5}}\,(M_5 + Y_5\,\Sigma)\,\mathbf{5} + \mathrm{h.c.}\,.$$

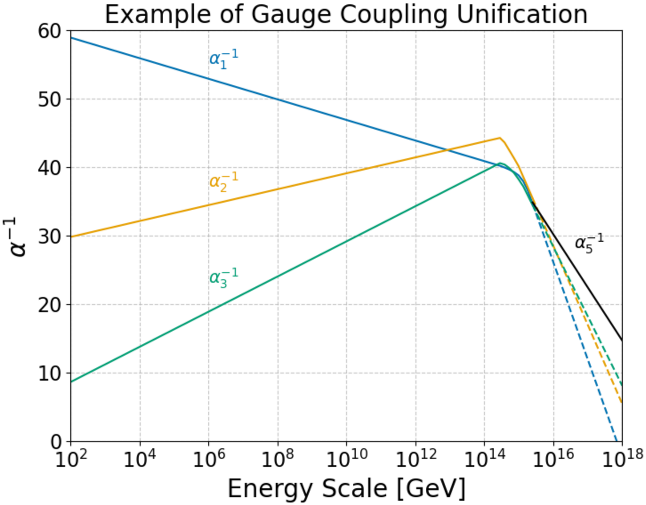
Assign masses from Gaussian distributions

$$(M_{10,5})_{ij} = \frac{1}{\sqrt{2}}\,(\mathcal{N}(0,1) + i\mathcal{N}(0,1)) \times M_0$$

$$(Y_{10,5})_{ij} = \frac{1}{\sqrt{2}}(\mathcal{N}(0,1) + i\mathcal{N}(0,1))\ .$$

We suppose $M_0 \sim M_{\mathrm{GUT}}$.

Result: Gauge Coupling Unification



- An example of gauge coupling unification at $n_{10} = n_5 = 15$.
- Gauge coupling unification cannot be achieved if $n_{10,5}$ is too small.
- $M_X \sim 10^{15.5}\,\mathrm{GeV}$

  $\leftrightarrow \; M_X|_{minimal} \sim 10^{13.6}\,\mathrm{GeV}$

Result: Suppression of Proton Decay

Main Suppression Factors

- Increase in the X boson mass M_X
- Suppression by $\kappa^{(1,2)}$

Proton decay operator:

$$\mathcal{L} = -\frac{g_5^2}{M_X^2}\kappa^{(1,2)}_{\alpha\beta\gamma\delta}\mathcal{O}^{(1,2)}_{\alpha\beta\gamma\delta} + \mathrm{h.c.}$$

$$\mathcal{O}^{(1)}_{\alpha\beta\gamma\delta} = \varepsilon^{abc}\varepsilon^{pq}\left(\hat{\bar{u}}^{\dagger}_{a\beta}\hat{\bar{d}}^{\dagger}_{b\delta}\right)\left(\hat{q}_{cp\alpha}\hat{\ell}_{q\gamma}\right)\ ,$$
$$\mathcal{O}^{(2)}_{\alpha\beta\gamma\delta} = \varepsilon^{abc}\varepsilon^{pq}\left(\hat{q}_{ap\alpha}\hat{q}_{bq\beta}\right)\left(\hat{\bar{u}}^{\dagger}_{c\gamma}\hat{\bar{e}}^{\dagger}_{\delta}\right)\ .$$

$\kappa^{(1,2)}$ are coefficient of proton decay operater.

- $M_X \sim 10^{15.5}\,\mathrm{GeV}$
  $\leftrightarrow\ M_X|_{minimal} \sim 10^{13.6}\,\mathrm{GeV}$
- Proton Lifetime $\propto M_X^4$

Result: Suppression of Proton Decay

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- Increase in the X boson mass M_X
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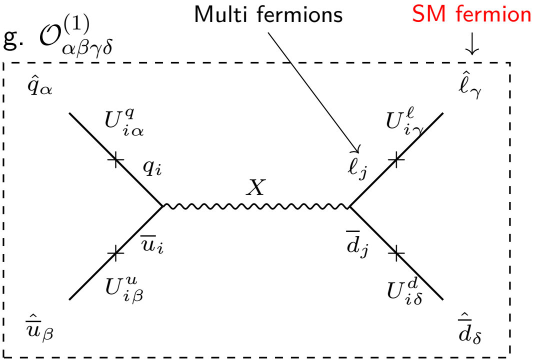
e.g. $\mathcal{O}^{(1)}_{\alpha\beta\gamma\delta}$



For an $n \times n$ random unitary matrix,

$$U_{ij} = \mathcal{O}\left(\frac{1}{\sqrt{n}}\right), \quad (U^\dagger U')_{ij} = \mathcal{O}\left(\frac{1}{\sqrt{n}}\right).$$

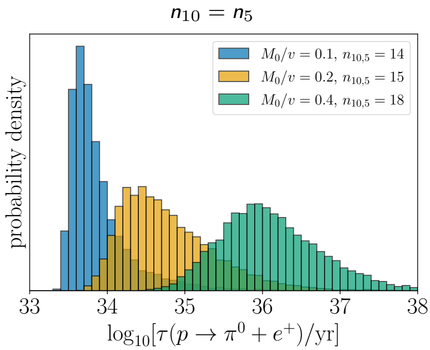
Coefficient $\kappa^{(1,2)}$ for large $n_{10,5}$:

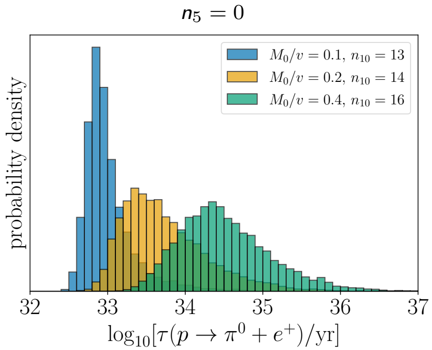
$$\kappa^{(1)}_{\alpha\beta\gamma\delta} \sim \frac{1}{\sqrt{n_{10}}} \times \frac{1}{\sqrt{n_5}},$$

$$\kappa^{(2)}_{\alpha\beta\gamma\delta} \sim \frac{1}{\sqrt{n_{10}}} \times \frac{1}{\sqrt{n_{10}}},$$

Result: Proton Lifetime Predictions

- With $n_{10,5} \sim 15$, proton lifetime potentially observable.
- Other channels are also longer than the experimental limits.





$$\mathrm{SK}: \quad \tau(p \to \pi^0 + e^+) \gtrsim 2.4 \times 10^{34}\,\mathrm{yr}\,, \quad \mathrm{HK}: \quad \tau(p \to \pi^0 + e^+) \gtrsim 7.8 \times 10^{34}\,\mathrm{yr}$$

Ref.: A. Takenaka *et al.* (Super-Kamiokande), Phys. Rev. D **102** (2020) 112011
K. Abe *et al.* (Hyper-Kamiokande), arXiv:1805.04163 (2018)

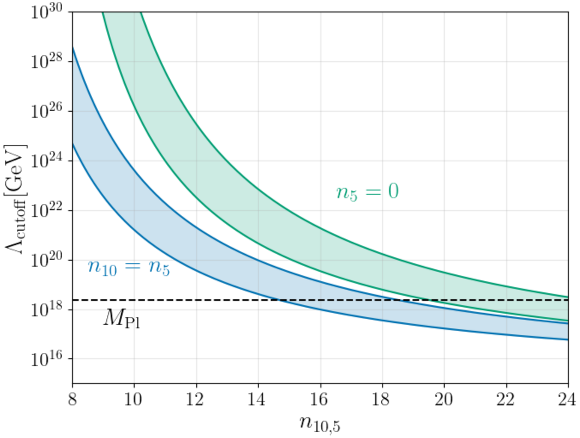
Summary and Future Work

- Multi vector-like fermions can solve minimal $\mathrm{SU}(5)$ GUT problems, gauge coupling unification, and proton decay.
- For $n_{10,5} \simeq 15$, the result becomes phenomenologically acceptable.
- As future work, we plan to extend this framework to $SO(10)$ or SUSY GUT models.

End

Upper Limit on the Number of Heavy Fermions

Good unification $\neq$ unlimited fermions: Landau poles set an upper bound.

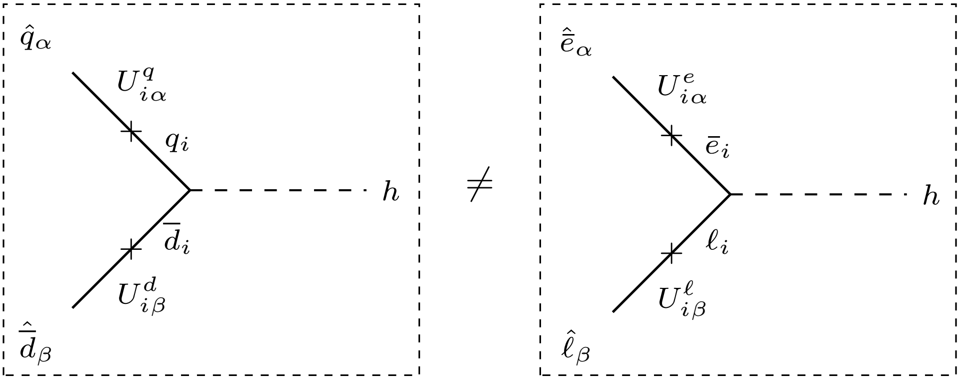


- Many fermions accelerate the divergence of the coupling g_5.
- For $n_5 = n_{10} \lesssim 18$ or $n_5 = 0, n_{10} \lesssim 24$, no landau pole appears.

$g_5\,(\Lambda_{\rm cutoff}) = 4\pi$
$g_5(10^{15}\,\mathrm{GeV}) = 0.48\text{–}0.55$

Vector-Like Effects on Yukawa Couplings

With the extra fermions, the Yukawa couplings are given by $(n_{10,5} + 3) \times (n_{10,5} + 3)$ matrices.



The bottom-tau unification problem can be avoided (note that $UYU' = \mathcal{O}(Y)$).