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QUANTUM EFFECTS ON NEUTRINO PARAMETERS FROM A FLAVORED GAUGE BOSON

GENERATING NEUTRINO MASSES VIA ONE-LOOP RGE RUNNING

2nd Hokkaido Workshop on Particle Physics at Crossroads

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YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS
京都大学 KYOTO UNIVERSITY
KEK 加速器だから見える世界。 Insight through Accelerators.
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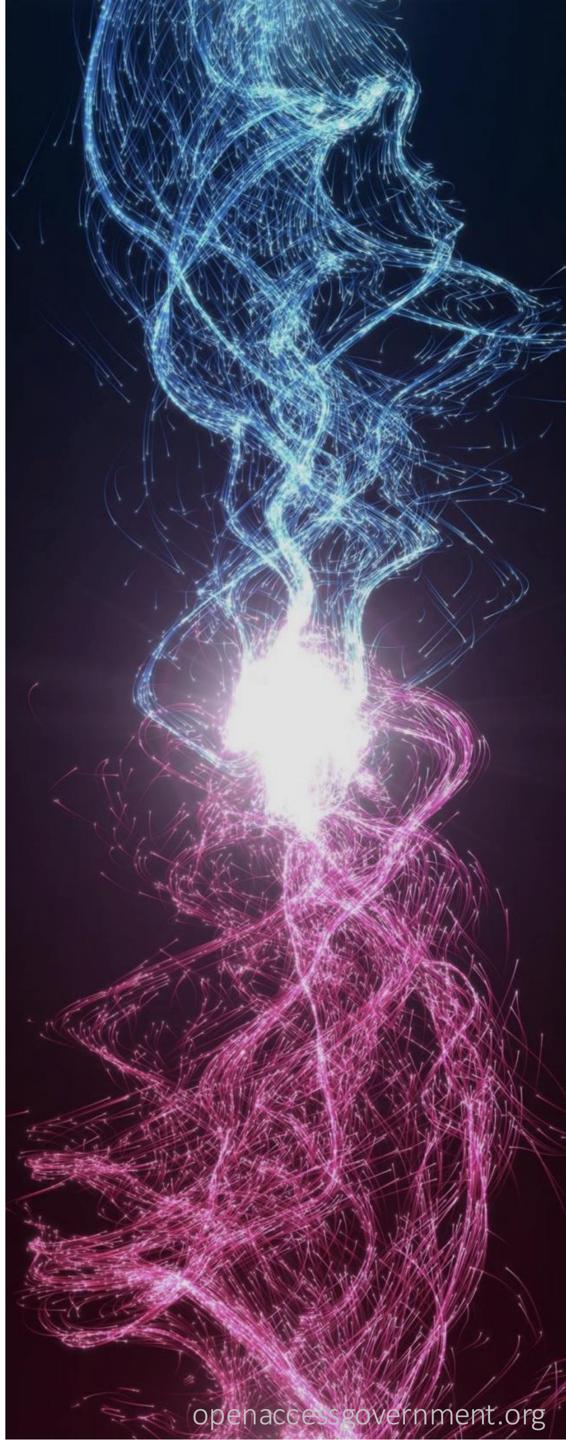
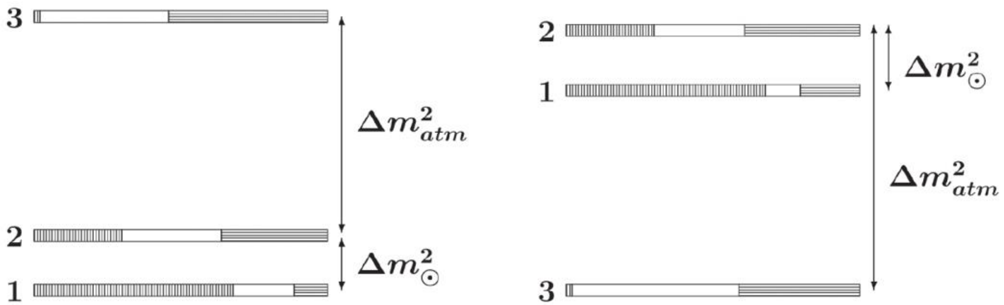
TUM



INTRODUCTION

- We know neutrinos have mass from oscillation data, don't know their hierarchy

P. Langacker, 2020

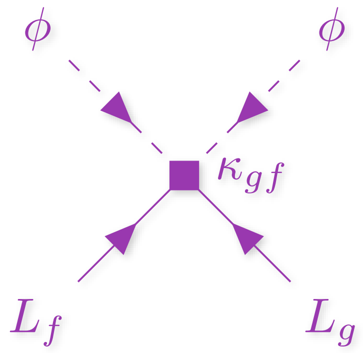


"Normal hierarchy" "Inverted hierarchy"

- Describe using dim. 5 effective SM interaction

$$\mathcal{L}_\kappa \sim \kappa_{gf}\, L_g\, L_f\, \phi\, \phi \rightsquigarrow \underbrace{\kappa_{gf}\, v^2}_{\sim\, m_\nu^{gf}}\, \nu_g\, \nu_f$$

Weinberg, 1979



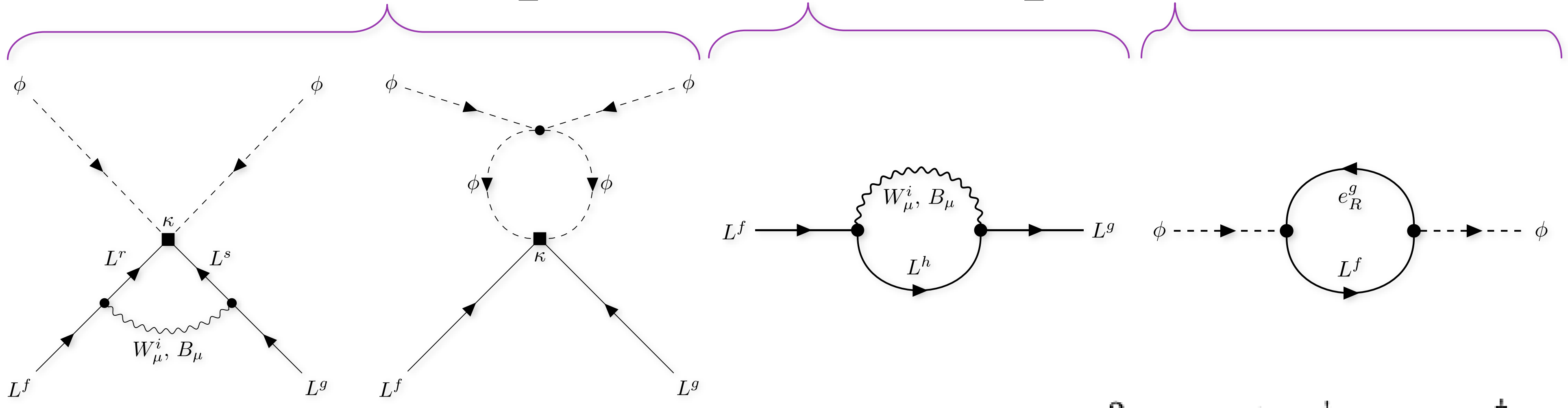
- Loop effects may be important; different scales involved in mass generation, creation, detection → <u>Running!</u>

BETA FUNCTION THE WEINBERG OPERATOR

- General formula: $\beta_\kappa = \delta\kappa_{,1} - \frac{1}{2}\left(\delta Z_{\phi,1} + \delta Z_{l,1}\right)^T \kappa - \frac{1}{2}\kappa\left(\delta Z_{\phi,1} + \delta Z_{l,1}\right)$

Babu+, 1992; Casas+, 2000; Antusch+, 2001

e.g.



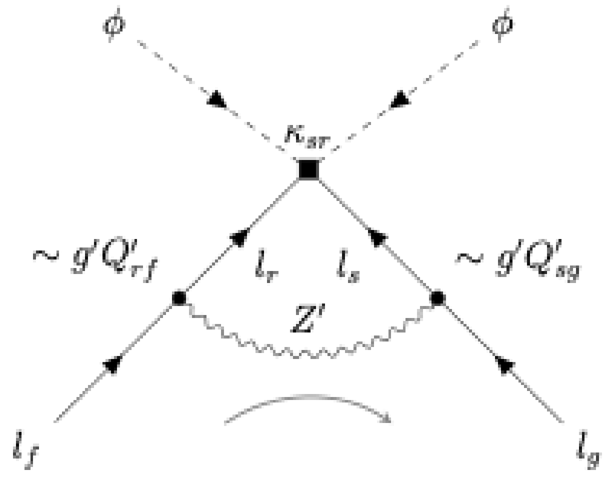
$$\alpha = \lambda - 3g_2^2 + 2\mathrm{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e)$$

$$P = -\frac{3}{2}(Y_e^\dagger Y_e)$$

- → Has structure $\beta_\kappa^{(\mathrm{SM})} = \alpha\,\kappa + P^T\,\kappa + \kappa\,P$
- We consider effects of flavor-dependent gauge couplings, such as $U(1)_{L_\mu - L_\tau}$
- Motivations: oscillation data, $g_\mu - 2$, dark matter, anomaly freedom,…

GENERATING MASSES VIA RGE RUNNING

Vertex renormalization



$$\frac{\mathrm{d}\kappa}{\mathrm{d}t} = \alpha\kappa + P^T\kappa + \kappa P + G^T\kappa G$$

$G = \sqrt{6}g'Q'$

SM: $\alpha\kappa + P^T\kappa + \kappa P$

$U(1)_{L_\mu - L_\tau}$: $G^T\kappa G$

$$\frac{\mathrm{d}\kappa_i}{\mathrm{d}t} = \alpha\,\kappa_i + 2(U\,P\,U^T)_{ii}\,\kappa_i + \sum_{k=1}^{n}\left[(U\,G\,U^T)_{ki}\right]^2\kappa_k$$

If initial $\kappa_i = 0$, stays 0

If one initial $\kappa_k \neq 0$, generates n hierarchical masses

At one loop, flavor-dep. gauge interaction only way to get $\beta_\kappa \supset G^T\,\kappa\,G$ and raise rank of mass matrix

MIXING ANGLE CONDITION FOR DEGEN. EIGENVALUES

$$\frac{\mathrm{d}\kappa_i}{\mathrm{d}t} = \alpha\,\kappa_i + 2(U\,P\,U^T)_{ii}\,\kappa_i + \sum_{k=1}^{n}\left[(U\,G\,U^T)_{ki}\right]^2\kappa_k$$

$$\tilde{A} = U\,A\,U^\dagger$$

$$\frac{\mathrm{d}U}{\mathrm{d}t} = T\,U$$

$$16\pi^2 T_{ij} = \begin{cases} i\sum_{k=1}^{n}\mathrm{Im}[\tilde{G}^2_{ki}]\,\frac{\kappa_k}{2\,\kappa_i} & \text{for } i=j, \\ \frac{\kappa_i+\kappa_j}{\kappa_i-\kappa_j}\mathrm{Re}[\tilde{P}_{ij}] + \boxed{\sum_{k=1}^{3}\frac{\kappa_k}{\kappa_i-\kappa_j}\mathrm{Re}[\tilde{G}_{ki}\,\tilde{G}_{kj}]} & \\ +\,i\left(\frac{\kappa_i-\kappa_j}{\kappa_i+\kappa_j}\mathrm{Im}[\tilde{P}_{ij}] + \sum_{k=1}^{3}\frac{\kappa_k}{\kappa_i+\kappa_j}\mathrm{Im}[\tilde{G}_{ki}\,\tilde{G}_{kj}]\right) & \text{for } i\neq j, \end{cases}$$

EXAMPLE: NORMAL HIERARCHY

For $\kappa_1 = \kappa_2 \ll \kappa_3$ we find

$$\left|\kappa_1|_{M_{Z'}}\right| \simeq \min_{i=1,2}\left\{\left|(a - bW_{1i}^2 - bW_{2i}^2)\kappa_2 - bW_{3i}^2\kappa_3\right|\right\},$$

$$\left|\kappa_2|_{M_{Z'}}\right| \simeq \max_{i=1,2}\left\{\left|(a - bW_{1i}^2 - bW_{2i}^2)\kappa_2 - bW_{3i}^2\kappa_3\right|\right\},$$

$$\left|\kappa_3|_{M_{Z'}}\right| \simeq \left|\kappa_3\right|$$

$$a = 1 - \alpha/(16\pi^2)\log(M_1/M_{Z'})$$

$$b = 6g'^2/(16\pi^2)\log(M_1/M_{Z'})$$

$$W_{ij} = \widetilde{G}_{ij}/(\sqrt{6}g')$$

$$W_{31} \simeq 0: \qquad \sin\theta_{13} \simeq -\frac{\sin\theta_{12}\cos\theta_{12}\tan\theta_{23}}{1+\sin^2\theta_{12}}$$

For $\kappa_1 = \kappa_2 = 0$, at leading log

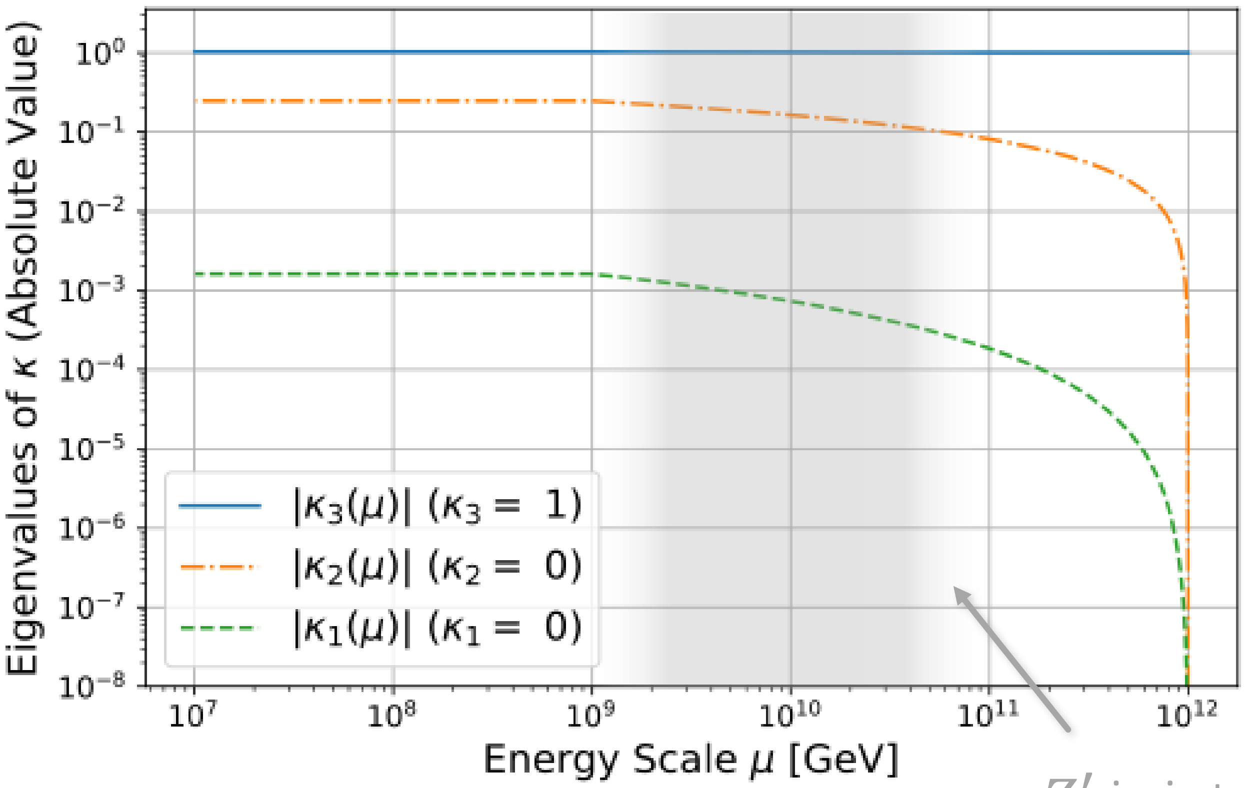
$$\kappa_1|_{M_{Z'}} \simeq 0$$

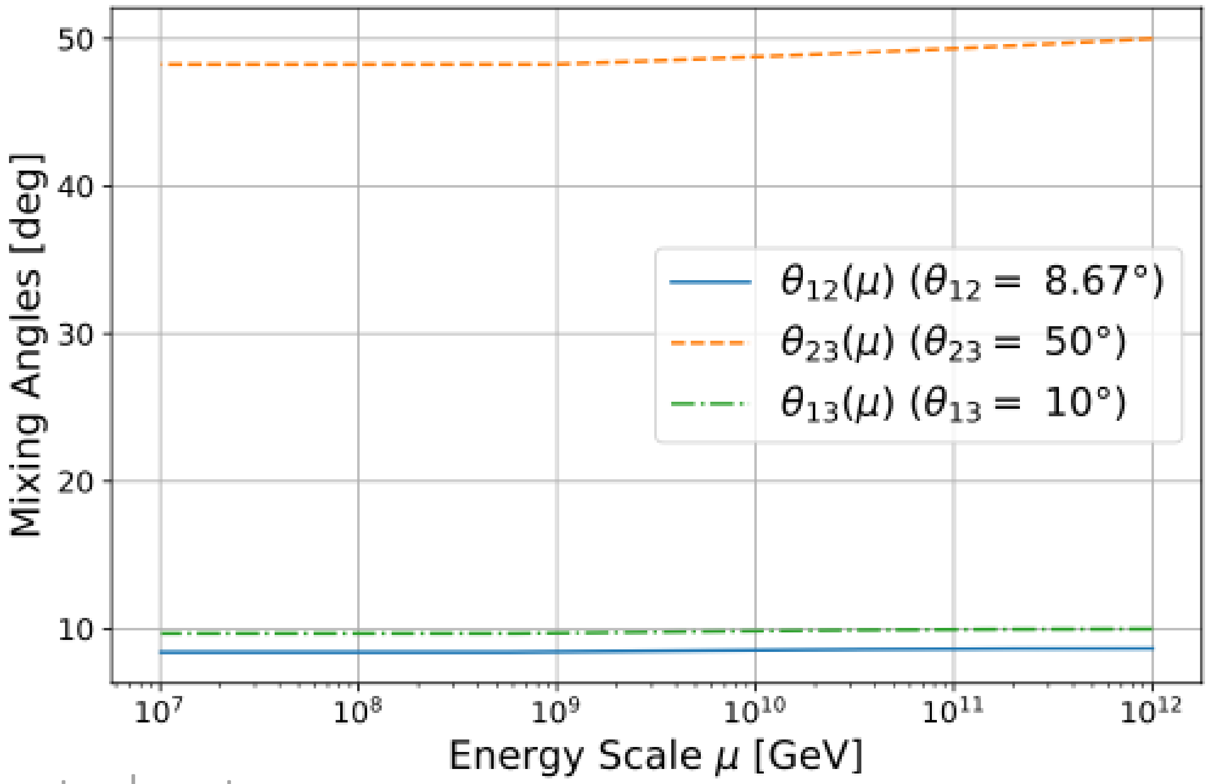
$$\kappa_2|_{M_{Z'}} \simeq -bW_{32}^2\kappa_3$$

$$\kappa_3|_{M_{Z'}} \simeq \kappa_3$$

$\kappa_1|_{M_{Z'}}$ generated at subleading log level $\sim b^2$

EXAMPLE: NORMAL HIERARCHY





Z' is integrated out
→ running stops

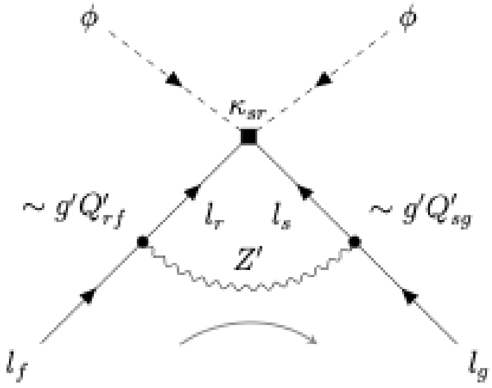
Generate <u>two new masses</u> via one-loop RGE
Can reproduce exp. splittings

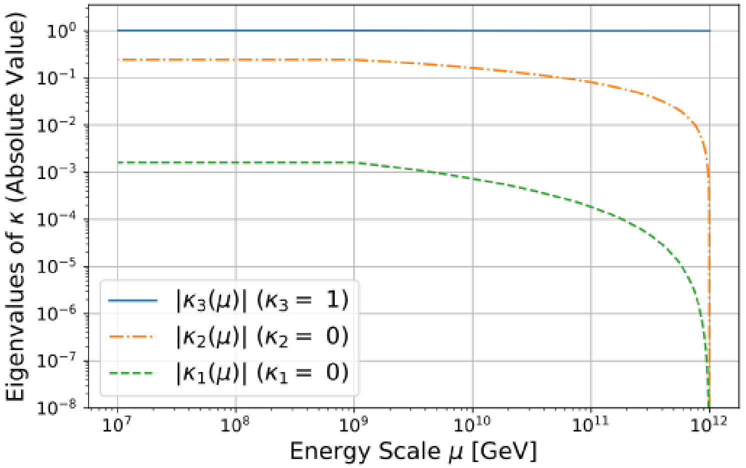
Including CP, Majorana phases can reproduce exp. angles

CONCLUSION

- One-loop RGE of effective mass operator in flavor-dependent gauge interaction vs. SM

$$\beta_\kappa = \alpha\,\kappa + P^T\,\kappa + \kappa\,P + G^T\,\kappa\,G \implies \frac{\mathrm{d}\kappa_i}{\mathrm{d}t} = \#\kappa_i + \sum_k \#\kappa_k$$





$$\sin\theta_{13} \simeq -\frac{\sin\theta_{12}\cos\theta_{12}\tan\theta_{23}}{1+\sin^2\theta_{12}}$$

Generate masses, explain mixing angles, predict mass splittings and hierarchy!

BACKUP

UV COMPLETION: EXAMPLE

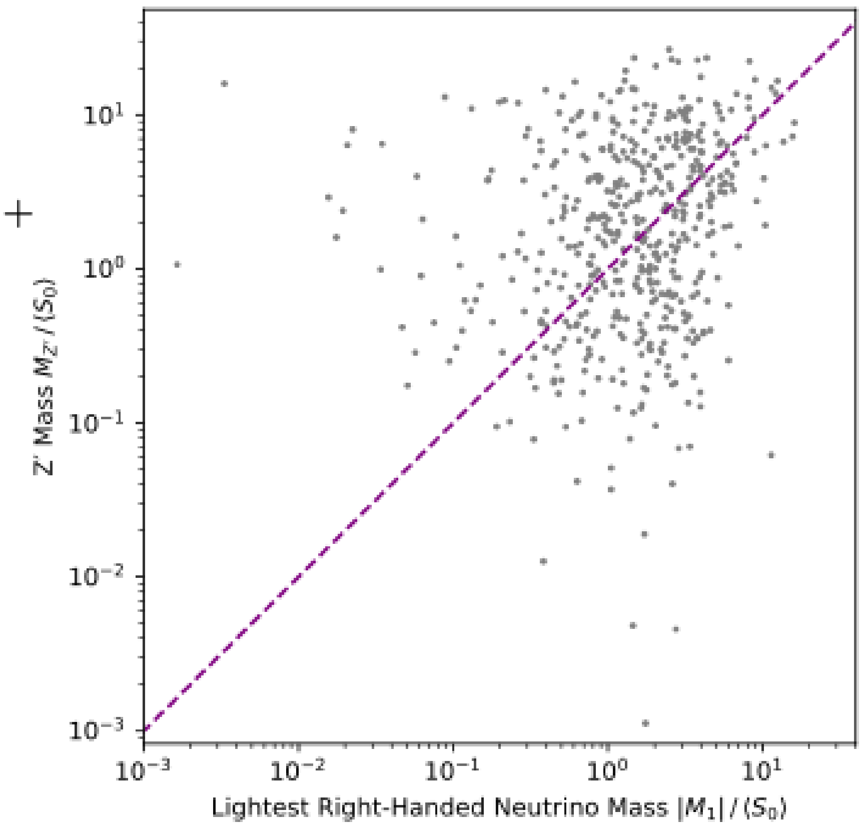
Field	$U(1)_{L_\mu - L_\tau}$ charge	Field	$U(1)_{L_\mu - L_\tau}$ charge
N_1	0	S_0	0
N_2	1	S_1	-1
N_3	-1	S_2	-2

$$2\mathcal{L}_{\text{mass}} = M_{11}\overline{N_1^C}N_1 + M_{23}\overline{N_2^C}N_3 + \text{h.c.},$$

$$2\mathcal{L}_{\text{Yuk}} = 2Y_{\nu,e1}\bar{l}_e\epsilon\phi^* N_1 + 2Y_{\nu,\mu 2}\bar{l}_\mu\epsilon\phi^* N_2 + 2Y_{\nu,\tau 3}\overline{l_\tau}\epsilon\phi^* N_3 + \lambda_{11}S_0\overline{N_1^C}N_1 + \lambda_{23}S_0\overline{N_2^C}N_3 + \\ + \lambda_{12}S_1\overline{N_1^C}N_2 + \lambda_{13}S_1^\dagger\overline{N_1^C}N_3 + \lambda_{22}S_2\overline{N_2^C}N_2 + \lambda_{33}S_2^\dagger\overline{N_3^C}N_3 + \text{h.c.}$$

$$M_R = \begin{pmatrix} M_{11} + \lambda_{11}\langle S_0\rangle & \lambda_{12}\langle S_1\rangle & \lambda_{13}\langle S_1\rangle \\ \lambda_{12}\langle S_1\rangle & \lambda_{22}\langle S_2\rangle & M_{23} + \lambda_{23}\langle S_0\rangle \\ \lambda_{13}\langle S_1\rangle & M_{23} + \lambda_{23}\langle S_0\rangle & \lambda_{33}\langle S_2\rangle \end{pmatrix}$$

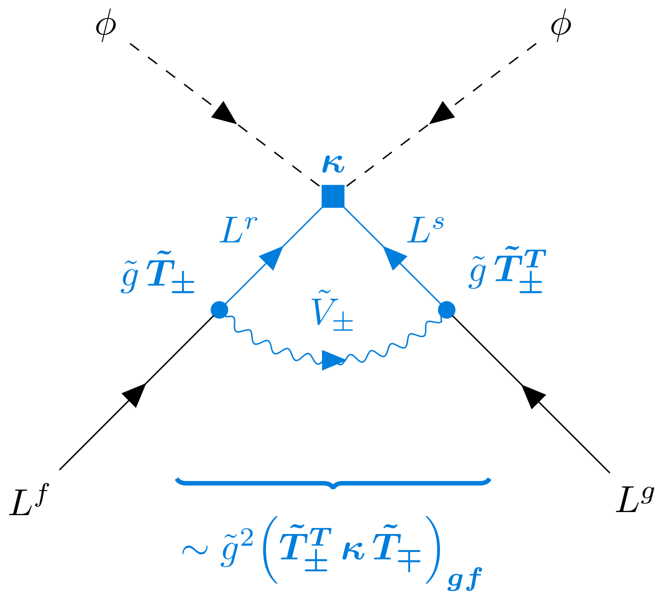
$$M_{Z'} = \sqrt{2}g' \cdot \sqrt{\langle S_1\rangle^2 + 4\langle S_2\rangle^2}$$



NON-ABELIAN GAUGE GROUPS

$$\beta_\kappa = \alpha\,\kappa + P^T\,\kappa + \kappa\,P + G^T\,\kappa\,G + \frac{1}{2}\left[G_+^T\,\kappa\,G_- + G_-^T\,\kappa\,G_+\right]$$

$$\frac{\mathrm{d}\kappa_i}{\mathrm{d}t} = \alpha\,\kappa_i + 2\tilde{P}_{ii}\,\kappa_i + \sum_{k=1}^{n} \mathrm{Re}\left\{\left[\tilde{G}_{ki}\right]^2 + \left[\tilde{G}_{+,ki}\,\tilde{G}_{-,ki}\right]\right\}\kappa_k$$



Enhanced running!