Inflationary Universe
- my personal perspective -

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Introduction

What is Inflation?

Brout, Englert & Gunzig '77, Starobinsky '79, Guth '81, Sato '81, …

- Inflation is a quasi-exponential expansion of the Universe at its very early stage; perhaps at $t \sim 10^{-35}$ sec.

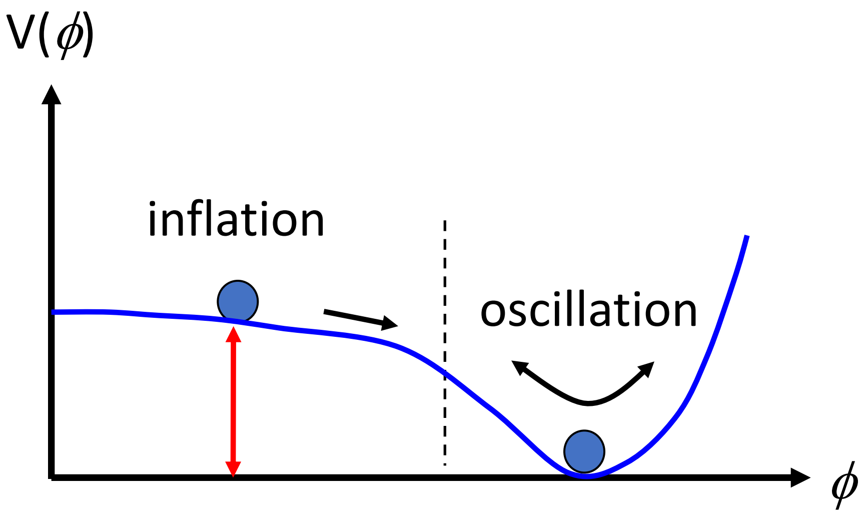
- It was meant to solve the initial condition (singularity, horizon & flatness, etc.) problems in Big-Bang Cosmology:
- if any of them can be said to be solved depends on precise definitions of the problems.

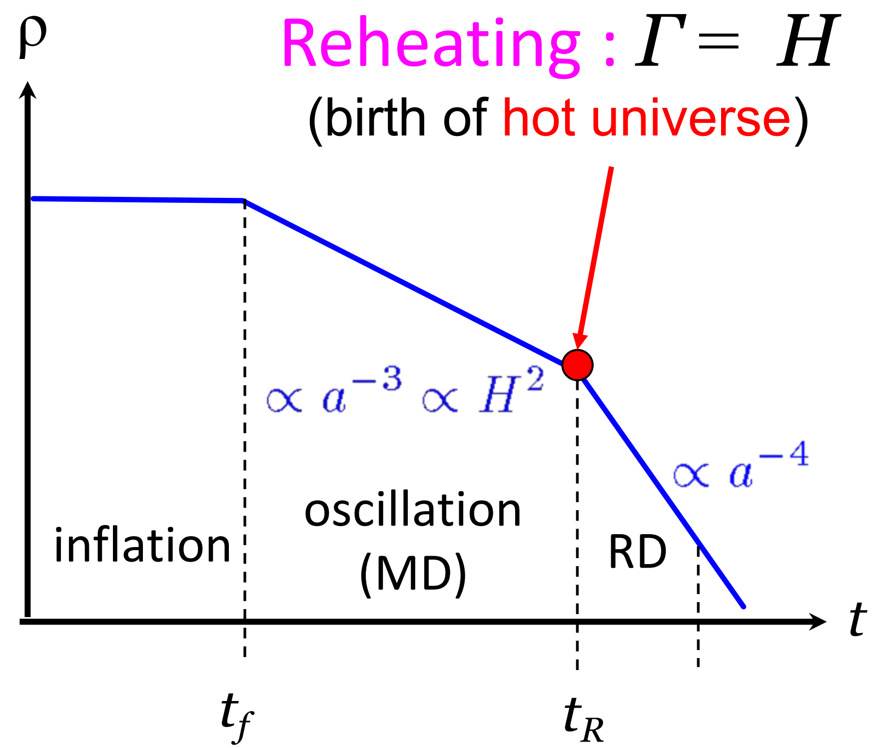
- Quantum vacuum fluctuations during inflation turn out to play the most important role. They give the initial condition for all the structures in the Universe.

- Cosmic gravitational wave background is also generated.

From inflation to bigbang

After inflation, vacuum energy is converted to thermal energy (called "re"heating) and hot Bigbang Universe is realized.





more on ^the meaning of^ inflation

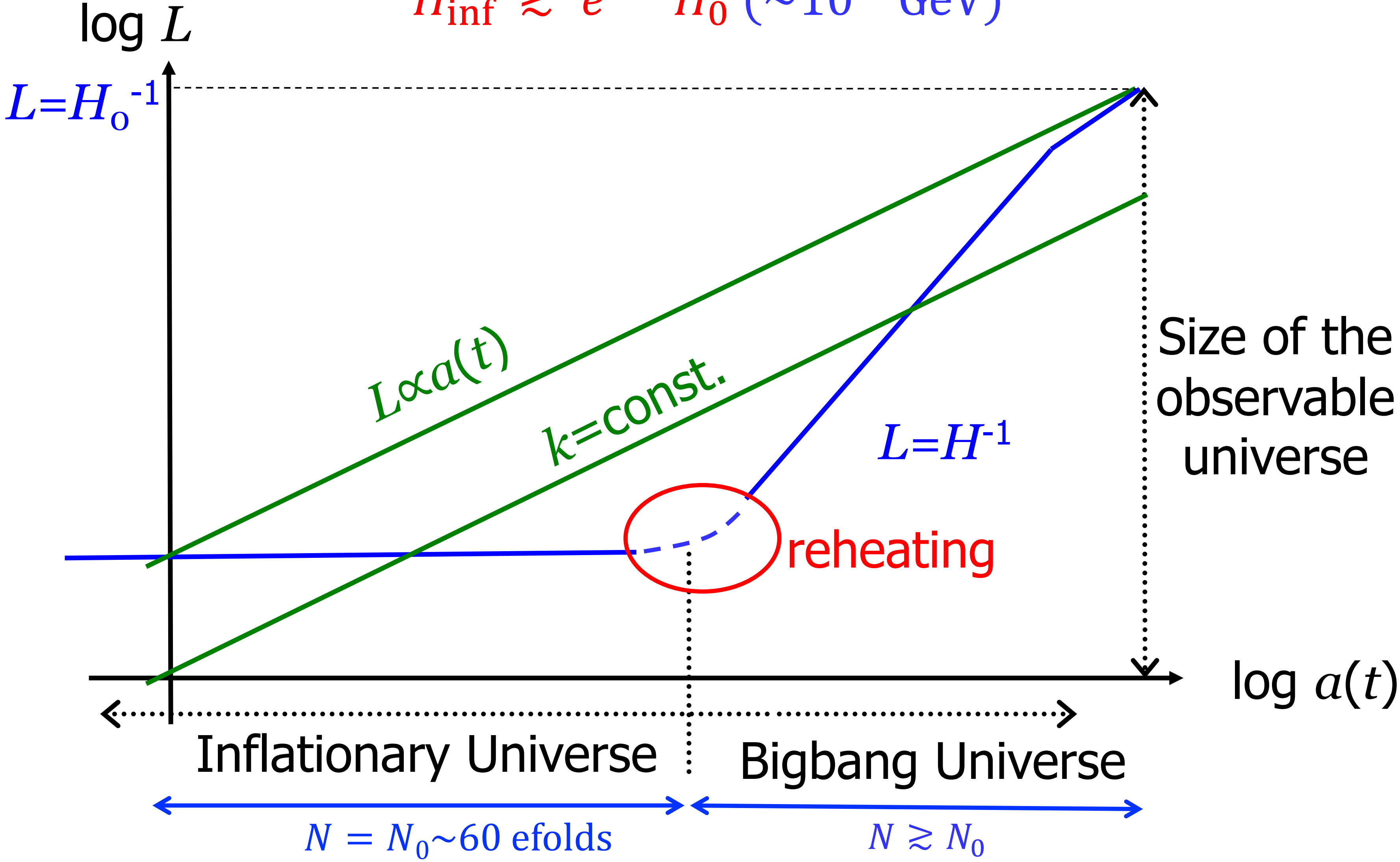
1. Homogeneity and isotropy are the (most important) assumptions, NOT a consequence of inflation.
2. Quasi-exponential expansion in the "Einstein frame": conformal invariant definition.
3. At least 50-60 e-folds before the end of inflation: solving "horizon problem"
4. Don't care what happened before inflation: predictions are almost independent of initial conditions.

1 & 2: basic assumptions/definition of inflation

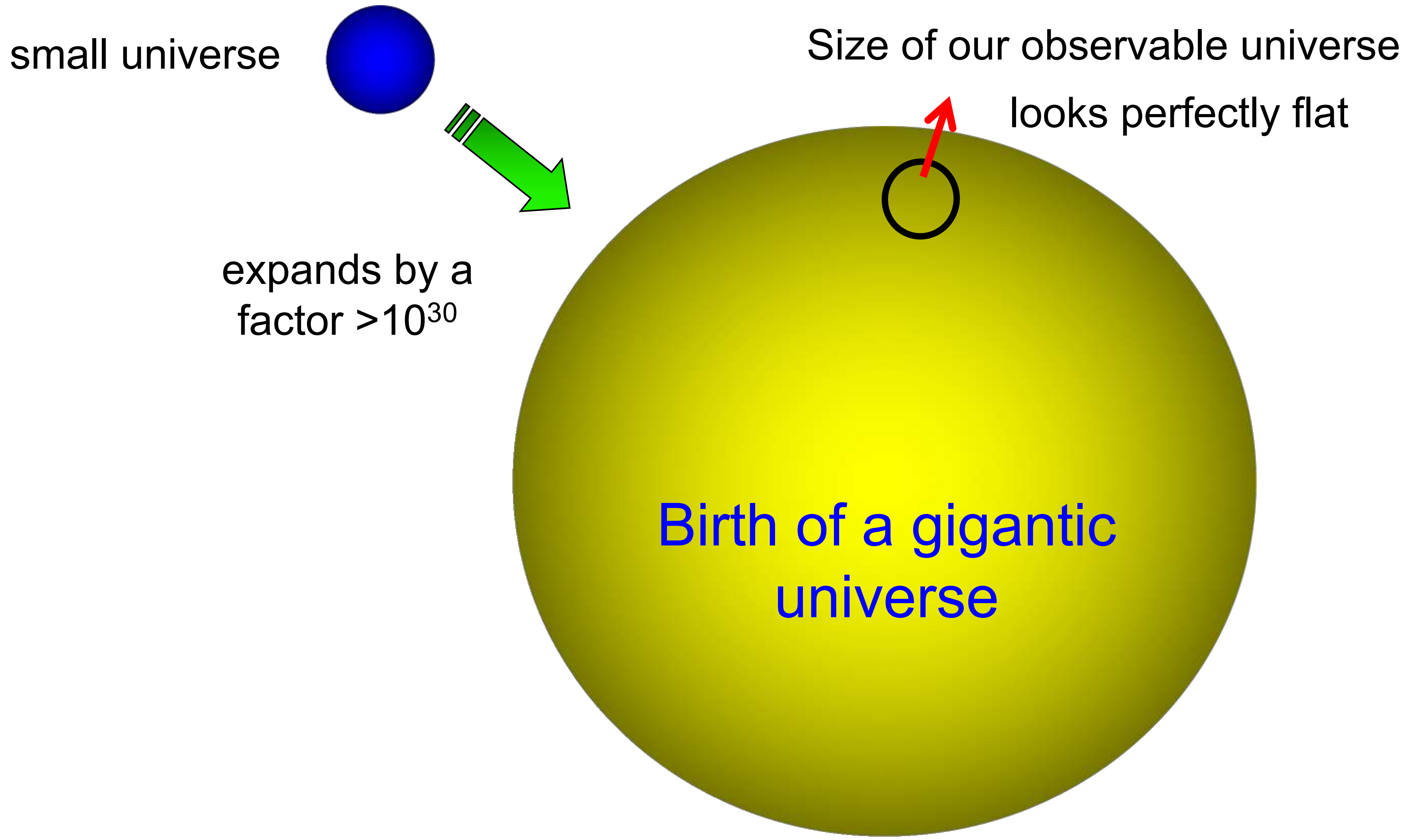
Causal structure

length scales of the inflationary universe

$H_{\text{inf}} \gtrsim e^{120} H_0$ ($\sim 10^{10}$GeV)



Flatness



Flatness can be explained by Inflation

NB: Inflation may not always imply flatness

Dynamics

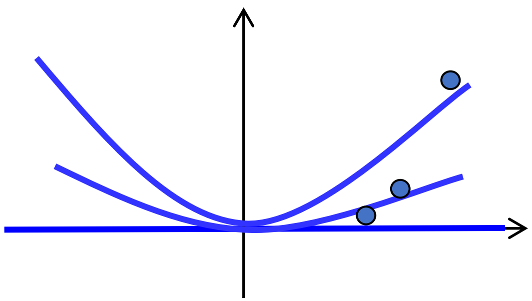
Seed of cosmological perturbations

Mukhanov & Chibisov '81, ….

Zero-point (vacuum) fluctuations of ϕ: $\delta\phi = \sum_k \delta\phi_k(t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \omega^2(t)\delta\phi_k = 0; \quad \omega^2(t) = \frac{k^2}{a^2(t)}$$

harmonic oscillator with friction term and time-dependent ω



$\delta\phi_k \rightarrow$ const.

$\cdots$ frozen when $\omega < H$
(on superhorizon scales)

tensor (gravitational wave) modes also satisfy the same eq.

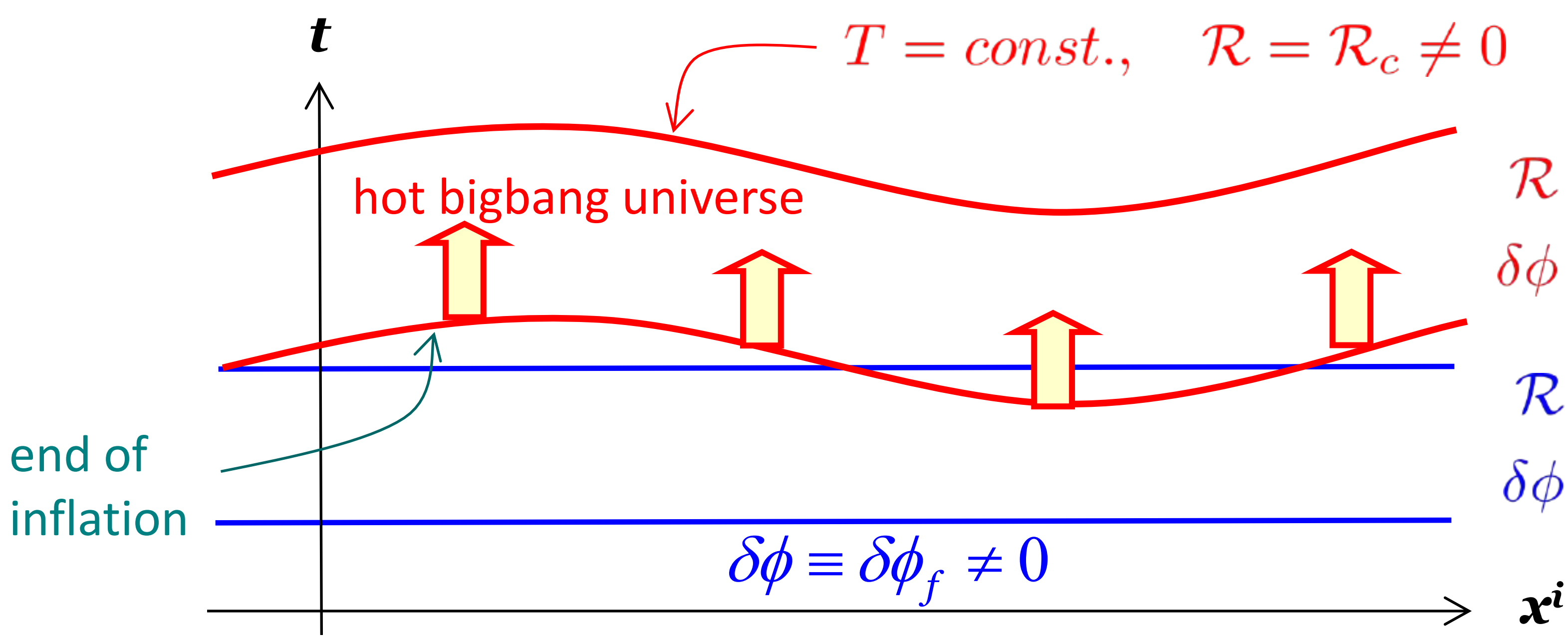
Starobinsky '79

Generation of Curvature Perturbation

curvature (potential) perturbation $\mathcal{R}$: $\delta R^{(3)} = -\frac{4}{a^2}\nabla^2\mathcal{R}$

curvature perturbation on comoving ($\delta\phi$ =0) slices $\mathcal{R}_c$ ~ Newton potential

- $\delta\phi$ is frozen on "flat" ($\mathcal{R}$ =0) 3-surface (t=const. hypersurface) $\mathcal{R}_c = -\frac{H}{\dot{\phi}}\delta\phi_f$
- Inflation ends/damped osc starts "comoving" (ϕ =const.) on 3-surface.



generic predictions of ∨ inflation

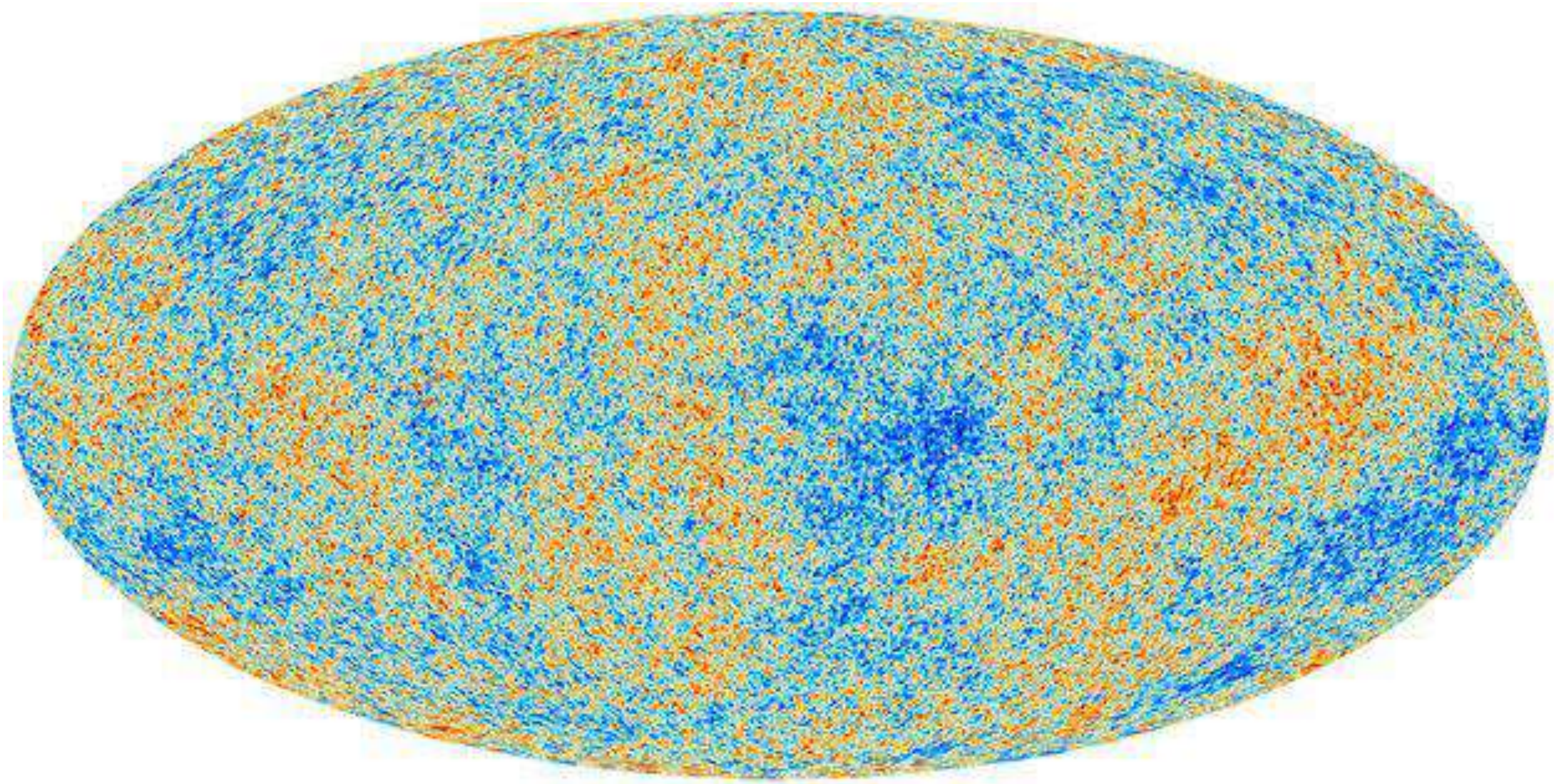
single-field slow-roll

- Spatially flat universe
- Almost scale invariant, adiabatic, Gaussian primordial scalar (curvature) perturbations
- Almost scale invariant, Gaussian primordial tensor (gravitational wave) perturbations

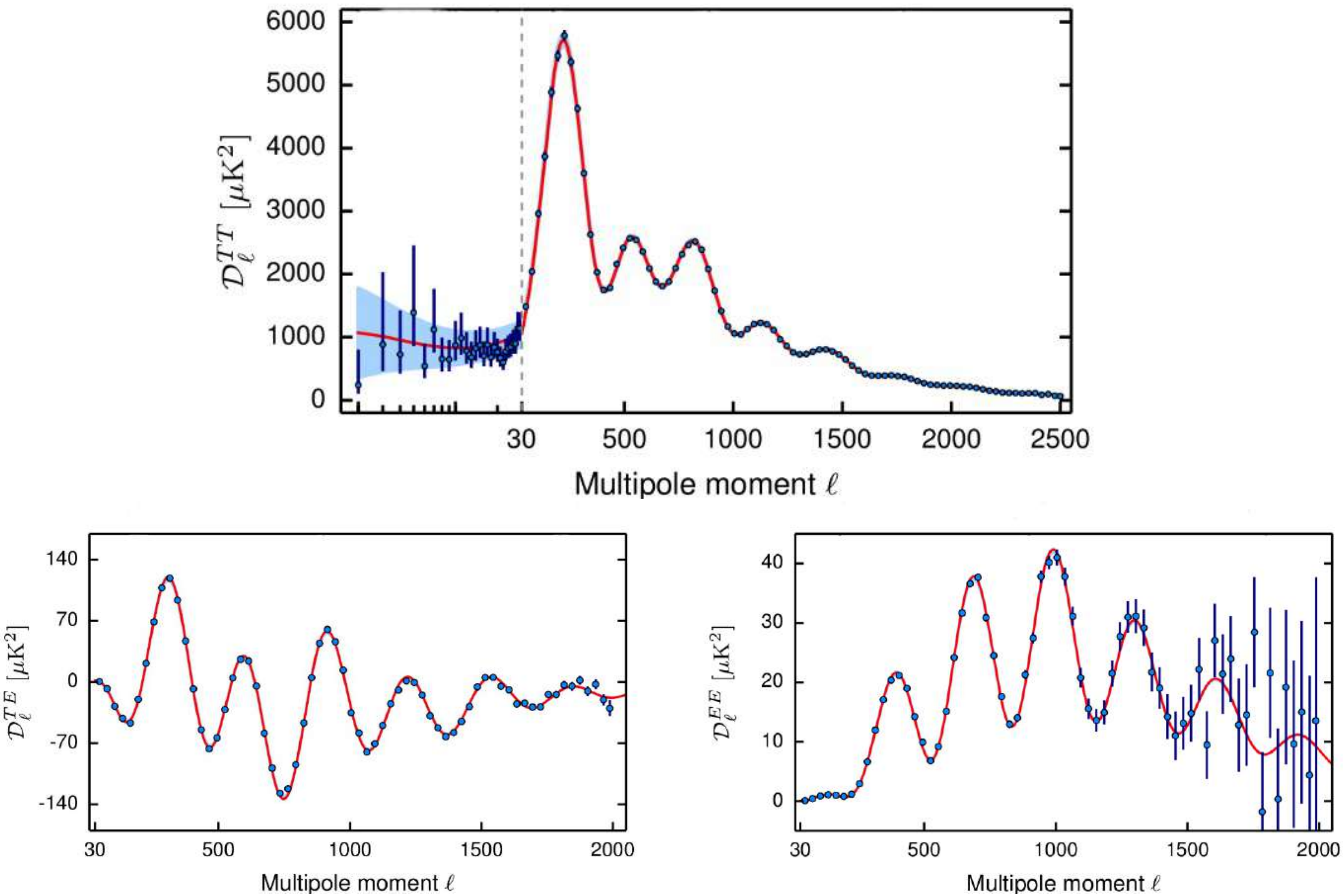
⇨ Generates CMB anisotropy
Origin of galaxies, stars, …

Observational results

CMB Full Sky Map by PLANCK



Planck TT, TE & EE spectrum



- Amplitude of curvature perturbation:

$$\mathcal{R}_c = \left.\frac{H^2}{2\pi\dot{\phi}}\right|_{k/a=H}$$

Mukhanov (1985), MS (1986)

$$\mathcal{R}_{c,\text{obs}} \sim 10^{-5} \;\Rightarrow\; V^{\frac{1}{4}}(\phi) \sim 10^{16}\,\text{GeV?}$$

- Power spectrum index:

$$M_P \equiv \frac{1}{\sqrt{8\pi G}} \sim 2.4\times 10^{18}\,\text{GeV: Planck mass}$$

$$\frac{4\pi k^3}{(2\pi)^3} P_S(k) = \left[\frac{H^2}{2\pi\dot{\phi}}\right]^2_{k/a=H} = Ak^{n_S-1}\;;\;\; n_S - 1 = M_P^2\left(2\frac{V''}{V} - 3\frac{V'^2}{V^2}\right)$$

Stewart-Lyth (1993)

$$n_{S,\text{Planck}} - 1 = -0.0355 \pm 0.0049 \;\Leftrightarrow\; n_S - 1 \sim -0.04$$ for a typical model

- Tensor (gravitational wave) spectrum:

Mukhanov & Chibisov (1981)

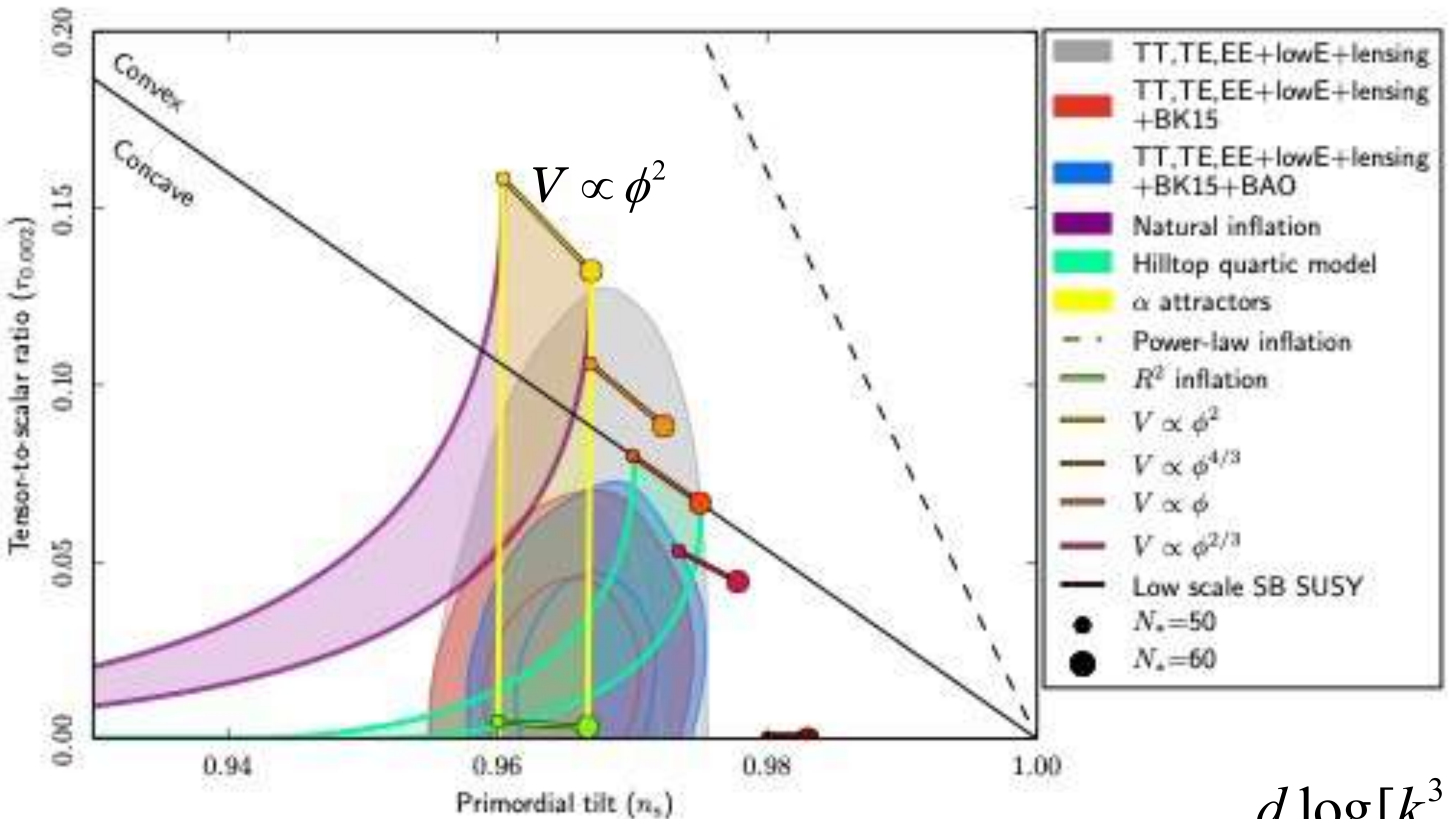
$$\frac{4\pi k^3}{(2\pi)^3} P_T(k) = Ak^{n_T}\;;\;\; n_T = -\frac{1}{8}\frac{P_T(k)}{P_S(k)} \equiv -\frac{r}{8}$$

Liddle-Lyth (1992)

to be observed by LiteBIRD !

Planck constraints on inflation

Planck 2018 results X



$$n_s - 1 \equiv \frac{d \log[k^3 P_S(k)]}{d \log k}$$

- scalar spectral index: n_s ~ 0.965 (0.974?)
- tensor-to-scalar ratio: r < 0.03
- simplest $V \propto \phi^2$ model is excluded

$$r \equiv \frac{P_T(k)}{P_S(k)}$$

ACT DR6

(Atacama Cosmology Telescope)

Implications

The most important message is:

Inflation as the Origin of All Structures in the Universe

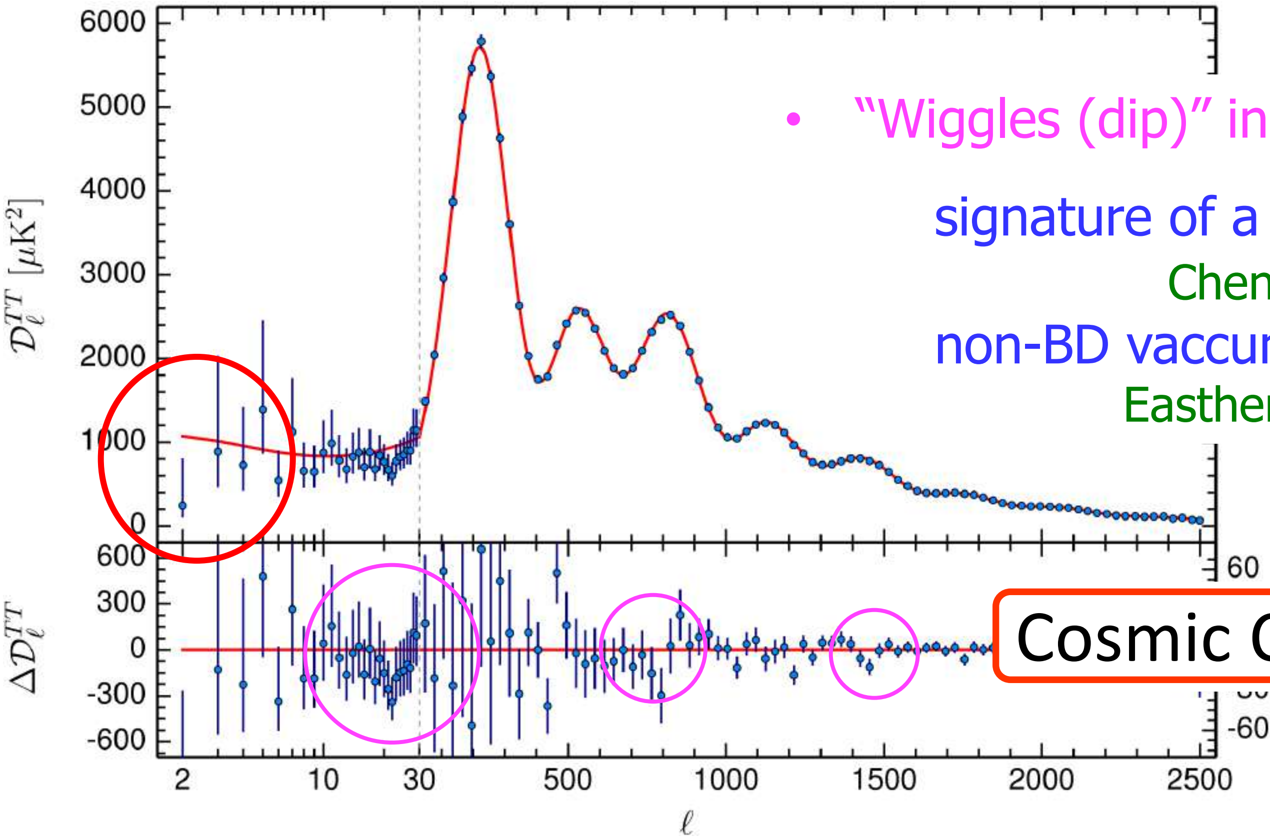
Current status

- scalar spectral index: $n_s<1$ at ~ 8 σ
- tensor/scalar ratio: $r < 0.03$ implies $E_{inflation} < 10^{16}$ GeV
- simple, canonical models are almost excluded
  ($m^2\phi^2$ model excluded at > 3 σ)
- R^2 (Starobinsky) model seems to fit best. But why?
  (large R^2 correction but negligible higher order terms)
- f_{NL}^{local} <O(1) suggests (effectively) single-field slow-roll
  (but non-slow-roll models with f_{NL}^{local} =O(1) not excluded)

elements of non-canonicality seem necessary

Beyond
(standard model of)
Inflation

Anomalies on (very) large scales?



- "Wiggles (dip)" in the power spectrum

  signature of a heavy field?

  Chen '11, …

  non-BD vaccum?

  Easther et al. '02,….

Cosmic Collider Physics

- Suppressed large scale fluctuations

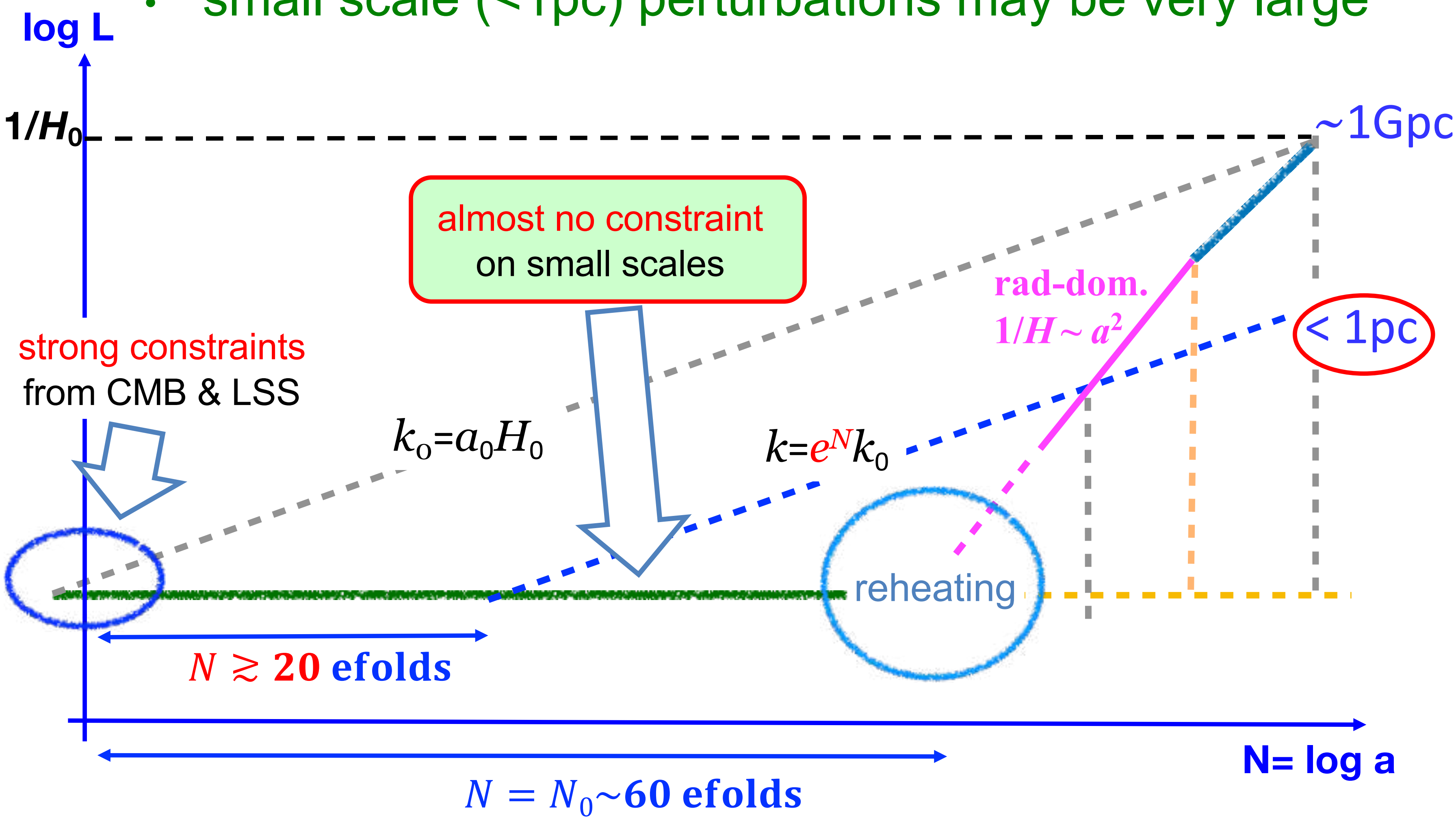
  quantum tunneling, open inflation?

  Linde, MS & Tanaka '99, ….

String Landscape?

New physics on small scales?

- we have no clues about the late stage of inflation
- small scale (<1pc) perturbations may be very large

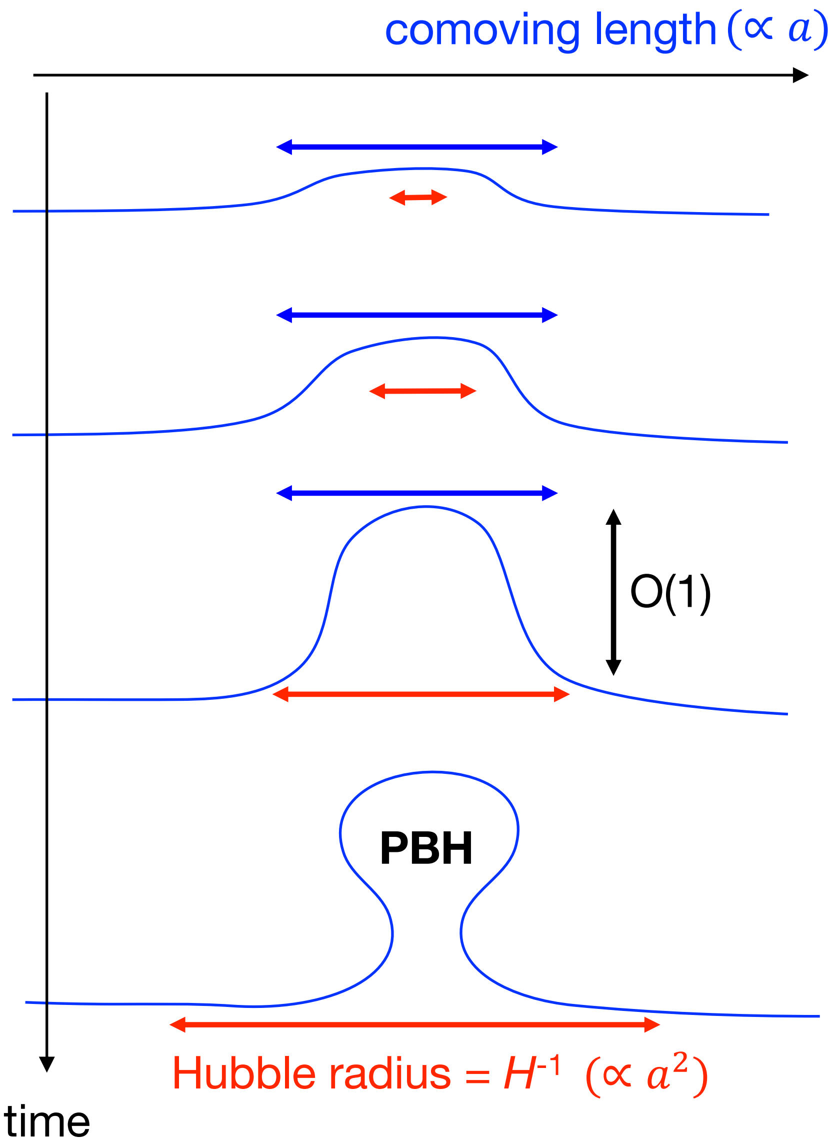


Primordial Black Holes!

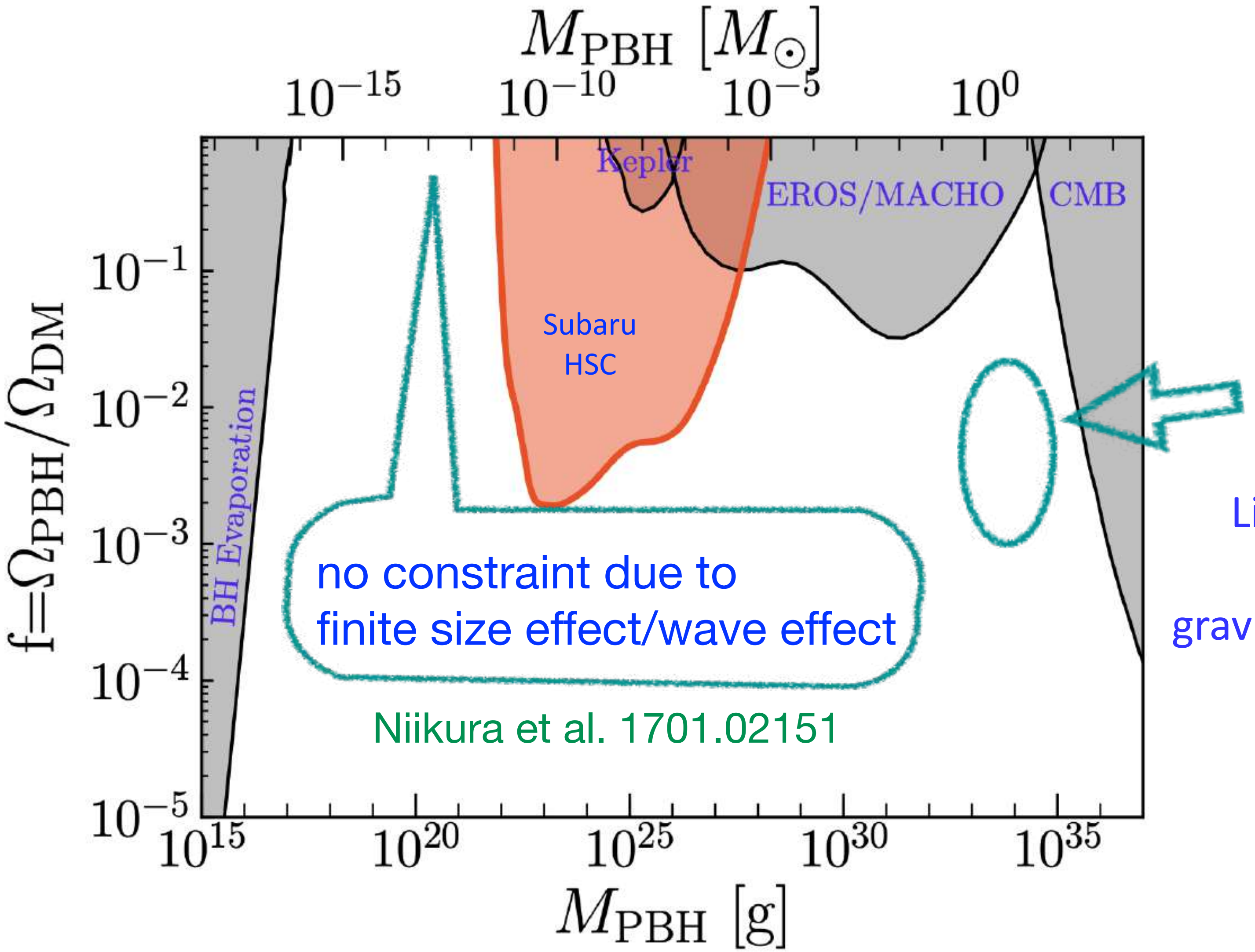
- Primordial Black Holes (PBHs) are those formed in the very early universe, conventionally when the universe was radiation-dominated.
- Presumably they originate from a large positive curvature perturbation produced during inflation (which hence should be a rare event).
- For a BH to form during radiation dominance, the perturbation must be O(1) on the Hubble horizon scale.

$$\boldsymbol{M_{\mathrm{PBH}} \sim M_{\mathrm{horizon}}}$$

$$\sim \left(\frac{100\mathrm{MeV}}{T}\right)^2 M_\odot \sim \left(\frac{\ell}{1\mathrm{pc}}\right)^2 M_\odot$$



observational constraints

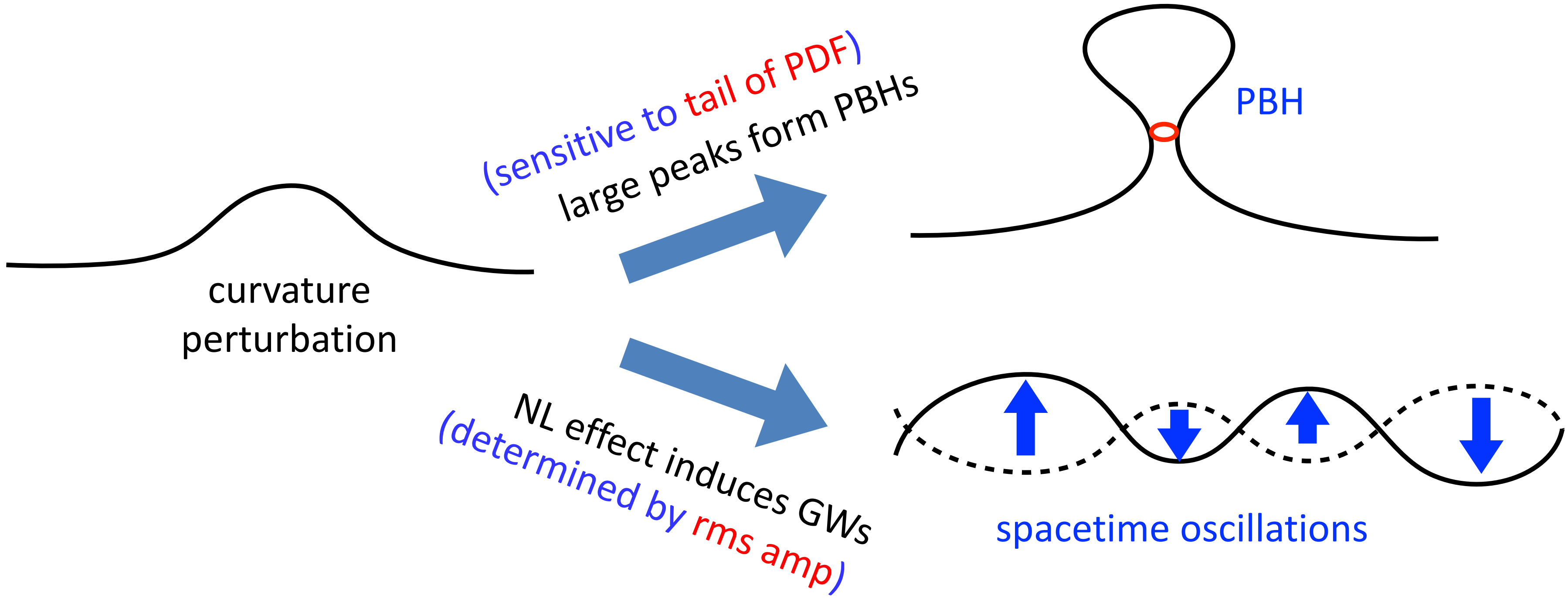


LVK BHs?

Ligo-VIRGO-KAGRA gravitational wave detectors

big window at $M_{\rm PBH} \approx 10^{17} - 10^{22}$g ⟺ $T_{\text{re-entry}} \sim 10^4$ -10^8 GeV

GWs can capture PBHs!



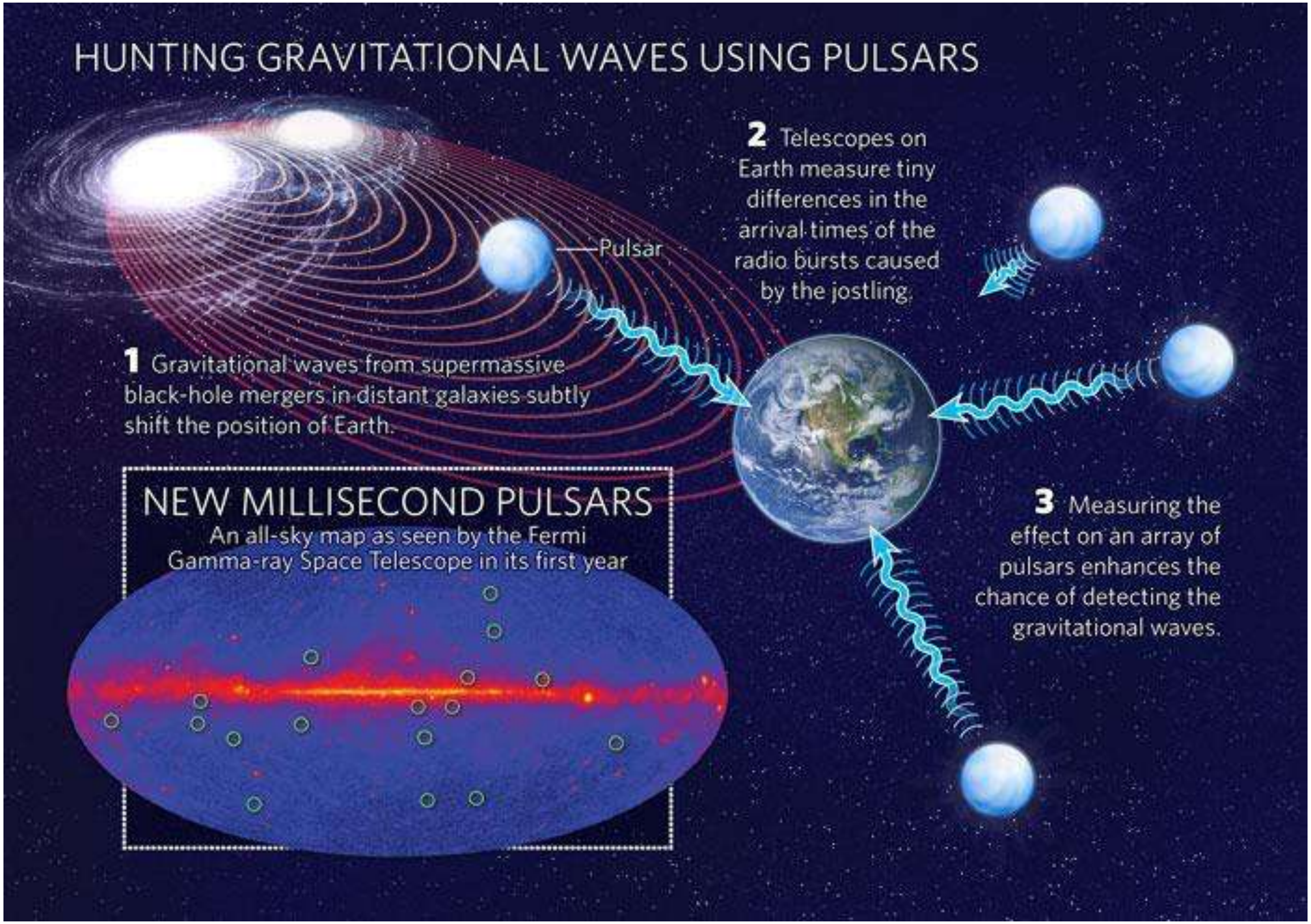
PBHs = CDM with $M_{PBH} \sim 10^{21}$g
generates GWs with $f \sim 10^{-3}$ Hz

⇨ Background GWs at LISA band

PBHs =LVK BHs with $M_{PBH} \sim 10 M_{\odot}$
generates GWs with $f \sim 10^{-8}$ Hz

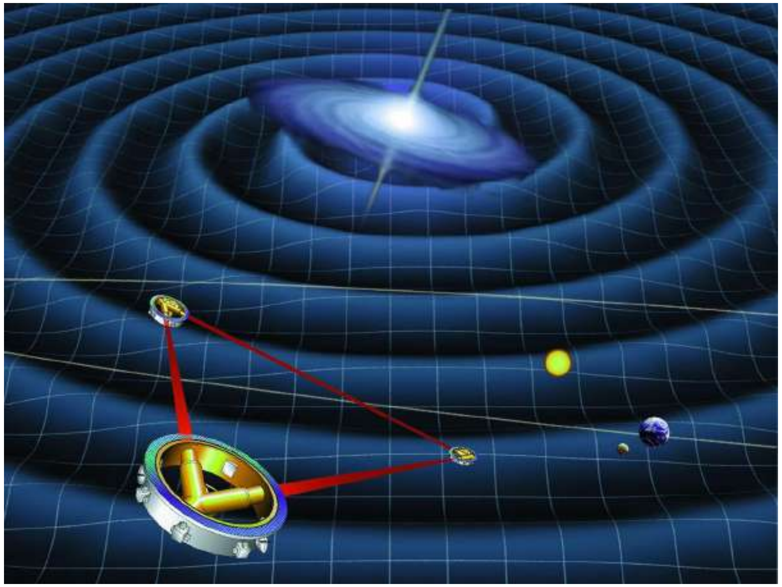
⇨ Background GWs at PTA band
(Pulsar Timing Array)

PTAs can detect Gravitational Waves



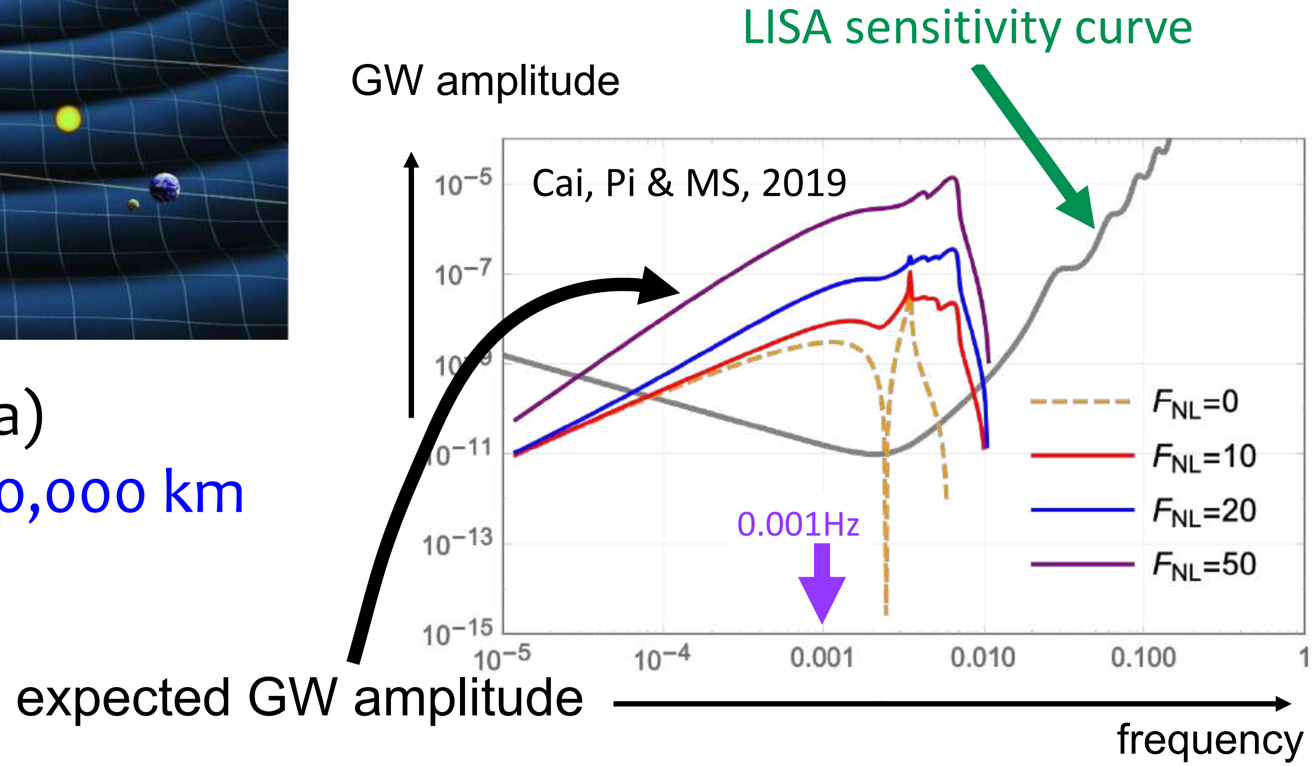
https://www.physics.unlv.edu/~jeffery/astro/relativity/gravitational_waves_low_frequency.html

GW observatories in space



LISA 2035? (ESA+NASA)
arm length: 5,000,000 km

Taiji 203X? (China)
arm length: 3,000,000 km



LISA/Taiji will prove/disprove PBH=CDM scenario

Inflation as the base for exploring
Physics of the Early Universe

Era of

Observational/Experimental
Inflationary Cosmology!

Thank you!