Axion-Mediated Dark Matter

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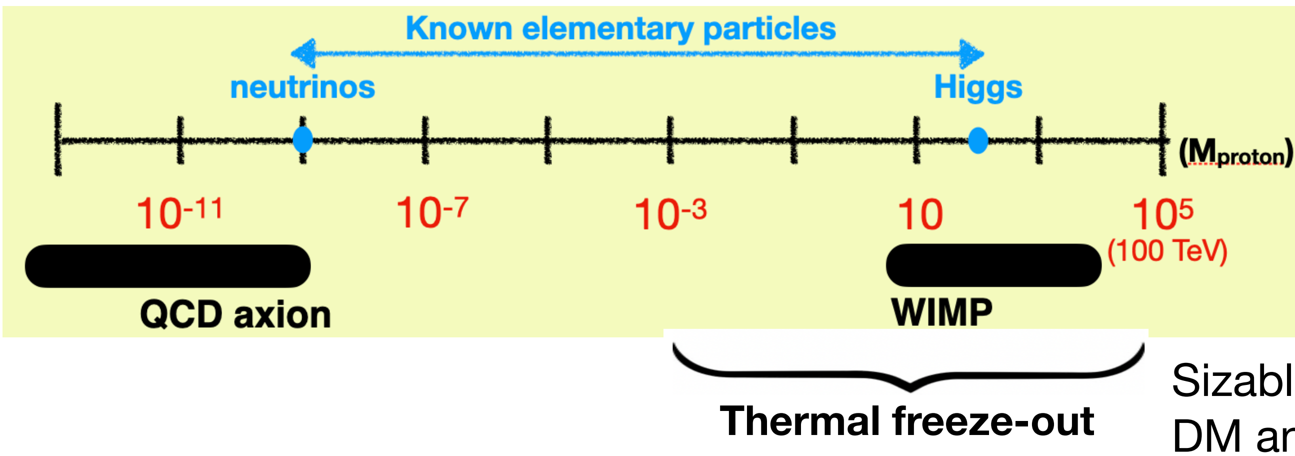
What is Dark Matter (DM)?

Weakly-Interacting-Massive-Particles (WIMPs) and the QCD axion have been the two leading DM candidates in the past several decades.

So far, not found

We do not know what the DM energy scale is:



Sizable coupling between DM and the Standard Model (SM).

Generically, need a light "mediator" between DM and the SM if $m_{DM} < O(10GeV)$

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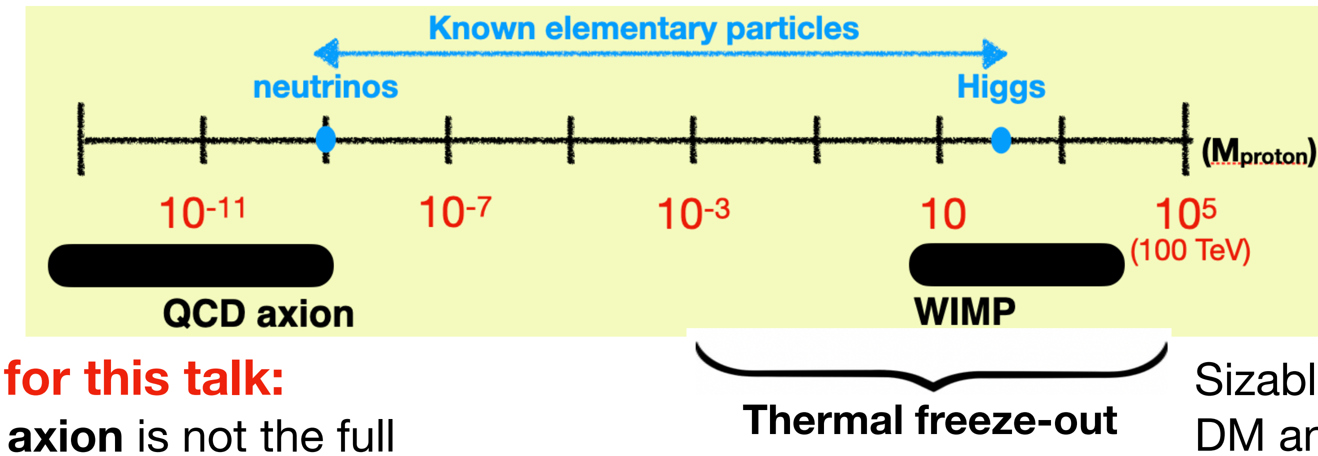

Question for this talk:
If the **QCD axion** is not the full story for DM, can it be the mediator between the SM and DM? What about **other axions beyond the QCD axion**?

Sizable coupling between DM and the Standard Model (SM).

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This talk:

1. Cosmology of axion-mediated DM models
2. Cosmological signatures
   Searches at laboratory-based experiments
   (searches for the axion + DM direct detection)

Dror, SG, Munbodh, 2306.03145SG,
SG, Knapen, Lin, Munbodh, Suter, 2506.11191

The strong CP problem and the QCD axion

Strong CP problem:
why is the QCD $\bar{\theta}$ parameter so small? $\mathcal{L}_{\rm QCD} \supset \theta \frac{g^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$

This operator generates a contribution to the neutron EDM:

$$|d_n| \sim 2 \times 10^{-16}\ \bar{\theta}\ e\ \mathrm{cm}$$

$\Rightarrow$ $\bar{\theta} \lesssim 10^{-10}$

$\bar{\theta} = \theta + \arg(\det(Y_u Y_d))$ (basis invariant quantity)

QCD axion: elegant way to address this problem.

$$\Phi = \Phi_0 e^{i\frac{a}{f_a}}$$

$U(1)_{PQ}$, Peccei-Quinn symmetry breaking scale

Instantons induce a periodic potential for the axion.

$$\underbrace{\left(\theta + \frac{a}{f_a}\right)} \frac{g^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

At the minimum of the potential this coefficient = 0,

$\Rightarrow$ EDM constraints are 

By-product:
Axions can easily be a Dark Matter (DM) candidate for

$$f_a \gtrsim \mathcal{O}(10^{10} - 10^{11})\ \mathrm{GeV}$$

- pre inflation: misalignment
- post inflation: cosmic string and domain wall decay

Caveat: lower f_a could lead to the measured relic abundance in other scenarios: e.g., kinetic misalignment,
Ko, Hall, Harigaya, 1910.14152

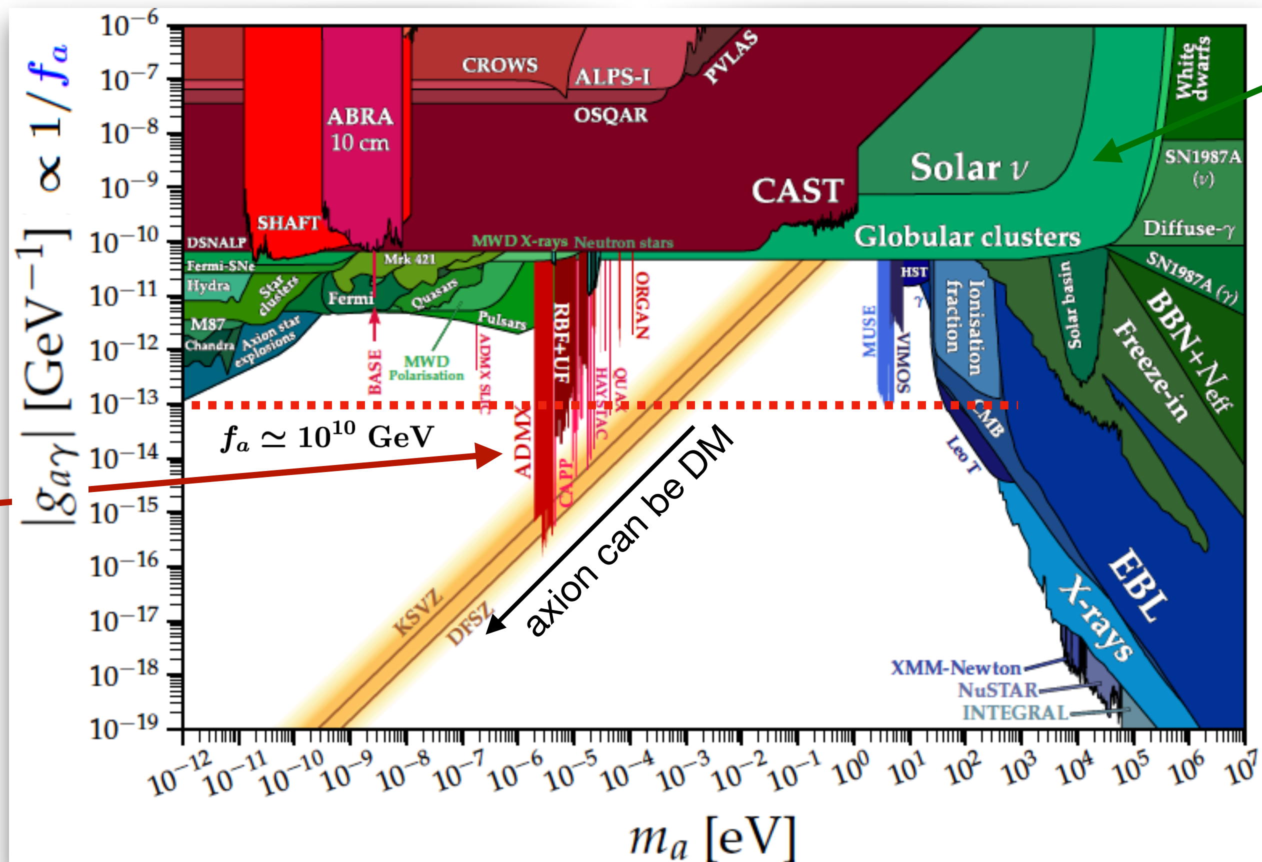
Discovering the QCD axion. Present and future

The QCD axion mass is set by its decay constant, f_a: $m_a f_a \sim f_\pi m_\pi$
The generic expectation is that the QCD axion couples to the Standard Model $\sim 1/f_a$

Adams at al., 2203.14923

Star cooling bounds

ADMX & HAYSTAC:
axion "haloscopes" detect axion DM by converting them into detectable photons



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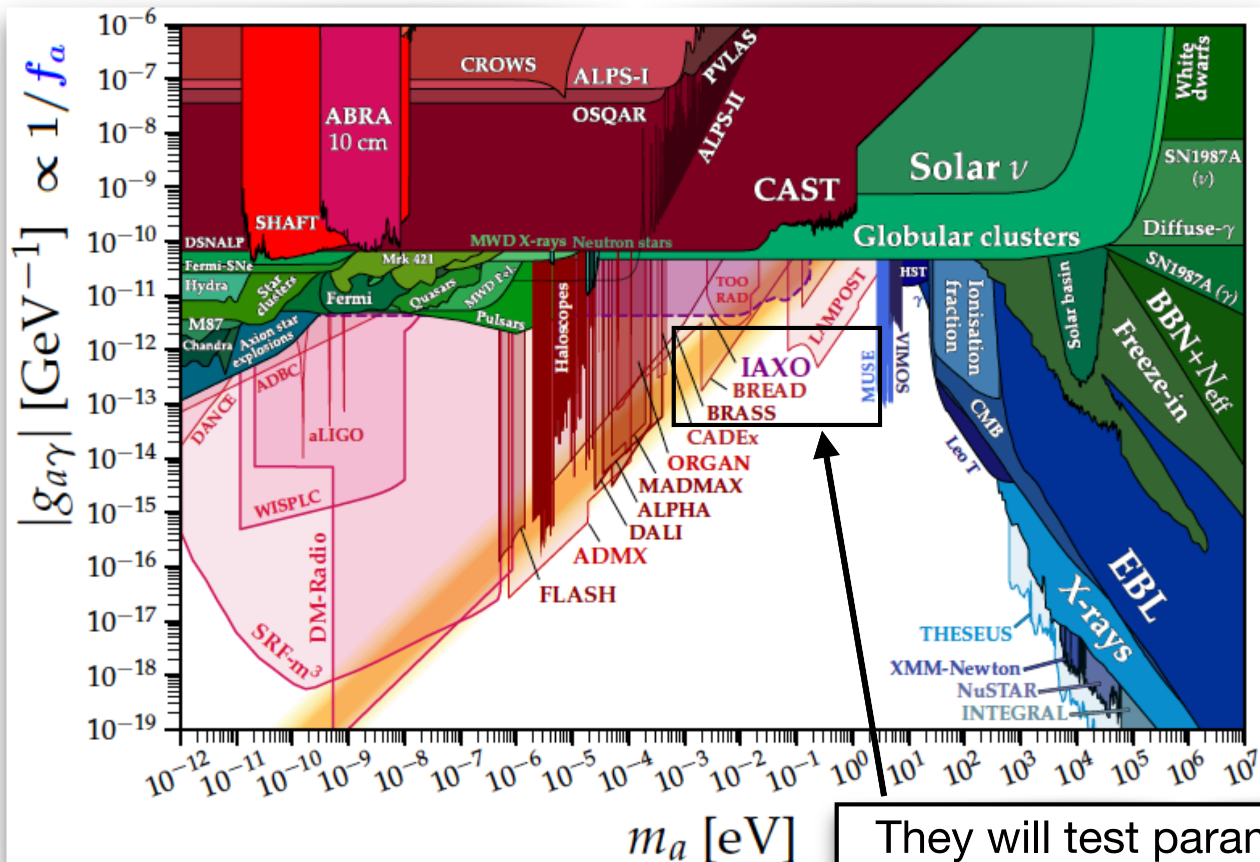


Star cooling bounds

They will test parameter space where the axion is typically a DM sub-component

The QCD axion as DM-SM mediator

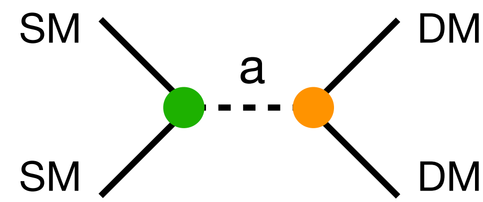
The “axion portal”

- The axion couples to SM particles with low-dimensional “portals” (dim 5)

$$\mathcal{L} \supset -\frac{g_{ag}}{4}\, a\, G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - \frac{g_{aW}}{4}\, a\, W^a_{\mu\nu}\tilde{W}^{a\mu\nu} - \frac{g_{aB}}{4}\, a\, B_{\mu\nu}\tilde{B}^{\mu\nu} + i g_{af}(\partial_\mu a)(\bar{f}\gamma^\mu\gamma_5 f)$$

- We can couple it to fermionic DM as $\mathcal{L} \supset \frac{c_\chi}{2 f_a}\partial_\mu a \bar{\chi}\gamma^\mu\gamma_5\chi$

(or eventually scalar operator, if CP is broken)



A minimal model:
KSVZ or DFSZ QCD axion coupled to fermionic DM

Dror, SG, Munbodh, 2306.03145

- A small set of free parameters fixes the cosmology and phenomenology of the model:

$$f_a,\ m_\chi,\ g_{a\chi} \equiv \frac{c_\chi m_\chi}{f_a},\ T_{\rm RH}$$

Bounds from DM self-interaction:

$$c_\chi \frac{m_\chi}{f_a} \equiv g_{a\chi} \lesssim 0.21 \left(\frac{m_\chi}{1\ \mathrm{MeV}}\right)^{3/4}$$

Cooling bounds.
The most stringent one for the DFSZ axion is from the **red giants** (bound on the axion-electron interaction)

$$f_a \gtrsim 10^8\ \mathrm{GeV}$$

The QCD axion as DM-SM mediator

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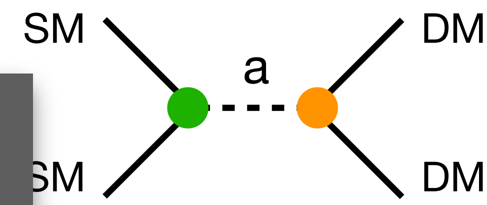

A minimal model:
KSVZ or DFSZ QC

Dror, SG, Munbodh, 2306.03145

* A small set of fr ... ology of the model:

The well - motivated region of the QCD axion parameter space is bounded:

$$\mathcal{O}(10^8\ \mathrm{GeV}) \lesssim f_a \lesssim \mathcal{O}(10^{11}\ \mathrm{GeV})$$

astrophysical bounds

subdominant DM component

Bounds from DM self-interaction:

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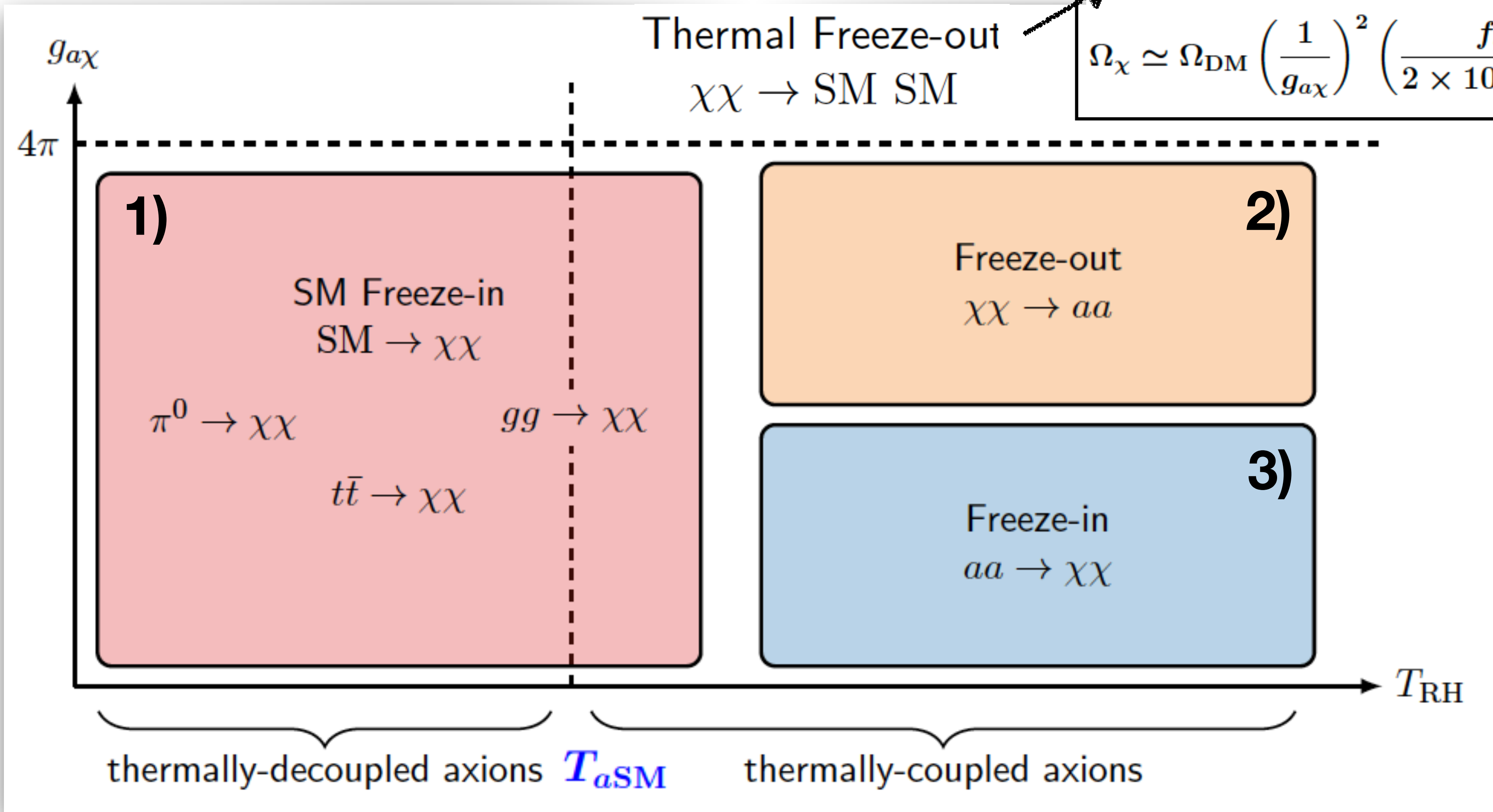
A bird's-eye view of possible cosmologies

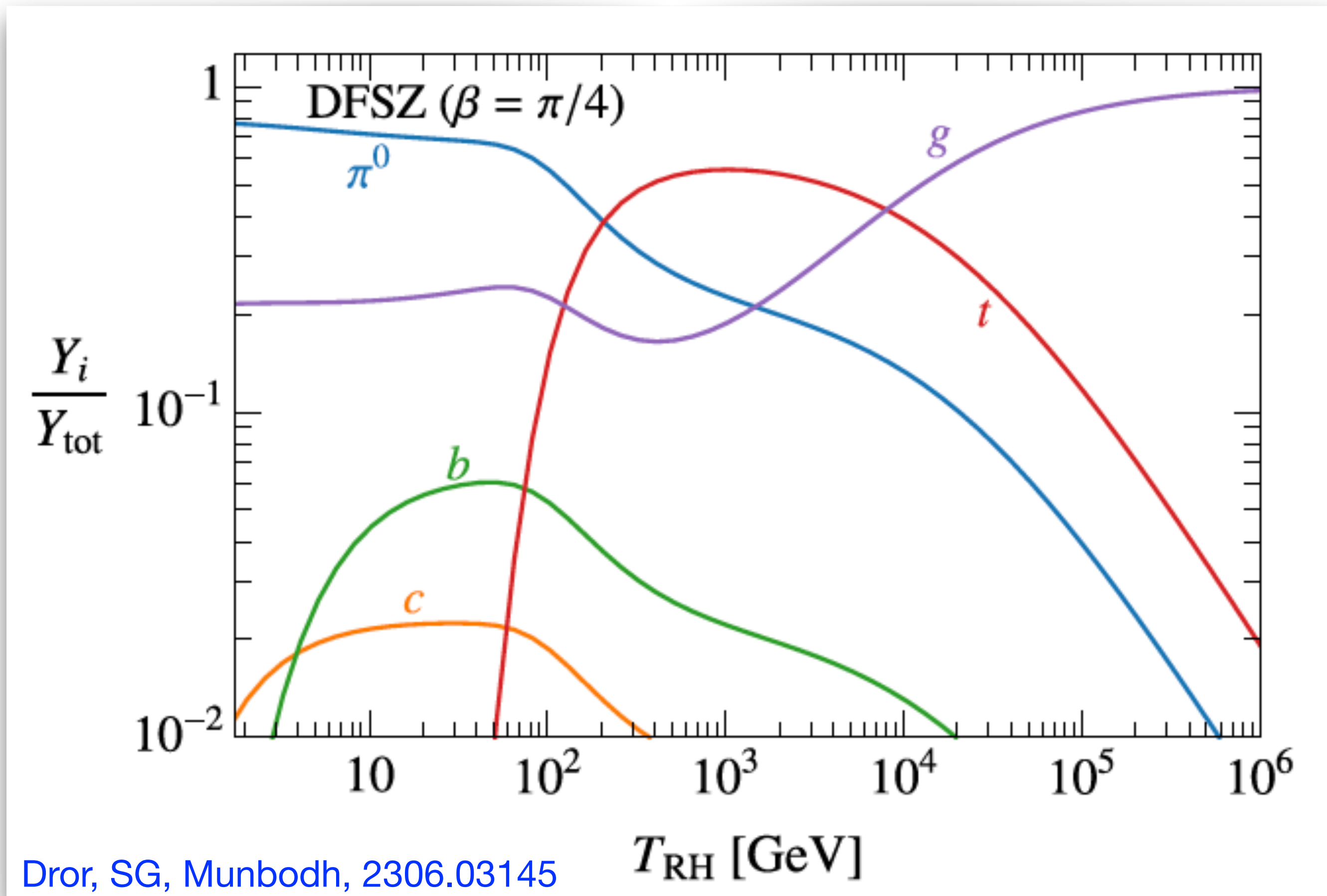
Experimentally not allowed:

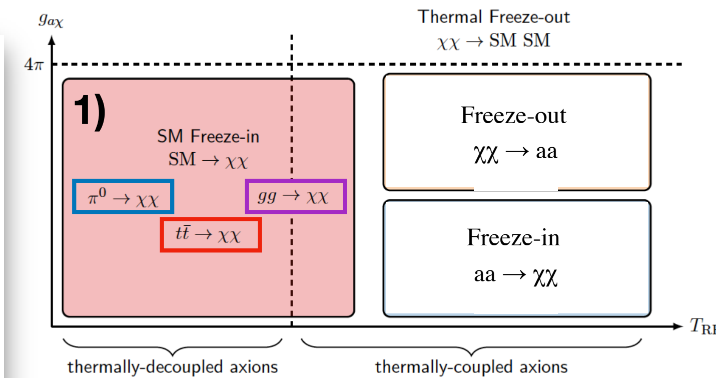
$$\Omega_\chi \simeq \Omega_{\rm DM}\left(\frac{1}{g_{a\chi}}\right)^2\left(\frac{f_a}{2\times10^3\,\mathrm{GeV}}\right)^2$$



Dror, SG, Munbodh, 2306.03145

1) Dark Matter from SM freeze-in





If $T_{RH} < T_{\chi SM}$ and $T_{RH} < T_{aSM}$

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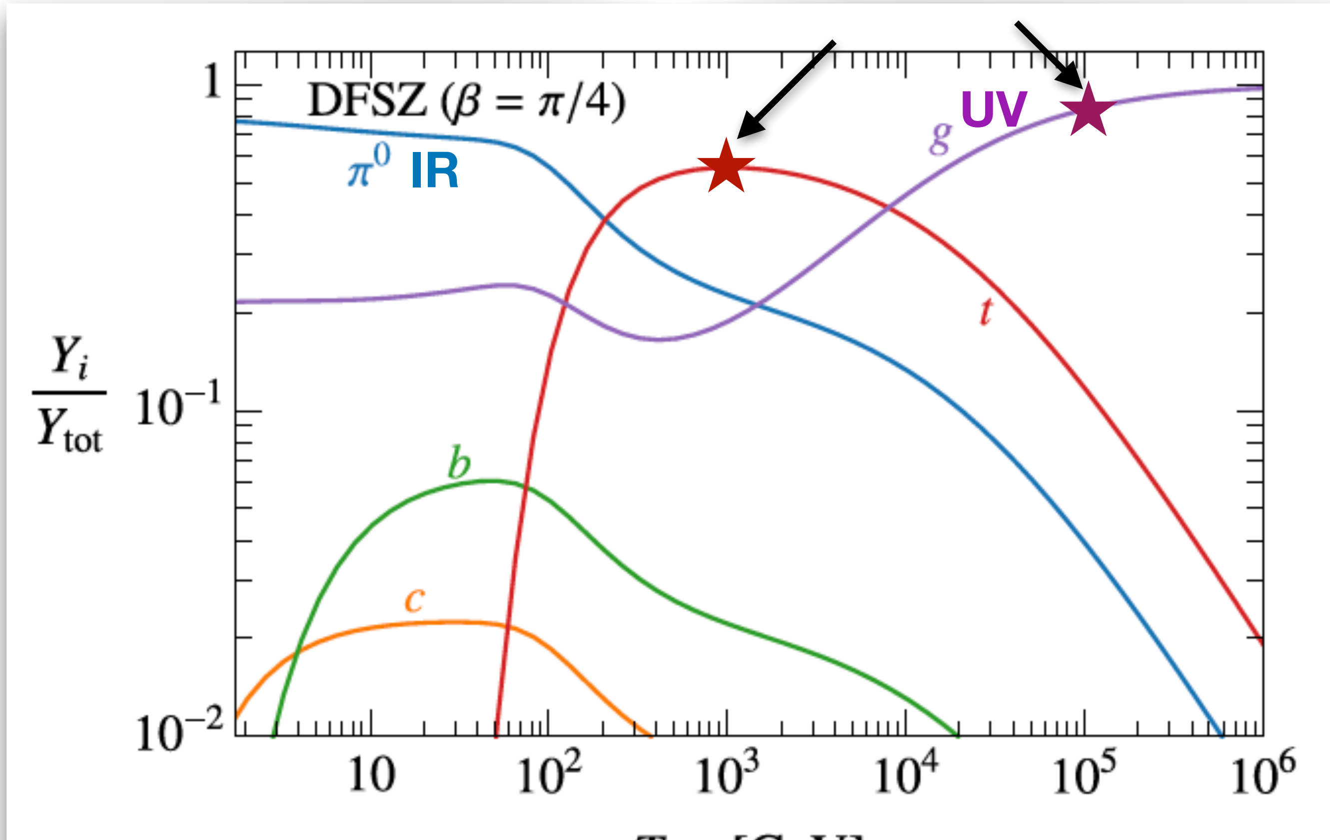

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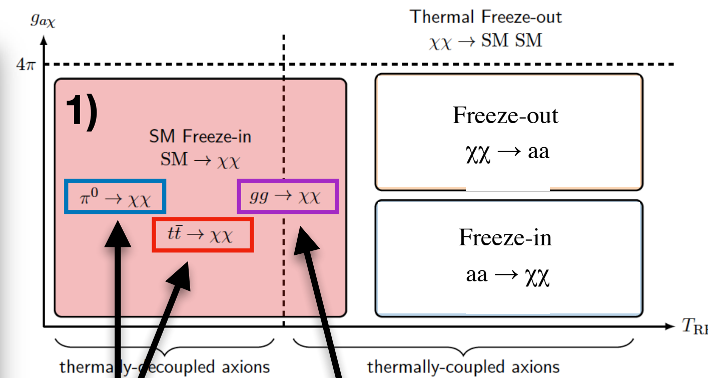


IR freeze-in

UV freeze-in

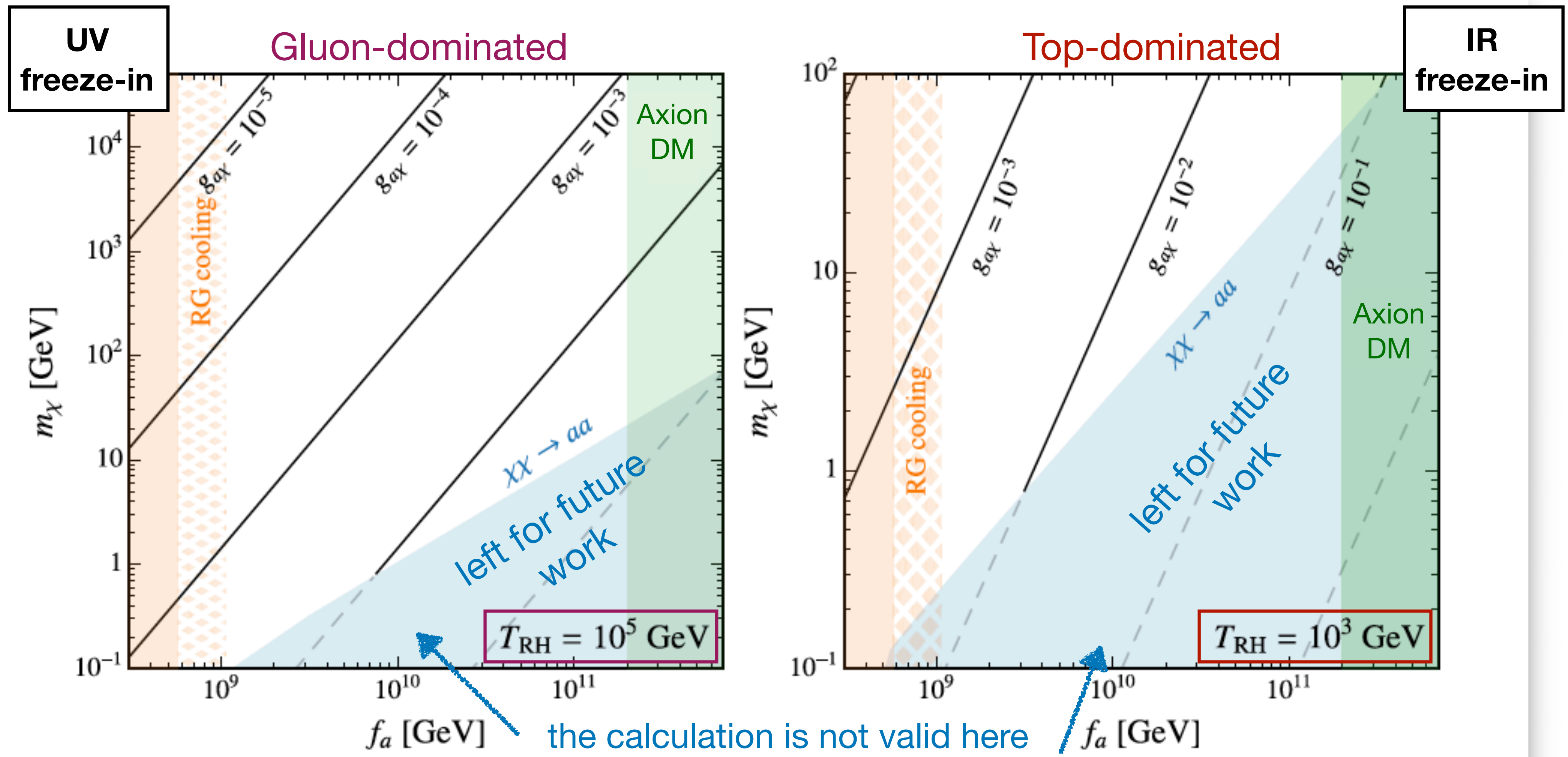
All these processes arise from dim-5 operators. However,

~insensitive to the UV cutoff, $T_{\rm RH}$, since regulated by the mass of the scattering- or decaying-particles

If $T_{\rm RH} < T_{\chi \rm SM}$ and $T_{\rm RH} < T_{a \rm SM}$

1) SM freeze-in: the relic abundance

$g_{a\chi}$ needed to get the measured relic abundance



the calculation is not valid here
since DM can freeze-out to axions

$$n_\chi \sigma_{\chi\bar{\chi}\to aa} \gtrsim H$$

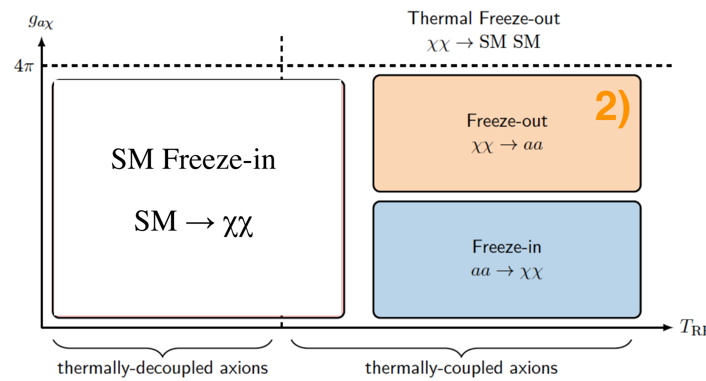
Decoupled freeze-out regime
in Bharucha et al., 2209.03932

Typically not natural regime since $g_{a\chi} \equiv \frac{c_\chi m_\chi}{f_a}$

2), 3) Dark Matter from axion freeze-out or freeze-in

No coupling with the SM is necessary

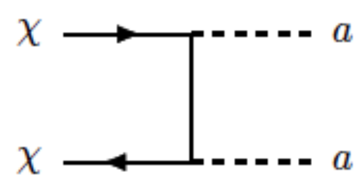
In general, challenging to test experimentally



$T_{\rm RH} \gtrsim T_{\chi \rm SM}$ **secluded freeze-out**

2) DM in thermal contact with the SM in the early universe.

$$\frac{\Omega_\chi}{\Omega_{\rm DM}} \sim \left(\frac{m_\chi}{1\rm GeV}\right)^2 \left(\frac{4.4\times 10^{-2}}{g_{a\chi}}\right)^4$$ **unnatural regime**



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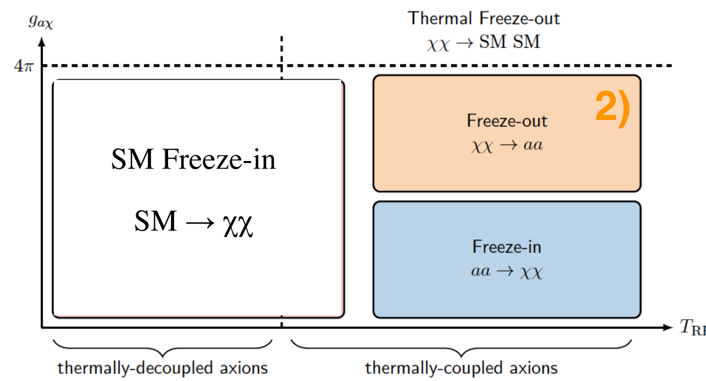

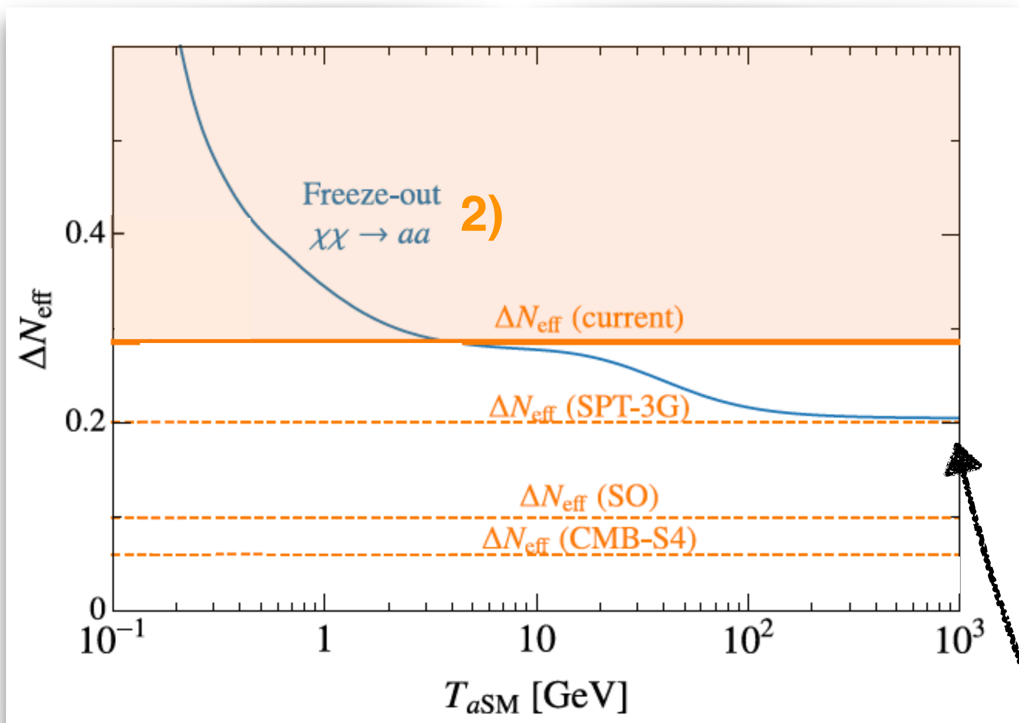

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DM annihilates into QCD axions, which remain relativistic today

➡ significant source of dark radiation.

Future experiments will be able to **completely probe** the freeze-out scenario

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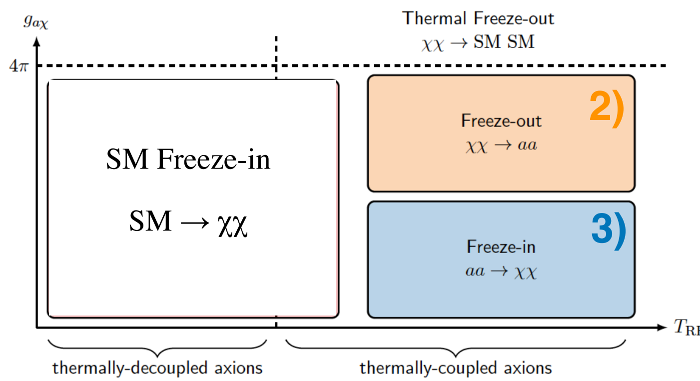

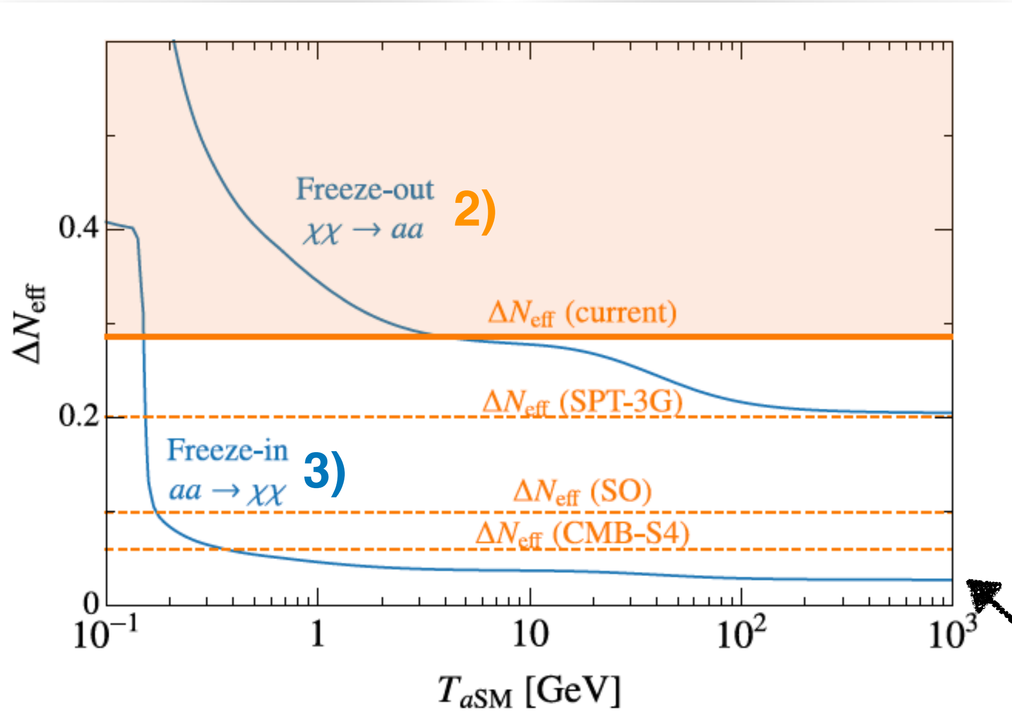

secluded freeze-in

$$T_{a\rm SM} \lesssim T_{\rm RH} \lesssim T_{\chi\rm SM} \text{ and } T_{\rm RH} \gtrsim T_{a\chi}$$

3) axion is thermalized in the early universe

$$\frac{\Omega_\chi}{\Omega_{\rm DM}} \sim \left(\frac{g_{a\chi}}{3\times 10^{-6}}\right)^4$$

natural regime

The QCD axions will contribute to $N_{\rm eff}$, but now the DM does not have a sizable energy density ➡ the evolution of the dark sector temperature will be different than in the freeze-out case

The freeze-in scenario is **pretty hidden**, even to future experiments

Take home message from this first part

QCD axion-mediated DM models are a minimal extension of the SM.

Interesting interplay between UV and IR physics.

Astrophysical and cosmological data lead to interesting constraints on the model.

It is challenging to test these models at lab-based experiments.
(this is due to the stringent constraints on the QCD axion-SM couplings. Large f_a!)

Beyond the QCD axion mediator

What if we extend this framework to much heavier axions/axion-like-particles?

Recently, there has been a revival of models addressing the strong CP problem with **multiple heavier axions**.
(Dimopoulos et al., 1606.03097; Agrawal, Howe, 1710.04213; Foster, Kumar, Safdi, Soreq, 2208.10504, …)

In these models, it is easier to address the **axion quality problem**. I.e., smaller shift in the $\bar{\theta}$ parameter in case of PQ breaking at some UV scale, Λ_{UV}, by a D-dimensional operator:

$$\delta\bar{\theta} \sim \frac{f_a^{D-2}}{m_a^2 \Lambda_{\mathrm{UV}}^{D-4}}$$

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Example model:

$$\mathcal{L} \supset c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f_a} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

and with a generic mass,
$m_a f_a \neq f_\pi m_\pi$

DM freeze-out can arise from several processes depending on the ALP-DM mass spectrum: $\chi\bar{\chi} \to aa$

$\chi\bar{\chi} \to a \to \mathrm{SM}$

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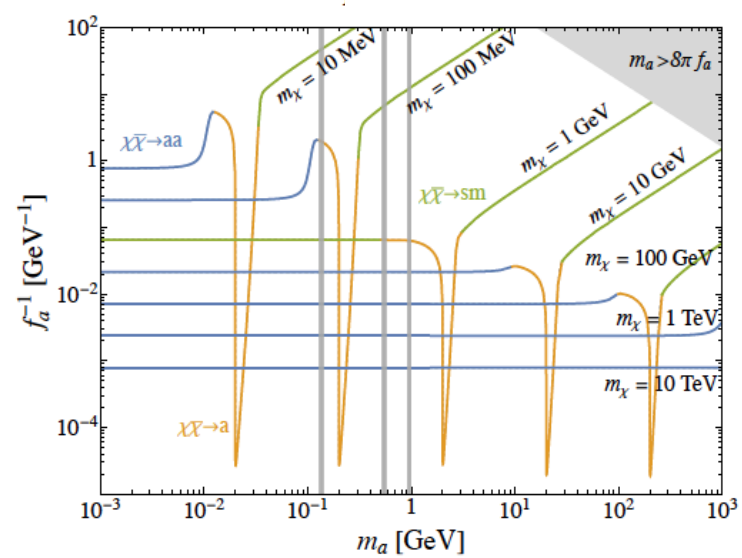
This minimal model can lead to the measured DM relic abundance through **freeze-out**



Fitzpatrick et al, 2306.03128

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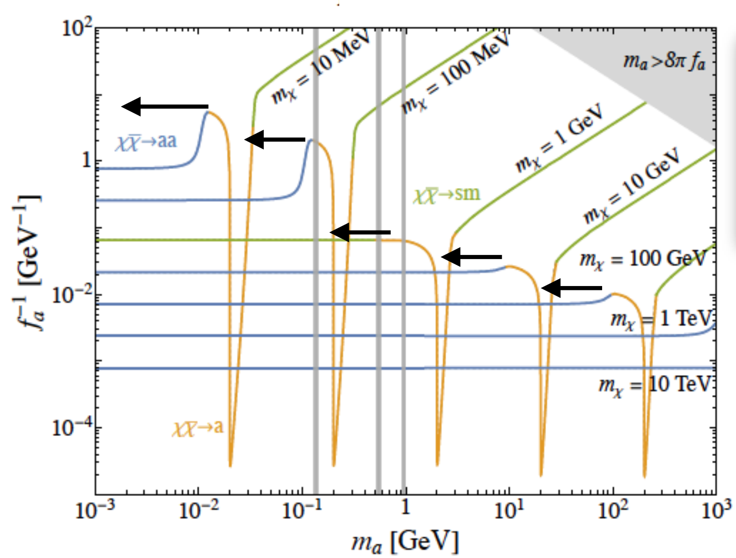


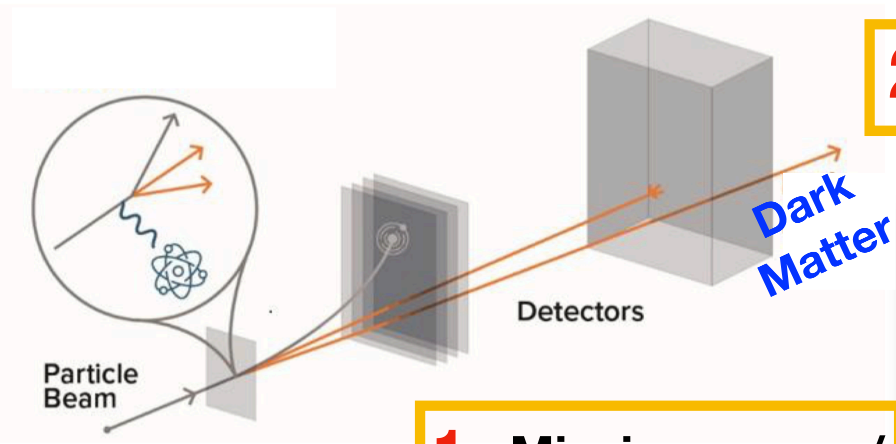
Lab-tests for $m_a < m_{\rm DM}$?

SG, Knapen, Lin, Munbodh, Suter, 2506.11191

Fitzpatrick et al, 2306.03128

Classes of high-intensity experiments for DM/dark sectors

Production of dark matter

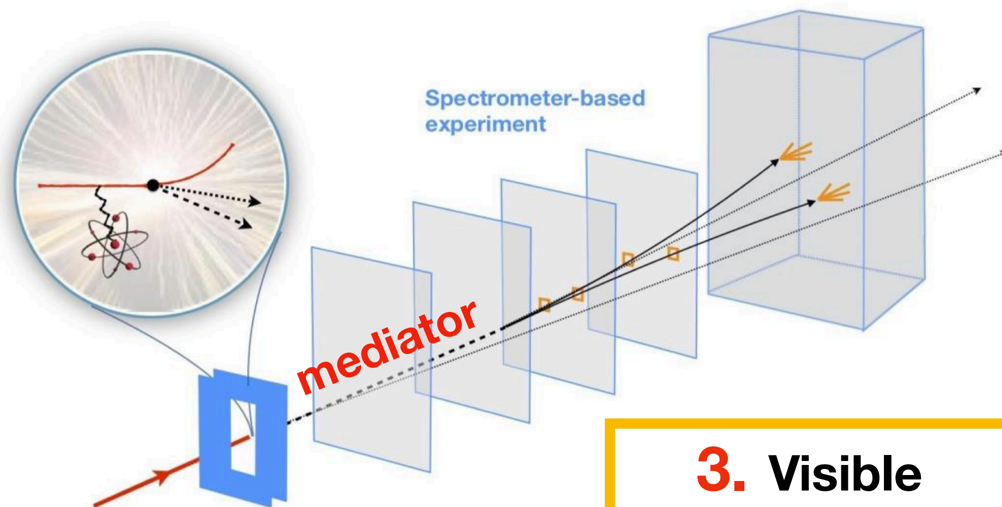


2. Re-scattering

1. Missing energy/ momentum

Fixed target experiments

Production of the mediator



3. Visible decay products

Flavor factories

Several experiments are running and will be running in the coming years (B, kaon, and pion factories)

4. DM and mediators produced from meson decays
(e.g., $B \to K a$)

https://science.osti.gov/-/media/hep/pdf/Reports/Dark_Matter_New_Initiatives_rpt.pdf

** The production does not depend much on the nature of DM/mediator.
** Production of relativistic DM/mediators

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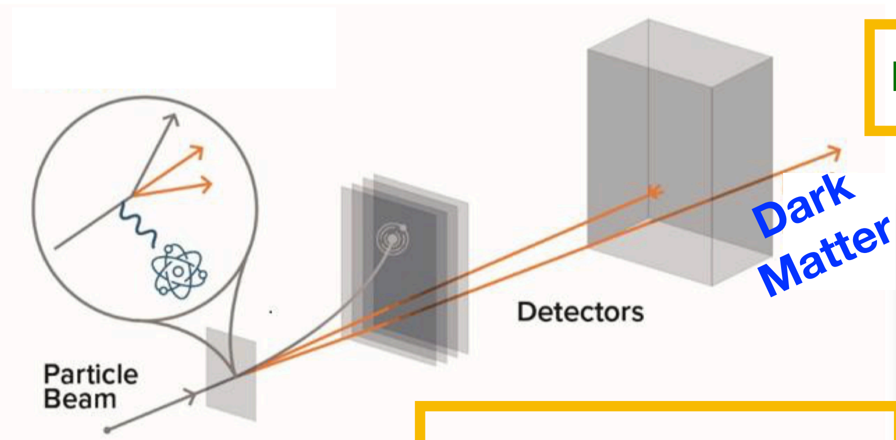
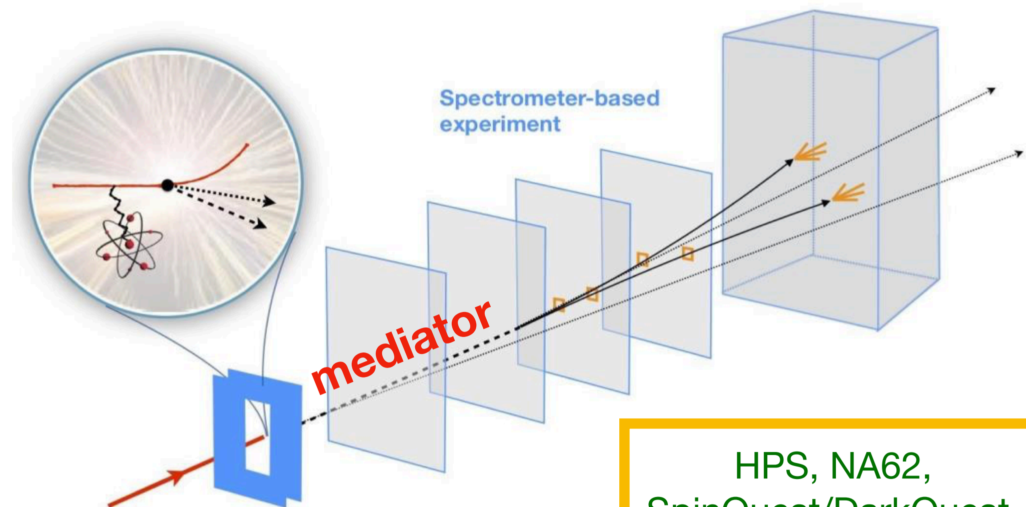


Neutrino experiments

NA64, LDMX, M³…

Production of the mediator



HPS, NA62, SpinQuest/DarkQuest, SHiP, DarkLight

https://science.osti.gov/-/media/hep/pdf/Reports/Dark_Matter_New_Initiatives_rpt.pdf

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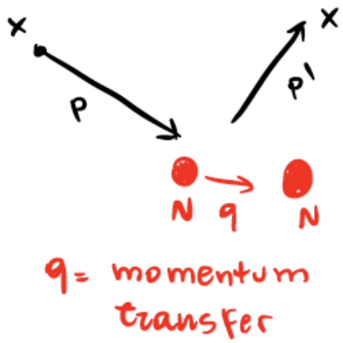
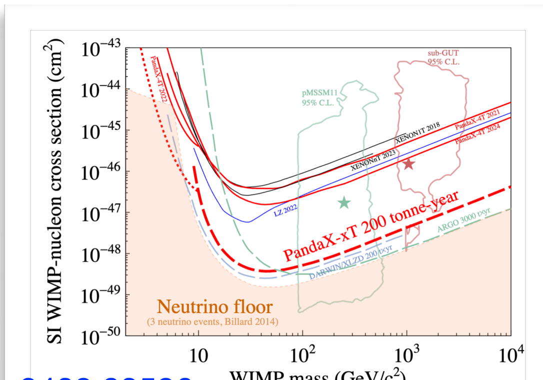
Belle II
NA62, KOTO
PIONEER

DM at direct detection experiments

"Classic" direct detection:





2402.03596

$$E_R^{\max} = \frac{q^2_{\max}}{2m_N} = \frac{2\mu^2_{\chi N} v^2}{m_N}$$

DM velocity

Reduced mass of the DM-nucleus system: $\mu_{\chi N} = \frac{m_\chi m_N}{m_\chi + m_N} \simeq m_\chi$

E.g., m_X= 1GeV → E_R^{max} ~ 0.01 keV

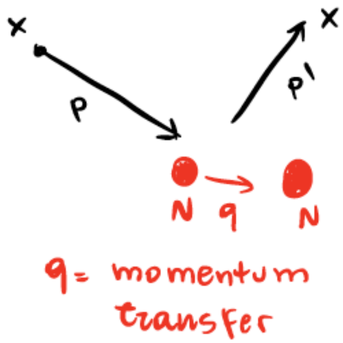
$E_R \geq$ O(keV) in most experiments

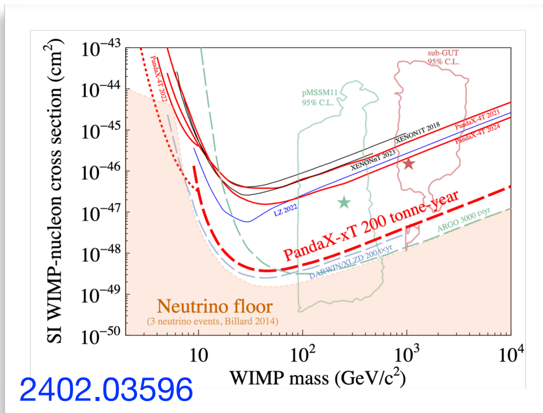
This gives a ~lower bound on the DM masses we can probe.

➡ **Challenging to probe m_{DM} < O(GeV)**

DM at direct detection experiments

"Classic" direct detection:





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Several techniques developed in the past few years:

DM-electron scattering

Very low energy thresholds.

It allows to transfer O(1) amount of DM kinetic energy

Migdal effect

Nuclear recoil + delayed electron signal (excitation / ionization of the atom)

Low-threshold detectors

For a review: Kahn, Lin, 2108.03239

The cross section strongly depends on the DM nature

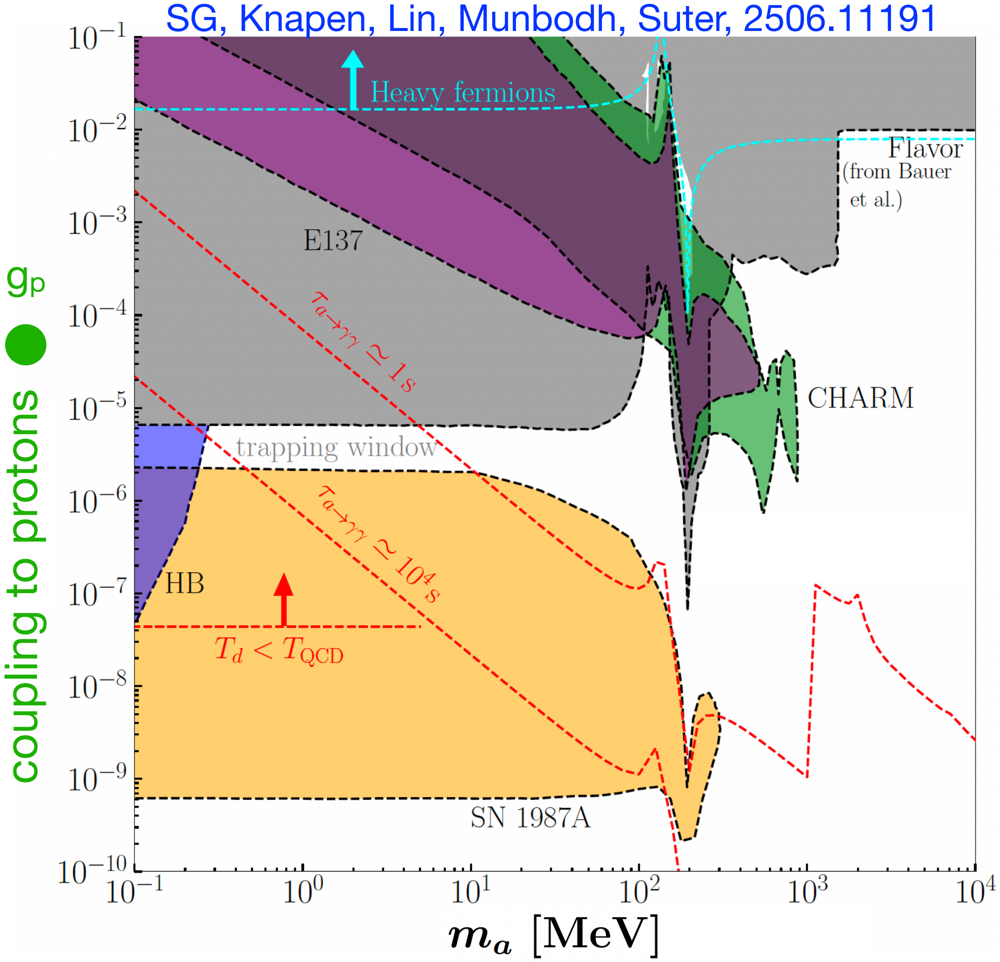
E.g., velocity suppression if DM = Majorana fermion and the mediator is a dark photon

DM is non-relativistic!

Testing the mediator (high intensity+astrophysics)

Going back to our axion model… $\mathcal{L} \supset c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f_a} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$

SG, Knapen, Lin, Munbodh, Suter, 2506.11191



Note: the mediator is lighter than DM → it decays back to the SM

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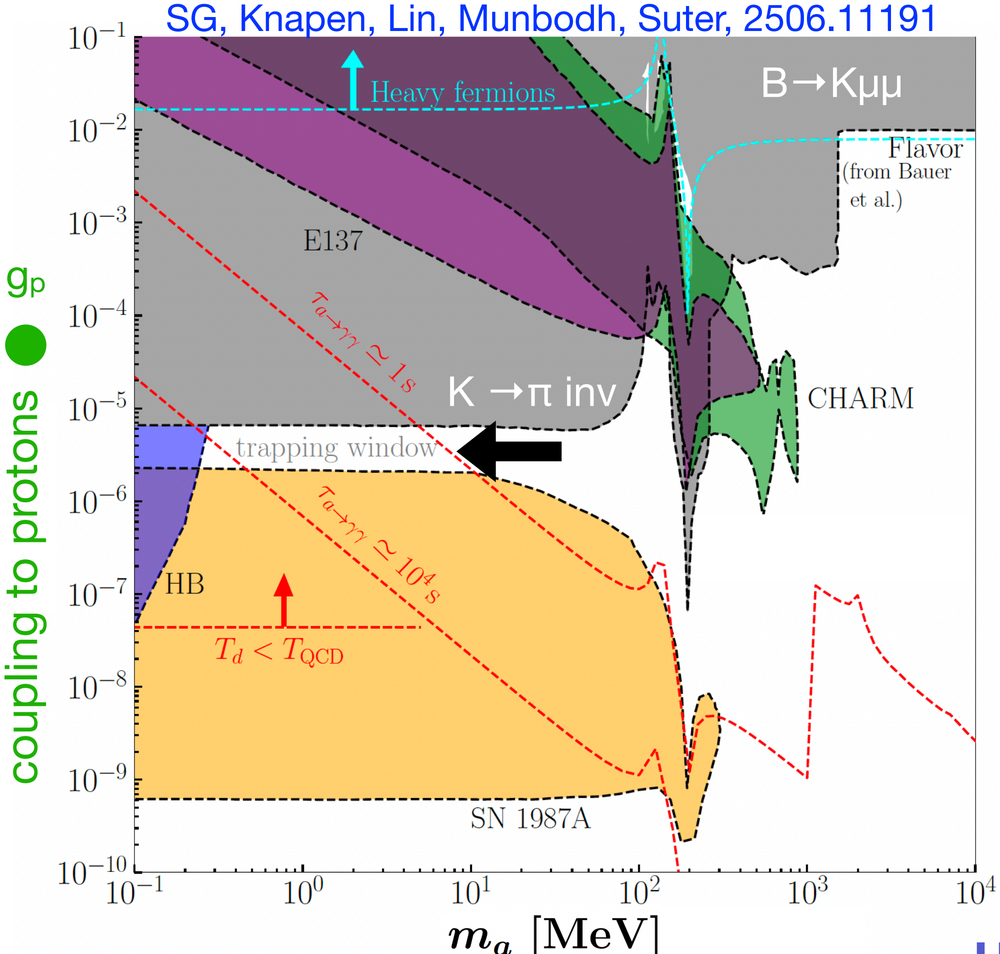


Flavor probes:
K →π inv, B→Kμμ

Fixed target experiments:
E137 (primakoff + strahlung production)
CHARM (axion from meson decays)

Supernova (SN1987a) bounds:
axion emission leads to a modified neutrino spectrum observed in the Kamiokande II detector

Trapping window: the axion cannot free stream out of the protoneutron star

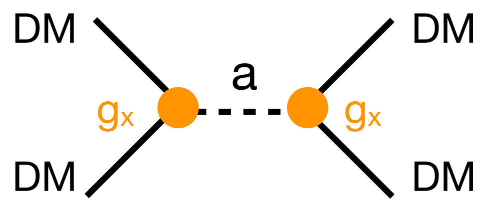
Horizontal branch (HB):
the axion reduces the ratio of horizontal branch stars to red giant branch stars in globular clusters

What about future fixed targets (DarkQuest), LHC auxiliary detectors, and Kaon/B factories?

Testing the DM: self-interactions

Viscosity cross section: $\sigma_V \equiv \int d\Omega \frac{d\sigma}{d\Omega}(1-\cos^2\theta) \lesssim (1.1\mathrm{cm}^2/g) m_\chi$

v ≃ 0.005c
(galaxy group)



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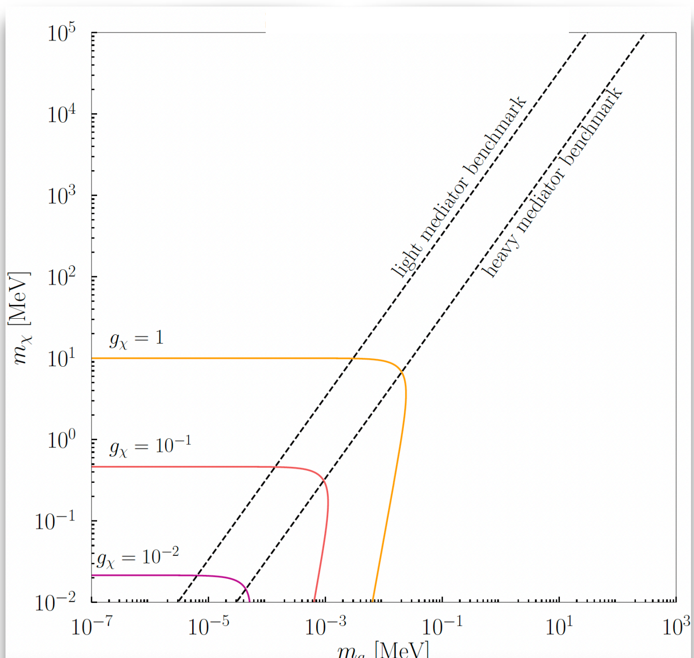
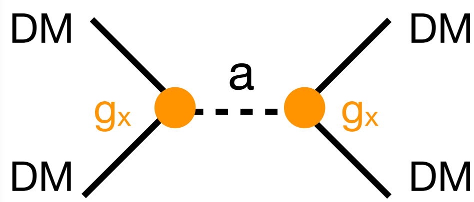
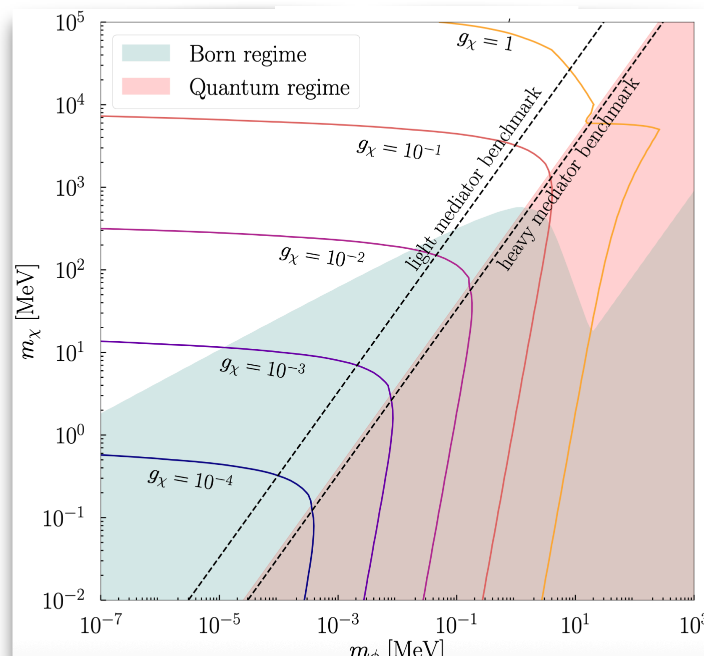
(galaxy group)

Most natural
No Sommerfeld enhancement
for **pseudoscalar** coupling with DM

(Less straightforward calculation,
if the ALP has a **scalar** coupling with DM







Relatively weak constraints

Stronger constraints)

- light mediator: $m_a = 0.3\, q_0 = 0.3\, m_x v_0$
- heavy mediator: $m_a = 3\, q_0 = 3\, m_x v_0$

DM Direct detection: nuclear recoil and phonon excitation

Basic idea

For light DM, the deBroglie wavelength > interatomic spacing.
“Billiard ball” nuclear recoil not applicable.
The DM sees more than one atom at a time.

→ Phonon excitations
(many coupled harmonic oscillators)

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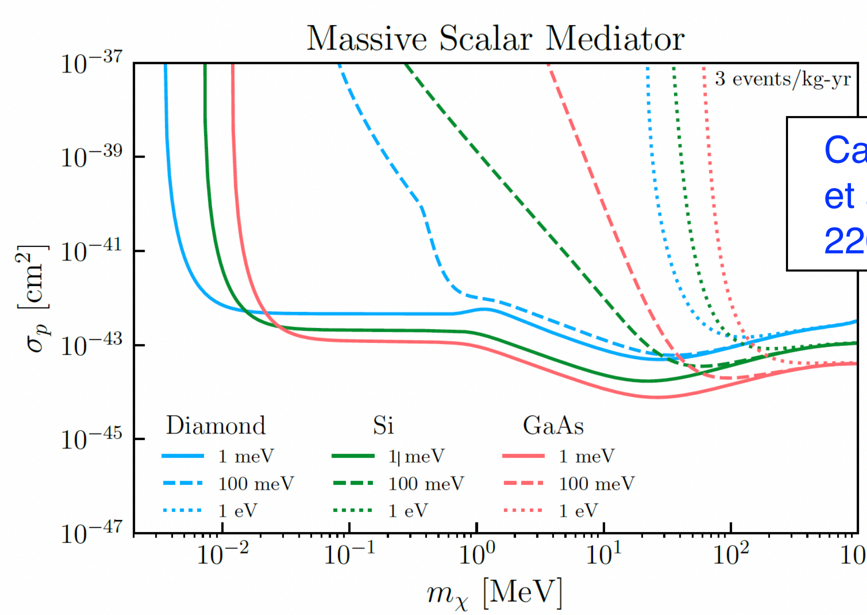
Expansion in the number of final state phonons, where each phonon comes with a factor of $\frac{q}{\sqrt{2 m_d \bar{\omega}_d}}$ (m_d and ω_d are the mass and average oscillation frequency of the atom at position d)

Knapen et al., 1712.06598,
Griffin et al., 1807.10291
Campbell-Deem et al., 1911.03482,
Kahn et al., 2011.09477, …

DM spin-independent cross section



Campbell-Deem et al., 2205.02250

One phonon approximation ← → multiphonon excitation → Nuclear recoil

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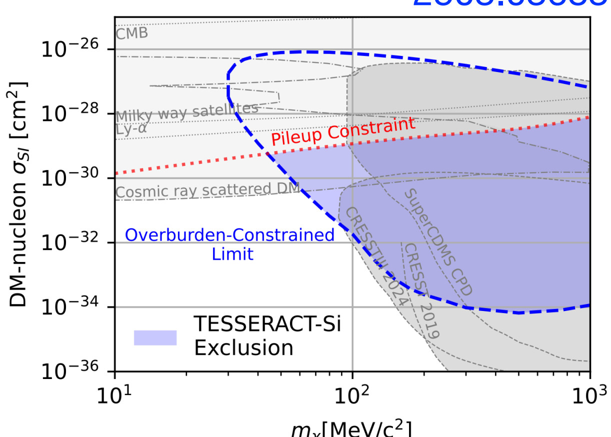
NEW: TESSERACT experiment at Berkeley started setting bounds using this technique (threshold energy=1.5eV; 0.233 g × 12 hours)

2503.03683



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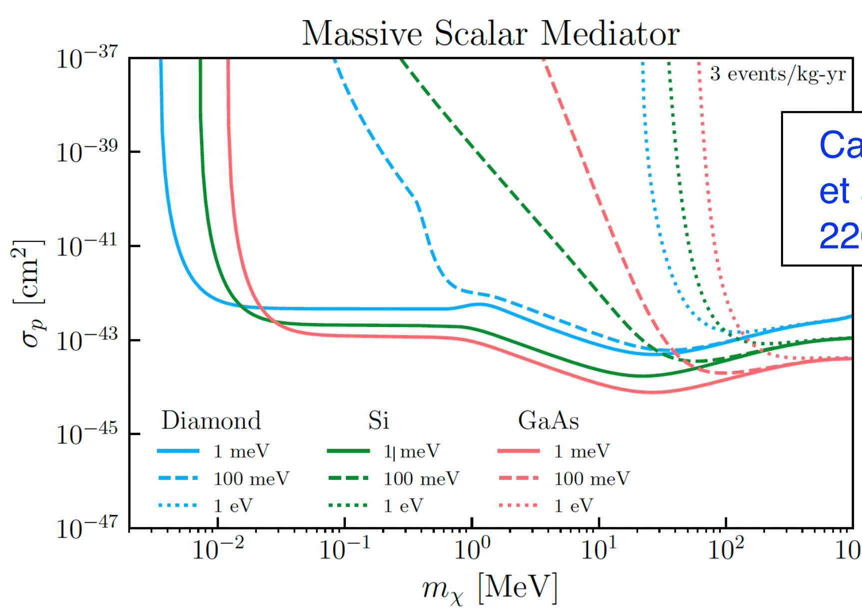

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One phonon approximation | multiphonon excitation | Nuclear recoil

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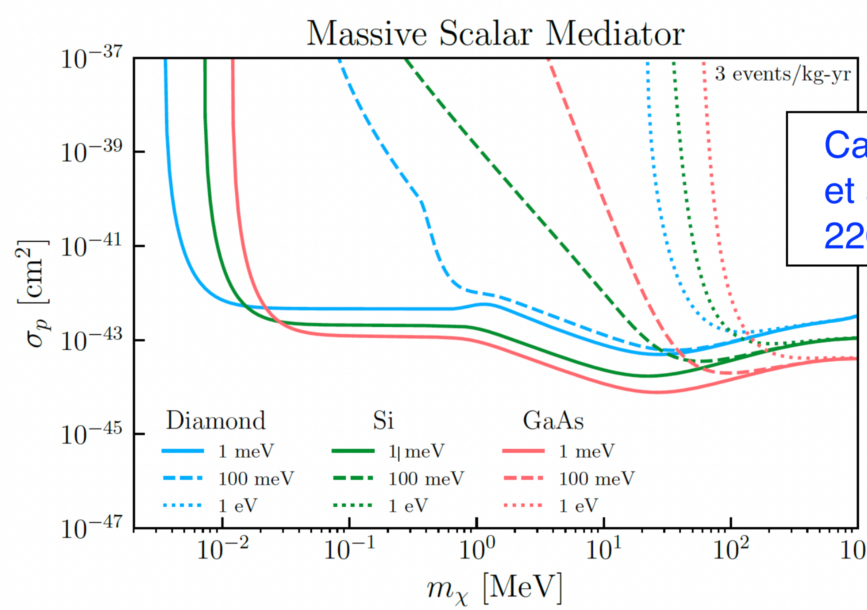
Our model predicts spin-dependent interactions

Can we use these novel ideas for this scenario?

Not much has been done so far for spin-dependent interactions…

DM spin-independent cross section



Campbell-Deem et al., 2205.02250

One phonon approximation ← → multiphonon excitation ← → Nuclear recoil →

Spin-dependent DM interactions

These interactions are generically more challenging to probe

In fact, spin-dependent scattering loses the coherent enhancement.
The present bounds on the scattering cross sections are ~6-7 orders of magnitude weaker than for spin-independent interactions

Our axion model predicts a spin-dependent transition

$$\begin{cases} \mathcal{L}_\phi = \phi\left[g_\chi \bar{\chi}\chi + g_p \bar{p}\gamma^5 p + g_n \bar{n}\gamma^5 n\right] \\ \mathcal{L}_a = a\left[g_\chi \bar{\chi}\gamma_5\chi + g_p \bar{p}\gamma^5 p + g_n \bar{n}\gamma^5 n\right] \end{cases} \Longleftarrow \quad \mathcal{L} \supset c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}$$

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How to compute…

DM-**nucleon** interaction Hamiltonian in the non-relativistic limit:

$$\mathcal{H}_\phi = -\frac{g_\chi g_p}{m_p}\frac{\mathrm{q}\cdot \mathrm{S}_p}{\mathrm{q}^2 + m_\phi^2}e^{i\mathrm{q}\cdot\mathrm{r}}$$ velocity suppression

$$\mathcal{H}_a = -\frac{g_\chi g_p}{m_p m_\chi}\frac{(\mathrm{q}\cdot \mathrm{J}_\chi)(\mathrm{q}\cdot \mathrm{S}_p)}{\mathrm{q}^2 + m_a^2}e^{i\mathrm{q}\cdot\mathrm{r}}$$ additional velocity suppression

$$\mathcal{H} = -\frac{g_\chi g_p}{q_0^2 + m_{\rm med}^2}F_{\rm med}(\mathrm{q})\,\mathcal{O}\left(\mathrm{J}_\chi, \mathrm{q}\right)\cdot \mathrm{S}_p\, e^{i\mathrm{q}\cdot\mathrm{r}}$$

$$F_{\rm med}(\mathrm{q}) = \frac{q_0^2 + m_{\rm med}^2}{q^2 + m_{\rm med}^2}$$

$q_0 = m_\chi v_0$,
$v_0 = 220$ km/s

DM-**nucleus** interaction Hamiltonian :

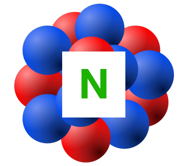
$$\mathcal{H}^N = -\frac{g_\chi g_N}{q_0^2 + m_{\rm med}^2}F_{\rm med}(\mathrm{q})\,\mathcal{O}\left(\mathrm{J}_\chi, \mathrm{q}\right)\cdot \mathrm{J}\, e^{i\mathrm{q}\cdot\mathrm{r}} \qquad g_N \equiv g_p f_p + g_n f_n$$

Using the Wigner-Eckart theorem: $f_{p/n} = \frac{\langle JJ|S^{\mathrm{tot},z}_{p/n}|JJ\rangle}{J}$ nuclear form factors

From nuclei to crystals

Nuclei, N

$$\mathcal{H}^N = -\frac{g_\chi g_N}{q_0^2 + m_{\rm med}^2} F_{\rm med}(\mathbf{q})\, \mathcal{O}\left(\mathbf{J}_\chi, \mathbf{q}\right) \cdot \mathbf{J}\, e^{i\mathbf{q}\cdot\mathbf{r}}$$

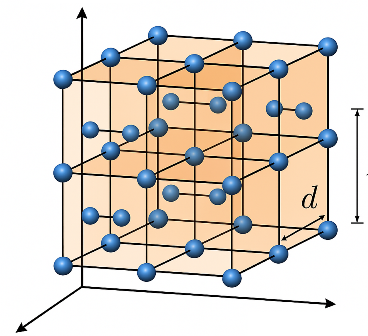


matching

Crystal

$$\mathcal{H}^c(q) = -\frac{g_\chi g_p}{q_0^2 + m_{\rm med}^2} F_{\rm med}(\mathbf{q}) \sum_{\ell,d} \lambda_{\ell,d} \mathcal{O}\left(\mathbf{J}_\chi, \mathbf{q}\right) \cdot \mathbf{J}_{\ell,d}\, e^{i\mathbf{q}\cdot\mathbf{r}_{\ell,d}}$$

$$\lambda_{\ell,d} = \frac{g_{N_{\ell,d}}}{g_p}$$



random spin states

$$\Gamma = \frac{2\pi}{V} \sum_{i,f} \sum_{i_\chi, f_\chi} w_i w_{i_\chi} \int \frac{d^3 q}{(2\pi)^3} |\langle f, f_\chi | \mathcal{H}^c(q) | i, i_\chi \rangle|^2 \delta(E_f - \omega - E_i)$$

ω = energy deposited by DM
i (f) = initial (final) state of the crystal
i_χ, f_χ = initial (final) state of DM

$$|i\rangle = |i_s\rangle \otimes |0\rangle$$

we consider an unpolarized crystal

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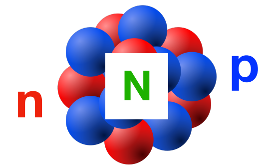
$$\mathcal{H}^N = -\frac{g_\chi g_N}{q_0^2 + m_{\rm med}^2} F_{\rm med}(\mathbf{q})\, \mathcal{O}\left(\mathbf{J}_\chi, \mathbf{q}\right) \cdot \mathbf{J}\, e^{i\mathbf{q}\cdot\mathbf{r}}$$



matching

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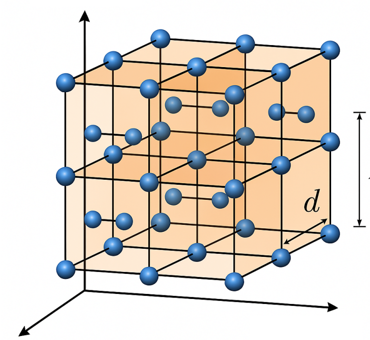

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i_χ, f_χ = initial (final) state of DM

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we consider an unpolarized crystal

$$\begin{cases} \Gamma = \dfrac{g_\chi^2 g_p^2}{(q_0^2 + m_{\rm med}^2)^2} \dfrac{N}{V} \displaystyle\int \dfrac{d^3\mathbf{q}}{(2\pi)^3} |F_{\rm med}(\mathbf{q})|^2 G(\mathbf{q}) S(\mathbf{q}, \omega) \\ G(\mathbf{q}) \equiv \dfrac{1}{3} \displaystyle\sum_{i_\chi} w_{i_\chi} \langle i_\chi | \mathcal{O}\left(\mathbf{J}_\chi, \mathbf{q}\right) \cdot \mathcal{O}\left(\mathbf{J}_\chi, \mathbf{q}\right) | i_\chi \rangle \\ S(\mathbf{q}, \omega) = \displaystyle\sum_d \overline{\lambda_d^2 J_d (J_d + 1)} C_{\ell,d}(\mathbf{q}, \omega), \quad C_{\ell,d}(\mathbf{q}, \omega) \equiv \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \langle 0 | e^{-i\mathbf{q}\cdot r_{\ell,d}(0)} e^{i\mathbf{q}\cdot r_{\ell,d}(t)} | 0 \rangle \end{cases}$$

DM part of the rate

Crystal part of the rate

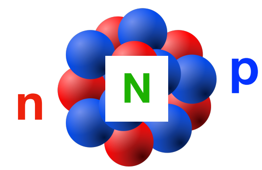
different than in the spin-independent case

computed for spin-independent interactions (expansion in the number of phonons)

From nuclei to crystals

Nuclei, N

$$\mathcal{H}^N = -\frac{g_\chi g_N}{q_0^2 + m_{\rm med}^2} F_{\rm med}(\mathbf{q})\, \mathcal{O}\left(\mathbf{J}_\chi, \mathbf{q}\right) \cdot \mathbf{J}\, e^{i\mathbf{q}\cdot\mathbf{r}}$$



matching

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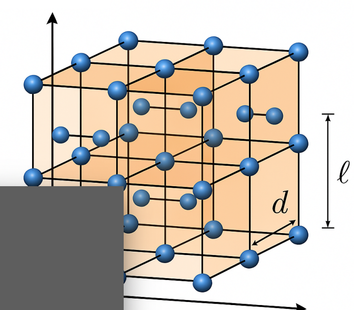


random spin states

$$\Gamma = \frac{2\pi}{V} \sum_{i,f} \ldots$$

$|i\rangle = |i_s\rangle \ldots$

...sider an
...ed crystal

What is the expected number of events in the parameter space that remains unexplored by other measurements?
(i.e., searches for the axion mediator + DM self-interaction)

$$\Gamma = \frac{g \ldots}{(q_0^2 + \ldots}$$

$$G(\mathbf{q}) \equiv \frac{1}{3} \sum_{i_\chi} w_{i_\chi} \langle i_\chi | \mathcal{O}\left(\mathbf{J}_\chi, \mathbf{q}\right) \cdot \mathcal{O}\left(\mathbf{J}_\chi, \mathbf{q}\right) | i_\chi \rangle$$

DM part of the rate

Crystal part of the rate

$$S(\mathbf{q}, \omega) = \sum_d \overline{\lambda_d^2 J_d (J_d + 1)} C_{\ell,d}(\mathbf{q}, \omega), \quad C_{\ell,d}(\mathbf{q}, \omega) \equiv \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \langle 0 | e^{-i\mathbf{q}\cdot r_{\ell,d}(0)} e^{i\mathbf{q}\cdot r_{\ell,d}(t)} | 0 \rangle$$

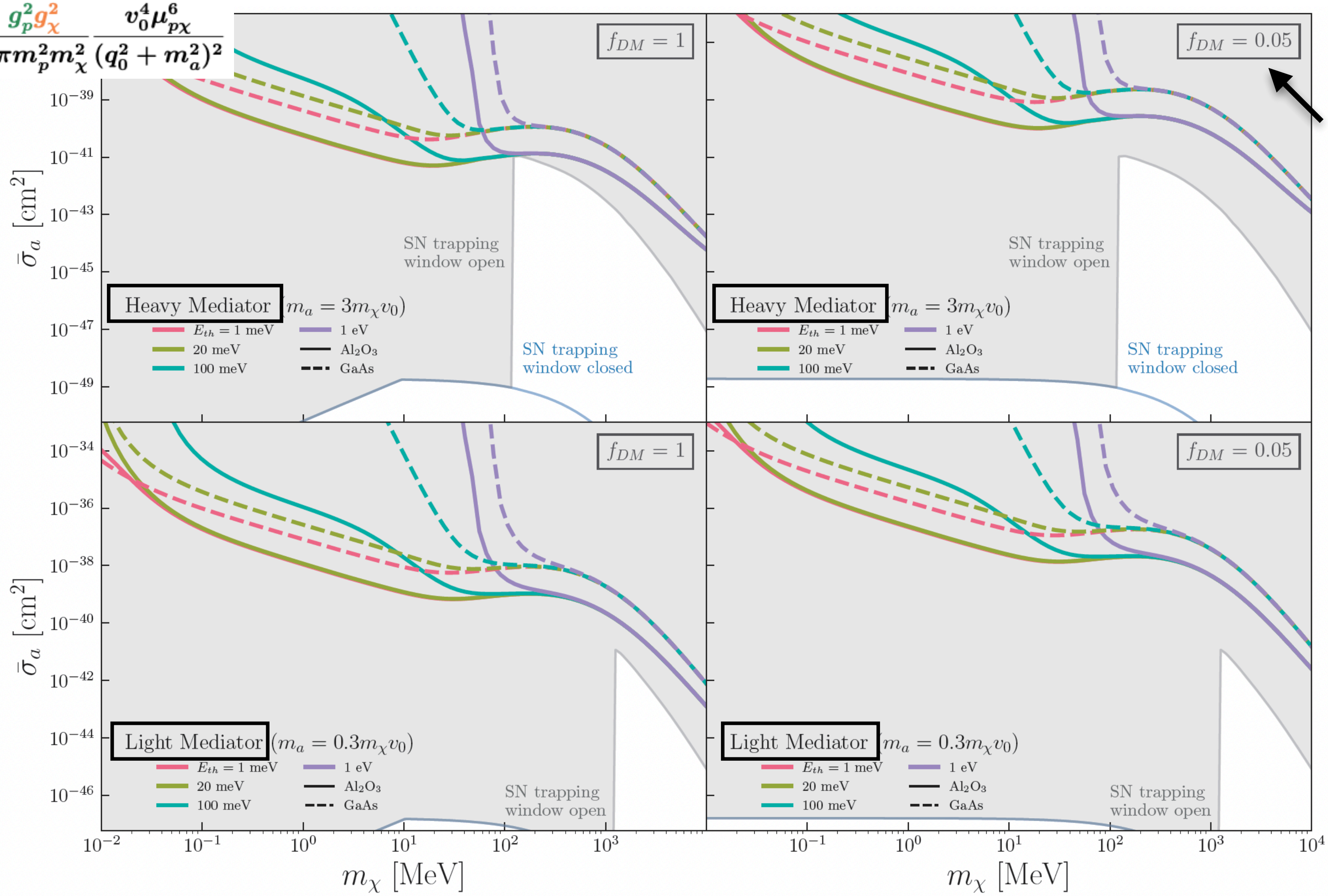
different than in the spin-independent case

computed for spin-independent interactions (expansion in the number of phonons)

How many events at direct detection? a

Taking the max allowed g_χ and g_p for each value of m_χ:

$$\bar{\sigma}_a = \frac{g_p^2 g_\chi^2}{3\pi m_p^2 m_\chi^2} \frac{v_0^4 \mu_{p\chi}^6}{(q_0^2 + m_a^2)^2}$$



DM subcomponent
no constraints from SIDM

different threshold energies

different materials

3 events / kg / year

$$\mathcal{L}_a = a\left[g_\chi \bar{\chi}\gamma_5\chi + g_p \bar{p}\gamma^5 p + g_n \bar{n}\gamma^5 n\right]$$

SG, Knapen, Lin, Munbodh, Suter, 2506.11191

How many events at direct detection? ϕ

Taking the max allowed g_χ and g_p for each value of m_χ:

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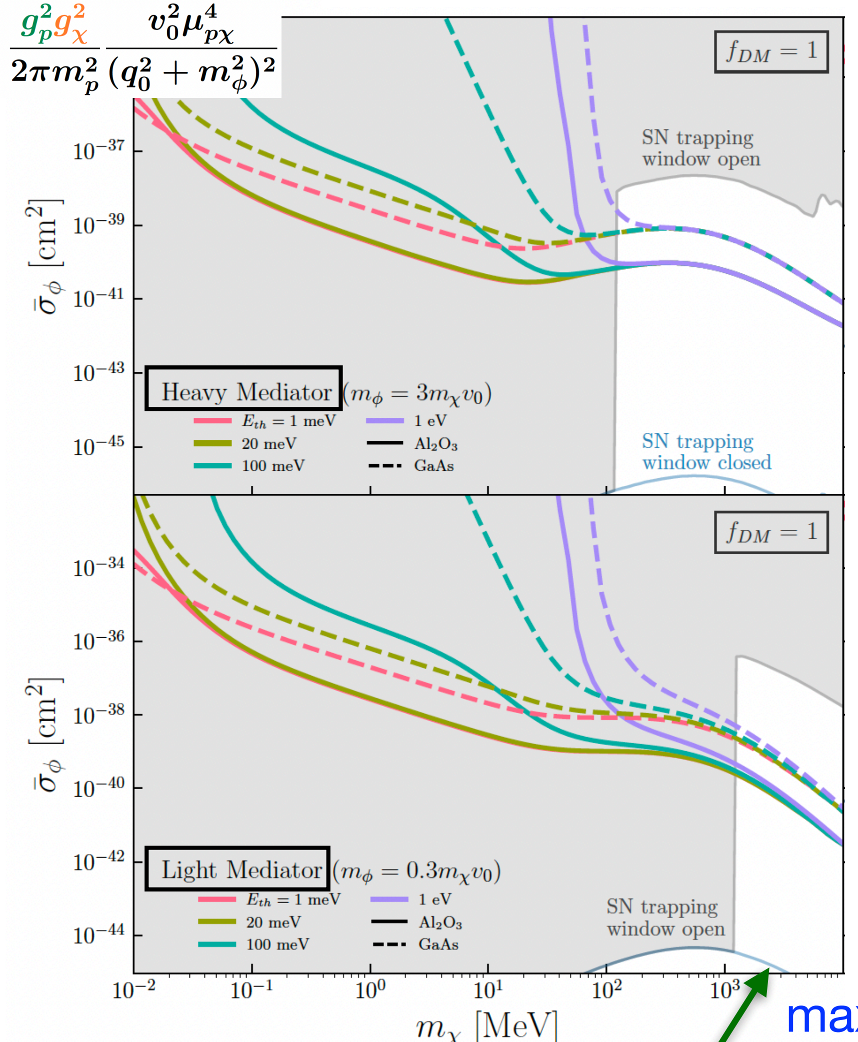

max n. ~3000

3 events/kg-year

3 events/kg-year

max n. ~16000

different threshold energies

different materials

Bound if the supernova "trapping window" gets closed in the future

$$\mathcal{L}_\phi = \phi \left[g_\chi \bar{\chi}\chi + g_p \bar{p}\gamma^5 p + g_n \bar{n}\gamma^5 n \right]$$

How many events at direct detection? ϕ

Taking the max allowed g_χ and g_p for each value of m_χ:

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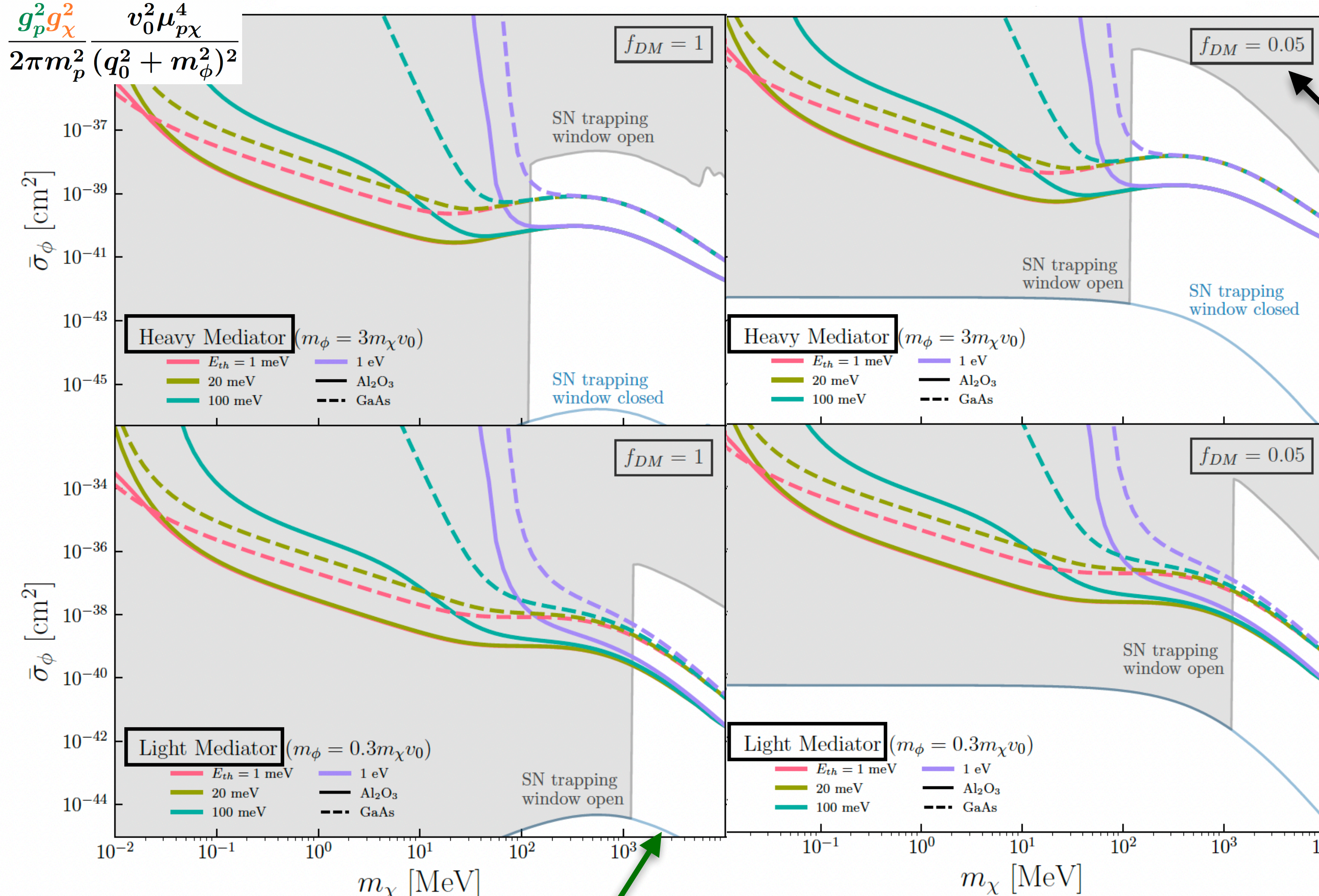


DM subcomponent
no constraints from SIDM
(in general larger $g_{a\chi}$)

max n. ~6000

max n. ~110000

different threshold energies

different materials —— ....

Bound if the supernova "trapping window" gets closed in the future

$$\mathcal{L}_\phi = \phi\left[g_\chi \bar{\chi}\chi + g_p \bar{p}\gamma^5 p + g_n \bar{n}\gamma^5 n\right]$$

Outlook

Axion or ALP mediated Dark Matter (DM) models are simple and theoretically very well motivated.

Several cosmological histories can lead to the measured DM abundance.

Complementarity between searches for the axion or ALP mediator (astrophysics + high-intensity experiments) and searches for DM (self-interaction + direct detection).

Spin-dependent interactions between DM and the Standard Model are less explored and they arise in axion or ALP mediated models.

Interesting prospects to search for these interactions using direct detection experiments.
One or multiple phonon excitations in crystals.

Bounds on QCD axion-mediated DM

A minimal model:
let's take the QCD axion (either KSVZ or DFSZ) and let's couple it to a singlet Dirac fermion DM candidate:

$$\boxed{\mathcal{L} \supset \frac{c_\chi}{2f_a}\partial_\mu a \bar{\chi}\gamma^\mu\gamma_5\chi}$$

A small set of free parameters fixes the cosmology of the model: $f_a,\ m_\chi,\ g_{a\chi} \equiv \frac{c_\chi m_\chi}{f_a},\ T_{\rm RH}$
(eventually tanβ in DFSZ)

Couplings	KSVZ	DFSZ
Gluons	$\frac{\alpha_s}{8\pi f_a}$	
Photons	$-\frac{\alpha}{8\pi f_a}(1.924)$	$\frac{\alpha}{8\pi f_a}\left(\frac{8}{3}-1.924\right)$
Quarks	Loop suppressed	up : $\frac{\cos^2\beta}{6f_a}$, down : $\frac{\sin^2\beta}{6f_a}$
Leptons	Loop suppressed	Type I : $\frac{\sin^2\beta}{6f_a}$, Type II : $-\frac{\cos^2\beta}{6f_a}$

Bounds from DM self-interaction:

$$c_\chi \frac{m_\chi}{f_a} \equiv g_{a\chi} \lesssim 0.21\left(\frac{m_\chi}{1\text{ MeV}}\right)^{3/4}$$

$$f_a \gtrsim \begin{cases} 3.9\times 10^8\ \text{GeV} & \text{(KSVZ)} \\ 1.2\times 10^9\ \text{GeV}\ \sin^2\beta & \text{(DFSZ-I)} \\ 1.2\times 10^9\ \text{GeV}\ \cos^2\beta & \text{(DFSZ-II)} \end{cases}$$

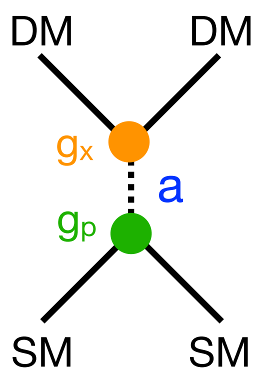
supernova cooling bounds (axion-nucleon interaction)

cooling bounds on red giant (axion-electron interaction)

The rates

Results implemented in DarkELF
https://github.com/tongylin/DarkELF

$$R_a \sim \frac{\bar{\sigma}_a}{\sum_d m_d} \frac{\rho_\chi}{m_\chi} \frac{m_p^2 m_\chi^2}{v_0^4 \mu_{\chi p}^6} J_\chi (J_\chi + 1) \int d^3\mathrm{v}\, f(\mathrm{v}) \int \frac{d^3 q}{(2\pi)^3} |F_a(\mathrm{q})|^2 \frac{|q|^4}{m_p^2 m_\chi^2} S(q,\omega)$$



$$\bar{\sigma}_a = \frac{g_p^2 g_\chi^2}{3\pi m_p^2 m_\chi^2} \frac{v_0^4 \mu_{p\chi}^6}{(q_0^2 + m_a^2)^2}$$

Reference cross section
(it reproduces the DM-proton cross section in the limit, $m_a >> q$)

Our approximations:

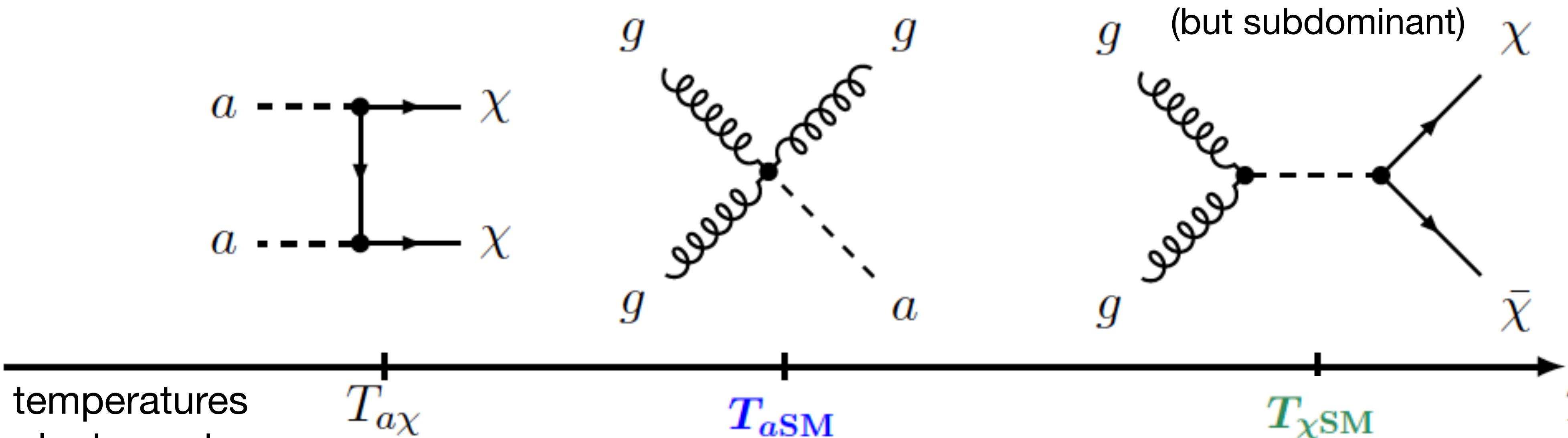
- Harmonic Approximation: Decompose into a sum of harmonic oscillators weighted by the phonon density of states (that enters the $C_{l,d}$ function)
- Isotropic crystal with randomly distributed nuclear spins
- Nucleus spins are spherically symmetric

How many events we can get in the parameter space that is not probed by other experiments?

Dark Matter thermal history

The thermal history depends on three classes of processes:

also fermion scattering (but subdominant)



temperatures only depend on

$f_a,\ m_\chi,\ g_{a\chi}$

(mild)

$$T_{a\rm SM} \simeq 2\times 10^4 \mathrm{GeV}\left(\frac{f_a}{10^9\mathrm{GeV}}\right)^2$$

$$\boxed{T_{\chi\rm SM} \gg T_{a\rm SM}}$$

$$T_{\chi\rm SM} \simeq 4\times 10^7 \mathrm{GeV}\left(\frac{f_a}{10^9\mathrm{GeV}}\right)^2\left(\frac{1}{g_{a\chi}}\right)^2$$

(this is shown in the figure above)

3 possible hierarchies:

$T_{a\chi} \ll T_{a\rm SM} \ll T_{\chi\rm SM}$

$T_{a\rm SM} \ll T_{a\chi} \ll T_{\chi\rm SM}$

$T_{a\rm SM} \ll T_{\chi\rm SM} \ll T_{a\chi}$

see also Salvio et al., 1310.6982

Freeze-in and freeze-out in the dark sector

2) $\chi\bar{\chi} \to aa$ freeze-out in the dark sector: $T' = \left(\frac{(7/8) g_\chi + g_a}{g_a} \frac{g_{\star,S}(T)}{g_{\star,S}(T_{a\mathrm{SM}})} \right)^{1/3} T$

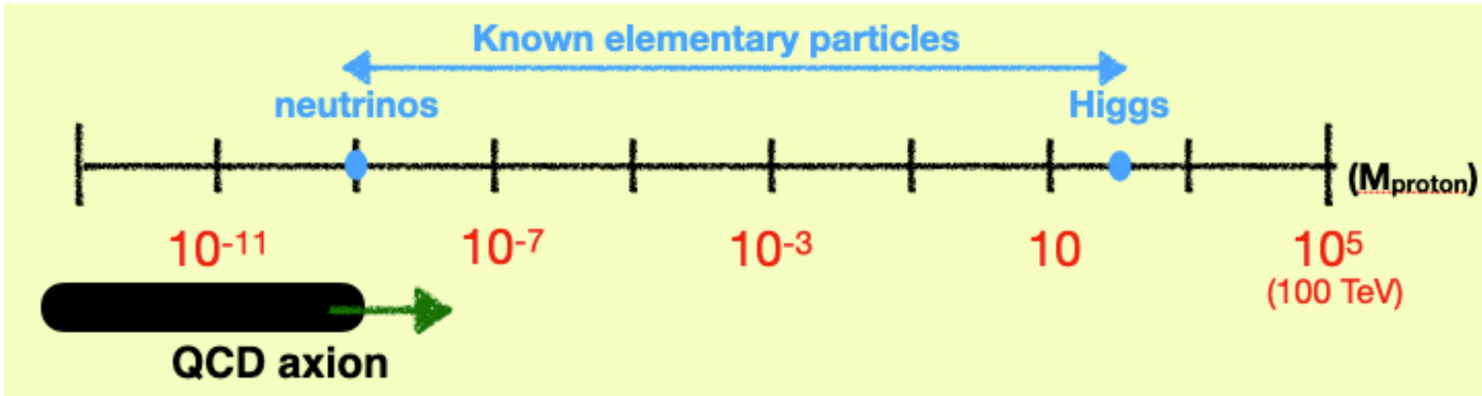
$$\Omega_\chi \simeq \Omega_{\mathrm{DM}} \left[\frac{g_{\star,S}(T_{a\mathrm{SM}})}{g_{\star,S}(T_f)} \right]^{1/3} \left(\frac{m_\chi / T_f}{10} \right)^2 \left(\frac{g_\star(T_f)}{15} \right)^{\frac{1}{2}} \left(\frac{15}{g_{\star,S}(T_f)} \right) \left(\frac{m_\chi}{1\ \mathrm{GeV}} \right)^2 \left(\frac{4.4 \times 10^{-2}}{g_{a\chi}} \right)^4$$

even freeze-out can work
for perturbative couplings

3) $aa \to \chi\bar{\chi}$ freeze-in in the dark sector: $T' = \left(\frac{g_{\star,S}(T)}{g_{\star,S}(T_{a\mathrm{SM}})} \right)^{1/3} T$

$$\Omega_\chi \simeq \Omega_{\mathrm{DM}} \left[\frac{g_{\star,S}(m_\chi)}{g_{\star,S}(T_{a\mathrm{SM}})} \right]^{5/3} \left(\frac{10^3}{g_{\star,S}(m_\chi)\sqrt{g_\star(m_\chi)}} \right) \left(\frac{g_{a\chi}}{3 \times 10^{-6}} \right)^4$$

Heavier axions and the strong CP problem



Models where two or more axions naturally cooperate to address the strong CP problem.

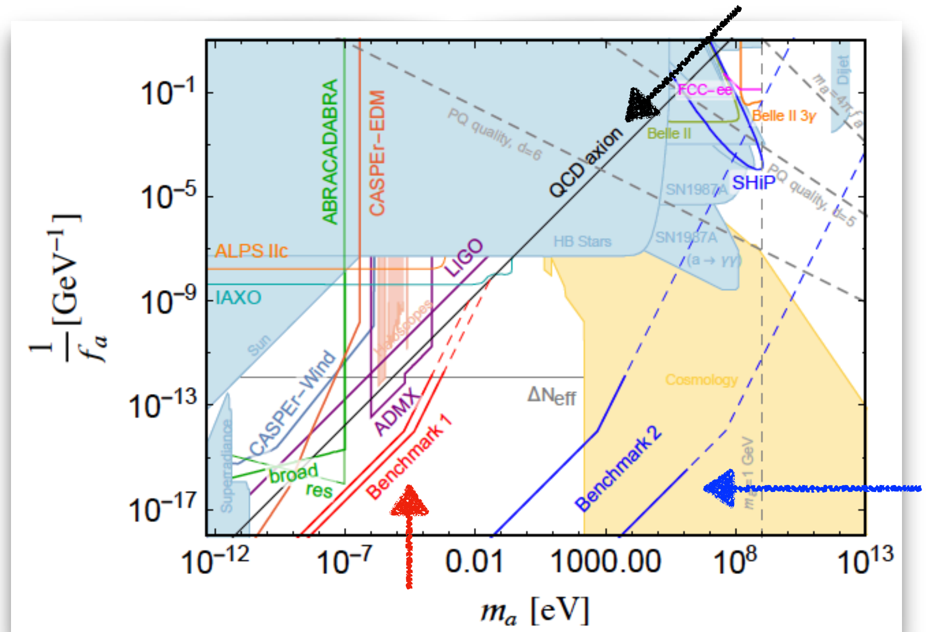
(Some references:
Agrawal, Howe, 1710.04213;
Foster, Kumar, Safdi, Soreq, 2208.10504, …)

$$SU(3) \times SU(3) \rightarrow SU(3)_D = SU(3)_c$$

Lot of freedom in the (m_a-f_a) plane.
Also different hierarchies in axion-SM couplings.

Axion quality problem alleviated for heavy axions

if PQ symmetry broken by dimension D operators at Λ_{UV}:
$$\delta\bar{\theta} \sim \frac{f_a^{D-2}}{m_a^2 \Lambda_{UV}^{D-4}}$$



Agrawal, Howe, 1710.04213

Phenomenology of the ALP mediator

$$\boxed{\mathcal{L} \supset c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f_a} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}}$$

It generates couplings to nucleons (protons and neutrons) as well as photons

$$\mathcal{L}_{n,p} = a(g_p \, \bar{p}\gamma_5 p + g_n \, \bar{n}\gamma_5 n)$$

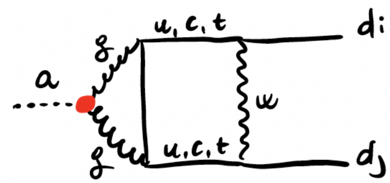
$$\mathcal{L}_{\gamma\gamma} = c_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\begin{cases} g_p \approx \dfrac{m_p}{f_a} c_{GG} \left(g_0 + g_A \dfrac{m_\pi^2}{m_a^2} A(m_a) \right) \\ g_n \approx \dfrac{m_p}{f_a} c_{GG} \left(g_0 - g_A \dfrac{m_\pi^2}{m_a^2} A(m_a) \right) + \delta_{RG} c_{GG} \\ c_{\gamma\gamma} \approx -c_{GG}(1.92 + A(m_a)) \end{cases}$$

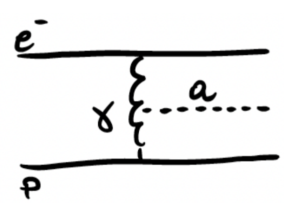
RGE running

Production at

- meson factories: K →π a, B→K a



- Proton and electron beam dump experiments



- LHC auxiliary detectors

Decay into photons or hadrons (+ leptons)

Advantages of high intensity experiments compared to high energy colliders

- high luminosity and therefore large productions
- forward kinematics
- cleaner detector environment

ideal for sub-GeV particles

Approximations in the calculation of rates

- Harmonic Approximation: Decompose into a sum of harmonic oscillators weighted by the phonon density of states (that enters the $C_{l,d}$ function)
- Isotropic crystal with randomly distributed nuclear spins
- Nucleus spins are spherically symmetric

Results implemented in DarkELF
https://github.com/tongylin/DarkELF

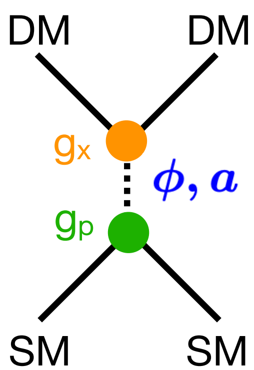
The rates

We can compute the number of events we expect at a direct detection experiment:

$$R_\phi \sim \frac{\bar{\sigma}_\phi}{\sum_d m_d}\frac{\rho_\chi}{m_\chi}\frac{m_p^2}{v_0^2\mu_{\chi p}^4}\int d^3\mathrm{v}\, f(\mathrm{v})\int\frac{d^3q}{(2\pi)^3}|F_\phi(\mathrm{q})|^2\frac{|q|^2}{m_p^2}S(q,\omega)$$

$$R_a \sim \frac{\bar{\sigma}_a}{\sum_d m_d}\frac{\rho_\chi}{m_\chi}\frac{m_p^2 m_\chi^2}{v_0^4\mu_{\chi p}^6}J_\chi(J_\chi+1)\int d^3\mathrm{v}\, f(\mathrm{v})\int\frac{d^3q}{(2\pi)^3}|F_a(\mathrm{q})|^2\frac{|q|^4}{m_p^2 m_\chi^2}S(q,\omega)$$

further velocity suppression



$$\bar{\sigma}_\phi = \frac{g_p^2 g_\chi^2}{2\pi m_p^2}\frac{v_0^2\mu_{p\chi}^4}{(q_0^2+m_\phi^2)^2}$$

$$\bar{\sigma}_a = \frac{g_p^2 g_\chi^2}{3\pi m_p^2 m_\chi^2}\frac{v_0^4\mu_{p\chi}^6}{(q_0^2+m_a^2)^2}$$

Reference cross sections.
They reproduce the DM-proton cross section in the limit, $m_{\phi,a} >> q$

How many events can we get in the parameter space that is not probed by other measurements?
(i.e., searches for the axion mediator + DM self-interaction)

DM-crystal interaction rate (more details)

$$\Gamma = \frac{2\pi}{V}\sum_{i,f}\sum_{i_\chi,f_\chi} w_i w_{i_\chi}\int\frac{d^3q}{(2\pi)^3}|\langle f, f_\chi|\mathcal{H}^c(q)|i,i_\chi\rangle|^2\delta(E_f-\omega-E_i)$$

energy deposited by DM into the crystal

Fourier transform:

$$\Gamma = \frac{1}{V}\int_{-\infty}^{+\infty}dt\int\frac{d^3q}{(2\pi)^3}\sum_{i,f}\sum_{i_\chi,f_\chi} w_i w_{i_\chi}\langle i,i_\chi|\mathcal{H}^{c\dagger}|f,f_\chi\rangle\langle f,f_\chi|e^{-iE_ft}\mathcal{H}^c e^{iE_it}|i,i_\chi\rangle e^{i\omega t}$$

$$= \frac{1}{V}\int_{-\infty}^{+\infty}dt\,e^{i\omega t}\int\frac{d^3q}{(2\pi)^3}\sum_{i,i_\chi} w_i w_{i_\chi}\langle i,i_\chi|\mathcal{H}^c(0)^\dagger\mathcal{H}^c(t)|i,i_\chi\rangle$$

eigenvalues of the crystal Hamiltonian

$$\Gamma = \frac{1}{V}\frac{g_\chi^2 g_p^2}{(q_0^2+m_{\rm med}^2)^2}\int_{-\infty}^{+\infty}dt\,e^{i\omega t}\int\frac{d^3q}{(2\pi)^3}|F_{\rm med}(\mathrm{q})|^2\times$$
$$\sum_{\alpha,\beta=x,y,z}\left[\sum_{i_\chi} w_{i_\chi}\langle i_\chi|\mathcal{O}(\mathrm{J}_\chi,\mathrm{q})^\alpha\,\mathcal{O}(\mathrm{J}_\chi,\mathrm{q})^\beta|i_\chi\rangle\right]\times$$
$$\left[\sum_i w_i\sum_{\ell,d;\ell',d'}\lambda_{\ell,d}\lambda_{\ell',d'}\langle i|J^\alpha_{\ell,d}e^{-iq\cdot r_{\ell,d}(0)}J^\beta_{\ell'd'}e^{iq\cdot r_{\ell'd'}(t)}|i\rangle\right]$$

- nuclear spin orientations are isotropically distributed;
- $\ell\neq\ell'$, $d\neq d'$ average to zero when summed over all initial nuclear spin configurations

$$J^\alpha_{\ell,d}J^\beta_{\ell'd'}\to\frac{1}{3}\delta^{\alpha\beta}\delta_{\ell\ell'}\delta_{dd'}J^2_{\ell,d}$$

$$\Gamma = \frac{g_\chi^2 g_p^2}{(q_0^2+m_{\rm med}^2)^2}\frac{N}{V}\int\frac{d^3\mathrm{q}}{(2\pi)^3}|F_{\rm med}(\mathrm{q})|^2 G(\mathrm{q})S(\mathrm{q},\omega)$$

$$S(q,\omega) = \frac{1}{N}\sum_{\ell,d}\lambda^2_{\ell,d}C_{\ell,d}(q,\omega)\sum_{i_s}w_{i_s}\langle i_s|J^2_{\ell,d}|i_s\rangle = \sum_d\overline{\lambda_d^2 J_d(J_d+1)}C_{\ell,d}(\mathrm{q},\omega)$$

$$C_{\ell,d}(q,\omega) \equiv \int_{-\infty}^{+\infty}dt\,e^{i\omega t}\langle 0|e^{-iq\cdot r_{\ell,d}(0)}e^{iq\cdot r_{\ell,d}(t)}|0\rangle$$

DM-crystal interaction rate (more details)

$$\Gamma = \frac{2\pi}{V} \sum_{i,f} \sum_{i_\chi, f_\chi} w_i w_{i_\chi} \int \frac{d^3q}{(2\pi)^3} |\langle f, f_\chi | \mathcal{H}^c(q) | i, i_\chi \rangle|^2 \delta(E_f - \omega - E_i)$$

energy deposited by DM into the crystal

Fourier transform:

$$\Gamma = \frac{1}{V} \int_{-\infty}^{+\infty} dt \int \frac{d^3q}{(2\pi)^3} \sum_{i,f} \sum_{i_\chi, f_\chi} w_i w_{i_\chi} \langle i, i_\chi | \mathcal{H}^{c\dagger} | f, f_\chi \rangle \langle f, f_\chi | e^{-iE_f t} \mathcal{H}^c e^{iE_i t} | i, i_\chi \rangle e^{i\omega t}$$

eigenvalues of the crystal Hamiltonian

$$\frac{1}{3}\delta^{\alpha\beta}\delta_{\ell\ell'}\delta_{dd'}J^2_{\ell,d}$$

Correlation function computed in Campbell-Deem et al., 2205.02250 (in the context of spin-independent transitions)

$$C_{\ell,d}(q,\omega) = 2\pi e^{-2W_d(q)} \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{q^2}{2m_d}\right)^n \prod_{i=1}^{n} \int d\omega_i \frac{D_d(\omega_i)}{\omega_i} \delta\left(\sum_{i=1}^{n} \omega_i - \omega\right)$$

Debye-Waller factor: $W_d(q) \equiv \frac{q^2}{4m_d} \int d\omega' \frac{D_d(\omega')}{\omega'}$

$$\Gamma = \frac{g_\chi^2 g_p^2}{(q_0^2 + m_{\rm med}^2)^2} \frac{N}{V} \int \frac{d^3\mathrm{q}}{(2\pi)^3} |F_{\rm med}(\mathrm{q})|^2 G(\mathrm{q}) S(\mathrm{q}, \omega)$$

$$S(q,\omega) = \frac{1}{N} \sum_{\ell,d} \lambda_{\ell,d}^2 C_{\ell,d}(q,\omega) \sum_{i_s} w_{i_s} \langle i_s | J_{\ell,d}^2 | i_s \rangle = \sum_d \overline{\lambda_d^2 J_d(J_d+1)} C_{\ell,d}(\mathrm{q},\omega)$$

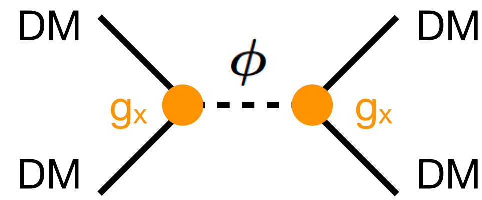
$$C_{\ell,d}(q,\omega) \equiv \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \langle 0 | e^{-iq\cdot r_{\ell,d}(0)} e^{iq\cdot r_{\ell,d}(t)} | 0 \rangle$$

Testing the DM: self-interactions, ϕ

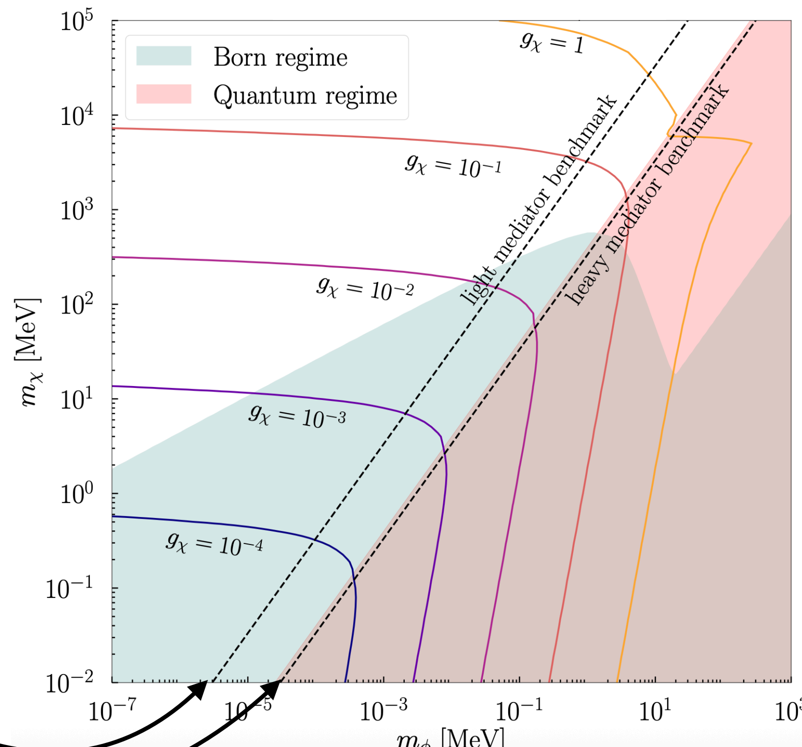
Viscosity cross section:

$$\sigma_V \equiv \int d\Omega \frac{d\sigma}{d\Omega}(1-\cos^2\theta) \lesssim (1.1\mathrm{cm}^2/g)m_\chi$$

v ≃ 0.005c
(galaxy group)



Less straightforward calculation, if the ALP has scalar coupling with DM: $g_\chi \phi \bar{\chi}\chi$



* Perturbative calculation valid in the Born approximation

$$\frac{g_\chi^2 m_\chi}{4\pi m_\phi} \ll 1$$

* Quantum regime (non-perturbative), Hulthen potential

$$\frac{m_\chi v}{m_\phi} \ll 1$$

Colquhoun et al., 2011.04679
Tulin et al., 1302.3898
Tulin et al., 1210.0900

2 benchmark models:

- light mediator: $m_\phi = 0.3\, q_0 = 0.3\, m_\chi v_0$
- heavy mediator: $m_\phi = 3\, q_0 = 3\, m_\chi v_0$

More details on matrix element calculations

From nucleon to nucleus matrix element:

$$\langle J m_f | S^{\mathrm{tot},z}_{p/n} | J m_i \rangle = f_{p/n} \langle J m_f | J | J m_i \rangle \Rightarrow f_{p/n} = \frac{\langle J J | S^{\mathrm{tot},z}_{p/n} | J J \rangle}{J}$$

Wigner-Eckart theorem

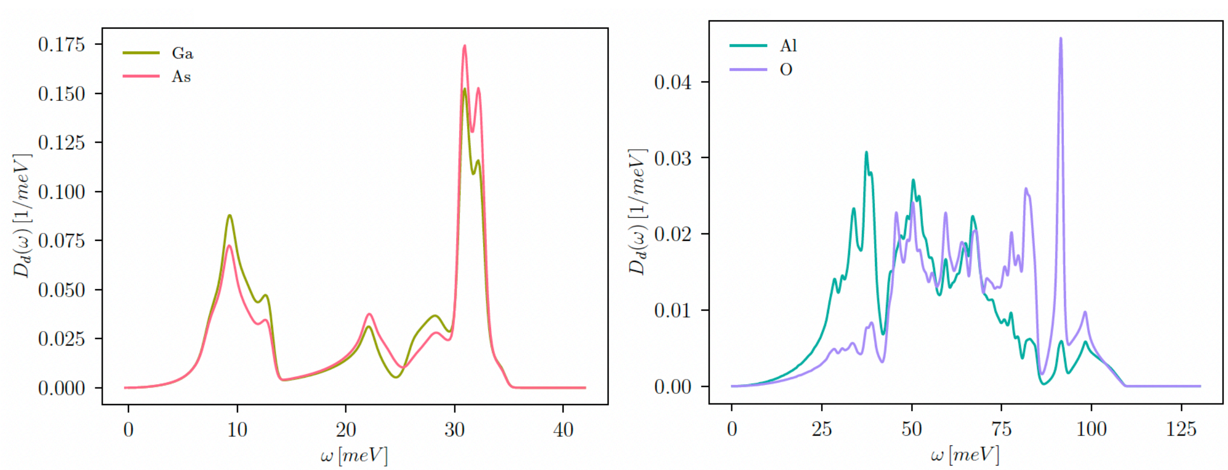
Computed using Shell-model or Odd-Group Model

Phonon part of the matrix element. Summing over phonon excitations, n:

$$C_{\ell,d}(q,\omega) = 2\pi e^{-2W_d(q)} \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{q^2}{2m_d} \right)^n \prod_{i=1}^{n} \int d\omega_i \frac{D_d(\omega_i)}{\omega_i} \delta \left(\sum_{i=1}^{n} \omega_i - \omega \right)$$

Sum converges to a Gaussian in the regime where q^2/m_d is much larger than the typical phonon frequency

phonon partial density of states, D_d:

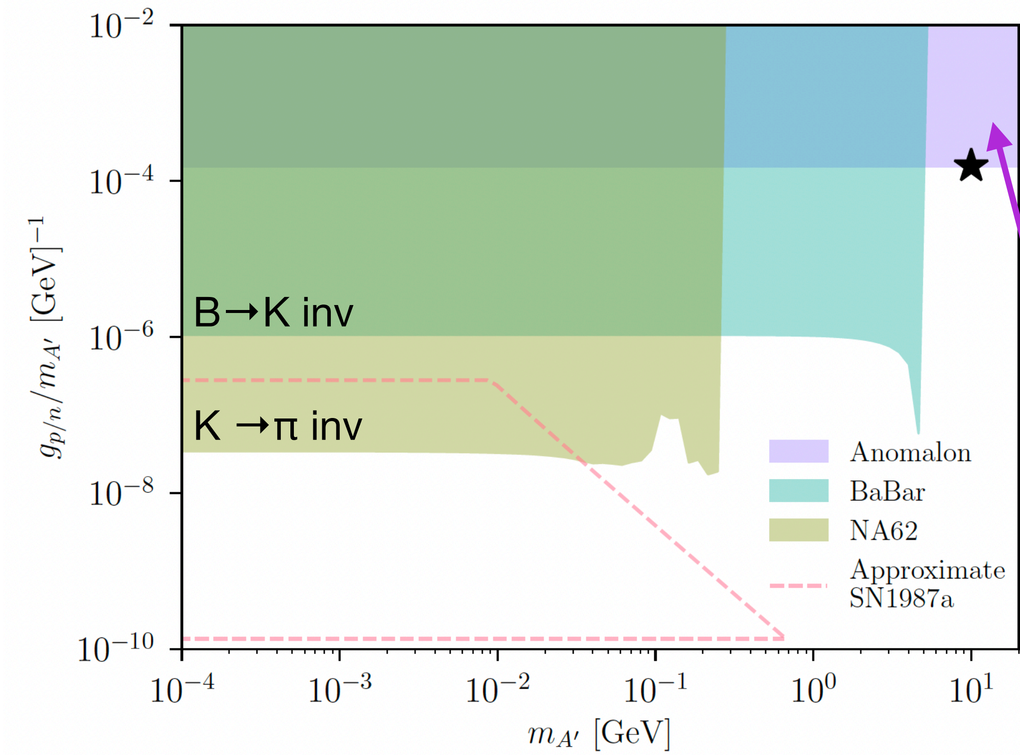


Access to lower energy DM states

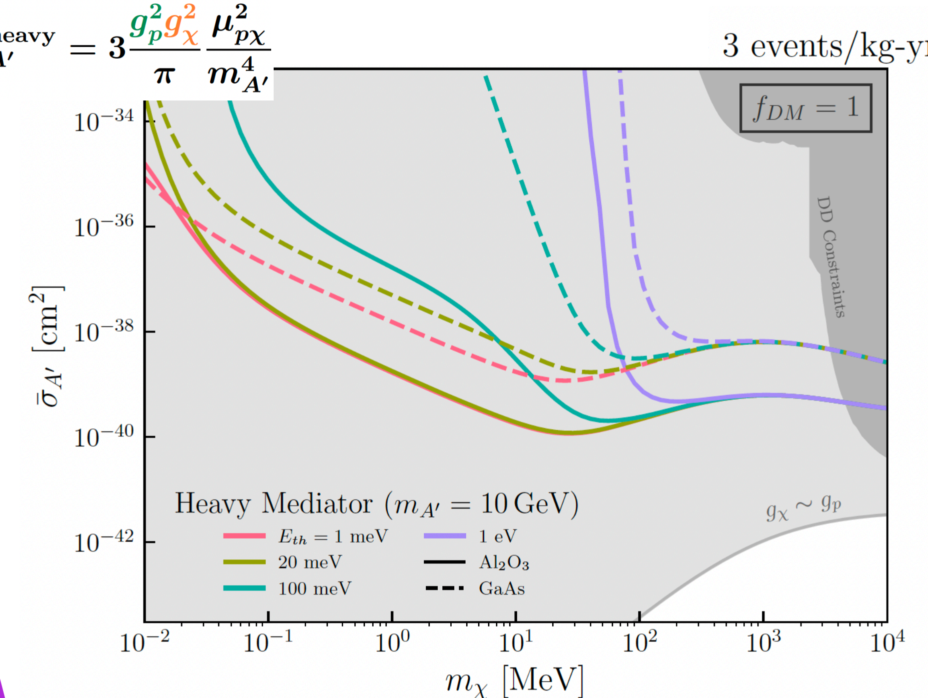
Axial vector model

$$\mathcal{L} \supset g' A'^{\mu} \left[\sum_q \bar{q}\gamma_\mu\gamma_5 q - 2\bar{\nu}\gamma_\mu P_R \nu - \frac{1}{2}\bar{\chi}\gamma_\mu\gamma_5\chi \right]$$

Bounds on the mediator:



$$\bar{\sigma}_{A'}^{\text{heavy}} = 3\frac{g_p^2 g_\chi^2}{\pi}\frac{\mu_{p\chi}^2}{m_{A'}^4}$$



additional quarks are required
to cancel anomalies