ON GRAVITATIONAL RADIATION FROM COSMIC STRING SEGMENTS

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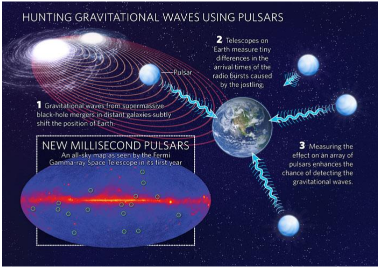
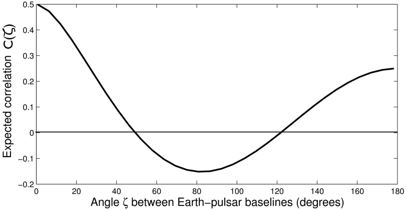
Pulsar Timing Array (PTA)



(NASA/DOE/Fermi LAT Collaboration via Nature)

The Hellings–Downs curve: angular correlation (ζ_{ij}: angle between pulsars)

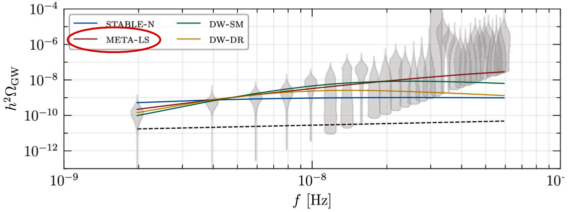


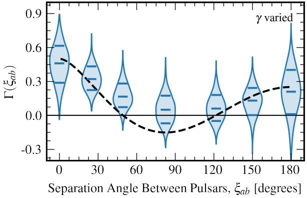
[Hellings, Downs, ApJL. 265, L39 (1983)]

- ✓ **Millisecond pulsars**: stable clocks in space
- ✗ Individual pulsar timing data can be noisy (e.g., interstellar medium, pulsar rotational irregularities)
- ✓ **GWs cause correlated timing variations** in multiple pulsar signals
- ✓ By combining data from multiple pulsars, PTA increases the sensitivity to detect GWs (67 pulsars for 15 years)
- ✓ The **Helling-Downs curve** provides strong evidence for GW!

Gravitational Wave Signal?

NANOGrav 15yr results [2306.16213, 2306.16219]





✓ Signal strength in **nHz** range:

$$\Omega_{\rm GW}h^2 = 9.3^{+5.8}_{-4.0} \times 10^{-9}\ , \quad \left(\Omega_{\rm GW} = \frac{1}{\rho_{\rm cr}}\frac{d\rho_{\rm GW}}{\log f}\right)$$

✓ GW from metastable cosmic string fits well!

String tension (mass per length): $\mu \sim (10^{15}\text{GeV})^2$–$(10^{17}\text{GeV})^2$.

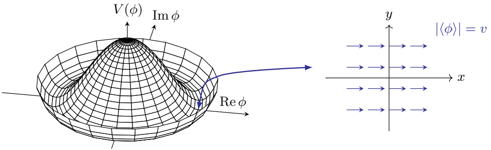
String breaking time in the Universe: $t_d \simeq 10^{3\text{–}6}$ sec.

Review of Metastable Cosmic String

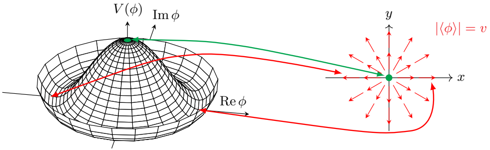
Cosmic String

U(1) symmetry breaking is associated with cosmic strings !

✓ Trivial vacuum configuration of in broken phase
= phase in ϕ is aligned in the same direction [e.g. $(\mathrm{Re}\,\phi, \mathrm{Im}\,\phi) = (v, 0)$]



✓ Non-trivial vacuum configuration due to the finite causality length
= phase of ϕ depends on the spatial direction [e.g. $(\cos\varphi, \sin\varphi)$]

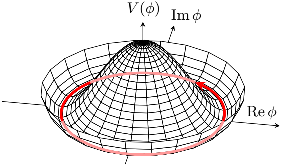


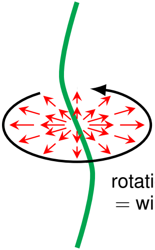
U(1) symmetry is restored at the core of the configuration!
$\rightarrow$ energy concentration at the core = **topological defect**

Cosmic String

Energy concentrations connected in one dimension = **Cosmic (ANO) String**

[Abrikosov,Sov.Phys.JETP5,1174(1957), Nielsen, Olese, Nucl.Phys.B61,45(1973),302]





rotation of $\vec{\phi}$

$=$ winding number

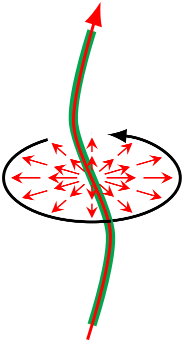
✓ String tension = mass per unit length (= relativistic string)

String-width : $\rho_{\rm str} \simeq m_\phi^{-1} \sim \dfrac{1}{\lambda^{1/2} v}$

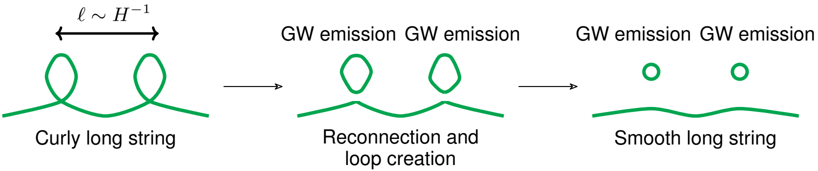
Tension : $\mu \sim$ [cross-section] $\times V(\phi = 0) \sim \pi v^2$

$\rightarrow$ String Mass : $\mu \times$ [Length]

Key Cosmological Features of Cosmic Strings



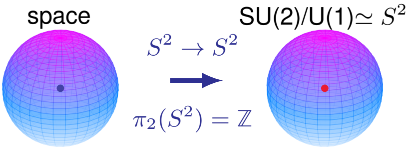
- ✓ **U(1) Magnetic flux** goes through the string
  $\Rightarrow$ Long strings are stable
- ✓ Long strings keep generating loop strings through **reconnections** and **self-intersections**
- ✓ Loop strings shrink by emitting **GWs** and eventually disappear
- ✓ **String Network** [$\rho_{\rm str} \sim G_N\mu \times \rho_{\rm tot}$]
  Long strings remain in the Universe
  Loops are constantly generated



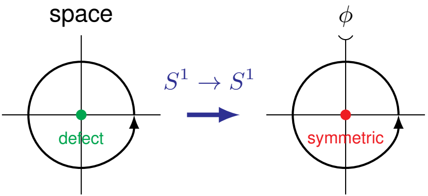
Metastable String?

Two Step Model : SU(2) $\to$ U(1) $\to$ Nothing [Preskill, Vilenkin, Phys.Rev.D47,2324(1993)]

✓ Magnetic monopole = topological defect associated with SU(2) $\to$ U(1)



✓ Cosmic string = topological defect associated with U(1)$\to$Nothing

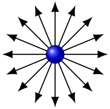


Defect extends 1D perpendicular

✓ π_1(SU(2)) = π_2(SU(2)) = empty $\to$ No stable defects exist!
$\to$ String is metastable at best!

Metastable String?

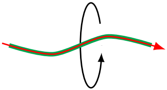
✓ Magnetic monopole at SU(2) → U(1) [by SU(2) triplet]



Magnetic flux :

$$\int d\vec{S}\vec{B} = \frac{4\pi}{g}$$

✓ Magnetic flux in string for U(1)→Nothing [by SU(2) doublet]



$$\int d^2 x B_z = \oint_{r\to\infty} A_\varphi d\varphi = \frac{4\pi}{g}$$

Magnetic monopoles can become the **endpoints** of cosmic strings

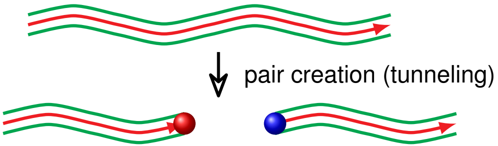
Anti-Monopole

Monopole

Metastable String?

- ✓ Assume SU(2)→U(1) breaking takes place **before inflation**
  → **monopoles are diluted away**
- ✓ Assume U(1) breaking takes place **after reheating**
  → **A cosmic string network is formed**
- ✓ A long string becomes **metastable** for a **large monopole** mass $M_m \gg \mu^{1/2}$
  [For $M_m \ll \mu^{1/2}$ string is unstable (W−boson melting)]
- ✓ Metastable string eventually decay **at later time**

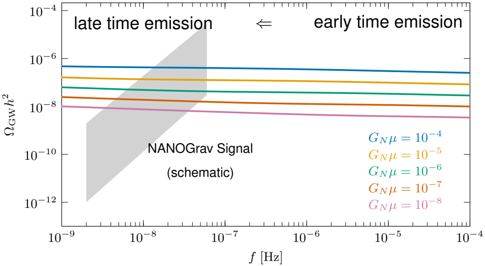
Monopole and antimonopole pair creation inside cosmic string!



Gravitational Wave from Metastable Cosmic Strings

Gravitational Waves from Stable String

✓ GWs from **stable strings** [see e.g. Gouttenoire et al., JCAP 07 (2020) 032]



✓ The amplitude has uncertainties

✓ The flat spectrum shape is rather robust in this frequency range

GW radiation power of a string loop with a length ℓ

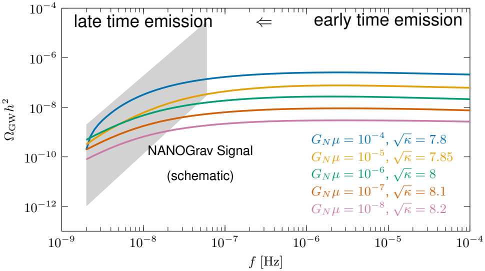
$$P \simeq \sum_n P_n \simeq 50 \times G_N \mu^2 \ , \ \left(f^{(n)}_{\text{emit}} = \frac{2n}{\ell} \ , \ P_n \propto n^{-4/3} \right)$$

$$f^{(n)}_{\text{obs}} \sim 3\,\text{Hz} \times n \left(\frac{10^{-9}}{G_N \mu} \right) \left(\frac{T_{\text{emit}}}{\text{GeV}} \right) .$$

GWs from stable strings do not fit the PTA signal

Gravitational Waves from Stable String

✓ GWs from **metastable strings**



✓ Metastable strings disappears at late Universe

✓ The GW spectrum is suppressed at low frequencies

Decay rate per string length is

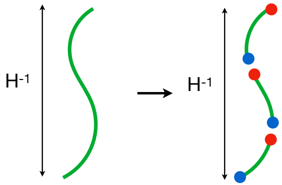
$$\Gamma_d = \frac{\mu}{2\pi} e^{-\pi\kappa}, \quad \sqrt{\kappa} \sim \frac{M_m}{\mu^{1/2}}, \text{ (decay time } t_d \sim \Gamma_d^{1/2})$$

GWs from metastable strings fit the PTA signal well!

Gravitational Waves from Monopole Oscillation

Gravitational Waves from Oscillating Monopoles

- ✓ Strings break into segments ended by **monopole-antimonopole pairs**
- ✓ Monopoles and antimonopoles are accelerated by the string tension μ, and **begin to oscillate**.
- ✓ **Oscillating monopoles** emit **GWs** (segment contribution)
- ✓ Segment contributions alter the **low-frequency tale** as well as the **high-frequency region of the GW spectrum**
  [Buchmulle, Domcke, Schmitz JCAP 12 (2021) 12]



We revisit the segment contribution and find that monopole oscillations do not lead to GWs in the PTA or GW interferometer frequency ranges

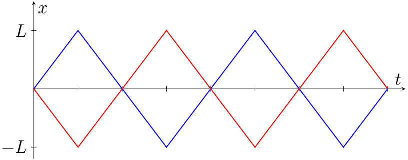
Gravitational Waves from Oscillating Monopoles

GWs radiation from a single segment [Martin and Vilenkin Phys. Rev. D 55, 6054]

✓ Consider a straight segments ended by monopole and antimonopole.

✓ The acceleration of this motion is **constant**

$$a = \frac{\mu}{M_m} = \text{constant} \; .$$



GW radiation power from a straight segment with a length L

$$P \simeq \frac{G_N \mu^2}{n} \text{ for } n\text{-th harmonic mode } f = \frac{n}{L} \; ,$$

$$P_n = \sum_{n}^{n_{\text{max}}} \simeq 8 \log \gamma_m \times G_N \mu^2 \; , \quad (\gamma_m : \text{maximum bost factor})$$

Gravitational Waves from Oscillating Monopoles

Applying to the PTA-favored parameters?

✓ When the segment shortens by a length ℓ, the monopole is boosted to

$$\gamma_m(\ell) = \frac{\mu}{M_m}\,\ell$$

✓ For $\ell = \mathcal{O}\big(H^{-1}\big)$ at $T = 1\,$MeV, $M_m \simeq \mu^{1/2} \simeq 10^{16}\,$GeV,

$$\gamma_m \simeq 10^{40} \Rightarrow E_m \simeq 10^{56}\,\text{GeV?}$$

Is the monopole really accelerated that much?
This may be due to overly simplified assumptions?
Is there any force that could counteract this acceleration?

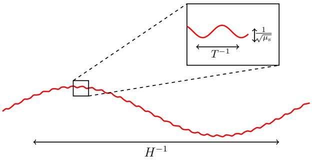
✗ The monopole reaches this acceleration even for strings with curvature radius of order H^{-1}.

Fluctuations on Cosmic Strings

- ✓ Realistic cosmic strings are always accompanied by **small-scale fluctuations**
- ✓ For a locally straight piece along z, the Nambu–Goto action gives a 2D massless field (= transverse zero-mode) on the worldsheet

$$S \supset -\frac{\mu}{2}\int d\eta\, dz\, a(\eta)^2\, \left[(\partial_\eta \vec{x}_\perp)^2 - (\partial_z \vec{x}_\perp)^2\right]$$

ex) Thermal fluctuation on the cosmic string



Thermal distribution:

$$f(p) = \frac{1}{e^{p/T} - 1}$$

Typical wavelength $\lambda_{\rm th} \sim T^{-1}$

Amplitude $\delta x_\perp \sim \mu^{-1/2}$

Monopole–Fluctuation Collisions (Drag Force)

- ✓ Assume the cosmic string sector is thermalized with the SM bath up to a high temperature T_*
- ✓ Redshifted distribution on the string at later times:

$$f(p) \simeq \frac{1}{e^{p/T_s} - 1} \, , \qquad T_s := \frac{a_*}{a} T_*$$

Monopoles pulled by the string continuously scatter with thermal fluctuations!

- ✓ Center-of-mass energy for a typical collision:

$$\sqrt{s} \simeq 2\sqrt{E_m p} = 2\sqrt{\gamma_m M_m p} \, , \quad (p = \mathcal{O}(T_s))$$

- ✓ Energy loss of a monopole per collision

$$\Delta E(p) \simeq \frac{\left(\gamma_m + \sqrt{\gamma_m^2 - 1}\right) M_m p}{\left(\gamma_m - \sqrt{\gamma_m^2 - 1}\right) M_m + 2p} \to \begin{cases} 4\gamma_m^2 p & (\gamma_m \ll \frac{M_m}{p}) \\ \gamma_m M_m & (\gamma_m \gg \frac{M_m}{p}) \end{cases}$$

in the rest frame of thermal bath

Monopole–Fluctuation Collisions (Drag Force)

✓ Energy loss rate from repeated collisions (= **Drag Force**):

$$\left.\frac{dE_m}{dt}\right|_{\text{coll}} = -\int_0^\infty \frac{dp}{\pi} f(p) \mathit{\Delta} E(p)(1+\beta_m)$$
$$\sim \begin{cases} -\dfrac{\pi}{6}(1+\beta_m)^3 \gamma_m^2 T_{\text{s}}^2 & (\gamma_m \ll \frac{M_m}{T_{\text{s}}}) \\ -\dfrac{2}{\pi}\gamma_m T_{\text{s}} M_m \ln \dfrac{4\gamma_m T_{\text{s}}}{M_m} & (\gamma_m \gg \frac{M_m}{T_{\text{s}}}) \end{cases} .$$

✓ When $|dE_m/dt|_{\text{coll}} \gtrsim \mu$, the drag force win and the string tension can no longer accelerate the monopole

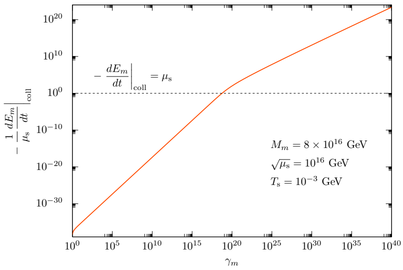
✓ The monopole reaches a **terminal velocity**:

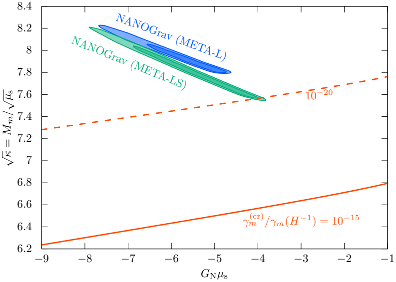
$$\beta_m^{(cr)} = \sqrt{\frac{\gamma_m^{(\text{cr})2} - 1}{\gamma_m^{(\text{cr})}}} , \quad \gamma_m^{(\text{cr})} \simeq \max\left[\frac{\mu}{M_m T_s} , \frac{\sqrt{\mu}}{T_s}\right]$$

[The center-of-mass energy for a monopole–fluctuation collision reaches $\sqrt{s^{(\text{cr})}} \simeq \sqrt{\mu}$]

Monopole–Fluctuation Collisions (Drag Force)

✓ What happens for PTA-favored parameters?





- ✓ Terminal velocity is reached before monopole starts oscillation.
- ✓ The terminal boost factor is much smaller than $\gamma_m(H^{-1}) \sim 10^{40}$. $\gamma_m^{(cr)}/\gamma_m(H^{-1}) \ll 10^{-20}$
- ✓ The energy $\mu\ell \sim \mu H^{-1}$ of the string segment is lost to excited transverse oscillations (and massive string excited modes)
- ✓ Excitations of the string eventually decay into particles of the U(1)-breaking sector particles ($m \sim \sqrt{\mu}$)

Summary

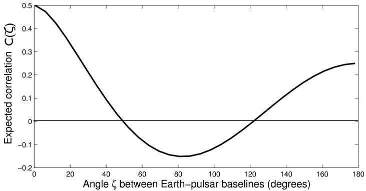
- ✓ **Metastable cosmic strings** can fit the PTA GW signal.
- ✓ Previous studies considered **oscillating monopole** with an enormous boost factor γ_m.
- ✓ Realistic strings contain small-scale fluctuations (wiggles).
- ✓ Scattering with these fluctuations induces a **drag force**, and for PTA-favored parameters the monopole reaches **terminal velocity before oscillation**.
- ✓ $\Rightarrow$ **No GW contribution from monopole oscillations** in the PTA or GW interferometer frequency.

Microscopic behavior of the final segment decay and cosmological constraints from $\sim \mu^{1/2}$ particles are left for future work.

Backup

Pulsar Timing Array (PTA)

✓ The Hellings-Downs curve : the angular correlation of gravitational wave signals between different pulsars (ζ_{ij} : angle between pulsars) [Hellings, Downs, Astrophys.J.Lett.265,L39(1983)]



$$C(\zeta_{ij}) := \left\langle \frac{\nu_i}{\nu_i} \frac{\nu_j}{\nu_j} \right\rangle / \langle h^2 \rangle$$

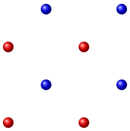
$$C(\zeta_{ij}) = \frac{1-\cos\zeta_{ij}}{2} \log \frac{1-\cos\zeta_{ij}}{2} - \frac{1}{6} \frac{1-\cos\zeta_{ij}}{2} + \frac{1}{3}$$

The Helling-Downs curve provides strong evidence for the detection of GW!

Metastable String?

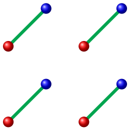
✓ At phase transition SU(2) $\to$ U(1) at $T \sim$ [SU(2) breaking scale]

$\to$ Monopole-Animonopole pairs are created



✓ At phase transition of U(1) breaking at $T \sim$ [U(1) breaking scale]

$\to$ Cosmic strings are generated between monopole antimonopole



✓ Monopoles and antimonopoles annihilate and **nothing left**

consistent with $\pi_{1,2}$(SU(2)) = [empty] !

How to calculate breaking rate ?

✓ Cosmic ray breaking by monopole-antimonopole pair creation

= tunneling process

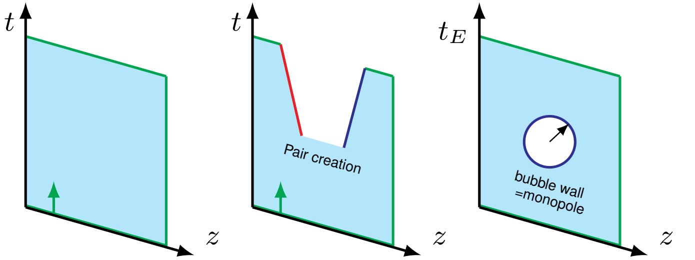
✓ **Infinitely thin cosmic string limit** [Preskill, Vilenkin, Phys.Rev.D47,2324(1993)]

→ Cosmic string along z-axis = 2D theory with (t, z) Lorentz symmetry

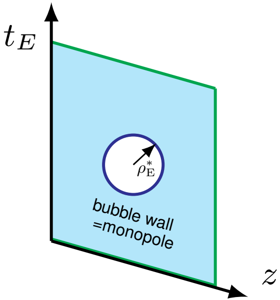
→ Metastable string = **false vacuum on 2D theory**

→ String breaking = bubble formation of the true (without string) vacuum

→ Rate is given by Euclidean path integration ($t = -it_{\rm E}$)
[c.f. WKB approximation of tunneling = imaginary momentum p]



How to calculate breaking rate ?



- ✓ Bubble radius $\rho_{\rm E}^*$ is determined to maximize the effective action,

$$S_B = m_M \int_{\text{worldline}} dx - \mu \int_{\text{hole area}} d^2S$$
$$= 2\pi \rho_{\rm E}^* m_M - \pi \rho_{\rm E}^{*2} \mu \quad [\, m_M = \text{monopole mass}]$$

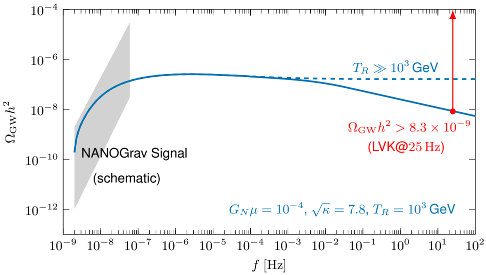
- ✓ The bubble radius and the bounce action :

$$\rho_{\rm E}^* = \frac{m_M}{\mu} \quad \to \quad S_B^{(\rm P.V.)} = \frac{\pi m_M^2}{\mu} =: \pi\kappa \quad \to \quad \Gamma_d = \frac{\mu}{2\pi} e^{-\pi\kappa}$$

High-Frequency Region

✓ GWs from metastable strings can fit the PTA signal well
[GWs emitted at $T_{emit} \sim 100$ keV]

✓ Higher frequency spectrum from the string emitted at $T_{emit} \gg \mathcal{O}(100)$ GeV, depends on UV assumption
[e.g., T_R, $g_*(T)$ etc.]



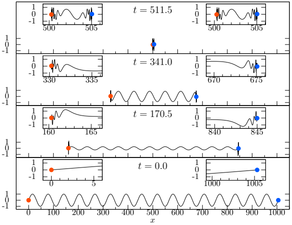
GWs Metastable string can evade the constraint set by LIGO–Virgo–KAGRA

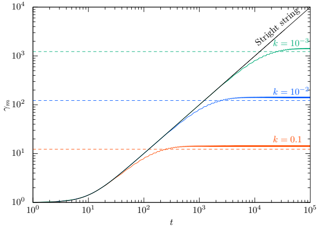
Numerical Simulation

- ✓ Segment action [Martin and Vilenkin Phys. Rev. D 55, 6054] :

$$S = -\sum_{i=m,\overline{m}} M_m \int d\zeta^0 \sqrt{-g_{\mu\nu}\dot{X}_i^\mu \dot{X}_i^\nu} - \mu \int d\zeta^0 \int_{\sigma_m(\zeta^0)}^{\sigma_{\overline{m}}(\zeta^0)} d\zeta^1 \sqrt{\det(-g_{\mu\nu}X^\mu{}_{,a}X^\nu{}_{,b})}$$

- ✓ String in 4D coordinate : $X^\mu(\zeta^a)$, $\mu = 0, 1, 2, 3$, $a = 0, 1$
- ✓ Worldsheet coordinate : (ζ^0, ζ^1)
- ✓ Endpoint (anti)monopole : $X_i^\mu = X^\mu(\zeta^0, \sigma_i(\zeta^0))$ $i = m, \overline{m}$
- ✓ Initial condition ($k \sim T$): $X^\mu(t, \zeta) = (0, \zeta, \sin(k\zeta), 0)$, $\dot{X}^\mu = (1, 0, 0, 0)$





The monopole reach the terminal boost factor $\gamma_m \propto k^{-1} \sim T^{-1}$

Time evolution (movie)