Nambu-Goldstone Theorem, QYBE and Black Hole information

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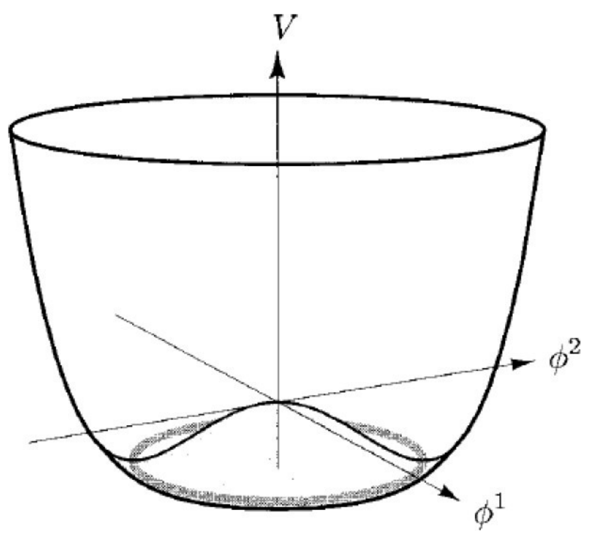
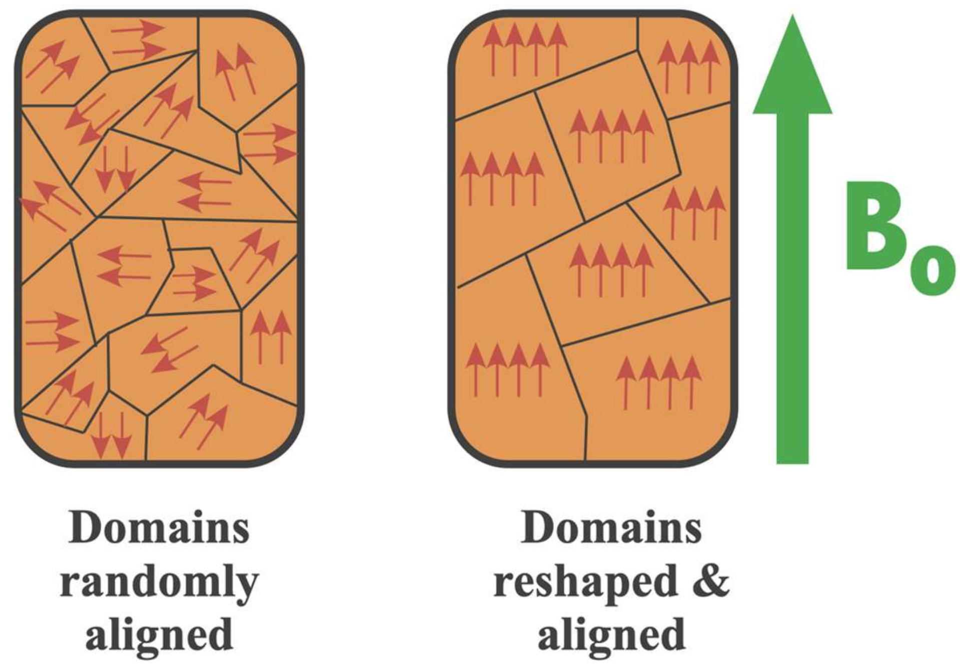
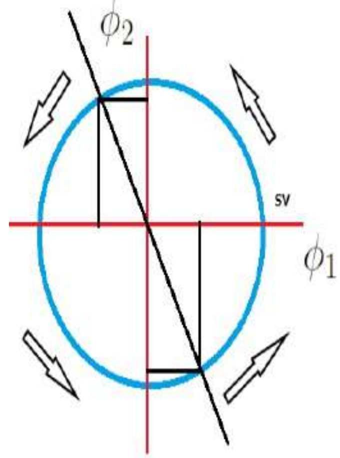
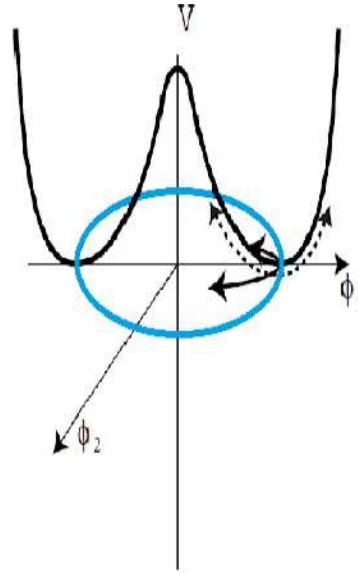
SPONTANEOUS SYMMETRY BREAKING





SUM OF VACUUMS IS TRIVIAL





THE SUM OF VACUUMS IS TRIVIAL

$$\sum_{0} < 0_{DV}|\phi_1|0_{DV} >= \bar{\phi}_1 - \bar{\phi}_1 + \bar{\phi}_2 - \bar{\phi}_2 + ... + \bar{\phi}_n - \bar{\phi}_n = \sum_{n=1}^{N} \bar{\phi}_i = 0.$$

$$< 0_{SV}|[Q, \phi_2]|0_{SV} >= i < 0_{SV}|\phi_1|0_{SV} > .$$

$$\sum_{0} < 0_{DV}|[Q, \phi_2]|0_{DV} >= 0.$$

Why can we sum over different vacuums which repreesnt in principle different Hilbert spaces?

EACH VACUUM IS ORTHOGONAL TO EACH OTHER AT THE THERMODYNAMIC LIMIT

$$|\theta >_0 \rightarrow U_{\theta'}|\theta >_0 = |\theta + \theta' > .$$

$$U_\theta = e^{i\theta Q} \qquad\qquad | < \theta'|\theta >_0 | \rightarrow 0.$$

$$| < \theta'|\theta >_0 | = e^{-\Omega|\theta - \theta'|^2},$$

SPONTANEOUS SYMMETRY BREAKING

$$(\phi_0^i)^2 = \frac{\mu^2}{\lambda}.$$

Vacuum condition

$$\phi_0^i = (0, 0, ..., 0, v), \ v = \mu/\sqrt{\lambda}.$$

Arbitrary selection of the direction

$$\phi(x) = (\pi^k(x), v + \sigma(x)),$$

Vacuum redefinition

$$k = 1, ..., N - 1.$$

$$\pounds = \frac{1}{2}(\partial_\mu \pi^k)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\lambda}\mu\sigma^3 - \sqrt{\lambda}\mu(\pi^k)^2\sigma - \frac{\lambda}{4}\sigma^4 - \frac{\lambda}{2}(\pi^k)^2\sigma^2 - \frac{\lambda}{4}[(\pi^k)^2]^2.$$

Nambu-Goldstone Theorem

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi - \phi_0)^a(\phi - \phi_0)^b \left(\frac{\partial^2}{\partial\phi^a\partial\phi^b}V\right)_{\phi_0} + \ldots,$$

$$\partial V/\partial\phi^a = 0 \qquad \left(\frac{\partial^2}{\partial\phi^a\partial\phi^b}V\right)_{\phi_0} = m^2_{ab} \geq 0.$$

Nambu et al. 1961

Vacuum condition plus mass matrix

$$\pounds = (kinetic\ \ terms) - V(\phi),$$

$$G: \quad \phi_0^{a'} = U(g)\phi_0^a \neq \phi_0^a,$$

$$H: \quad \phi_0^{a'} = U(h)\phi_0^a = \phi_0^a,$$

Vacuum is not invariant under the action of the whole group.

Nambu-Goldstone Theorem

$$U(g) = e^{T^a \alpha} \approx \hat{I} + \alpha T^a \to U(h)\phi_0^a = \phi_0^a + \alpha T^a(\phi_0),$$

$$T^a(\phi_0)T^b(\phi_0)\frac{\partial^2}{\partial\phi^a\partial\phi^b}V(\phi) = T^a(\phi_0)T^b(\phi_0)m_{ab}^2 = 0.$$

We can define a conserved current as

$$j_\mu^a(x) = \frac{\partial \pounds}{\partial(\partial^\mu\phi)}\frac{\delta\phi(x)}{\delta\alpha^a}, \qquad Q^a(x) = \int d^3x j_0^a(x).$$

Nambu-Goldstone Theorem

$[Q^a, Q^b] = C^{abc} Q^c,$ Lie algebra structure

$U = e^{iQ^a \alpha^a}.$ $Q^a|0>= 0.$ Standard definition of vacuum

$U \neq e^{iQ^a \alpha^a},$ $Q^a|0>\neq 0.$ Degenerate vacuum condition

$[Q^a, \phi'(x)] \backsim \phi(x).$ Order parameter

$< 0|Q^a \phi'(x) - \phi'(x) Q^a|0 >\neq 0.$ Nambu-Goldstone field commuted with broken charge

Nambu-Goldstone Theorem

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2).$$

Convenient separation

$$< 0_{SV}|\phi_1|0_{SV} >= \pm\mu/\sqrt{\lambda}.$$

Example of order parameter for the case of $U(1)$ symmetry.

$$\sum_n \int d^3y \, [< 0|j_0^a(y)|n >< n|\phi'(x)|0 > - < 0|\phi'(x)|n >< n|j_0^a(y)|0 >]_{x^0=y^0} \neq 0,$$

$$(2\pi)^3 \sum_n \delta^{(3)}(\vec{p}_n) \left[< 0|j_0^a(0)|n >< n|\phi'(x)|0 > e^{iM_n y_0} - < 0|\phi'(x)|n >< n|j_0^a(0)|0 > e^{-iM_n y^0}\right]_{x^0=y^0},$$

Spacetime translational invariance

Time-independence guarantees gapless condition

$$\frac{\partial}{\partial y_0}\langle 0|[Q^a, \phi'(x)]|0\rangle = \frac{\partial}{\partial y_0}\int d^3y\,\langle 0|[j_0^a(y), \phi'(x)]|0\rangle$$

$$= -\int d^3y\,\langle 0|[\nabla\cdot\mathbf{j}^a(y), \phi'(x)]|0\rangle$$

$$= -\int d\mathbf{S}\cdot\langle 0|[\mathbf{j}^a(y), \phi'(x)]|0\rangle$$

DISCREPANCIES BETWEEN TYPE I AND II NGB

1) Dispersion relation.

2) Counting rule. $n_{\mathrm{BS}} - n_{\mathrm{NG}} = \frac{1}{2}\,\mathrm{rank}\,\rho,$ **Watanabe and Murayama**.

$< 0_{SV}|\phi_{a,b}(x)|0_{SV} >\neq 0.$ Generic form of order parameter

$\phi_{a,b}(\vec{x}) = \phi^{m,l}_{a,b}\epsilon_m \otimes \epsilon_l$ $[Q_{m,k}(y), \phi_{a,b}(x)] \backsim \phi'_{a,b}(x).$

$< 0_{SV}|[Q_{m,k}(y), \phi_{a,b}(x)]|0_{SV} >\neq 0.$ $Q_{m,l} = Q^{p,k}_{m,l}\epsilon_p \otimes \epsilon_k.$

QUANTUM YANG BAXTER EQUATIONS

$$< 0_{SV}| \, [\phi_{b,a}(x), [Q_{p,k}(y), Q_{l,m}(z)]] \, |0_{SV} > \neq 0.$$

$$Q_{k,p}(y) = e^{-ipy} Q_{k,p}(0) e^{ipy}$$

$$\begin{aligned}
&\sum_{n,n'} < 0_{SV}|\phi_b(x)|n >< n| \, Q_p(0)|n' >< n'|Q_l(0)|0_{SV} > e^{-i(p_n - p_{n'})y} e^{-i\tilde{p}_{n'} z} \\
&- < 0_{SV}|\phi_b(x)|n' >< n'| \, Q_l(0)|n >< n|Q_p(0)|0_{SV} > e^{-i(\tilde{p}_{n'} - \tilde{p}_n)z} e^{-ip_n y} \\
&- < 0_{SV}| \, Q_p(0)|n >< n|Q_l(0)|n' >< n'|\phi_b(x)|0_{SV} > e^{-i(\tilde{p}_n - \tilde{p}_{n'})z} e^{ip_n y} \\
&+ < 0_{SV}|Q_l(0)|n' >< n'| \, Q_p(0)|n >< n|\phi_b(x)|0_{SV} > e^{-i(p_{n'} - p_n)y} e^{i\tilde{p}_{n'} z} \neq 0.
\end{aligned}$$

QUANTUM YANG BAXTER EQUATIONS

$Q_{p,k} = Q_{p,m} = Q_{m,p} = Q_{k,p} = Q_p$; $Q_{l,m} = Q_{l,a} = Q_{a,l} = Q_{m,l} = Q_l$, and $\phi_{b,a} = \phi_{b,k} = \phi_{k,b} = \phi_{a,b} = \phi_b$

$$
\begin{aligned}
R^{0,n'}_{m,l} &=< 0_{DV}|Q_{m,l}(y)|n' >, \quad R^{n,k}_{p,n'} =< n'|Q_{k,p}(z)|n >, \\
R^{a,b}_{n,0} &=< n|\phi_{a,b}(x)|0_{DV} >, \quad R^{n,0}_{p,m} =< n|Q_{p,m}(z)|0_{DV} >, \\
R^{a,n'}_{n,l} &=< n'|Q_{l,a}(y)|n >, \quad R^{b,k}_{0,n'} =< 0_{DV}|\phi_{b,k}(x)|n' > .
\end{aligned}
$$

$R_{(1,2)}R_{(1,3)}R_{(2,3)} = R_{(2,3)}R_{(1,3)}R_{(1,2)}$, $R : M \otimes V = R^{a,b}_{c,d} m_a \otimes m_b$ $R^{s_2,s_3}_{j,k} R^{s_1,c}_{i,s_3} R^{a,b}_{s_1,s_2} = R^{r_1,r_2}_{i,j} R^{a,r_3}_{r_1,k} R^{b,c}_{r_2,r_3}$.

$$
R^{0,n'}_{m,l} R^{n,k}_{p,n'} R^{a,b}_{n,0} = R^{n,0}_{p,m} R^{a,n'}_{n,l} R^{b,k}_{0,n'},
$$

QUANTUM YANG BAXTER EQUATIONS

$$\sum_{0,n,n'} < 0_{DV}|Q_{m,l}(0)|n' >< n'|Q_{k,p}(0)|n >< n|\phi_{a,b}(x)|0_{DV} >= \sum_{0,n,n'} < 0_{DV}|\phi_{b,k}(x)|n' > \times$$
$$< n'|Q_{l,a}(0)|n >< n|Q_{p,m}(0)|0_{DV} > ($$

In simplified notation

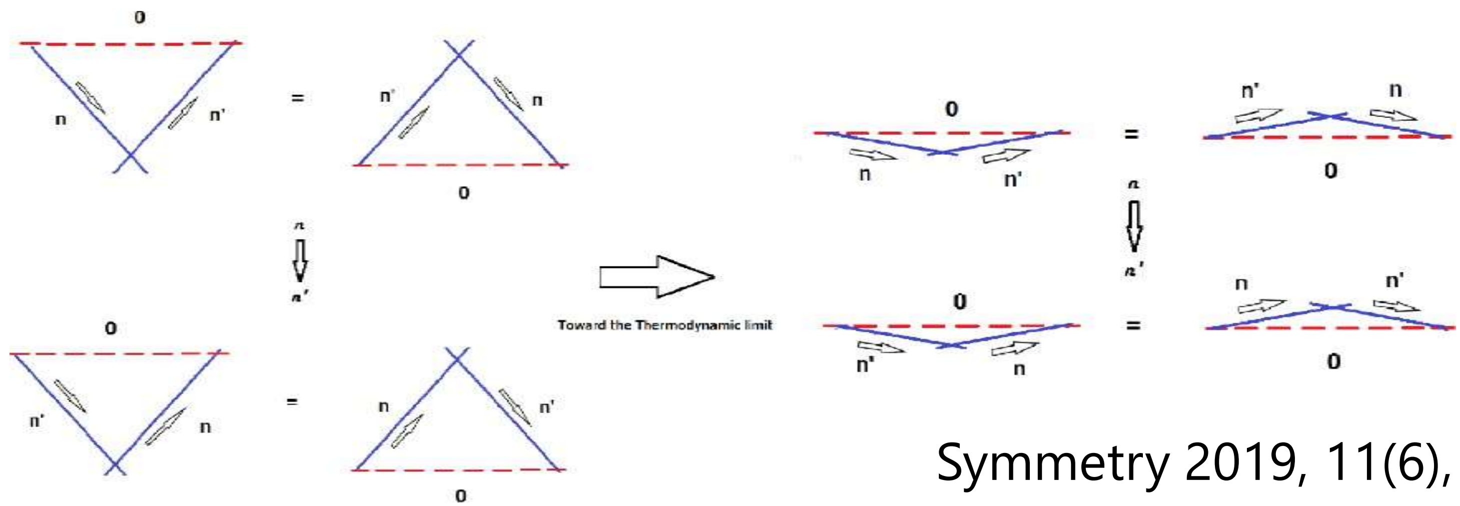
$$\sum_{0} < 0_{DV}|Q_l(0)Q_p(0)\phi_b(x)|0_{DV} >= \sum_{0} < 0_{DV}|\phi_b(x)Q_l(0)Q_p(0)|0_{DV} > .$$

QUANTUM YANG BAXTER EQUATIONS

$n \rightarrow n'$ Exchange of particles

$$R^{0,n}_{m,p} R^{n',a}_{l,n} R^{k,b}_{n',0} = R^{n',0}_{l,m} R^{k,n}_{n',p} R^{b,a}_{0,n}, \qquad R_{(3,2)} R_{(3,1)} R_{(2,1)} = R_{(2,1)} R_{(3,1)} R_{(3,2)}.$$

Witten 2016, Kostello 2013



Symmetry 2019, 11(6), 803

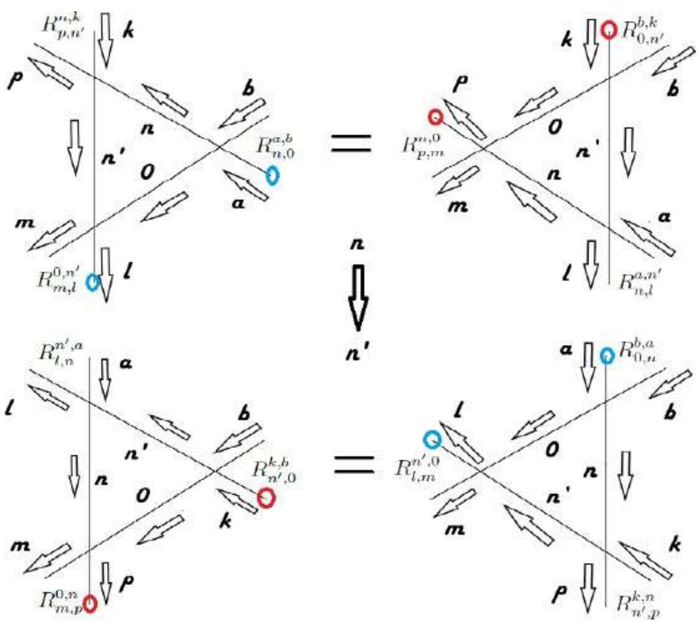
QUANTUM YANG BAXTER EQUATIONS

$$\sum_{0,n,n'} < 0_{DV}|Q_{m,p}(0)|n >< n|Q_{a,l}(0)|n' >< n'|\phi_{k,b}(x)|0_{DV} >= \sum_{0,n,n'} < 0_{DV}|\phi_{b,a}(x)|n > \times$$
$$< n|Q_{p,k}(0)|n' >< n'|Q_{l,m}(0)|0_{DV} >($$

In simplified notation we get

$$\sum_{0} < 0_{DV}|Q_p(0)Q_l(0)\phi_b(x)|0_{DV} >= \sum_{0} < 0_{DV}|\phi_b(x)Q_p(0)Q_l(0)|0_{DV} > .$$

QUANTUM YANG BAXTER EQUATIONS



M. Jimbo 1989,

QUANTUM YANG BAXTER EQUATIONS: TYPE B NGB

$$\sum_{0,n,n'} < 0_{DV}|\phi_b(x)|n >< n|\ Q_p(0)|n' >< n'|Q_l(0)|0_{DV} > 2e^{-i\tilde{E}_{n'}z_0} cos\,(\tilde{\mathbf{p}}_n \cdot \mathbf{z})$$
$$- < 0_{DV}|\ Q_p(0)|n >< n|Q_l(0)|n' >< n'|\phi_b(x)|0_{DV} > 2e^{iE_n y_0} cos\,(\mathbf{p}_n \cdot \mathbf{y}) = 0.$$

$p_n = p_{n'}$ and $\tilde{p}_n = \tilde{p}_{n'}$ $\quad n = n'$ $\quad p_n y = E_n y_0 - \vec{p_n} \cdot \vec{y},$

$$2\sum_{0,n,n'} < 0_{DV}|\phi_b(x)|n >< n|\ Q_p(0)|n' >< n'|Q_l(0)|0_{DV} > (e^{-i\tilde{E}_{n'}z_0} cos\,(\tilde{\mathbf{p}}_n \cdot \mathbf{z}) \times$$
$$-e^{iE_n z_0} cos\,(\mathbf{p}_n \cdot \mathbf{y})) = 0,$$

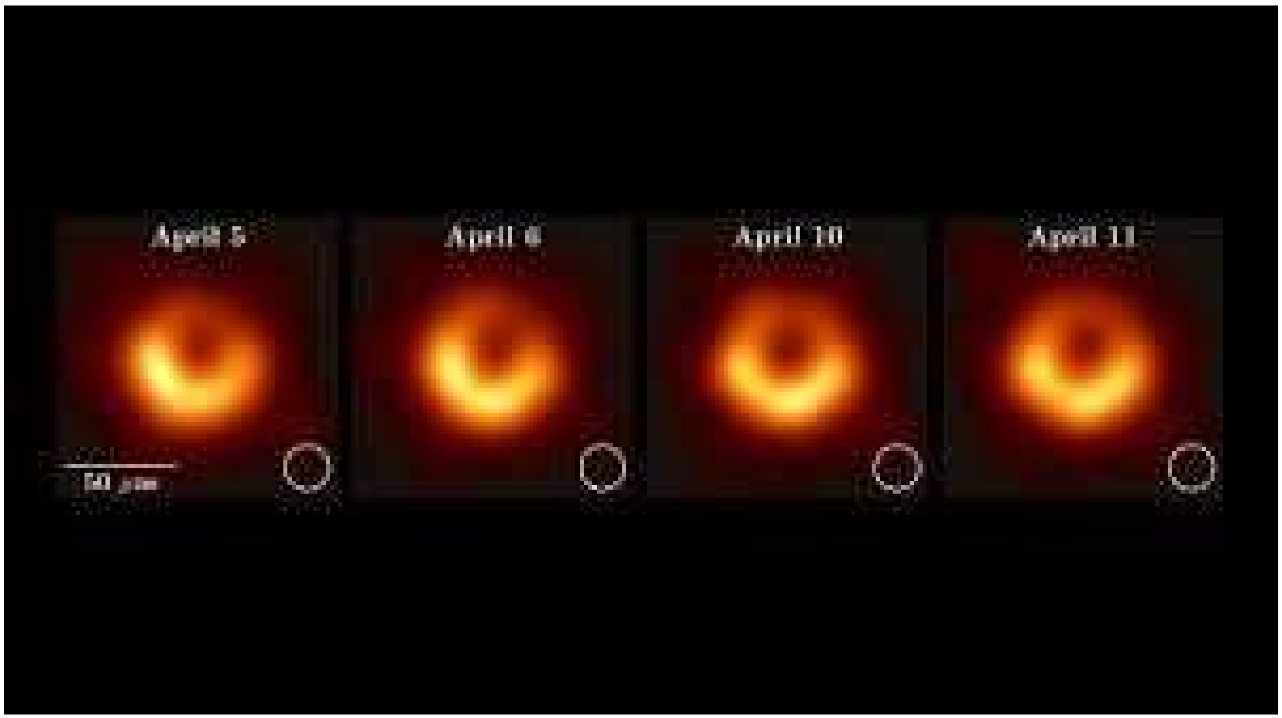
Spatial integration

$E_n \rightarrow 0$ $\qquad p_n \rightarrow 0$

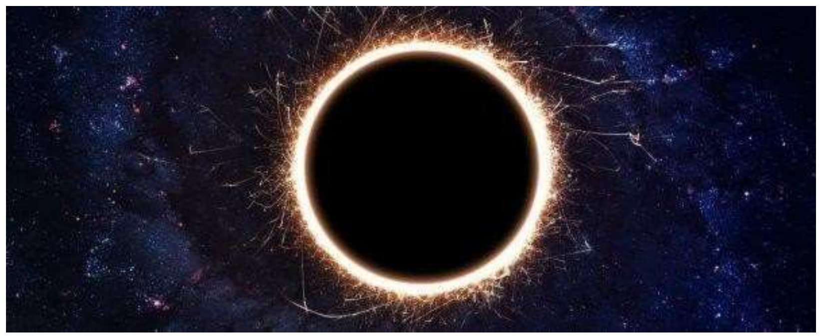
APPLICATIONS

- 1). Color superconductors.
- 2). Black Holes.
- 3). Neutrino oscillations?

NGB IN GRAVITY: BLACK HOLES



NGB IN GRAVITY

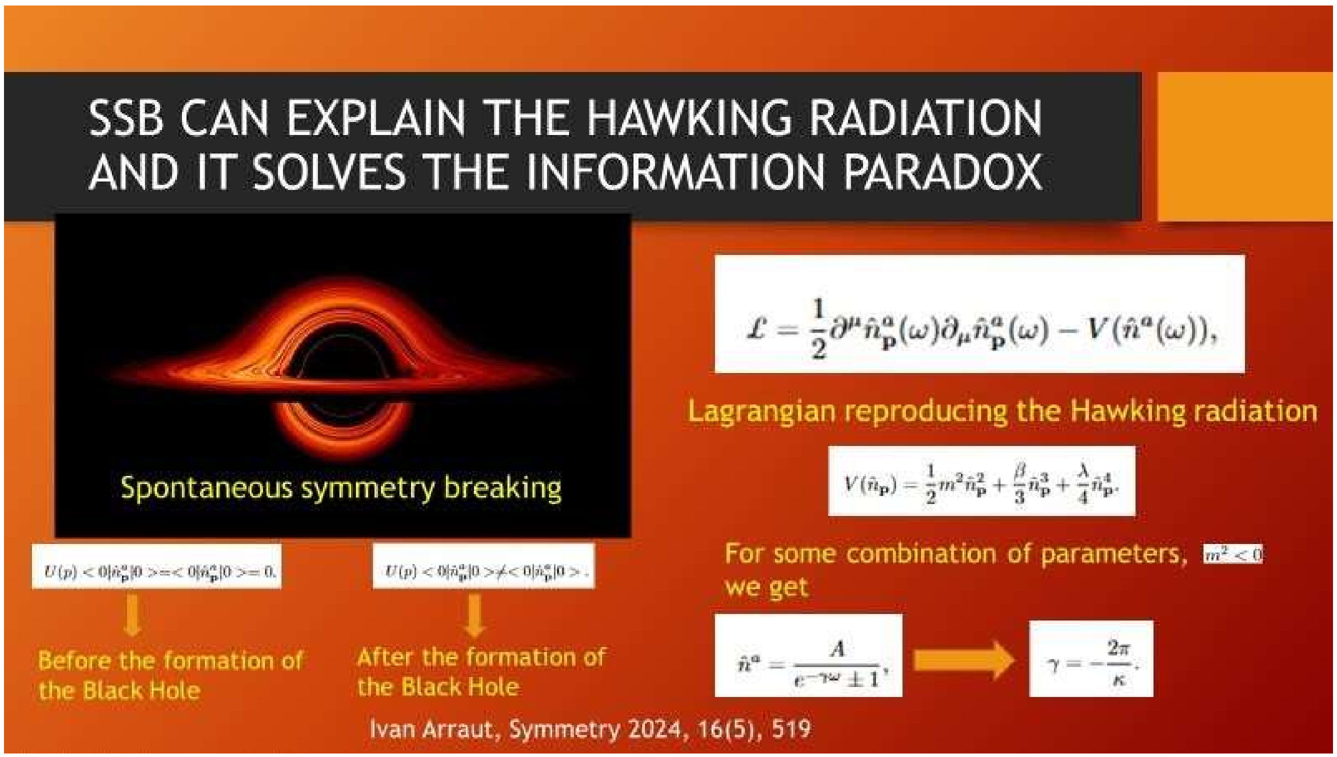


Dvali et al, 2011

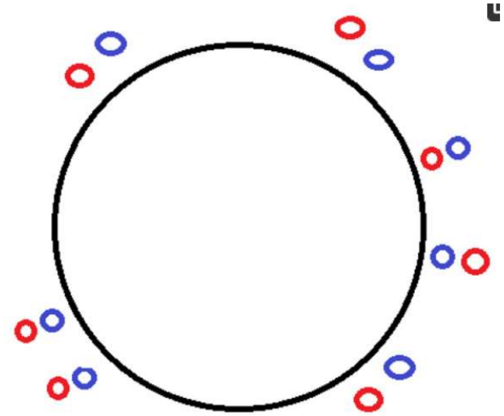
The Area of the event horizon measures the entropy.

HAWKING RADIATION AS A MANIFESTATION OF SSB

Symmetry 2024, 16(5), 519



SSB conditions over a Black Hole



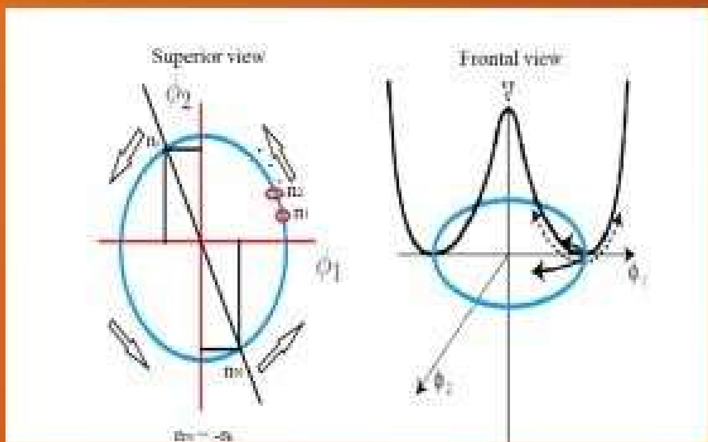
$$U(p) < 0|\hat{n}^a_{\mathbf{p}}|0> = < 0|\hat{n}^a_{\mathbf{p}}|0> = 0.$$

$$U(p) < 0|\hat{n}^a_{\mathbf{p}}|0> \neq < 0|\hat{n}^a_{\mathbf{p}}|0>.$$

$$< 0|\hat{n}_1|0>_1 + < 0|\hat{n}_2|0>_2 + ... + < 0\left|\hat{n}_N\right|0>_N = 0.$$

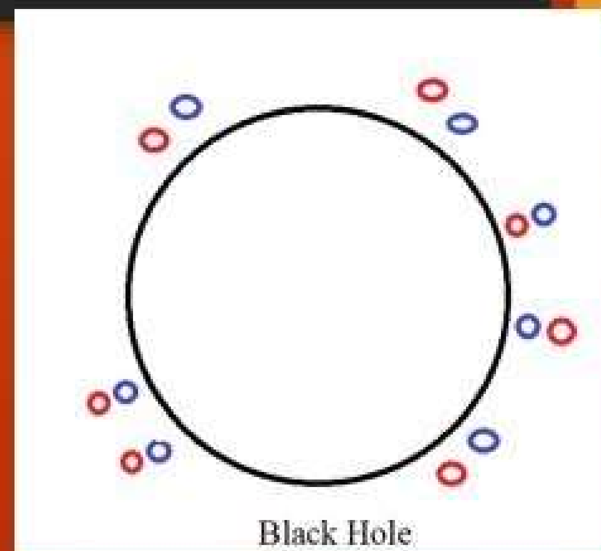
What is the interpretation of negative particle number operators? Antiparticles? Particles eaten by the Black Hole?

Symmetry arguments might solve the information paradox in black holes



Picture of Hawking radiation:
Spontaneous Symmetry Breaking

$$< 0|\hat{n}_1|0 >_1 + < 0|\hat{n}_2|0 >_2 + ... + < 0|\hat{n}_N|0 >_N = 0.$$

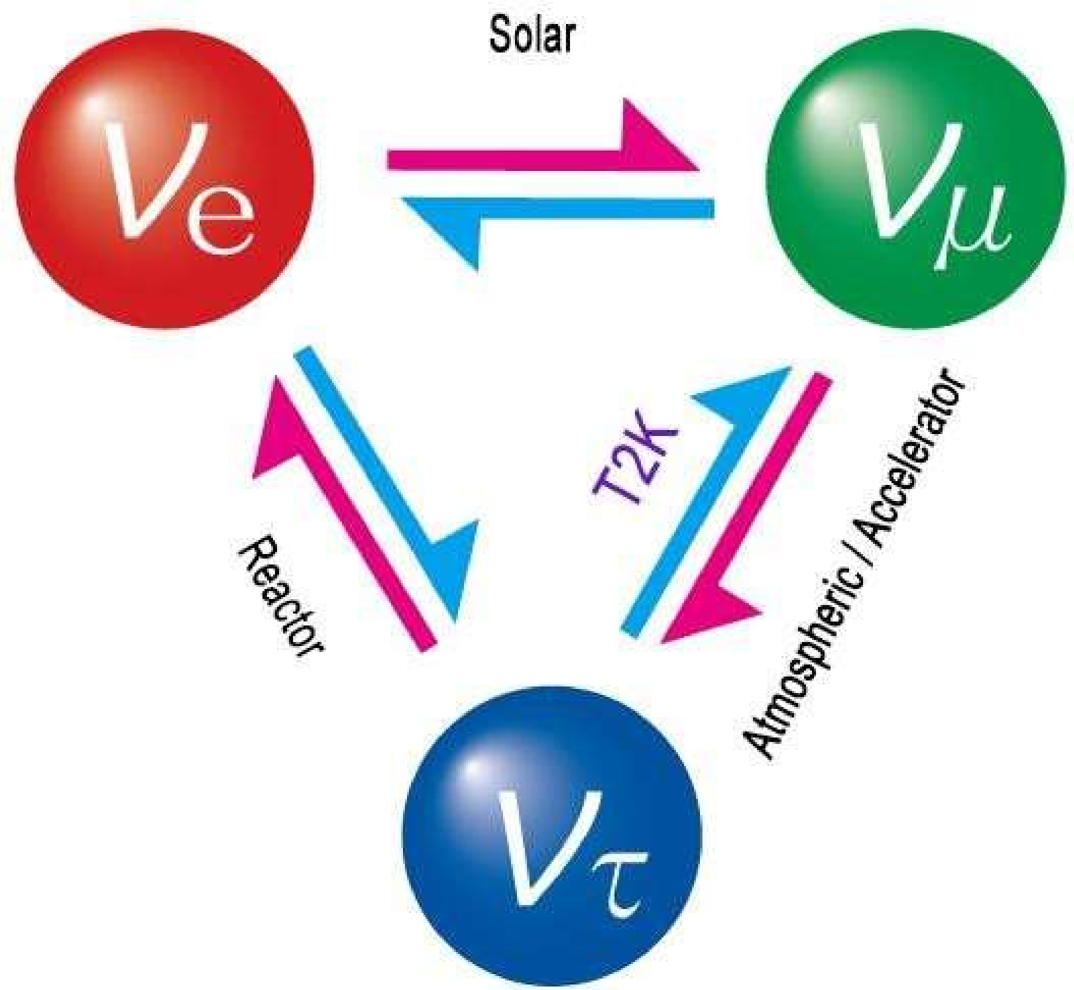


Picture of Hawking radiation:
Pair production

The black holes must emit partricles and antiparticles at different instants for solving the information paradox.

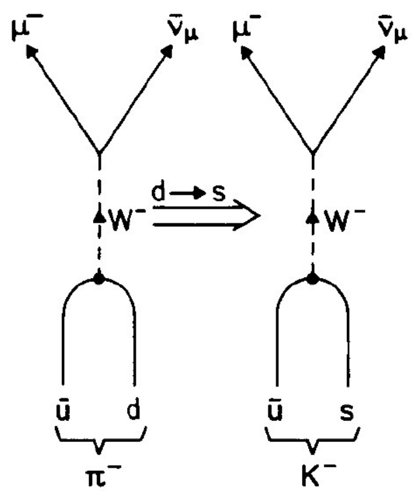
Ivan Arraut, AppliedMath 2025, 5(1), 4

Yang Baxter and the neutrino mass problem



Neutrino oscillation between three generations

FLAVOR CHANGES ARE NOT STRANGE INSIDE THE ELECTROWEAK THEORY



However, it only occurs when charged currents are involved. It is not the neutrino case.

Origin of mass of particles inside the Electroweak theory

$$\mathcal{L}_{\phi-F} = -g_e[\overline{L}\Phi e_R + \overline{e}_R\Phi^\dagger L]$$

Electron, muon, tau, get mass via Higgs mechanism with
Terms introduced by hand

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\binom{0}{v}$$

$$-\frac{g_e}{\sqrt{2}}\left[(\overline{\nu}_L, \overline{e}_L)\begin{pmatrix} 0 \\ v \end{pmatrix} e_R + \overline{e}_R(0, v)\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}\right]$$

$$= -\frac{g_e v}{\sqrt{2}}(\overline{e}_L e_R + \overline{e}_R e_L) = -\frac{g_e v}{\sqrt{2}}\overline{e}e$$

BOSONS Z AND W

Get mass via Higgs mechanism but this time with terms emerging from a gauge principle.

$$
\begin{aligned}
(D_\mu \Phi)^\dagger D^\mu \Phi &= \left\{ \left(\partial_\mu + \frac{ig}{2} \vec{\tau} \vec{W}_\mu \right) \Phi \right\}^\dagger \left(\partial^\mu + \frac{ig}{2} \vec{\tau} \vec{W}^\mu \right) \Phi \\
&= \frac{g^2}{4} \Phi^\dagger (\vec{\tau} \vec{W}_\mu)^\dagger (\vec{\tau} \vec{W}^\mu) \Phi + \dots \\
&= \frac{g^2}{4} \sum_{ij} W^i_\mu W^{j\mu} \Phi^\dagger \cdot \underbrace{\tau_i \tau_j}_{} \cdot \Phi + \dots \\
&\qquad\qquad\qquad = \begin{cases} -\tau_j \tau_i & \text{for } i \neq j \\ 1 & \text{for } i = j \end{cases} \\
&= \frac{g^2}{4} \sum_{i} W^i_\mu W^{i\mu} \Phi^\dagger \Phi + \dots \\
&= \frac{g^2}{8} \vec{W}_\mu \vec{W}^\mu (0, v + \eta) \begin{pmatrix} 0 \\ v + \eta \end{pmatrix} + \dots \\
&= \frac{g^2 v^2}{8} \vec{W}_\mu \vec{W}^\mu + \frac{g^2 v}{8} \vec{W}_\mu \vec{W}^\mu 2\eta + \dots
\end{aligned}
$$

WHAT ABOUT THE NEUTRINOS? SUPERPOSITION OF VACUUMS

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i .$$

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{bmatrix}$$

$$\hat{a}_e^+|0_e >= |\nu_e >, \quad \hat{a}_e|0_e >= 0.$$

$$\hat{a}_i|0 >_i = 0. \qquad |0 >_i = \sum_\alpha a_\alpha |0 >_\alpha ,$$

DEGENERATE VACUUM

arXiv:2409.00560 **[hep-ph]**

$$\bar{c}_{12} = cos\left(\theta_{12} - \frac{\pi}{2}\right)$$

$$\bar{s}_{12} = sin\left(\theta_{12} - \frac{\pi}{2}\right).$$

$$\begin{aligned} |0\rangle_e &= c_{12}c_{13}|0\rangle\rangle_1 + s_{12}c_{13}|0\rangle\rangle_2 + s_{13}|0\rangle\rangle_3, \\ |0\rangle_\mu &= (-s_{12}c_{23} - c_{12}s_{23}s_{13})|0\rangle_1 + \\ & (c_{12}c_{23} - s_{12}s_{23}s_{13})|0\rangle_2 + s_{23}c_{13}|0\rangle_3, \\ |0\rangle_\tau &= (s_{12}s_{23} - c_{12}c_{23}s_{13})|0\rangle_1 + \\ & (-c_{12}s_{23} - s_{12}c_{23}s_{13})|0\rangle_2 + c_{23}c_{13}|0\rangle_3. \end{aligned}$$

By using the QYBE

$$|0\rangle_e + |0\rangle_\mu + |0\rangle_\tau = 0.$$

This convert into the following expressions

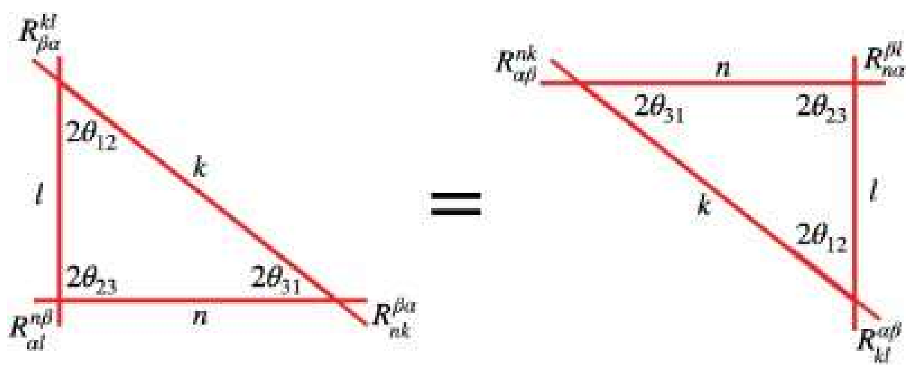
$$\begin{aligned} c_{12}c_{13} - s_{12}c_{23} - c_{12}s_{23}s_{13} + s_{12}s_{23} - c_{12}c_{23}s_{13} = 0, \\ s_{12}c_{13} + c_{12}c_{23} - s_{12}s_{23}s_{13} - c_{12}s_{23} - s_{12}c_{23}s_{13} = 0, \\ s_{13} + s_{23}c_{13} + c_{23}c_{13} = 0. \end{aligned}$$

The first two equations are redundant

$$\bar{c}_{12}c_{13} - \bar{s}_{12}c_{23} - \bar{c}_{12}s_{23}s_{13} + \bar{s}_{12}s_{23} - \bar{c}_{12}c_{23}s_{13} = 0.$$

The results match with the observations!!! Could be coincidence?

$$tan(\theta_{13}) = -sin(\theta_{23}) - cos(\theta_{23}).$$



arXiv:2406.02641

CONCLUSIONS

- 1). Symmetries govern the interactions.
  2). The Nambu-Goldstone theorem emerges from constraints emerging from the QYBE, when we analyze the interaction of pairs of Nambu-Goldstone bosons.
- 3). Hawking radiation is a mechanism of spontaneous symmetry breaking.
- 4). The neutrino mixing angles follow an approximate triangular pattern, interpreted via QYBE.