Consideration on application of neural networks to realize 3D beam injection

3次元らせんビーム入射の実現に向けた ~~機械学習~~ Neural network の活用検討

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@ 4th Acc ML workshop

Abstract

- For a precise muon g-2 measurement in J-PARC, we want to store relativistic muon in a compact ring.
  To realize this, a new injection scheme, called “3D beam injection”, is being developed.

- Design method for this non-standard optics must be developed.
  - consideration on performance (injection efficiency), robustness of the design, and commissioning strategy.
- The beam optics of 3D injection has specific features:
  - Strong focusing: skew component of fringe field (f~0.03m) + Drift length (L~30m)
  - Intrinsic non-linearity: Non-linear B-field distribution, Small radius effect

- Conventional tool to tackle this system is Lie map and truncated polynomials. Representing it by neural network may give us a guideline for beamline design.

This is still an “idea phase”. Any suggestions from experts are very welcome.

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Muon g-2 measurement in a compact ring

- Muon g-2 is a powerful tool to validate possible contribution from BSM.
  - (in my opinion) the situation on muon g-2 is "ambiguous"
    - Measurement of ee->had cross section is inconsistent with lattice QCD calculation.
    - Additonal data from experimental side is needed.

- Previous experiments (at CERN/ BNL/FNAL) : "magic-γ approach"

$$\vec{\omega}_a + \vec{\omega}_\eta = -\frac{\mathrm{e}}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

= 0 at γ=29.3, P=3.1GeV/c

- New experiment at J-PARC : "zero E-field approach"

$$\vec{\omega}_a + \vec{\omega}_\eta = -\frac{\mathrm{e}}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

= 0 at $\vec{E}$=0 = 0 at $\vec{E}$=0

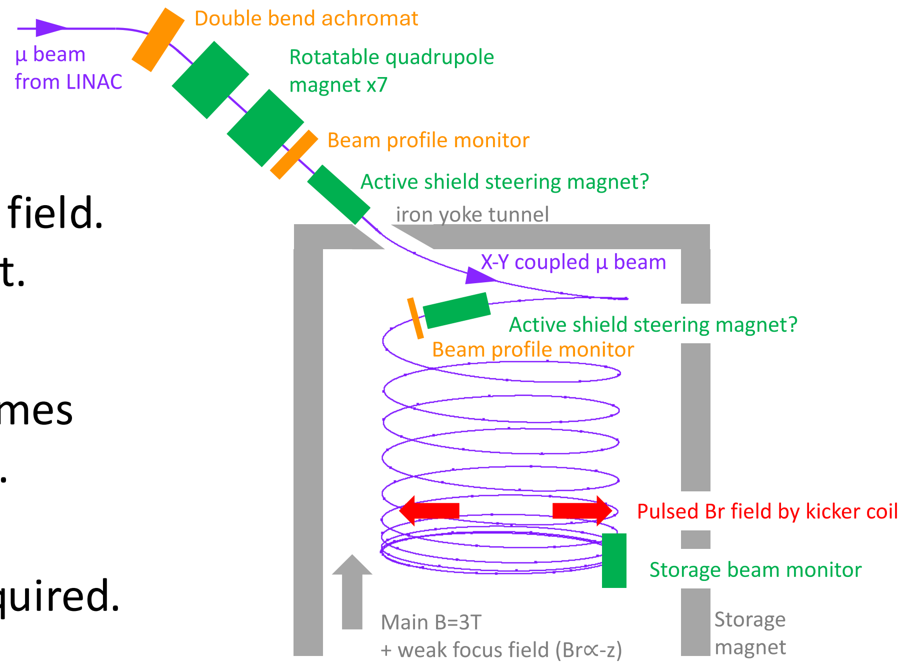
  - Key concept: Measurement of lower muon momentum with more compact setup.

- Injection of relativistic beam into small radius ring is needed.
  - Muon beam of 300 MeV/c (γ=3.0), Ring radius r = 0.33m at B = 3.0 T.
- We need to store almost all the muons in the "good field region".
  - The experiment relys on the uniformity of B-field where muon is stored.

How we can inject beam into compact ring?

- The conventional beam injection scheme will not work. Too fast and too strong kicker system is demanded.
  - Smaller cyclotron period requires fast kicker pulse.
  - Smaller radius requires larger kick angle = (septum thickness/ring radius).
  - In our case, a kick of BL ~ 30 mT in 7 ns is needed. This is out of technical reach.

- We are developing a new injection scheme, called "3D beam injection".
  - Beam follows helical orbit by solenoid field. Kicker field is gradually applied along it.
  - Advantage: Required kicker field becomes within the technical reach.
  - Challenges: Precise beam control in a complicated optics is required.



What we want to achieve

Goal

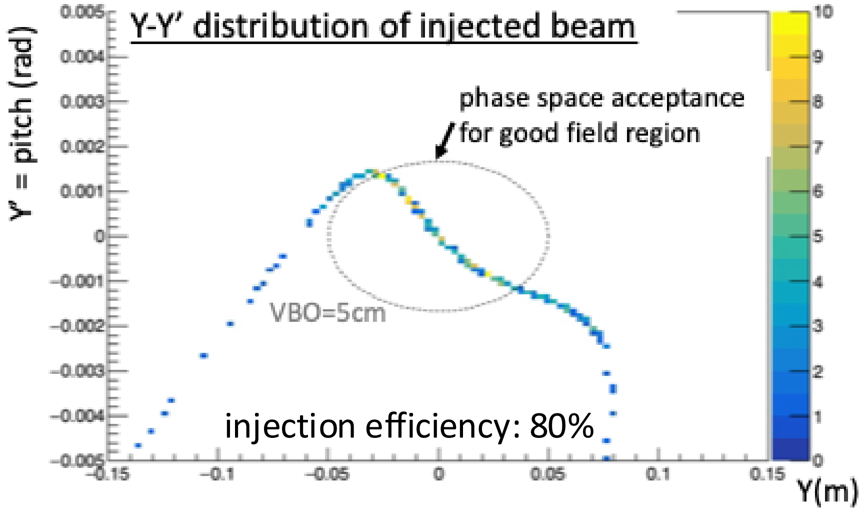
- We need to design this injection system having
  - high injection efficiency (>80%)
  - sufficient robustness for the error on B-field and incoming beam
  - realistic commissioning plan (B-field probe, beam monitor, tuning knob)

Method

- Design method for this non-standard optics must be developed.
  - It looks like much different from the standard beamlines,
    the framework of beam dynamics should be still applicable and useful.

Status

- A configuration which achieves 80% injection efficiency was found.
  - Based on the linear optics approximation of this system.
  - non-linear effect is observed.



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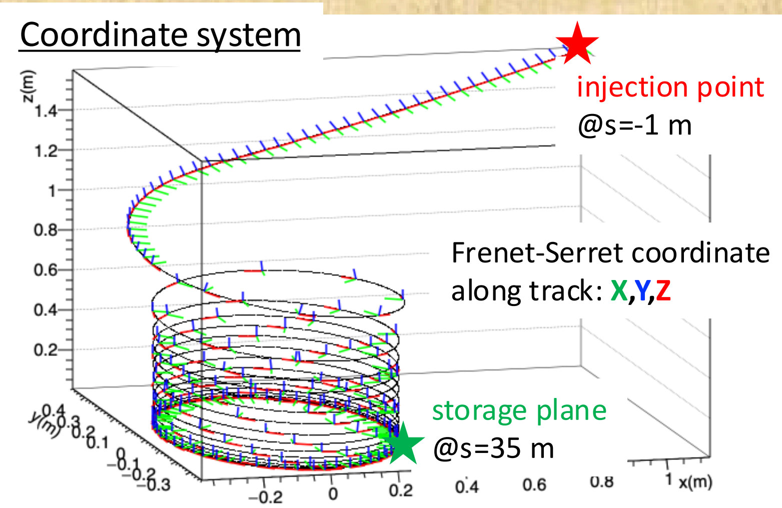
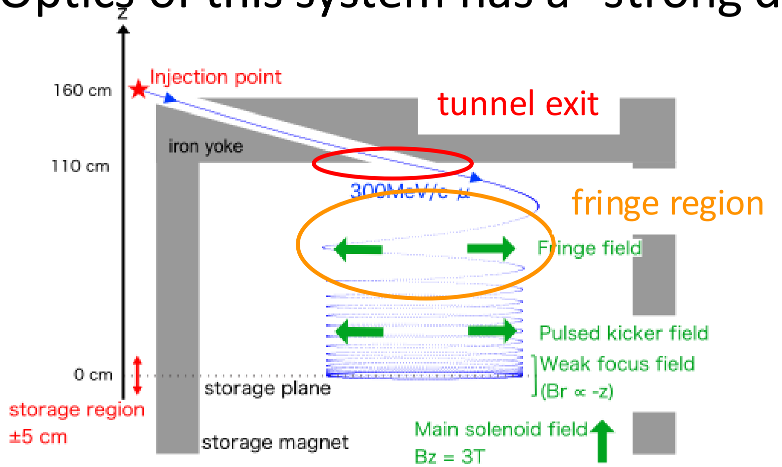
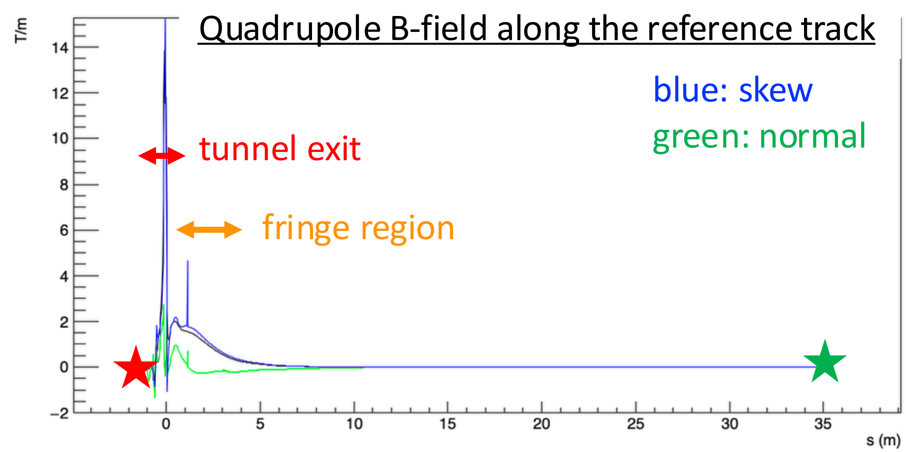
Feature of 3D injection: “strong defocusing with XY-coupled way"

- Frenet-Serret coordinate is defined along the reference track to treat this in beam dynamics.
- The beam feels strong skew quadrupole field by the fringe of 3T storage magnet.
  -> focal length of 3cm for X-Y coupled way ※



- It is followed by drift length of ~30m before reaching the storage plane.
- Optics of this system has a “strong defocusing effect with XY-coupled way”

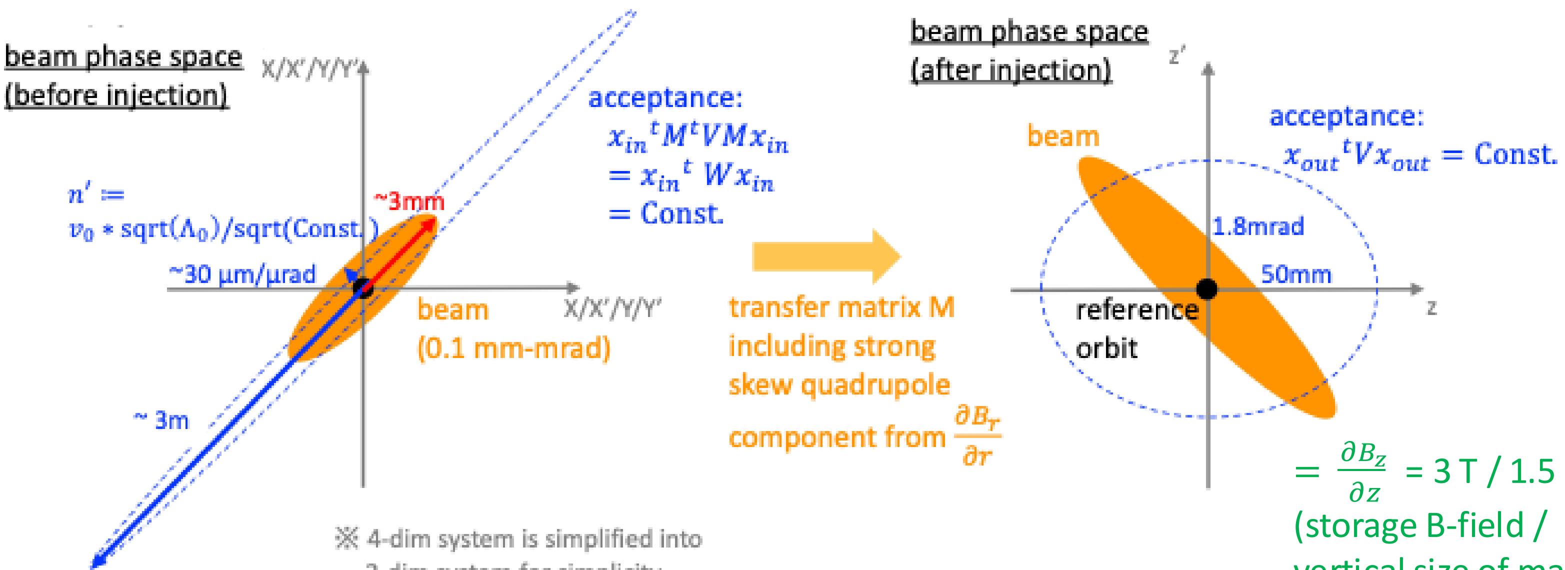




※ In additon, longitudinal field (Bz) also contribute X-Y coupling.

Feature of 3D injection: “strong defocusing with XY-coupled way"

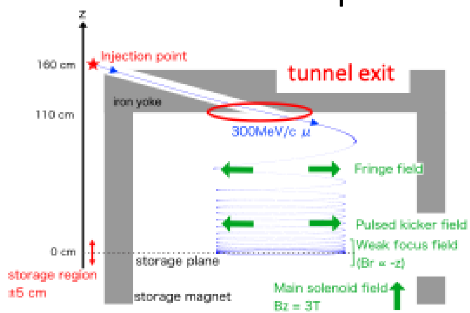
- The strong skew quadrupole field is inevitable.
  - $\frac{\partial B_r}{\partial r}$ at finite radius is induced from $\frac{\partial B_z}{\partial z}$, as we inject beam form B_z = 0 T to B_z = 3 T.
- Due to this strong defocusing effect, a tiny shift of the injecting beam is magnified to a large shift.
  - magnification factor: 1000 (= “drift length ~ 30m”/”focal length ~ 3cm”)
- This requires us a precise beam control at the injection point in 4-dimensional beam phase space(X,X’,Y,Y’).
  - This is achieved by beam transport line having XY-coupled optics (rotated Q-mags).

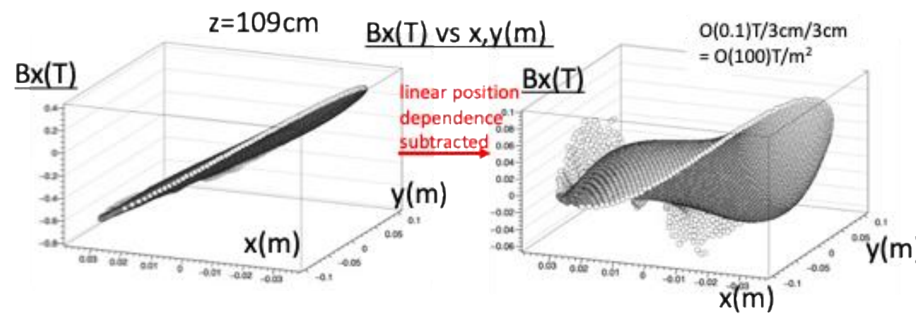


$= \frac{\partial B_z}{\partial z}$ = 3 T / 1.5 m (storage B-field / vertical size of magnet)

Intrinsic non-linearity

- Additionally, inevitable non-linearity effect can happen.
  - some of them can be enhanced by maginification term in liear optics.
- Source of non-linearity No.1 : **Non-linear positional dependence of B-field**
  - B-field around tunnel region has strong positional dependence, including non-linearity
  - Kicker field also has non-linear positional dependence





- Source of non-linearity No.2 : **Small ring effect**
  - Our compact ring setup may make the “small ring effect” non-negligible.

$$H(x, y, z, \tilde{p}_x, \tilde{p}_y, \delta; s) = -\left(1 + \frac{x}{\rho}\right)\left[(1+\delta) - \frac{\tilde{p}_x^2 + \tilde{p}_y^2}{2(1+\delta)} + \tilde{A}\right]$$

= “beam size ~ 1 cm”/ “radius~33 cm” = 0.03

- We need to build a robust system to these non-linearity.
- Hopefully, we may improve the injection efficiency by correcting them.

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Conventional tool to tackle this system

- Conventional tool to treat non-linear beam optics is “Lie map”.

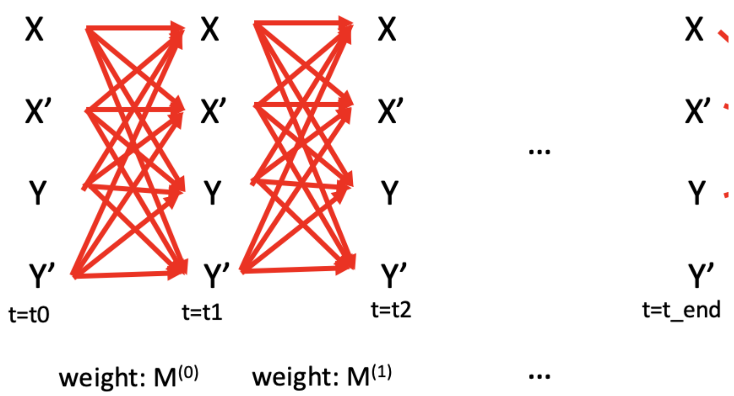
- Time evolution of phase space parameters under Hamiltonian H:
  $\frac{dx}{ds} = -[H, x]$, where $[A, B] \equiv \frac{\partial A}{\partial q}\frac{\partial B}{\partial p} - \frac{\partial B}{\partial p}\frac{\partial A}{\partial q}$
- The transfer map $\mathcal{M}$ ($x(s = ds) = \mathcal{M}\ x(s = 0)$) from one point to another point along the beamline can be evaluated as:
  - $\mathcal{M} = \exp(: -ds: H:)$,
    where $: A: B \equiv [A, B], \exp(: -ds: H:) \equiv \sum_{k=0}^{\infty} \frac{(-ds)^k}{k!}\ (: H:)^k$

- Thus, once the B-field is given, H is determined, then exp(H) can be calculated.
- In the computation, the infinite series is truncated at some dimension.

→ Transfer map is representated by truncated polynomials or tensor map.

Neural network as a transfer map

- Representing this Lie map by neural network is a recently proposed topic.
  - Time evolution of each step is represented as a map between layers.
    - For example, for the linear optics $x_{out} = \prod_l M^{(l)} x_{in}$, the weight of the network $w_{ij}^{(l)}$ is transfer matrix of each step $w_{ij}^{(l)} = M_{ij}^{(l)}$.
    - Non-linear term can also be added, of course.
- This is a physically interpretable network, and a new representation of the Lie map.

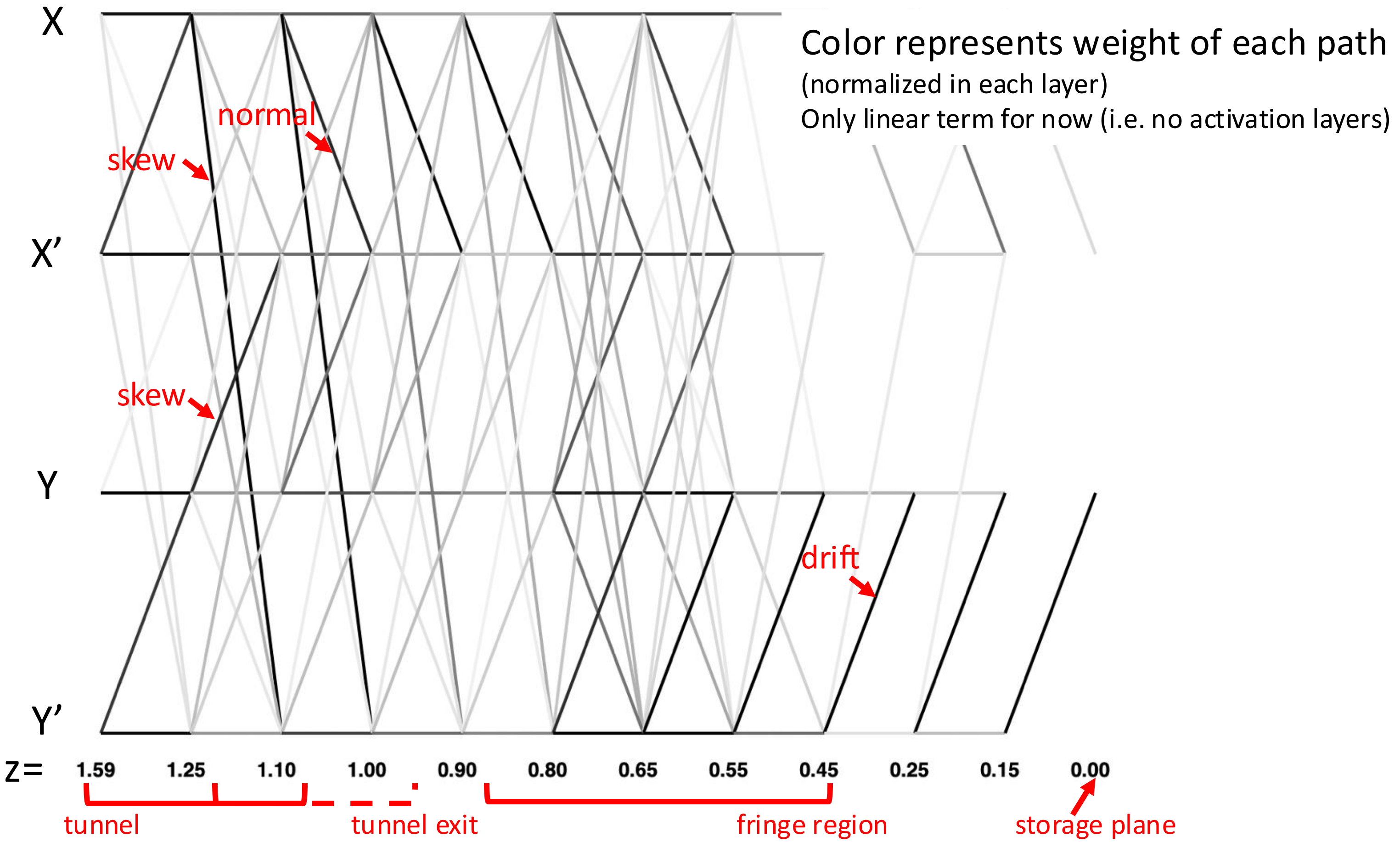


- Previous works utilized this representation for beam optics correction in real machines.
  - Polynomial Neural Networks [1]
    - Implement taylor map as neural network.
    - Utilized in PETRA in DESY.
  - Deep Lie Map Network [2]
    - Implement Lie map as neural network.
    - Utilized in SIS18 in GSI

[1] Ivanov Andrei and Agapov Ilya, Phys. Rev. Accel. Beams, 23, 074601 (2020)
[2] Caliari Conrad and Oeftiger Adrian and Boine-Frankenheim Oliver, Phys. Rev. Accel. Beams, 28, 024601(2025)

Neural network as a transfer map

- Representation of 3D injection optics. (only linear term for now)



- (Though this is not an essencial benefit, but technically)
  This gives us a graphical intuition on our optics than a series of the transfer matrix.

Augmentation of Lie map by neural network representation

This representation of the Lie map by neural network seems to have several advantages for the system having complicated optics like 3D injection.

<u>No.1: Auto-differentiability of the model</u>

- In NN, back propagation gives us $\frac{\delta(\text{eval. func.})}{\delta(\text{input beam})}, \frac{\delta(\text{eval. func.})}{\delta(\text{weight} \sim \text{B}-\text{field})}$.
- This is the "robustness" and "required precision" of the system.
  -> Basis for beamline design including errors.
- If we add non-linear devices on the transport line,
  this tells us its optimal configuration.

<u>No.2: "Learning" in the real commissioning</u>

- Based on the model optics in the design,
  we can train the model with real data in the commissioning.
  - as is already done in previous works ([1][2] in P.13)
- We may include this as a "default plan" from the beamline design.

Summary

- We would like to store relativistic muons in a compact ring.
- A new 3D beam injection scheme is being developed for this.

- Its beam optics has intrinsic strong defocusing, and intrinsic non-linearity.
- Design method for this non-standard optics must be developed.

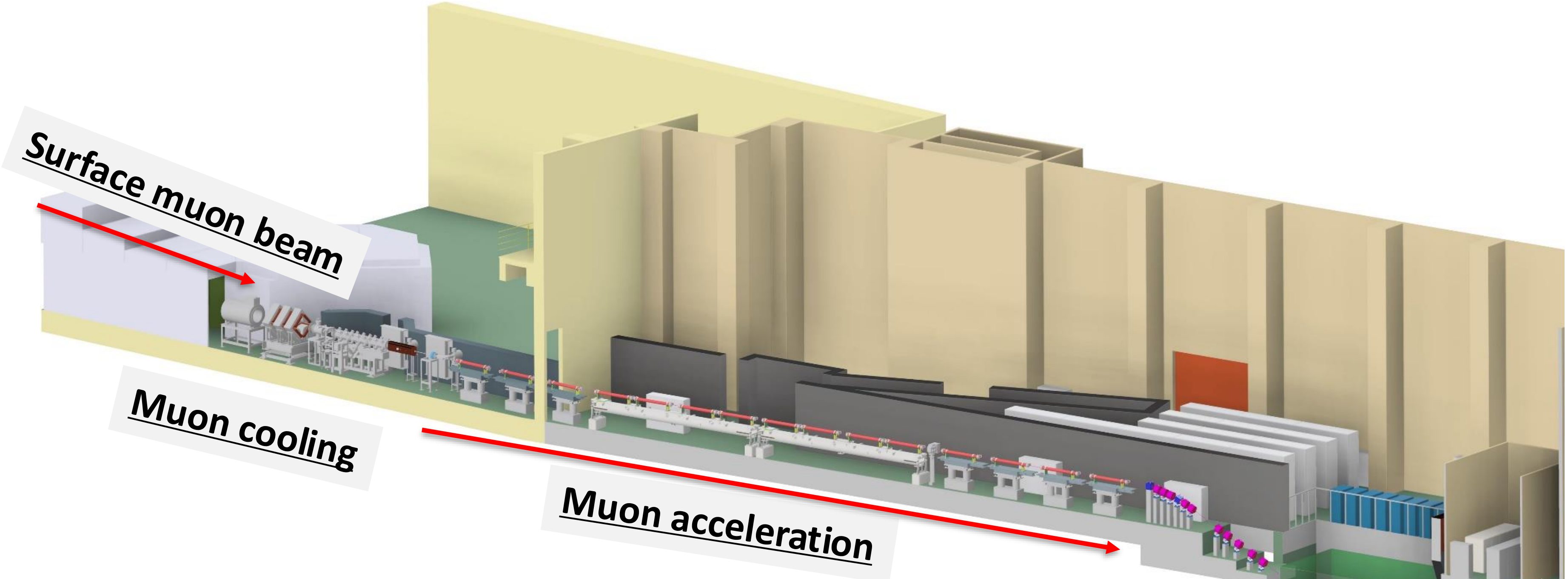
- Augmentation of the conventional Lie map for non-linear optics
  by neural network representation can benefit the design of 3D injection.
  - Especially its auto-differentiability.

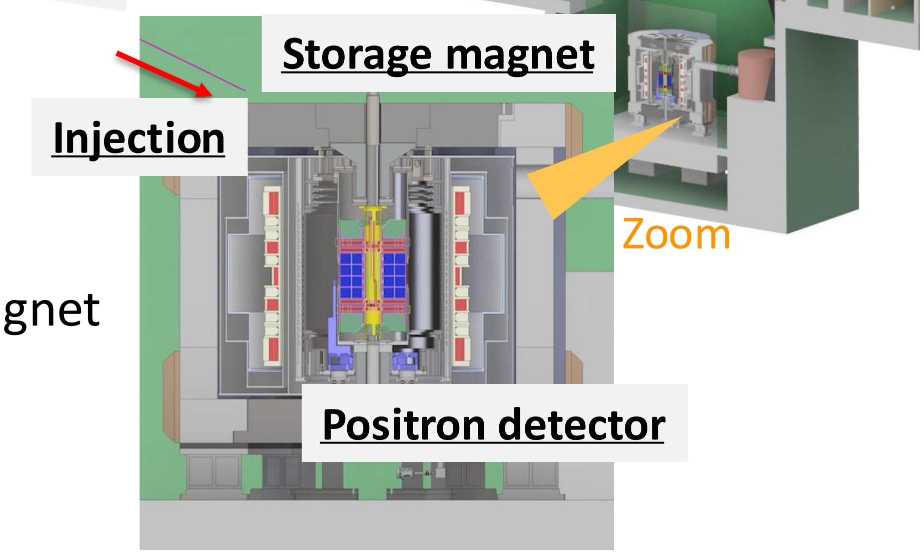
- Planned next step is try building a model.
  - B-field map from FEM -> Hamiltonian -> Transfer map neural network
    -> Robustness of the system, Interpretation of the system, beam monitor design, etc…

Any comments, suggestions are very welcome.

BACKUP

Experimental setup overview





1. High intensity muon beam from MLF
2. Low emittance beam
   by muon cooling & reacceleration.
3. Dedicated beam injection scheme.
4. Quite uniform B-field by MRI-like magnet
5. Compact detector
   with high hit rate tolerances

Three-dimensional spiral beam injection

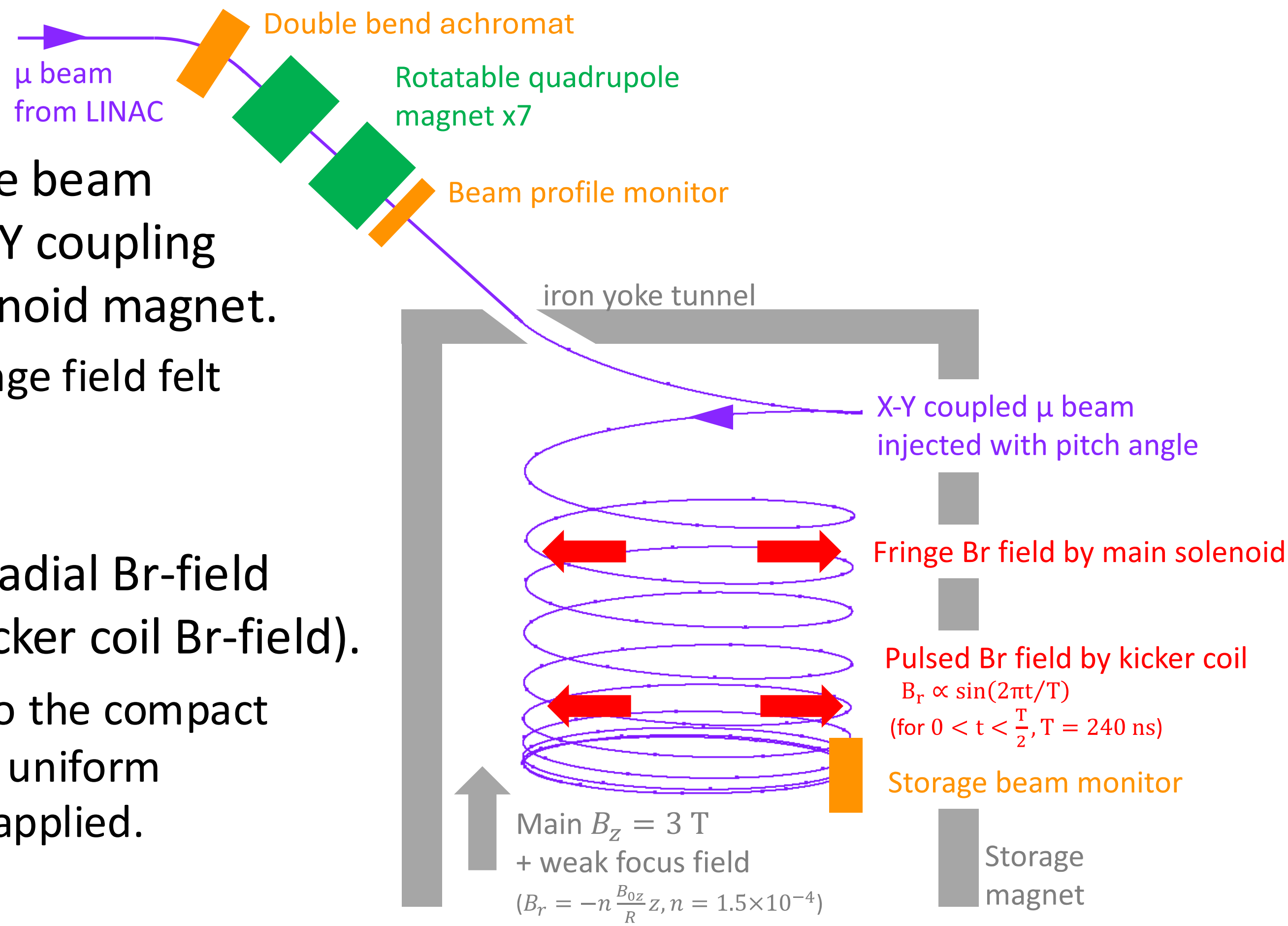
- Low emittance muon beam (300MeV, 0.3π mm-mrad) will be injected into compact storage orbit (Bz=3.0T, R=33.3cm), and stored without electric focusing with good injection efficiency.

<u>Key points</u>

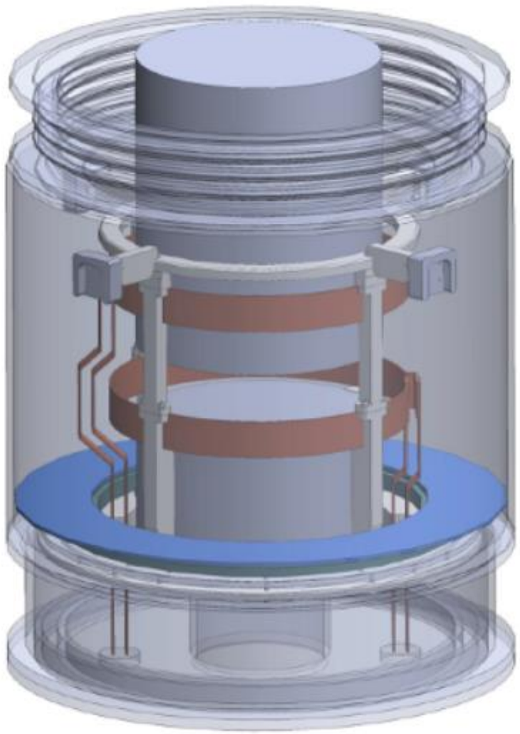
1. Inject low emittance beam with appropriate X-Y coupling is injected into solenoid magnet.
   – to compensate fringe field felt by each muon

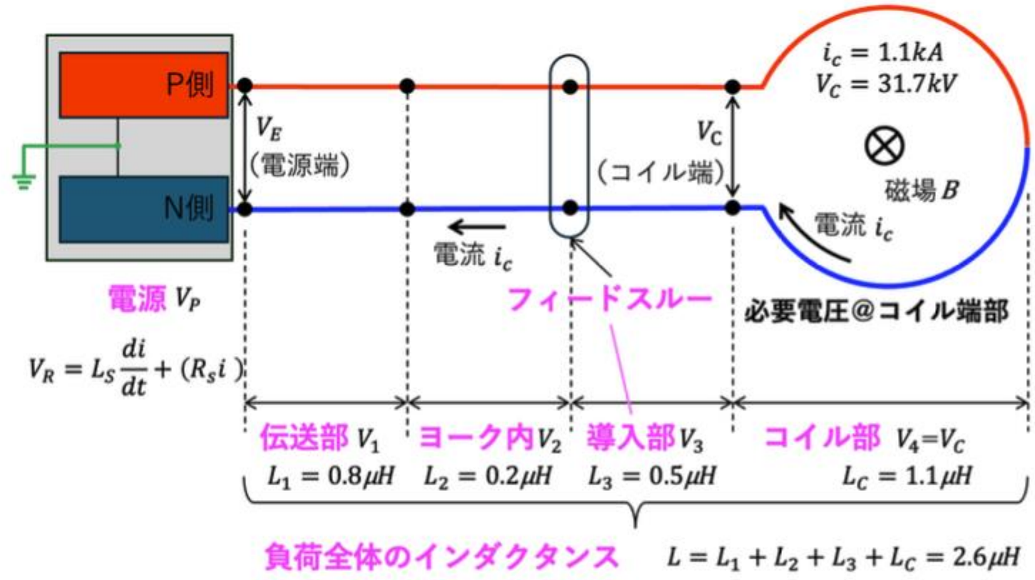
2. Apply appropriate radial Br-field (Fringe Br-field + kicker coil Br-field).
   – to guide muons to the compact region where the uniform magnetic field is applied.

3. Store muon beam by weak focusing.



Kicker system





Study topics to realize 3D beam injection

seems to be world-first
not world-first, but rare

A: Clarification of requirement from physics

- What is required from g-2/EDM measurement in compact storage ring?

B: Design method and overall design of 3D beam injection

- 3D beam injection is a brand new injection scheme which includes complicated beam dynamics.

C: Kicker system

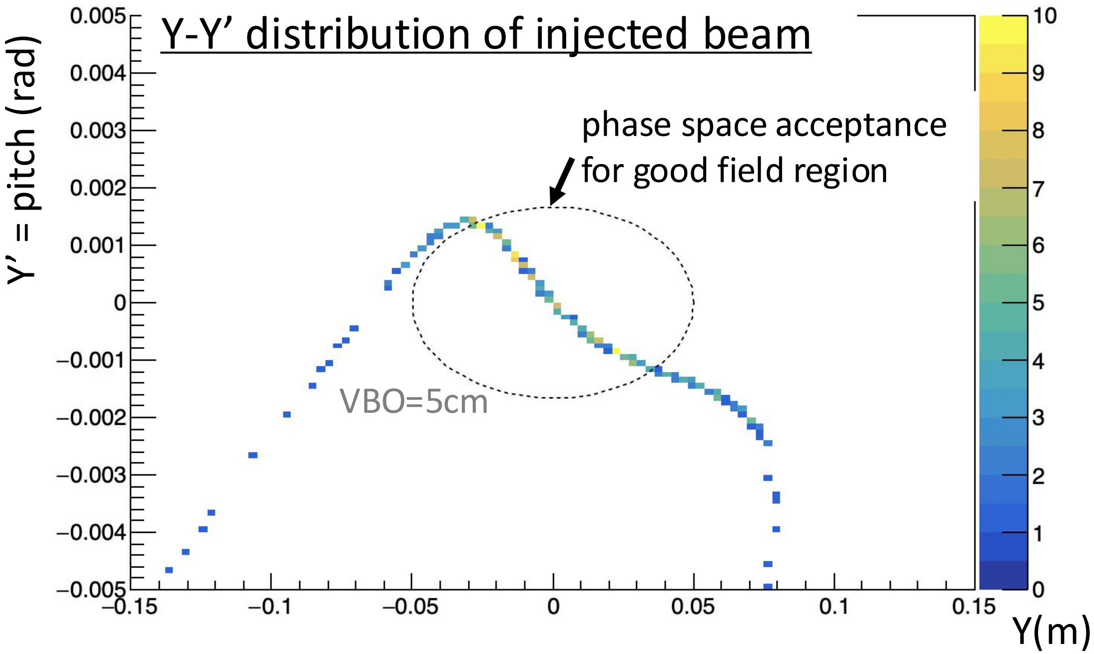
- High power kicker system with little timing jitter and little residual field

D: Transport beam line

- Beam orbit and phase distribution control in strongly XY-coupled system

E: Beam transport inside storage magnet

- Beam orbit and phase distribution control in complicated beam dynamics system


Y-Y' distribution of injected beam
phase space acceptance
for good field region
VBO=5cm
Y' = pitch (rad)
Y(m)
0.005
0.004
0.003
0.002
0.001
0
−0.001
−0.002
−0.003
−0.004
−0.005
−0.15
−0.1
−0.05
0
0.05
0.1
0.15
10
9
8
7
6
5
4
3
2
1
0

Augmentation of Lie map by neural network representation

No.1: Auto-differentiability of the model

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  - as is already done in previous works ([1][2] in P.13)
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No.3: Introduction of concepts in neural network

- For our case, it is not clear which effect in the optics is important?
-> Likely, this can be connected to the "importance of each path in NN".