Enhancing the Dynamic Range of Quantum Sensing via Quantum Circuit Learning

Yuichiro Matsuzaki

Chuo University

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Back grounds

2005/4- 2008/12 **Waseda University** (bachelor degree)

2005/4- 2008/12 **University of Tokyo** (Master degree), supervised by Akira Shimizu

Research topic: Quantum computation

2008/1- 2011/1 **University of Oxford**, supervised by Simon Benjamin

Research topic: Quantum computation and quantum sensing

2011/2 -2011/3 **Aalto University** (postdoctor), supervised by Mikko Mottonen

Research topic: Quantum computation

2011/4 – 2018/12 **NTT basic research laboratories (staff)**

Research topic: superconducting qubits, nitrogen vacancy centers in diamond

2019/1 –2023/4 **Advanced Industrial Science and Technology (Staff)**

Research topic: quantum annealing, NISQ computing, quantum thermodynamics

2023/4 – Chuo University (associate professor)

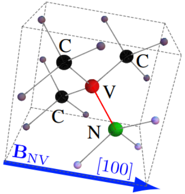
collaborators

- Hideaki Kawaguchi (Keio University)
- Yuichiro Mori (Chuo University)
- Takahiko Satoh (Keio University)

Background

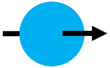
Nitrogen vacancy (NV) centers in diamond C. Degen et al., Rev. Mod. Phys. 89.3 (2017)

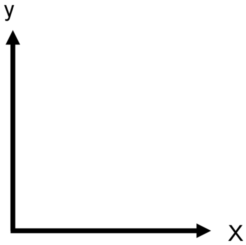
- Two electron spins are trapped to form a triplet states
- Long coherence time as a few milliseconds at room temperature
- Manipulation of the spin by microwave pulses
- Initialization by green laser
- Readout of the spin from the photoluminescence
- Coupling with the magnetic field
  $\Longrightarrow$application to magnetic field sensors



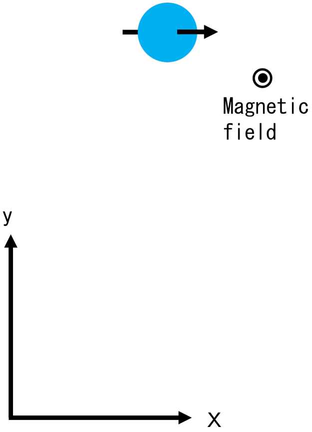
Quantum Sensing Flow with qubits 1

1. Prepare a spin state along x direction



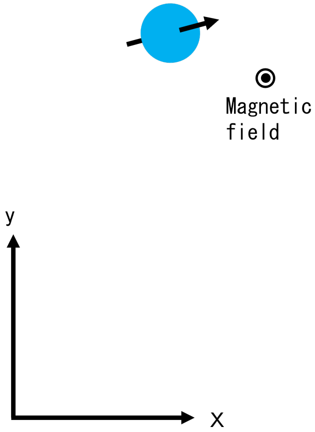


Quantum Sensing Flow with qubits 2



1. Prepare a spin state along x direction
2. Applying magnetic field along z direction

Quantum Sensing Flow with qubits 2



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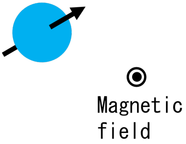
Quantum Sensing Flow with qubits 2



1. Prepare a spin state along x direction
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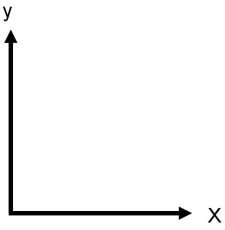
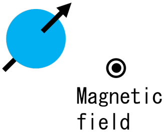
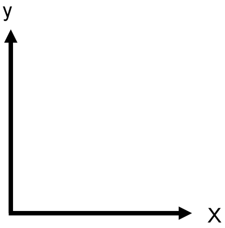




1. Prepare a spin state along x direction
2. Applying magnetic field along z direction
3. Readout the spin state



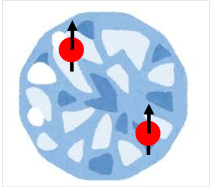
Quantum Sensing Flow with qubits (summary)

Estimating the value of $\omega = g\mu_b B$ in the Hamiltonian of $H = \frac{\omega}{2}\sigma_z$

1. State Preparation by using a green laser and microwave pulse. $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
2. Time Evolution with the Hamiltonian
3. Readout by photoluminescence
4. Repeat 1–3, and obtain measurement results.
5. From the measurement results, estimate the value of ω

Sensitivity increases as we increase the number of repetitions (or number of qubits.)

Small diamond (nm) with a low density

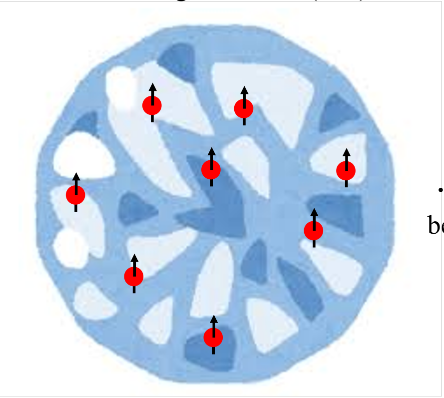


- Spatial resolution is good such as a few nano meters
- Sensitivity is bad because of a small number of spins

P. Maletinsky, et al. *Nature nanotechnology* 7.5 (2012): 320-324.

Approach 2

Large diamond (mm) with a low density



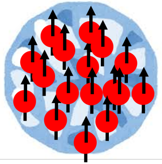
- Spatial resolution is bad
  such as a few milli meter

- Sensitivity is good
  because of a large number of spins

M. Fujiwara et al. *APL Photonics* 8.3 (2023).

Approach 3

small diamond (nm) with a high density



・Spatial resolution is good
such as milli meter

・Sensitivity is good
because of a large number of spins

・Controllability is bad due to
the strong coupling between spins

H. Zhou et al., Physical Review Letters 131, 220803(2023)

Motivation

	Conventional 1	Conventional 2	Conventional 3	Our target
Diamond	· Small volume · Low density	· Large volume · Low density	· Small volume · High density	· Small volume · High density
Sensitivity	Bad	Good	Good	Good
Spatial resolution	Good	Bad	Good	Good
Dynamic Range	Good	Good	Bad	Good

Dynamic range is the span of values we can measure without ambiguity.

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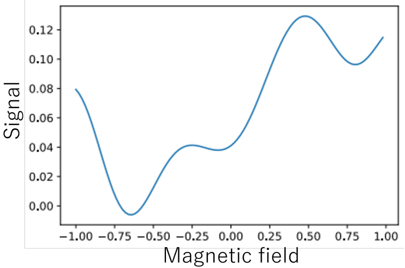
Ex. Bad dynamic range (We cannot uniquely specify the magnetic field)



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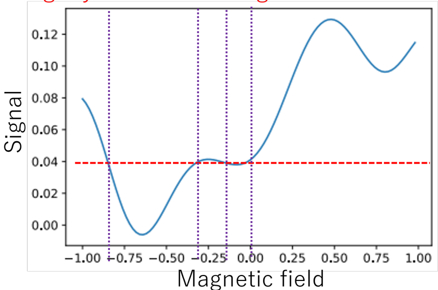
Dynamic Range	Good	Good	Bad	Good

Ex. Bad dynamic range (We cannot uniquely specify the magnetic field)

Ambiguity to estimate magnetic field value exists!



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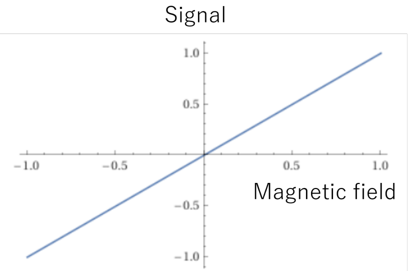
Ex. Good dynamic range (We can uniquely specify the magnetic field)



Approaches to use small diamond with dense NV centers

· The previous approach uses Floquet engineering, which works when the pulse operations are done faster than the inverse of the coupling strength ($\tau_{\mathrm{pulse}} \ll 1/g$).

H. Zhou et al., Physical Review X 10, 031003 (2020), H. Zhou et al., Physical Review Letters 131, 220803 (2023),

· As an alternative approach, we theoretically propose to use quantum circuit learning, which could , in principle, be applied to more general circumstances.

H. Kawaguchi, Y. Mori, T. Satoh, Y. Matsuzaki (2025). arXiv:2505.04958.,

Quantum metrology to measure magnetic fields

Hamiltonian for a single qubit

$$H = \frac{\omega}{2}\hat{\sigma}_z,$$

$\omega = g\mu_B B$: Zeeman energy.

Expectation values

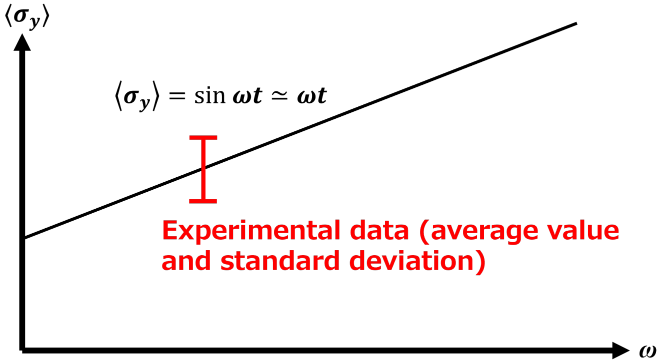
$$\langle\phi(t)|\hat{\sigma}_y|\phi(t)\rangle = \sin(\omega t).$$

where $|\phi(t)\rangle = e^{-iHt}|+\rangle$ and $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

Uncertainty (inverse of the sensitivity)

$$\delta\omega = \frac{\sqrt{\langle\phi(t)|(\delta\hat{\sigma}_y)^2|\phi(t)\rangle}}{\left|\frac{d\langle\phi(t)|\hat{\sigma}_y|\phi(t)\rangle}{d\omega}\right|\sqrt{M}} = \frac{1}{\sqrt{M}t}$$

Uncertainty of the estimation



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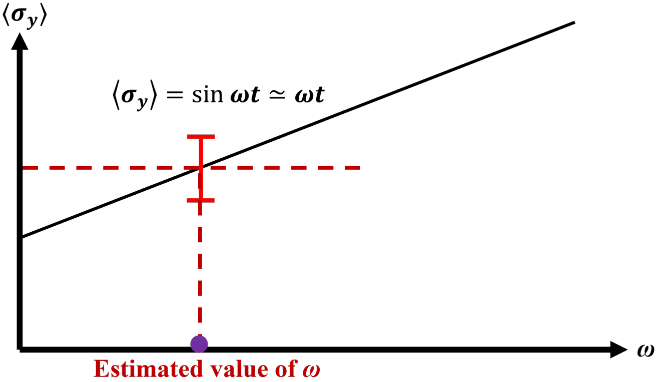
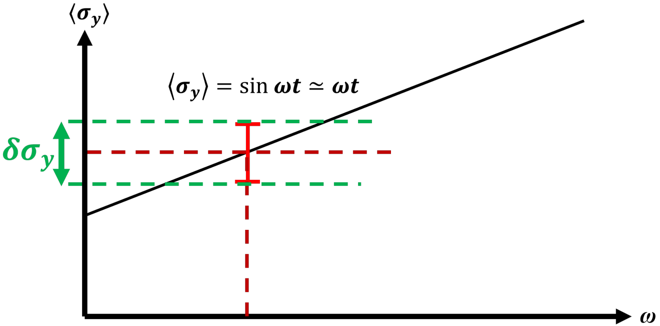
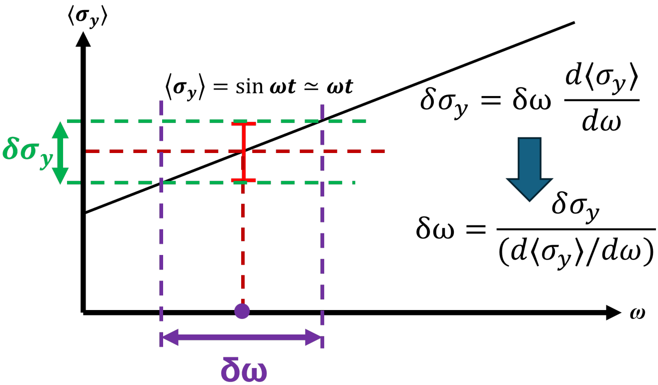


Uncertainty of the estimation

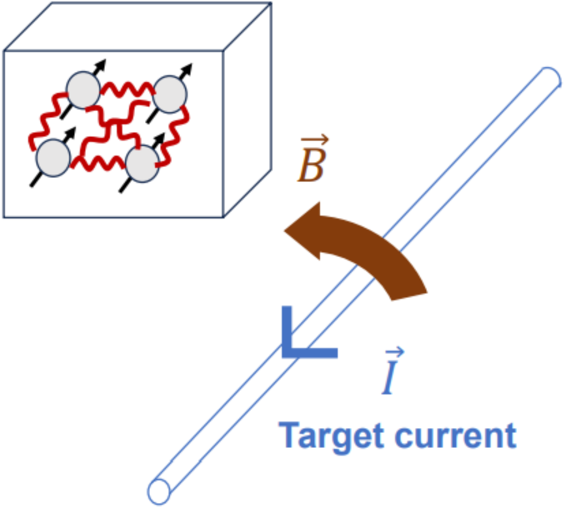


Uncertainty of the estimation



Setup

Using an ensemble of qubits, we measure magnetic fields from electric currents and aim to estimate their strength.



Quantum metrology to measure electric current

Hamiltonian for an ensemble of qubits

$$H = \sum_{j=1}^{L} \frac{h_j I}{2} \hat{\sigma}_z^{(j)}, \quad (1)$$

h_j: Relative coupling strength, I: Electric current,

Expectation values

$$\langle \phi(t) | \hat{M}_y | \phi(t) \rangle = \sum_{j=1}^{L} \sin(h_j I t). \quad (2)$$

where $|\phi(t)\rangle = e^{-iHt}|++\cdots+\rangle$ and $\hat{M}_y = \sum_{j=1}^{L} \hat{\sigma}_y^{(j)}$

Sensitivity

$$\delta I = \frac{\sqrt{\langle \phi(t) | (\delta \hat{M}_y)^2 | \phi(t) \rangle}}{\left| \frac{d\langle \phi(t) | \hat{M}_y | \phi(t) \rangle}{dI} \right| \sqrt{M}} = \frac{\sqrt{\sum_{j=1}^{L} (1 - \sin^2(h_j I t))}}{\left| \sum_{j=1}^{L} h_j t \cos(h_j I t) \right| \sqrt{M}}, \quad (3)$$

Quantum circuit learning (QCL)

Setup for supervised learning

- Training data set $(x_i, y_i)_{i=1}^{N}$ is given with samples N
- Relationship such as $y = \tilde{f}(x)$ between x and y is assumed
- Function f_θ is used to approximate $\tilde{f}$ by minimizing the following

$$L(\boldsymbol{\theta}) = \sum_{i=1}^{N} (f_{\boldsymbol{\theta}}(x_i) - y_i)^2. \tag{4}$$

where θ is the parameter.

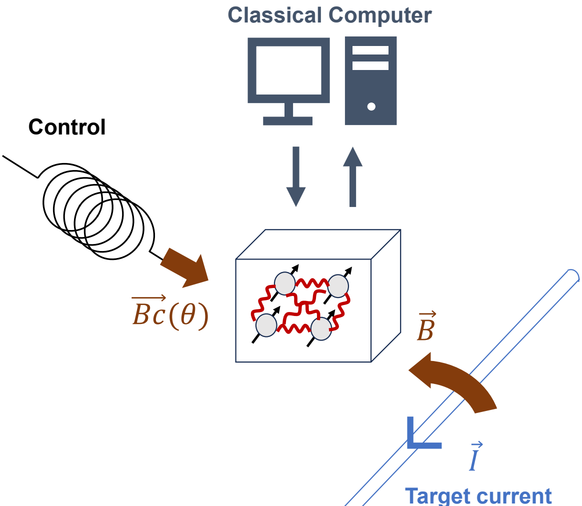
Quantum circuit learning K. Mitarai, Kosuke, et al. " Physical Review A 98.3 (2018): 032309.

- The input state is $e^{-ix_i H}|00\cdots 0\rangle$
- A parametrized unitary operator U_θ is applied with the input state
- By using an observable $\hat{M}$, the learning model is defined as

$$f_{\boldsymbol{\theta}}(x) = \langle 0...0| \, e^{ixH} U^\dagger(\boldsymbol{\theta}) \hat{M} U(\boldsymbol{\theta}) e^{-ixH} \, |0...0\rangle \, . \tag{5}$$

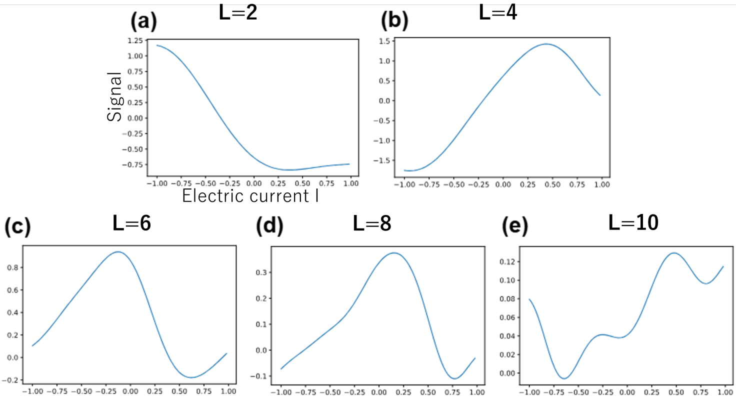
Setup for quantum metrology with QCL

By using an ensemble of qubits, we aim to estimate the strength of the electric currents. However, at high qubit densities, inter-qubit interactions induce multiple oscillations in the signal, which reduces the dynamic range.



Low dynamic range

As the number of qubits increases, the region where the expectation value changes monotonically becomes narrower, indicating a reduction in the dynamic range.



Hamiltonian and input state

Hamiltonian

$$H_{data} = H_I + \sum_{j=1}^{L} \frac{h_j I}{2} \hat{\sigma}_y^{(j)}, \quad (6)$$

$$H_I = \sum_{i,j=1}^{L} J_{ij} (\hat{\sigma}_x^{(i)} \hat{\sigma}_x^{(j)} + \hat{\sigma}_y^{(i)} \hat{\sigma}_y^{(j)} + \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(j)}), \quad (7)$$

· H_I is the interaction Hamiltonian and J_{ij} denotes the strength of the interaction
· $h_j = \frac{1}{2} + 2r_j$, where r_j is sampled from the uniform distribution on $[0, 1)$.
· $J_{ij} = -1 + 2s_{ij}$ where s_{ij} is drawn from a uniform distribution on $[0, 1)$.

Input state

$$|\phi_{input}\rangle = e^{-iH_{data}t} |00 \cdots 0\rangle, \quad (8)$$

Single-qubit rotation

$$R_x(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix},$$

$$R_y(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix},$$

$$R_z(\theta) = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}.$$

Parametrized unitary

$$U_x(\theta_1) = \left(\bigotimes_{i=1}^{L} R_x^{(i)}(\theta_1)\right) e^{-itH_I},\ U_y(\theta_2) = \left(\bigotimes_{i=1}^{L} R_y^{(i)}(\theta_2)\right) e^{-itH_I},$$

$$U_z(\theta_3) = \left(\bigotimes_{i=1}^{L} R_z^{(i)}(\theta_3)\right) e^{-itH_I},$$

$$H_{gx} = \sum_{j=1}^{L} B_j^x \hat{\sigma}_x + H_I,\ H_{gy} = \sum_{j=1}^{L} B_j^y \hat{\sigma}_y + H_I, H_{gz} = \sum_{j=1}^{L} B_j^z \hat{\sigma}_z + H_I$$

where $B_j^x = B_j^y = B_j^z = B_0 j$ and $B_0 = 1$. The total unitary is as follows

$$U(\boldsymbol{\theta}) = \prod_{d=1}^{D} U^{(d)}(\boldsymbol{\theta^{(d)}}), \tag{9}$$

where $U^{(d)}(\boldsymbol{\theta^{(d)}}) = e^{-iH_{gz}t} U_z(\theta_3^{(d)}) \cdot\ e^{-iH_{gy}t} U_y(\theta_2^{(d)}) \cdot\ e^{-iH_{gx}t} U_x(\theta_1^{(d)})$.

Training inputs and target functions

· Let $\{I_i\}_{i=1}^{N}$ denote the set of training inputs, where we set $N = 200$.

· The inputs $\{I_i\}$ are generated by uniformly sampling from the interval $[-1, 1]$.

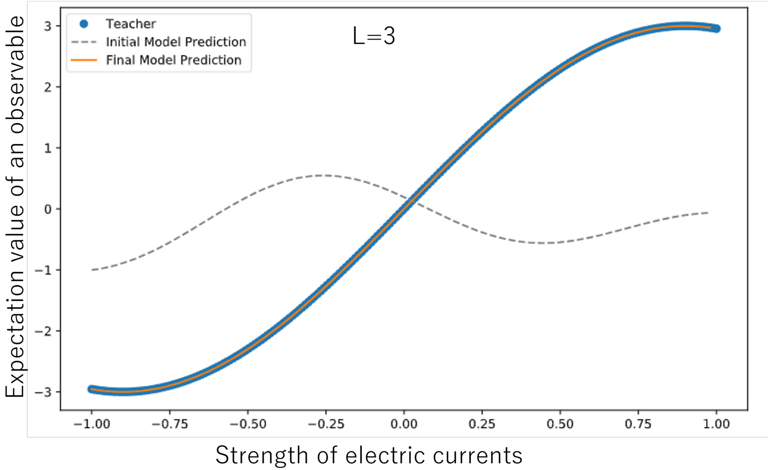
· The target function $f(I)$ is defined as follows:

$$f(I) = A \cdot L \cdot \sin\left(\frac{\sum_j h_j I t}{B \cdot L}\right), \tag{10}$$

where we set $A = B = 1$.

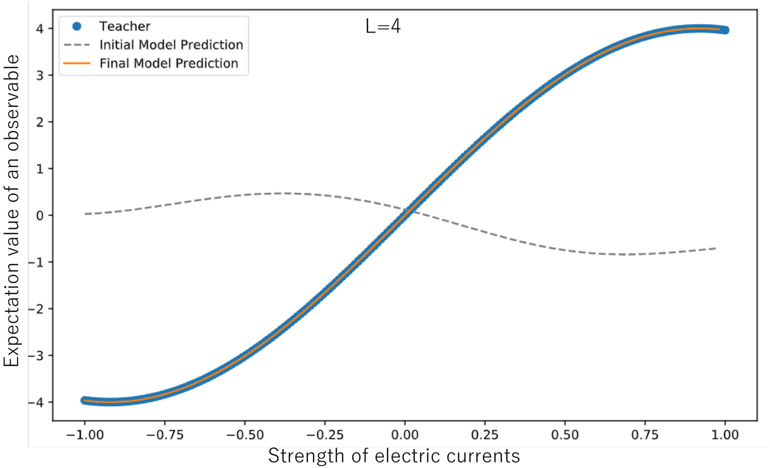
Results with $L = 3$

Dynamic range improvement of quantum sensors through QC



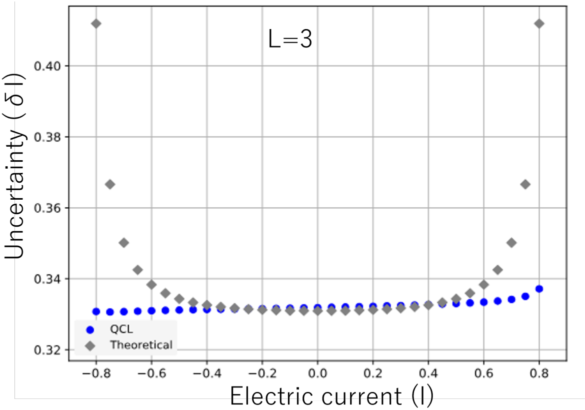
Results with $L = 4$

Dynamic range improvement of quantum sensors through QC



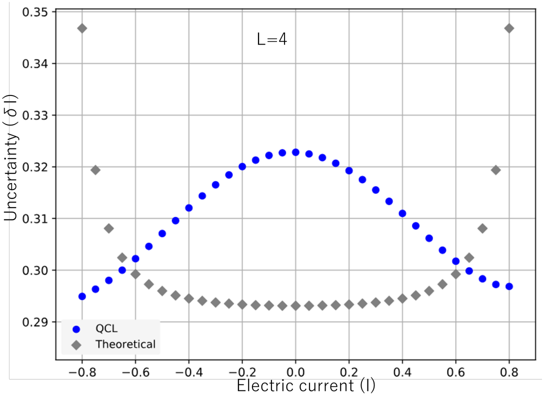
Comparison of the sensitivity (L=3)

Let us compare the sensitivity of our method with that of other cases. As a comparison, we consider an imaginary scenario with negligible inter-qubit coupling strength and calculate the sensitivity (theoretical) with separable states as $\delta I = \frac{\sqrt{\langle\phi(t)|(\delta\hat{M}_y)^2|\phi(t)\rangle}}{\left|\frac{d\langle\phi(t)|\hat{M}_y|\phi(t)\rangle}{dI}\right|\sqrt{M}} = \frac{\sqrt{\sum_{j=1}^{L}(1-\sin^2(h_j It))}}{\left|\sum_{j=1}^{L} h_j t\cos(h_j It)\right|\sqrt{M}}$



Comparison of the sensitivity (L=4)

Let us compare the sensitivity of our method with that of other cases. As a comparison, we consider an imaginary scenario with negligible inter-qubit coupling strength and calculate the sensitivity (theoretical) with separable states as $\delta I = \frac{\sqrt{\langle\phi(t)|(\delta\hat{M}_y)^2|\phi(t)\rangle}}{\left|\frac{d\langle\phi(t)|\hat{M}_y|\phi(t)\rangle}{dI}\right|\sqrt{M}} = \frac{\sqrt{\sum_{j=1}^{L}(1-\sin^2(h_j It))}}{\left|\sum_{j=1}^{L} h_j t\cos(h_j It)\right|\sqrt{M}}$



Conclusion and perspective

Conclusion

· We propose to improve the dynamic range of quantum sensing at high qubit densities by using quantum circuit learning
· From numerical simulations, we confirm that the dynamic range is improved without a significant reduction of the sensitivity

Perspective

· We will use a more realistic setup where the Hamiltonian is determined from the positions of qubits and the current lines.
· Sensitivity can be further improved by an entanglement if we consider a more sophisticated cost function

H. Kawaguchi, Y. Mori, T. Satoh, Y. Matsuzaki (2025). arXiv:2505.04958.,