

The **BAO-CMB tension** and *implications for inflation*

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*Frontiers of Precision Cosmology:
From the Early Universe to Observational Tensions*

04/March/2026

Statistical challenges in cosmology

Exploring the growth of parameter spaces, statistical effects, data combination and their impact on cosmological inference and tensions in cosmology

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Statistical methods in cosmology

Cosmology as a precision science owes much of its progress to the large and precise cosmological data sets



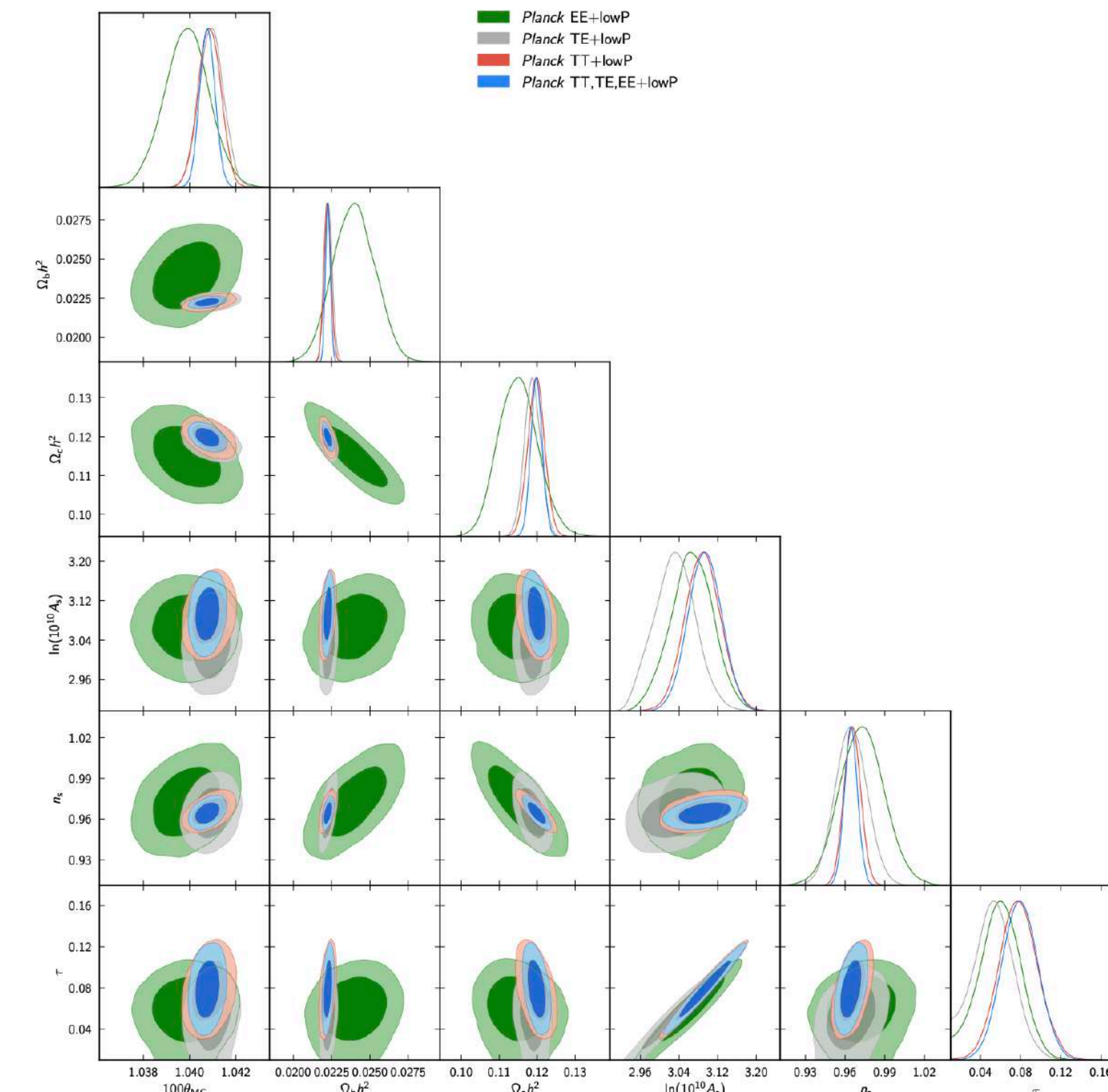
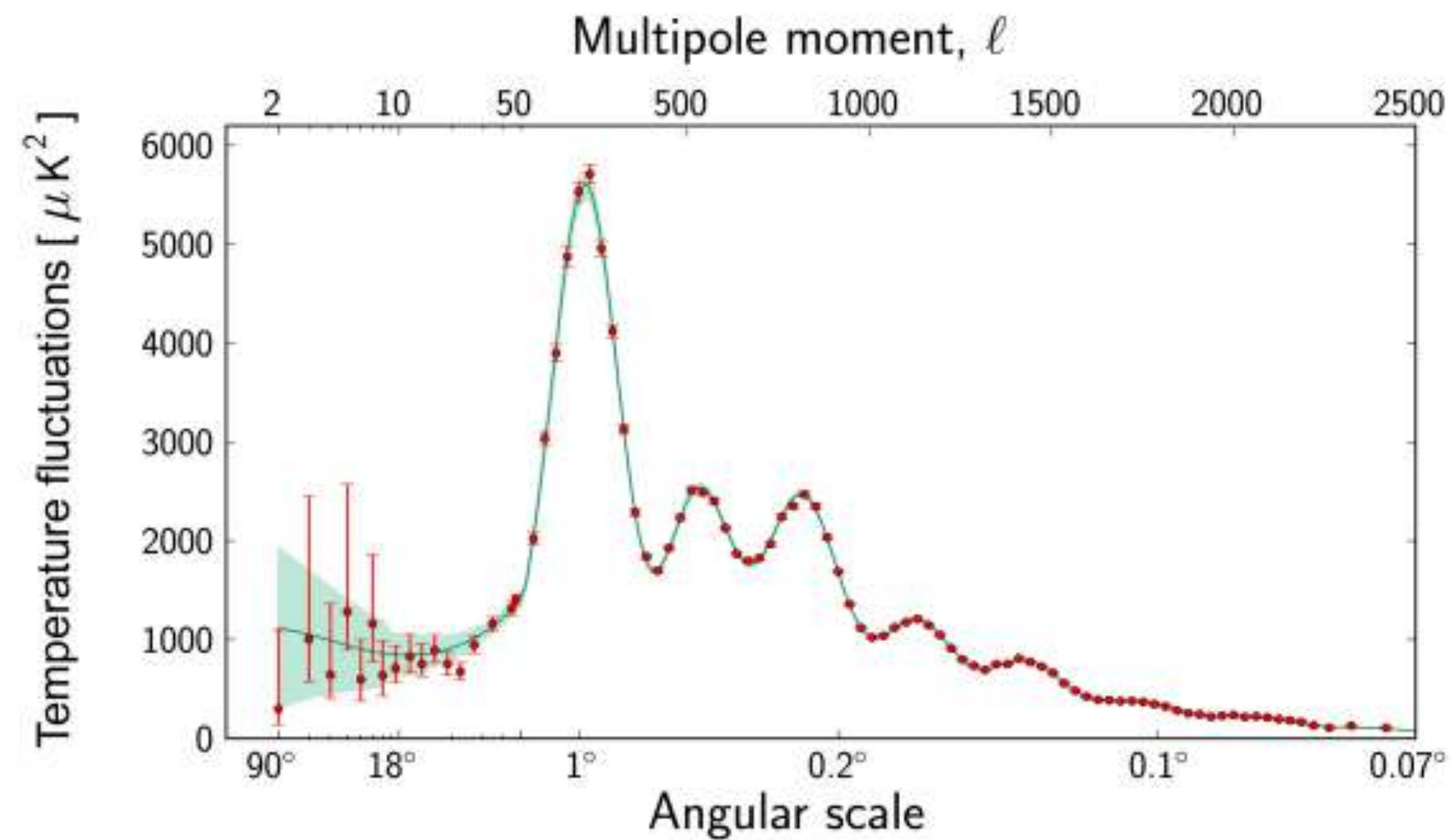
Bayesian statistics;
MCMC techniques

Statistical methods in *cosmology*

Cosmology as a precision science owes much of its progress to the large and precise cosmological data sets

ΛCDM →

Bayesian statistics;
MCMC techniques



Planck analysis: **~30–40 parameters** (6 cosmological + ~25 nuisance)

Statistical methods in cosmology

Cosmology as a precision science owes much of its progress to the large and precise cosmological data sets

LCDM
→

Bayesian statistics;
MCMC techniques

Next decade: growth of statistical power of cosmological experiments

(Now!)

New probes

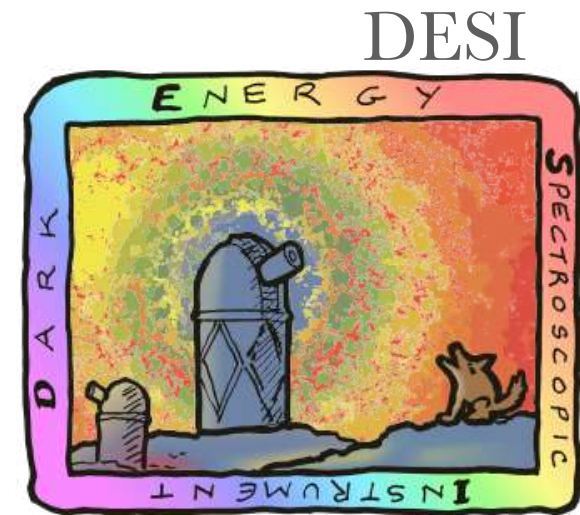
→

Stress test LCDM,
tensions, new physics, ...

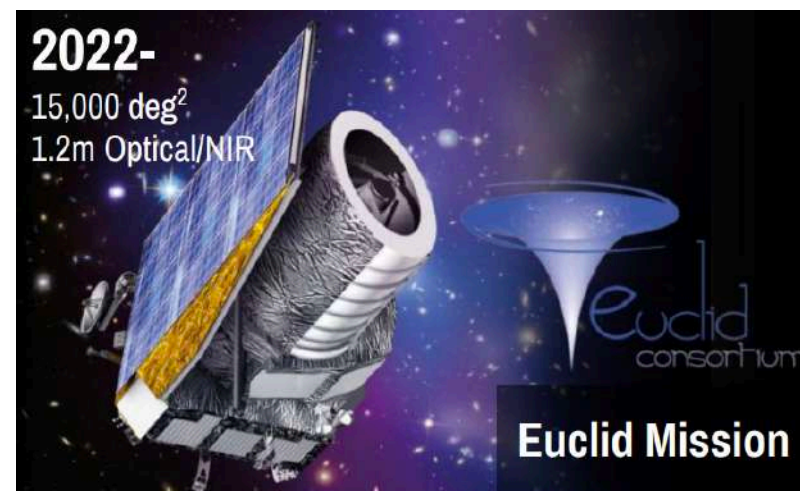
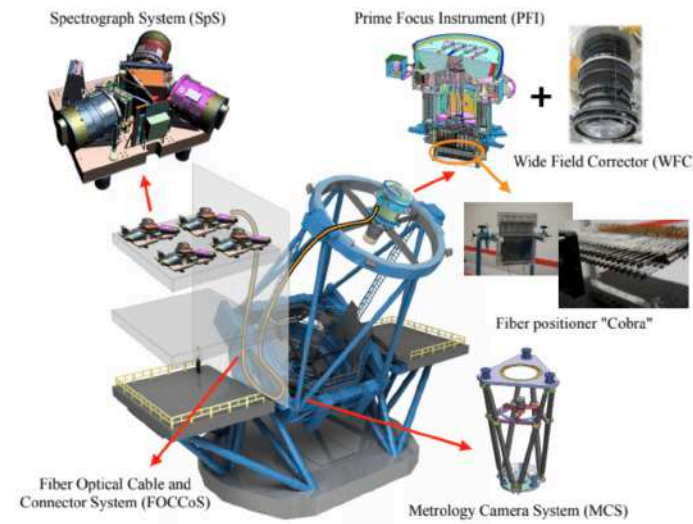
Observations - Stage 3 and 4

Huge amount of data coming now and in the future: incredible precision, new probes, small scales, polarization, ...

Photometric and spectroscopic surveys



Prime Focus Spectrograph (PFS)



CMB



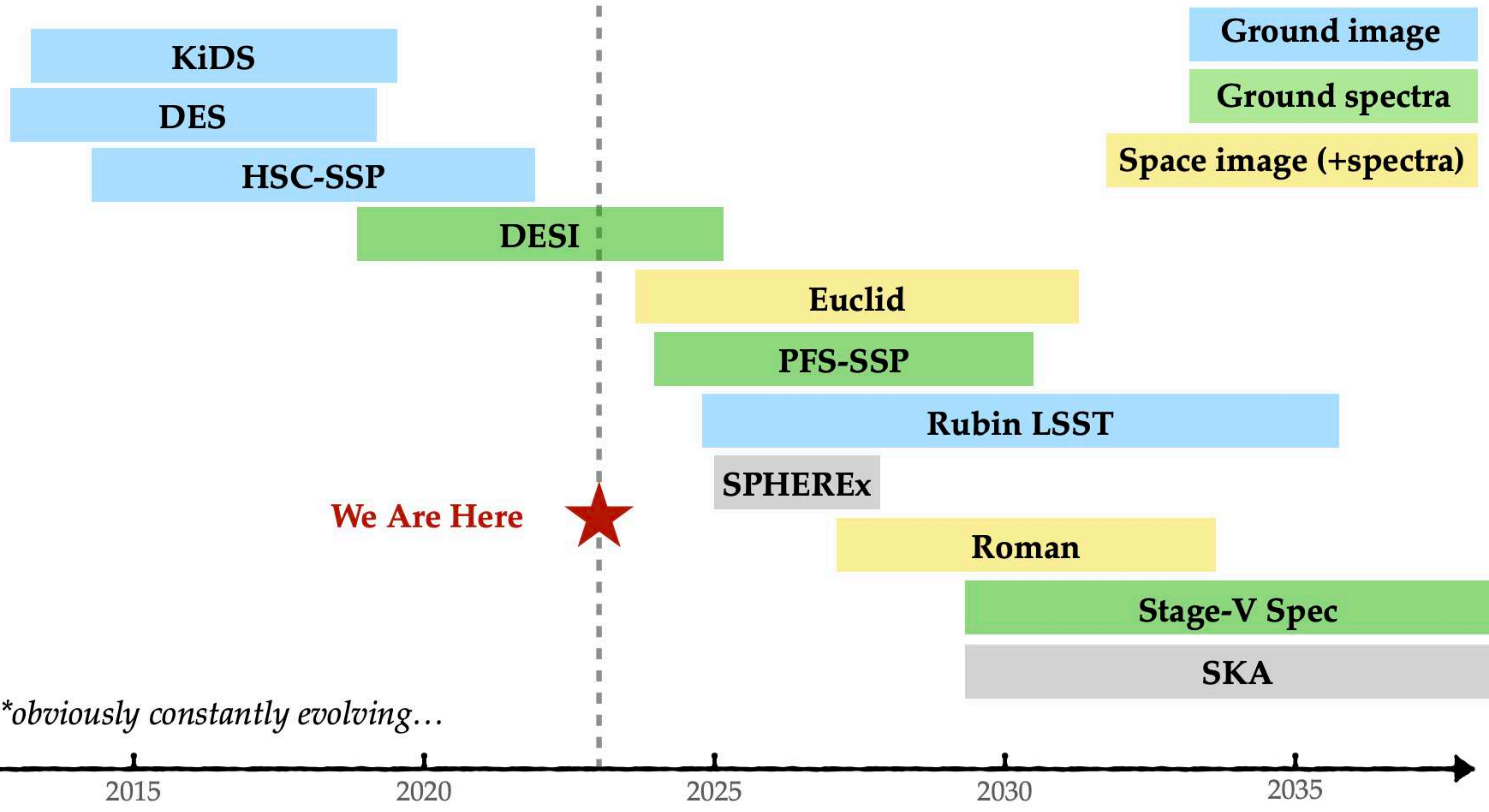
GWs



CMB-S4

Next Generation CMB Experiment

21cm



Statistical methods in cosmology

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Next decade: growth of statistical power of cosmological experiments
New probes

→

Stress test LCDM,
tensions, new physics,
break degeneracies, ...

However...

Increased statistical power and complexity from new observational, theoretical, and systematic uncertainties introduce additional **nuisance parameters**, making statistical inference more challenging.

Particularly important with the increase of high quality data, where systematics will play important role: stage IV (even stage III), cosmological tensions, ...

LSS surveys: large number of observational and “astrophysical systematics”

Statistical methods in cosmology

LSS surveys: large number of observational and “astrophysical systematics”

Photometric surveys - avenue for addressing important questions in cosmology

Ex: galaxy clustering + weak gravitational lensing (DES) -

- 6 cosmological parameters (Λ CDM and the total neutrino mass)
- 25 nuisance parameters

Parameter	Prior
Cosmological Parameters	
Ω_m	U[0.1, 0.9]
A_s	U[0.5, 5.0] $\times 10^{-9}$
Ω_b	U[0.03, 0.07]
n_s	U[0.87, 1.07]
h	U[0.55, 0.91]
$\Omega_\nu h^2$	U[0.6, 6.44] $\times 10^{-3}$
w	U[-2, -0.333]
Calibration Parameters	
m_1	$\mathcal{N}(-0.0063, 0.0091)$
m_2	$\mathcal{N}(-0.0198, 0.0078)$
m_3	$\mathcal{N}(-0.0241, 0.0076)$
m_4	$\mathcal{N}(-0.0369, 0.0076)$
Δz_1	$\mathcal{N}(0.0, 0.018)$
Δz_2	$\mathcal{N}(0.0, 0.015)$
Δz_3	$\mathcal{N}(0.0, 0.011)$
Δz_4	$\mathcal{N}(0.0, 0.017)$
Intrinsic Alignment Parameters	
a_1	U[-5, 5]
a_2	U[-5, 5]
η_1	U[-5, 5]
η_2	U[-5, 5]
b_{TA}	U[0, 2]
Shear Ratio Parameters	
Δz_1^{lens}	$\mathcal{N}(-0.009, 0.007)$
Δz_2^{lens}	$\mathcal{N}(-0.035, 0.011)$
Δz_3^{lens}	$\mathcal{N}(-0.005, 0.006)$
$\delta_{z,1}^{\text{lens}}$	$\mathcal{N}(0.975, 0.062)$
$\delta_{z,2}^{\text{lens}}$	$\mathcal{N}(1.306, 0.093)$
$\delta_{z,3}^{\text{lens}}$	$\mathcal{N}(0.870, 0.054)$
b_g^{1-3}	U[0.8, 3]

Statistical methods in *cosmology*

Cosmology as a precision science owes much of its progress to the large and precise cosmological data sets

ΛCDM
→

Bayesian statistics;
MCMC techniques

Increased statistical power and complexity from new observational, theoretical, and systematic uncertainties introduce additional **nuisance parameters**, making statistical inference more challenging.

This **inflation of parameters** can lead to statistical artefacts or inefficient data analysis that can **bias** the cosmological parameter inference, affecting how powerful these data sets can be

Statistical tools at center stage! *Bayesian, frequentist, ML, ...*

Questions:

*What are those effects and how important/how much they affect *parameter inference*?*

*Can we combine data sets to improve *parameter constraints*?*

*Cosmological *tensions*?*

Statistical methods in cosmology - challenges for current and next generation of experiments

Combining data sets & tension in cosmology

Can we combine data sets to improve parameter constraints?

Cosmological tensions?

- *The BAO-CMB Tension and Implications for Inflation, EF, et al - **PRD Editor's suggestion***
- *The Shifting Spectrum of n_s constraints from DESI and CMB data, E McDonough, EF*

Volume effects & profile likelihood in cosmology

Parameter inflation

*What are those effects and how important/
how much they affect parameter inference?*

Challenges for new experiments

- *Laura Herold, EF 2210.1629*
- *Laura Herold, EF and Eiichiro Komatsu 2112.12140,*
- *Reeves, L. Herold, S. Vagnozzi, B. Sherwin, EF 2207.01501*
- *Profile Likelihoods in Cosmology: When, Why and How illustrated with Λ CDM, Massive Neutrinos and Dark Energy, L. Herold, EF, L. Heinrich, 2025*
- *E.Holm, L. Herold, T. Simon, EF, S. Hannestad, V. Poulin, T. Tram 2023*

*The **BAO-CMB tension** and Implications for Inflation*

- *The BAO-CMB Tension and Implications for Inflation, **EF**, E McDonough, L Balkenhol, R Kallosh, L Knox, and A Linde, 2025 - **Editor's suggestion***
- *The Shifting Spectrum of n_s constraints from DESI and CMB data, E McDonough, **EF**, 2025*

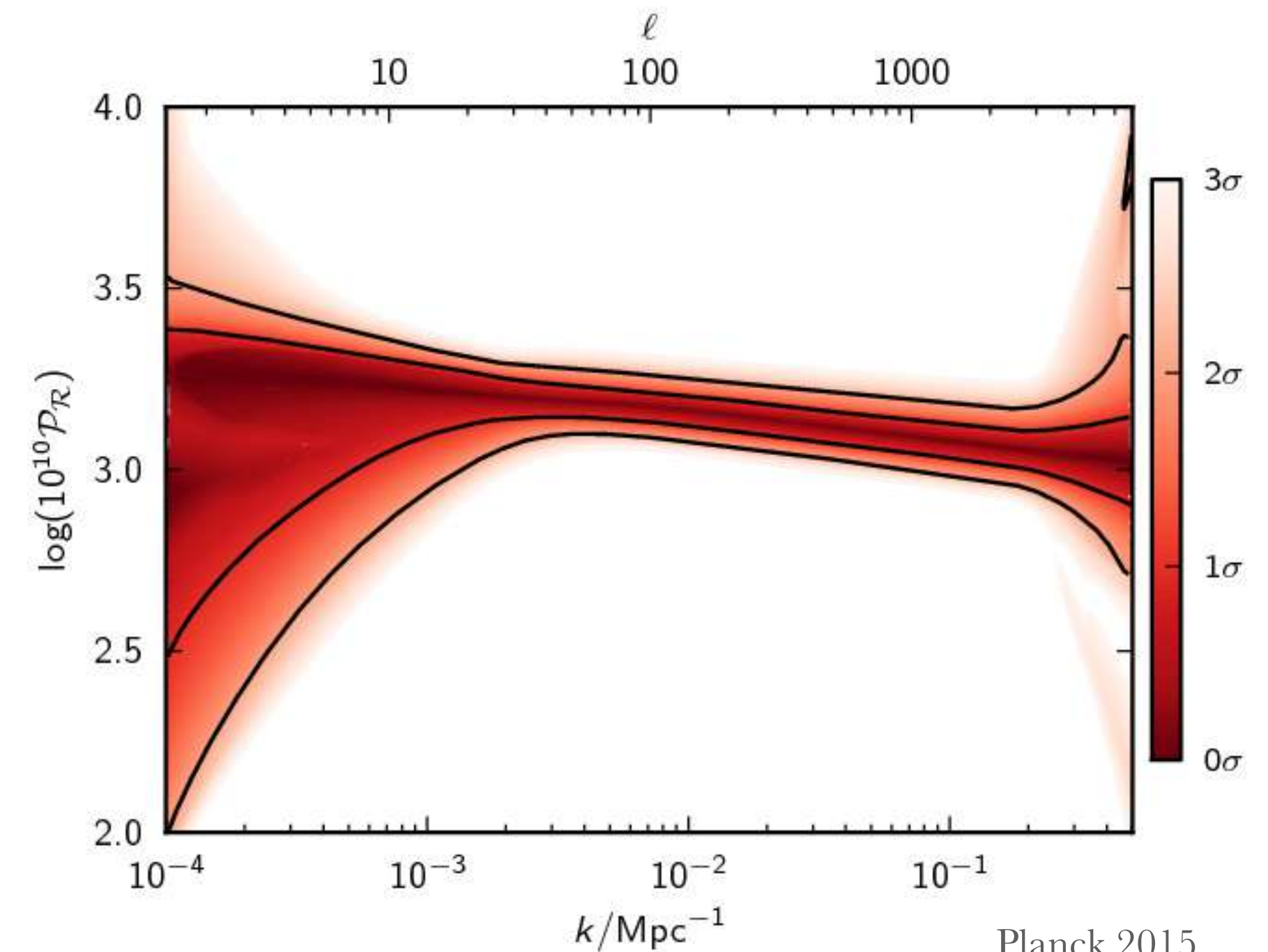
Early universe and the *spectral index*

Prediction from inflation: primordial density fluctuations that are nearly Gaussian and adiabatic and are described by a nearly scale-invariant power spectrum

- Confirmed by observations (CMB and LSS)
- Scale invariant excluded!

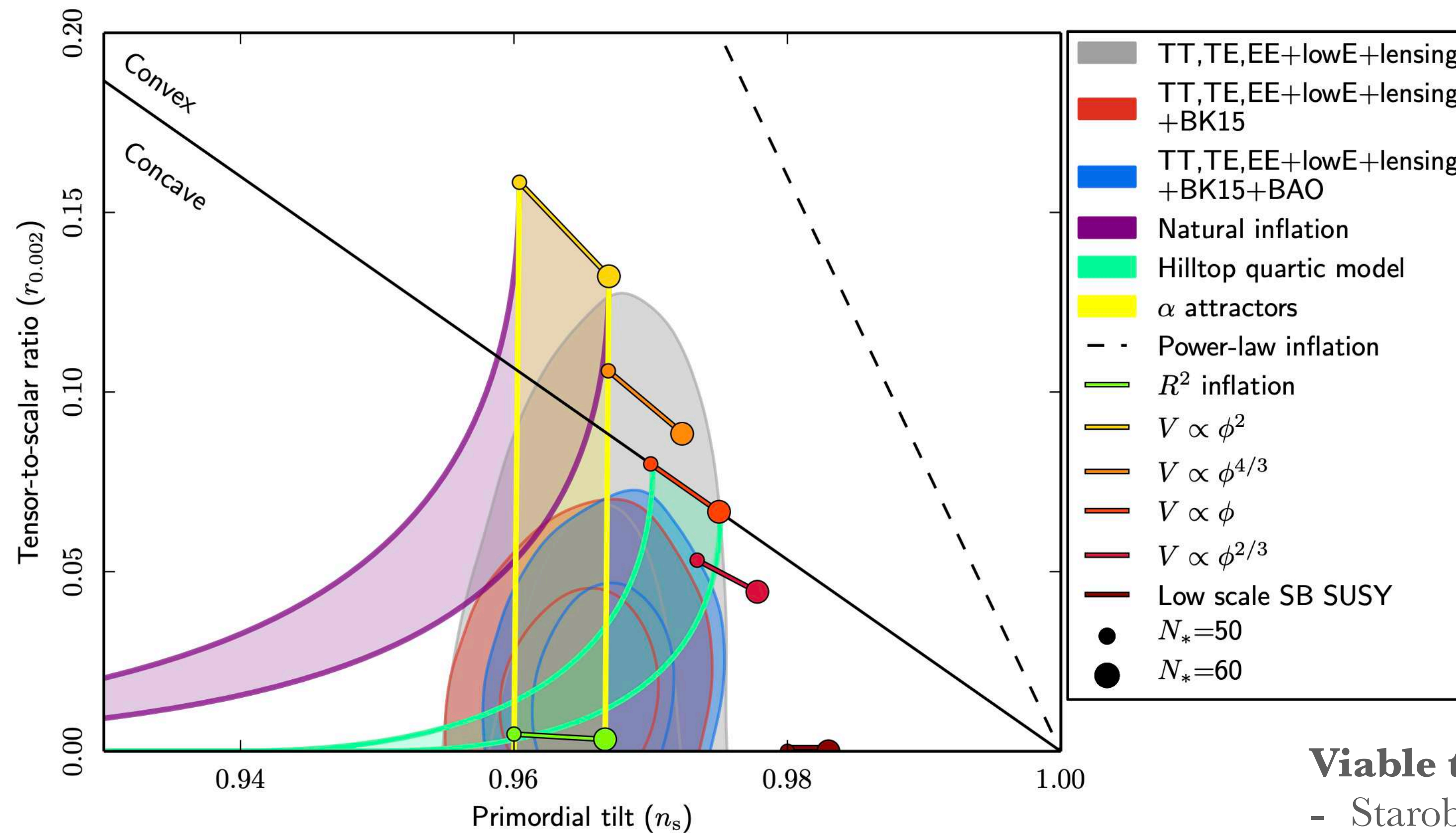
$$P(k) = A_S \left(\frac{k}{k_*} \right)^{n_S - 1}$$

Value of n_s : great discriminator of inflationary (and early universe) models



Early universe and the *spectral index*

Current status:



Planck 2018

$$n_s = 0.9626 \pm 0.0057 \quad (68\% \text{ CL, Planck TT+lowE})$$

$$n_s = 0.9649 \pm 0.0044 \quad \tau = 0.052 \pm 0.008$$

(68 % CL, Planck TT,TE,EE+lowE)

Planck 2018 + lensing

$$n_s = 0.9634 \pm 0.0048$$

(68 % CL, Planck TT+lowE+lensing),

$$n_s = 0.9649 \pm 0.0042$$

(68 % CL, Planck TT,TE,EE+lowE+lensing).

Planck 2018 + lensing + BAO (BOSS)

$$n_s = 0.9665 \pm 0.0038$$

(68 % CL, Planck TT,TE,EE+lowE+lensing+BAO).

A_s and n_s determined with great precision ($< 1\%$)!

Viable targets (Planck/BICEP/Keck/ACT/SPT data):

- Starobinsky model ,
- Higgs inflation and
- α -attractors (exponential - T and E α -attractor)
- Polynomial, ...

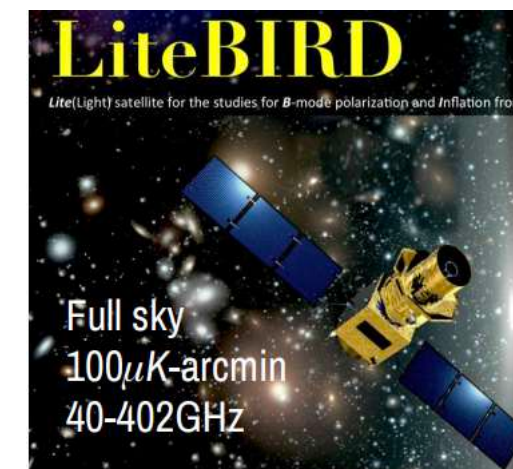
Early universe and the *spectral index*

Given that we determined A_s and n_s with great precision, ...

Next steps for early universe cosmology:

- Primordial gravitational waves
- Non-Gaussianity
- Features in the power spectrum
- New phenomenology

Main goal of the next generation of experiments:



- + LSS experiments
- + GW wxperiments
- + New probes

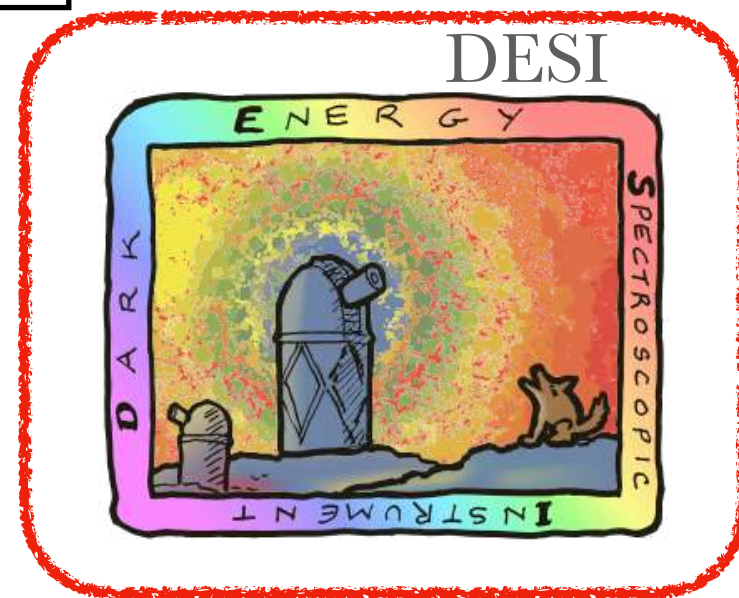
BUT...

In 2025, n_s comes back to the light and a new discrepancy arises

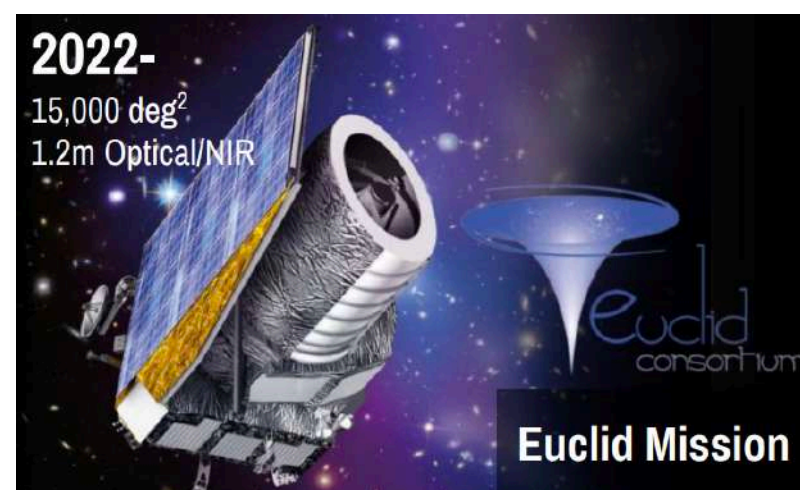
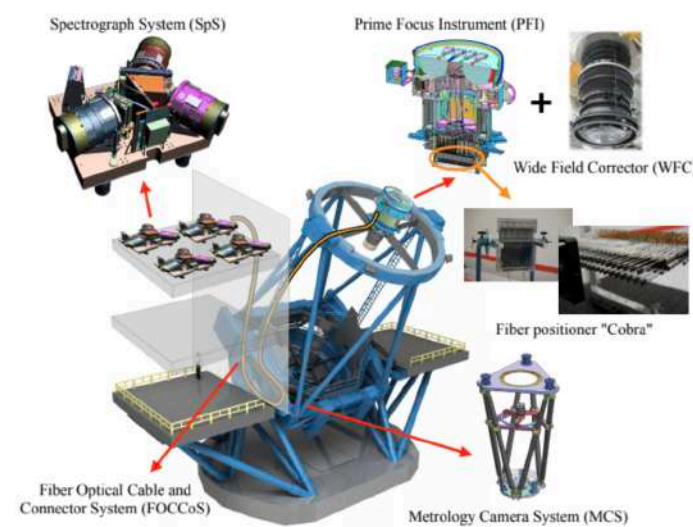
Observations - Stage 3 and 4

From the ones we already have results, we already learned a lot of new things!!!

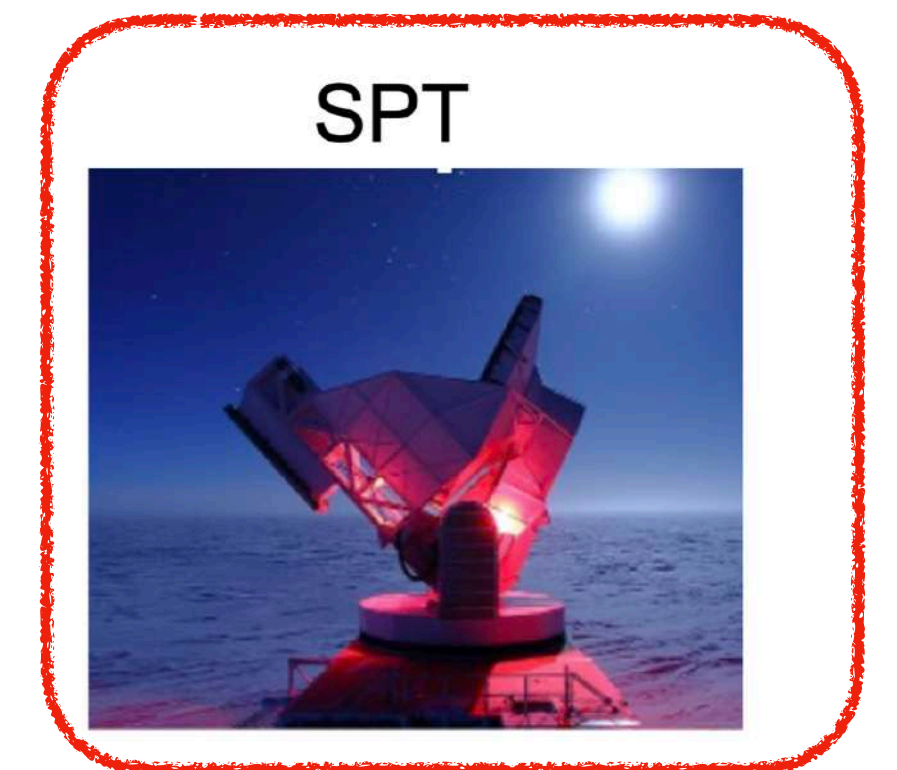
Photometric and spectroscopic surveys



Prime Focus Spectrograph (PFS)



CMB



GWs



CMB-S4

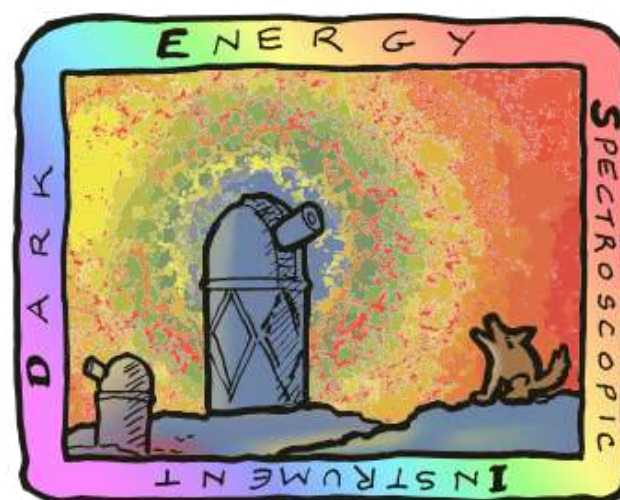
Next Generation CMB Experiment

21cm



The Dark Energy Spectroscopic Instrument (DESI)





The Dark Energy Spectroscopic Instrument - *DESI*

DESI galaxy samples: 40M galaxy redshifts in 5 years of observations

QSO: 3M (SDSS: 500k)

Lya $1.8 < z$

Tracers $0.8 < z < 2.1$

ELG: 16M (SDSS: 200k)

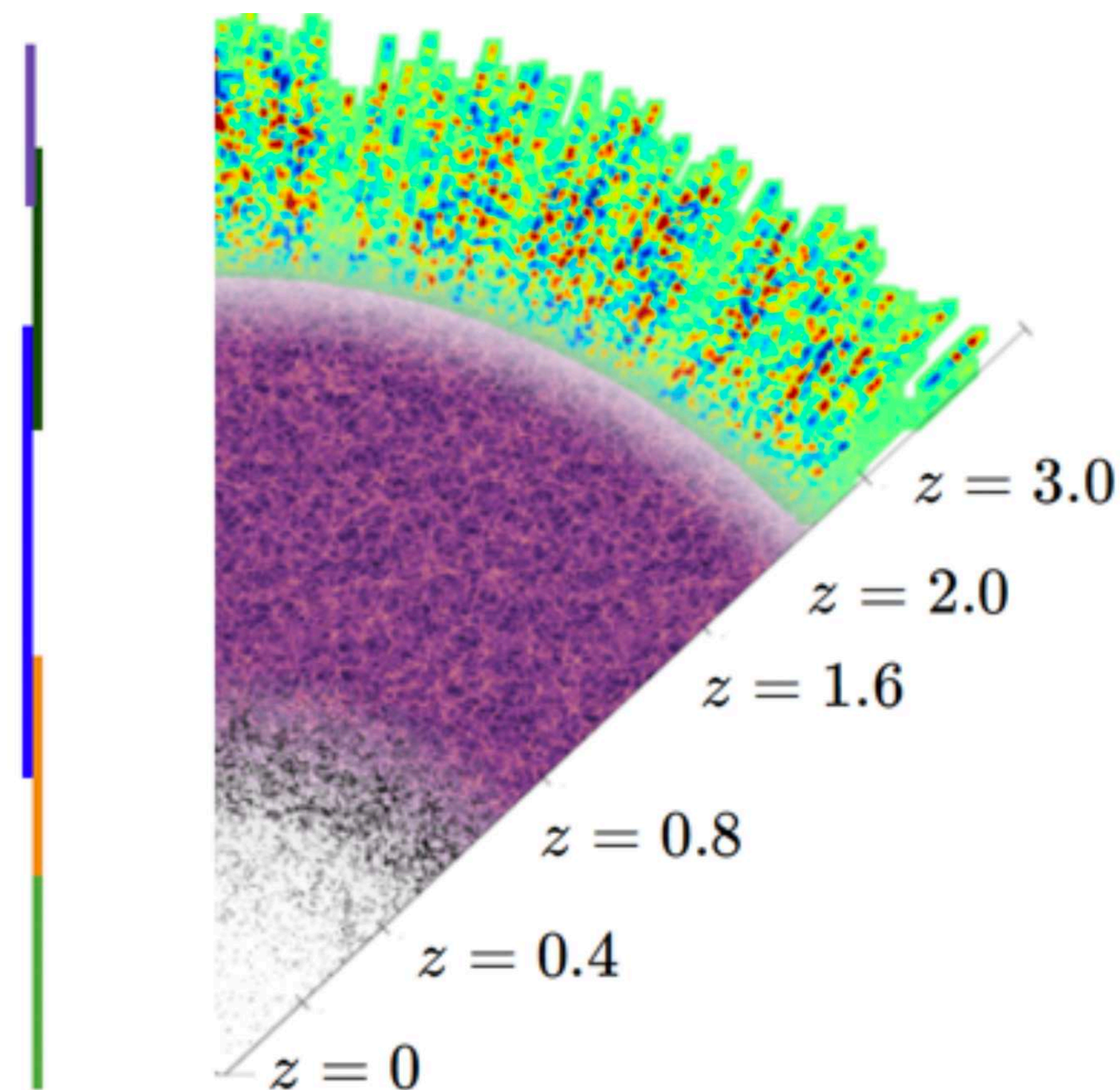
$0.6 < z < 1.6$

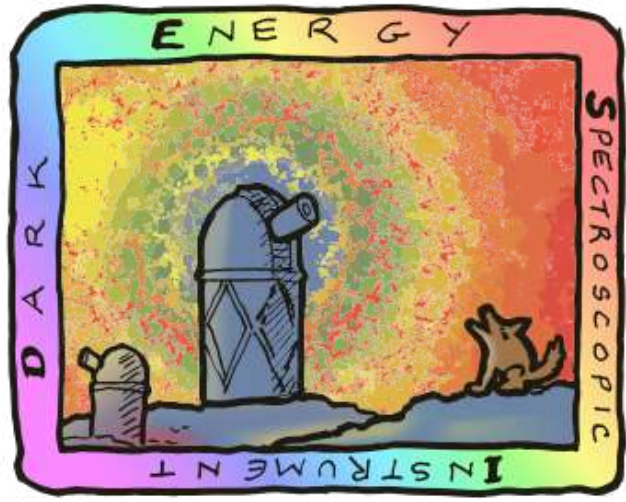
LRG: 8M (SDSS: 1M)

$0.4 < z < 0.8$

Bright Galaxies: 14M
(SDSS: 600k)

$0 < z < 0.4$





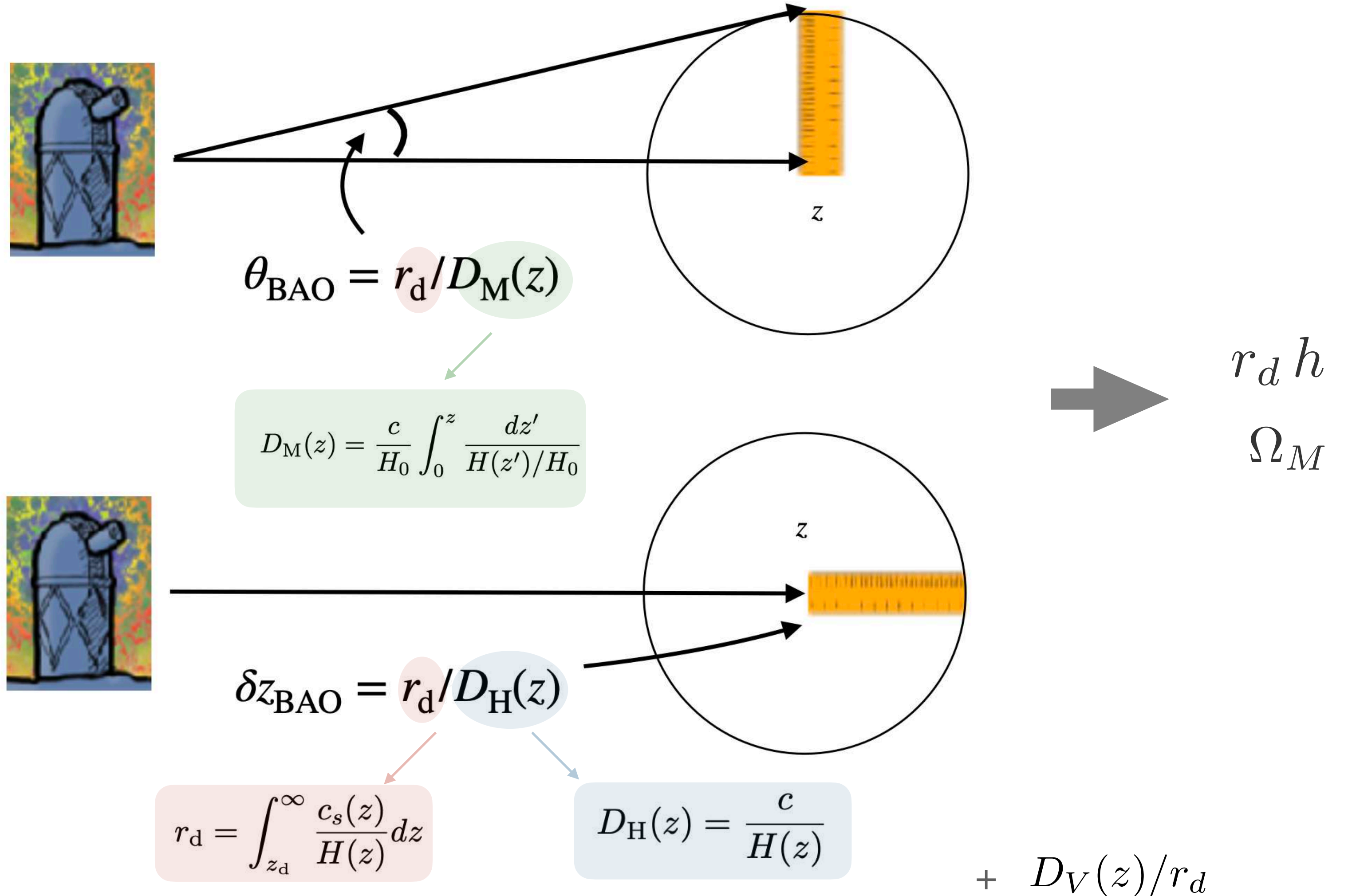
The Dark Energy Spectroscopic Instrument - *DESI*

What DESI BAO measures

$$\omega_X = \Omega_X h^2$$



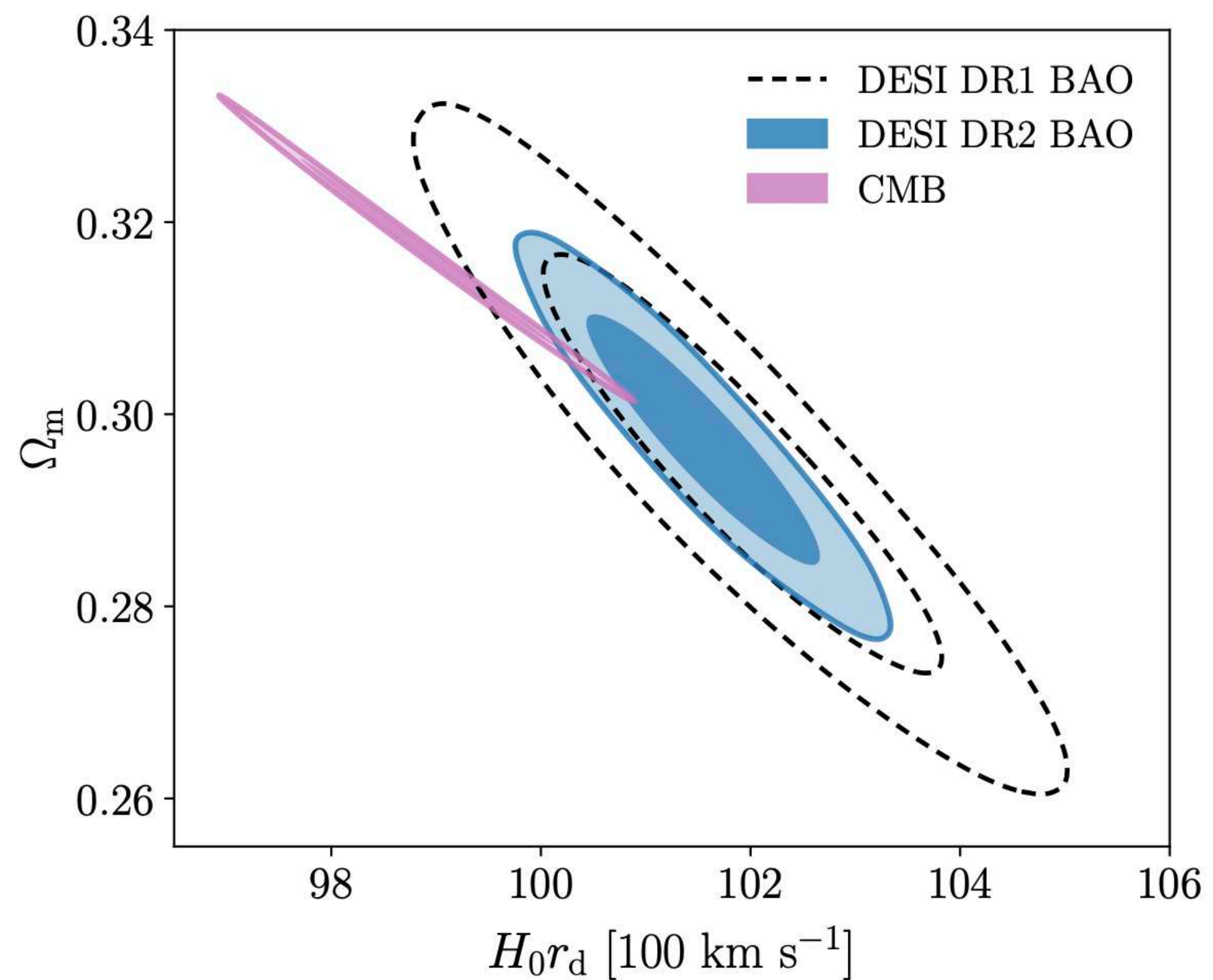
$$r_d = 147.05 \text{ Mpc} \times \left(\frac{\omega_b}{0.02236}\right)^{-0.13} \left(\frac{\omega_{bc}}{0.1432}\right)^{-0.23} \left(\frac{N_{\text{eff}}}{3.04}\right)^{-0.1}$$





The Dark Energy Spectroscopic Instrument - *DESI*

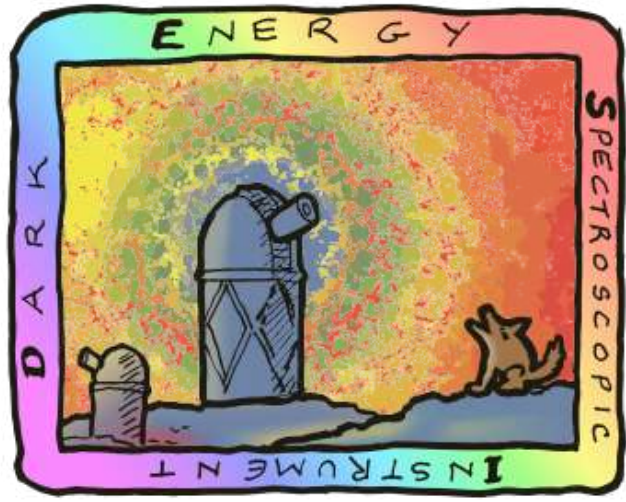
Results: LCDM



- DR2: 40% Improvement in the precision on and compared to DR1

$$\left. \begin{aligned} \Omega_m &= 0.295 \pm 0.015, \\ r_d h &= (101.8 \pm 1.3) \text{ Mpc}, \end{aligned} \right\} \text{ DESI BAO}$$

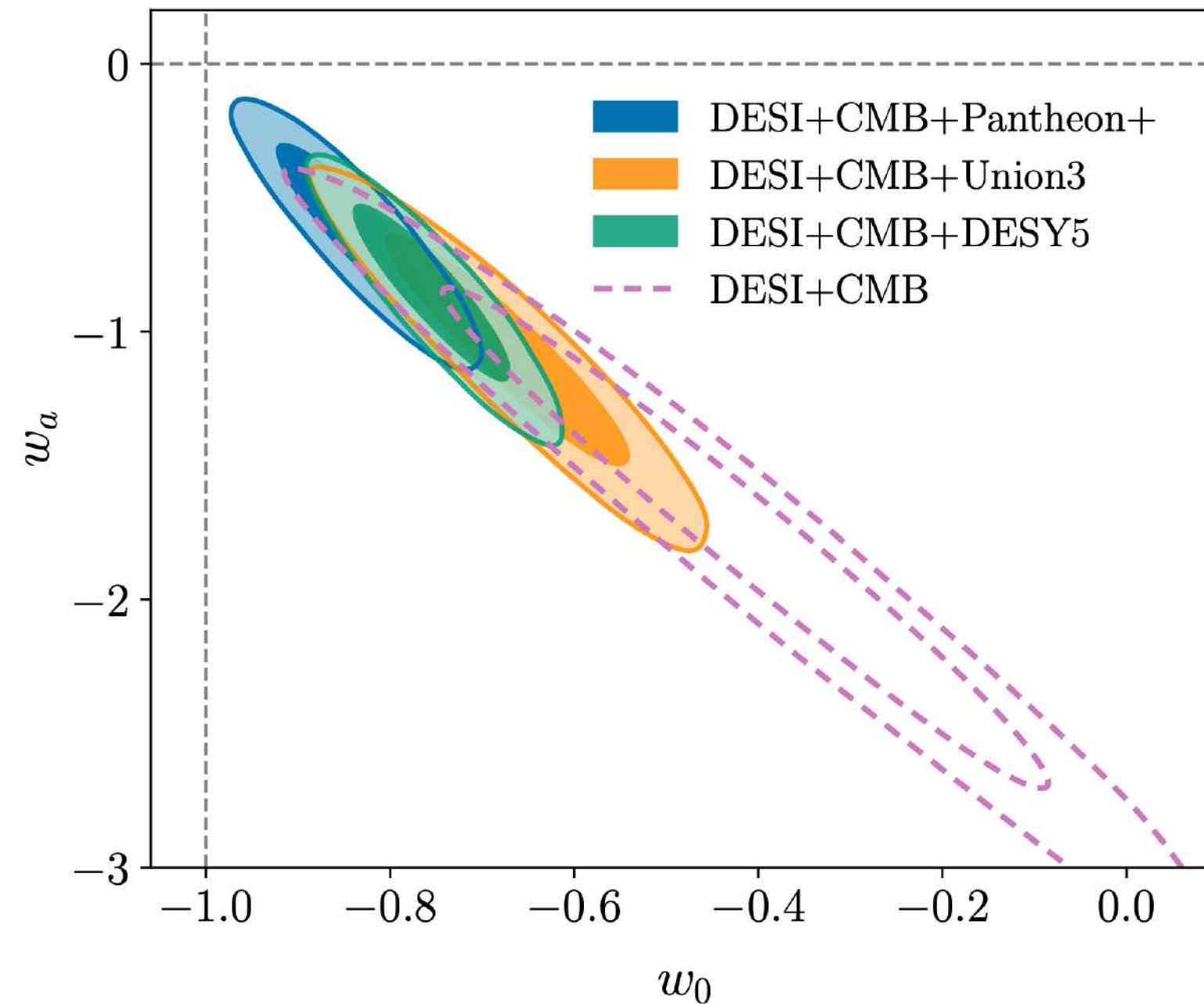
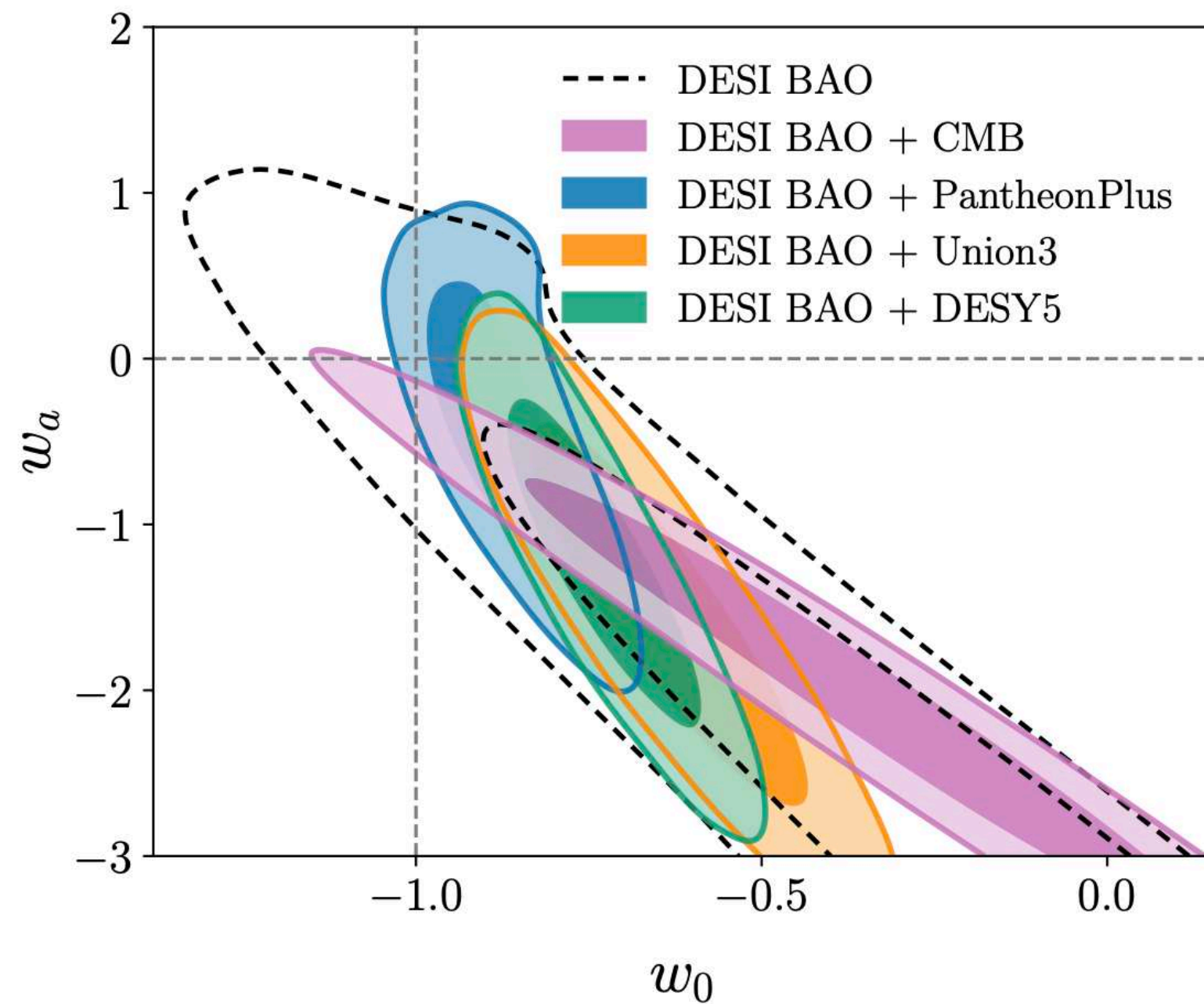
- Constraint: $\sim 2.3\sigma$ away from *Planck* (*DR1*: $\sim 1.9\sigma$)



The Dark Energy Spectroscopic Instrument - *DESI*

Results: $w_0 w_a$ model

DR1



- DESI alone cannot constrain w_0 and w_a

$$\left. \begin{array}{l} w_0 = -0.48^{+0.35}_{-0.17} \\ w_a < -1.34 \end{array} \right\} \begin{array}{l} \text{DESI BAO,} \\ \text{DR2} \end{array}$$

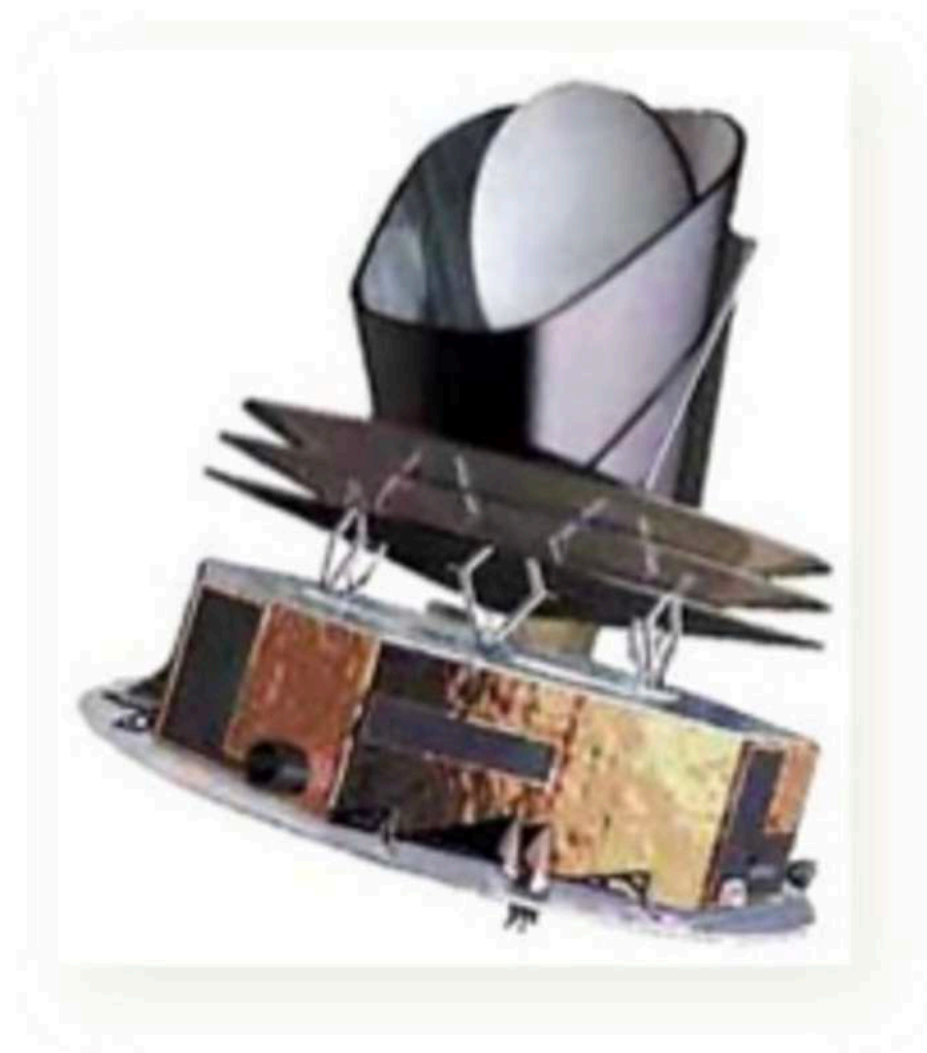
- Combining with other data sets, hint for dynamical DE

$$\left. \begin{array}{l} w_0 = -0.42 \pm 0.21 \\ w_a = -1.75 \pm 0.58 \end{array} \right\} \begin{array}{l} \text{DESI+CMB} \\ \text{DR2} \end{array}$$

$$\left. \begin{array}{l} w_0 = -0.838 \pm 0.055 \\ w_a = -0.62^{+0.22}_{-0.19} \end{array} \right\} \begin{array}{l} \text{DESI+CMB+} \\ \text{Pantheon+,} \end{array}$$

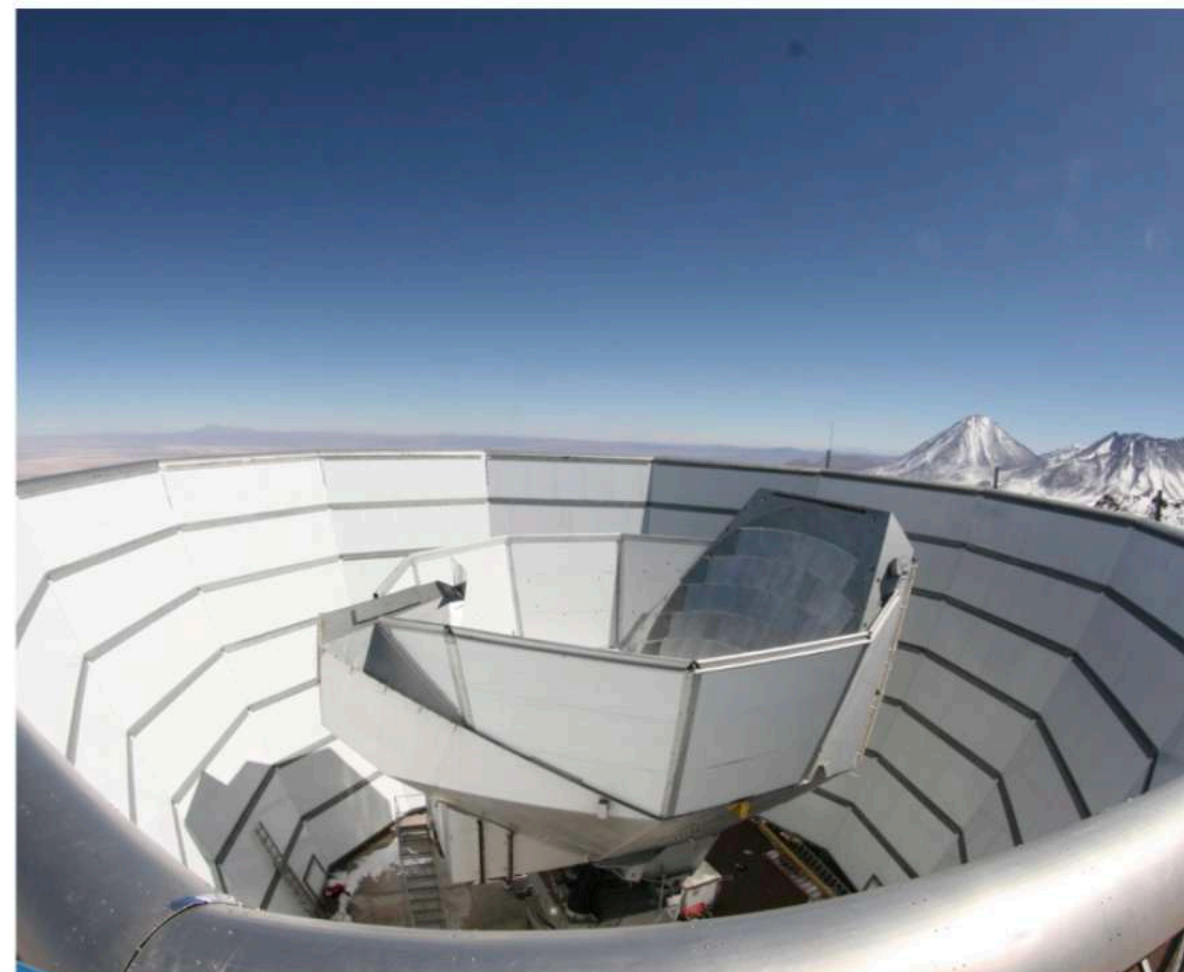
Stage 3 CMB experiments

Planck



Final release **2018**
 $f_{\text{sky}}=70\%$ for science
5-10 arcmin resolution
9 bands from 30-857 GHz

ACT



Observations **2008-2022**
 $f_{\text{sky}}\sim 30\%$ for science
5x Planck resolution
 $\frac{1}{3}$ Planck white noise RMS
Precise polarization measurements

March 2025- DR6 Biggest data improvement since Planck!

SPT



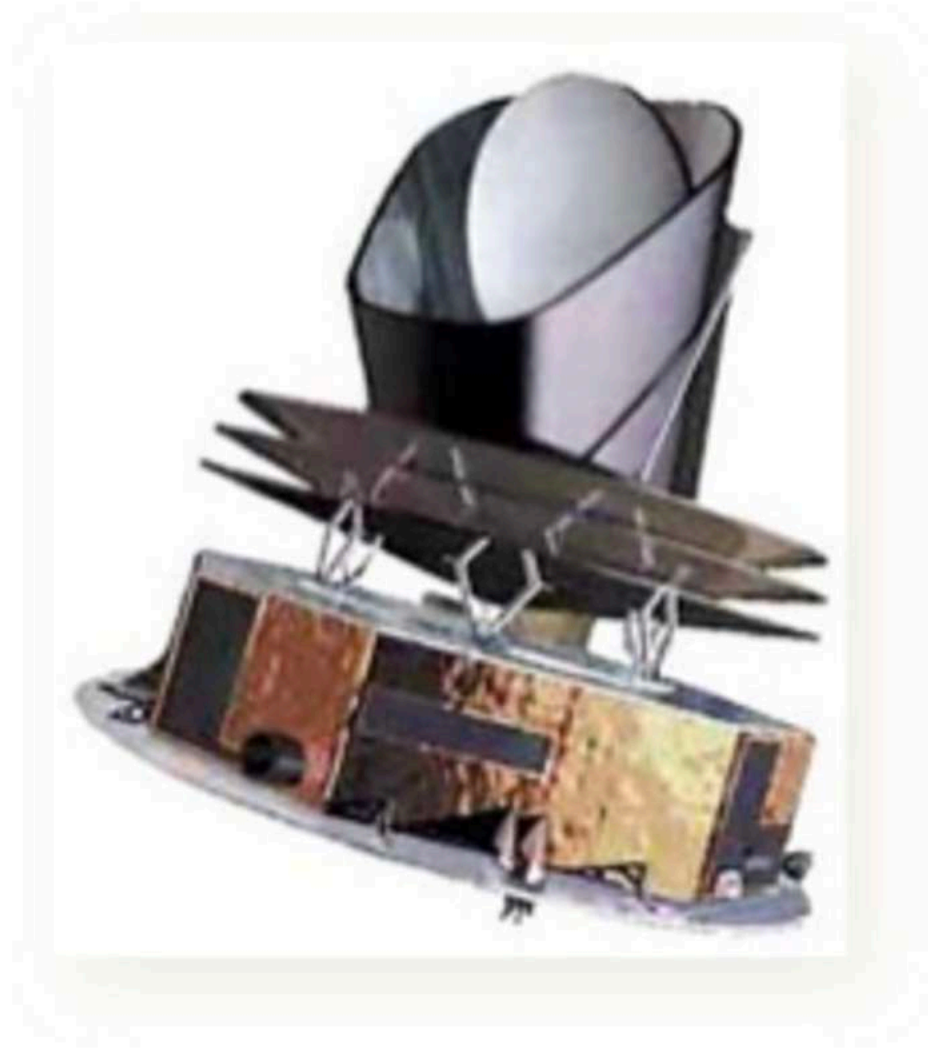
Observations **2007-2024(27)**
 $f_{\text{sky}}\sim 4-10\%$ for science
Noise 6-17x lower than Planck
1-1.6 arcmin resolution
90, 150, 220 GHz



Atacama Cosmology Telescope (ACT)

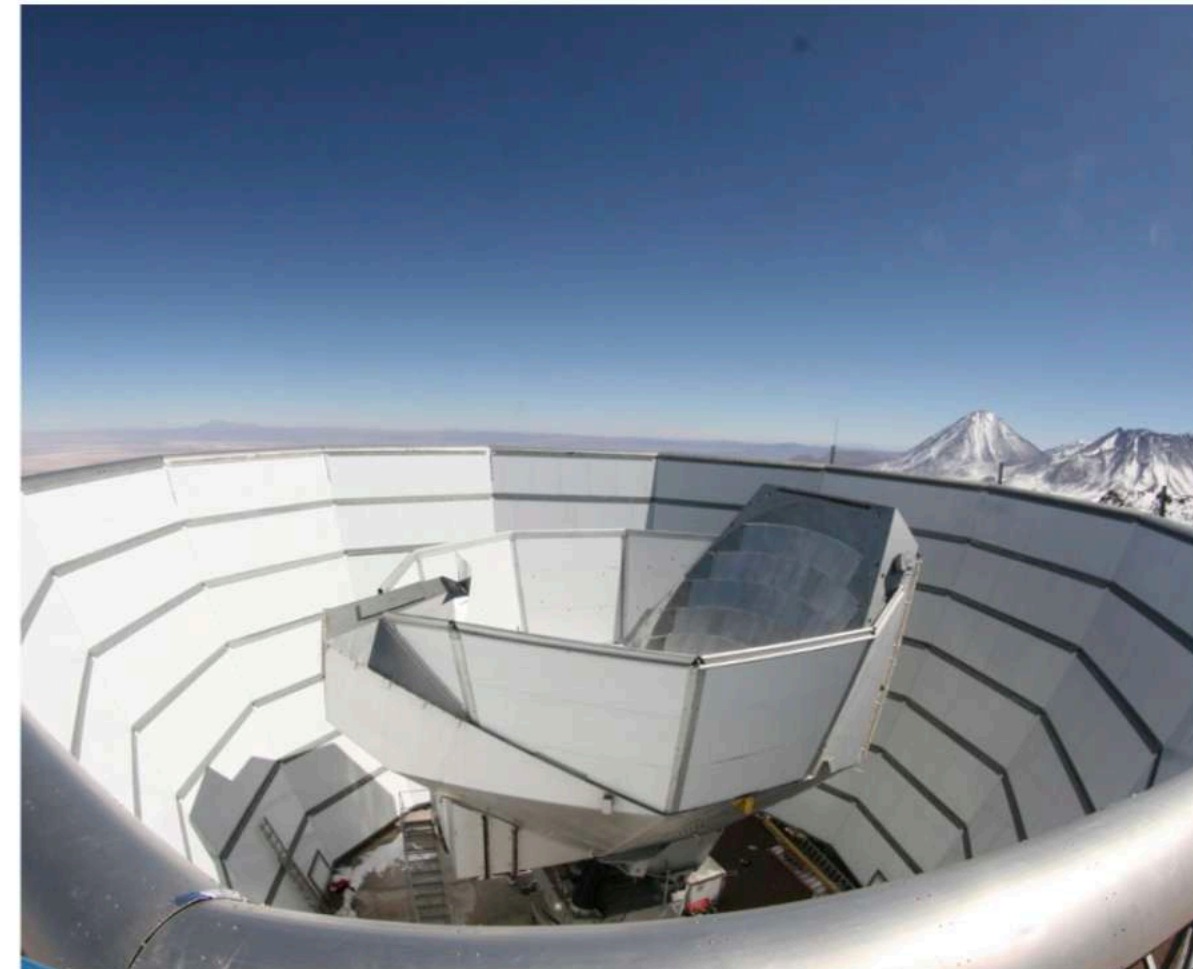
Atacama Cosmology Telescope - *ACT*

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Noise 3-6x lower than Planck
1-2 arcmin resolution
98, 150, 220 GHz (+30, 40 GHz)

- 5x Planck resolution
- $\frac{1}{3}$ Planck white noise RMS
- Small scales!!
- Precise polarization measurements

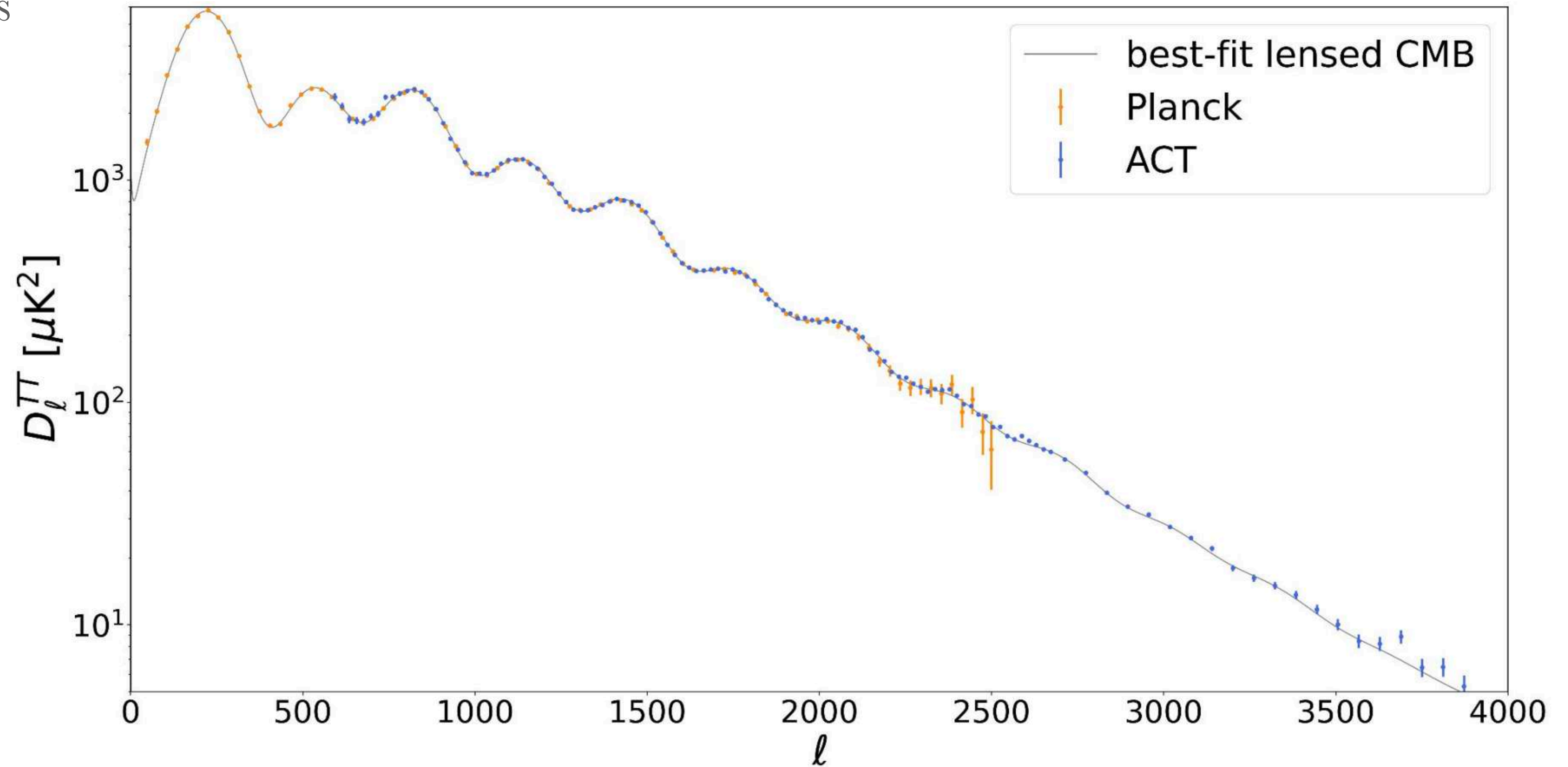


March 2025- DR6

Biggest data
improvement since
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Atacama Cosmology Telescope - *ACT*

What ACT measures

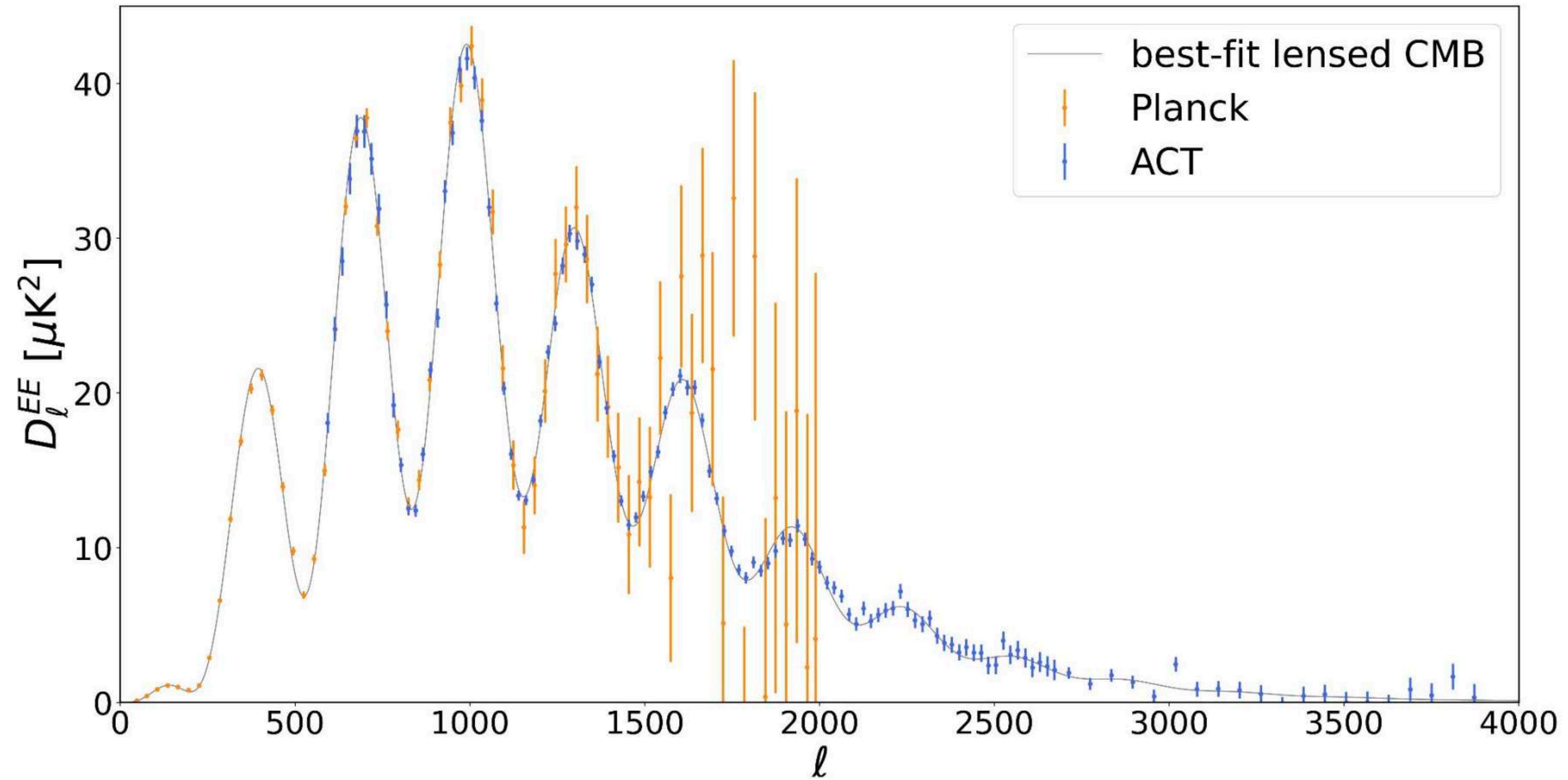


ACT data

- 1) Extend Planck measurement to small angular scales
- 2) Consistent with Planck data on overlapping angular scales

Atacama Cosmology Telescope - *ACT*

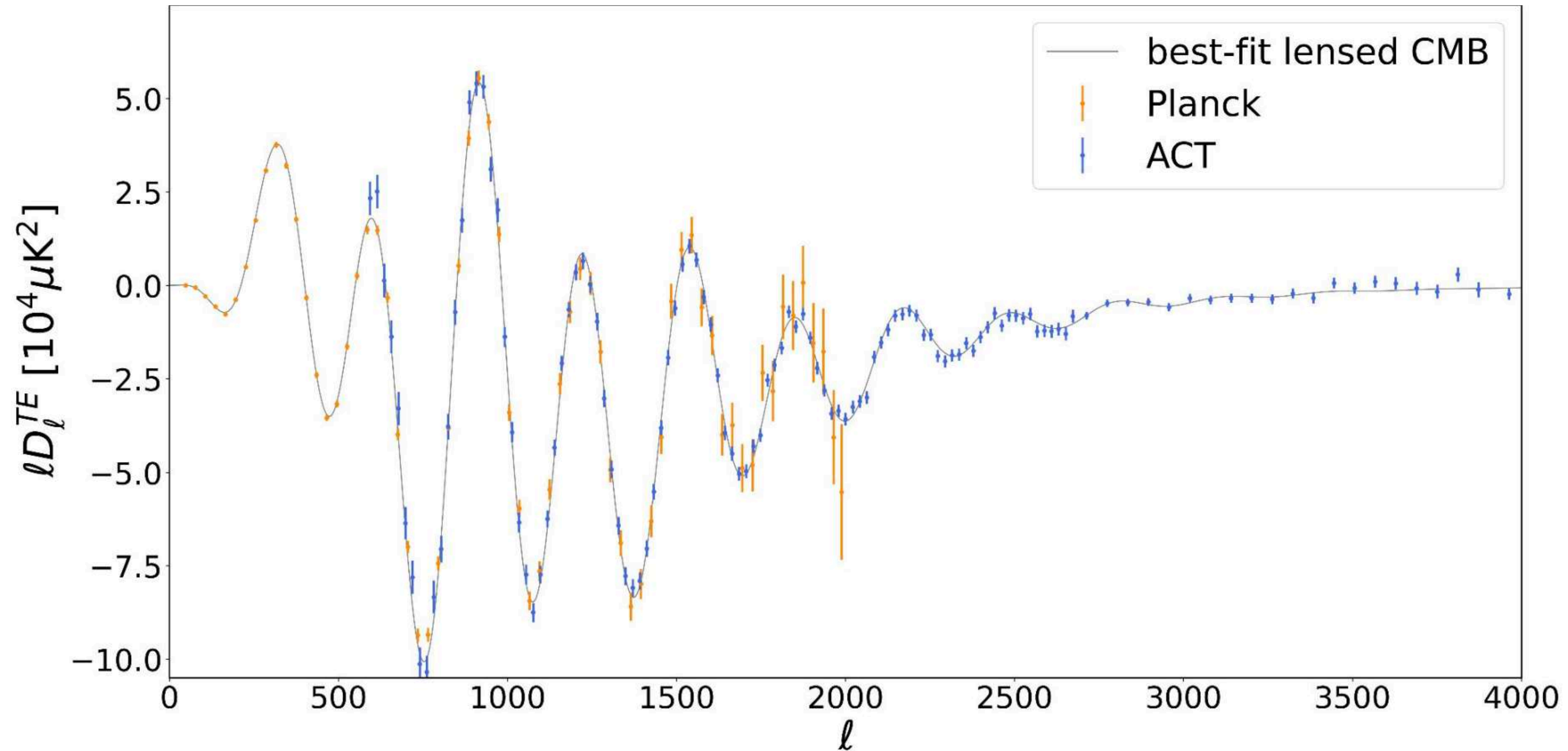
What ACT measures



ACT DR6 EE is more sensitive than Planck for multipoles $l > 600$

Atacama Cosmology Telescope - *ACT*

What ACT measures



1.1 % constraint on H_0 from TE alone

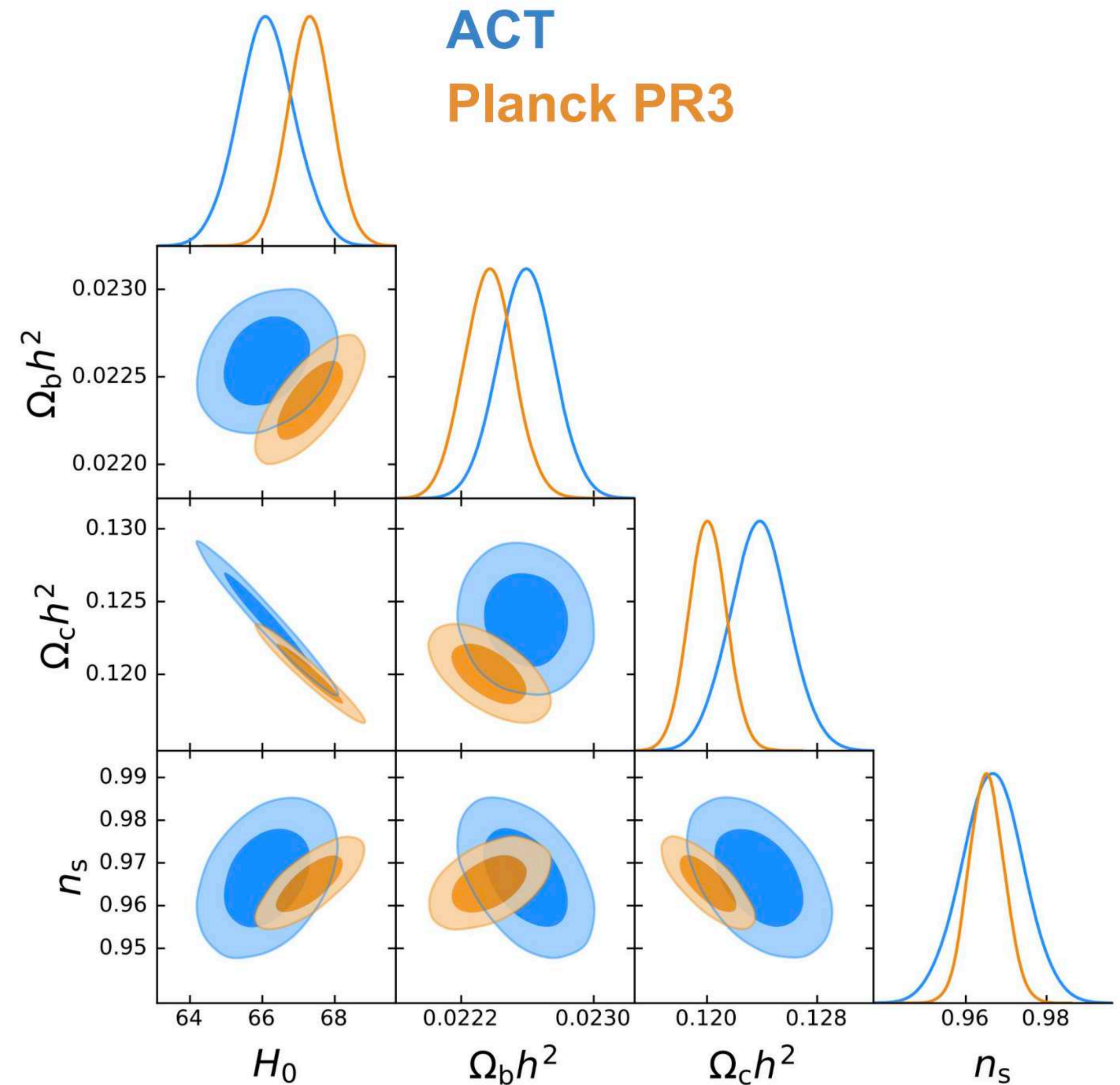
Atacama Cosmology Telescope

- ACT

LCDM

Concordance with Planck (PR3): 1.6σ

	ACT
<i>Parameter</i>	
<i>Sampled</i>	
θ_{MC}	0.0104056 ± 0.0000031 ..
$\Omega_b h^2$	0.02259 ± 0.00017
$\Omega_c h^2$	0.1238 ± 0.0021
$\log(10^{10} A_s)$	3.053 ± 0.013
n_s	0.9666 ± 0.0077
τ	$0.0562^{+0.0053}_{-0.0063}$
<i>Derived</i>	
H_0	66.11 ± 0.79
Ω_m	0.337 ± 0.013
σ_8	0.8263 ± 0.0074



*How collaborations assess the compatibility between **data sets***

Ex.: ACT and Planck

- Calibration and transfer-function checks:

ACT maps were cross-correlated with Planck to measure scale-dependent calibration and power-loss. After applying conservative ℓ -cuts and updated calibration factors, ACT and Planck agree at the ≈ 0.3 % level.

- Power-spectrum consistency tests:

ACT \times ACT, ACT \times Planck, and Planck \times Planck TT/TE/EE spectra were compared on the same sky region. Residuals and PTEs show no significant discrepancies.

- Checks for residual tilts or scale dependence:

Ratios of ACT and Planck spectra to best-fit Λ CDM models show no significant relative tilt or scale-dependent mismatch once the transfer-function cuts are applied.

- Parameter-shift tests:

ACT-only and Planck-only posteriors overlap well. A single Λ CDM model fits both datasets with high goodness of fit, indicating statistical compatibility.

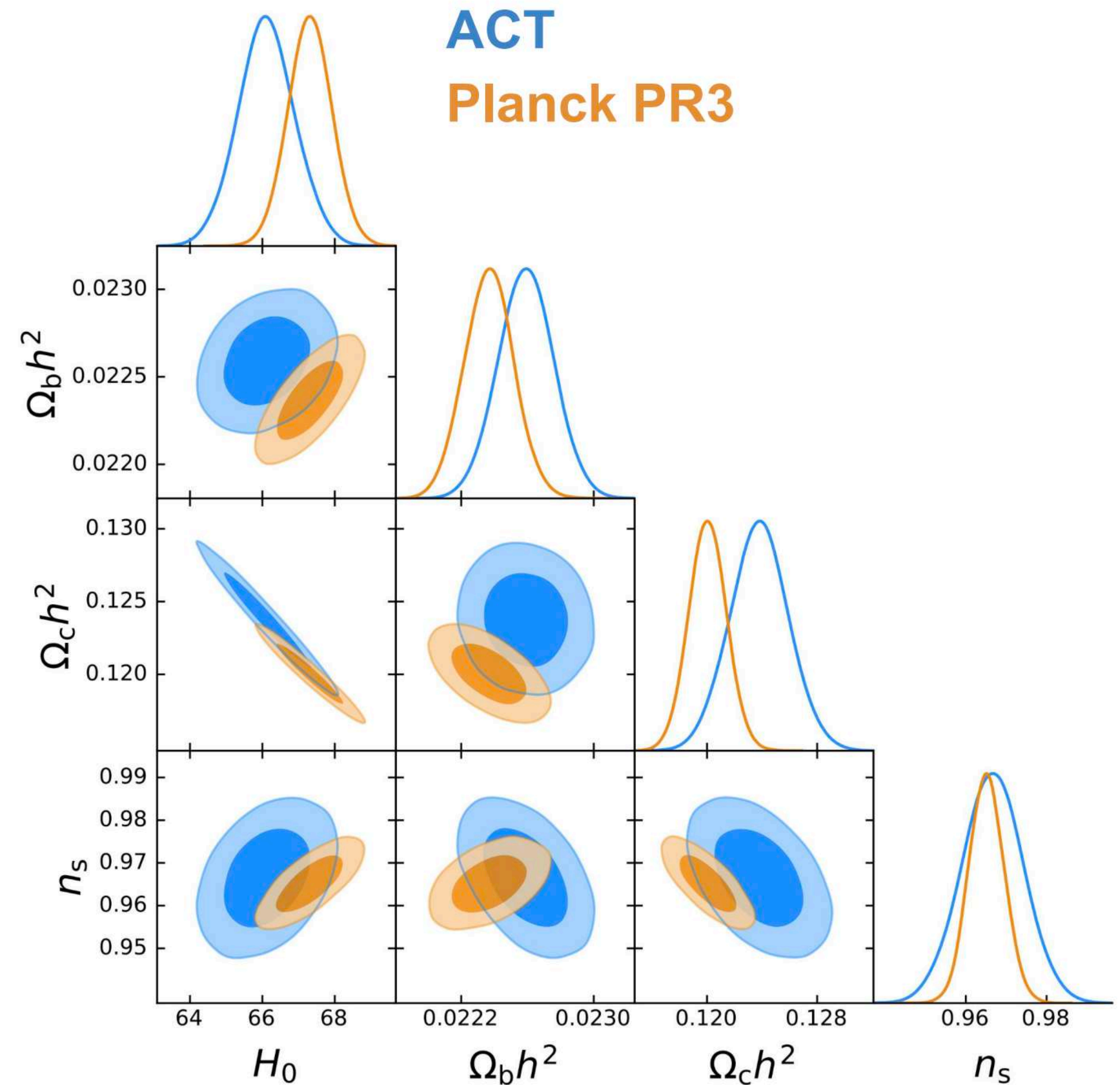
Atacama Cosmology Telescope

- ACT

LCDM

Concordance with Planck (PR3): 1.6σ

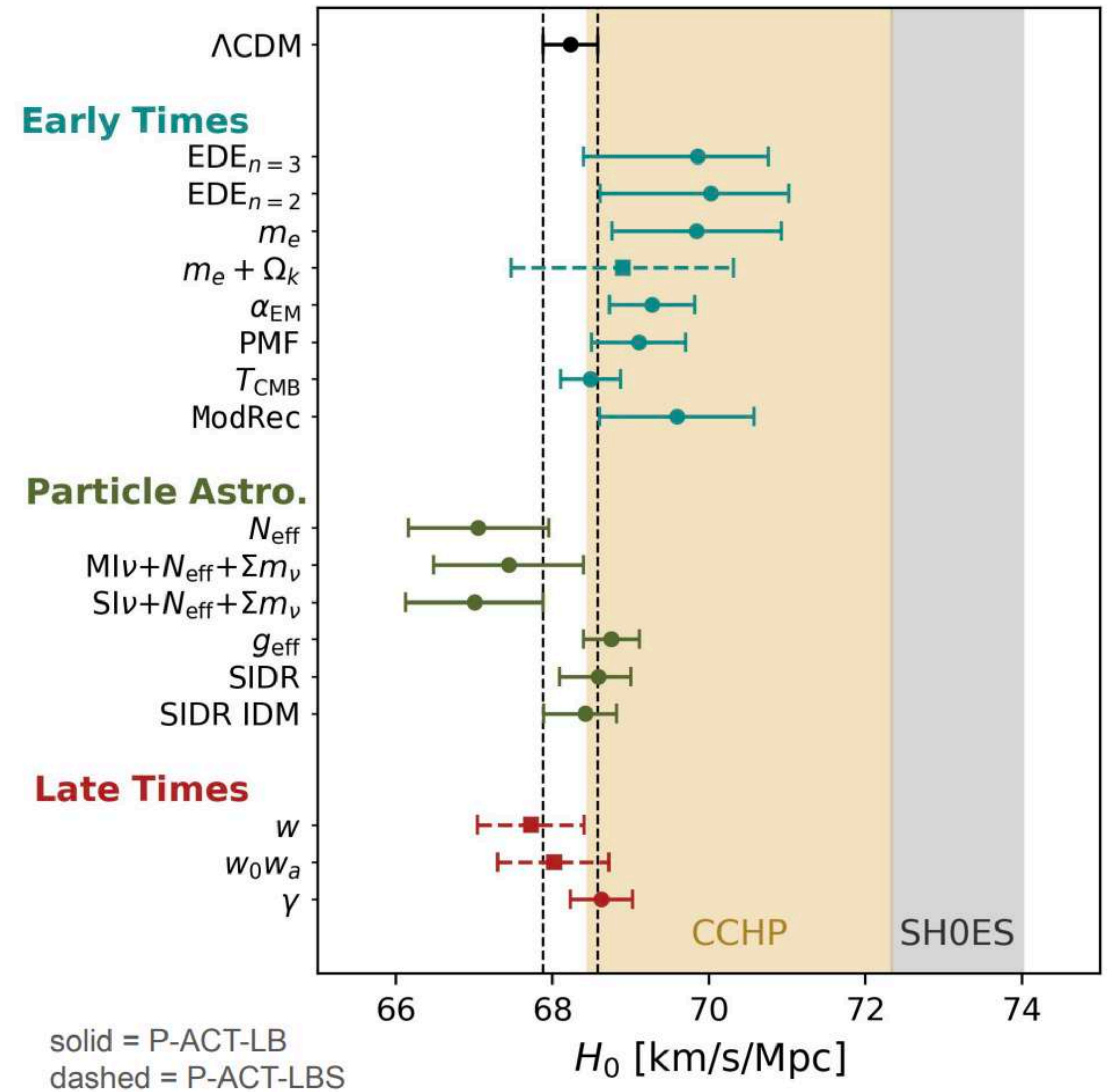
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Ω_m	0.337 ± 0.013
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Atacama Cosmology Telescope - ACT

Extensions to LCDM:

More than 30 extension of LCDM tested:
no preference over LCDM!



South Pole Telescope - SPT3G



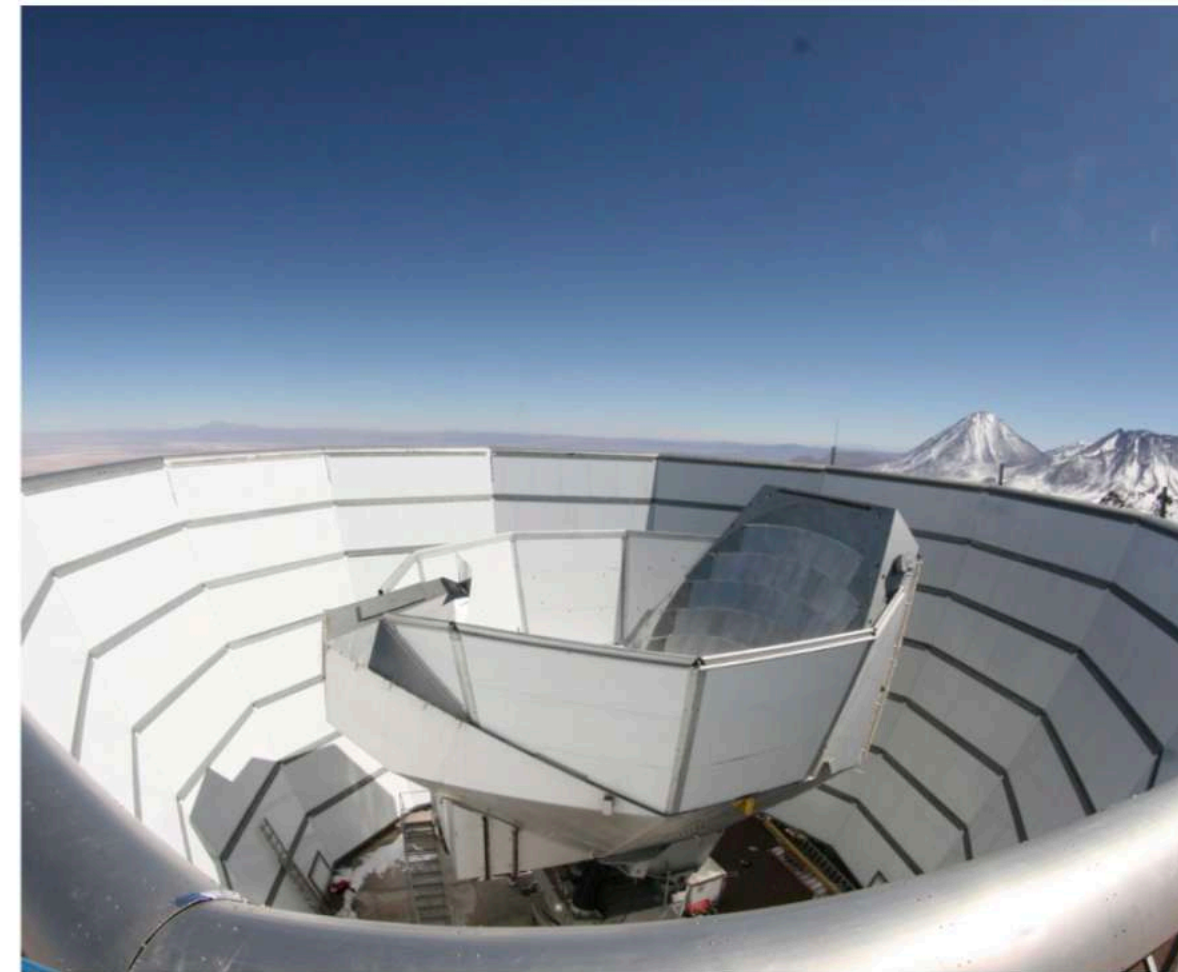
Stage 3 *CMB* experiments

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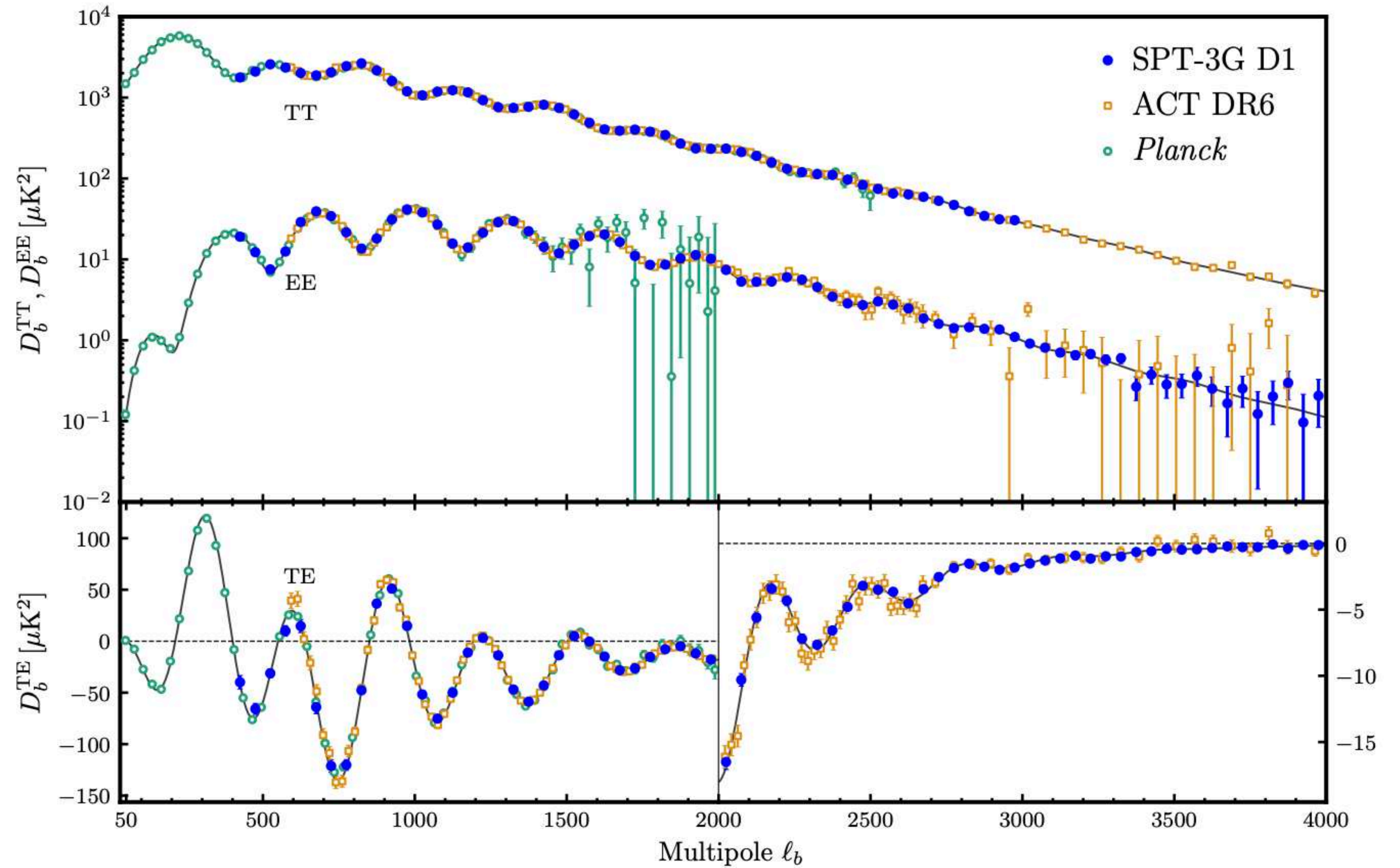
SPT



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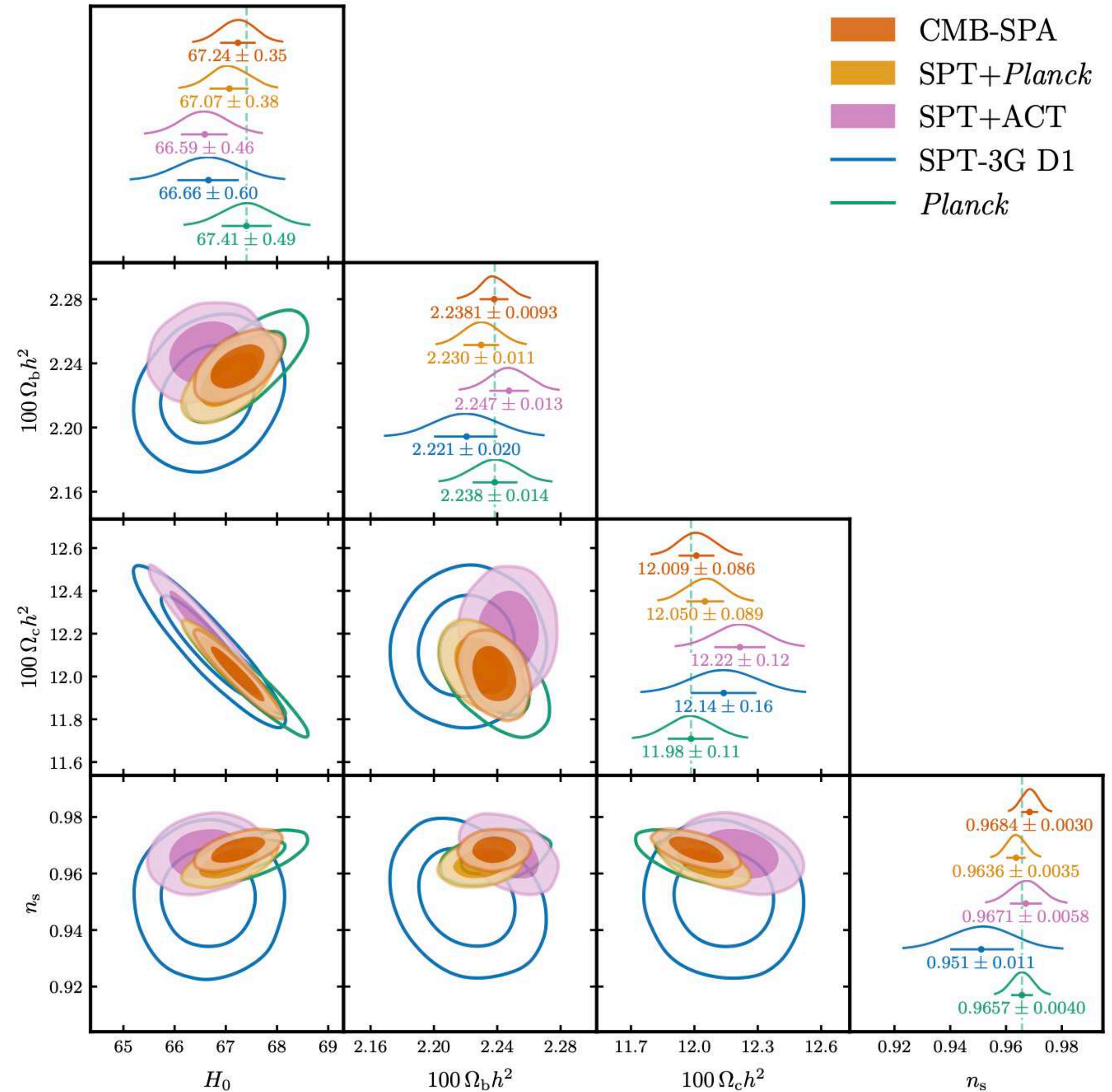
South Pole Telescope - *SPT3G*

June 2025: Latest data release



South Pole Telescope - *SPT3G*

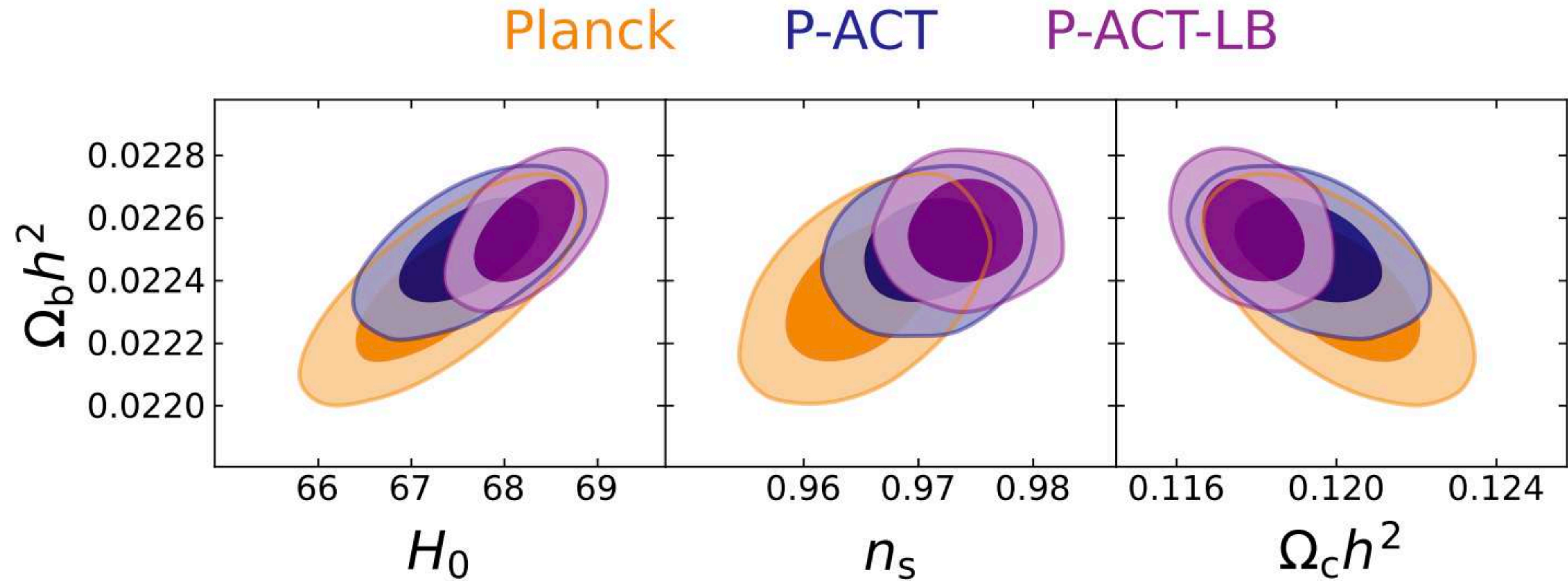
Concordance with Planck and ACT



Next obvious step:

combine these data sets

Combining CMB and *LSS* data



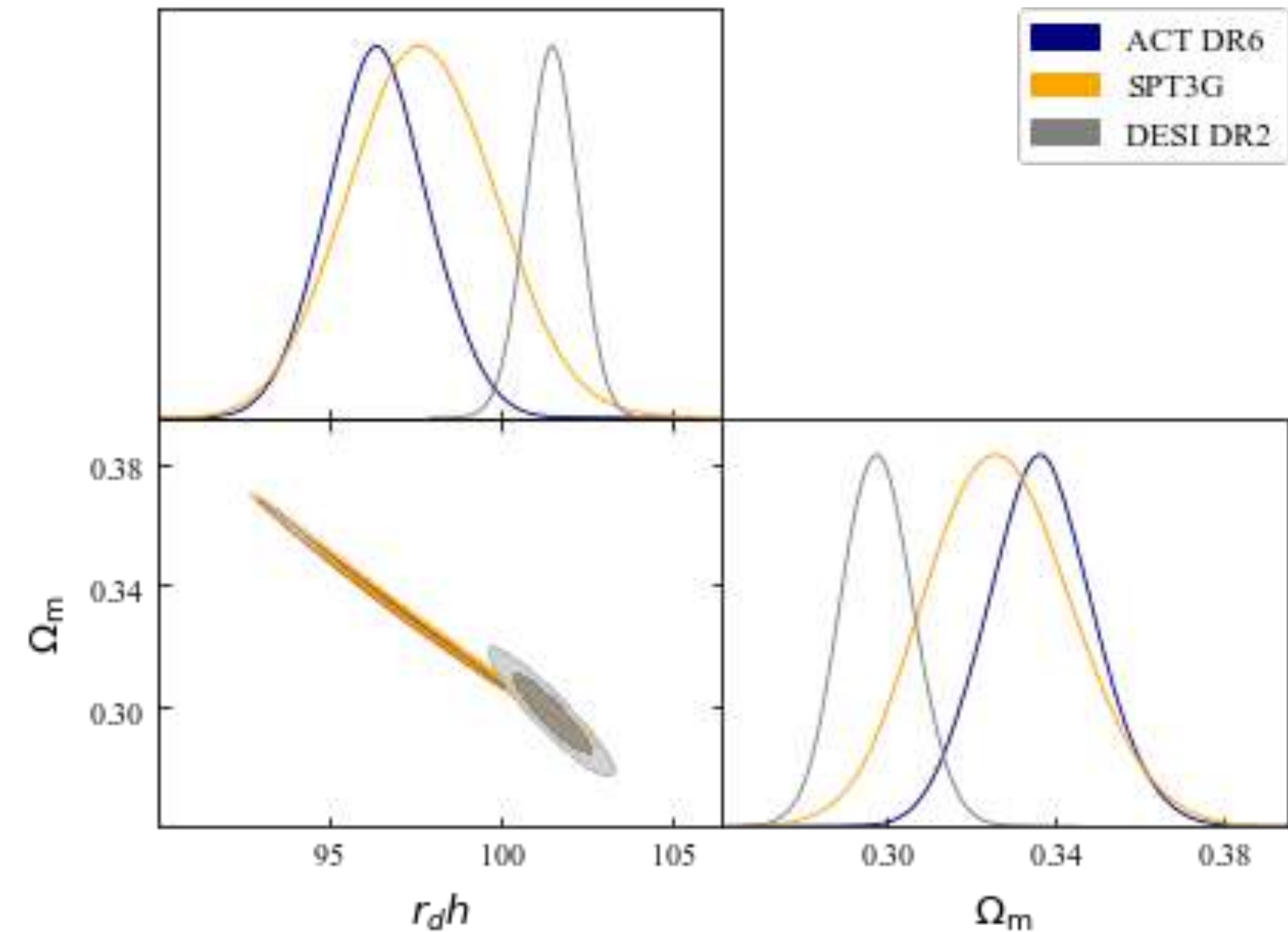
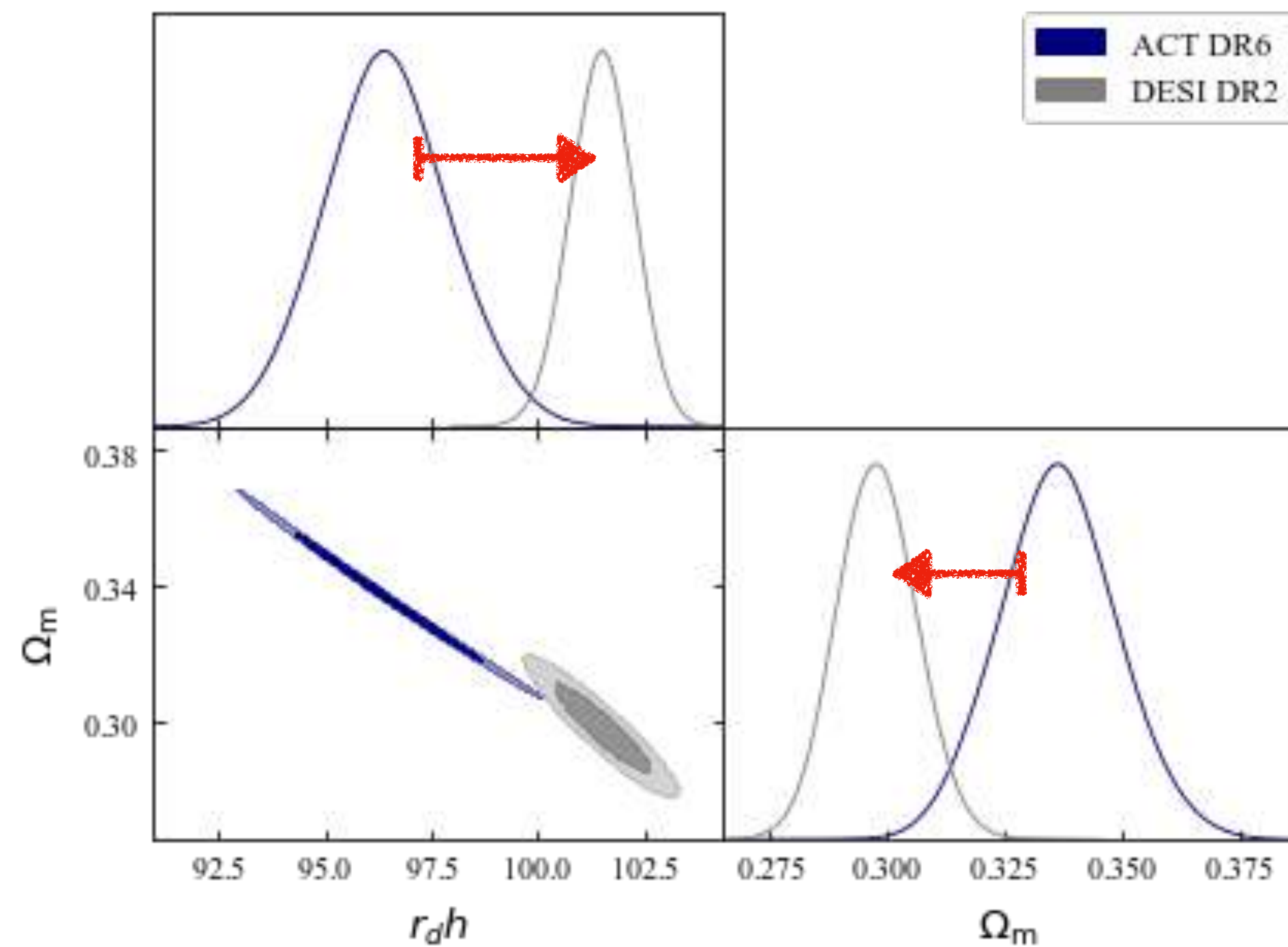
P: Planck
L: ACT lensing
B: BAO from DESI

BAO - CMB tension

The BAO-CMB Tension and Implications for Inflation
EF, E McDonough, L Balkenhol, R Kallosh, L Knox, and A Linde,
2507.12459

Are **ACT** and **SPT** compatible with **DESI** ?

(or BAO-CM tension or Ω_M tension)



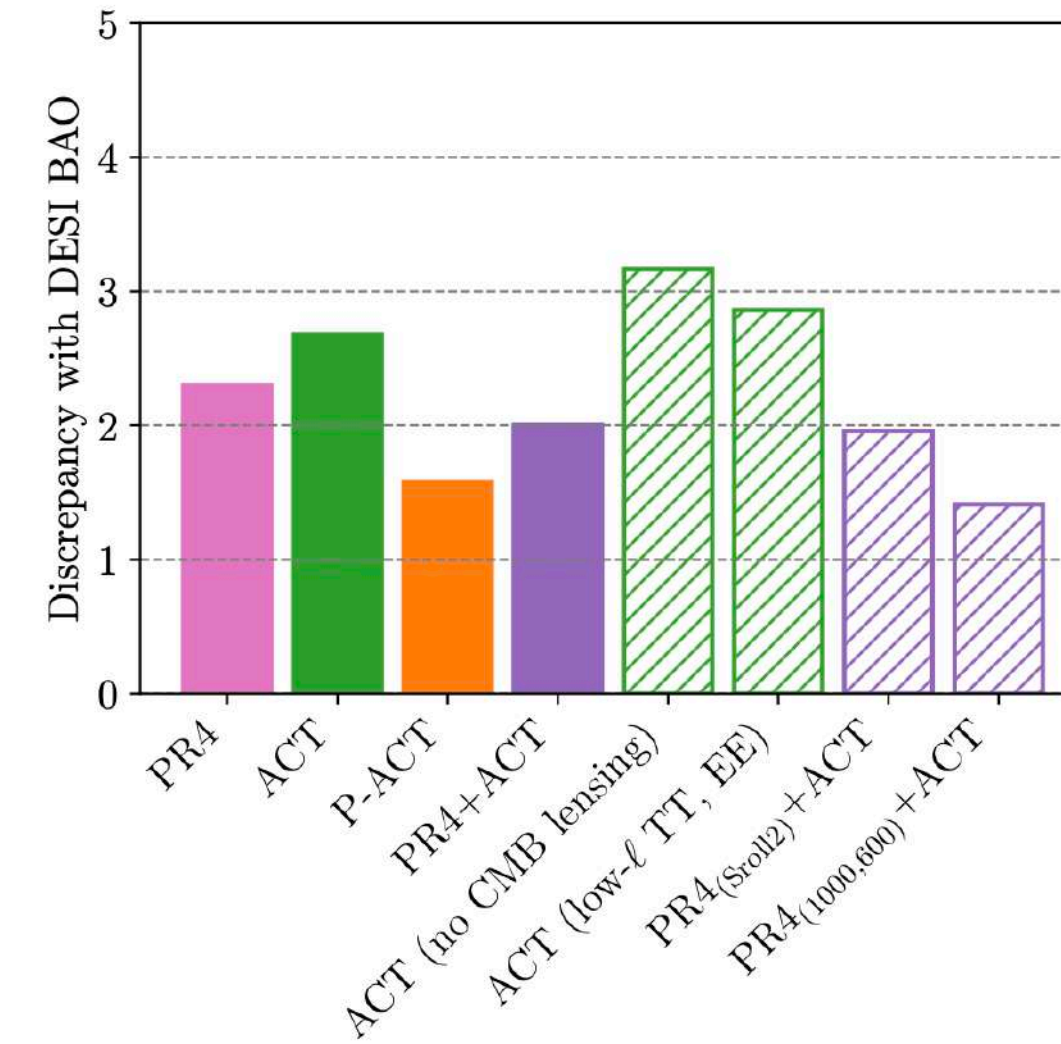
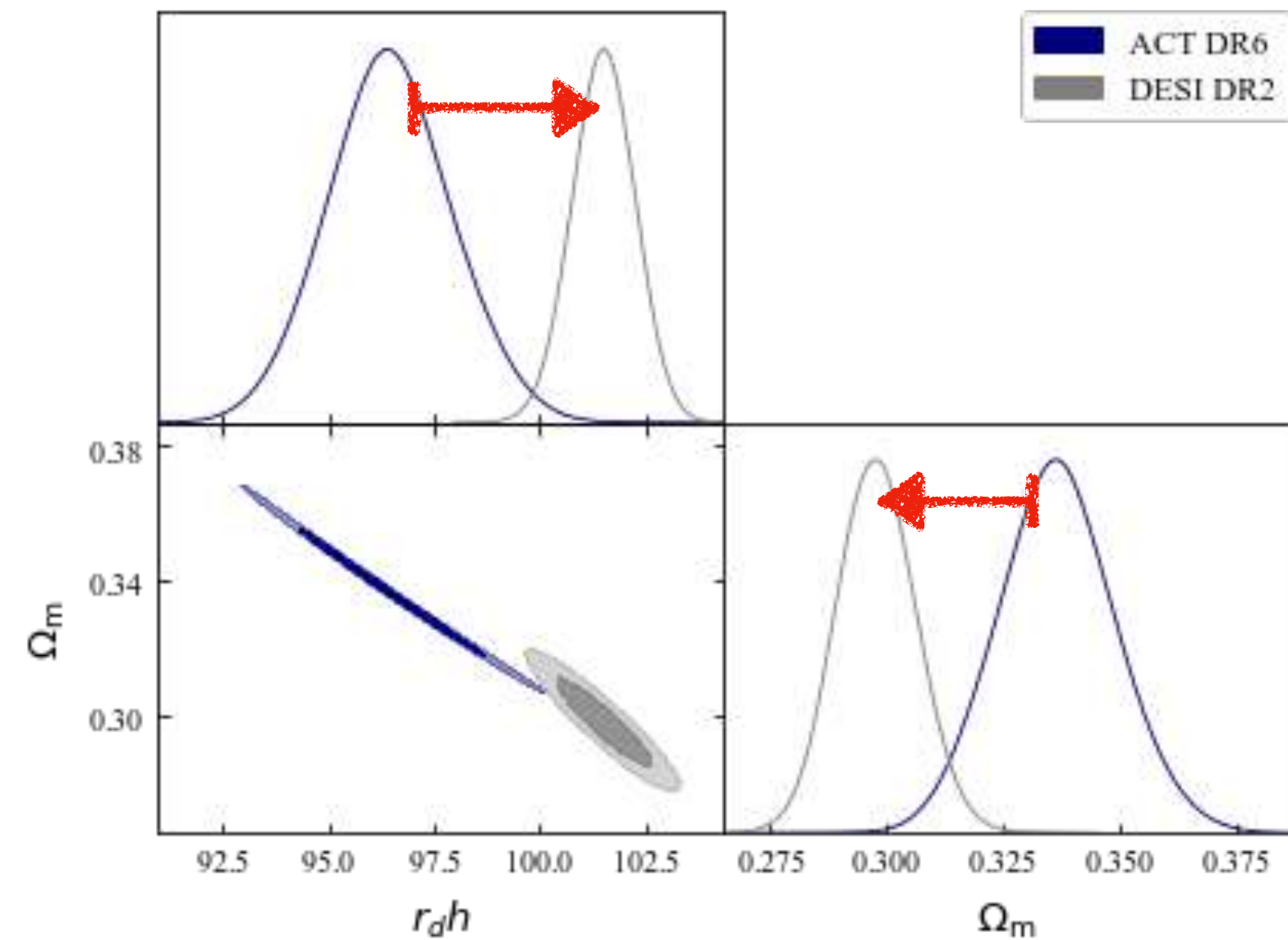
- 2d posteriors show small overlap
- 1d posterior shifted

Discrepancy (or tension) in the values of $r_d h$ and Ω_M measured from CMB and DESI BAO!

BAO - CMB tension

Discrepancy or tension in the values of $r_d h$ and Ω_M measured from CMB and DESI BAO!

The BAO-CMB Tension and Implications for Inflation
 Elisa G.M. Ferreira et al, 2507.12459



Garcia-Quintero et al
 (DESI collaboration) 2024

ACT and SPT present parameter-level offset wrt to DESI !

Translating the differences in the Ω_m and $r_d h$ constraints from CMB and DESI data to equivalent statistical significances for a one-dimensional Gaussian distribution:

	$100 \Omega_m$	$h r_d$ [Mpc]	Distance to DESI
CMB-SPA	31.66 ± 0.50	98.89 ± 0.63	2.8σ
SPT+ACT	32.77 ± 0.72	97.51 ± 0.87	3.7σ
SPT+Planck	31.89 ± 0.54	98.63 ± 0.67	3.0σ
ACT DR6	33.0 ± 1.0	97.2 ± 1.2	3.1σ
SPT-3G D1	32.47 ± 0.91	97.9 ± 1.1	2.5σ
Planck	31.45 ± 0.67	99.18 ± 0.84	2.0σ
DESI	29.76 ± 0.87	101.52 ± 0.73	

2 – 4 σ discrepancy between CMB and BAO datasets

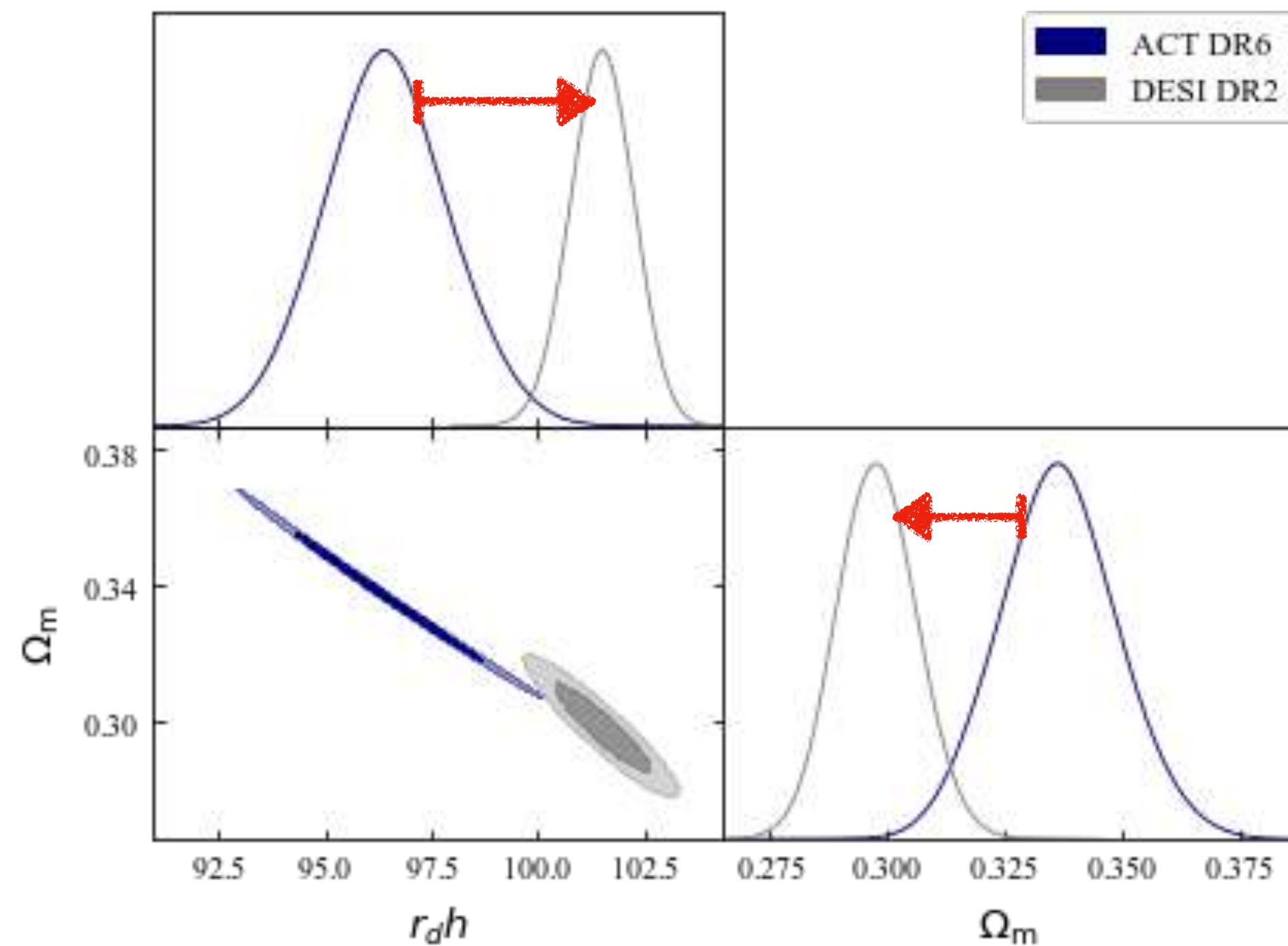
Camphuis et al (SPT collaboration) 2025

⇒ Implication for other cosmological parameters

BAO - CMB discrepancy

The BAO-CMB Tension and Implications for Inflation
 Elisa G.M. Ferreira et al, 2507.12459

Discrepancy or tension in the values of $r_d h$ and Ω_M measured from CMB and DESI BAO!

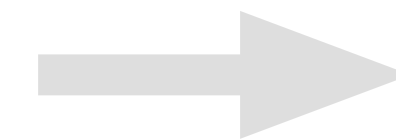


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Camphuis et al (SPT collaboration) 2025

ACT and SPT might not be compatible with DESI!

2 - 4 σ discrepancy between CMB and BAO datasets



Is it even valid to combine them?

SPT collaboration doesn't combine their data with DESI for this reason

Caution required!

How collaborations assess the compatibility between *data sets*

ACT and DESI

- Parameter-shift test or parameter level consistency tests: compare Λ CDM posteriors (ACT vs DESI BAO)
- BAO distance comparison: ACT-inferred distances vs DESI BAO measurements; ACT+Planck fully consistent.
- Cross-correlation study between ACT DR6 CMB-lensing maps and DESI Luminous Red Galaxies (LRGs) (redshift range $\sim 0.4-1.0$) detects the signal at $\sim 38\sigma$ (ACT only) and $\sim 50\sigma$ (ACT + Planck Collaboration PR4) level.

Caveats:

- Parameter-shift test:
 - Shift in the BAO and other parameters
 - Posterior 2d overlap (or one-dimensional posterior)
 - Goodness of fit
- Covariance

PS: Other cosmological parameters might present smaller shift than the BAO ones, which might lead to not noticing this discrepancy.

- The BAO-CMB Tension and Implications for Inflation

Elisa G.M. Ferreira et al, 2025

- Cosmological implications of DESI DR2 BAO measurements in light of the latest ACT DR6 CMB data, DESI collaboration 2025

"While ACT alone exhibits a tension with DESI exceeding 3σ within the Λ CDM model, this discrepancy is reduced when ACT is analyzed in combination with Planck."

- Camphuis et al (SPT collaboration) 2025

"We forego reporting the combination of DESI with SPT+ACT as it does not meet our 3σ consistency requirement; if we were to do so, differences in the favored Ω_m and h_{rd} values would also lead to sizeable shifts in other cosmological parameters in the joint constraints compared to the CMB-preferred values."

"...while we find that the results from SPT-3G, Planck, and ACT DR6 are in excellent agreement and consistent with Λ CDM, we report a growing inconsistency with the BAO results from DESI DR2"

Consequences for *the spectral index n_s*

The BAO-CMB Tension and Implications for Inflation
Elisa G.M. Ferreira et al, 2507.12459

The BAO-CMB tension has important consequences for n_s

Consequences for *the spectral index n_s*

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The BAO-CMB tension has important consequences for n_s

- From the correlation between Ω_m and $r_d h$, and the correlation of each of these with n_s :

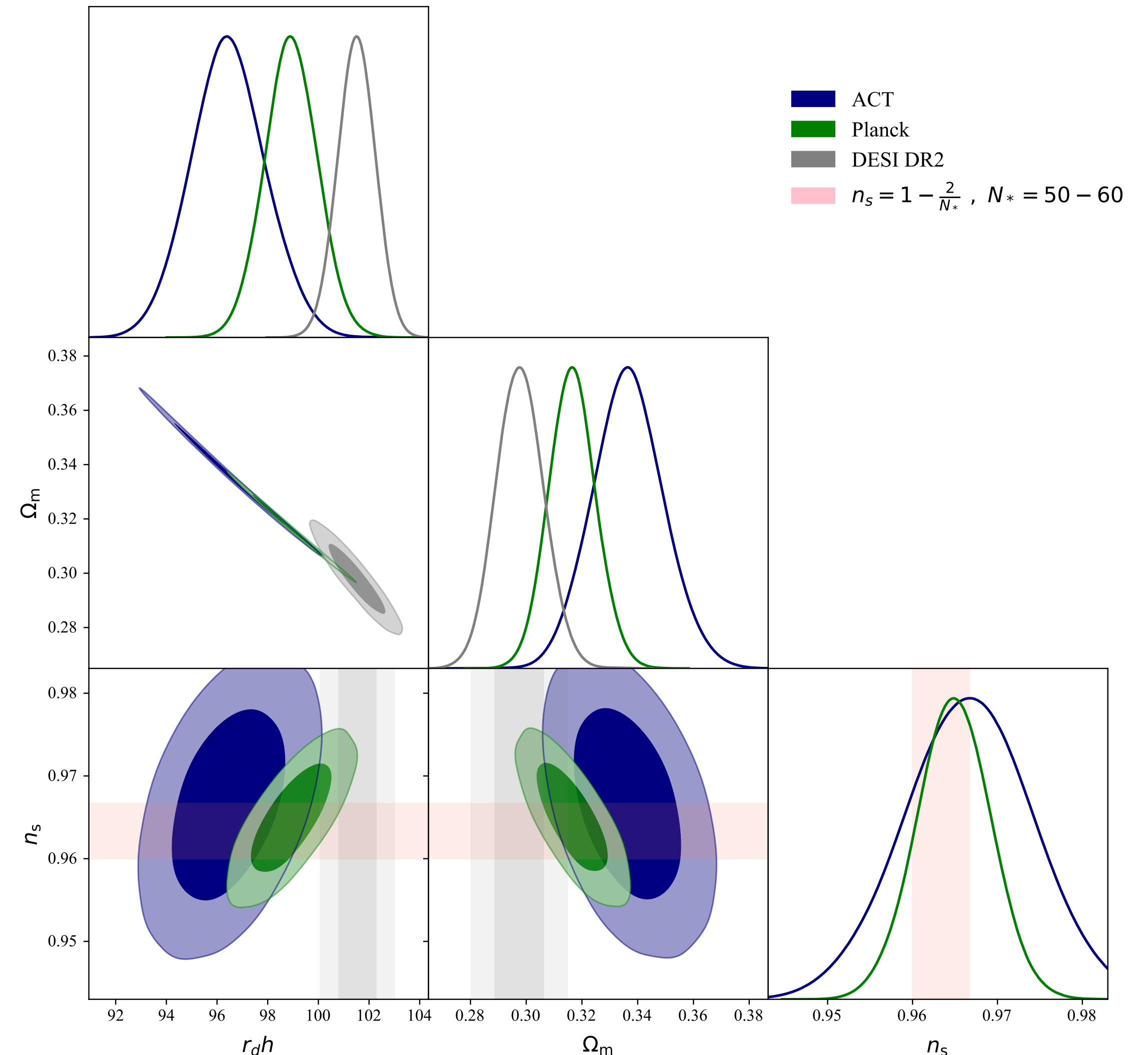
Moving Ω_m and $r_d h$, towards the DESI-preferred region, while maintaining the fit to the data, requires a **equivalent increase in n_s** .

Pink band:

To help contextualize we compare with the predicted n_s from inflation (Mukhanov & Chibisov 1981; Starobinsky 1983) compatible with 'benchmark' models

$$n_s = 1 - 2/N_*$$

$$\Rightarrow N_* = [50, 60], n_s = [0.9600, 0.9667]$$



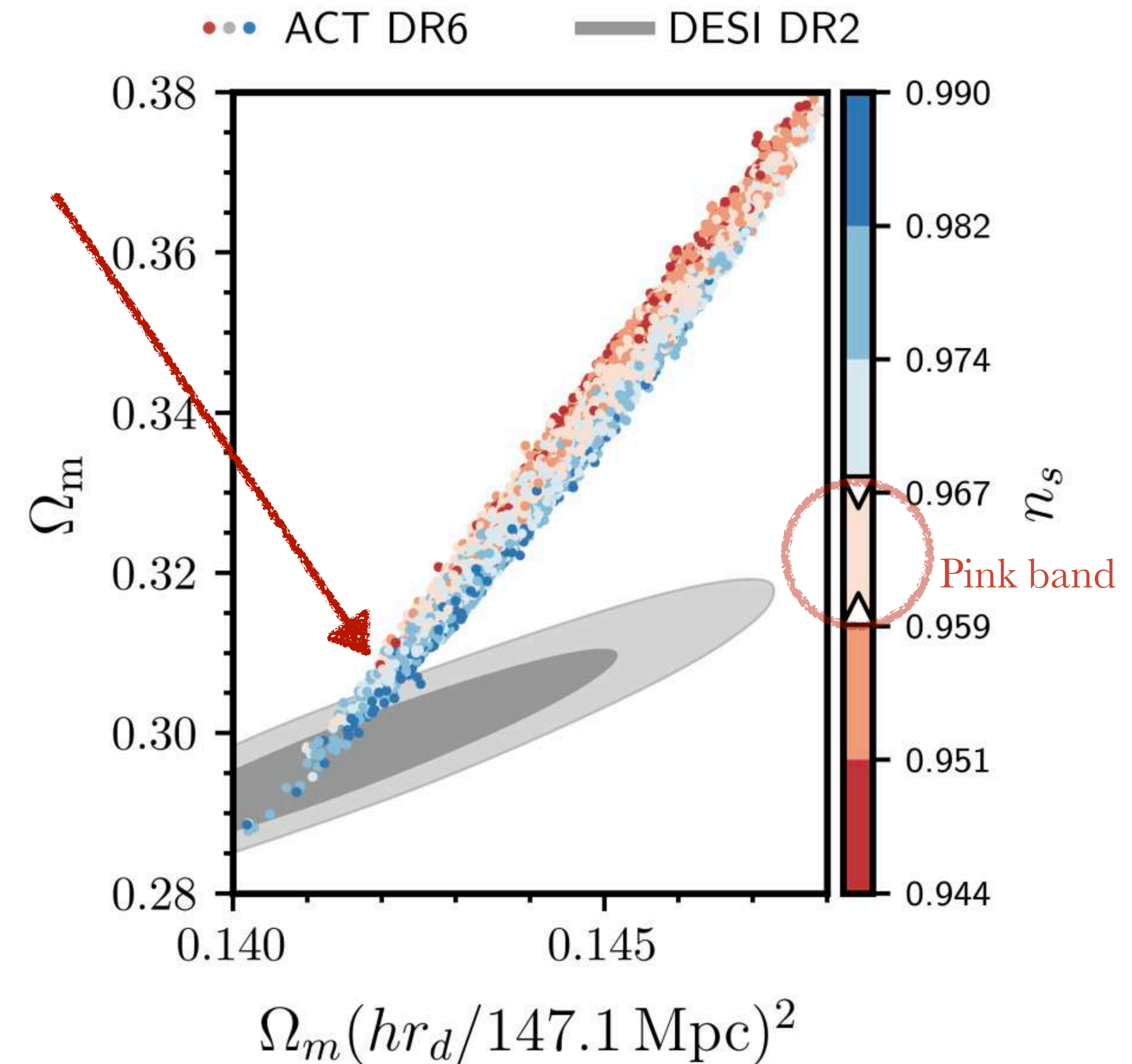
Consequences for *the spectral index n_s*

The BAO-CMB Tension and Implications for Inflation
Elisa G.M. Ferreira et al, 2507.12459

The BAO-CMB tension has important consequences for n_s

- To illustrate, I show constraints on Ω_m and $r_d h$, from ACT DR6 and DESI DR2.
 - ACT barely enters DESI, and when it does $n_s > 0.97$
 - DESI only overlaps with ACT for large values of n_s

In the $n_s - r_d h$ plane: there is **no simultaneous overlap** between the ACT 2σ contour, the 2σ DESI constraint on $r_d h$, and the band of n_s values predicted by benchmark inflation scenarios!



Consequences for *the spectral index n_s*

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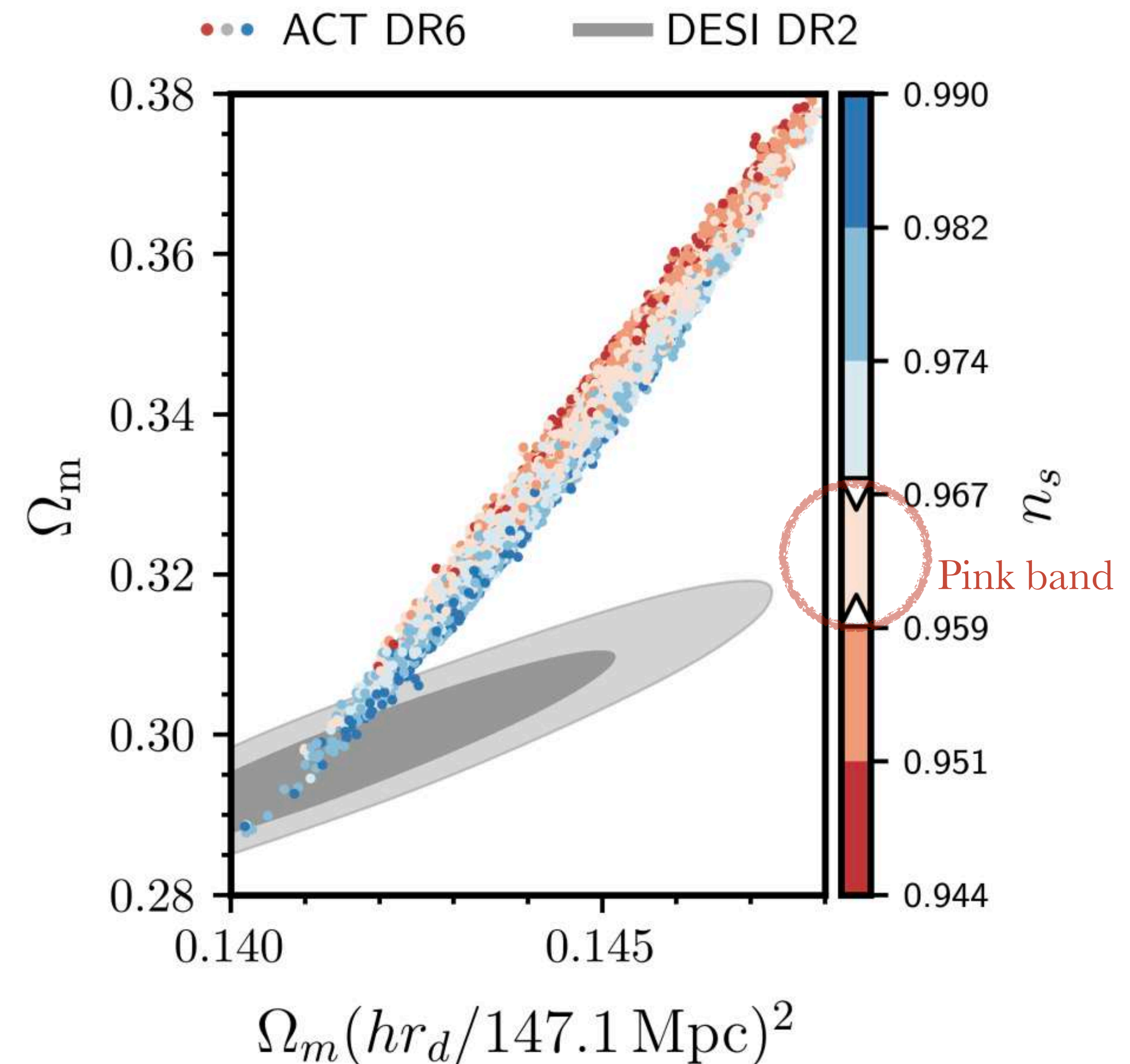
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 - ACT barely enters DESI, and when it does $n_s > 0.97$
 - DESI only overlaps with ACT for large values of n_s

In the $n_s - r_d h$ plane: there is **almost no simultaneous overlap** between the

- ACT 2σ contour,
- the 2σ DESI constraint on rdh , and
- the band of n_s values predicted by benchmark inflation scenarios!

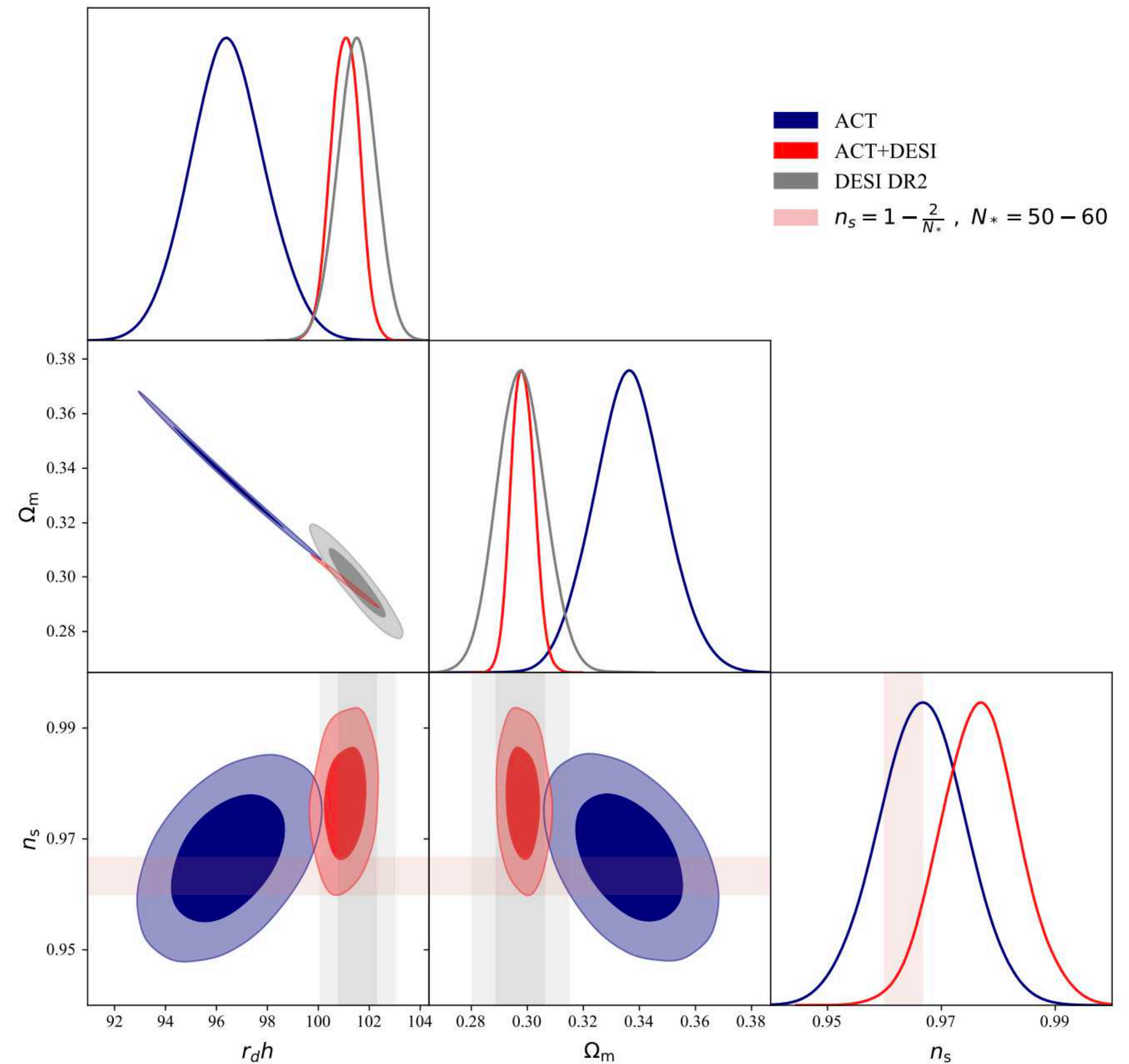
*This **discrepancy** suggests that there is little evidence that **ACT** and **DESI** constraints are **both right**, while at the same time Λ CDM with a benchmark inflation scenario holds true; **something has to give**.*



Consequences for *the spectral index n_s*

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Combining ACT DR6 and DESI DR2



Consequences for *the spectral index n_s*

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Combining ACT DR6 and DESI DR2

- Significant shift in the BAO parameters Ω_m and $r_d h$,
- corresponding shift of n_s along the degeneracy direction of the fit to ACT alone

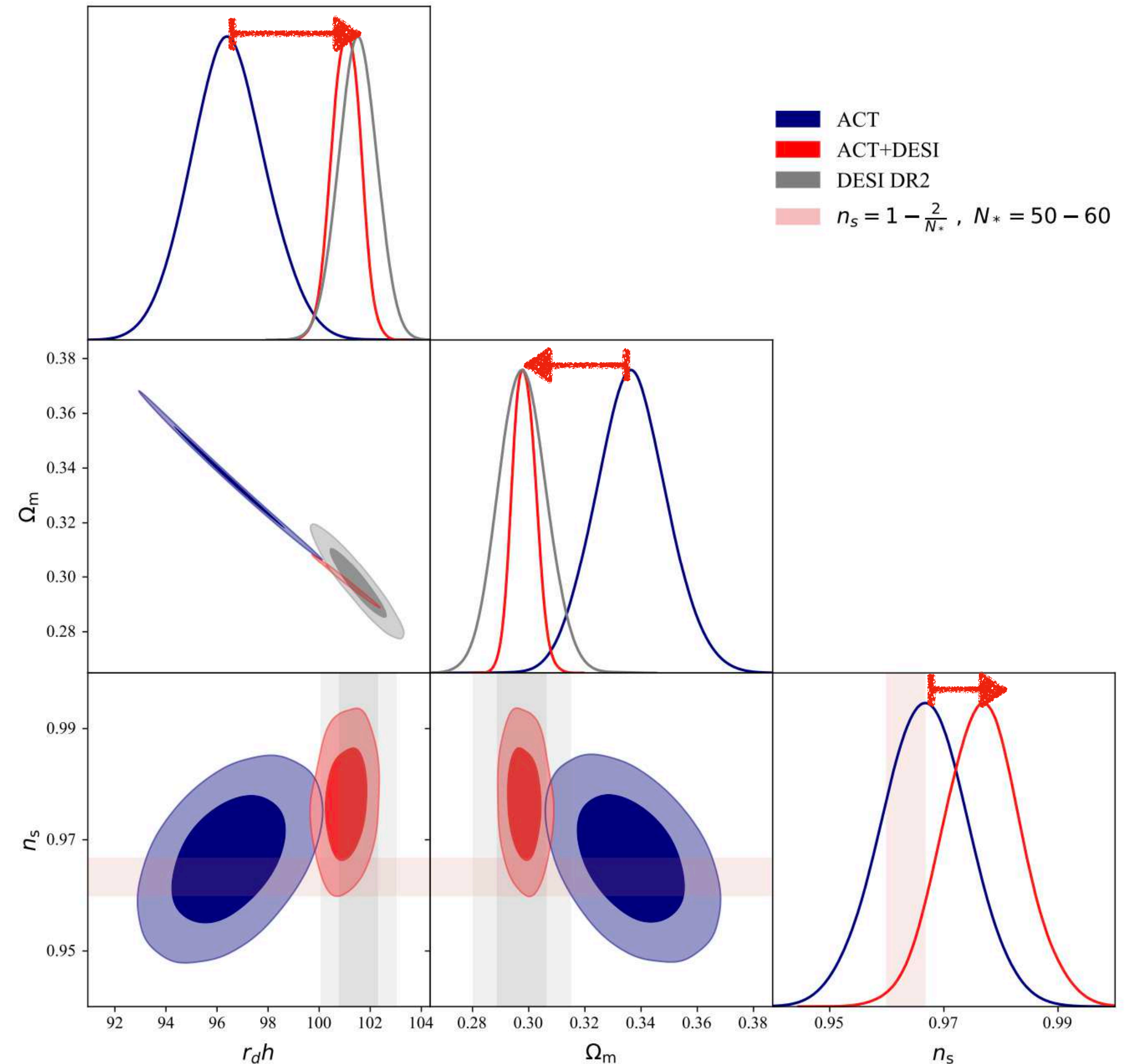
Constraints from ACT DR6 CMB and DESI DR2 BAO Data

Parameter	ACT	ACT + DESI
n_s	0.9666 (0.9664) \pm 0.0076	0.9770 (0.9754) \pm 0.0070
$r_d h$ [Mpc]	96.5 (96.27) \pm 1.5	101.04 (101.05) \pm 0.54
Ω_m	0.337 (0.338) \pm 0.013	0.2999 (0.2996) \pm 0.0040

~ 3.34 σ distance

(translating into a one-dimensional Gaussian difference)

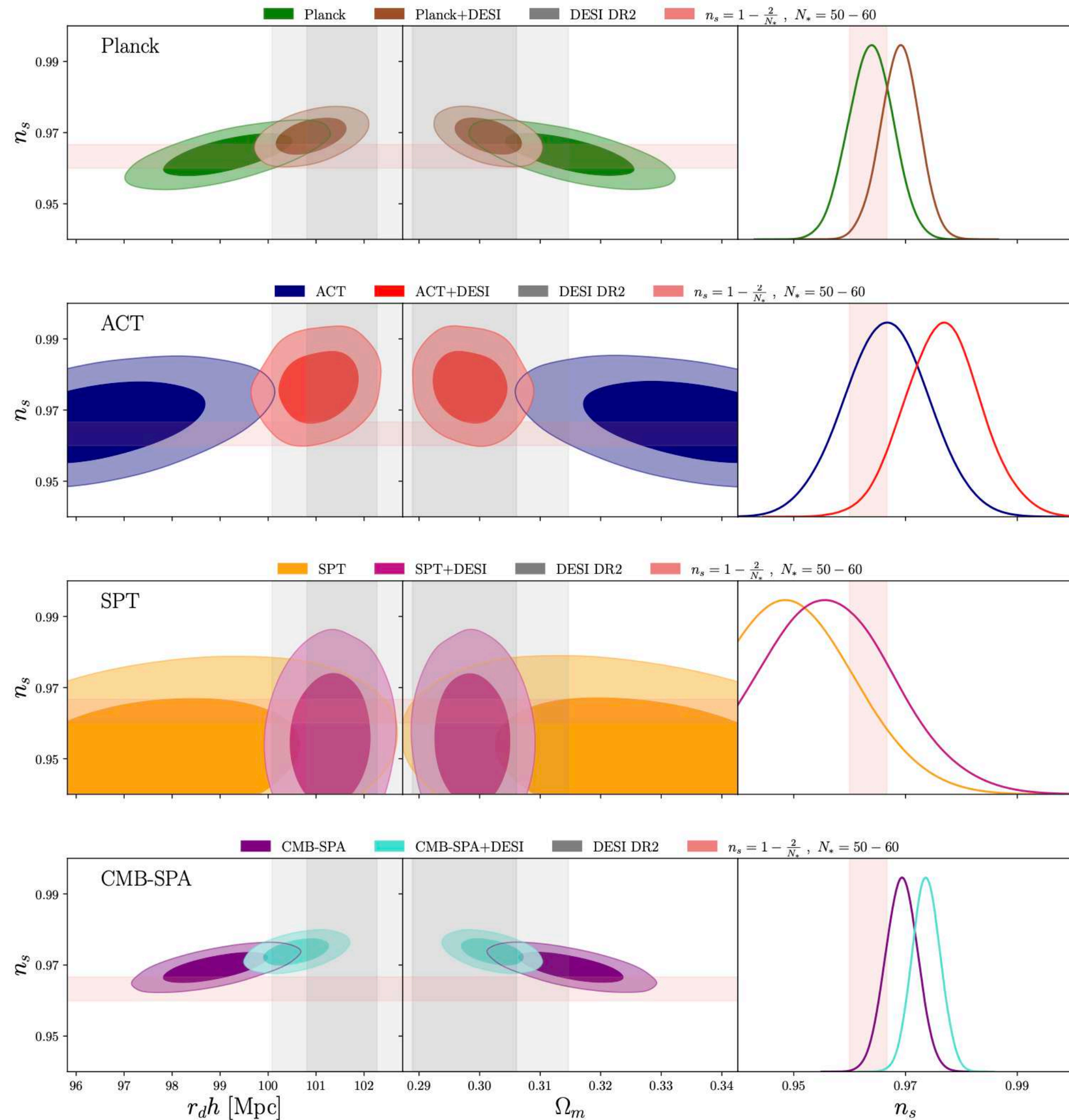
For n_s alone, the discrepancy is from ~ 3.5 σ
 (Correlated data - dependent on the metric used)



highly significant shift

Consequences for the spectral index n_s - SPT

*The Shifting Spectrum of n_s constraints from DESI and CMB data, E McDonough, **EF**, R Kallosh, and A Linde, to appear*



Planck + DESI

- Planck has the **tightest** constraint on n_s and the **strongest correlation** between n_s and BAO parameters.
- Planck and DESI differ at the level of $\sim 2.0\sigma$.
- Adding DESI shifts n_s **slightly upward**, but Planck remains **fully compatible** with pink band
- Even after adding DESI, n_s stays **strongly correlated** with BAO parameters due to Planck's constraining power.

SPT + DESI (full SPT likelihood, no lensing)

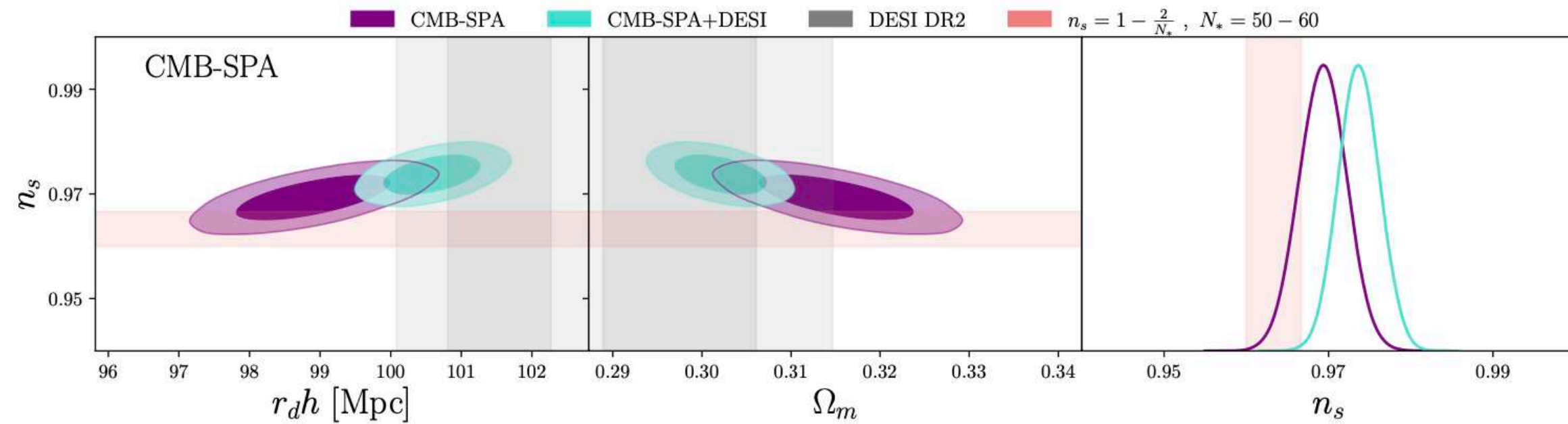
- SPT has a **weaker intrinsic correlation** between n_s and the BAO parameters, so the upward shift in n_s is **smaller** than for ACT.
- SPT prefers a **lower n_s** than ACT, so DESI acts to move SPT **toward** the pink band
- As with ACT, DESI causes a **re-orientation** and **decorrelation** of the n_s - BAO contours.

Data set	n_s	r_dh [Mpc]	Ω_m	τ
DESI	–	101.54 ± 0.73	0.2975 ± 0.0086	–
Planck	0.9638 ± 0.0040	99.16 ± 0.88	0.3149 ± 0.0070	0.0516 ± 0.0078
Planck + DESI	0.9690 ± 0.0035	100.90 ± 0.49	0.3014 ± 0.0037	0.0554 ± 0.0076
ACT	0.9666 ± 0.0076	96.5 ± 1.5	0.337 ± 0.013	$0.0562^{+0.0053}_{-0.0063}$
ACT + DESI	0.9767 ± 0.0068	101.05 ± 0.55	0.2984 ± 0.0041	0.0636 ± 0.0064
SPT	0.948 ± 0.012	97.6 ± 2.1	0.328 ± 0.017	0.0506 ± 0.0059
SPT + DESI	0.955 ± 0.012	101.30 ± 0.56	0.2988 ± 0.0042	0.0529 ± 0.0059
CMB-SPA	0.9693 ± 0.0029	98.90 ± 0.72	0.3151 ± 0.0057	0.0557 ± 0.0036
CMB-SPA + DESI	0.9737 ± 0.0025	100.59 ± 0.45	0.3021 ± 0.0034	0.0575 ± 0.0036

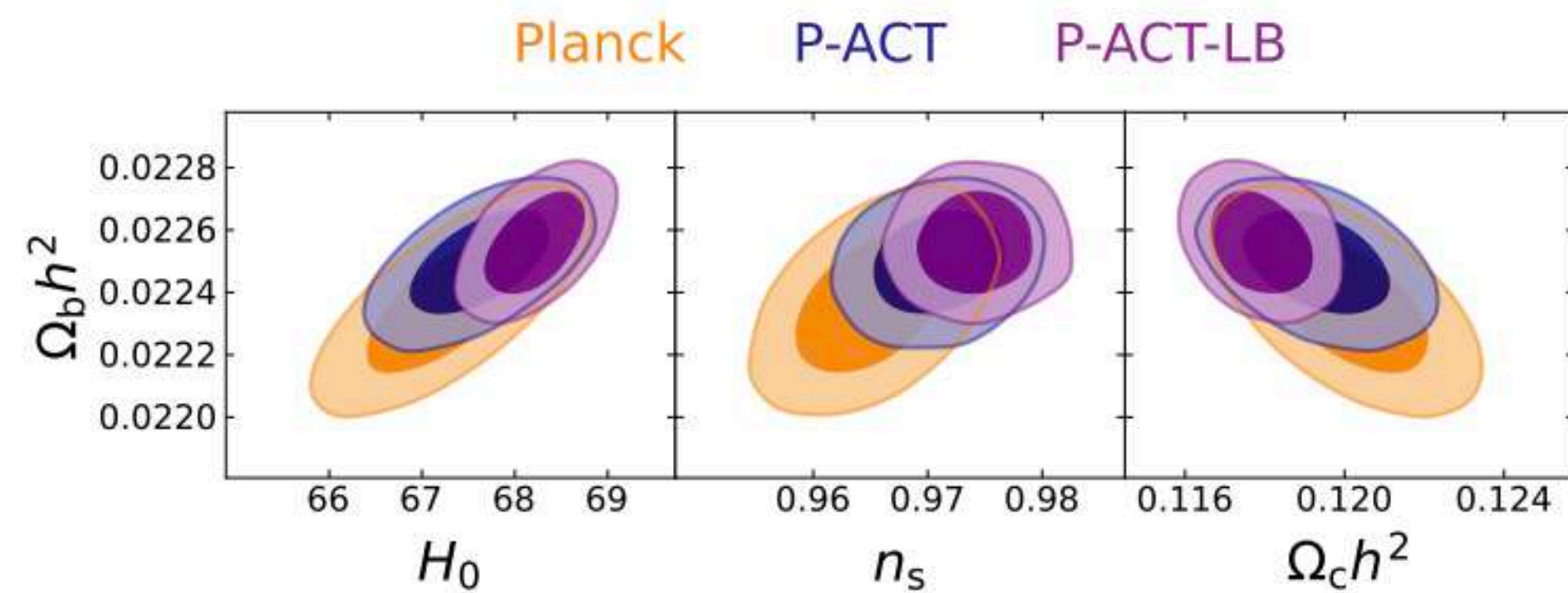
SPA: SPT+Planck+ACT

Consequences for the spectral index n_s - Planck, ACT, SPT

The Shifting Spectrum of n_s constraints from DESI and CMB data, E McDonough, **EF**, R Kallosh, and A Linde, to appear



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$$\begin{aligned}
 n_s &= 0.9666 \pm 0.0077(\text{ACT}) \\
 &= 0.9709 \pm 0.0038(\text{P} - \text{ACT}) \\
 &= 0.9743 \pm 0.0034(\text{P} - \text{ACT} - \text{LB})
 \end{aligned}$$

SPA: SPT+Planck+ACT

- CMB-SPA shows excellent alignment with Planck (and ACT) in the structure of the n_s correlations.
- Adding DESI to CMB-SPA mirrors the behavior seen in the other data sets: an upward shift in n_s and reduced correlation with BAO parameters.

The “high” value of n_s comes only when combined with DESI

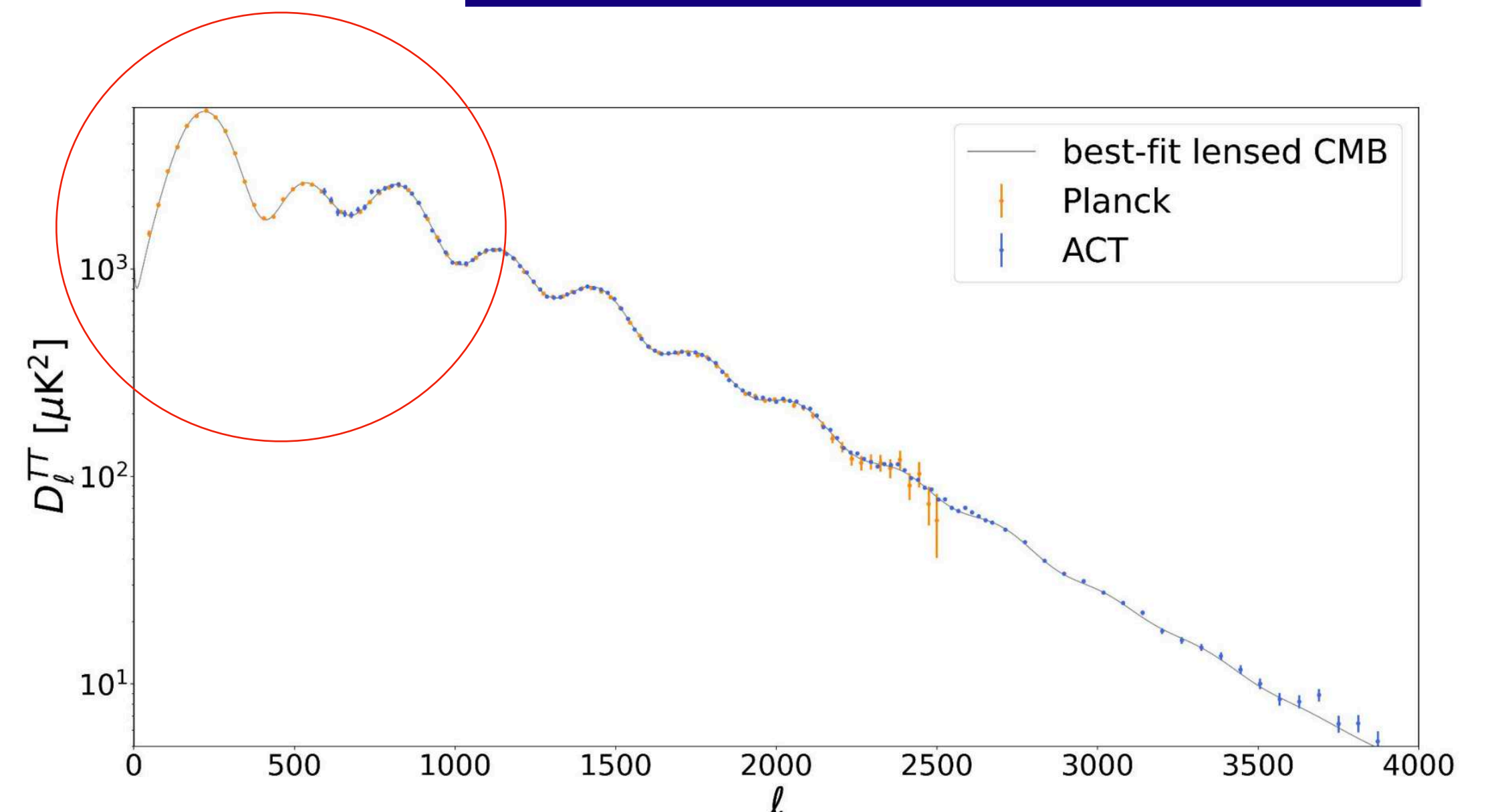
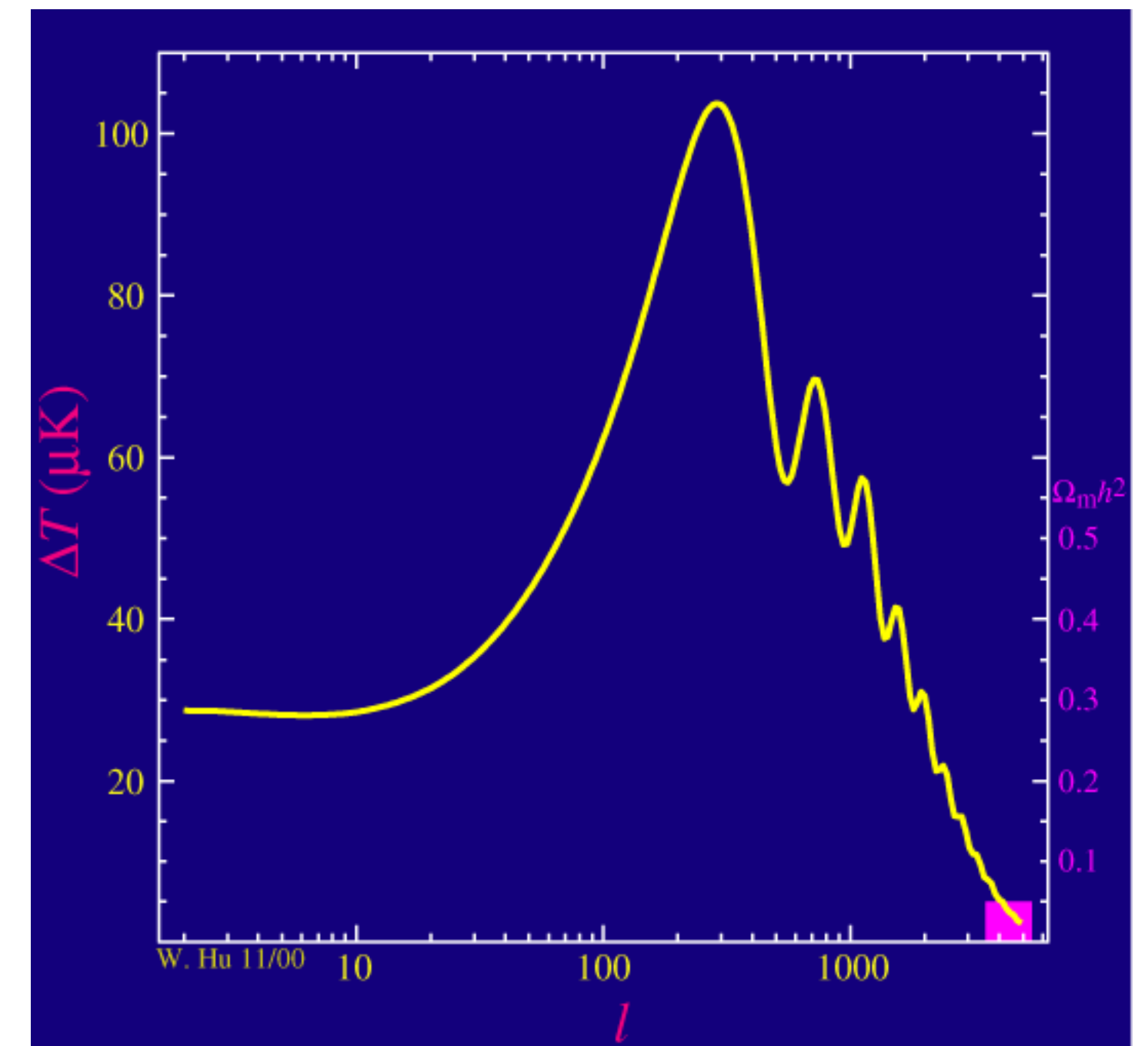
Consequences for *the spectral index n_s*

Understanding these shifts and significance:

Why ACT shifts so much when including DESI?

- Smaller BAO-CMB discrepancy for Planck
- Different degree of correlation of BAO parameters with n_s for these data sets. Correlation structure depends non-trivially on:
 - **Angular scales** covered by the power spectrum measurements:
 - Experiment with more of its information coming from large angular scales (Planck) has the higher correlation: more information about $\Omega_m h^2$
 - Weighting of the data towards **temperature or polarization set by the noise level**
 - Relative weight of **gravitational lensing** information: gravitational lensing is more important on smaller scales, and is another source of information about $\Omega_m h^2$, one we expect to be relatively uncorrelated with n_s .

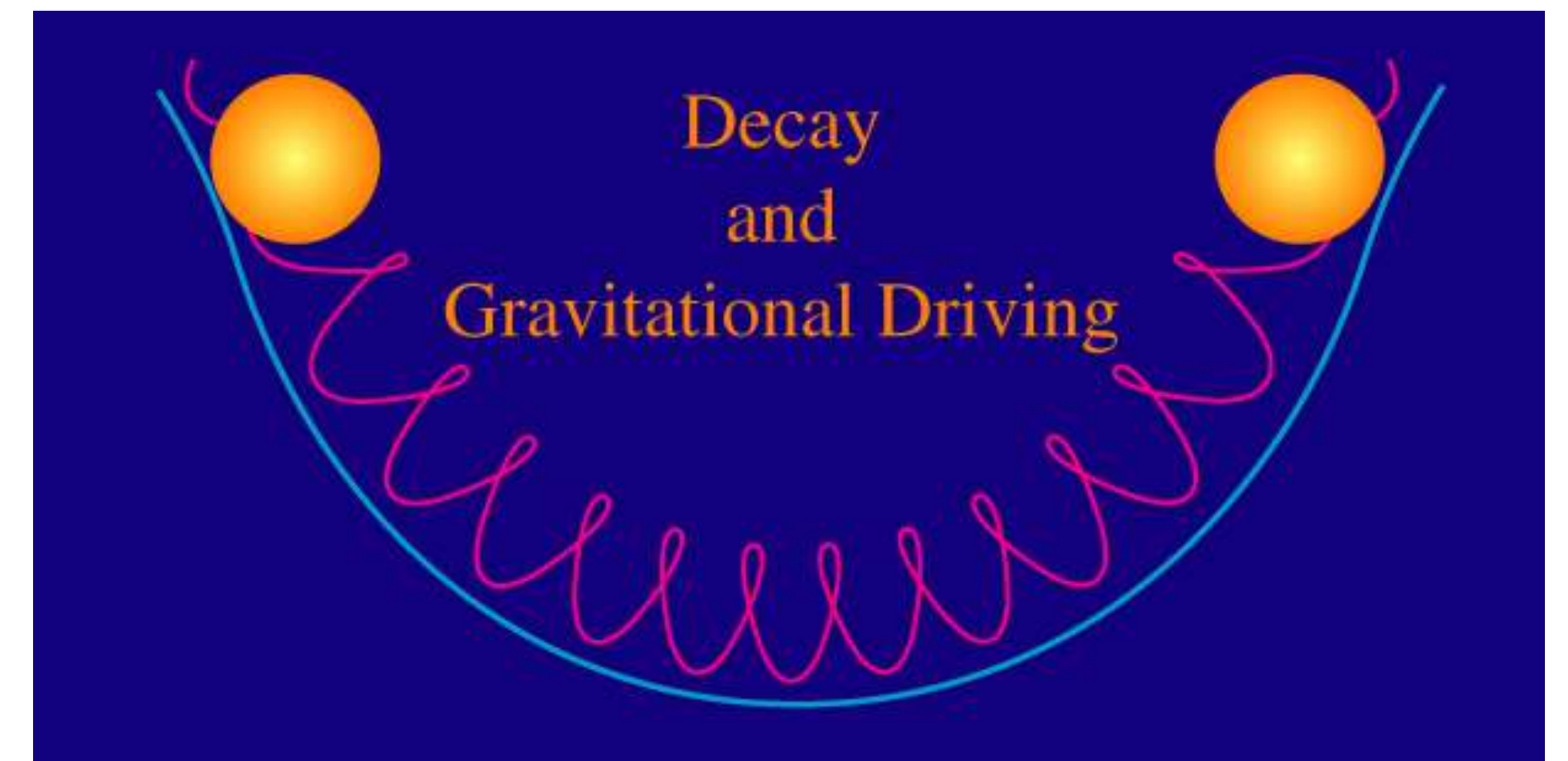
ACT allows more variation of Ω_m than Planck!



BAO parameters in the CMB

BAO parameters $r_d h$ and Ω_m can also be inferred from CMB data

- $\omega_m = \Omega_m h^2$ is determined primarily by the imprint of the ‘radiation driving’ effect
 - ‘Radiation driving’: boosting of acoustic oscillation amplitude due to potential decay at horizon crossing
- The amplitude depends on the matter-to-radiation ratio at horizon crossing and hence changes smoothly over a wide range of angular scales, with some similarity to changes in n_s
 - Leads to a partial degeneracy between n_s and ω_m .
- Determination of Ω_m comes from combining ω_m and the angular size of the sound horizon, θ_s (\rightarrow best measure quantity in cosmology)
- Ω_m determination is correlated with n_s since $\Omega_m h^2$ is correlated with n_s and θ_s is not



Implications to *inflation*

The BAO-CMB Tension and Implications for Inflation
 Elisa G.M. Ferreira et al, 2507.12459

Does this shift has consequences for inflationary models?

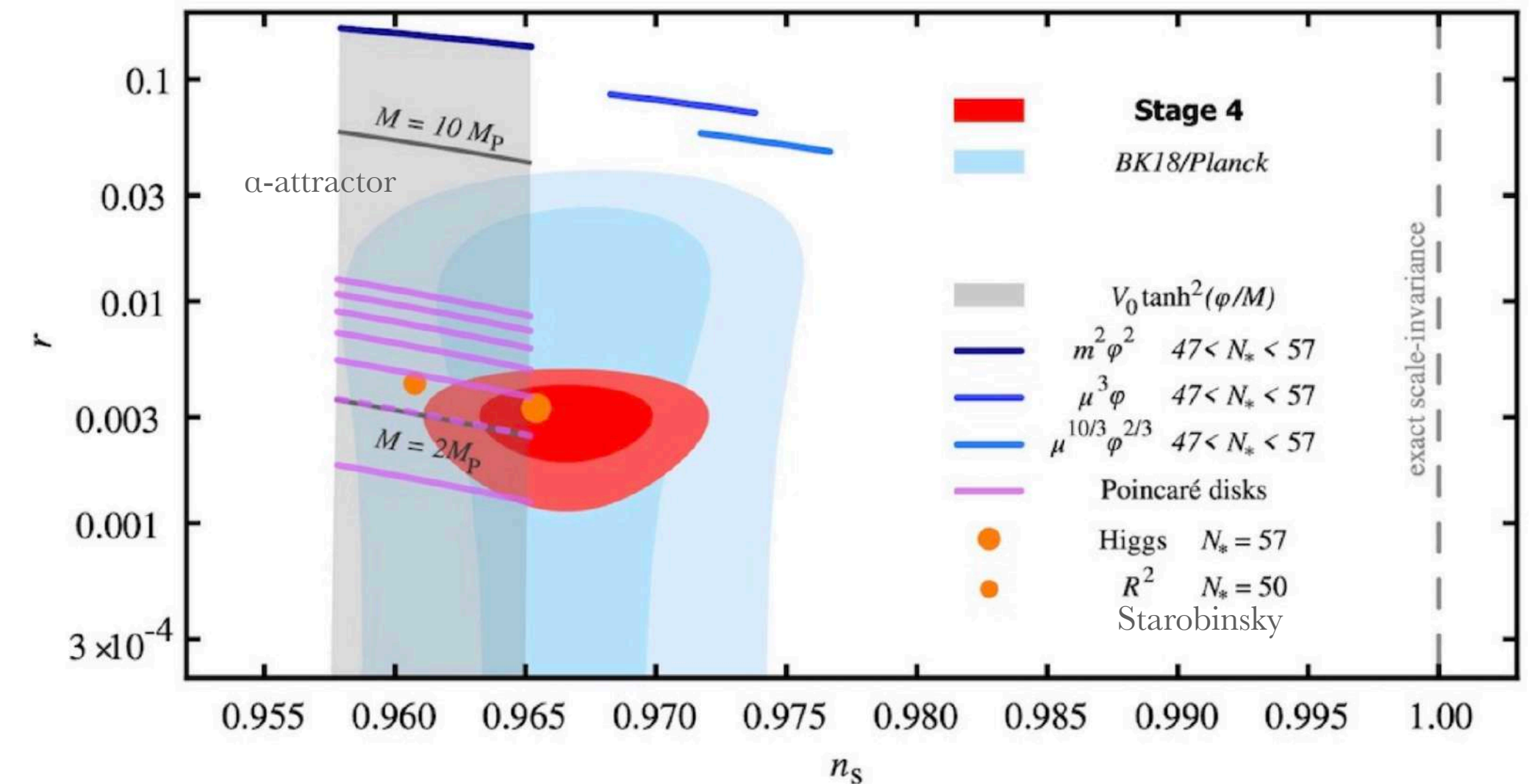
Lessons from *Planck*:

Plateau potentials are preferable to monomial potentials. Moreover, over the last 10 years, the data have supported models approaching the plateau exponentially

Viable targets (Planck/BICEP/Keck/ACT/SPT data):

- Starobinsky model ,
- Higgs inflation and
- α -attractors (exponential - T and E α -attractor)

From CMB White Paper and LiteBIRD



No viable targets at $n \gtrsim 0.967$

The predictions are for $47 < N^* < 57$.

If one plots the model predictions in the often used range $50 < N^* < 60$, they would shift to the right by about 0.002

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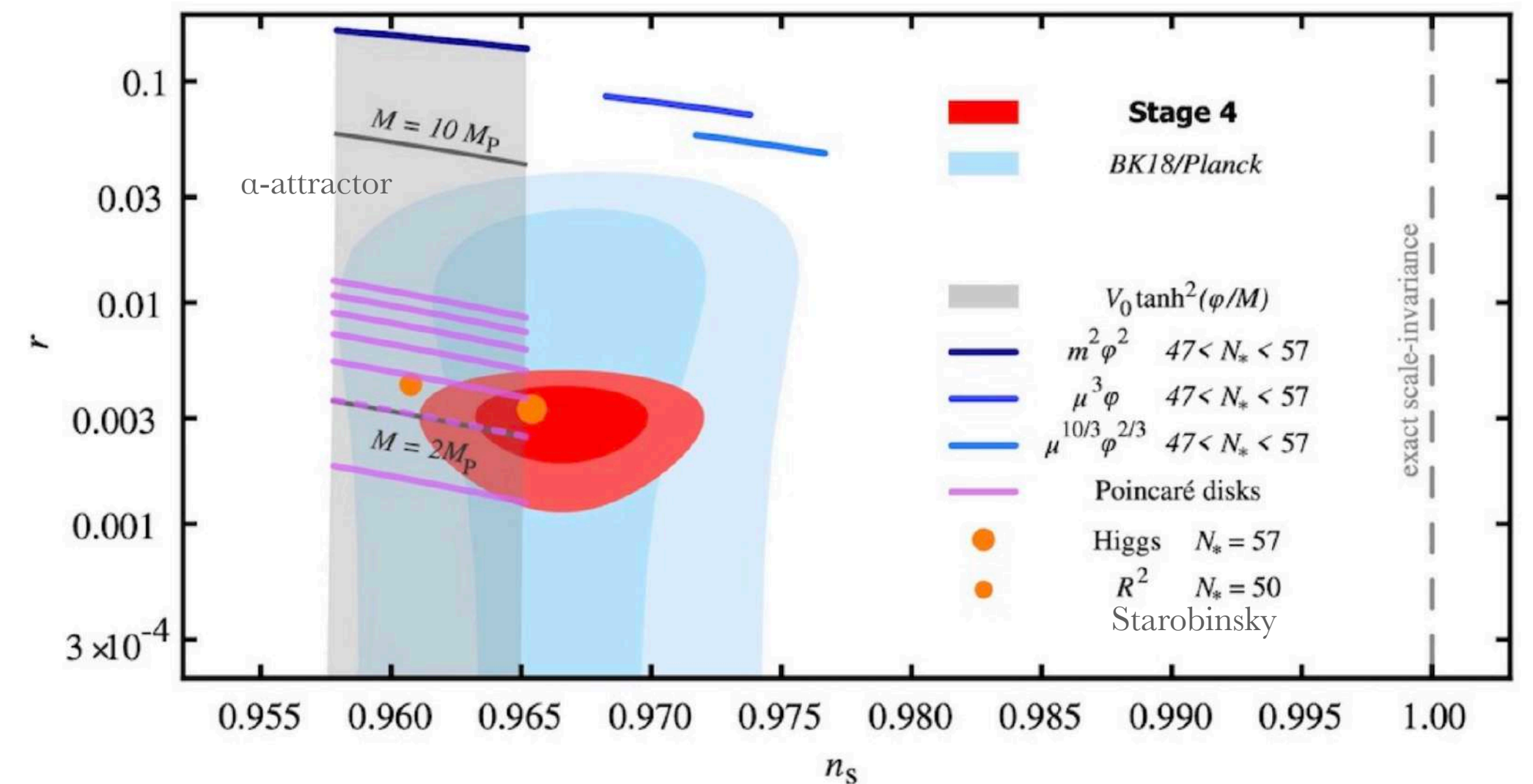
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- Higgs inflation and
- α -attractors (exponential - T and E α -attractor)

Hard to change conditions of these models to have $n \gtrsim 0.967$ - e.g. changing *reheating*

- Starobinsky model: not very efficient $n_s \sim 0.962 - 0.963$
- Higgs inflation: very efficient $n_s \sim 0.966$
- α -attractors: $\max \Delta n_s \sim 0.006$

From CMB White Paper and LiteBIRD



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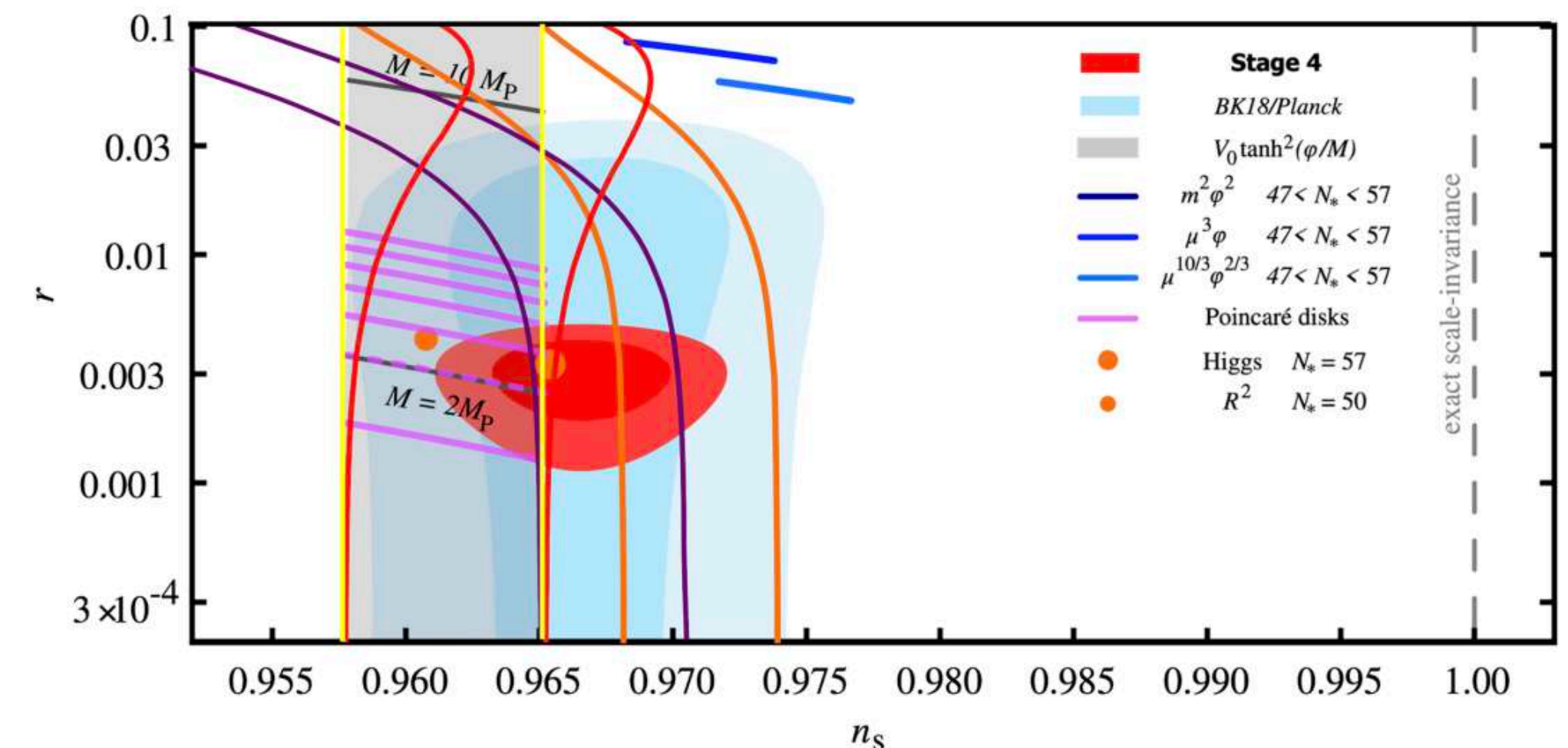
Taking the high value of n_s from ACT+DESI (or P-ACT-LB result $n_s = 0.9739 \pm 0.0034$) **at face value**:

- Starobinsky model, Higgs inflation and α -attractors **NOT viable** anymore!
- Would need models that allow $n_s > 0.97$

Ex.: simple inflationary models with polynomial potentials- e.g. pole inflation, D-brane inflation, KKLT models, polynomial α -attractors, chaotic inflation with nonminimal coupling to gravity ($N_* = 60$)

- Polynomial attractors with $k = 1, 2$ compatible with P-ACT-LB

If we confirm that data prefers $n_s \sim 0.973 - 0.974$:
data prefer potentials approaching the plateau polynomially rather than exponentially



Red lines represent the E-model α -attractors, while the **purple** and **orange** lines show the predictions of the polynomial attractors with $k = 4$ and $k = 2$. Predictions of the simplest models with $k = 4$ and $k = 2$ in the small r area have $0.965 < n_s < 0.974$ for $47 < N_* < 50$

Implications to *inflation*

The BAO-CMB Tension and Implications for Inflation
Elisa G.M. Ferreira et al, 2507.12459

- Large value of n_s only appears for the data combinations that add BAO DESI data to CMB (ACT or SPT, with or without Planck)
- Shift comes from the BAO-CMB discrepancy (within LCDM); **can we rely in these combinations?**
- CAUTION when interpreting these constraints until the BAO-CMB discrepancy is resolved
- These shifts in n_s can strongly impact the viable inflationary scenarios in accordance to data

Remember: many of the current hints are only present when *combining datasets* e.g. *dynamical DE*

At this point, we **don't know** what is the final value of n_s

Since we don't know what this tension is indicating - systematics, new insights on the analysis, new physics - we need to be careful when combining data sets

* Interesting interplay between *tau*, the CMB-BAO tension and n_s .

Interplay of n_s and τ

Sailer et al, 2504.16932;
Jhaveri et al 2504.21813;
Liu et al 2510.14957

τ - optical depth to reionization

CMB-BAO tension: inability to change the cold dark matter density $\Omega_c h^2$ given the measurements of the CMB lensing once the optical depth to reionization τ is constrained by the Planck low ℓ polarization data

Optical depth constraint prevents the amplitude A_s of the curvature power spectrum, $A_s e^{-2\tau}$, from adjusting the lensing amplitude to fit the data and instead fixes $\Omega_c h^2 \rightarrow$ fixes $\Omega_m (D_m)$

If it is not systematics, what would happen if these constraints on τ can be somewhat **relaxed** in extended reionization models or alternately by reducing the large scale primordial power spectrum from inflation

τ is measured from:

- 1. Low- ℓ E-mode polarization bump** from rescattering during reionization
 \rightarrow main observable; $C_\ell^{EE} \propto \tau^2$
- 2. Suppression of high- ℓ TT/TE/EE acoustic peaks**
 \rightarrow gives τ only via and is highly degenerate with A_s
- 3. Low- ℓ TE correlations**
 \rightarrow secondary constraint

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Sailer et al, 2504.16932;
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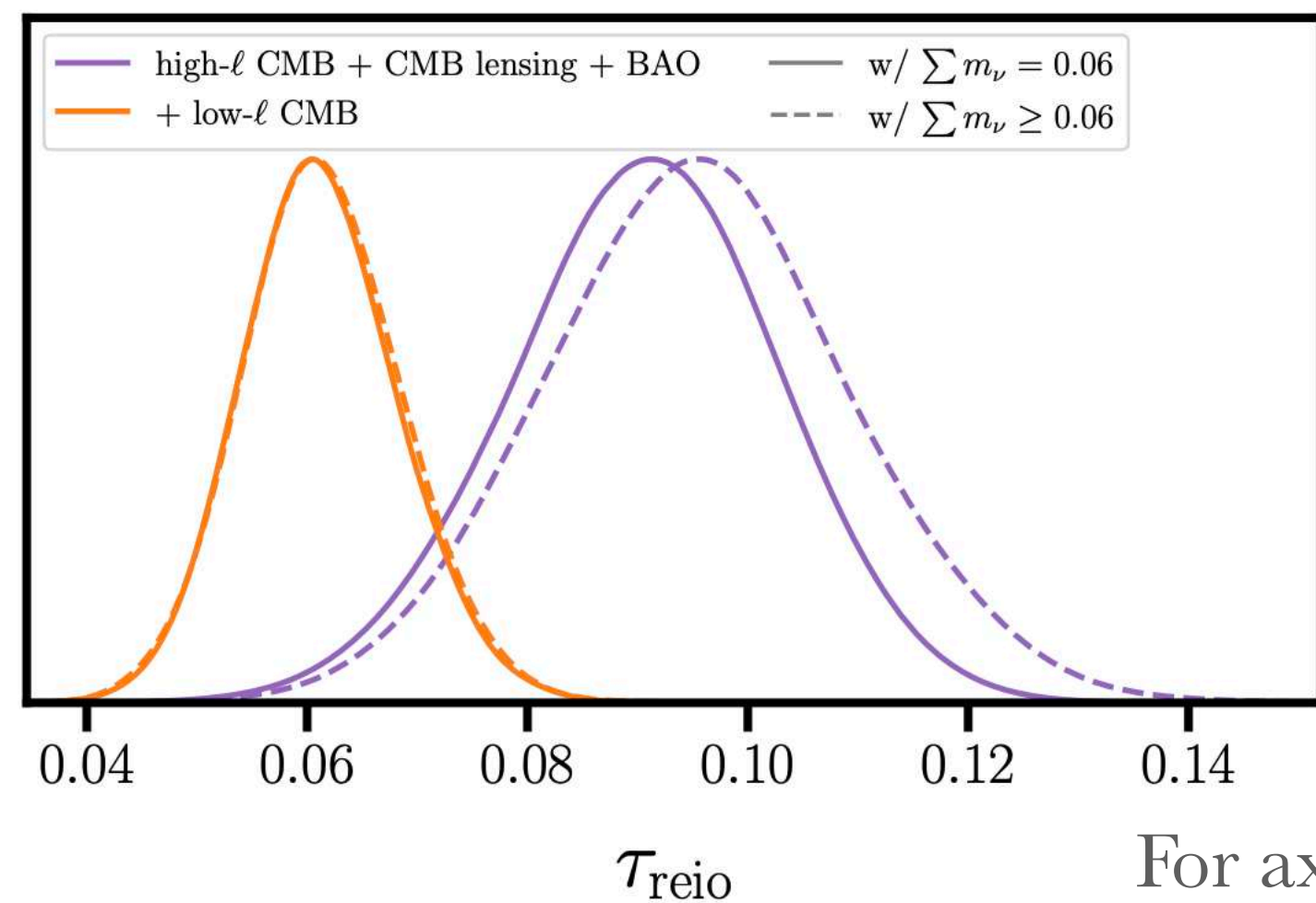
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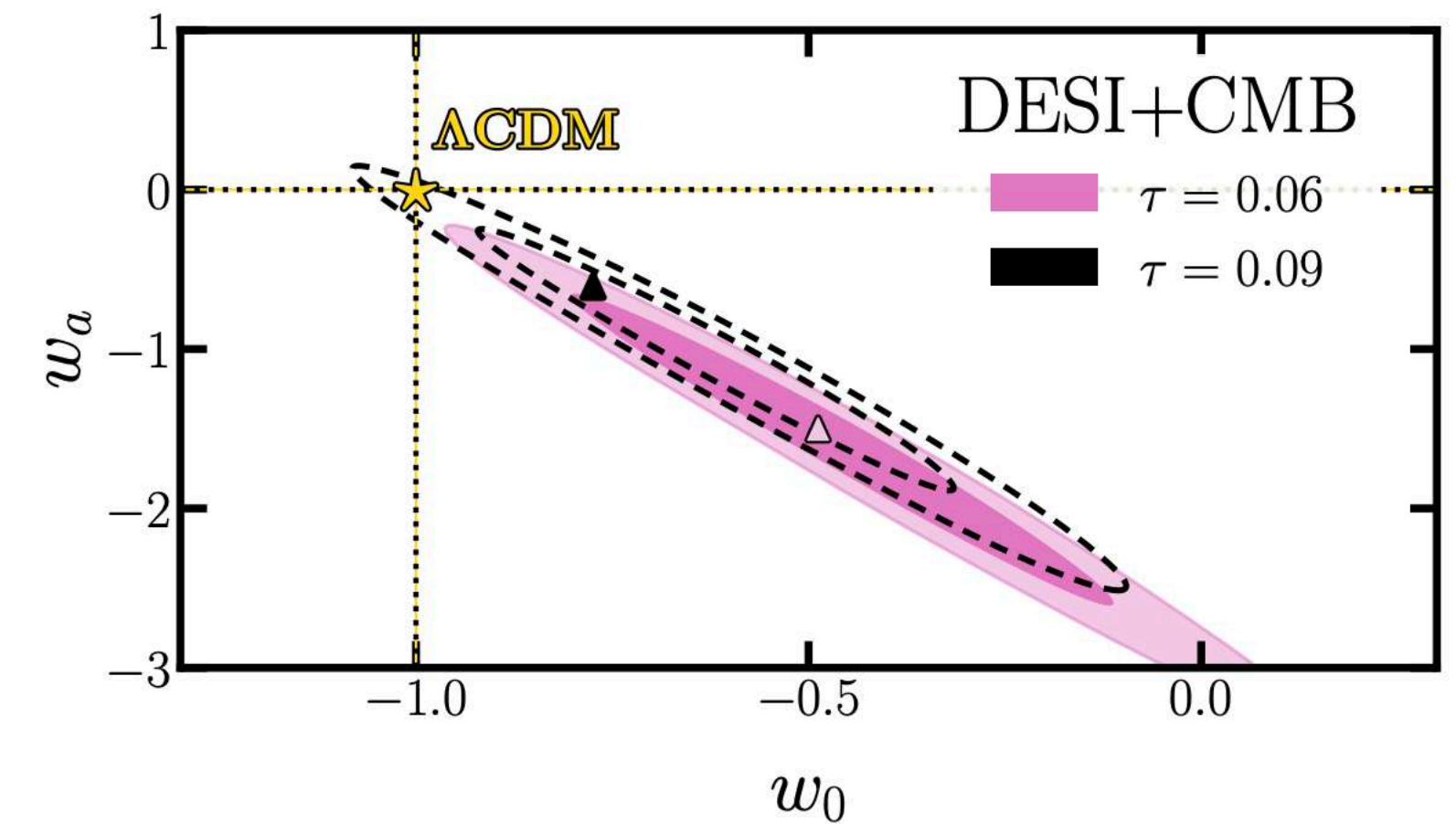
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Influence of τ for sum of neutrino mass and DE constraints

CMB-BAO discrepancy can be resolved by **raising** τ_{reio} from 0.054 to a higher value, $\sim 0.09 - 0.1$ (in the absence of low- ℓ EE data)



Reduces preference for dynamical DE from 3σ to 1.5σ



For axion DE, if one had $\tau = 0.1$, this would fit the data as well as $w_0 - w_a$ phantom dark energy

However, the required increase in τ is in $\approx 3 - 5\sigma$ tension with Planck low- ℓ E-mode polarization

Interplay of n_s and τ

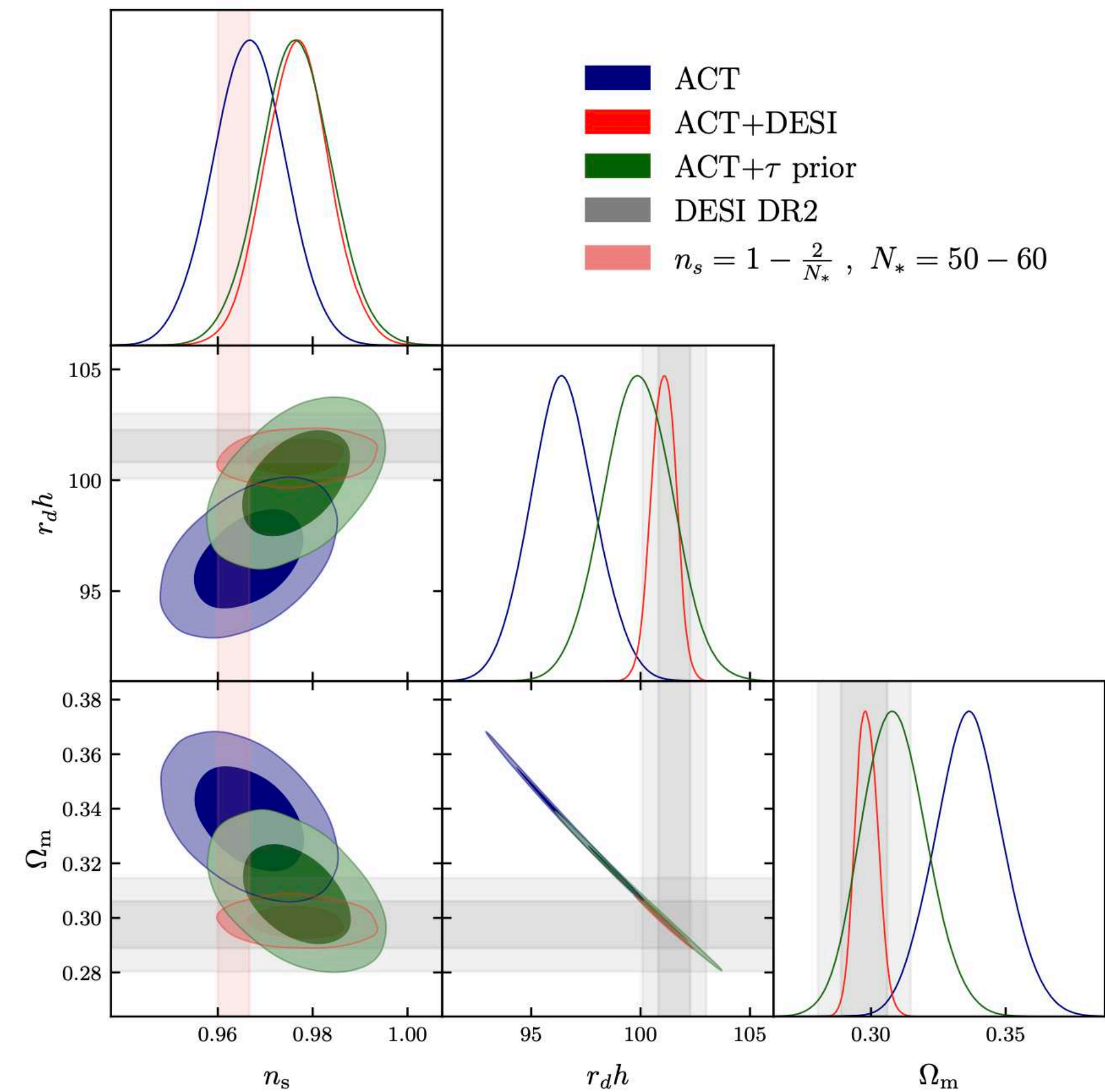
*The Shifting Spectrum of n_s constraints from DESI and CMB data, E McDonough, **EF**, 2512.05108*

Exercise:

Force τ to be large, ~ 0.1 , has the **same effect** in the constraints in the BAO parameters and n_s as combining with DESI

Expected: τ tightly correlated to A_s and n_s

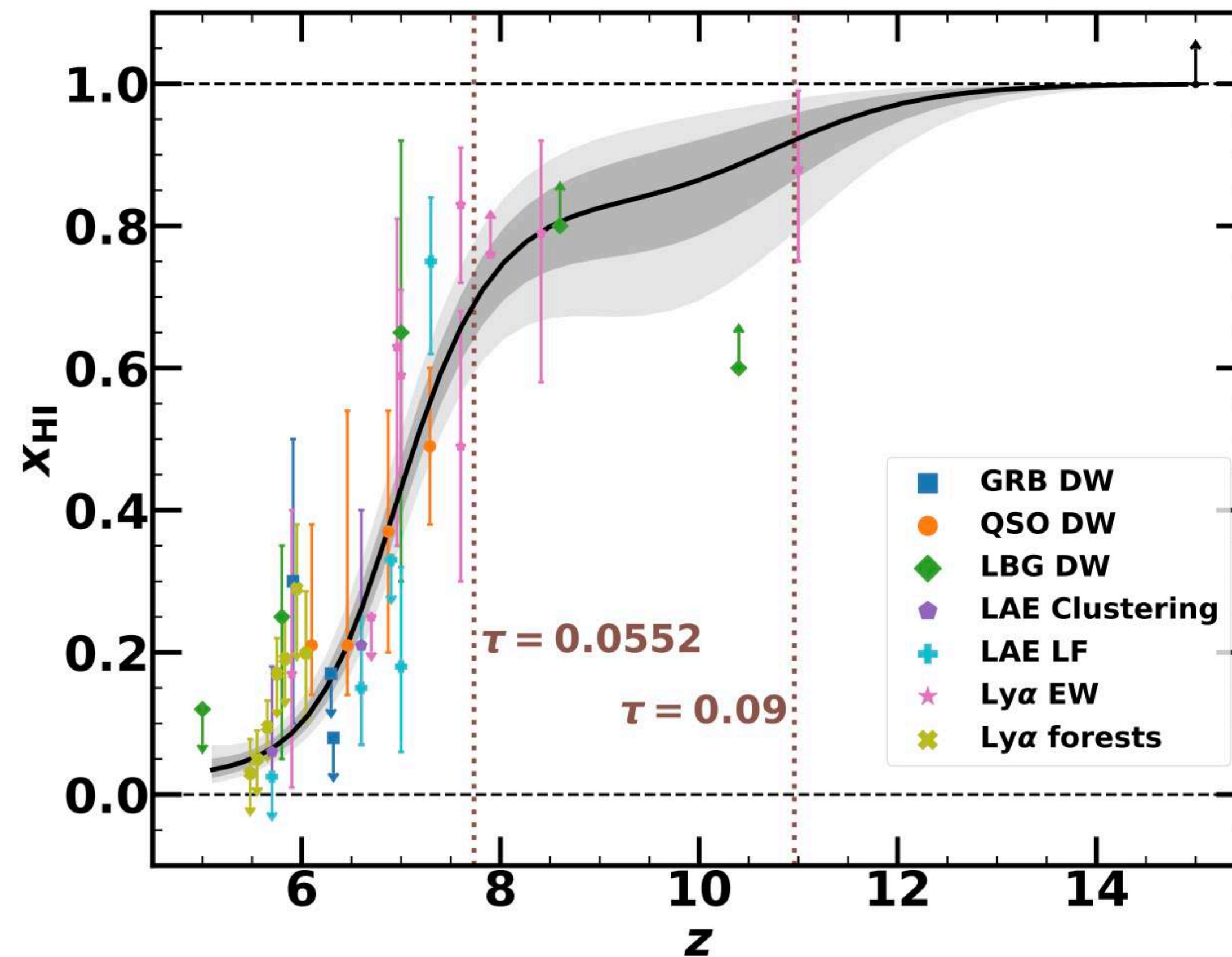
There might be systematics in the measurements of low l polarization and in the determination of τ - important goal of LiteBIRD



Interplay of n_s and τ

A New Constraint on the Optical Depth from the Reionization History Independent of CMB Large-Scale E-Mode Polarization, Y. Kageura, M. Ouchi, F. Naokawa, H. Umeda, A. Matsumoto, Y. Harikane, M. Nakane and T Thi Thai

Can we really have a large value for τ ?



- Ly α damping-wing absorption of GRBs at $z = 5.9 - 6.3$ (Totani et al. 2006, 2014; Fausey et al. 2025)
- Ly α damping-wing absorption of QSOs at $z = 6.0 - 7.5$ (Greig et al. 2022; Ďurovčíková et al. 2024)
- Ly α damping-wing absorption of LBGs at $z = 5 - 13$ (Umeda et al. 2025a)
- Clustering properties of Ly α emitters at $z = 5 - 7$ (Umeda et al. 2025b)
- Evolution of the Ly α luminosity function at $z = 5 - 7$ (Wold et al. 2022; Umeda et al. 2025b)
- Evolution of the Ly α EW distribution at $z = 5 - 14$ (Mason et al. 2019b; Jung et al. 2020; Whitley et al. 2020; Bolan et al. 2022; Kageura et al. 2025)
- Dark gaps in the Ly β forest at $z = 5.5 - 6.0$ (Zhu et al. 2022)
- Fraction of dark pixels in the Lyman-series forests at $z = 4.9 - 6.2$ (Davies et al. 2026)

*The resolution of the **BAO-CMB tension** remains an open question,
and so too does the fate of n_s !*

Little evidence that ACT and DESI constraints are both right, while at the same time Λ CDM with a benchmark inflation scenario holds true; something has to give.

*But this goes beyond inflation. It affects: dynamical DE, neutrinos, ...
Important!*

*Also alerts us for to be careful when **combining datasets***

Statistical methods in cosmology - challenges for current and next generation of experiments

Combining data sets & tension in cosmology

*Can we combine data sets to improve
parameter constraints?*

Cosmological tensions?

- *The BAO-CMB Tension and Implications for Inflation, EF, et al -
PRD Editor's suggestion*
- *The Shifting Spectrum of n_s constraints from DESI and CMB data, E
McDonough, EF*

Volume effects & profile likelihood in cosmology

Parameter inflation

*What are those effects and how important/
how much they affect parameter inference?*

Challenges for new experiments

- *Laura Herold, EF 2210.1629*
- *Laura Herold, EF and Eiichiro Komatsu 2112.12140,*
- *Reeves, L. Herold, S. Vagnozzi, B. Sherwin, EF 2207.01501*
- *Profile Likelihoods in Cosmology: When, Why and How illustrated with Λ CDM,
Massive Neutrinos and Dark Energy, L. Herold, EF, L. Heinrich, 2025*
- *E.Holm, L. Herold, T. Simon, EF, S. Hannestad, V. Poulin, T. Tram
2023*

Profile likelihood in cosmology

Statistical methods in *cosmology*

Cosmology as a precision science owes much of its progress to the large and precise cosmological data sets

ΛCDM →

Bayesian statistics;
MCMC techniques

Increased statistical power and complexity from new observational, theoretical, and systematic uncertainties introduce additional **nuisance parameters**, making statistical inference more challenging.

Particularly important with the increase of high quality data, where systematics will play important role: stage IV (even stage III), cosmological tensions, ...

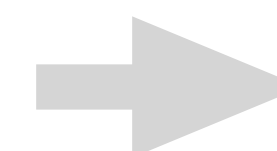
Statistical tools center stage! *Bayesian, frequentist, ML, ...*

Important:

MCMC effect: **Prior volume effect**

→

when large, affects the results



Profile likelihood

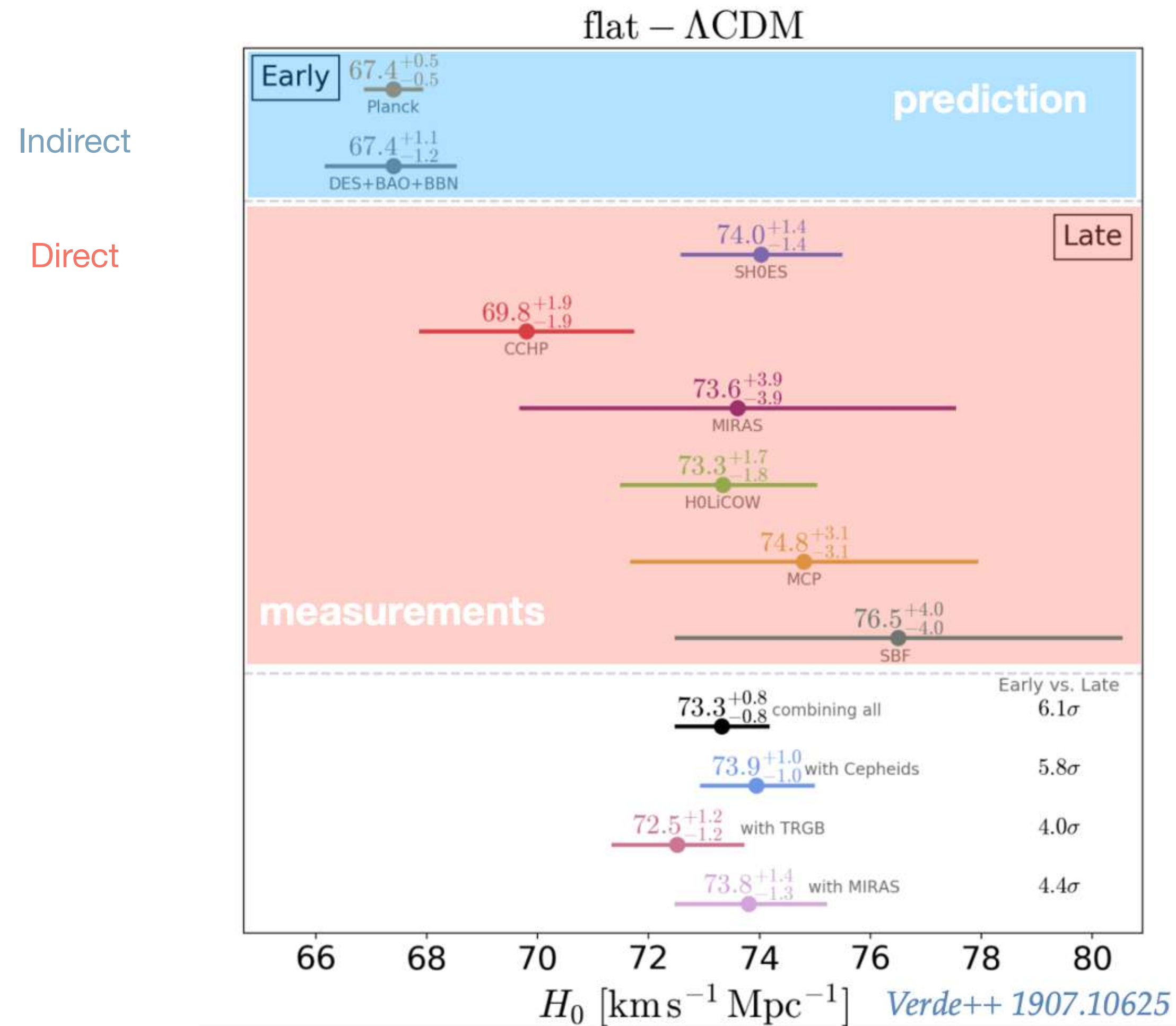
*Cosmological tensions and **early dark energy***

Based on Laura Herold, EF 2210.1629

& Laura Herold, EF and Eiichiro Komatsu 2112.12140,

& A. Reeves, L. Herold, S. Vagnozzi, B. Sherwin, EF 2207.01501

Hubble tension



Indirect

Direct

→ Depends on the cosmological model

New SHOES result (*Riess et al 2021*):

$$H_0 = 73.04 \pm 1.04 \text{ km/s/Mpc}$$

5 σ tension with Planck!



$$H_0 = 67.71 \pm 0.40 \text{ km/s/Mpc}$$

(Planck+BAO+Sn)

Systematics or new physics?

Early vs late time solutions to the *Hubble tension*

To increase H_0 :

Measured with a 0.03% precision by *Planck* 2018

$$\theta_s(z_*) = \frac{r_s(z_*)}{D_A(z_*)} = \frac{\int_{z_*}^{\infty} c_s(z) dz / H(z)}{\int_0^{z_*} c dz / H(z)}$$

r_s : pre-recombination physics \rightarrow depend on physical densities (b, r, cdm, ν)
 D_A : post-recombination physics \rightarrow information on H_0

$r_s(z_*) \rightarrow$ Sound horizon at lss
 $D_A(z_*) \rightarrow$ Angular diameter distance
 $\theta_s(z_*) \rightarrow$ Angular scale of the sound horizon

LATE UNIVERSE SOLUTIONS

Change only late time physics: early universe unaffected

Little room for changes in the physics (?)

Shown to not solve the tension, only alleviates!

EARLY UNIVERSE SOLUTIONS

Change only early time physics, late time almost unaffected

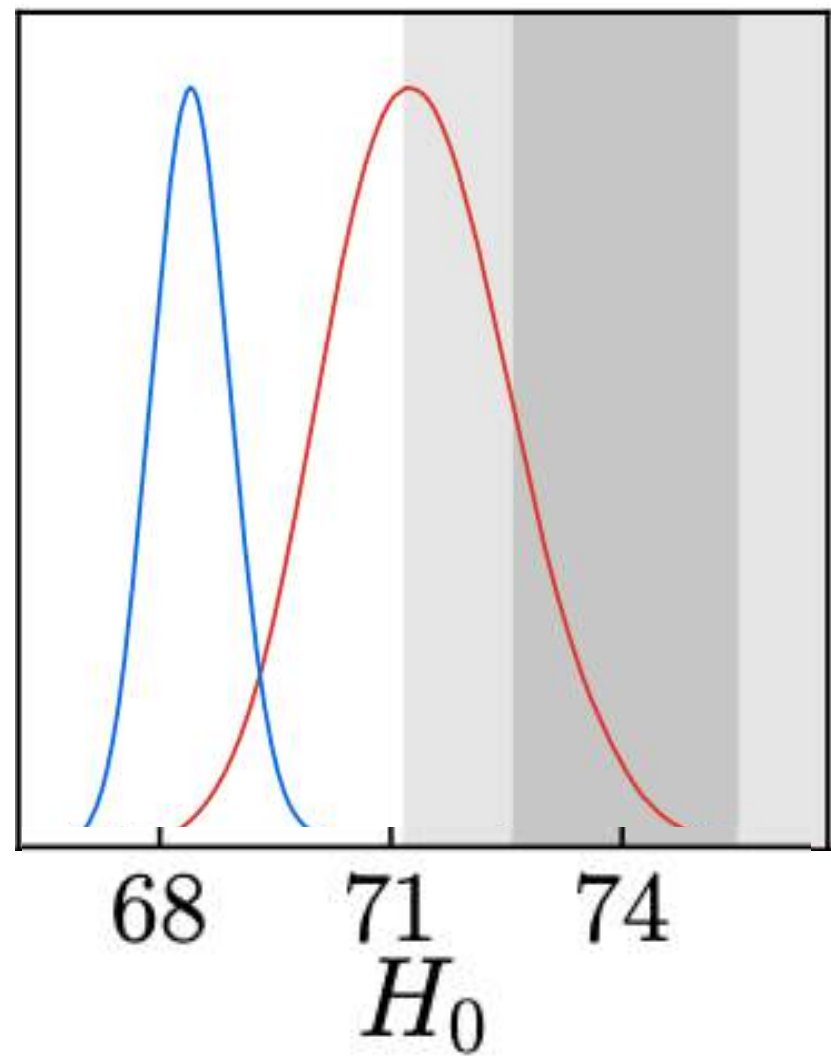
$\theta_s(z_*)$: decrease $r_s(z_*) \implies$ decreases $D_A(z_*)$

Fixed



Increases H_0

Early time solutions to the *Hubble tension*



ΛCDM

EDE

$\theta_s(z_*)$: decrease $r_s(z_*) \implies$ decreases $D_A(z_*) \implies$ Increases H_0
Fixed

\implies If we **decrease r_s by 10 Mpc** (keeping r_s/r_d and r_s/r_{eq} fixed) we can **solve the H_0 tension**

Many ideas on how to do this:

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{dz}{H(z)} c_s(t)$$

Change $H(z)$: increasing $H(z)$ before recombination with additional components

- Light Relics
- Early Dark Energy

Change z_* : change recombination physics - primordial B (Jedamzik + 2020); change m_e or m_e, Ω_k (Sekiguchi+ 2020)

Change c_s (e.g. Boddy + 2018)

The H_0 Olympics: A fair ranking of proposed models, Schöneberg + (2021)

	ΔN_p	
Majoron	3	🥉
primordial B	1	🥈
varying m_e	1	🥇
varying $m_e + \Omega_k$	2	🥇
EDE	3	🥉
NEDE	3	🥉
EMG	3	🥉

*The H_0 Olympics: A fair ranking
of proposed models,
Schöneberg + (2021)*

	ΔN_p	
Majoron	3	②
primordial B	1	③
varying m_e	1	①
varying $m_e + \Omega_k$	2	①
EDE	3	②
NEDE	3	②
EMG	3	②

Early dark energy

Based on Laura Herold, EF 2210.1629

& Laura Herold, EF and Eiichiro Komatsu 2112.12140,

& A. Reeves, L. Herold, S. Vagnozzi, B. Sherwin, EF 2207.01501

Early dark energy

Idea: add an extra component (to increase $H(z)$) that starts acting around equality, behaves as DE and dilutes faster than matter

↓
 ϕ_i

↓
Initially frozen
Slow-rolls

Early dark energy

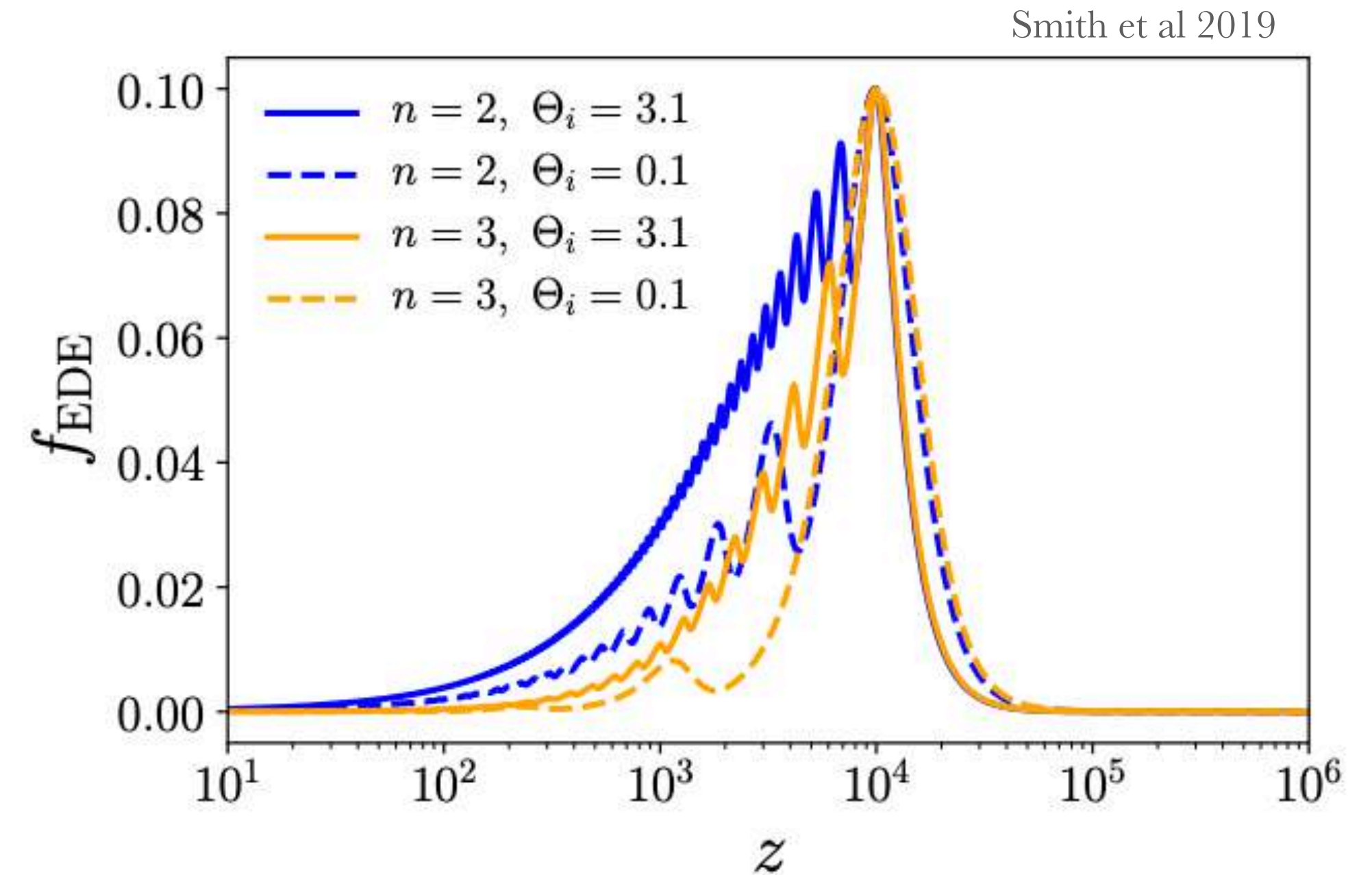
Idea: add an extra component (to increase $H(z)$) that starts **acting around equality**, behaves as DE and **dilutes faster than matter**

$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$

3 free parameters: $\{m, f, n\}$ + IC: ϕ_i

- V_0 or m ($V_0 = m^2 f^2$): the field is ultra-light $m \sim H(z_{eq}) \sim 10^{-27}$ eV
- f (spont. sym. breaking scale)
- n : controls the decay \rightarrow needs to be hidden at late times

$$n \geq 2 \quad (w \geq 1/3)$$



Phenomenological parameters $\{f_{\text{EDE}}(z_c), z_c, n, \theta_i = \phi_i/f\}$

$$f_{\text{EDE}}(z_c) \equiv \frac{\rho_{\text{EDE}}}{\rho_{\text{tot}}} \Big|_{z_c} = \frac{\rho_{\text{EDE}}}{(3M_{\text{pl}}^2 H^2)} \Big|_{z_c}$$

We usually fix $n = 3$

$\Rightarrow \{f_{\text{EDE}}(z_c), z_c, \theta_i\}$

Early dark energy

Does EDE really solves the H_0 tension?

Early dark energy can resolve the *Hubble tension*

V. Poulin, T. Smith, T. Karwal,
M. Kamionkowski, 2019

EDE from CMB

- For: *Planck* + *BOSS DR12 BAO/RSD* + *6dFGS* + *Pantheon*
+ *SHOES 2016*

$$H_0 = 71.49 \pm 1.20 \text{ km/s/Mpc}$$

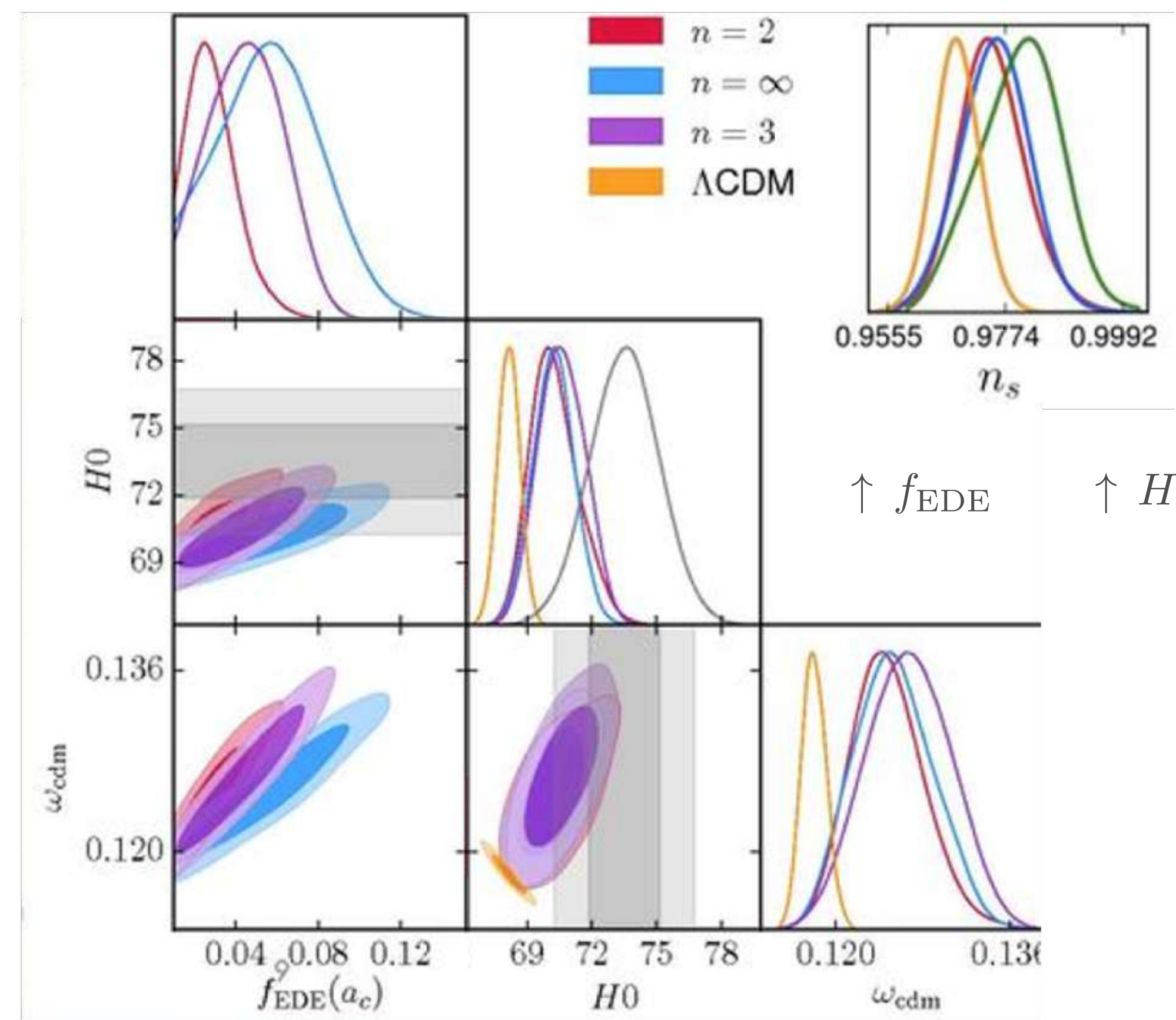
Solves the tension!

$\uparrow f_{\text{EDE}}$ $\uparrow H_0$

But with:

- More DM

- Higher n_s



$$w_{\text{cdm}} = \Omega_{\text{cdm}} h^2$$

Early dark energy does NOT restore cosmological concordance

EDE from **LSS**

2020

Use LSS to constrain EDE

- CMB: *Planck* 2018 TT, TE, EE
- LSS
 - *Planck* lensing
 - “Compressed” likelihood
 - BAO
 - Weak lensing from KIDS+VIKING-450 + HSC
- **FULL SHAPE OF THE PS**

Early Dark Energy Does Not Restore Cosmological Concordance

J. COLIN HILL,^{1,2} EVAN McDONOUGH,³ MICHAEL W. TOOMEY,⁴ AND STEPHON ALEXANDER³

Constraining Early Dark Energy with Large-Scale Structure

Mikhail M. Ivanov,^{1,2} Evan McDonough,³ J. Colin Hill,^{4,5} Marko Simonović,⁶
Michael W. Toomey,⁷ Stephon Alexander,⁷ and Matias Zaldarriaga⁸

Uniform prior in the phenomenological parameters

$$f_{\text{EDE}}, z_c, \theta_i$$

Early dark energy does NOT restore *cosmological concordance*

EDE from LSS

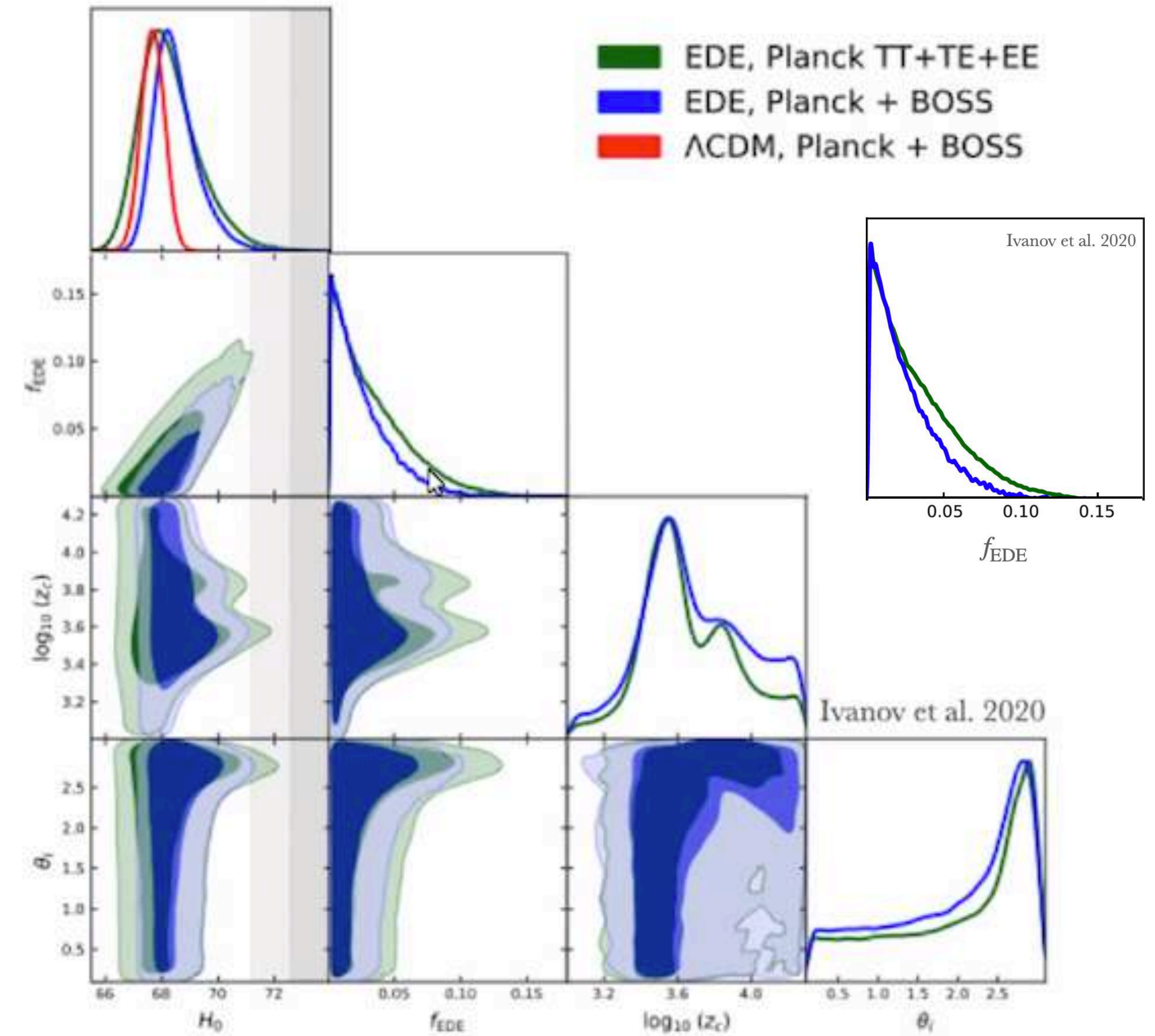
- For: *Planck* + *BOSS DR12 BAO/RSD* + ~~*SHOES 2016*~~ + *full-shape of PS*

Constraints from *Planck* 2018 data + BOSS DR12

Parameter	Λ CDM	EDE ($n = 3$)
f_{EDE}	—	< 0.072 (0.047)
H_0 [km/s/Mpc]	67.70 (67.56) ± 0.42	68.54 (68.83) $^{+0.52}_{-0.95}$
Ω_m	0.3105 (0.3112) $^{+0.0053}_{-0.0058}$	0.3082 (0.3120) $^{+0.0056}_{-0.0057}$

$H_0 = 68.54^{+0.52}_{-0.95}$ km/s/Mpc
 $f_{\text{EDE}} < 0.072$ (95 % CL)
 3.6 σ tension with SHOES !!

Adding S_8 prior $\longrightarrow f_{\text{EDE}} < 0.058$



Early dark energy does NOT solve **Hubble tension!**

Wait...

Not all groups agree with this result

Volume effects?

- Previous result can be a consequence of choice of priors of the EDE parameters

Volume effects: $f_{\text{EDE}} \rightarrow 0$, any value of $\log(z_c)$ and θ_i , degenerate with ΛCDM



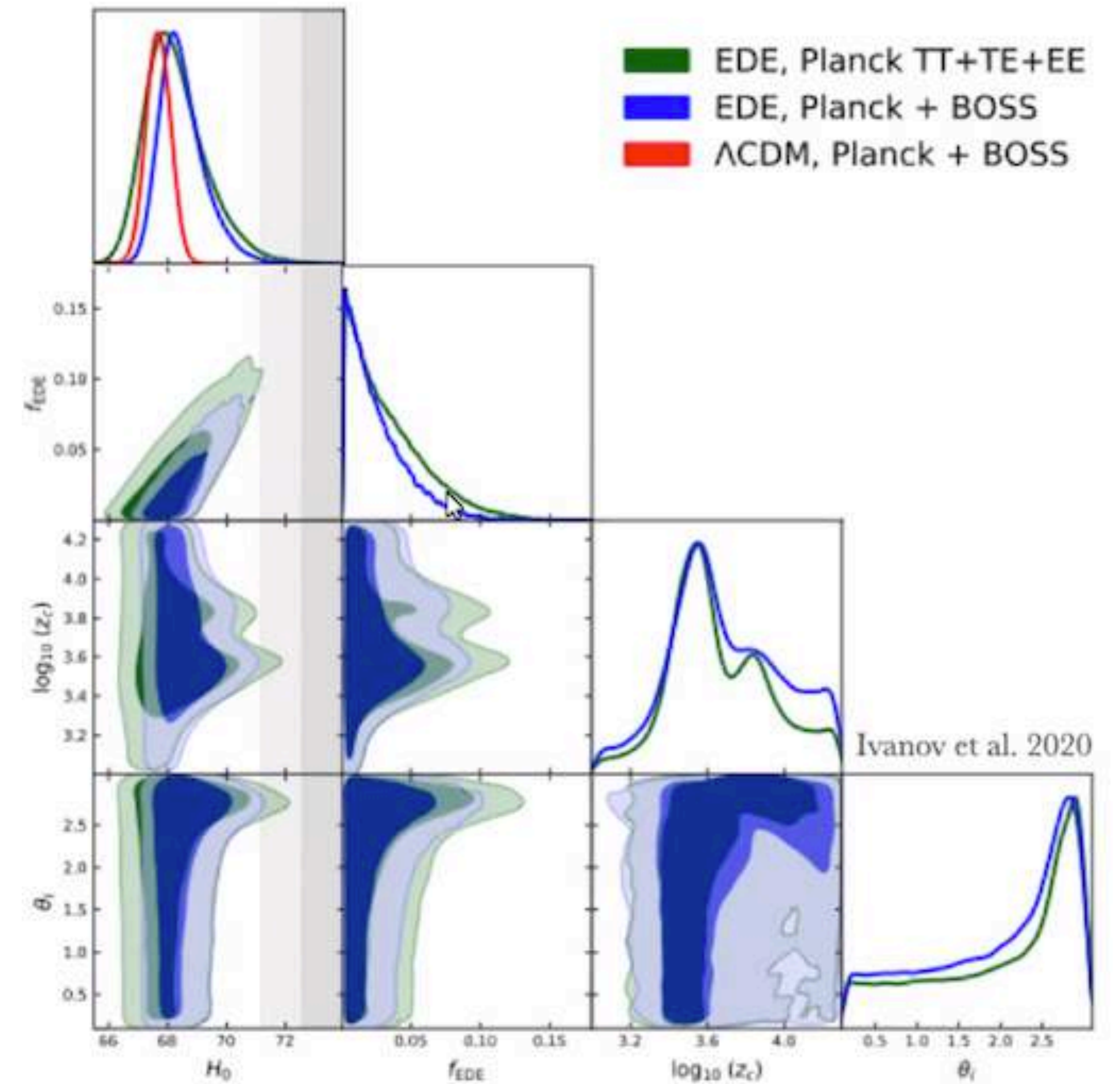
Marginalization: preference for $f_{\text{EDE}} \sim 0$

- $\log(z_c)$ and θ_i are not well constrained by data

Early dark energy is **NOT** excluded by current LSS data

T. Smith et al (2020)

Niedermann, Sloth (2019)



Prior volume *effects*

Bayesian marginalization of the full-dimensional posterior involves integrating out the nuisance dimensions and other cosmological parameters

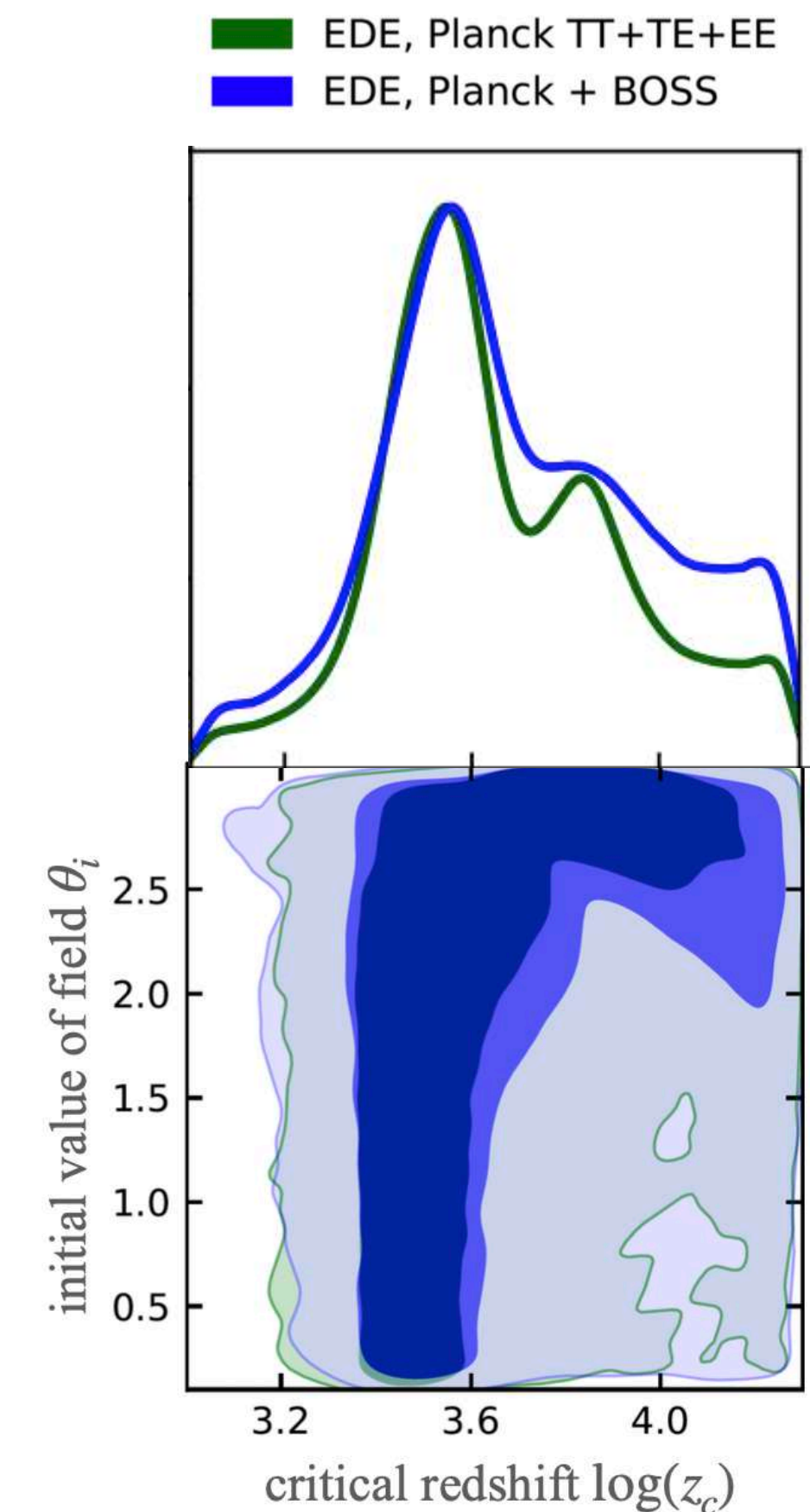
Since in addition to the value of the posterior, an integral is sensitive to the prior volume

→ **Large parameter regions** (of possibly non-maximal posterior values) are emphasized compared to **smaller regions** (of possibly larger posterior values).

Inescapable feature of the Bayesian method!!

(volume effect can occur even with flat priors)

And if this is true, does it bias the parameter so much that we don't know if the Hubble tension is solved or not by EDE.



Prior volume *effects*

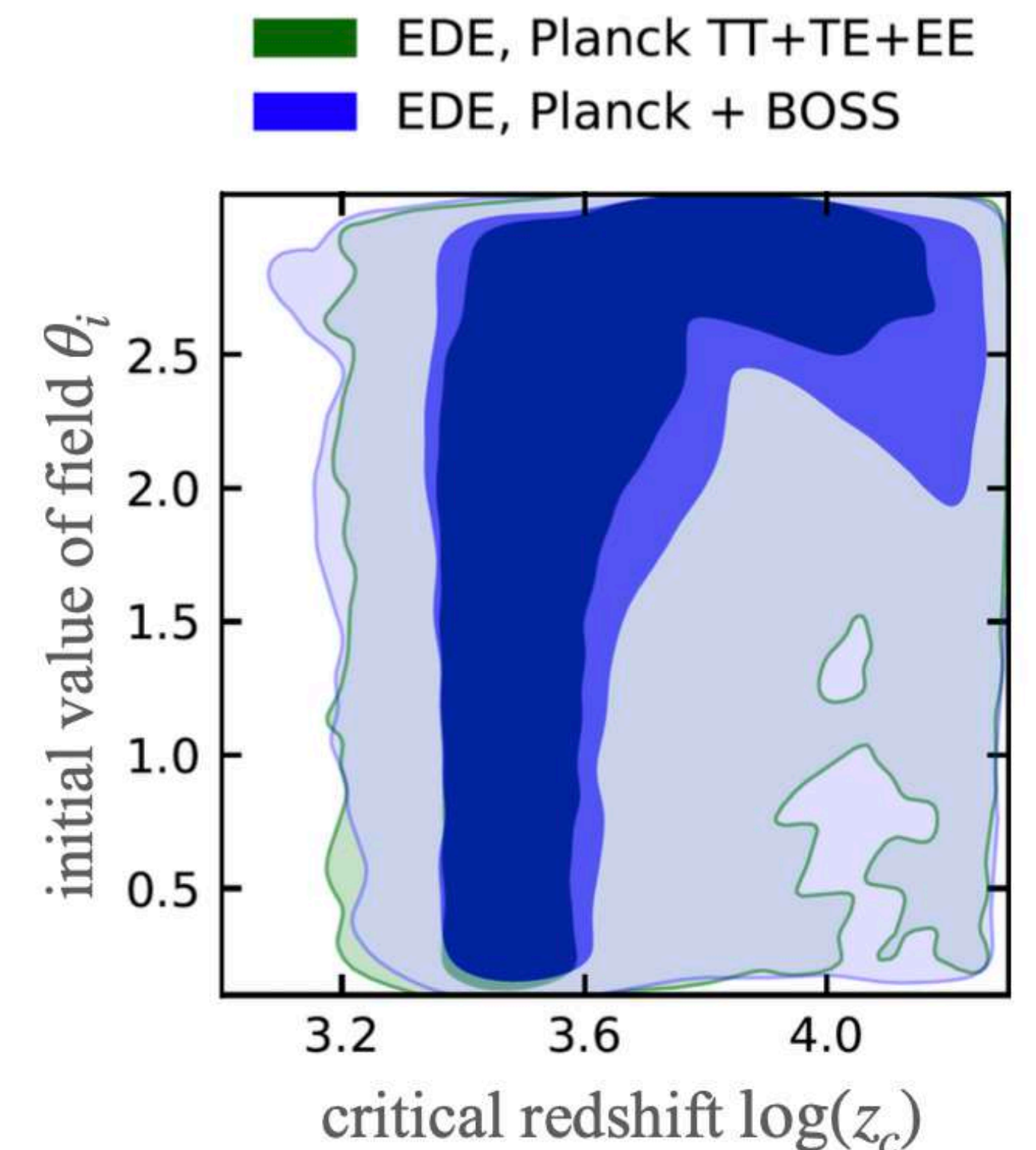
Prior volume effects or marginalization effects: ...appear if the posterior is dominated by the prior volume

When they appear:

- Model has too many parameters / data is not constraining.
- Posterior is very non-Gaussian.
- Parameter structure of the model generates large volume differences.

Leads to:

- **Bias/shifts** in the marginalized posterior
- Inefficiencies in sampling
- Incorrect assessment of the effect of errors, such as systematic errors

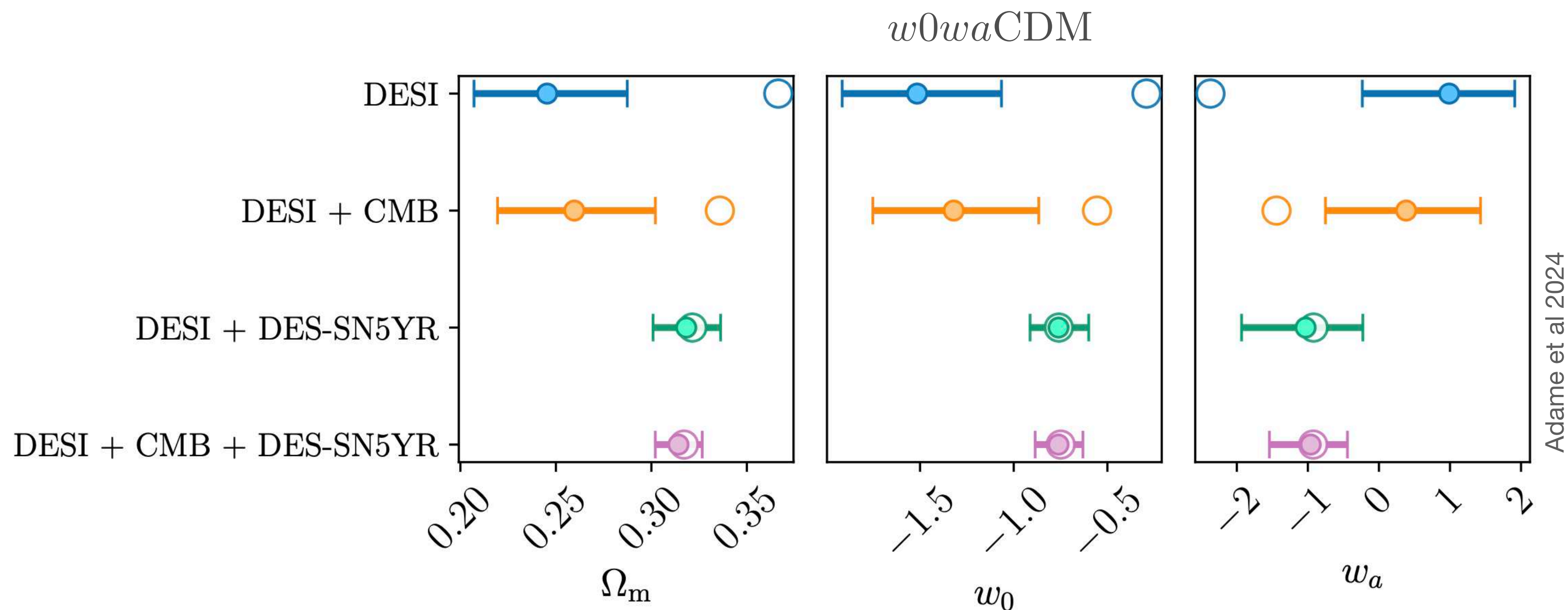


Elisa Ferreira to appear

Ivanov et al. 2020

Important to understand to what extent the results are affected by volume effects!

DESI DR1 full shape analysis



Adame et al 2024

Solid horizontal lines: 95% marginalised posteriors, •: mean
 ○ : MAP (maxima of the corresponding posteriors)

DESI collaboration *doesn't even quote* the DESI and DESI+CMB values since this result is dominated by prior volume effects!

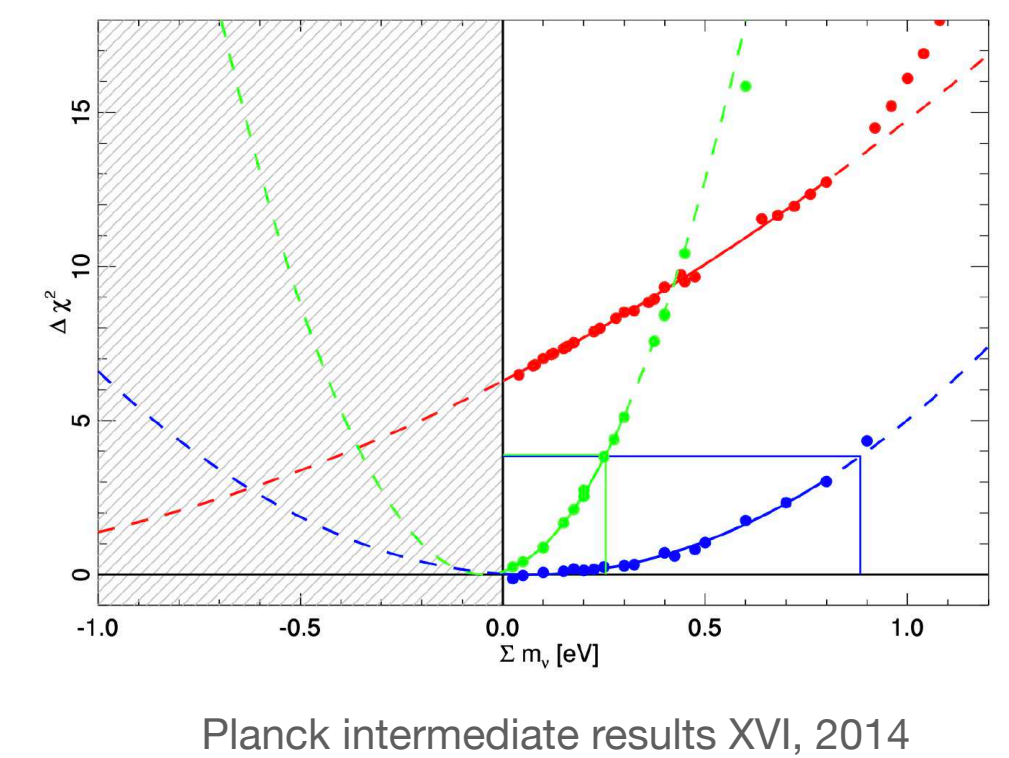
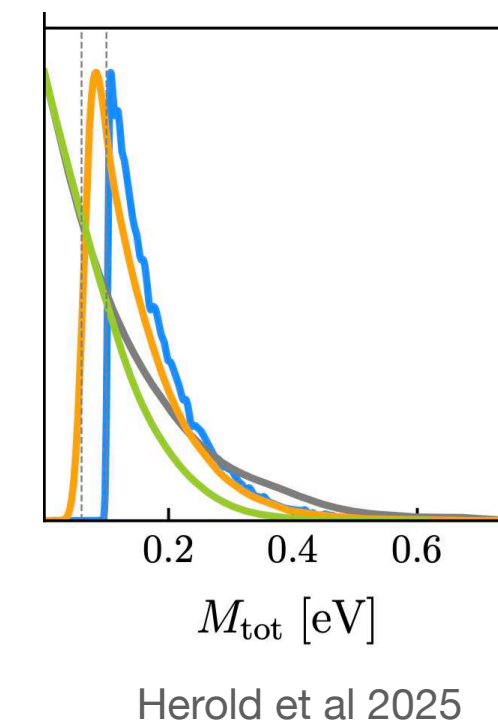
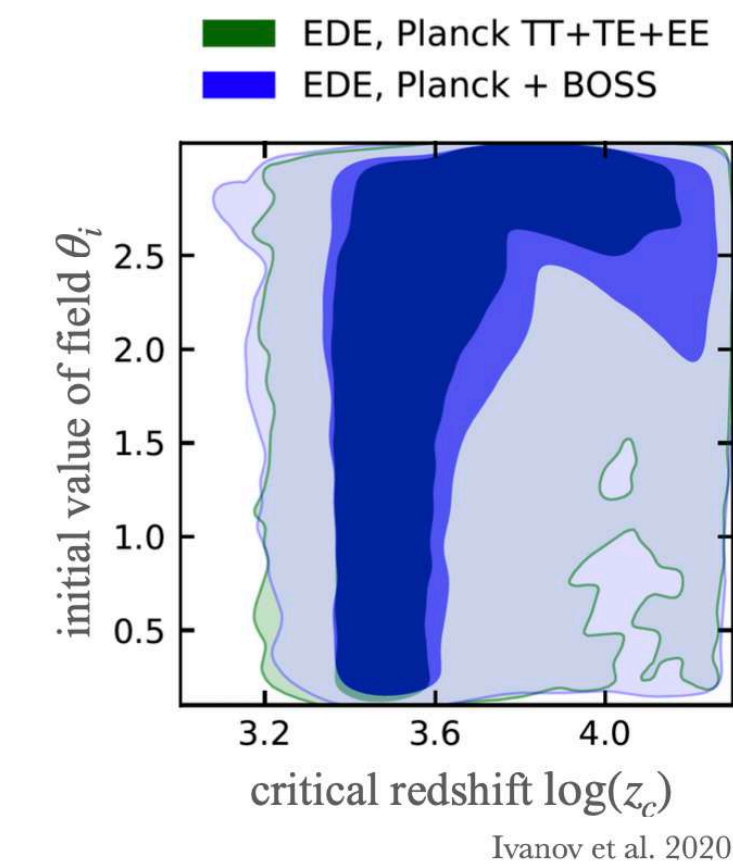
Challenges for data *analysis*

Statistical challenges for parameter inference:

- Prior volume/marginalization/projection effect
- Prior weight effect
- Parameters at the border of the prior volume
- Very non-Gaussian likelihoods
- ...

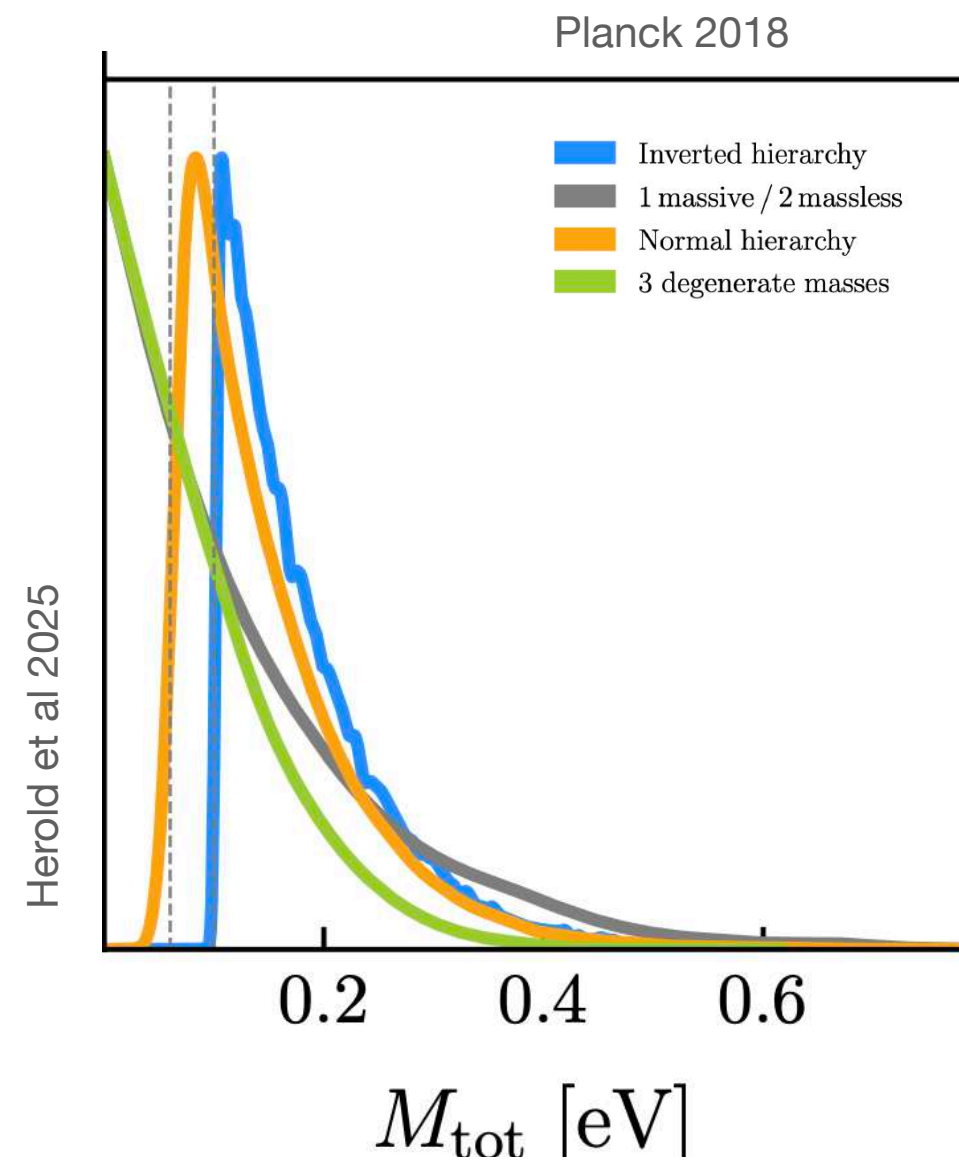
Marginalization

Prior

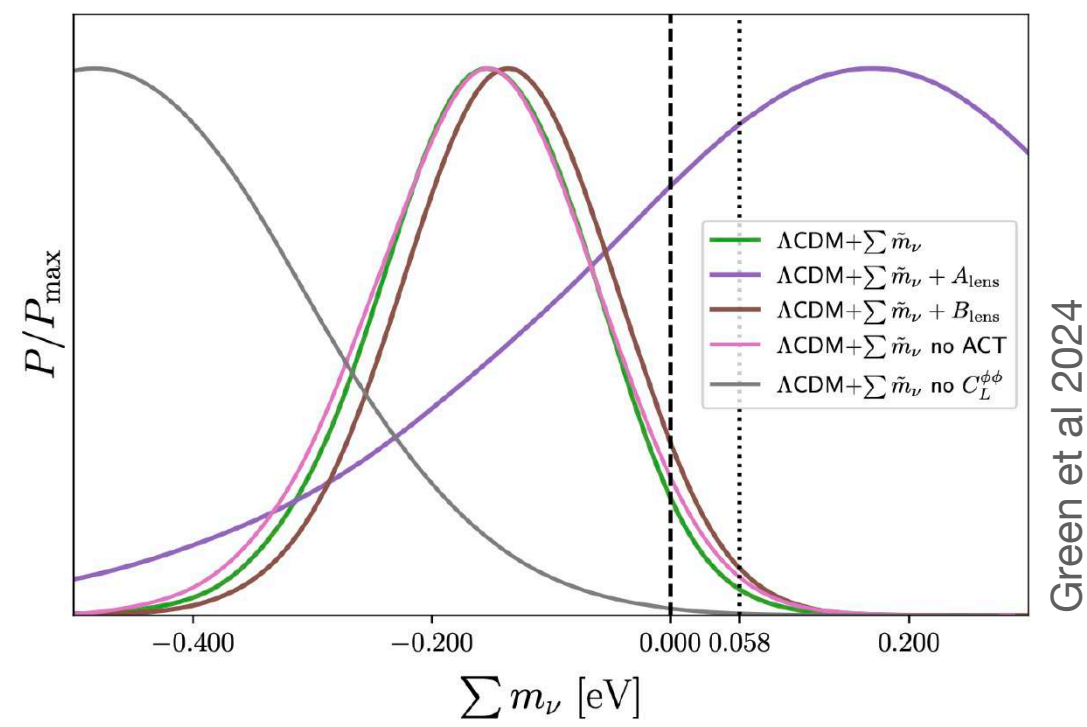


Parameters at the border of the *prior*

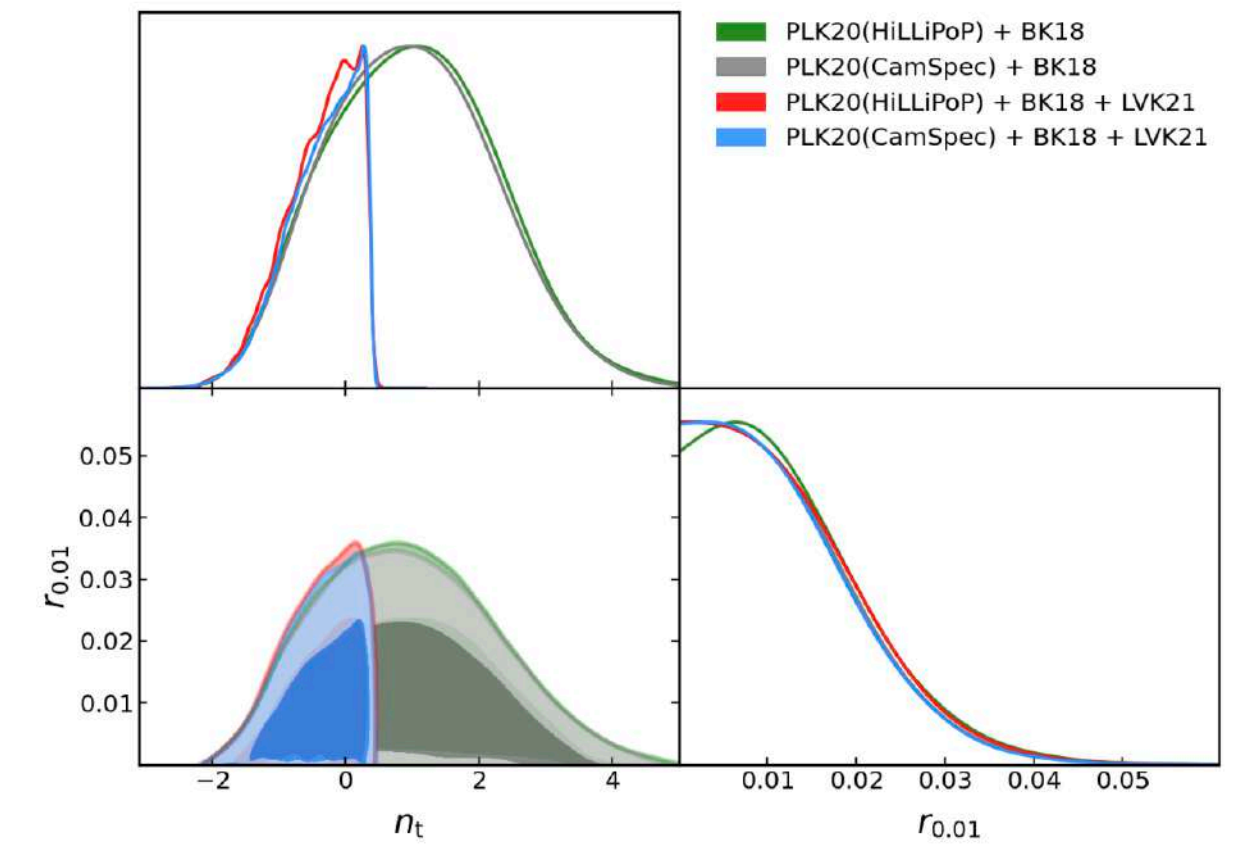
Neutrino mass:



Negative neutrino mass

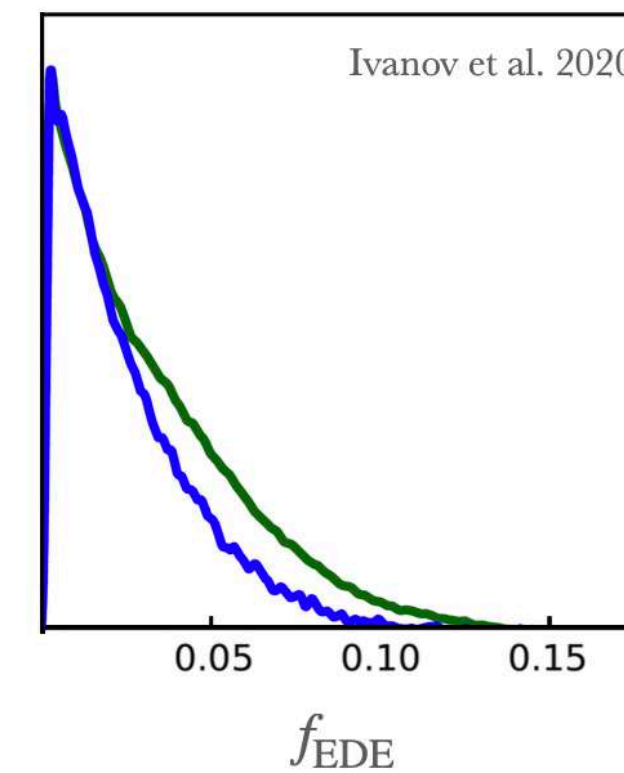


Tensor to scalar ratio (r):



f_{EDE} :

- EDE, Planck TT+TE+EE
- EDE, Planck + BOSS



Bayesian and frequentist

Bayesian statistics

Bayesian inference: derive the posterior probability as a consequence of two antecedents: a prior probability and a "likelihood function" derived from a statistical model for the observed data.

$$\text{Posterior } P(H | E) = \frac{\text{Likelihood } P(E | H) \cdot \text{Prior } P(H)}{P(E)}$$

H: hypothesis
E: evidence

- **Likelihood**: probability of observing E *given* H
- **Prior**: Probability of H *before* E is observed
- **Posterior**: probability of H *given* E, i.e., *after* E is observed. Probability of a hypothesis *given* the observed evidence.

Frequentist approach

$$\mathcal{L}(x|\theta)$$

data model
parameters

Frequentist statistics never uses or calculates the probability of the hypothesis, while Bayesian uses probabilities of data and probabilities of both hypothesis.

Frequentist methods do not demand construction of a prior and depend on the probabilities of observed and unobserved data.

Profile likelihood

Profile Likelihoods in Cosmology: When, Why and How
illustrated with Λ CDM, Massive Neutrinos and Dark Energy,
L. Herold, **EF**, L. Heinrich

Motivation:

Frequentist method for comparison with Bayesian to check for prior or marginalization effects

What is the profile likelihood $\chi^2(\theta)$?

- **Fix** the parameters of interested θ to different values
- **Maximize** the likelihood L (**minimize** $\chi^2 = -2 \ln L$) wrt to the other parameters for different values of the parameter of interest
- For **Gaussian** distribution this gives a **parabola** \rightarrow fit a parabola

Confidence interval:

- A confidence region, can be extracted from the likelihood ratio statistic:

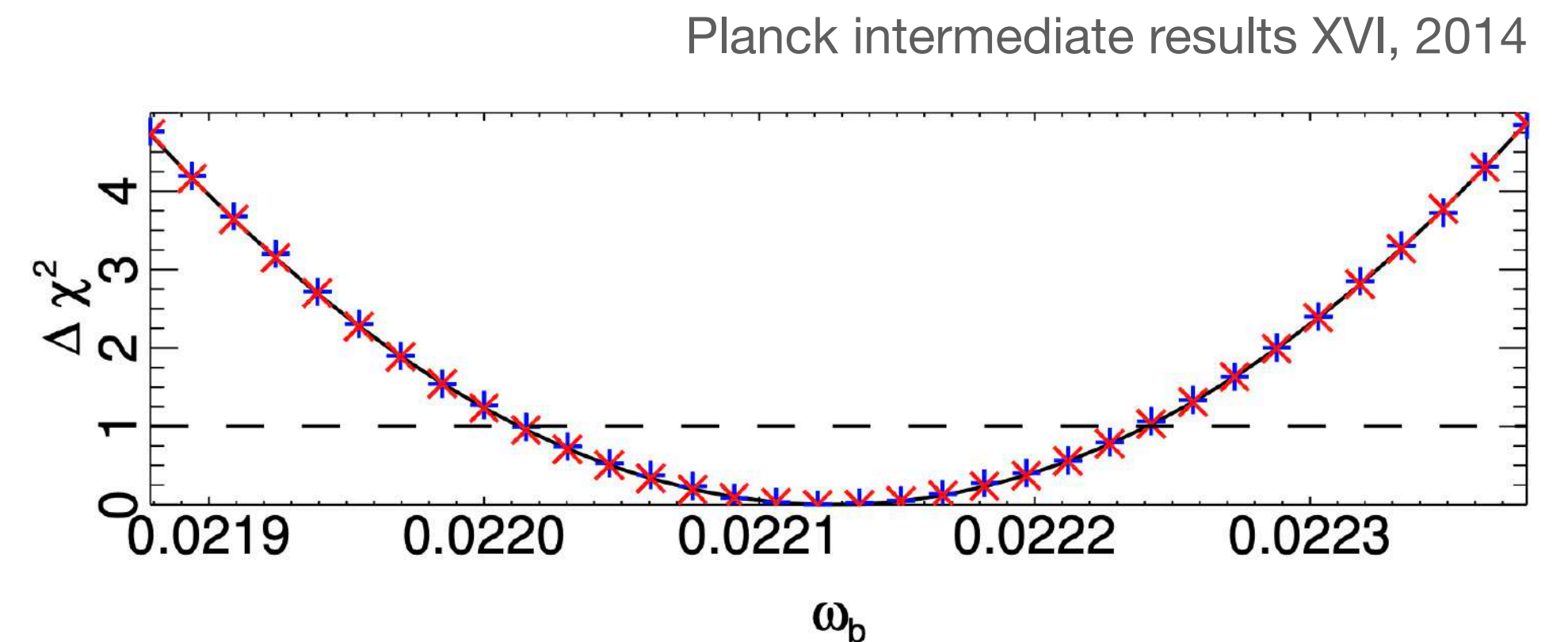
$$\Delta\chi^2(\theta) = \chi^2(\theta) - \chi_{\min}^2 = -2 \ln(\mathcal{L}/\mathcal{L}_{\max})$$

- For parabolic $\chi^2(\theta)$, and one dof, the c.i. is given by:

$$\Delta\chi^2 = 1, 2.7, 3.84 \text{ for } 68, 90 \text{ and } 95\%, \text{ respectively} \longrightarrow \text{Graphical profile likelihood method}$$

(Only valid in the asymptotic limit)

χ_{\min}^2 is obtained from global maximum likelihood estimate given the entire set of parameters



Profile *likelihood*

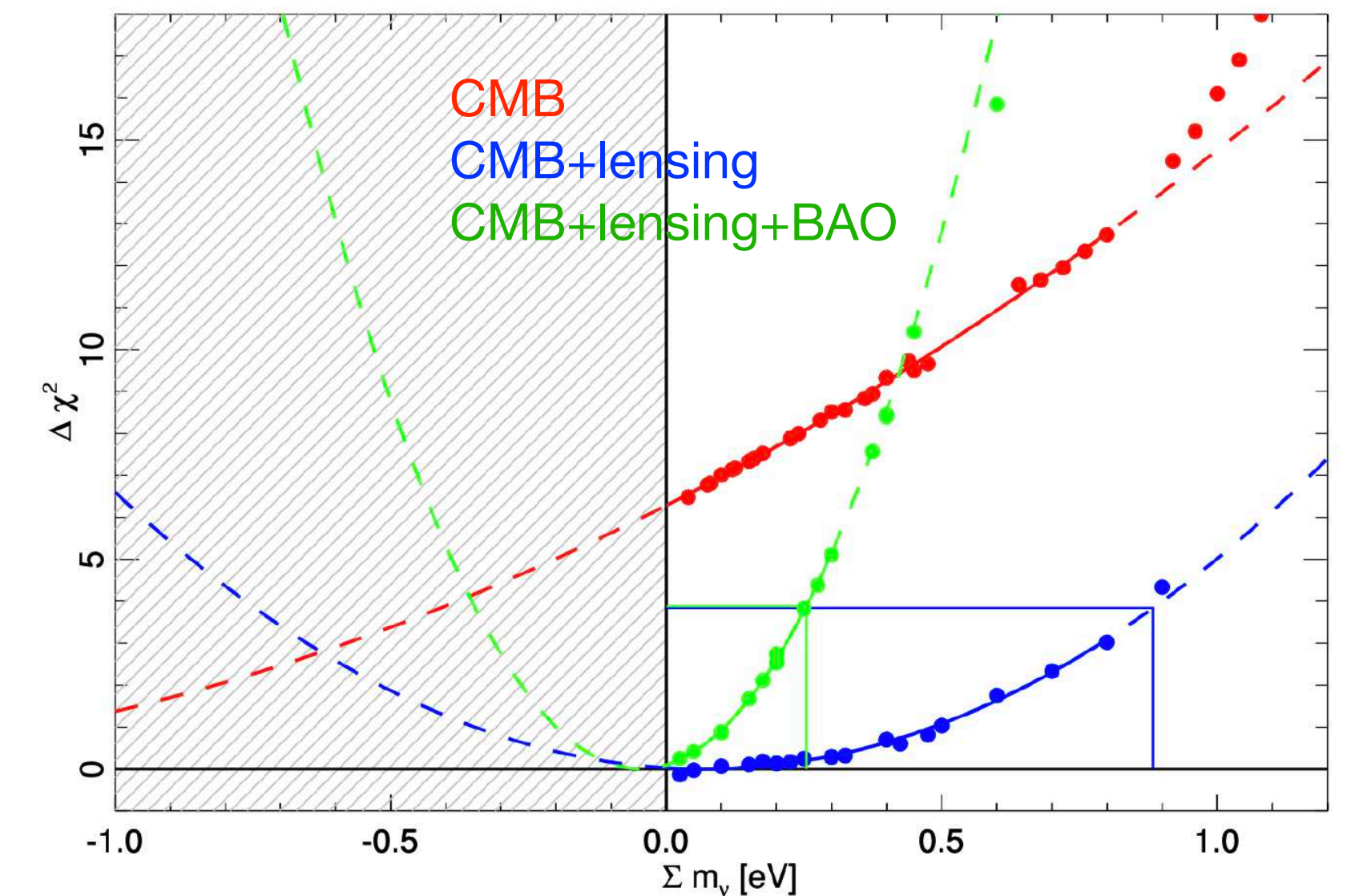
Advantages:

- Not affected by **volume effects** (no priors)
- Invariant under reparametrizations (since $L(\theta)$ is a MLE)
- Allows construction of confidence intervals close to boundaries

Disadvantages:

- Computationally expensive
- When can be used \rightarrow asymptotic limit
- Insensitive to parameter volume; prefer cosmology small parameters volume ("fine tuning")

Profile Likelihoods in Cosmology: When, Why and How illustrated with Λ CDM, Massive Neutrinos and Dark Energy,
L. Herold, EF, L. Heinrich

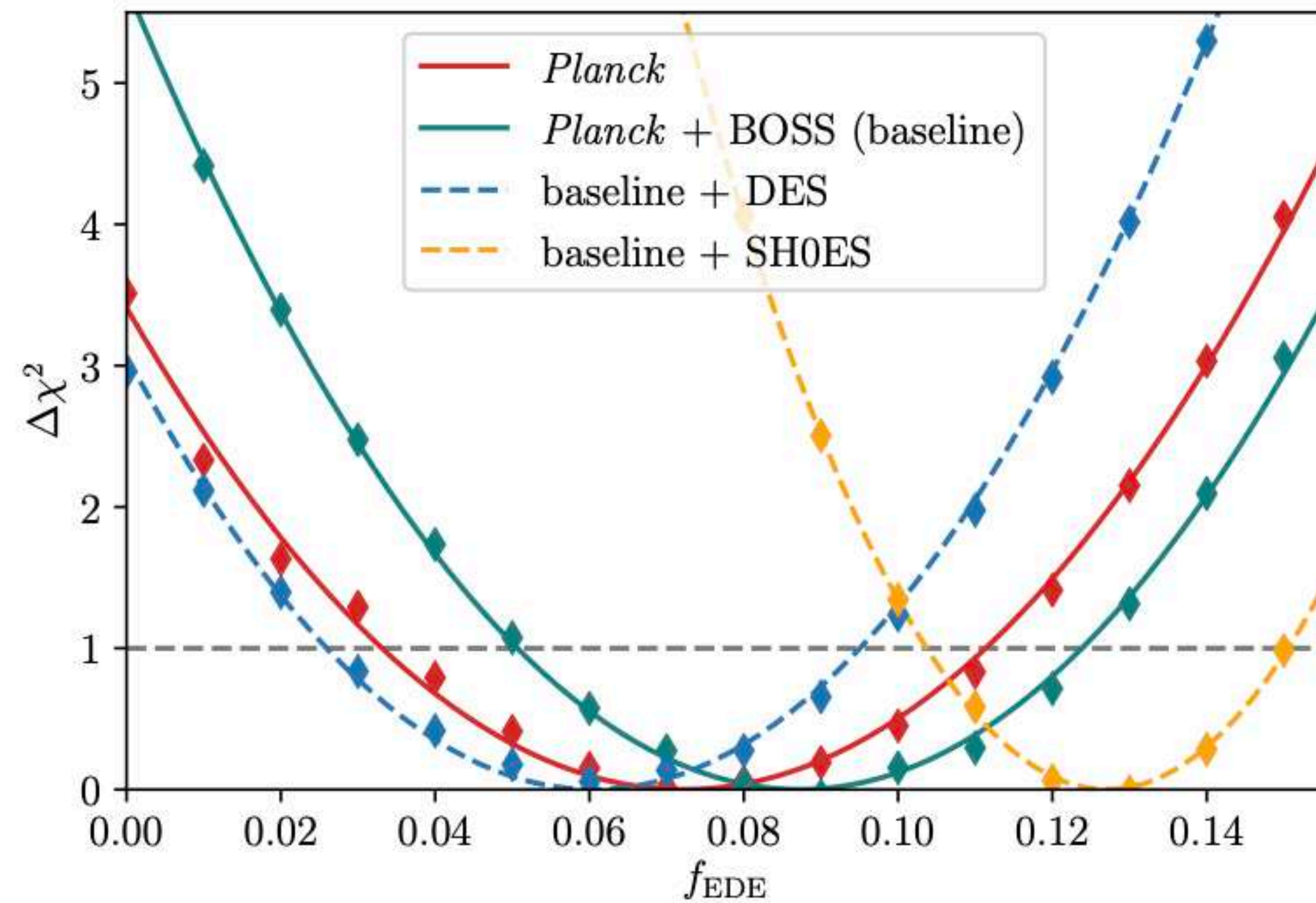


Planck intermediate results XVI, 2014

Going back to our case of *EDE* and the *H0* tension...

Profile likelihood for f_{EDE}

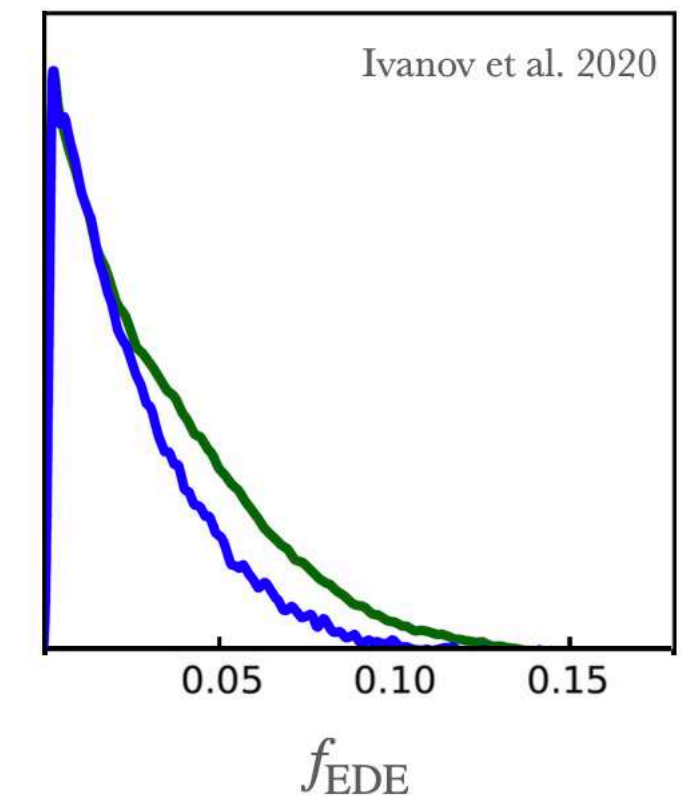
Check for *volume effects*:
comparison to previous results



Bayesian result:

$$f_{\text{EDE}} < 0.072 \quad (95\%CL)$$

■ EDE, Planck TT+TE+EE
■ EDE, Planck + BOSS



$$f_{\text{EDE}}^{(\text{base})} = 0.087 \pm 0.037$$

(68%CL)

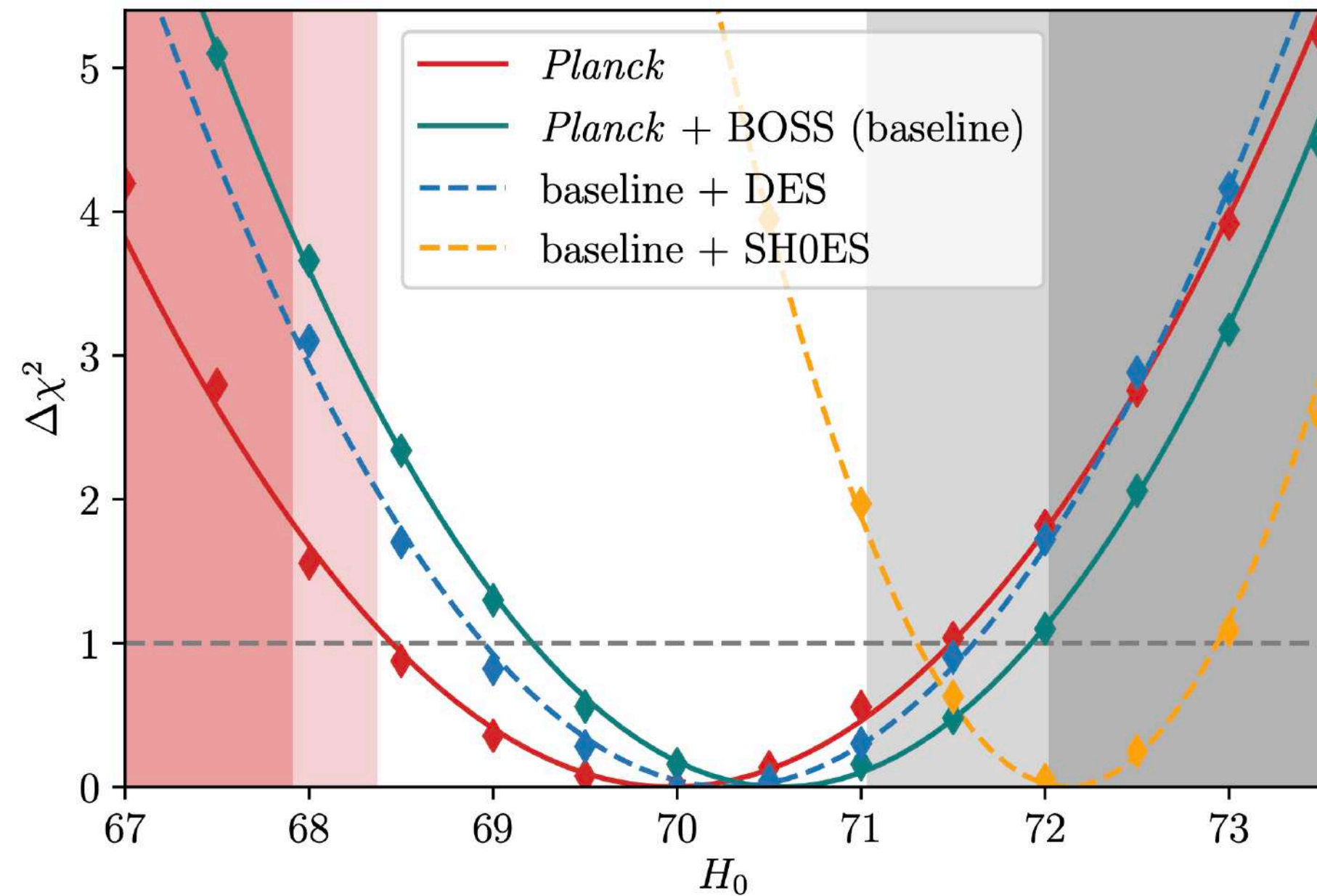
Baseline: Planck+BOSS FS

Profile likelihood for H_0

Laura Herold, EF 2210.1629

Laura Herold, EF and Eiichiro Komatsu 2112.12140

Baseline: Planck+BOSS FS



PL analysis: could show that there indeed is was a prior volume effect in the MCMC analysis.

$$H_0^{\text{EDE},(\text{base})} = 70.57 \pm 1.36$$

(68%CL)

Data set	$\chi^2(\Lambda\text{CDM})$	$\chi^2(\text{EDE})$	$\Delta\chi^2$	f_{EDE}	H_0 (consistency w. SHOES)
Planck	2774.24	2770.72	-3.52	0.072 ± 0.039	69.97 ± 1.52 (1.7σ)
Planck+BOSS (base)	3045.65	3039.98	-5.67	0.087 ± 0.037	70.57 ± 1.36 (1.4σ)
Baseline + DES	3052.06	3049.13	-2.93	$0.061^{+0.035}_{-0.034}$	70.28 ± 1.33 (1.6σ)
Baseline + SHOES	3068.44	3042.08	-26.36	0.127 ± 0.023	72.12 ± 0.82 (0.69σ)

Consistent with SHOES at $< 1.4\sigma$!

No H_0 tension with EDE
(For this dataset)

Solve the confusion in the field and proposed new method

Profile likelihood - advantages

Profile likelihood useful tool:

- Very informative for **diagnosis** and knowledge of your likelihood and parameter space
- Provides *complementary* statistical methods to deal with volume effects
- Useful and informative for parameters **close to a physical boundary** (e.g. Forecasts in LiteBIRD)
- Valuable when priors strongly influence results
- In some cases yields tighter constraints (e.g., neutrinos, r)

Profile likelihood: when, how, why

PL cookbook - practical guide

*Profile Likelihoods in Cosmology: When, Why and How
illustrated with Λ CDM, Massive Neutrinos and Dark Energy,*
L. Herold, **EF**, L. Heinrich

Why compute frequentist confidence intervals?

How to compute the profile likelihood and frequentist confidence intervals?

When does the graphical profile likelihood construction give constraints with correct coverage?

Goal to be the main guide for researcher on PL in cosmology

Profile likelihood: *when, how, why*

PL cookbook - practical guide

Profile Likelihoods in Cosmology: When, Why and How illustrated with Λ CDM, Massive Neutrinos and Dark Energy,
L. Herold, EF, L. Heinrich

Why compute frequentist confidence intervals?

Run the MCMC. If...

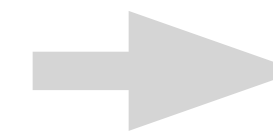
- Some of the parameters of the model are not well constrained; OR
- At a physical boundary

Check:

1. Assess the sensitivity of the results on the choice of prior
2. Compute point estimates like the maximum likelihood estimate (MLE) or maximum a posteriori (MAP)

Compute PL if:

1. Dependence of the results on the choice of prior
2. MLE/MAP strongly deviates from the mean



Hint that volume effects might be strong!

How to compute the profile likelihood and frequentist confidence intervals?

When does the graphical profile likelihood construction give constraints with correct coverage?

Profile likelihood: when, how, why

Profile Likelihoods in Cosmology: When, Why and How
illustrated with Λ CDM, Massive Neutrinos and Dark Energy,
L. Herold, EF, L. Heinrich

PL cookbook - practical guide

Why compute frequentist confidence intervals?

How to compute the profile likelihood and frequentist confidence intervals?

Assume that the Gaussian approximation or Wilks' theorem holds, and the graphical profile likelihood method is used
(*acknowledging that correct coverage might not be fulfilled*)

1. Compute a profile likelihood using an efficient minimizer (ex. pinc)
2. Construct a confidence interval. For that, it is relevant whether the parameter is near a physical boundary.
 - (i) If far away from boundary, one can use the simple graphical profile likelihood method
 - (ii) If the parameter is near a physical boundary, one needs to use the boundary-corrected graphical construction

When does the graphical profile likelihood construction give constraints with correct coverage?

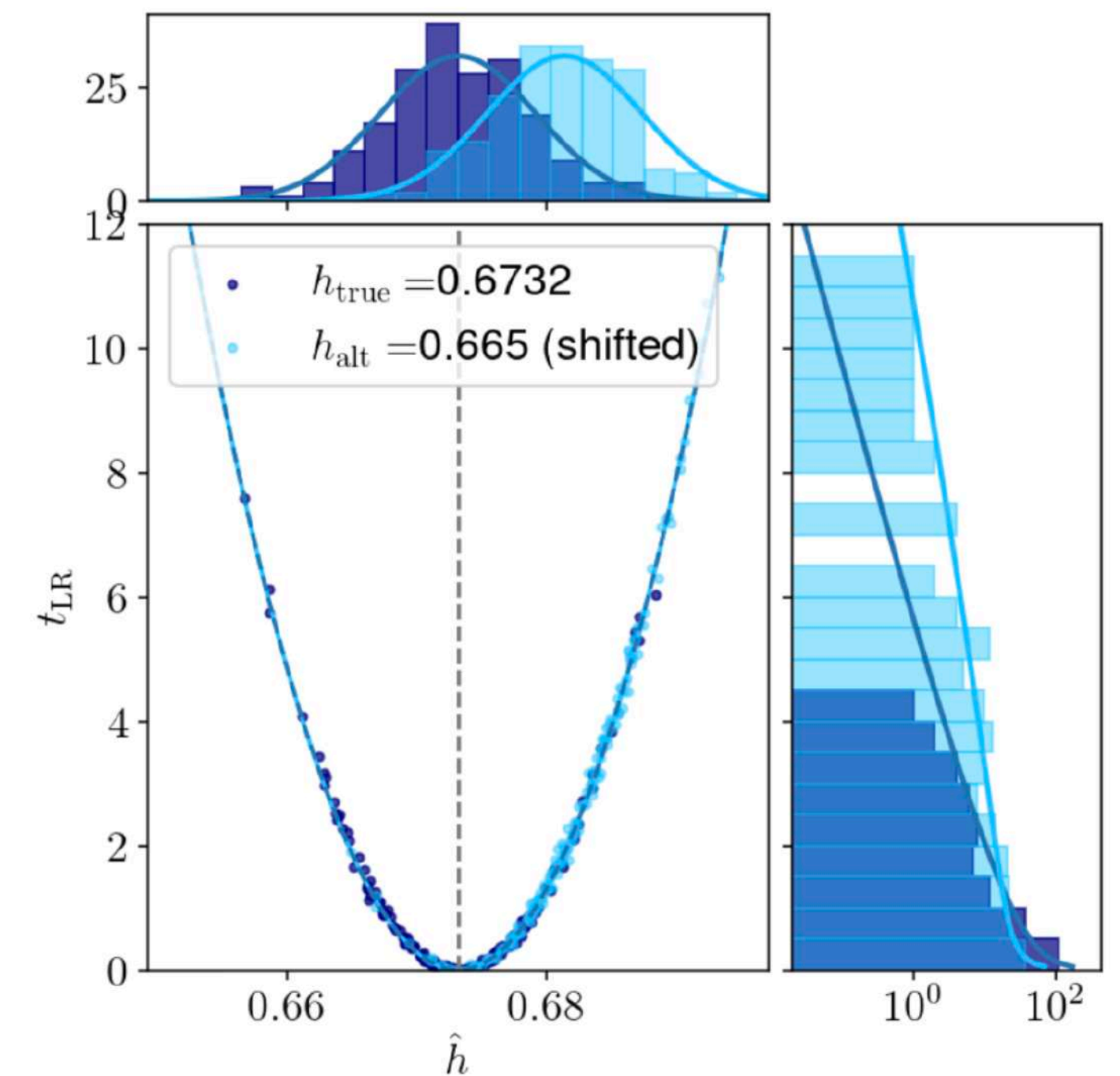
Profile likelihood: *when, how, why*

Profile Likelihoods in Cosmology: When, Why and How illustrated with Λ CDM, Massive Neutrinos and Dark Energy, L. Herold, EF, L. Heinrich

When does the graphical profile likelihood construction give constraints with correct coverage?

Verify whether the mocks follow the predictions in the asymptotic limit

- Λ CDM: tests indicate that asymptotic assumptions hold \rightarrow *graphical profile likelihood method*
- Λ CDM + M_ν : tests indicate consistency with Gaussian near boundary \rightarrow *boundary-corrected graphical method (Feldman-Cousins)*
- w CDM: tests indicate violation asymptotic assumptions \rightarrow *should use full Neyman construction*



Profile likelihood: codes

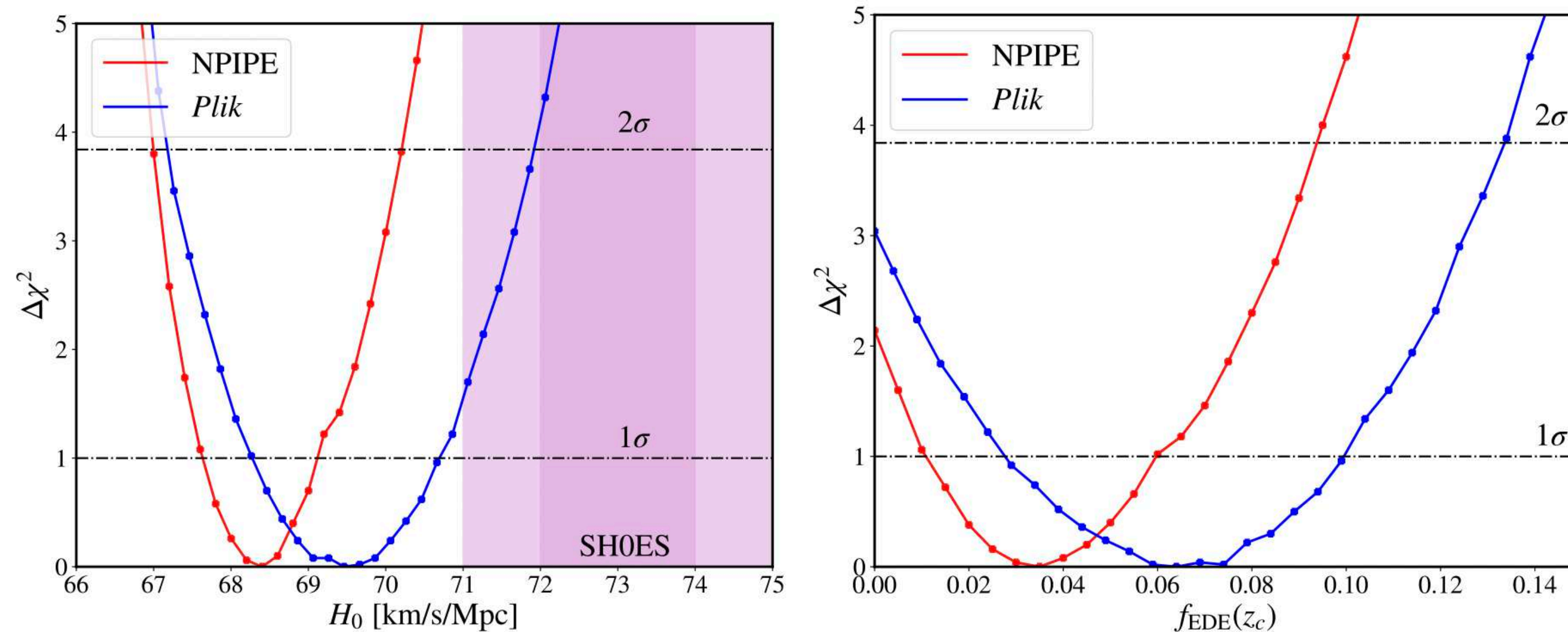
- CAMEL (Henrot-Versillé et al 2016)
- PROSPECT (Holm et al 2023)
- PROCOLI (Karwal et al 2024)
- **pinc (Herold, EF 2024)**
- CONNECT (Nygaard et al 2022)
- ...

Early dark energy - reanalysis

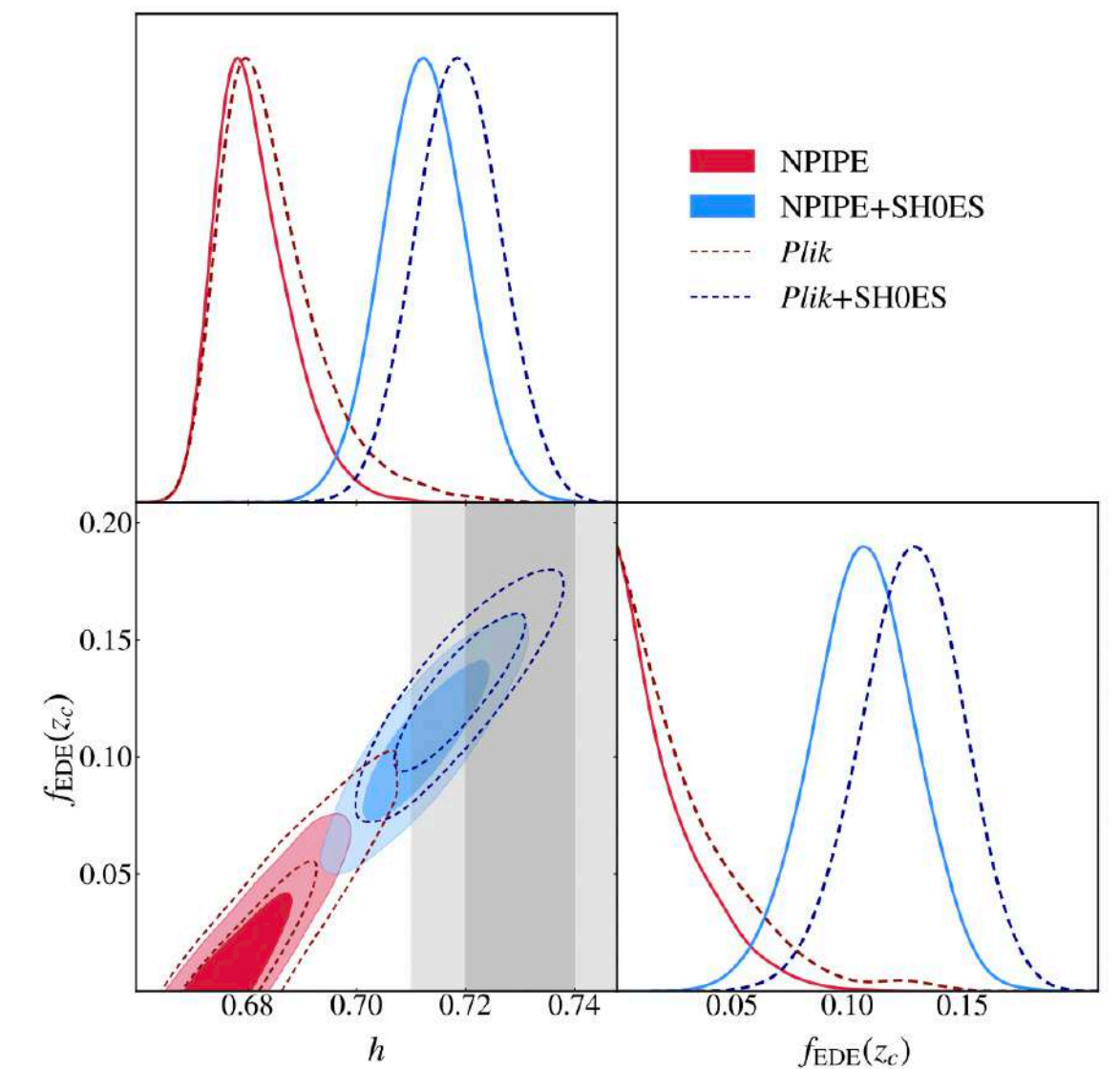
George Efstathiou, Erik Rosenberg, and Vivian Poulin 2023,
*Improved Planck constraints on axion-like early dark energy as a resolution
of the Hubble tension*

Reanalysis using **new** *Planck* likelihood (NPIPE)+CamSpec: different analysis choices, more data

Profile likelihood:



MCMC:



PL (MCMC) analysis: $H_0^{\text{EDE},(\text{NPIPE})} = 68.37 \pm 0.0075$ ($68.11^{+0.0047}_{-0.0082}$)

Distance from SHOES at 3.7σ

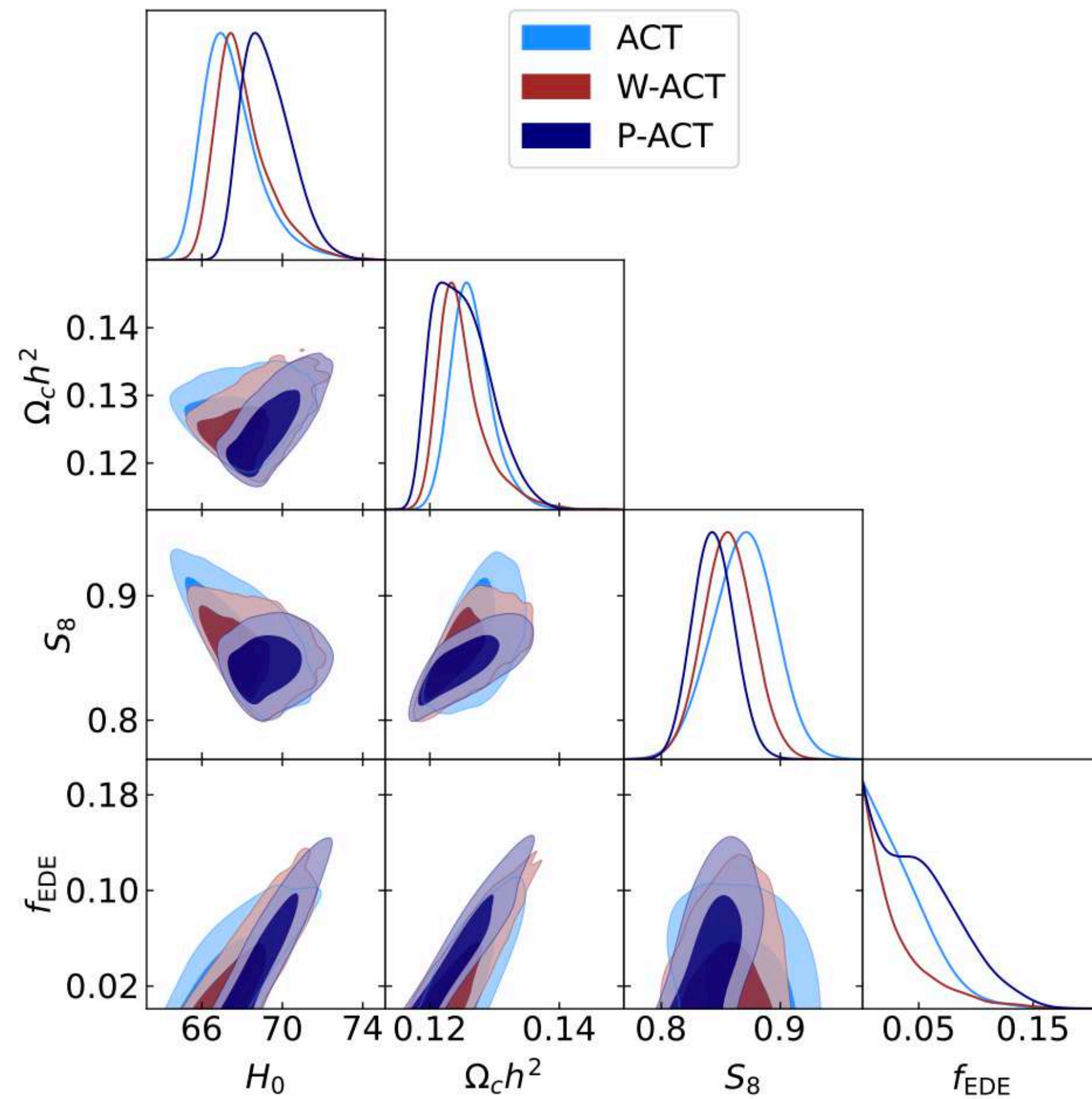
Volume effects present in the old likelihood.

NO volume effects with new likelihood!

ACT DR6

Early dark energy

Calabrese et al 2025



$$V(\phi) = \Lambda^4 [1 - \cos(\phi/f_a)]^3$$

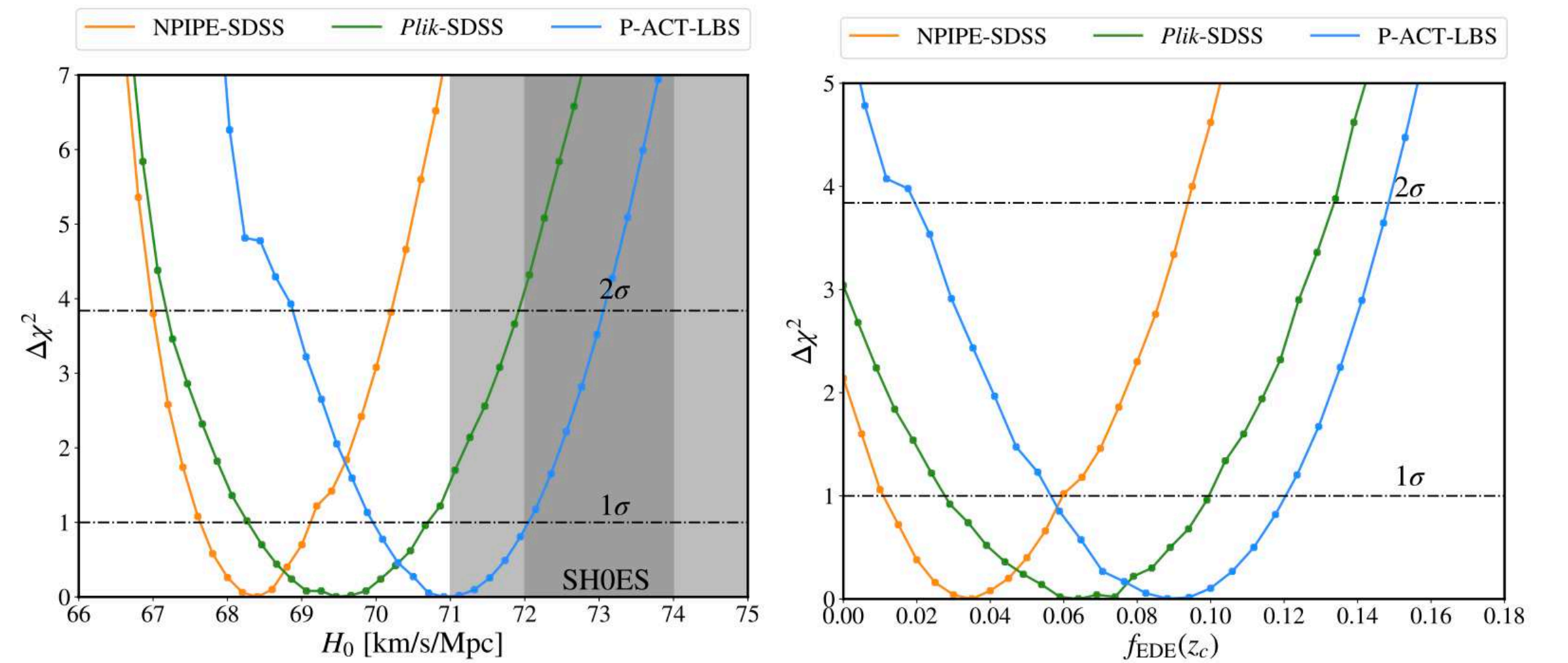
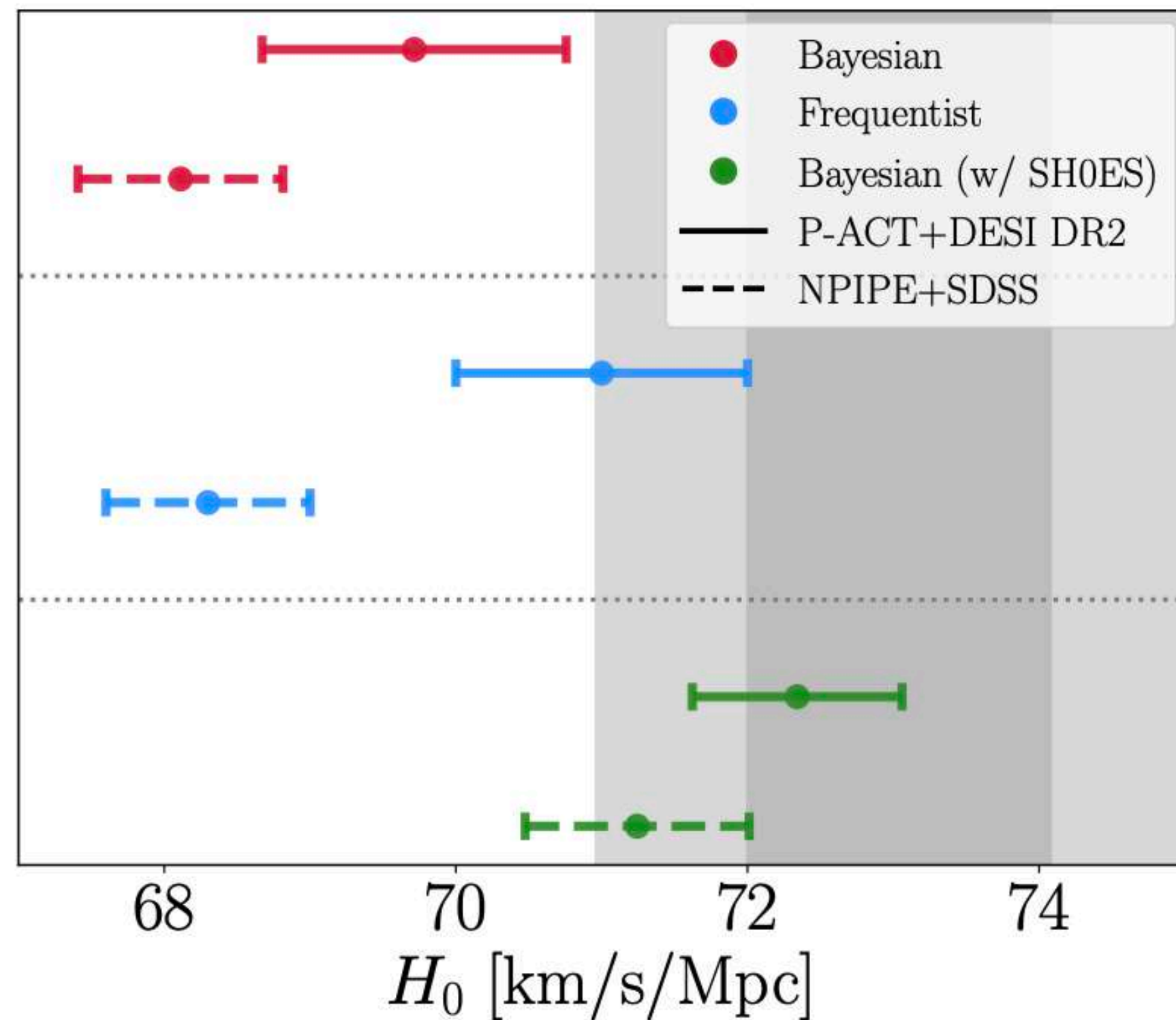
$$f_{\text{EDE}} < 0.088 \quad (95\%, \text{ACT}),$$
$$f_{\text{EDE}} < 0.12 \quad (95\%, \text{P-ACT}),$$

$$H_0 = 67.5^{+0.9}_{-1.7} \quad (68\%, \text{ACT}),$$
$$H_0 = 69.3^{+0.9}_{-1.5} \quad (68\%, \text{P-ACT}),$$

ACT DR6 - profile likelihood

Early dark energy

Poulin et al 2025



a residual tension with SH0ES of $\sim 2\sigma$ for the combination of *Planck* at $\ell < 1000$ + ACT DR6 + lensing + Pantheon-plus + DESI DR2, a significant decrease from 3.7σ for analyses that use NPIPE and SDSS BAO data. A profile likelihood analysis reveals significant prior-volume effects in Bayesian analyses which do not include SH0ES, with confidence intervals of $f_{\text{EDE}} = 0.09 \pm 0.03$ and $H_0 = 71.0 \pm 1.1$ km/s/Mpc. When including DESI data, the EDE model with $H_0 = 73$ km/s/Mpc provides a better fit than the Λ CDM model with $H_0 = 68.4$ km/s/Mpc. The inclusion of SH0ES data rises the preference well above 5σ , with $\Delta\chi^2 = -35.4$. Our work demonstrates that after ACT DR6 and DESI DR2, EDE remains a potential resolution to the Hubble tension.

Application to other cases

Prior effects in EFTofLSS analyses of full-shape BOSS and eBOSS data

E.Holm, L. Herold, T. Simon, **EF**, S. Hannestad, V. Poulin, T. Tram 2023

- Discrepancy between MCMC and PL results for both parametrizations of full-shape: WC and EC - **prior volume effects!**
- Profile likelihoods from WC and EC coincide - no more discrepancy

Strong prior dependence!

- Use this tool carefully!

Main tool for next generation of surveys

- Show that more constraining power reduces the difference between the Bayesian and frequentist approaches.

Sum of neutrino masses

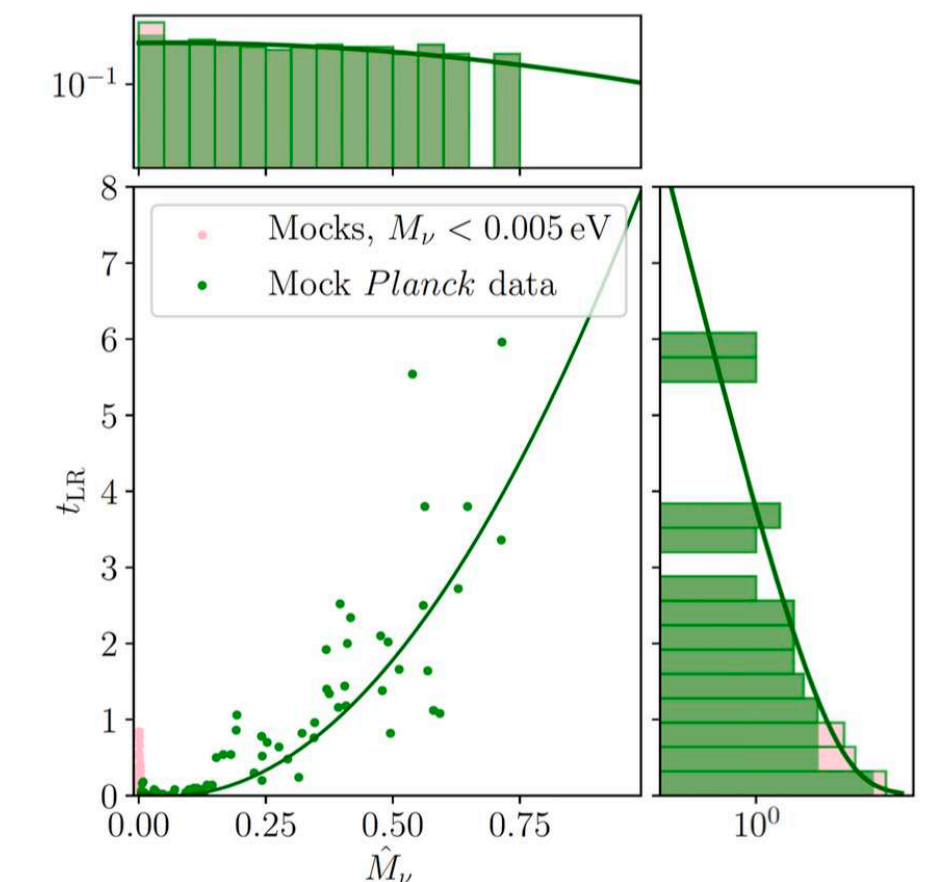
Profile Likelihoods in Cosmology: When, Why and How illustrated with Λ CDM, Massive Neutrinos and Dark Energy, L. Herold, **EF**, L. Heinrich

Sum of neutrino mass is close to a physical boundary
 $\sum m_\nu = M_\nu > 0 \rightarrow$ **strong prior dependence**

Perfect target for profile likelihood

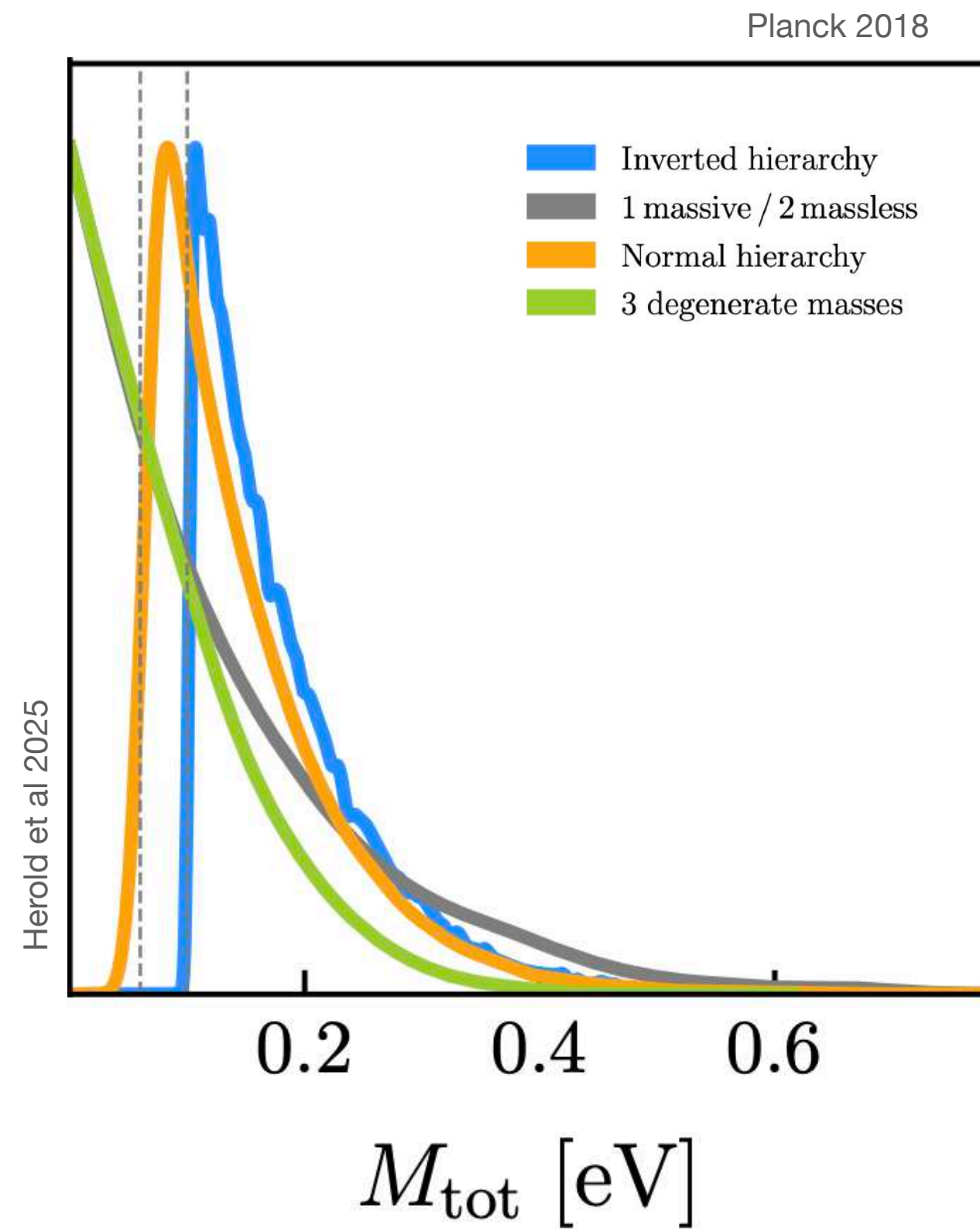
The presence of the physical border, $M_\nu \geq 0$, leads to an **accumulation of points** near $M_\nu = 0$.

Using PL can help yield correct results in this case!

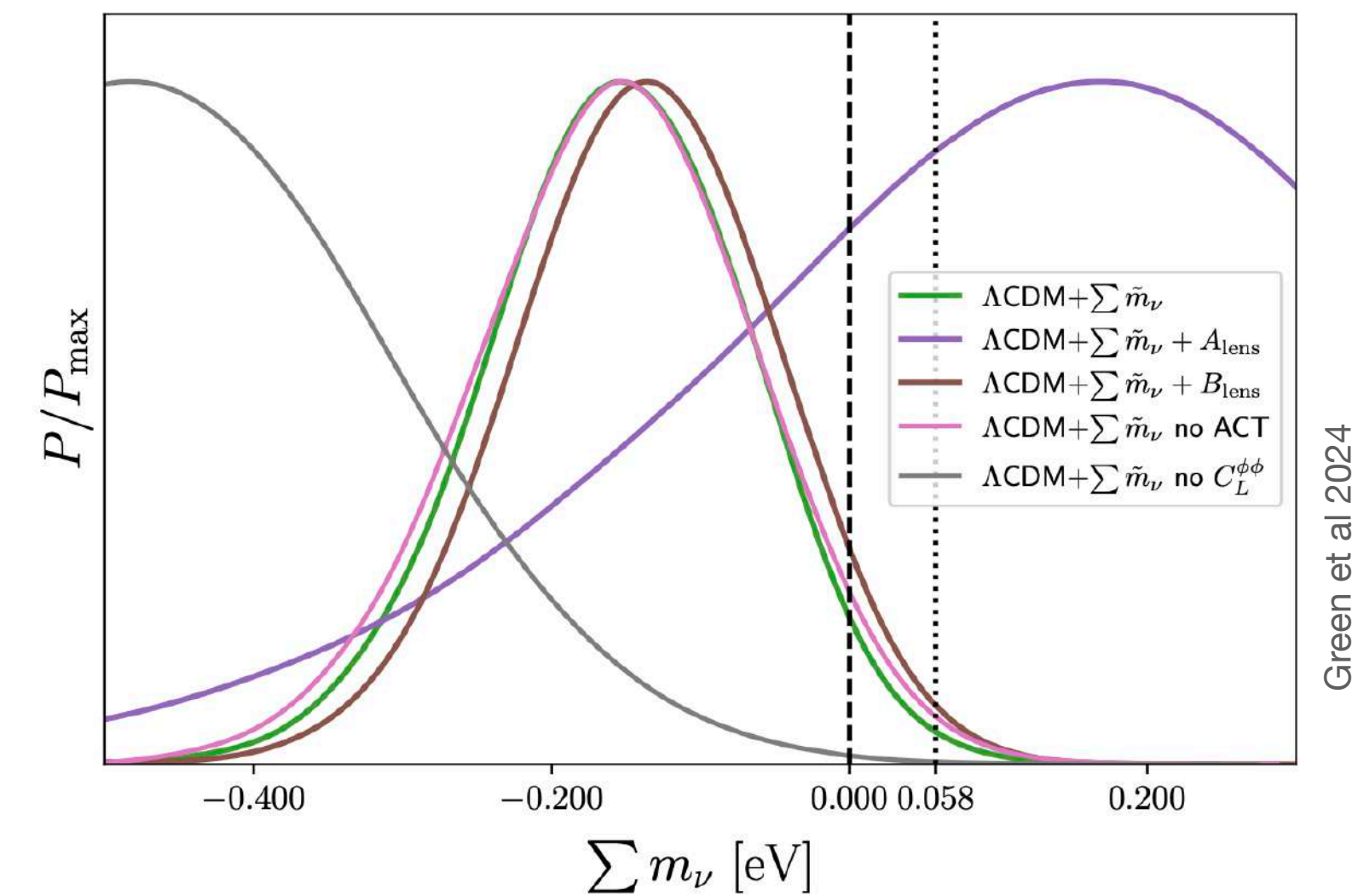


Neutrino mass

Sum of neutrino mass is close to a physical boundary $\sum m_\nu = M_\nu > 0 \rightarrow$ strong dependence on the prior



Negative neutrino mass



Neutrino mass

Profile Likelihoods in Cosmology: When, Why and How illustrated with Λ CDM, Massive Neutrinos and Dark Energy, L. Herold, EF, L. Heinrich

Sum of neutrino mass is close to a physical boundary $\sum m_\nu = M_\nu > 0 \rightarrow$ strong dependence on the prior

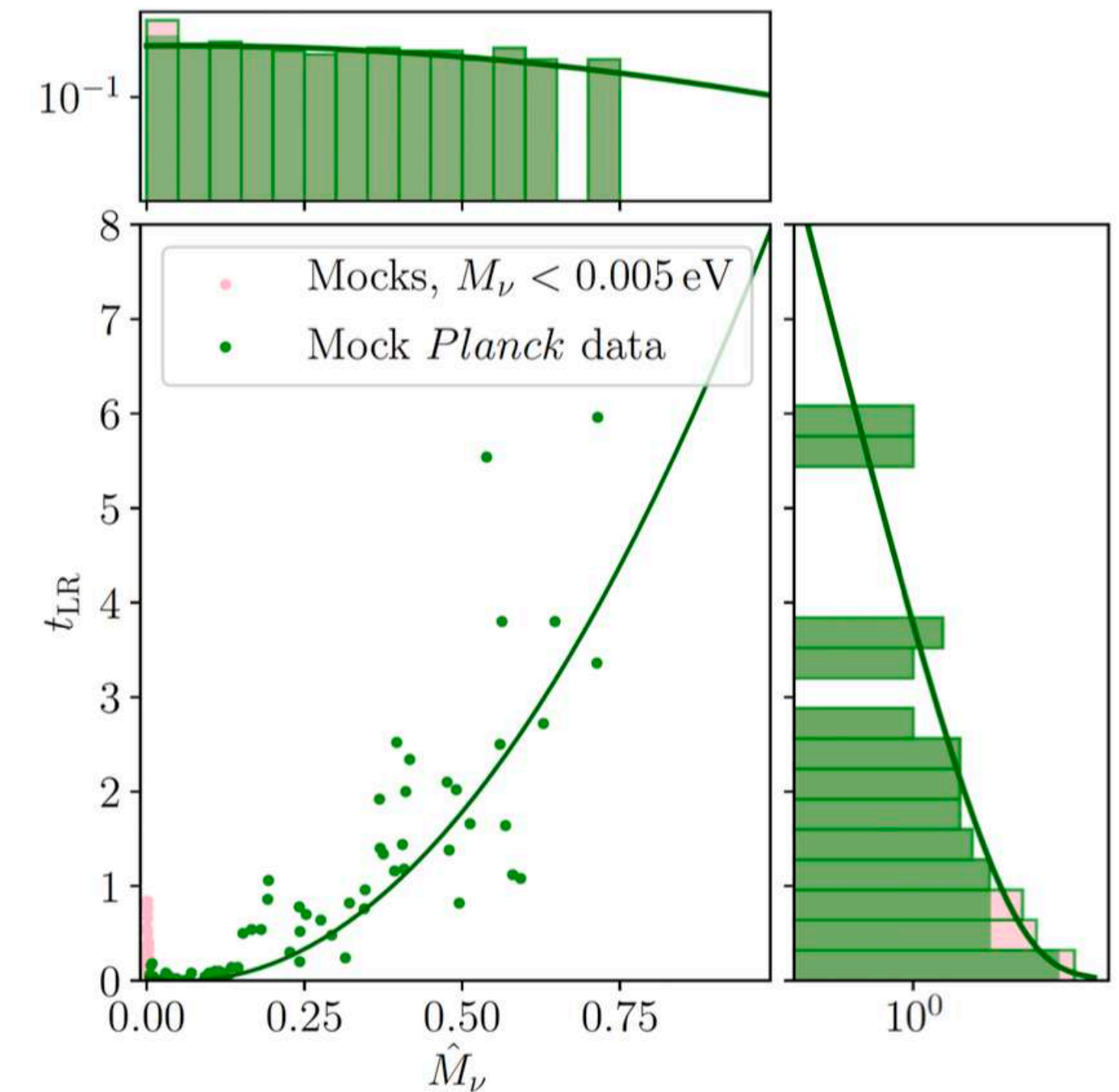
Perfect target for profile likelihood

The presence of the physical border, $M_\nu \geq 0$, leads to an **accumulation of points** near $M_\nu = 0$.

For all checks we did in Λ CDM+ M_ν :

- large scatter: more complex model or intrinsic? Distribution of M_ν deviates from Gaussian?
- Deviation from Wilks & Walds

\rightarrow Indicates that boundary-corrected graphical method (Feldman-Cousins construction) gives correct coverage \checkmark *ok to use PL!*



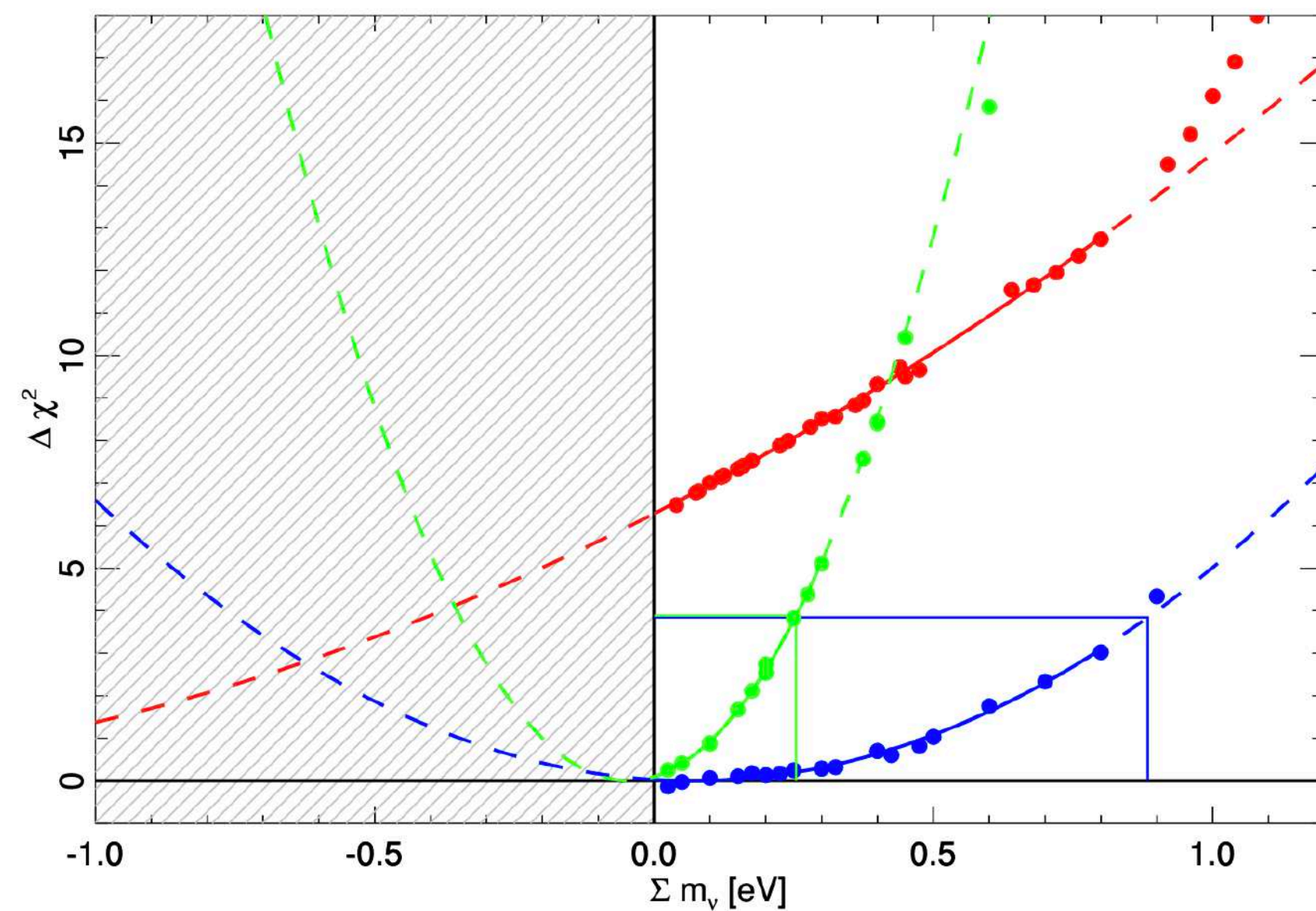
Neutrino mass

Planck intermediate results XVI, 2014
 Naredo-Tuero et al 2024
 Green et al 2024
 Noriega et al 2024
 Herold et al 2024
 Chebat et al 2025

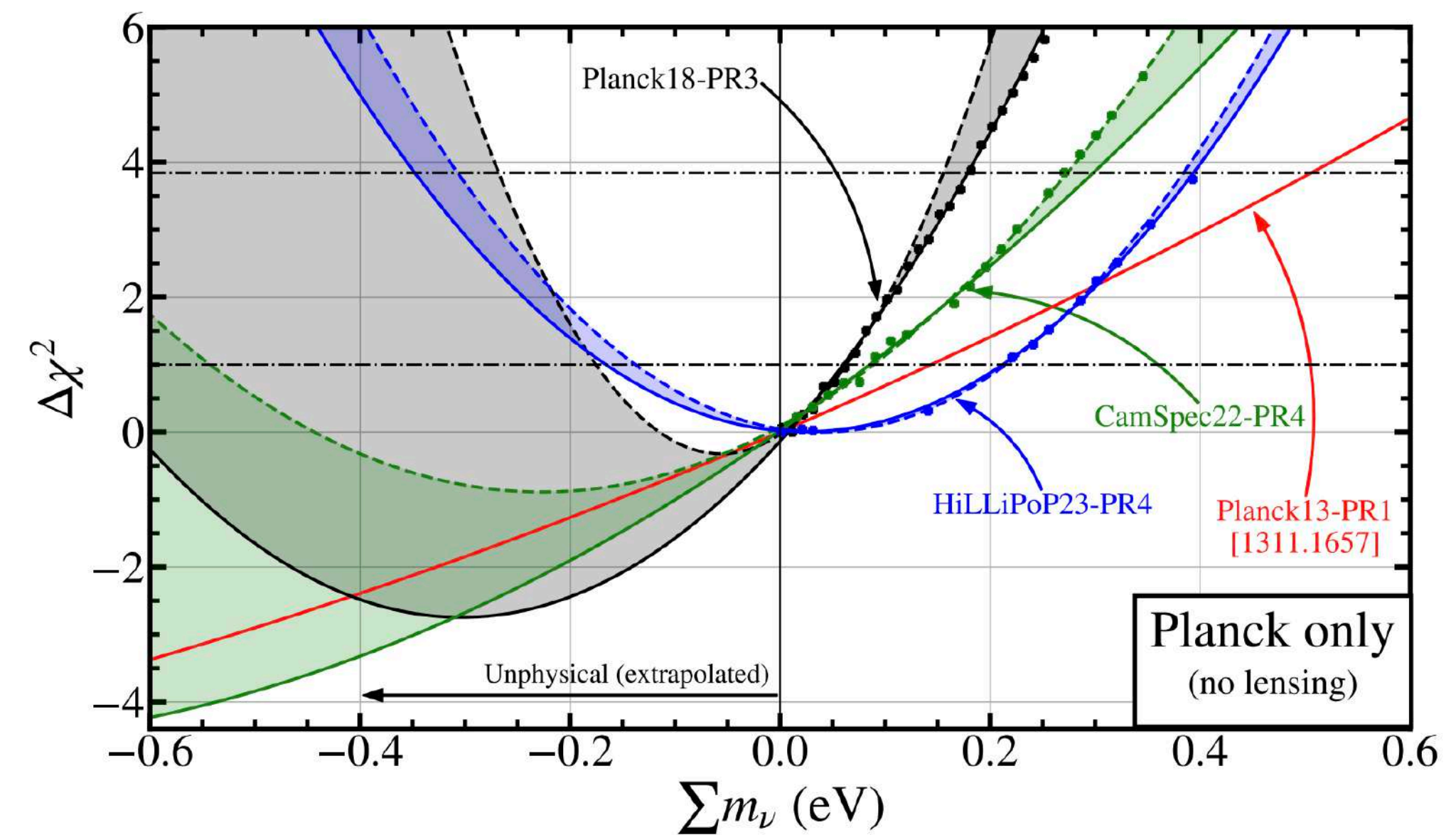
Sum of neutrino mass is close to a physical boundary $\sum m_\nu = M_\nu > 0 \rightarrow$ strong dependence on the prior

Perfect target for profile likelihood

Indeed, the PL has its minimum (or the Likelihood peaks) in negative values for all current data sets:



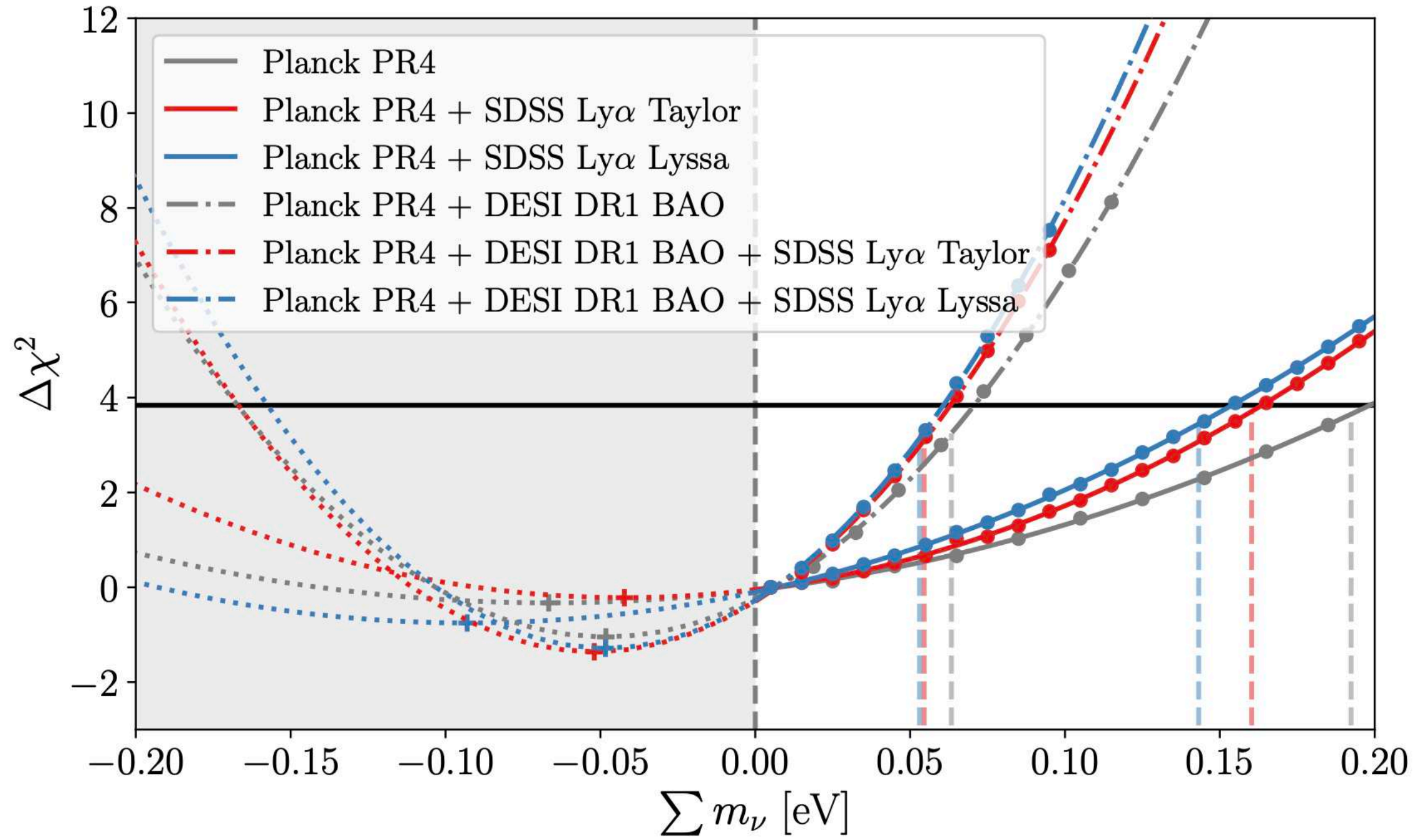
Planck intermediate results XVI, 2014



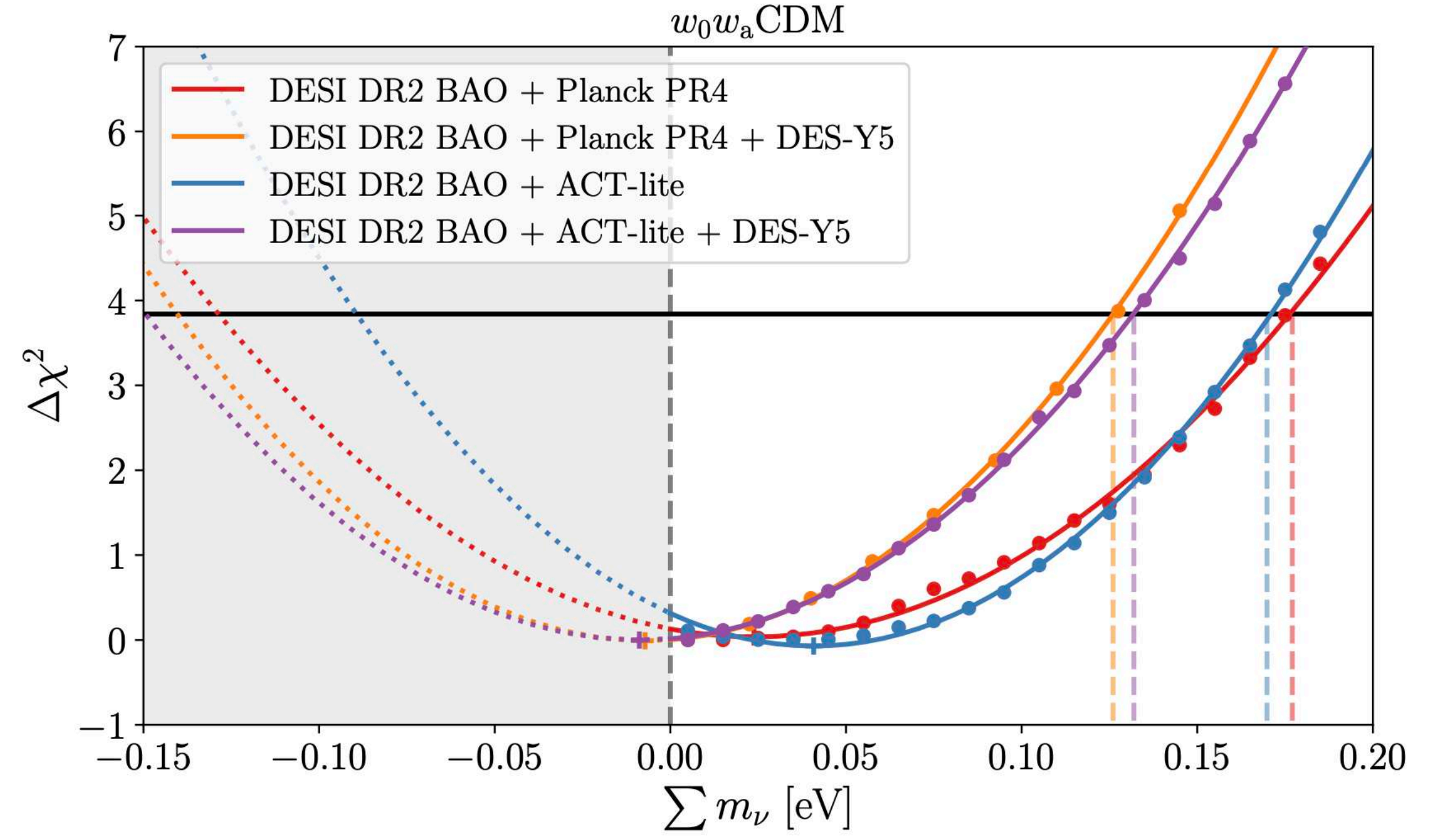
Naredo-Tuero et al 2024

Neutrino mass

Chebat et al 2025 - Cosmological neutrino mass: a frequentist overview in light of DESI



$$\Delta\chi^2\left(\sum m_\nu\right) = \chi^2\left(\sum m_\nu\right) - \chi^2\left(\sum m_\nu = 0.005 \text{ eV}\right)$$



	μ_0	σ	μ_{95}
Λ CDM			
DESI DR1 BAO + Planck PR4 (no lensing)	-0.068	0.067	0.074
DESI DR1 BAO + Planck PR4	-0.048	0.054	0.063
DESI DR2 BAO + Planck PR4	-0.036	0.043	0.053
DESI DR2 BAO + ACT-lite	-0.038	0.048	0.060
DESI DR1 BAO + Planck PR3 plik	-0.046	0.055	0.068
DESI DR1 FS+BAO + Planck PR3 plik	-0.063	0.053	0.053
Planck PR4	-0.067	0.130	0.192
Planck PR4 + SDSS Lyman- α Taylor	-0.042	0.102	0.160
Planck PR4 + SDSS Lyman- α Lyssa	-0.093	0.115	0.143
Planck PR4 + SDSS Lyman- α Taylor + DESI DR1 BAO	-0.052	0.050	0.055
Planck PR4 + SDSS Lyman- α Lyssa + DESI DR1 BAO	-0.048	0.048	0.053
w_0w_a CDM			
DESI DR2 BAO + Planck PR4	0.024	0.078	0.177
DESI DR2 BAO + Planck PR4 + DES-Y5	-0.007	0.068	0.126
DESI DR2 BAO + ACT-lite	0.041	0.066	0.171
DESI DR2 BAO + ACT-lite + DES-Y5	-0.009	0.077	0.132
$\Omega_K + \Lambda$ CDM			
DESI DR2 BAO + Planck PR4	-0.044	0.064	0.085

Addressing the challenges for data *analysis*

Prior volume effects, weight volume effects, ...

Many different ways to deal with them:

- More/better data
- Physical priors or prior/model reparametrization
- **Parameter reparametrization** (eg EFT of LSS: $b_n = \{b_1, b_2, b_{s^2}, b_{3nl}\} \longrightarrow \{b_1\sigma_8, b_2\sigma_8^2, b_{s^2}\sigma_8^2, b_{3nl}\sigma_8^3\}$)
- **Profile likelihood**
- **Hybrid Bayesian/frequentist methods**
- Approximate method to analytically marginalise over a large number of nuisance parameters (Hadzhyiska et al 2023)
- Informed prior to reduce volume effects (Marco Bonici +++, Guachalla et al 2024, Reeves et al 2025)
- Other computational methods
 - Hamiltonian Monte-Carlo approaches
- **Combining data**

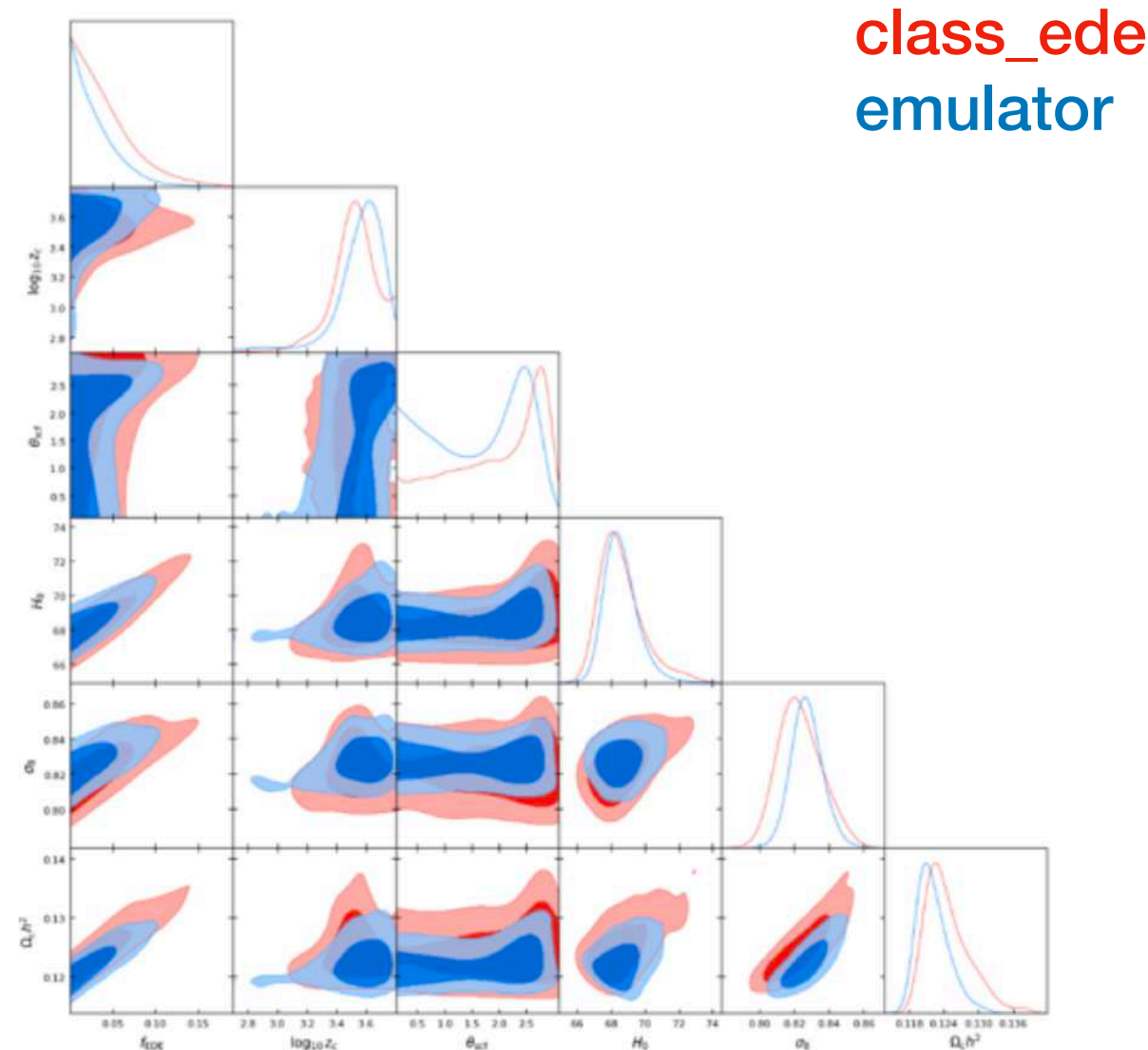
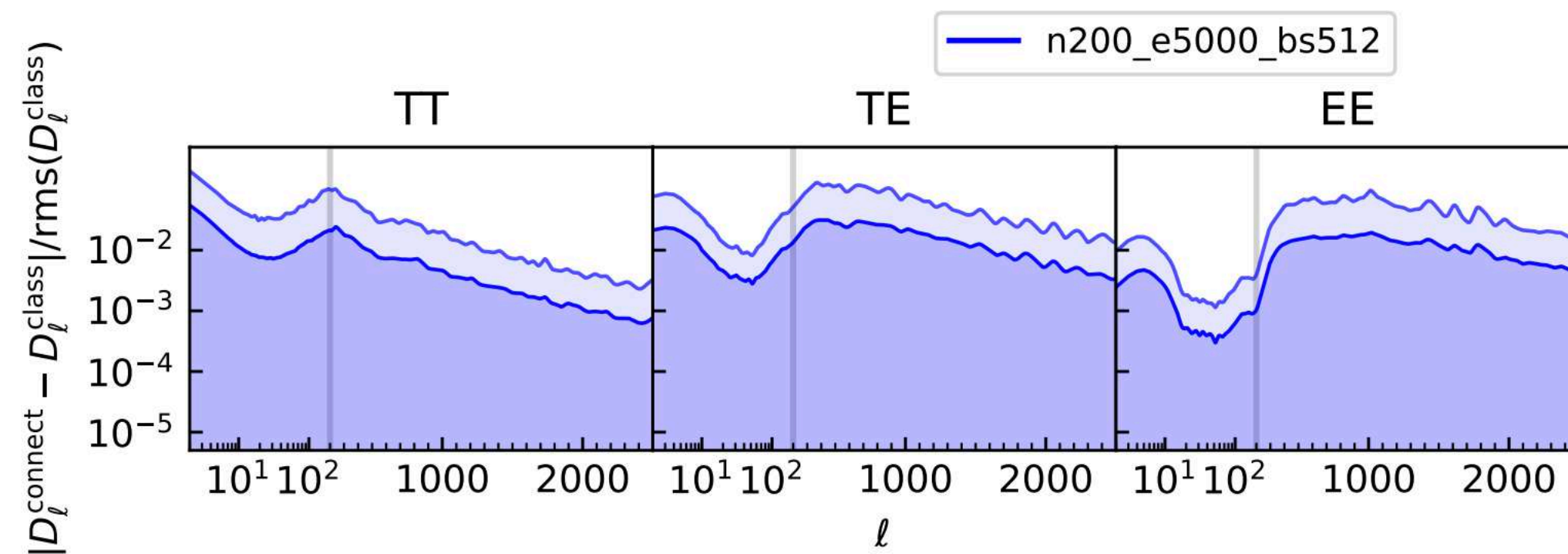
* **Work in progress**

Including inside collaborations, e.g. Euclid, LiteBIRD

But let's pay attention and deal with them!!

Relevant to study the extent to which an analysis is affected by those effects!

Emulator for *early dark energy*

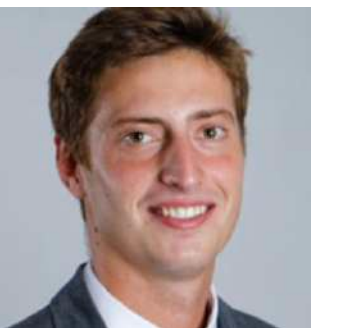


Linear predictions:

Boltzmann codes: axionCAMB, ClassEDE

Slow!! 1 - 3 min per spectrum

Parameter inference \rightarrow very hard!



(Work in progress) EDE emu

Theophile Laurent (2025 ILANCE student), **EF**, and Andreas Nygaard, 2604.XXXX

- Based on emulator CONNECT, it accelerates theoretical prediction.
- Optimizes MCMC and PL computations
- Publicly available (soon)

~~Bayesian vs Frequentist~~ Bayesian & Frequentist & ML & ...

Statistical methods are available to us to use. Each statistical method should be used in the appropriate situation.

No right or wrong, no better or worse, no preference.

Not use blindly

Arman Shafieloo: “Many statistical tools not used properly in cosmology”
or interpreted

Systematics dominated era?

Higher quality data → effects of systematic more important

Particularly serious for Stage IV experiments or cosmological tensions!

Careful to avoid misinterpretations
or “fake” new discoveries!

Prior/marginalization effects

Inflation in the number of nuisance parameters or beyond Λ CDM parameters that enter the statistical analysis leading to possible marginalization or prior volume effects in standard MCMC analysis

Complementary statistical methods necessary for current and future parameter inference analysis - **profile likelihood**

Next-generation cosmological surveys

Large volumes, different probes, different wavelengths

With statistical precision rapidly improving, systematic uncertainties will set the frontier

New, creative and complementary statistical analysis necessary to achieve this frontier

Opportunity for new analysis

Careful modelling of astrophysical systematics

Combining surveys and probes

Understand systematic effects

Opportunity to maximize constraining power

Careful! Need proper framework (avoid improper claims)

A lot work to be done...



Thank you!