KEK Cosmo 2025 workshop

Quantum Nature of Primordial Gravitational Waves in Hořava-Lifshitz Gravity

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[Phys. Rev. D 111, 104077]

Basic Idea

Detection of primordial gravitational waves using HBT interferometry

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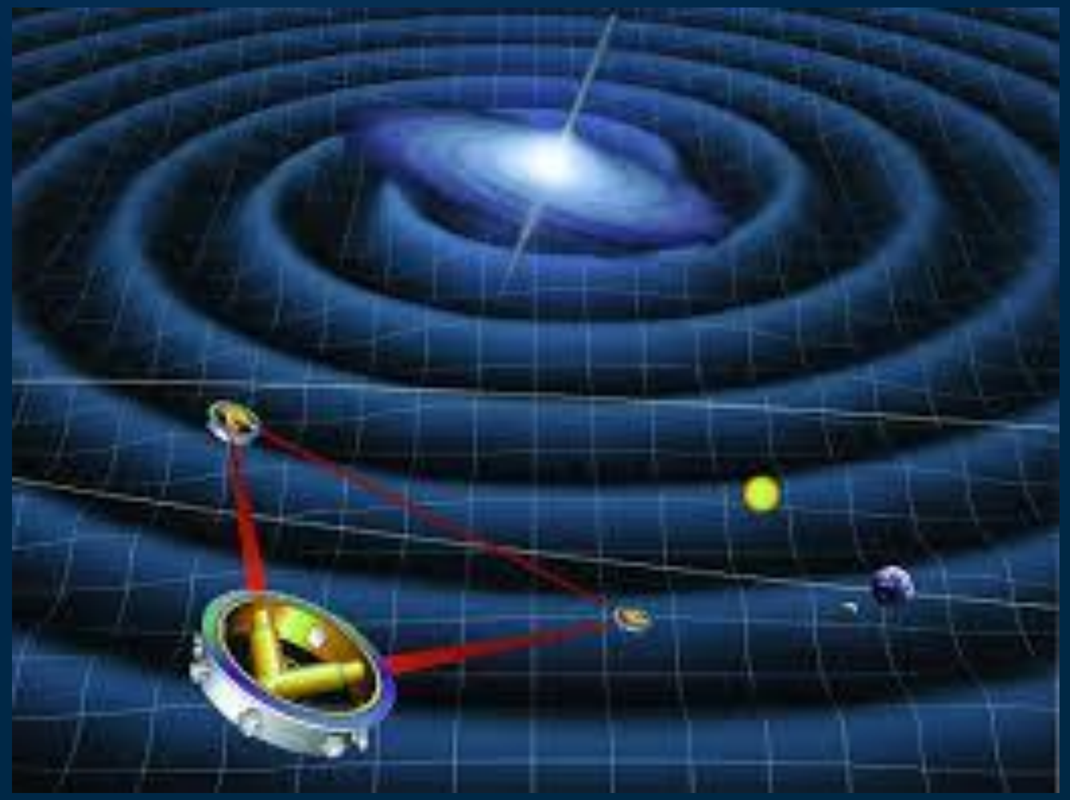
Hořava-Lifshitz Gravity

Gravity theories satisfying renormalizability/unitarity.



Outline of Our Work

Hořava–Lifshitz Gravity

Gravity theories satisfying renormalizability/unitarity

1. A candidate of quantum gravity theories
2. Cosmological prediction: Generating scale-invariant scalar/tensor perturbations without inflation (cosmic de-Sitter expansion)

$$P(k) = k^3 |h_k|^2 = \text{const.}$$

$$a \propto t^n, \quad n > 1/3$$

[S. Mukohyama, JCAP 06 (2009) 001]

Outline of Our Work

Hořava–Lifshitz Gravity

Gravity theories satisfying renormalizability/unitarity

Scale-invariant density perturbations/primordial gravitational waves

$$P(k) = k^3 |h_k|^2 = \text{const.}$$

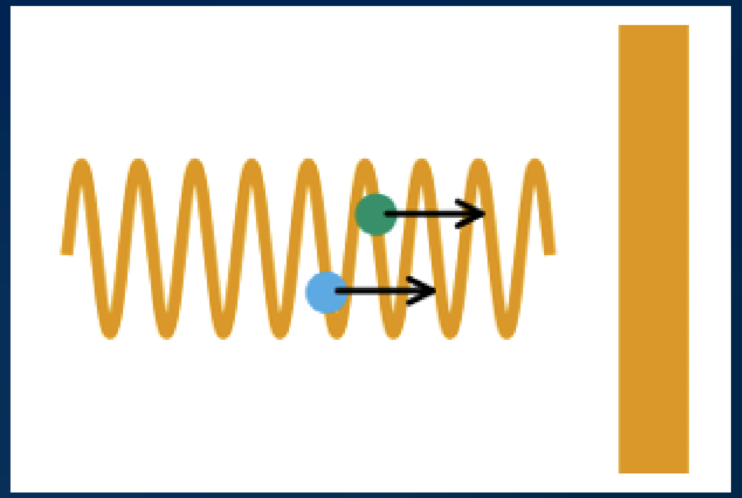
[S. Mukohyama, JCAP 06 (2009) 001]

Inflation Theory

Outline of Our Work

Hanbury Brown-Twiss (HBT)

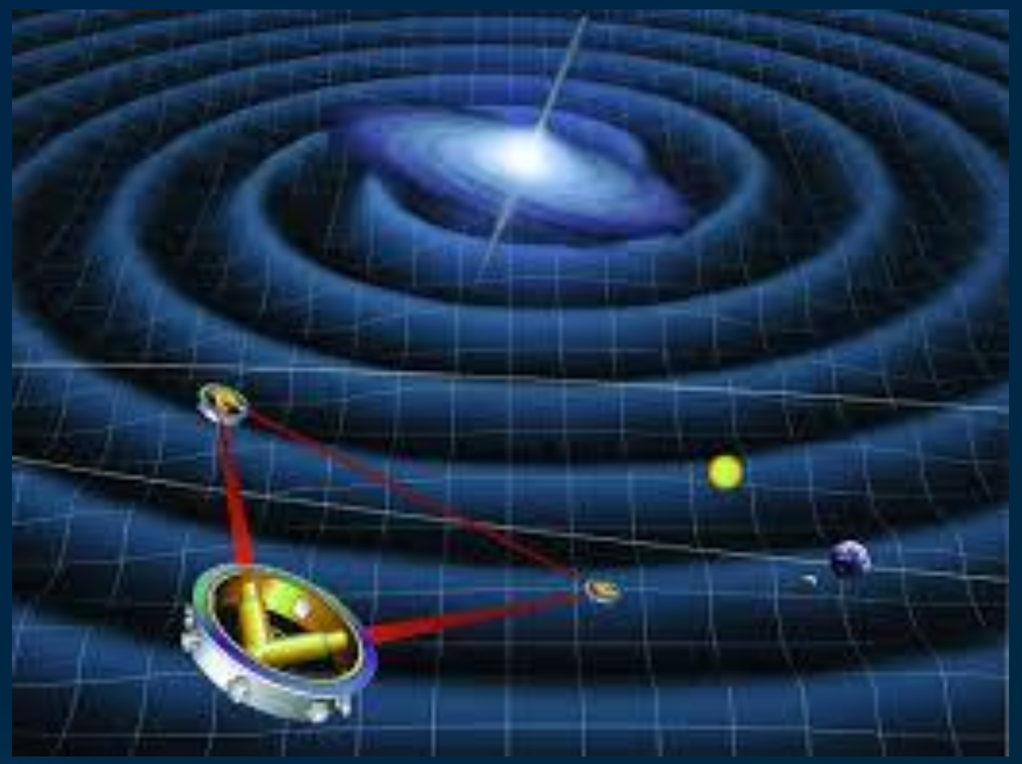
Quantum detection of primordial gravitational waves



Second-order coherence function

[S. Kanno and J. Soda, Phys. Rev. D 99 (2019) 084010]

$$g^{(2)}(\tau)=\frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle\langle I(t+\tau)\rangle}=\frac{\langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)\rangle}{\langle a^\dagger(t)a(t)\rangle\langle a^\dagger(t+\tau)a(t+\tau)\rangle}$$



It represents the correlation strength, and the value (τ=0) distinguishes between the statistical properties of light (classical, coherent, and single photon)

Outline of Our Work

Einstein Gravity

[S. Kanno and J. Soda, Phys. Rev. D 99 (2019) 084010]

$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \mathrm{d}^4 x \sqrt{-g}\,(R - 2\Lambda)$$

+ Inflation model

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Detection Limit for the Quantum Nature of PGWs

$$f > 10\ [\mathrm{kHz}]$$

Horava-Lifshitz Gravity

$$S_{\mathrm{HL}} = \frac{\mathcal{M}^2}{2} \int \mathrm{d}t\mathrm{d}^3 x N\sqrt{g}\left(K^{ij}K_{ij} - \lambda K^2 + c_g^2 R^{(3)} - 2\Lambda + \mathcal{O}_{z>1}\right)$$

$$\frac{\mathcal{O}_{z>1}}{2} = c_1 \nabla_i R^{(3)}_{jk} \nabla^i R^{(3)jk} + c_2 \nabla_i R^{(3)} \nabla^i R^{(3)} + c_3 R^{(3)j}_i R^{(3)k}_j R^{(3)i}_k + c_4 R^{(3)} R^{(3)j}_i R^{(3)i}_j + c_5 R^{(3)3} + c_6 R^{(3)j}_i R^{(3)i}_j + c_7 R^{(3)2}$$

$$f > 10^{-3}\ [\mathrm{Hz}]$$

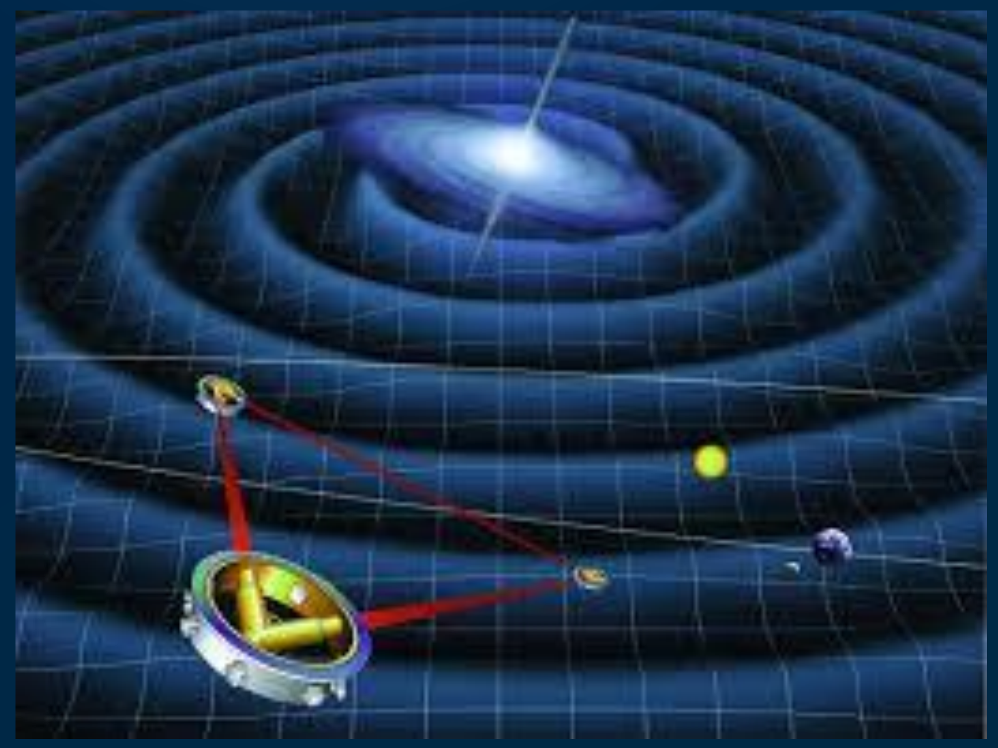
Detectable by the space-based gravitational wave telescope LISA

[S. Kanno, H. Matsui, S. Mukohyama, Phys. Rev. D 111, 104077]

Talk Plans

We discuss quantum nature of PGWs in the Horava-Lifshitz gravity framework. While inflationary models suggest that the non-classicality of PGWs is detectable by HBT interferometers, we demonstrate that the detectable frequency range exhibited by PGWs in HL gravity significantly exceeds that predicted by standard inflationary models.

1. Review of Horowitz-Lifshitz Gravity and Primordial Gravitational Waves
2. Review of HBT interferometers
3. Detection of the Quantum Nature of PGWs in Horowitz-Lifshitz Gravity



Horava-Lifshitz Gravity

Horava-Lifshitz Gravity

One candidate for quantum gravity theory: higher-order differential gravity theory satisfying renormalizability and unitarity (proposed by P. Horava in 2009).

Anisotropic scaling between space and time

[P. Horava, JHEP 03 (2009) 020, Phys. Rev. D 79 (2009) 084008]

$$\begin{cases} t \to b^{z} t \\ \vec{x} \to b \vec{x} \end{cases} \implies \text{Lorentz-invariance is broken at UV (for } z \neq 1)$$

Power counting renormalizability at UV (z=3)

$$\frac{1}{2}\int dt d^3x \left[\dot{h}^2 - h\left(-\Delta\right)^z h\right] \implies h \to b^{\frac{z-3}{2}} h, \; (z=3)$$

Higher derivative operators in spatial direction

$$c_1 \nabla_i R_{jk} \nabla^i R^{jk} + c_2 \nabla_i R \nabla^i R + c_3 R_i^j R_j^k R_k^i + c_4 R R_i^j R_j^i + c_5 R^3) + (c_6 R_i^j R_j^i + c_7 R^2)$$

Projectable Hořava-Lifshitz Gravity

$$S_{\mathrm{HL}} = \frac{\mathcal{M}^2}{2} \int \mathrm{d}t\mathrm{d}^3x N\sqrt{g} \left(K^{ij}K_{ij} - \lambda K^2 + c_g^2 R^{(3)} - 2\Lambda + \mathcal{O}_{z>1} \right)$$

$$\frac{\mathcal{O}_{z>1}}{2} = c_1 \nabla_i R^{(3)}_{jk} \nabla^i R^{(3)jk} + c_2 \nabla_i R^{(3)} \nabla^i R^{(3)} + c_3 R^{(3)j}_i R^{(3)k}_j R^{(3)i}_k + c_4 R^{(3)} R^{(3)j}_i R^{(3)i}_j + c_5 R^{(3)3} + c_6 R^{(3)j}_i R^{(3)i}_j + c_7 R^{(3)2}$$

Higher-order spatial differential terms provide renormalizability

ADM formalism

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + g_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$$

$$K_{ij} = \frac{1}{2N}(\partial_t g_{ij} - g_{jk}\nabla_i N^k - g_{ik}\nabla_j N^k)$$

$$N = N(t) \ , \quad N^i = N^i(t, \mathbf{x}) \ , \quad g_{ij} = g_{ij}(t, \mathbf{x})$$

Tensor perturbations (gravitational waves) on FLRW background

$$ds^2 = -dt^2 + a^2(t)(\gamma_{ij} + h_{ij})dx^i dx^j$$

$$S^{(2)}_{\mathrm{HL}} = \frac{\mathcal{M}^2}{8}\int dt d^3x a^3\left[\dot{h}^{ij}\dot{h}_{ij} + h^{ij}\Delta h_{ij} + \left(\frac{1}{\nu^2\mathcal{M}^2}\right)^2 h^{ij}\Delta^3 h_{ij}\right]$$

Fourier Expansion

$$h_{ij}(\eta, x^i) = \frac{\sqrt{2}}{\mathcal{M}}\frac{1}{\sqrt{V}}\sum_{\boldsymbol{k}}\sum_{s} h^s_{\boldsymbol{k}}(\eta)\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; p^s_{ij}(k)$$

Second-order perturbative action of tensor fields

$$S^{(2)}_{\mathrm{HL}} = \frac{1}{2}\int d\eta \sum_{\boldsymbol{k}}\sum_{s} a^2\left[h'^s_{\boldsymbol{k}} h'^s_{-\boldsymbol{k}} - k^2 h^s_{\boldsymbol{k}} h^s_{-\boldsymbol{k}} - \left(\frac{1}{\nu^2\mathcal{M}^2}\right)^2 \frac{k^6}{a^4} h^s_{\boldsymbol{k}} h^s_{-\boldsymbol{k}}\right]$$

Tensor perturbations (gravitational waves) on FLRW background

Second-order perturbative action of tensor fields

$$S^{(2)}_{\rm HL} = \frac{1}{2}\int \mathrm{d}\eta \sum_{\boldsymbol{k}} \sum_{s} a^2 \left[h'^{s}_{\boldsymbol{k}} h'^{s}_{-\boldsymbol{k}} - k^2 h^{s}_{\boldsymbol{k}} h^{s}_{-\boldsymbol{k}} - \left(\frac{1}{\nu^2 \mathcal{M}^2}\right)^2 \frac{k^6}{a^4} h^{s}_{\boldsymbol{k}} h^{s}_{-\boldsymbol{k}} \right]$$

Field Redefinition

$$\tilde{h}^{s}_{\boldsymbol{k}}(\eta) = a(\eta) h^{s}_{\boldsymbol{k}}(\eta)$$

EOM of PGWs

$$\tilde{h}''^{s}_{\boldsymbol{k}} + \left(\left(\frac{1}{\nu^2 \mathcal{M}^2}\right)^2 \frac{k^6}{a^4} + k^2 - \frac{a''}{a} \right) \tilde{h}^{s}_{\boldsymbol{k}} = 0$$

UV Solution and Scale Invariance

At the UV limit, the k^6 term is dominant

$$\frac{k}{a} \gg \nu \mathcal{M}$$

$$\tilde{h}''^{s}_{\boldsymbol{k}} + \left(\frac{k^6}{a^4 \nu^4 \mathcal{M}^4} - \frac{a''}{a} \right) \tilde{h}^{s}_{\boldsymbol{k}} = 0$$

UV-mode function

$$\tilde{h}^{s}_{\boldsymbol{k}}(\eta) = \frac{\nu \mathcal{M}}{\sqrt{2k^3}} a(\eta) \exp\left(-i \frac{k^3}{\nu^2 \mathcal{M}^2} \int^{\eta} \frac{d\eta'}{a^2(\eta')} \right)$$

Power spectrum is scale-invariant

$$P(k) = k^3 |h^{s}_{\boldsymbol{k}}|^2 \propto \nu^2 \mathcal{M}^2$$

[S. Mukohyama, JCAP 06 (2009) 001]

Squeezed Vacuum

Tensor field operator

$$\tilde{h}^s_{\boldsymbol{k}}(\eta) = a^s_{\boldsymbol{k}}\, u_k(\eta) + a^{s\dagger}_{-\boldsymbol{k}}\, u^*_k(\eta) = b^s_{\boldsymbol{k}}\, v_k(\eta) + b^{s\dagger}_{-\boldsymbol{k}}\, v^*_k(\eta)$$

Initial vacuum

$$a^s_{\boldsymbol{k}}|0\rangle_a = 0$$

Late vacuum

$$b^s_{\boldsymbol{k}}|0\rangle_b = 0$$

Bogoliubov transformation

$$b_{\boldsymbol{k}} = \alpha_k a_{\boldsymbol{k}} + \beta^*_k a^\dagger_{-\boldsymbol{k}}$$

Squeezing parameter

$$\alpha_k \equiv \cosh r_k$$

$$\beta_k \equiv e^{i\varphi} \sinh r_k$$

Initial vacuum is expressed by a two-mode squeezed state of the mode of k and -k

$$|0\rangle_a = \prod_{\boldsymbol{k}} \sum_{n=0}^{\infty} e^{in\varphi} \frac{\tanh^n r_k}{\cosh r_k} |n_{\boldsymbol{k}}\rangle_b \otimes |n_{-\boldsymbol{k}}\rangle_b$$

Squeezed Coherent Vacuum

Coherent state

$$|\xi_k\rangle_a = e^{-\frac{1}{2}|\xi_k|^2} \sum_{n=0}^{\infty} \frac{\xi_k^n}{\sqrt{n!}} |n_{\boldsymbol{k}}\rangle_a = \hat{D}^a(\xi_k)|0\rangle_a$$

Displacement operator

$$\hat{D}^a(\xi_k) = \exp\left[\xi_k\, a_{\boldsymbol{k}}^\dagger - \xi_k^*\, a_{\boldsymbol{k}}\right]$$

Time-evolution of operators

$$U = \exp\left[-i\int d\eta\, H_{\rm int}\right]$$

If the interaction Hamiltonian is linear form with each modes

$$H_{\rm int} = i\left[\xi_k\, a_{\boldsymbol{k}}^\dagger - \xi_k^*\, a_{\boldsymbol{k}}\right] \quad \dashrightarrow \quad U = \hat{D}^a(\xi_k)$$

(Squeezed) coherent state

$$|\xi_k\rangle_a = \exp\left[-i\int d\eta\, H_{\rm int}\right]|0\rangle_a = \prod_{\boldsymbol{k}}\prod_{s} \hat{D}^a(\xi_k)|0\rangle_a$$

Matter Interaction

Scalar field action

$$S_\phi = \frac{1}{2}\int \mathrm{d}t\mathrm{d}^3x N\sqrt{g}\left[\dot{\phi}^2 + \phi\Delta\phi + \left(\frac{1}{\nu_\phi^2\mathcal{M}^2}\right)^2 \phi\Delta^3\phi\right]$$

Fourier expansion

$$\phi(\eta, x^i) = \frac{1}{\sqrt{V}}\sum_{\boldsymbol{p}} \phi_{\boldsymbol{p}}(\eta)\, e^{i\boldsymbol{p}\cdot\boldsymbol{x}}$$

Interaction Hamiltonian

$$i\int \mathrm{d}\eta\, H_{\mathrm{int}} = -\sum_{\boldsymbol{k}}\sum_{s}\left[\xi_k^s\, a_{\boldsymbol{k}}^{s\dagger} - \xi_k^{s*}\, a_{\boldsymbol{k}}^s\right]$$

$$|\xi_{\boldsymbol{k}}\rangle_a = \exp\left[-i\int d\eta\, H_{\mathrm{int}}\right]|0\rangle_a$$
$$= \prod_{\boldsymbol{k}}\prod_{s}\exp\left[\xi_k^s\, a_{\boldsymbol{k}}^{s\dagger} - \xi_k^{s*}\, a_{\boldsymbol{k}}^s\right]|0\rangle_a$$

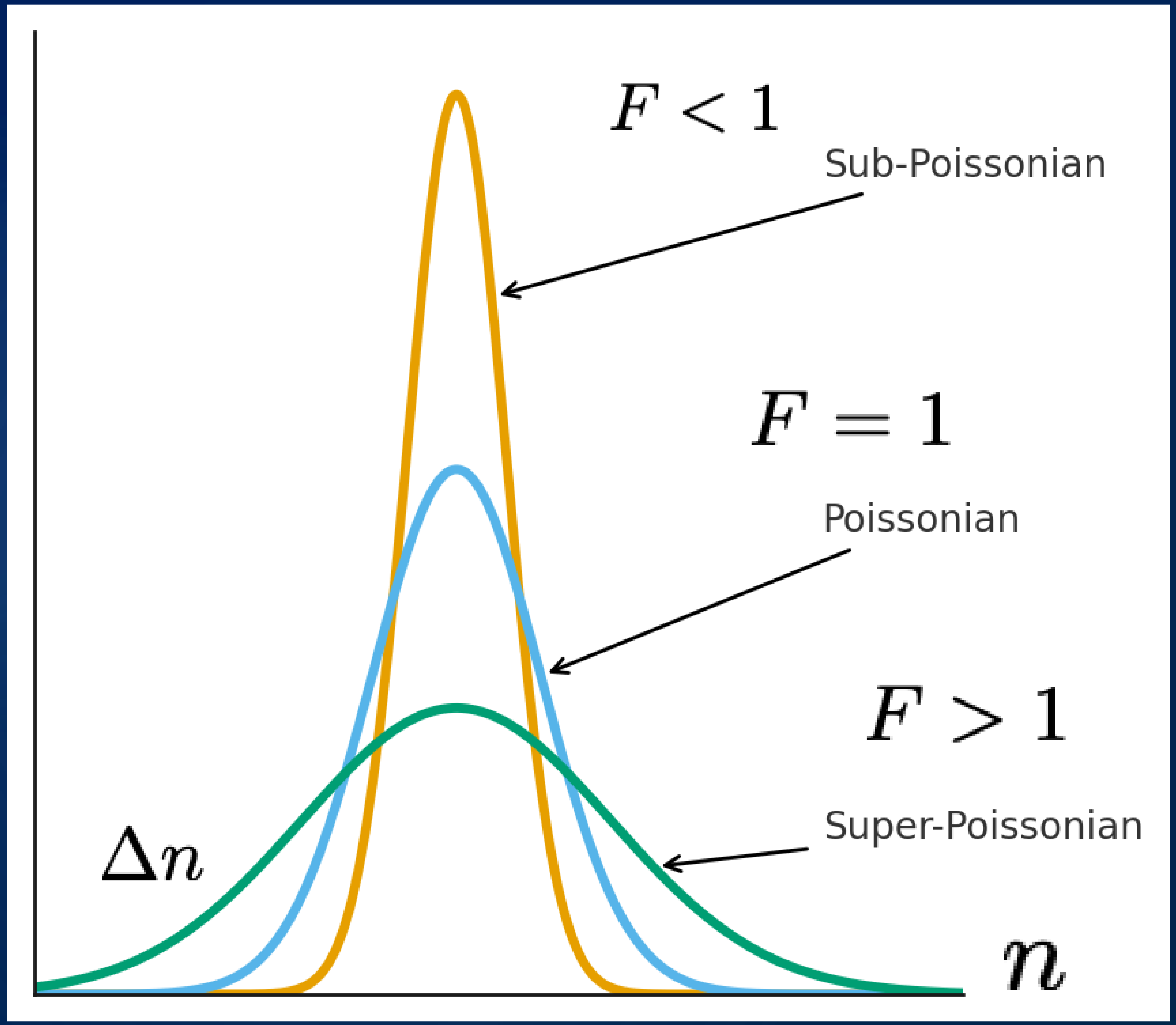
$$\xi_k^s = -\frac{i}{\mathcal{M}}\sqrt{\frac{2}{V}}\int \mathrm{d}\eta \sum_{\boldsymbol{p}}\left\{\frac{a}{2}\left(p^{ij,s}(-\boldsymbol{k})p_ip_j\right)u_k^*(\eta)\phi_{\boldsymbol{p}}\phi_{\boldsymbol{k}-\boldsymbol{p}}\right.$$
$$\left. + \frac{3}{2a^3\nu_\phi^4\mathcal{M}^4}\left(p^{ij,s}(-\boldsymbol{k})p_ip_j\right)\left(\gamma^{ij}p_ip_j\right)^2 u_k^*(\eta)\phi_{\boldsymbol{p}}\phi_{\boldsymbol{k}-\boldsymbol{p}}\right\}$$

Squeezed coherent state

HBT Interferometry and Graviton Statistics

Fano Factor and Graviton Statistics

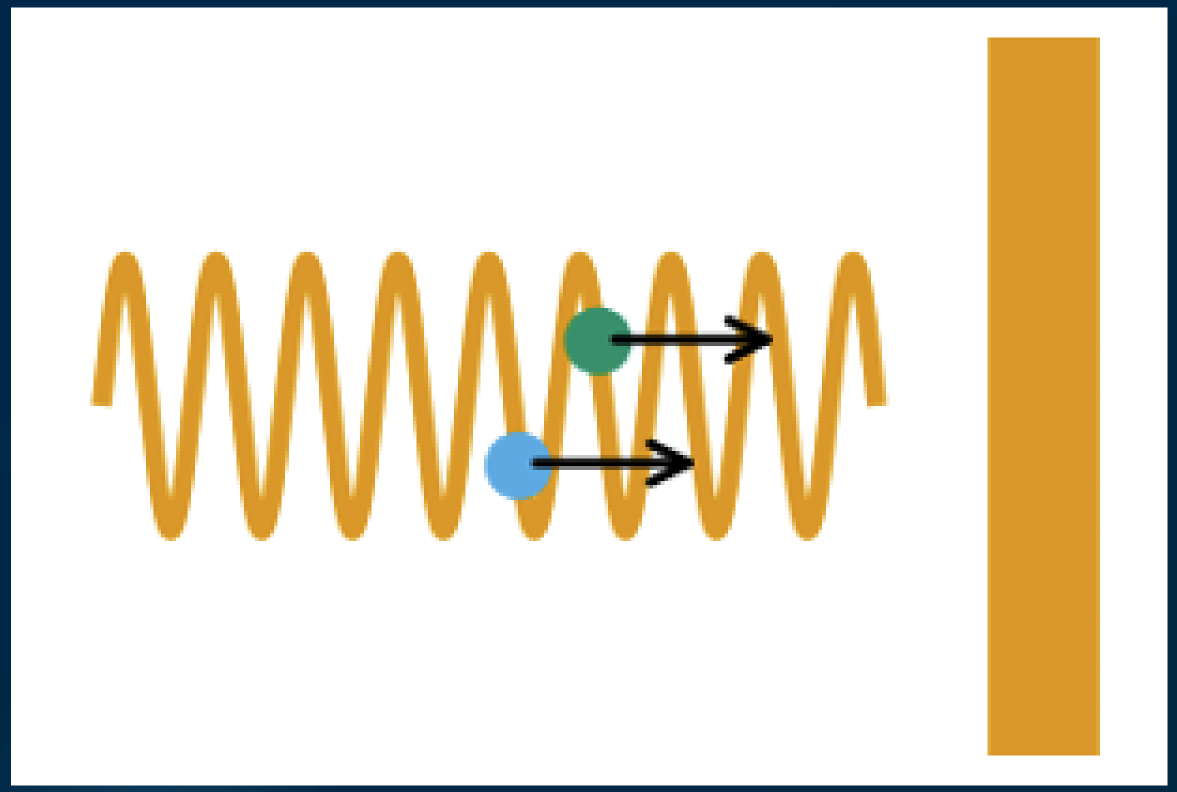
Probability of particle number



Classical fields cannot exhibit sub-Poissonian statistics; F≥1. Coherent state has exactly Poissonian statistics with F=1. Squeezed coherent state can be sub-Poissonian (F<1)

Fano Factor

$$F = \frac{(\Delta n)^2}{\langle n \rangle}$$



Fano Factor and Graviton Statistics

Fano Factor for Squeezed coherent state

$$F = \frac{|\xi_k|^2 e^{-4r_k} + 2\sinh^2 r_k + 2\sinh^4 r_k}{|\xi_k|^2 e^{-2r_k} + \sinh^2 r_k}$$

$$F = \frac{(\Delta n)^2}{\langle n \rangle}$$

Non-classicality condition (F<1)

$$|\xi_k|^2 \left(e^{-2r_k} - e^{-4r_k}\right) > \sinh^2 r_k + 2\sinh^4 r_k$$

Large squeezed limit

$$r_k \gg 1$$

$$\sinh^6 r_k < \frac{1}{8}|\xi_k|^2$$

Hanbury Brown-Twiss (HBT) Interferometry

Second-order coherence function

It represents the correlation strength, and the value τ =0 distinguishes between the statistical properties of light (classical, coherent, quantum)

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle\langle I(t+\tau)\rangle} = \frac{\langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)\rangle}{\langle a^\dagger(t)a(t)\rangle\langle a^\dagger(t+\tau)a(t+\tau)\rangle}$$

Zero-delay coherence function

[S. Kanno and J. Soda, Phys. Rev. D 99 (2019) 084010]

$$g^{(2)}(\tau \to 0) = 1 + \frac{(\Delta n)^2 - \langle n\rangle}{\langle n\rangle^2} = 1 + \frac{F-1}{\langle n\rangle}$$

$$F < 1 \Longrightarrow g^{(2)}(0) < 1$$

Primordial Gravitational Waves

Radiation-Dominated Universe

Scale factor

$$a(\eta) = C_r \eta$$

Mode functions

$$u_k(\eta) = \frac{\nu\mathcal{M}}{\sqrt{2k^3}} C_r \eta \exp\left(i \frac{k^3}{\nu^2 \mathcal{M}^2 C_r^2 \eta} \right) \qquad \frac{k}{a} > \nu\mathcal{M}$$

$$v_k(\eta) \equiv \frac{1}{\sqrt{2k}} e^{-ik\eta} \qquad \frac{k}{a} < \nu\mathcal{M}$$

EOM

$$\tilde{h}_{\boldsymbol{k}}^{\prime\prime s} + \left(\left(\frac{1}{\nu^2 \mathcal{M}^2} \right)^2 \frac{k^6}{a^4} + k^2 - \frac{a''}{a} \right) \tilde{h}_{\boldsymbol{k}}^s = 0$$

Matching condition

$$\eta = \eta_1 \qquad \nu\mathcal{M} = \frac{k}{C_r \eta_1}$$

Bogoliubov coefficients

Squeezing parameter

$$\alpha_k = \frac{e^{2ik\eta_1}\,(2k\eta_1 + i)}{2k\eta_1}, \qquad \beta_k = -\frac{i}{2k\eta_1} \qquad \sinh r_k = \left| \frac{1}{2\eta_1 k} \right|$$

Radiation-Dominated Universe

Particle number

$$N_f = |\beta_k|^2 = \frac{1}{4}\left(\frac{f_1}{f}\right)^4$$

$$f_1 \simeq 10^9 \sqrt{\frac{\nu \mathcal{M}}{10^{-4} M_{\rm pl}}} \quad [\mathrm{Hz}]$$

Amplitude of PGWs

$$h_0^2 \Omega_{\rm gw}(f) \simeq 10^{-13} \left(\frac{\nu \mathcal{M}}{10^{-4} M_{\rm pl}}\right)^2$$

Non-classicality condition

$$f > \left(\frac{1}{8}\right)^{\frac{1}{12}} |\xi_k|^{-\frac{1}{6}} f_1 = \left(\frac{1}{8}\right)^{\frac{1}{12}} 10^9 \, |\xi_k|^{-\frac{1}{6}} \sqrt{\frac{\nu \mathcal{M}}{10^{-4} M_{\rm pl}}} \quad [\mathrm{Hz}]$$

Radiation-Dominated Universe

Matter Interaction

$V = H_0^{-3}$ $\quad \nu \sim \nu_\phi \sim 10^{-4}$

$\mathcal{M} \sim M_{\rm pl}$

$$|\xi_k| \simeq \frac{k^{\frac{3}{2}}\sqrt{V}}{96\pi^3}\frac{\nu_\phi^2}{\nu}(k\eta_1) + \frac{3k^{\frac{3}{2}}\sqrt{V}}{32\pi^3}\frac{\nu^3}{\nu_\phi^2}(k\eta_1)\left(\frac{\eta_1}{\eta_0}\right)$$

$$f > 8.5 \times 10^4 \left(\frac{\nu}{\nu_\phi^2}\right)^{\frac{2}{19}} \left(\frac{\nu\mathcal{M}}{10^{-4}M_{\rm pl}}\right)^{\frac{8}{19}} \quad [\mathrm{Hz}]$$

Non-classicality condition

$$f > 10 \ [\mathrm{kHz}]$$

$$f > 7.2 \times 10^4 \left(\frac{\nu_\phi^2}{\nu^3}\right)^{\frac{2}{19}} \left(\frac{\eta_0}{\eta_1}\right)^{\frac{2}{19}} \left(\frac{\nu\mathcal{M}}{10^{-4}M_{\rm pl}}\right)^{\frac{8}{19}} \quad [\mathrm{Hz}]$$

[S. Kanno, H. Matsui, S. Mukohyama, Phys. Rev. D 111, 104077]

Radiation-Dominated Universe

[S. Kanno and J. Soda, Phys. Rev. D 99 (2019) 084010]

Einstein Gravity

$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \mathrm{d}^4x\sqrt{-g}\,(R - 2\Lambda)$$

+ Inflation model

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Detection Limit for the Quantum Nature of PGWs

$$f > 10\ [\mathrm{kHz}]$$

Frequency range: LISA/DECIGO (10^-4–1 Hz), ground interferometer (10 Hz–1 kHz)

Horava-Lifshitz Gravity

$$S_{\mathrm{HL}} = \frac{\mathcal{M}^2}{2} \int \mathrm{d}t\mathrm{d}^3xN\sqrt{g}\left(K^{ij}K_{ij} - \lambda K^2 + c_g^2 R^{(3)} - 2\Lambda + \mathcal{O}_{z>1}\right)$$

$$\frac{\mathcal{O}_{z>1}}{2} = c_1 \nabla_i R^{(3)}_{jk} \nabla^i R^{(3)jk} + c_2 \nabla_i R^{(3)} \nabla^i R^{(3)} + c_3 R_i^{(3)j} R_j^{(3)k} R_k^{(3)i}$$
$$+ c_4 R^{(3)} R_i^{(3)j} R_j^{(3)i} + c_5 R^{(3)3} + c_6 R_i^{(3)j} R_j^{(3)i} + c_7 R^{(3)2}$$

[S. Kanno, H. Matsui, S. Mukohyama, Phys. Rev. D 111, 104077]

Matter-Dominated Universe

Scale factor

$$a(\eta) = C_m \eta^2$$

Mode functions

$$u_k(\eta) = \frac{\nu \mathcal{M}}{\sqrt{2k^3}} C_m \eta^2 \exp\left(i \frac{k^3}{3C_m^2 \nu^2 \mathcal{M}^2 \eta^3} \right) \qquad \frac{k}{a} > \nu \mathcal{M}$$

$$v_k(\eta) = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta} \qquad \frac{k}{a} < \nu \mathcal{M}$$

EOM

$$\tilde{h}_{\boldsymbol{k}}^{\prime\prime s} + \left(\left(\frac{1}{\nu^2 \mathcal{M}^2} \right)^2 \frac{k^6}{a^4} + k^2 - \frac{a''}{a} \right) \tilde{h}_{\boldsymbol{k}}^s = 0$$

Matching condition

$$\eta = \eta_2 \qquad \nu \mathcal{M} = \frac{k}{C_m \eta_2^2}$$

Bogoliubov coefficients

Squeezing parameter

$$\alpha_k = \frac{e^{\frac{4ik\eta_2}{3}} (-3 + 2k\eta_2(k\eta_2 + 2i))}{2k^2 \eta_2^2}, \qquad \beta_k = \frac{e^{\frac{2ik\eta_2}{3}} (3 - 2ik\eta_2)}{2k^2 \eta_2^2} \qquad \sinh r_k = \left| \frac{3 - 2ik\eta_2}{2k^2 \eta_2^2} \right|$$

Matter-Dominated Universe

Squeezing parameter

$$\sinh r_k \simeq \frac{3}{2k^2\eta_2^2} = \frac{3}{2}\left(\frac{f_2}{f}\right)^3$$

$$f_2 \simeq \left(\frac{\nu\mathcal{M}}{10^{-4}M_{\rm pl}}\right)^{\frac{1}{3}} \ [\mathrm{Hz}]$$

Sub-Poissonian $F = \frac{(\Delta n)^2}{\langle n\rangle} < 1$

$$\sinh^6 r_k < \frac{1}{8}|\xi_k|^2 \qquad r_k \gg 1$$

Non-classicality condition

$$f > \left(\frac{9}{2}\right)^{\frac{1}{6}} |\xi_k|^{-\frac{1}{9}} f_2 = \left(\frac{9}{2}\right)^{\frac{1}{6}} |\xi_k|^{-\frac{1}{9}} \left(\frac{\nu\mathcal{M}}{10^{-4}M_{\rm pl}}\right)^{\frac{1}{3}} \ [\mathrm{Hz}]$$

Matter-Dominated Universe

Matter Interaction

$$V = H_0^{-3} \qquad \nu \sim \nu_\phi \sim 10^{-4}$$

$$\mathcal{M} \sim M_{\rm pl}$$

$$|\xi_k| \simeq \frac{k^{\frac{3}{2}}\sqrt{V}}{160\pi^3}\frac{\nu_\phi^2}{\nu}(k\eta_2) + \frac{k^{\frac{3}{2}}\sqrt{V}}{32\pi^3}\frac{\nu^3}{\nu_\phi^2}(k\eta_2)\left(\frac{\eta_2}{\eta_0}\right)^3$$

$$f > 1.2 \times 10^{-2}\left(\frac{\nu}{\nu_\phi^2}\right)^{\frac{1}{12}}\left(\frac{\nu\mathcal{M}}{10^{-4}M_{\rm pl}}\right)^{\frac{7}{24}} \ [\mathrm{Hz}]$$

Non-classicality condition

$$f > 10^{-3}\ [\mathrm{Hz}]$$

$$f > 1.0 \times 10^{-2}\left(\frac{\nu_\phi^2}{\nu^3}\right)^{\frac{1}{12}}\left(\frac{\eta_0}{\eta_2}\right)^{\frac{1}{4}}\left(\frac{\nu\mathcal{M}}{10^{-4}M_{\rm pl}}\right)^{\frac{7}{24}} \ [\mathrm{Hz}]$$

[S. Kanno, H. Matsui, S. Mukohyama, Phys. Rev. D 111, 104077]

Matter-Dominated Universe

Einstein Gravity

[S. Kanno and J. Soda, Phys. Rev. D 99 (2019) 084010]

$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \mathrm{d}^4 x \sqrt{-g}\,(R - 2\Lambda)$$

+ Inflation model

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Detection Limit for the Quantum Nature of PGWs

$$f > 10\ [\mathrm{kHz}]$$

Horava-Lifshitz Gravity

$$f > 10^{-3}\ [\mathrm{Hz}]$$

$$S_{\mathrm{HL}} = \frac{\mathcal{M}^2}{2} \int \mathrm{d}t\mathrm{d}^3 x N \sqrt{g} \left(K^{ij} K_{ij} - \lambda K^2 + c_g^2 R^{(3)} - 2\Lambda + \mathcal{O}_{z>1} \right)$$

$$\frac{\mathcal{O}_{z>1}}{2} = c_1 \nabla_i R^{(3)}_{jk} \nabla^i R^{(3)jk} + c_2 \nabla_i R^{(3)} \nabla^i R^{(3)} + c_3 R_i^{(3)j} R_j^{(3)k} R_k^{(3)i}$$
$$+ c_4 R^{(3)} R_i^{(3)j} R_j^{(3)i} + c_5 R^{(3)3} + c_6 R_i^{(3)j} R_j^{(3)i} + c_7 R^{(3)2}$$

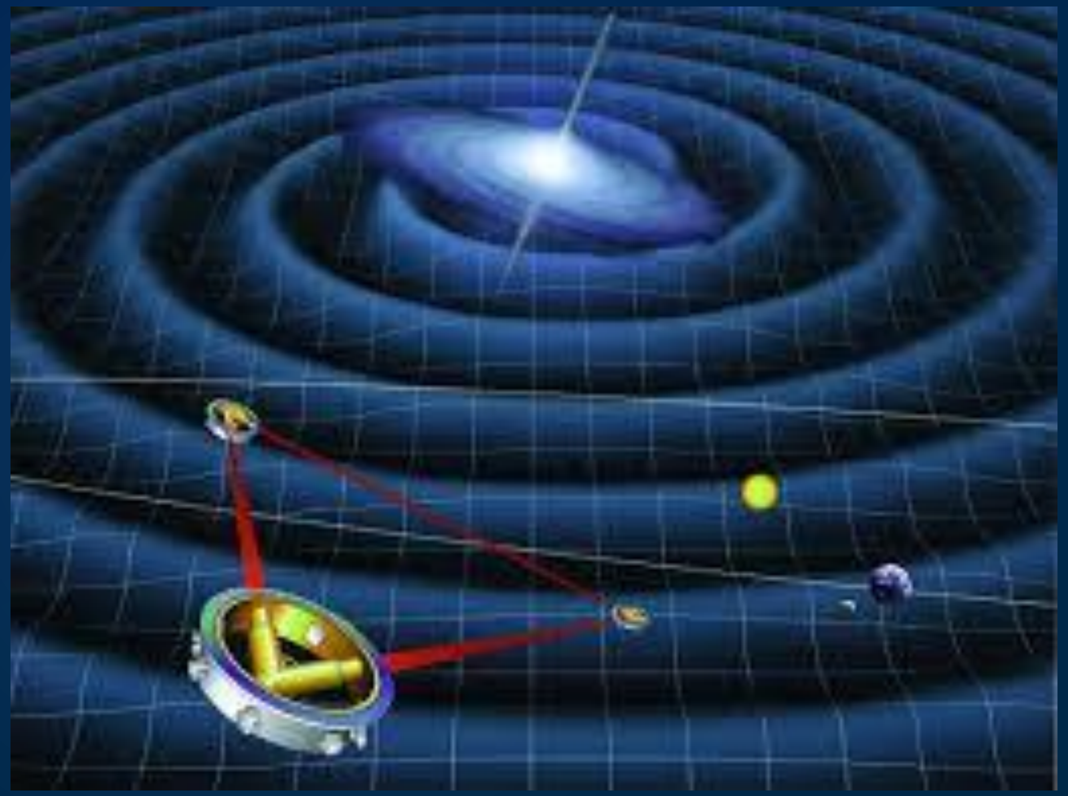
[S. Kanno, H. Matsui, S. Mukohyama, Phys. Rev. D 111, 104077]

Detection of quantum nature of PGWs

Horava-Lifshitz Gravity

$$S_{\rm HL} = \frac{\mathcal{M}^2}{2}\int {\rm d}t{\rm d}^3x N\sqrt{g}\left(K^{ij}K_{ij} - \lambda K^2 + c_g^2 R^{(3)} - 2\Lambda + \mathcal{O}_{z>1}\right)$$

$$\frac{\mathcal{O}_{z>1}}{2} = c_1 \nabla_i R^{(3)}_{jk}\nabla^i R^{(3)jk} + c_2\nabla_i R^{(3)}\nabla^i R^{(3)} + c_3 R^{(3)j}_i R^{(3)k}_j R^{(3)i}_k + c_4 R^{(3)} R^{(3)j}_i R^{(3)i}_j + c_5 R^{(3)3} + c_6 R^{(3)j}_i R^{(3)i}_j + c_7 R^{(3)2}$$



Frequency range: LISA/DECIGO (10^-4–1 Hz), ground interferometer (10 Hz–1 kHz)

Detection Limit for the Quantum Nature of PGWs

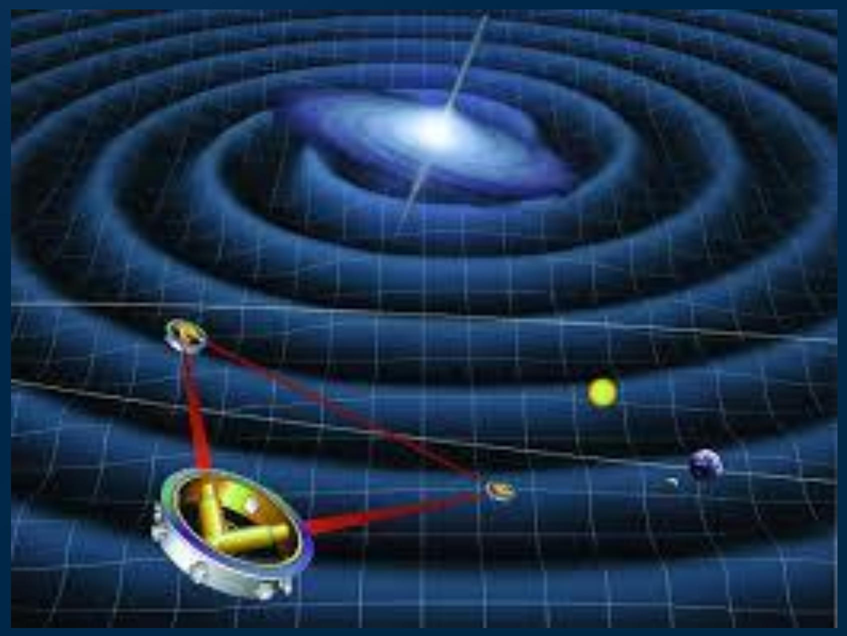
$$f > 10^{-3}\ [\mathrm{Hz}]$$

Detectable by the space-based gravitational wave telescope LISA

[S. Kanno, H. Matsui, S. Mukohyama, Phys. Rev. D 111, 104077]

Summary

1. Horava-Lifshitz gravity is one of the leading candidates for a quantum gravity theory that satisfies renormalizability and unitarity.
2. Horava-Lifshitz gravity generates scale-invariant fluctuations in a standard expanding universe without cosmic inflation.
3. The quantum nature of PGWs generated in the early universe can be verified using HBT interferometers.



Radiation-dominated universe → $f > 10$ kHz, same prediction as inflation theory. Matter-dominated universe → $f > 10^{-3}$ Hz, detectable in the future by the gravitational-wave observatory LISA.